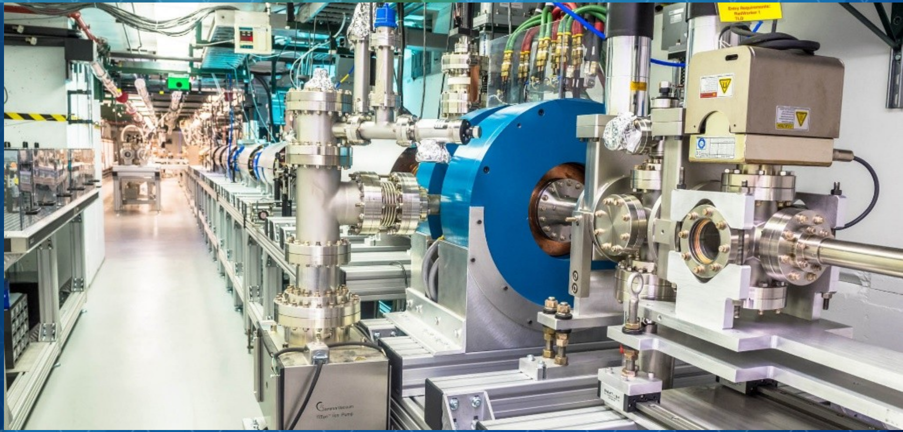
July 22nd 2024

AAC 2024

COHERENT SYNCHROTRON RADIATION SIMULATION METHODS USING CAVITY GREEN'S FUNCTIONS



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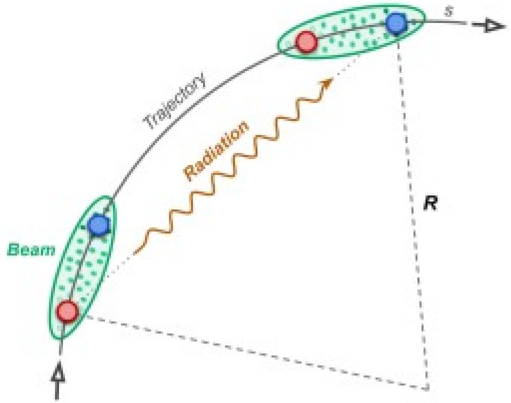
BACKGROUND AND MOTIVATION

- Degradation of phase space due to CSR is a major impediment to the generation of high brightness beams.
- Several mitigation strategies have been proposed (including shielding, longitudinal profile shaping).
- Many open problems exist, including understanding the limits of 1D CSR theory, CSR due to complex beams, shielding, etc.



- This work is part of a larger project that aims to experimentally probe some of these effects
(see poster 225, at 6PM today for more details on the experimental component).

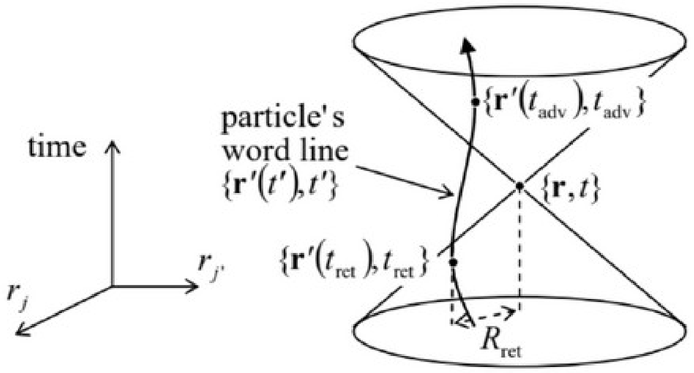
STATE-OF-THE-ART IN CSR SIMULATION

1D CSR Models	• Computationally Inexpensive • Shielding due to distant walls is a correction term. • Not appropriate for 2/3D beam shapes. • Shielding term blows up for very small gaps.
Lienard-Wiechert Solvers	• Exact field computation (valid for 2/3D) • Only valid in free space (except image theory for symmetric problems). • ‘Spikyness’ can be an issue (LW3D), though mixed kernel formulations exist.
PIC Style Methods	• Struggles with numerical dispersion, though some Discontinuous Galerkin methods exist in the literature. • Shielding is possible to resolve for general systems.

