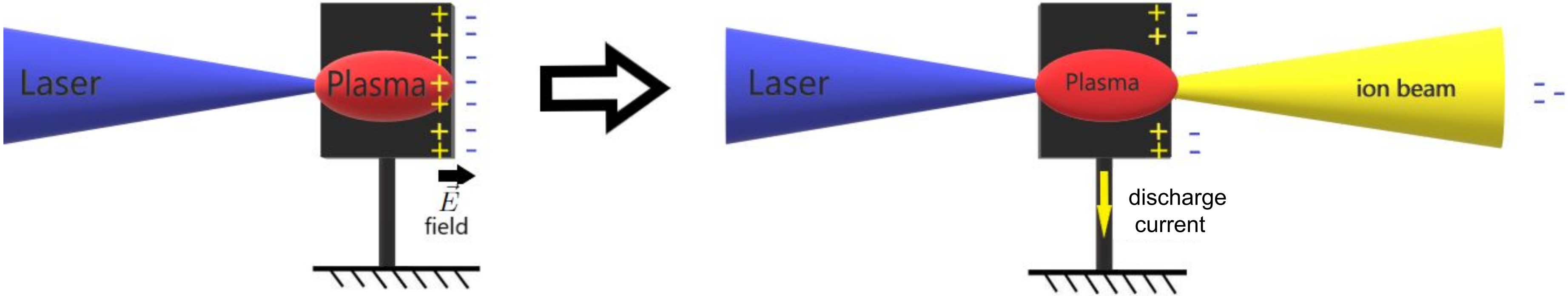
Theoretical model of post acceleration and focalization of protons produced with TNSA

CLC Lacoste[1,2,3], A Hirsch-Passicos[1,2], VT Tikhonchuk[2,4], P Antici[3], E D'Humières[2] et M Bardon[1,2]

1.CEA-CESTA, Le Barp 33114, France
2.CELIA, University of Bordeaux-CNRS-CEA, Talence 33405, France
3.INRS EMT, Varennes J3X 1P7, Canada
4.ELI beamlines, Dolni Brezany 251 42, Tchequie

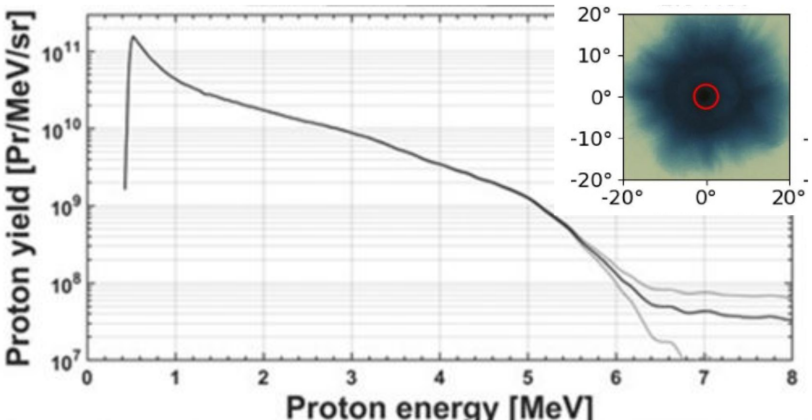
clement.lacoste@u-bordeaux.fr



1) Laser ASE/prepulse => plasma
2) Main pulse => electron acceleration and creation of the sheath

1) Hot electrons escape
2) The charge separation electric field accelerates some protons (TV/m) [1,2]
3) The charging of the target induces a discharge current

[1] RA Snavely and al. PRL,85(14) :2945, 2000
[2] S. Wilks and al. Phys.Plasmas, 8 :542–549, 2001.



__Problems for applications__

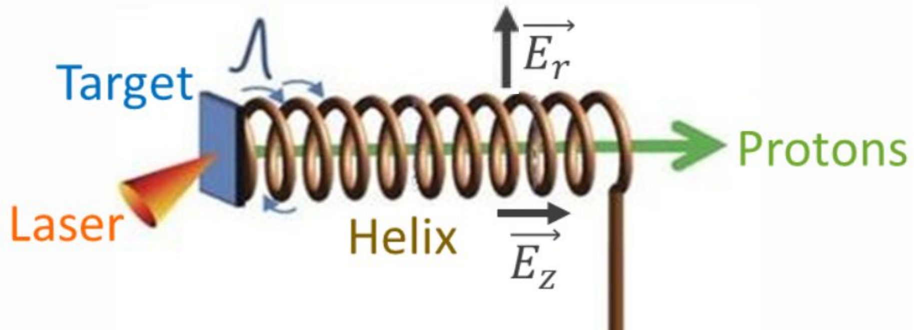
-Charge flux limited (dispersed due to opening angle)

-Maximum proton energy too low for certain applications (record ~100 MeV)

-Energy spectrum decreases exponentially

Adding a helical coil

-Creation of E and B fields which post-accelerate, focus and bunch the protons (GV/m)



[3] Satyabrata Kar et al. Nature communications, 7(1) :1–7, 2016.

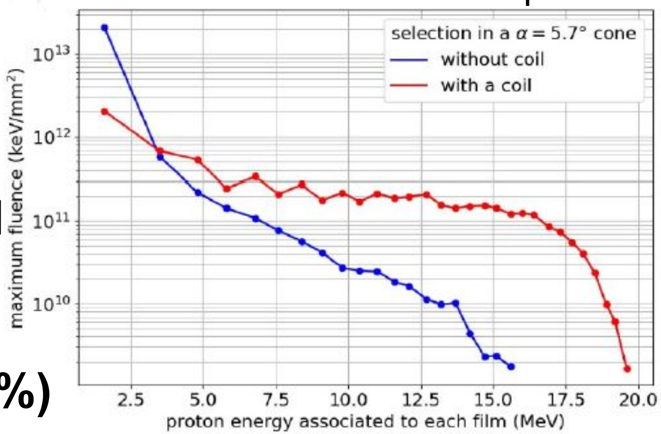
[4] M Bardon et al. Plasma Physics and Controlled Fusion, 62(12) :125019, 2020.

