UCLA Samueli
Electrical & Computer Engineering

Quasi-adiabatic plasma lens designs for the final focus of TeV electrons

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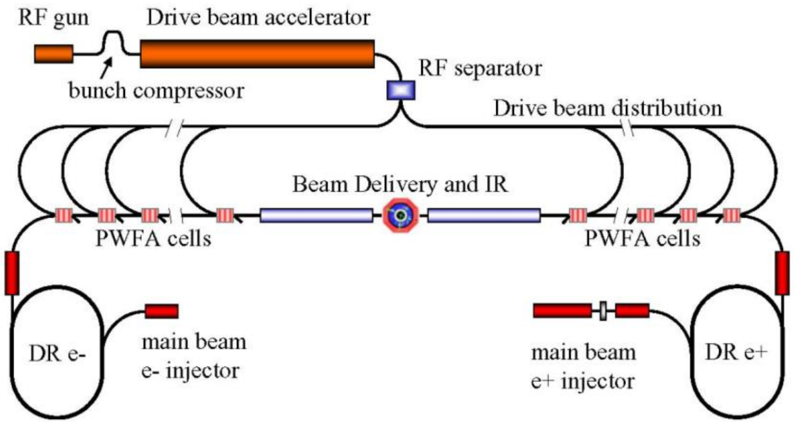
Work at UCLA supported by DOE through DE-SC0010064, and by NSF through award 2108970

The e+e- linear collider design requires tightly focused beams at the interaction point

Luminosity $L = f\ N^2/(4\pi\sigma_x\sigma_y)\ \ \sim 10^{34} cm^{-2} s^{-1}$

Interaction point :
Collision rate f ~ 100 Hz, bunch population, N ~ $10^9 \sim nC$, spot size $\sigma_x\sigma_y \sim (10\ nm)^2$

Beamstrahlung (Can be reduced by using flat beams, TLC spot size $\sigma_x \sim 100nm$, $\sigma_y \sim nm$)



Traditional beam delivery systems

- Long construction distance ~ km
- Chromatic effects requires low energy spread beams
- Oide limit of minimum beam size

Avoided by adiabatic plasma lens
But synchrotron radiation needs to be considered

A. Seryi, et. al. SLAC-PUB-13766, 2009
E. Adli, et. al., arXiv:1308.1145 [physics.acc-ph], 2013
T. Barklow et. al. Journal of Instrumentation, 18:09, 2023.
K. Oide, Physical Review Letters, 61:1713, 1988.
P. Chen, et. al.. Physical Review Letters, 64:1231--1234, 1990.
C. B. Schroeder, et. al. . Physical Review Accelerators and Beams, 13, 101301 (2010)

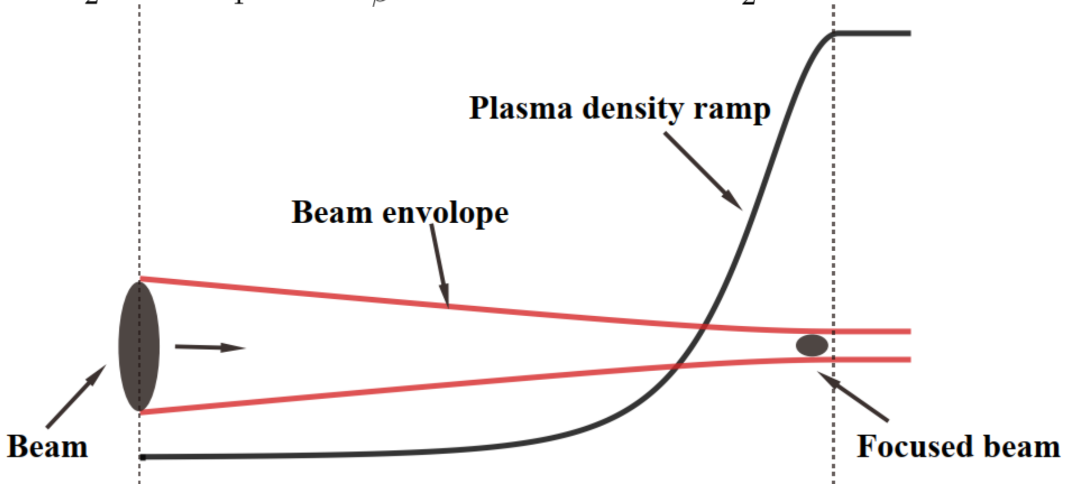
Quasi-adiabatic plasma lens: spot size remains matched as it is adiabatically focused in a density ramp