OUTLINE/GOALS

- Our primary goal in this work is to construct a CSR simulation technique that accounts for shielding while using an (almost) meshless approach.
- Our eventual method sets up exact image currents on the walls (in the form of a boundary element formulation), but we currently have a simplification that works for straight walls.
- The rest of this talk outlines the overall method and presents some preliminary results for shielding through parallel plates.
- The method will eventually be benchmarked against data obtained from a sequence of planned experiments at the AWA.

RETARDED TIME AND LIENARD WIECHERT POTENTIALS

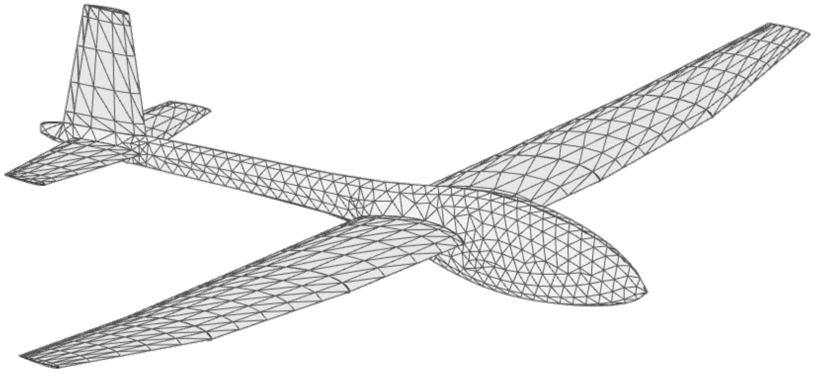


$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\mathbf{n})^3R^2} + \frac{\mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}\}}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^3cR}\right]_{\mathrm{ret}},$$

$$\mathbf{B} = \frac{\mathbf{n}_{\mathrm{ret}}\times\mathbf{E}}{c},$$

- In free space, the fields due to a moving source can be described entirely using Lienard-Wiechert potentials.
- The tricky part is computing the retarded time, since that generally requires a nonlinear curve-sphere intersection (simplifications exist).
- Shielding structures also invalidate the formula over longer timescales.

SHIELDING EFFECTS (TD-BEM)



$$\hat{n}\times\hat{n}\times\mathbf{E}^i(\mathbf{r},t) = -\hat{n}\times\hat{n}\times\mathbf{E}^s\circ\{\mathbf{J}(\mathbf{r},t)\}\ \forall\mathbf{r}\in\Omega$$

$$\mathbf{E}^s\circ\{\mathbf{J}(\mathbf{r},t)\} = -\partial_t\mathbf{A}\circ\{\mathbf{J}(\mathbf{r},t)\} - \nabla\Phi\circ\{\mathbf{J}(\mathbf{r},t)\}$$

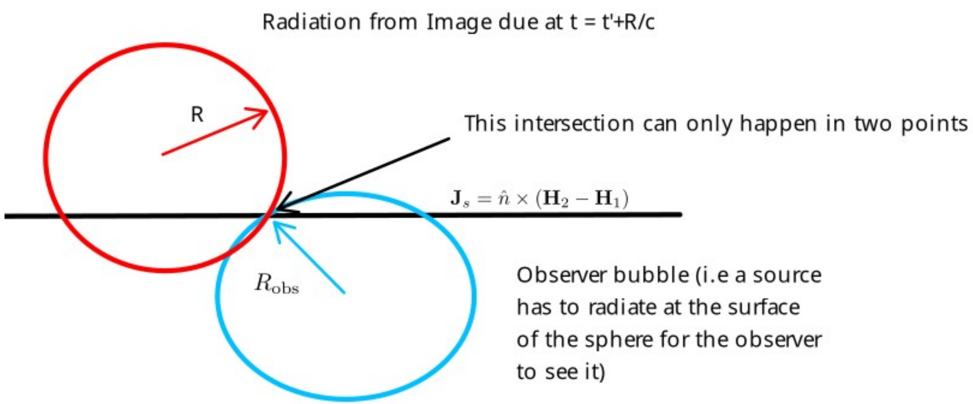
$$\mathbf{A}\circ\{\mathbf{J}(\mathbf{r},t)\} = \frac{\mu_0}{4\pi}\int_\Omega d\mathbf{r}'\frac{\mathbf{J}(\mathbf{r},\tau)}{R}$$