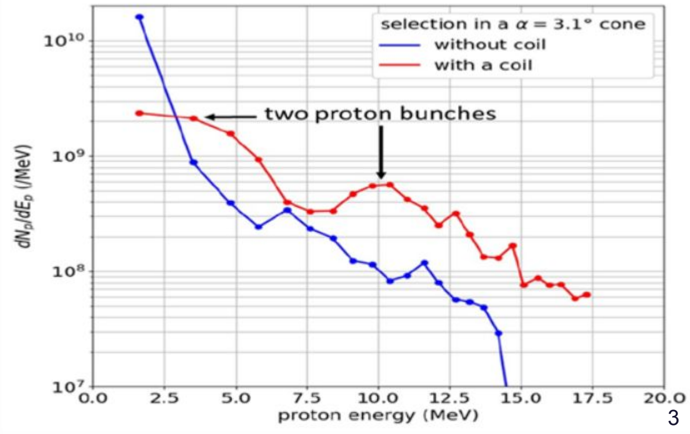
-Good focusing

-Post-acceleration and bunching is limited

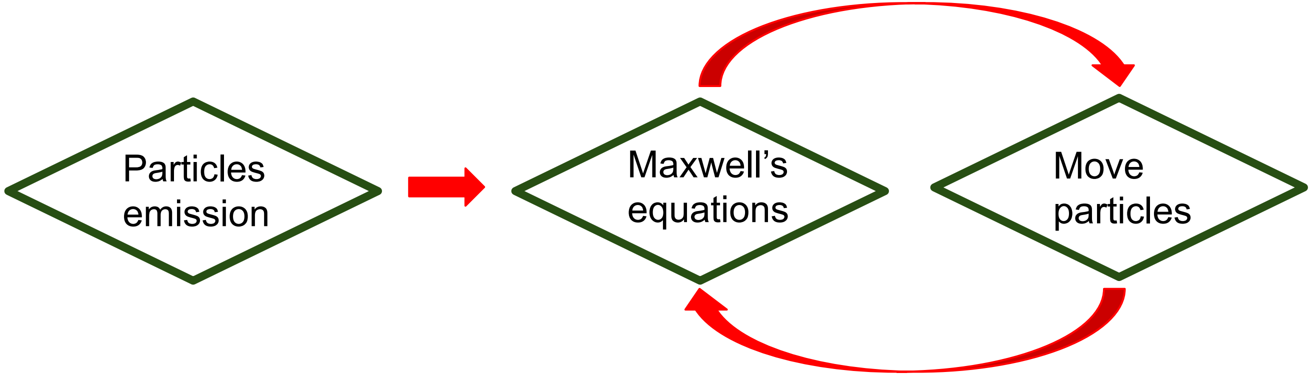
-Limited efficiency (15%)

PACMAN experiment at LULI2000 Facility [4]

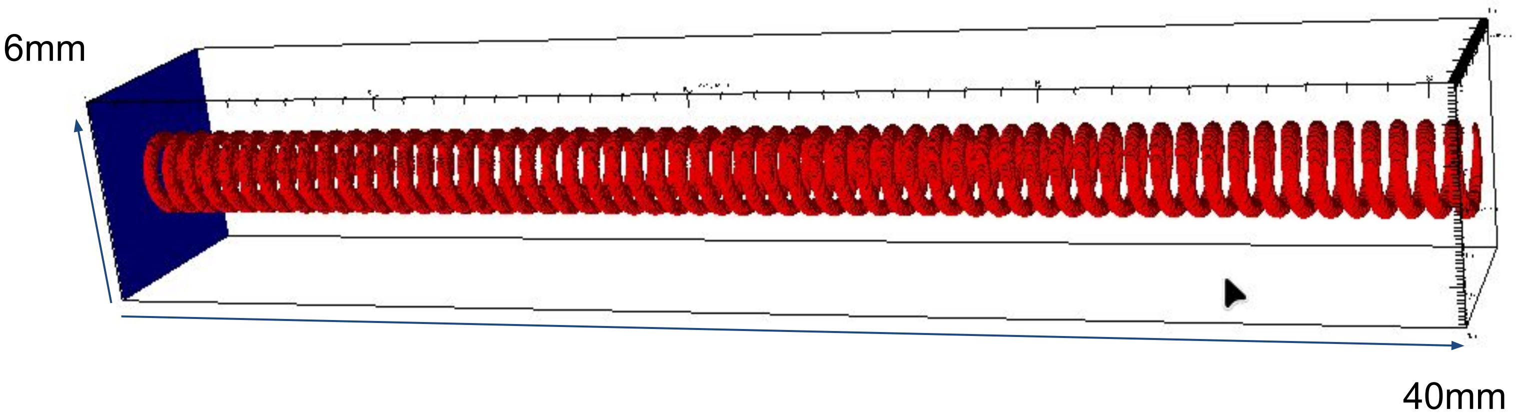




PIC simulation SOPHIE [5]



- **Wide area**
- **Boundary conditions permit the propagation of fields in matter**



-Reference code

-High precision

-High cost in time calculation

[5] Olivier Cessenat,. arXiv preprint arXiv:1301.4539, 2013

DoPPLIGHT (Dynamics of Particles Produced by Laser Interaction in Grounded Helical Targets)

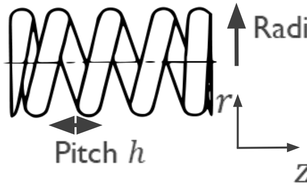
2D-axi
Time resolved
Not self-consistent

Inputs

Coil parameters → **Source term** → **Helix fields** [6] and [7] → **Space charge fields** → **Boris pusher**

$N_{particles}$

Coil parameters: a, h, L



Source term — For each particle species:

Q, q, m, R_0, z_0

$\frac{dN}{dt}, \frac{dN}{dE}, \frac{dN}{d\theta}, \frac{dN}{d\phi}$

Electric current

$I_{z=0}(t), t_0$

Helix fields:

$E_r^{helix}, E_\theta^{helix}, E_z^{helix}(r, z, t)$

$H_r^{helix}, H_\theta^{helix}, H_z^{helix}(r, z, t)$

Space charge fields:

$E_r^{SC}, E_z^{SC}\left(\frac{r}{\sigma_r}, \frac{z}{\sigma_z}, \frac{\sigma_r}{\sigma_z}\right)$

Boris pusher:

$\begin{pmatrix} r_i, \theta_i, z_i, \sigma_r, \sigma_z \\ Q_{Fly}, p_{x_i}, p_{y_i}, p_{z_i} \end{pmatrix}$

Elapsed time <= few minutes on laptop

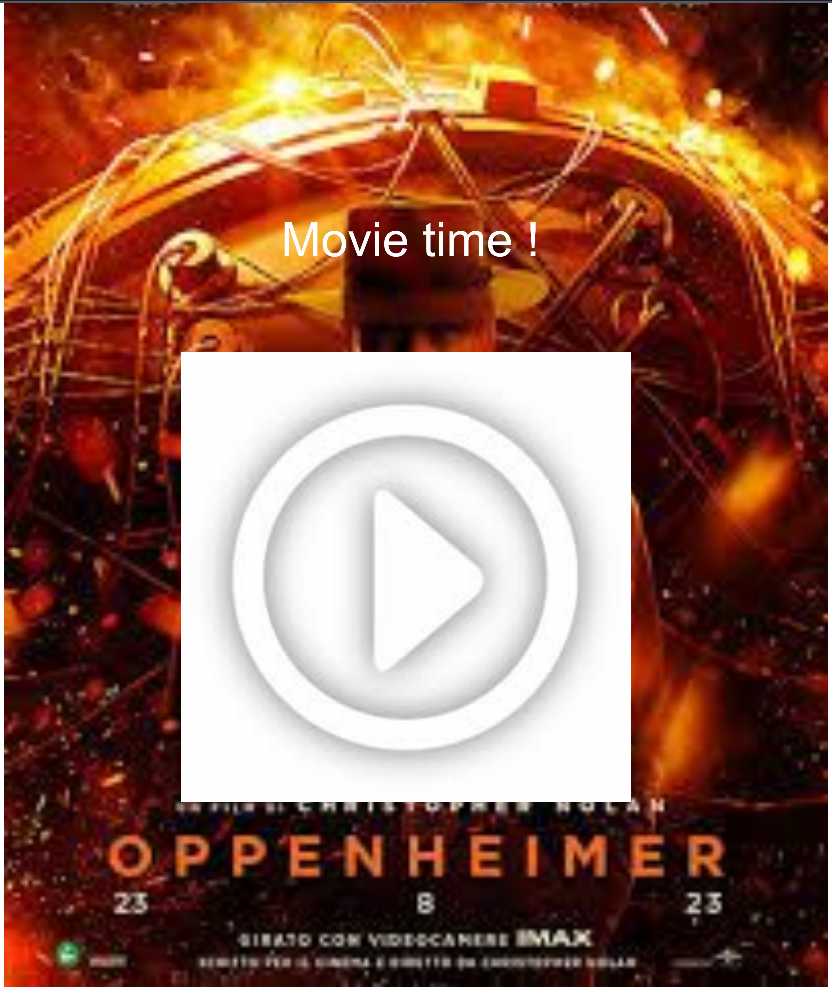
-Run in few minuts

-Less precise than SOPHIE

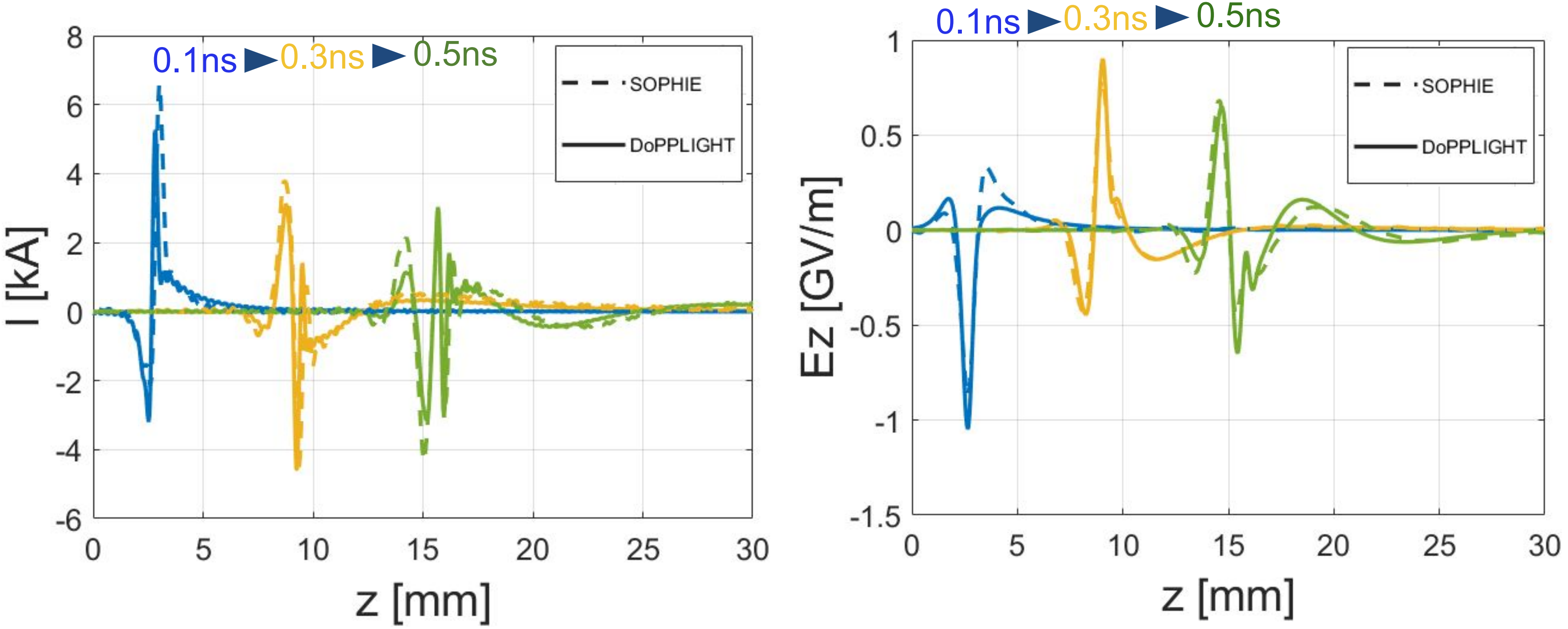
[6] John Robinson Pierce. The bell System technical journal, 29(2) :189–250, 1950.

[7] GS Kino and SF Paik. Journal of Applied Physics, 33(10) :3002–3008,1962.

Movie time !