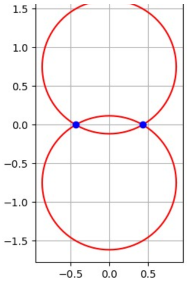
$$\Phi\circ\{\mathbf{J}(\mathbf{r},t)\} = \frac{1}{4\pi\epsilon_0}\int_\Omega d\mathbf{r}'\int_{-\infty}^{\tau} dt'\frac{\nabla'\cdot\mathbf{J}(\mathbf{r}',t')}{R}$$

$$\mathcal{Z}_0\mathcal{I}_j = \mathcal{F}_j - \sum_{i=1}^{j-1}\mathcal{Z}_i\mathcal{I}_{j-i} - \sum_{i=1}^{j-1}\tilde{\mathcal{Z}}_i\mathcal{C}_{j-i}$$

- The most rigorous way to compute surface wall currents is through a time domain integral equations method (EFIE, CFIE, etc.)
- Here, the shielding structures are discretized into a mesh, with the current unknowns typically lying on the edges.
- Then, for a given incident field, surface equivalence theorem can be used to compute the currents.
- Stable methods exist, but they tend to be relatively slow (involves a dense matrix inverse).

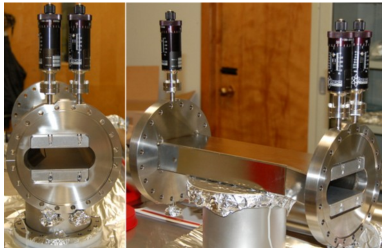
SHIELDING EFFECTS (IMAGE CURRENTS)





$$\mathbf{E}^s \circ \{\mathbf{J}(\mathbf{r},t)\} = -\partial_t \mathbf{A} \circ \{\mathbf{J}(\mathbf{r},t)\} - \nabla\Phi \circ \{\mathbf{J}(\mathbf{r},t)\}$$

Seen from above, the currents on the shielding wall due to an image source have to trace circles.

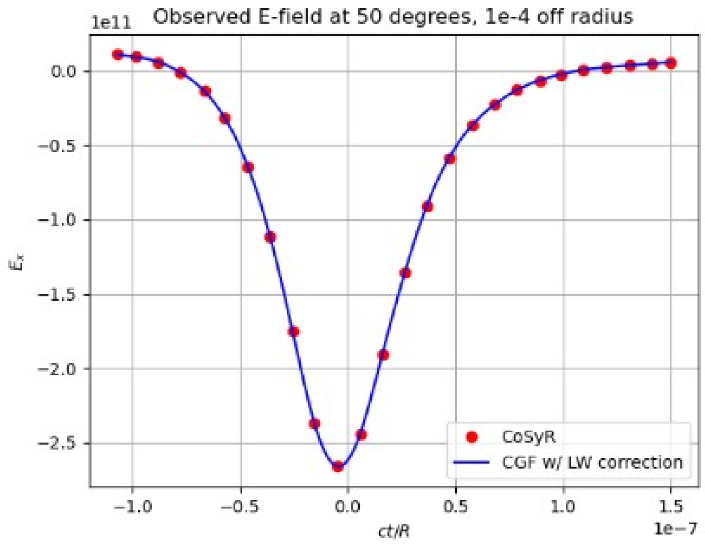


- For parallel plates, there is a simpler solution:
- Since the reflected field solution is already known through image theory, it is possible to directly compute the wall currents.
- These can be convolved with a free-space Green's function to get the reflected fields.
- Computing sphere intersection is a quadratic equation, as opposed to a nonlinear curve-sphere intersection.