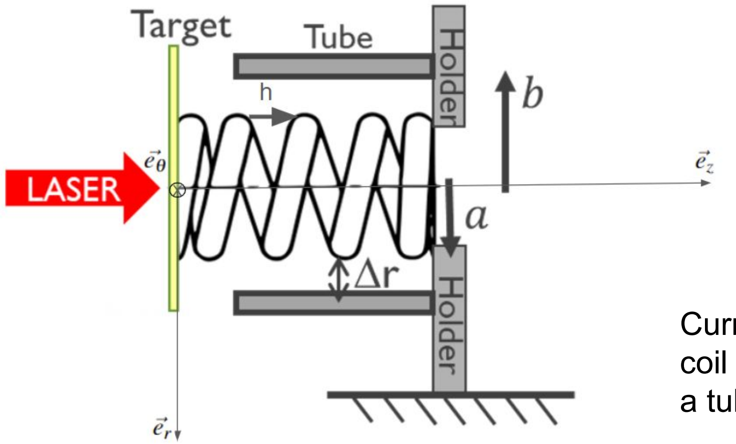


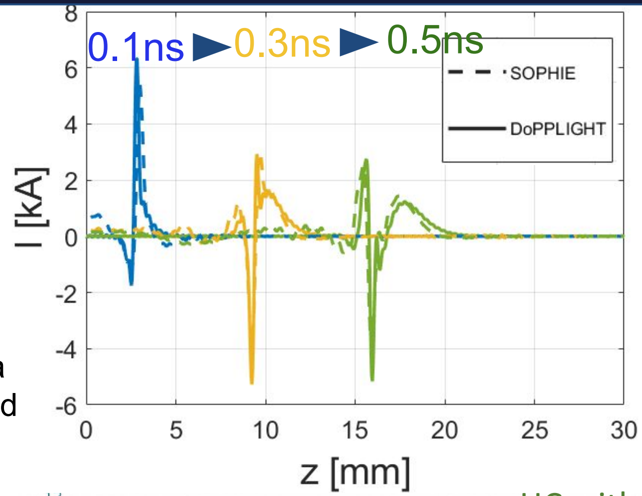
Current and longitudinal field at r=0.2 mm for a=0.5 mm h=0.35 mm



acceleration and deceleration alternation of electric field

Adding a tube around the helical coil





Current and proton spectra for a coil of a=0.5 mm h=0.35 mm and a tube of radius b=0.7mm

<u>*Helical coil with tube [8]:*</u>

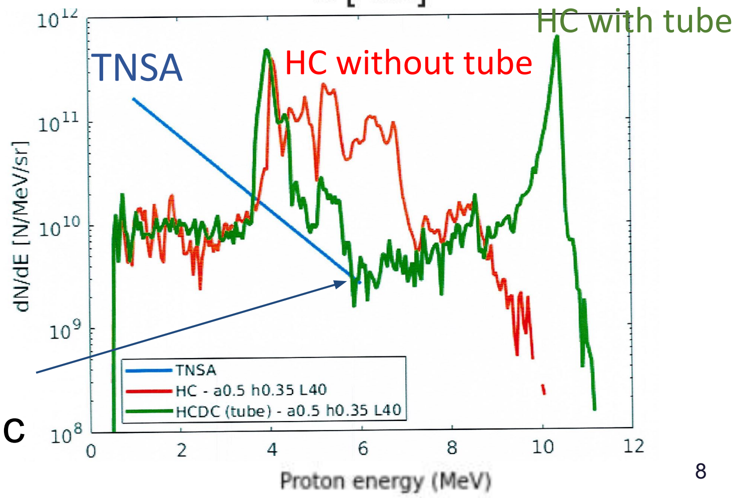
-Strongly reduces the current dispersion. [9-10]

- Creation of strong bunches on proton spectra (around the geometric energy)

[8] A Hirsch-Passicos, CLC Lacoste et al. PRE accepted 2024

[9] JP Freund et al. IEEE transactions on plasma science, 20(5) :543–553, 1992.

[10] Han S Uhm et al. Journal of Applied Physics, 53(12):8483–8488, 1982

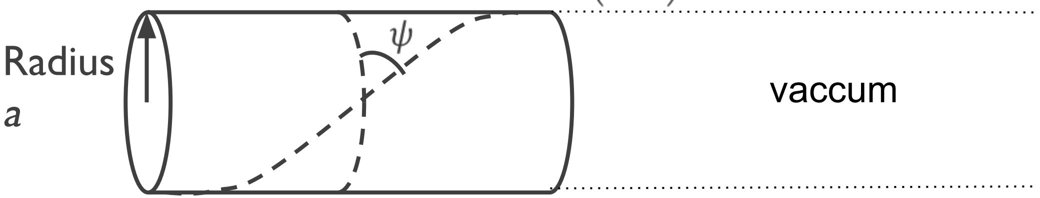


characteristic energy

Varying helical coil with tube - theoretical model

*mobility at ELI beamlines
with Vladimir Tikhonchuk

$$\psi = \arctan\left(\frac{h}{2\pi a}\right)$$



$$\begin{cases} E_z^-(r=a) = E_z^+(r=a) \\ B_\parallel^+ = B_\parallel^- \\ E_\parallel = 0 \\ B_\perp^+ - B_\perp^- = \dfrac{\mu_0 i(\omega,z)}{2\pi atan^2(\Psi)} \end{cases}$$

Boundary conditions

Continuity equation and propagation equation → i(z,t)

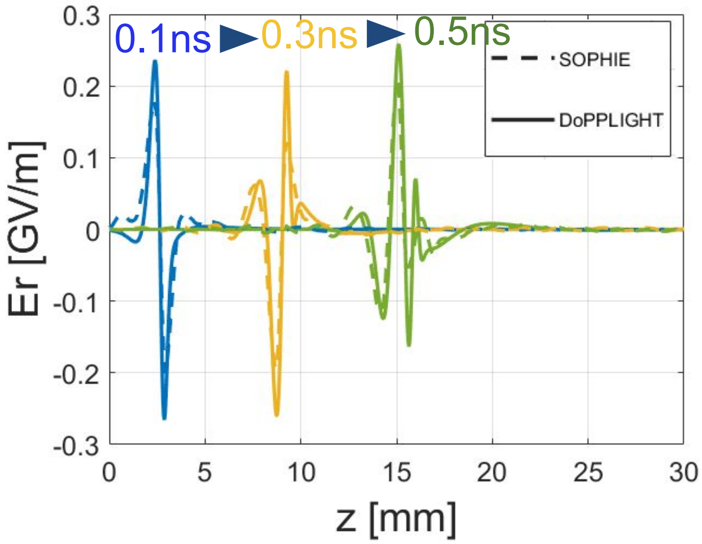
Boundary conditions → Constant of integration, dispersion relation and link between current and field

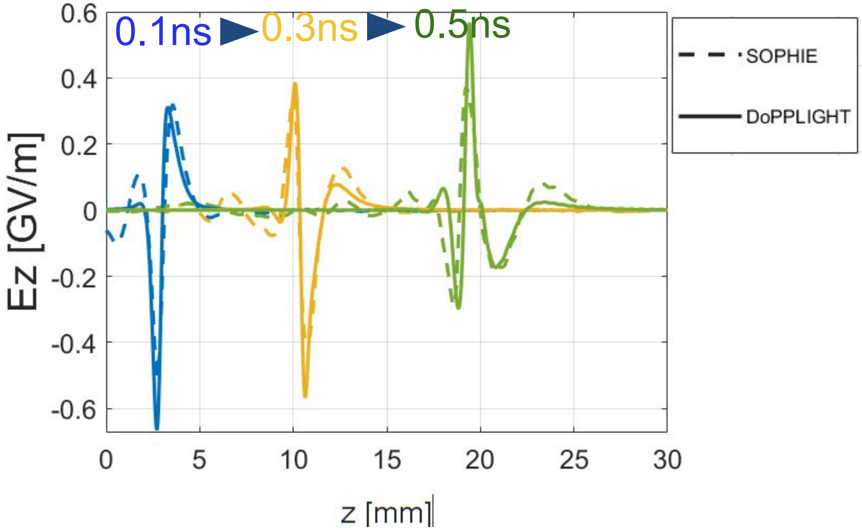
Maxwell's equations → E(r,z,t) and B(r,z,t)