PRELIMINARY RESULTS

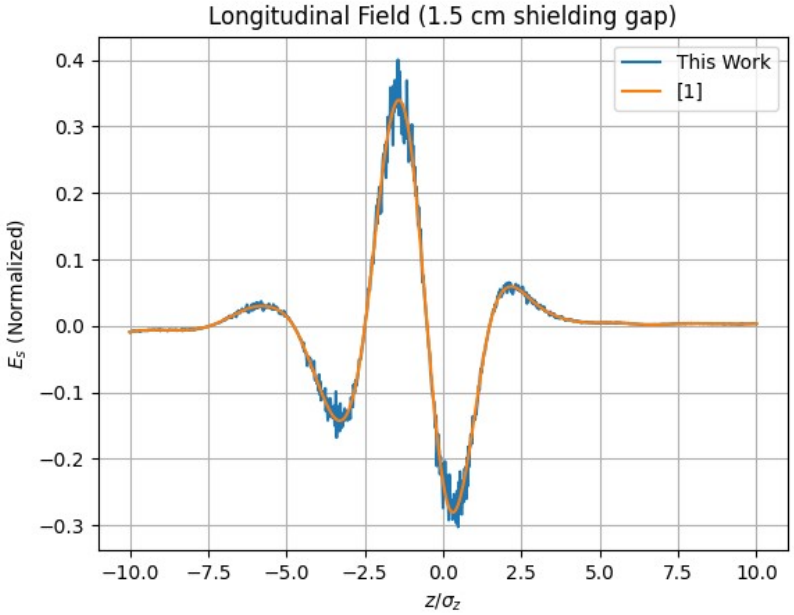
- We next go over preliminary CSR simulation results using the image current formulation described in the previous slides.
- This is primarily stressing the early and mid-time CSR effects.
- In each case, the external force was caused by a constant magnetic field, causing the bunch to trace an arc through the magnet.
- Extensions to arbitrary geometries and the inclusion of waveguide modes for long time simulation is currently under investigation.

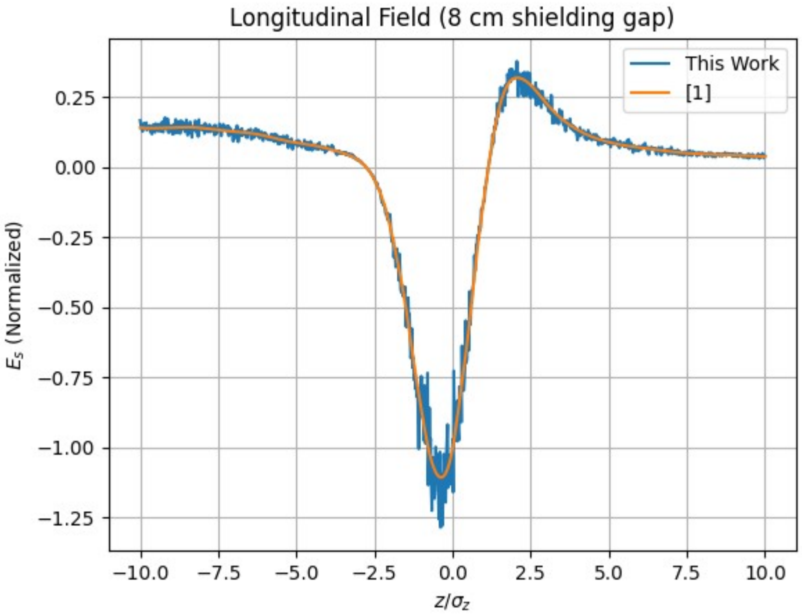
RESULTS: SINGLE ELECTRON CSR WAKE



- We validated the Lienerd Wiechert solver by looking at the time domain wake of a single electron moving in a circular orbit (radius 1, gamma=200)
- The observer was set at 50 degrees, and at a radius 0.1 mm larger than the trajectory.
- We note good agreement against the anaytical result.

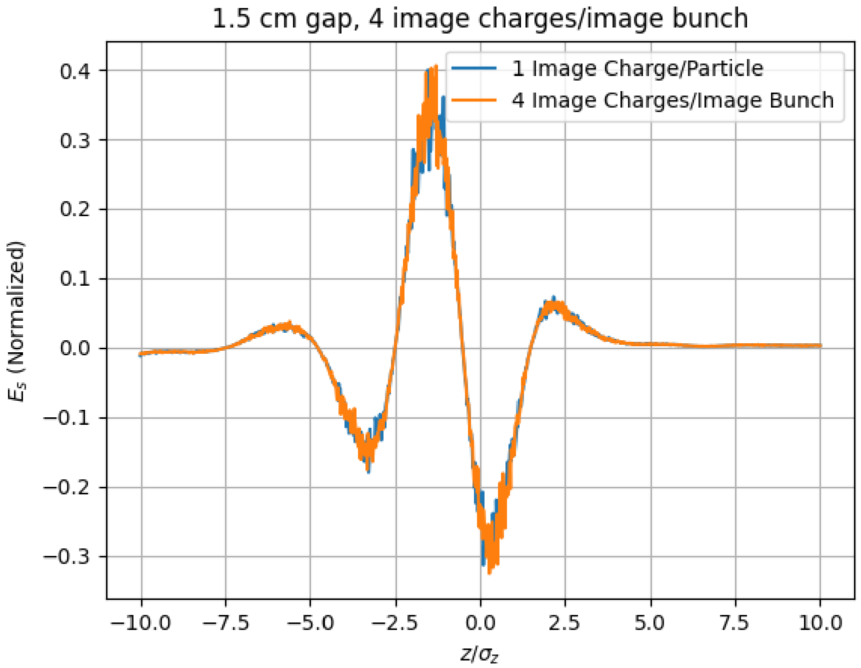
RESULTS: 1D CSR WAKE (SHIELDED)





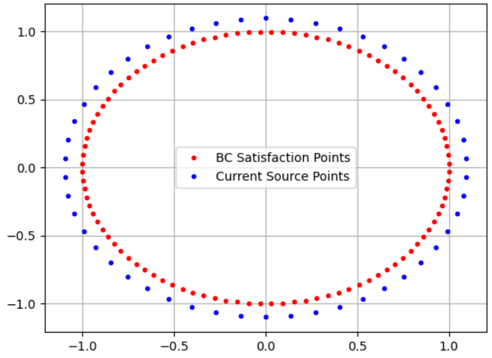
- 1D gaussian bunch (0.3 mm sigma), radius 10 m, 2.48 MeV
- The analytical results were taken from (Sagan et al. 2008).
- The noise is very likely due to the ‘spikyness’ of the LW fields.

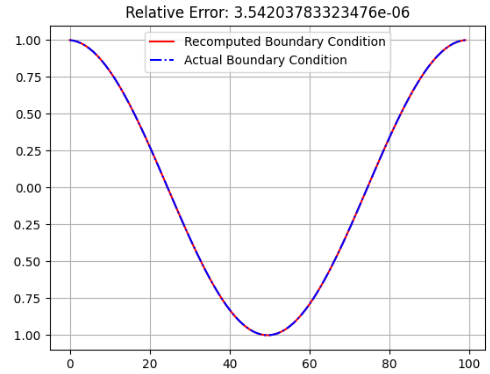
RESULTS: 1D CSR WAKE (SUBSAMPLED IMAGES)