→ Theoretical model

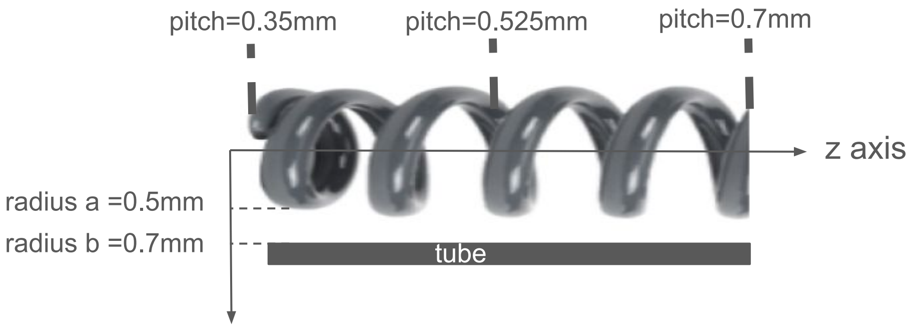
$$E_z(r,z,t) = \frac{\mu_0 c}{2\pi^2 a}\int_0^{+\infty} \frac{j i_0(\omega)\omega\sqrt{k(\omega,z=0)}\kappa(\omega)}{\alpha\sqrt{k(\omega,z)}}\,\frac{I_1(\alpha)[K_1(\alpha b)I_1(\alpha)-I_1(\alpha b)K_1(\alpha)]}{I_0(\alpha)[I_1(\alpha)I_1(\alpha b)K_0(\alpha)+I_0(\alpha)I_1(\alpha b)K_1(\alpha)]}\cot^3(\Psi)\cos(\Psi)I_0(\alpha r)e^{j\int_0^z k(\omega,z_\eta)dz_\eta-\omega t}d\omega$$

Varying helical coil with tube - Theoretical results





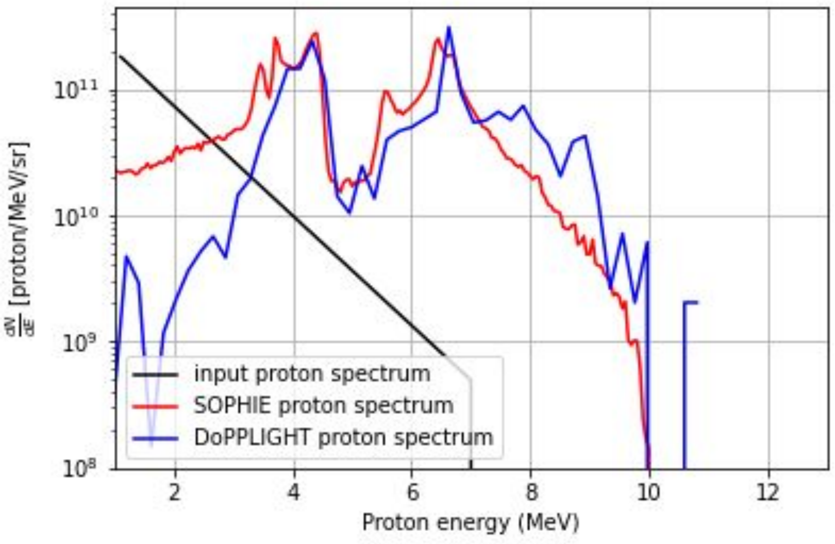
Length of helical coil L=40mm for ALLS facility



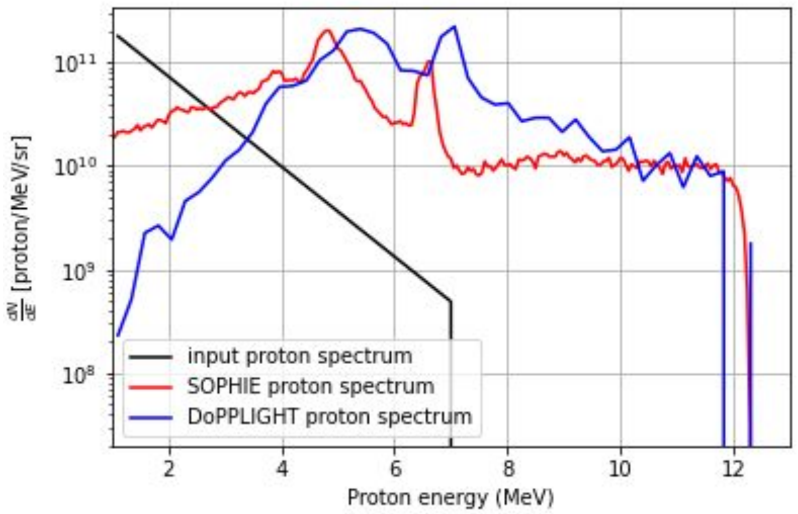
good agreement !

DoPPLIGHT vs SOPHIE (ALLS case)