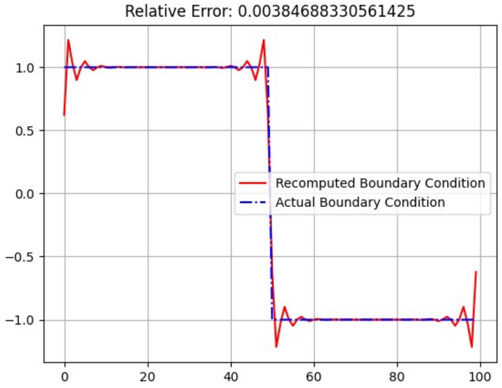


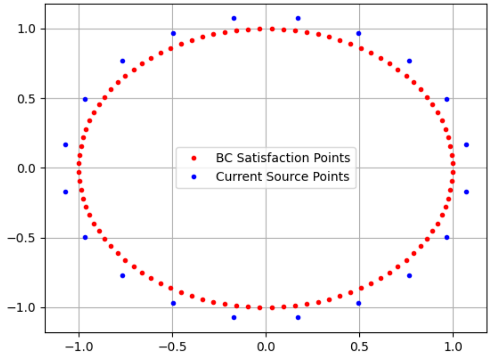
- Subsampling image charges seems to give very similar results.
- We can likely reduce the image charge computation cost by having a few macro-charges.

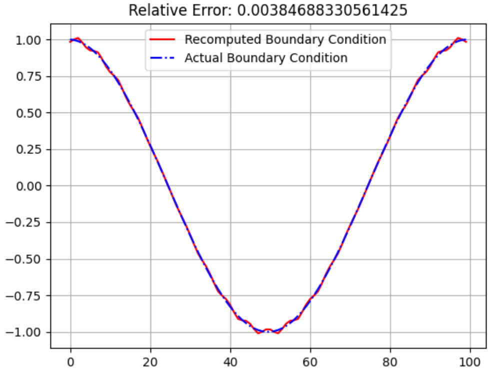
HOW MANY IMAGES DO WE NEED AT STEADY STATE?





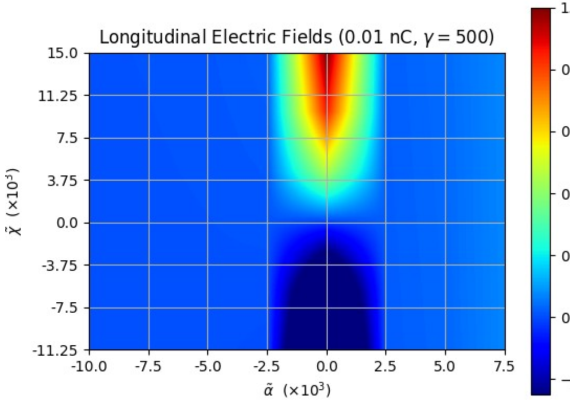


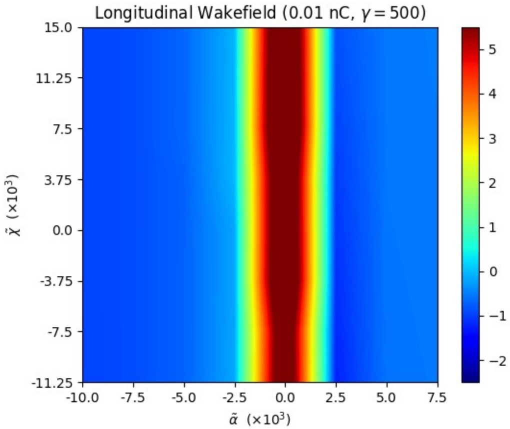


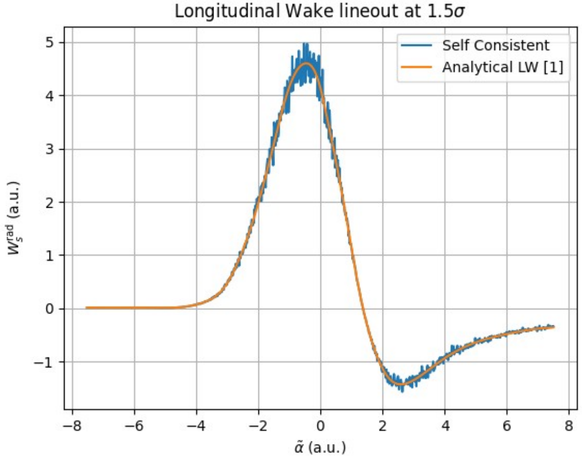


- We looked at the steady state static system in 2D (harder than early time).
- For smooth potentials, we can get away with very few images.
- But with sharp jumps, the errors on the edges get untenable.