Varying helical coil

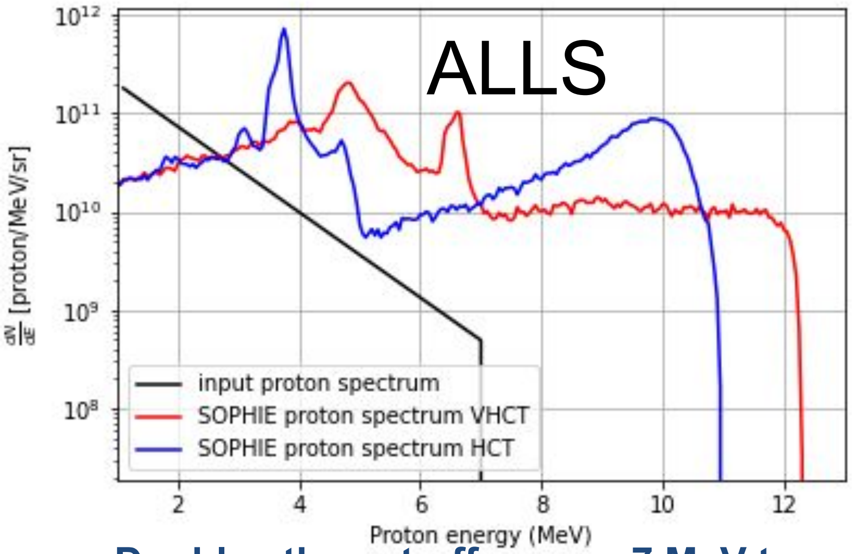


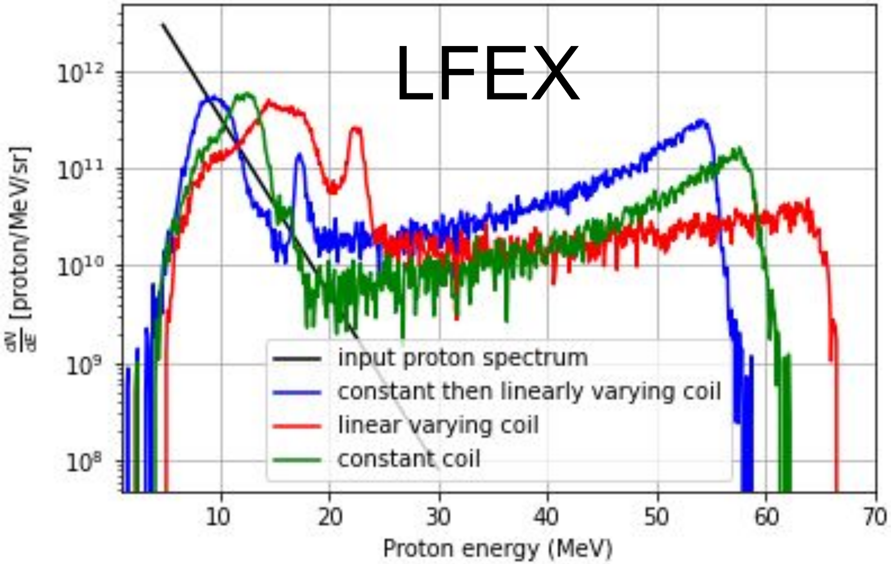
Varying helical coil with tube



Good agreement !

Some results





- **Doubles the cut-off energy 7 MeV to 12 MeV**
- **Increased number of protons per steradian E>3.5 MeV :**
  - *TNSA=10^{10} protons/sr
  - *Blue=3.7×10^{11} protons/sr
  - *Red=3×10^{11} protons/sr
- **High repetition rate**
- **Proof of concept**

- **Doubles the cut-off energy 30 MeV to 60 MeV**
- **Increased number of protons per steradian E>15 MeV:**
  - *TNSA=10^{11} protons/sr
  - *Blue=2.7×10^{12} protons/sr
  - *Red=2.7×10^{12} protons/sr
  - *Green=1.3×10^{12} protons/sr

Conclusions

Proton spectra simulation results

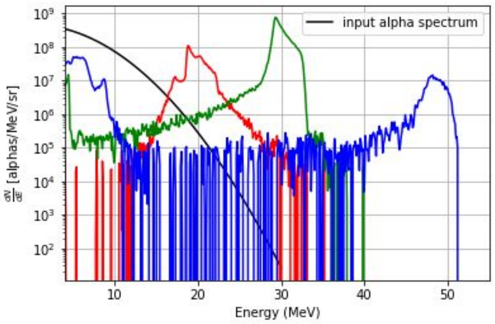
- Flux charge increased for E>3MeV
- Double the cut-off energy
- Bunch in energy of protons

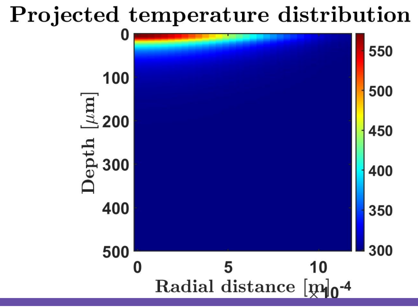
Computation

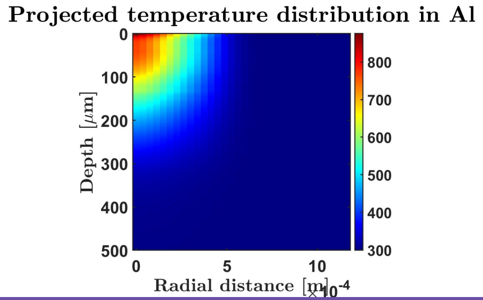
- Varying geometry developed and implement in DoPPLIGHT

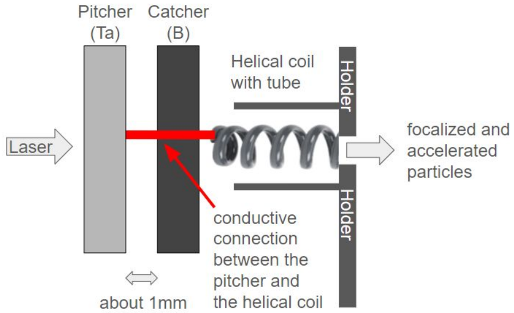
Work in progress

- Optimization of varying helical coil
- Increase the energy of bunch
- Increase the energy cut-off
- ***Experiment***
- Application to alpha particles
- Application to isochoric heating

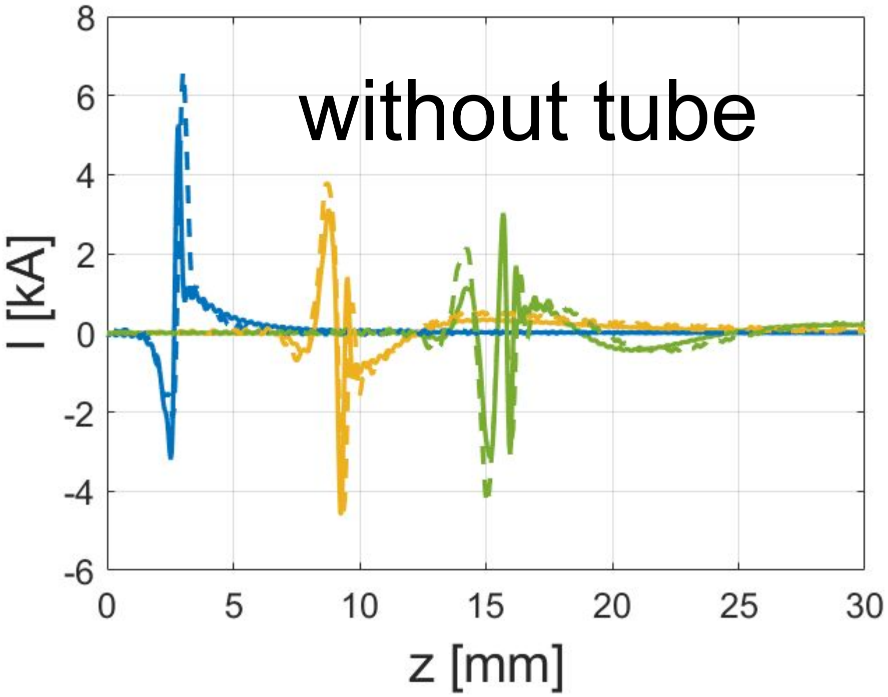


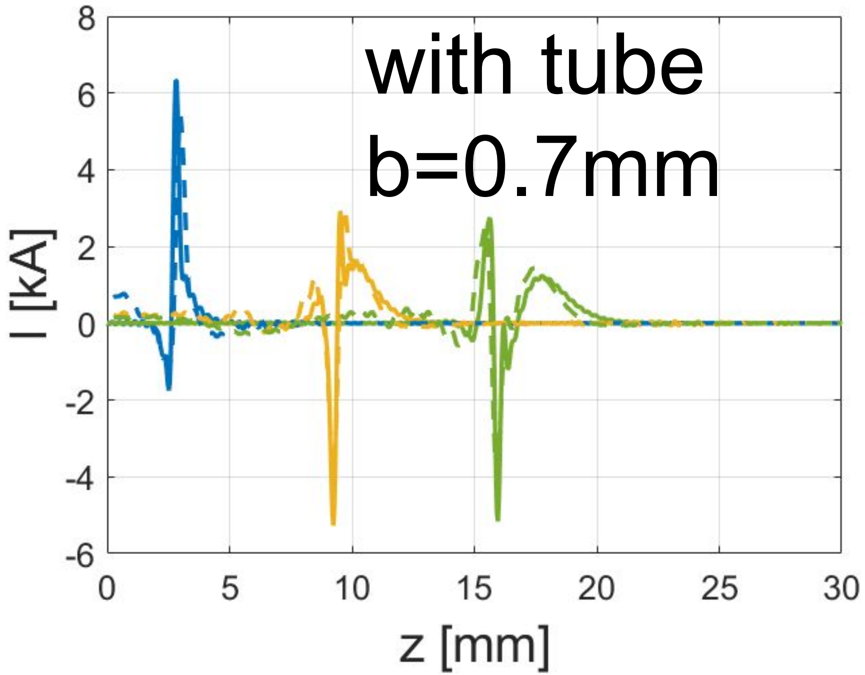






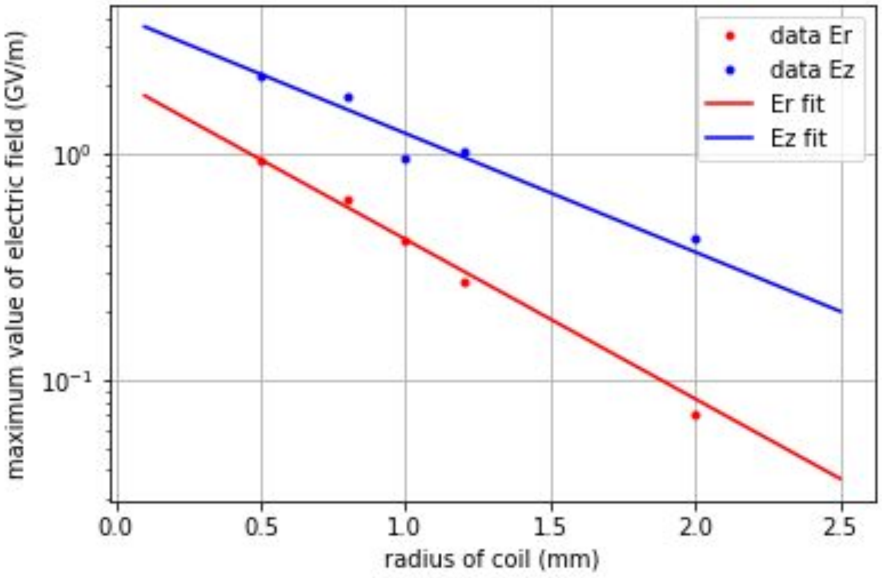
Current at r=0.2 mm for a=0.5 mm h=0.35 mm

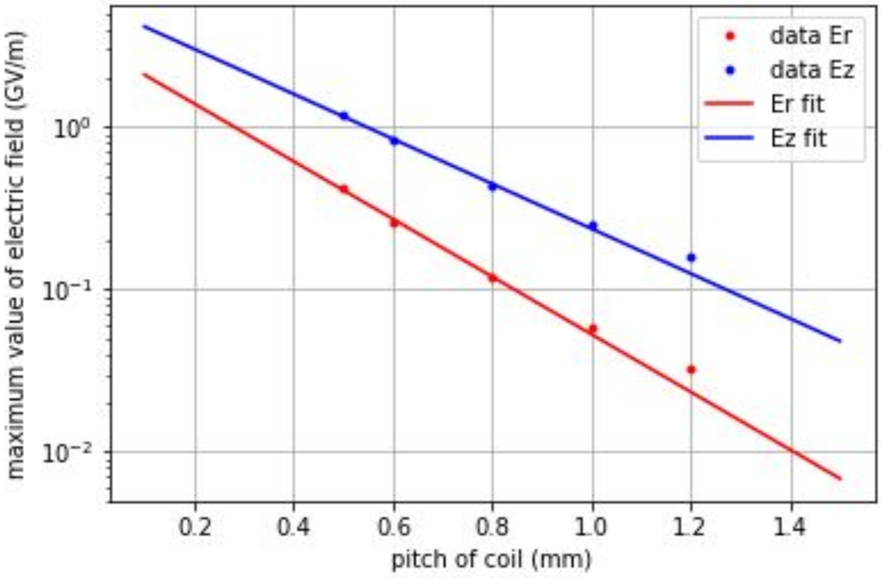




acceleration and deceleration alternation of electric field

Propagation of the current along Helical coil axis





Dans le cadre d'un circuit à résistance nulle l'équation différentielle du courant est :

$$\frac{\partial^2 j(\omega,z)}{\partial z^2}+k^2(\omega,z)j(\omega,z)=2j_0(\omega)\delta'(z)$$

Et considérant une solution de la forme :

$$j(\omega,z)=A(\omega,z)e^{i\phi(\omega,z)}$$

On obtient une solution similaire à l'approximation WKB:

$$j=j_0\frac{\sqrt{k_0}\kappa(\omega)}{\sqrt{k}}e^{i\int_0^z k(\omega,z_\eta)dz_\eta}$$