RESULTS: 2D CSR WAKE





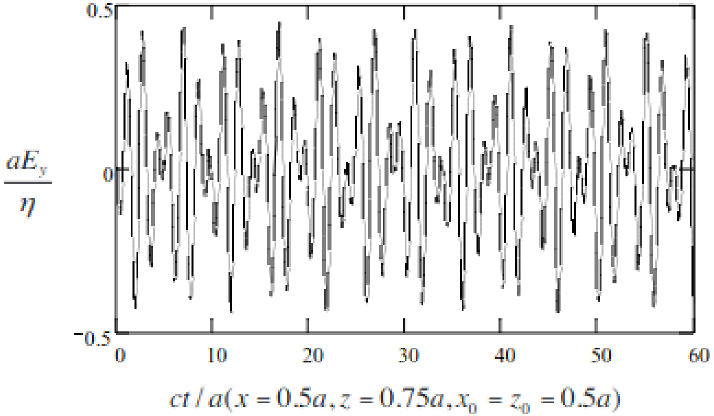


- Next, we considered a 2D Gaussian bunch (no shielding, 10 micron spot size), gamma=500.
- The longitudinal wakefields are compared against CoSyR (Huang et al. 2021) and shows good agreement.
- Once again, we see some noise due to the sharp peaks in the LW potentials.

LONG-TIME EFFECTS (FUTURE WORK)

- At long time, the wall reflections have enough time to distructively interfere away all of the evanescing contributions.
- This means that just the waveguide modes are sufficient to capture the fields over a long timescale (Wen, 2006).
- This would be a fast way to capture the effect of a long bunch going through a relatively short dipole chamber.

$$
\begin{aligned}
\boldsymbol{E}(\boldsymbol{r},t) &= \sum_n \boldsymbol{e}_n(\boldsymbol{r}) \int_V \boldsymbol{E}(\boldsymbol{r},t)\cdot \boldsymbol{e}_n(\boldsymbol{r})dv + \sum_v \boldsymbol{e}_v(\boldsymbol{r}) \int_V \boldsymbol{E}(\boldsymbol{r},t)\cdot \boldsymbol{e}_v(\boldsymbol{r})dv \\
&= \sum_n V_n(t)\boldsymbol{e}_n(\boldsymbol{r}) + \sum_v V_v(t)\boldsymbol{e}_v(\boldsymbol{r}) \\
\boldsymbol{H}(\boldsymbol{r},t) &= \sum_n \boldsymbol{h}_n(\boldsymbol{r}) \int_V \boldsymbol{H}(\boldsymbol{r},t)\cdot \boldsymbol{h}_n(\boldsymbol{r})dv + \sum_\tau \boldsymbol{h}_\tau(\boldsymbol{r}) \int_V \boldsymbol{H}(\boldsymbol{r},t)\cdot \boldsymbol{h}_\tau(\boldsymbol{r})dv \\
&= \sum_n I_n(t)\boldsymbol{h}_n(\boldsymbol{r}) + \sum_\tau I_\tau(t)\boldsymbol{h}_\tau(\boldsymbol{r})
\end{aligned}
$$



CONCLUSIONS

- The primary objective of this work is the construction of a 3D simulation method that incorporates wall effects to an underlying Lienerd Wiechert solver.
- This would allow for a robust characterization of shielding that is beyond the capability of current simulation methods.
- We show good agreement with analytical results (where present), and against similar simulation data in the literature.
- Future plans for this project involve
  - Extending to general conductor shapes with a TD-BEM.
  - Using coarser image current distributions to reduce computational cost.
  - Benchmark the predictions of the shielding methods against planned experiments at the AWA.

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