Pour les champs, les équations différentielles sont:

$$-\Delta\vec{E}+\frac{\vec{\nabla}\rho}{\epsilon_0}=-\frac{\partial(\mu_0\vec{J}+\frac{1}{c^2}\frac{\partial\vec{E}}{\partial t})}{\partial t}$$

$$-\Delta\vec{B}=\mu_0\vec{\nabla}\wedge\vec{J}-\frac{1}{c^2}\frac{\partial^2\vec{B}}{\partial t^2}$$

Par la méthode de séparation de variables :

$$E_z(\omega,r,z)=f(r)g(z)\text{ et }B_z=h(r)l(z)$$

On résout donc pour Ez et Bz, puis grâce à Maxwell-Gauss et Maxwell-Faraday, il vient:

$$\begin{cases}B_r(\omega,r,z)=\frac{-1}{r}\frac{\partial l(z)}{\partial z}\int_0^r rh(r)dr\\E_\theta(\omega,r,z)=\frac{j\omega}{r}l(z)\int_0^r rh(r)dr\\B_z(\omega,r,z)=h(r)l(z)\end{cases}$$

$$\begin{cases}E_r(\omega,r,z)=\frac{-1}{r}\frac{\partial g(z)}{\partial z}\int_0^r rf(r)dr\\B_\theta(\omega,r,z)=\frac{j}{r\omega}\frac{\partial^2 g(z)}{\partial z^2}\int_0^r rf(r)dr+\frac{j}{\omega}\frac{\partial f(r)}{\partial r}g(z)\\E_z(\omega,r,z)=f(r)g(z)\end{cases}$$

On résout pour f(r), h(r), g(z) et l(z), il vient alors :

$$\begin{cases} E_r(\omega,r,z) = \frac{\sqrt{k(\omega,z=0)}\sqrt{k(\omega,z)}}{\alpha}\Lambda(\omega,z)e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[-B_1I_1(\alpha r)+B_2K_1(\alpha r)] \\ B_\theta(\omega,r,z) = \frac{j\sqrt{k(\omega,z=0)}\omega}{\alpha\sqrt{k(\omega,z)}c}\aleph(\omega,z)e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[B_1I_1(\alpha r)-B_2K_1(\alpha r)] \\ E_z(\omega,r,z) = \frac{\sqrt{k(\omega,z=0)}}{\sqrt{k(\omega,z)}}e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[B_1I_0(\alpha r)+B_2K_0(\alpha r)] \end{cases}$$

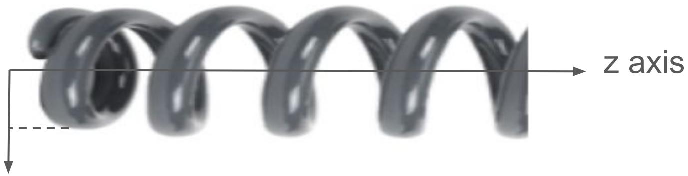
avec $\aleph(\omega,z) = -\frac{k''(\omega,z)}{2k(\omega,z)\omega^2} + \frac{3(k'(\omega,z))^2}{4k^2(\omega,z)\omega^2} - 1$

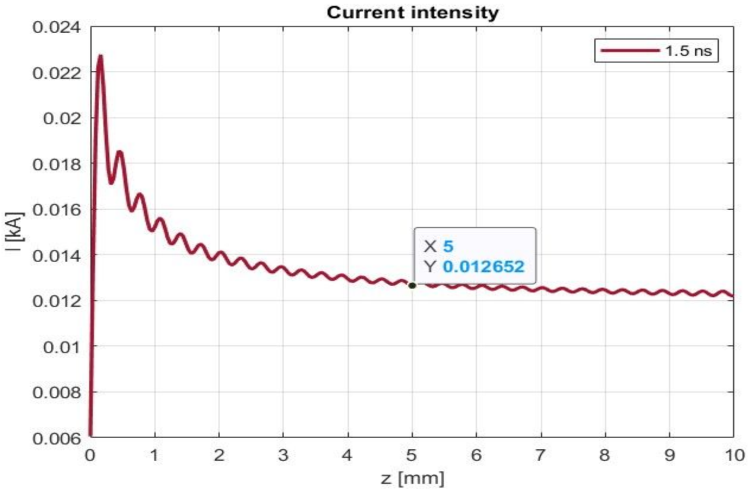
et $\Lambda(\omega,z) = j - \frac{k'(\omega,z)}{2k^2(\omega,z)}$

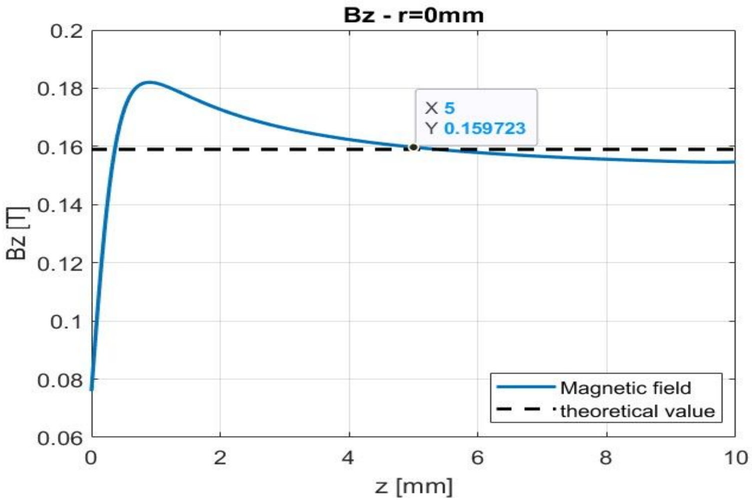
$$\begin{cases} B_r(\omega,r,z) = \frac{\sqrt{k(\omega,z=0)}\sqrt{k(\omega,z)}}{\alpha}\Lambda(\omega,z)e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[-B_3I_1(\alpha r)+B_4K_1(\alpha r)] \\ E_\theta(\omega,r,z) = j\frac{\omega\sqrt{k(\omega,z=0)}c}{\alpha\sqrt{k(\omega,z)}}e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[B_3I_1(\alpha r)-B_4K_1(\alpha r)] \\ B_z(\omega,r,z) = \frac{\sqrt{k(\omega,z=0)}}{\sqrt{k(\omega,z)}}e^{j\int_0^z k(\omega,z_\eta)dz_\eta}[B_3I_0(\alpha r)+B_4K_0(\alpha r)] \end{cases}$$

Varying helical coil with tube - theoretical validation

pitch=0.1mm   little pitch => approximation of a coil







Theoretical validation ; theoretical value of coil $B_z(r, z, t) = \frac{\mu_0 N I_{\theta_0}}{L}$