Linear focusing force for fully blown out plasma $F_\perp = -\frac{1}{2}\, r\,(m\,\omega_p^2)$ ~ plasma density $n_p(s)$

Particles transverse motion in a linear focusing system $x'' + k_\beta^2(s)x = 0$ $\quad k_\beta(s) = \frac{\omega_p}{c\sqrt{2\gamma}}$

Courant–Snyder (twiss) parameters β and α $\beta = \frac{\langle x^2 \rangle}{\epsilon}, \alpha = -\frac{\langle xx' \rangle}{\epsilon}$ $\quad \epsilon$: beam emittance

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + k_\beta^2\beta^2 = 1 \qquad \alpha = -\frac{1}{2}\beta'$$



Adiabatic condition

$$\frac{k'_\beta(s)}{k_\beta(s)} \lesssim k_\beta(s) \iff \alpha \lesssim 1$$

Beam matched at the entrance of plasma will stay matched ($\beta(s) = \frac{1}{k_\beta(s)}$) when propagate in the plasma ramp

X. L. Xu, et. al. Physical Review Letters, 116:124801, 2016.
Y. Zhao, et. al. Physical Review Accelerators and Beams, 23:01, 2020
R. Ariniello, et. al. Physical Review Accelerators and Beams, 22:04, 2019..
K. Floettmann, et. al. Physical Review Accelerators and Beams, 17:054402, 2014

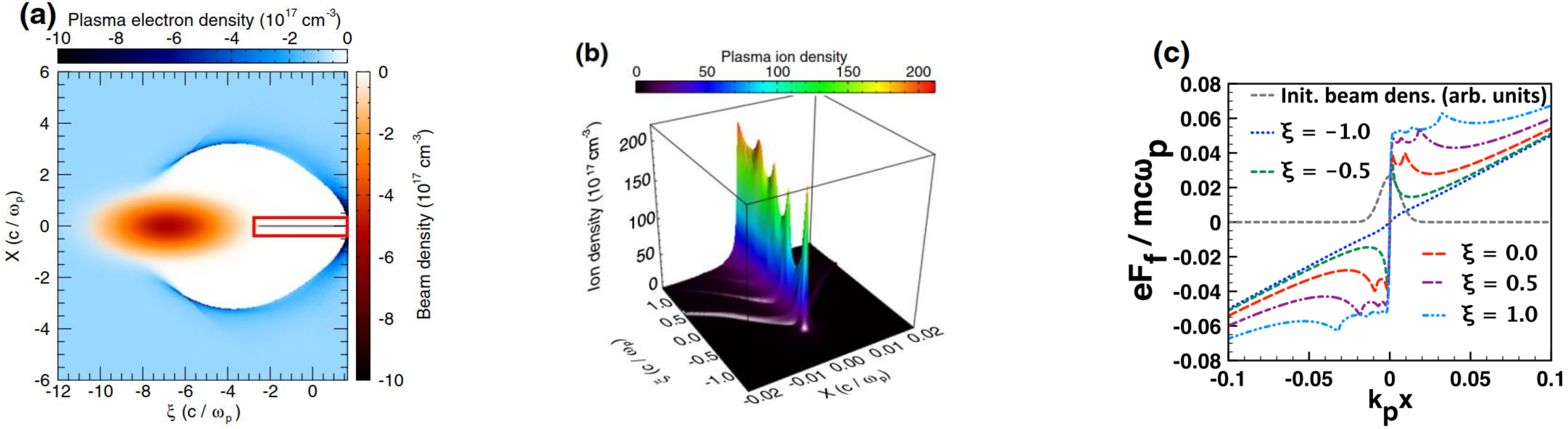
Fields of matched electron beams pull ions inward leading to nonlinear and axial dependent, but higher focusing forces

Ion motion cannot be ignored when $\frac{n_b}{n_0} \gtrsim \frac{m_i}{m_e} = 1836$ for hydrogen ions

Beam density with linear collider parameter $n_b \gtrsim \frac{10^9}{100\ nm\ nm\ um} \sim 10^{25} cm^{-3}$

Plasma frequency with beam density $\Omega_b = \sqrt{\frac{4\pi n_b Ze^2}{m_i}} \sim fs^{-1}$ for hydrogen ions and linear collider parameters

Ion motion occurred in the beam when $\frac{\boldsymbol{\Omega_b \sigma_z}}{\boldsymbol{c}} \gtrsim \mathbf{1}$



S. Lee, T. Katsouleas, AIP Conf. Proc. 472, 524–533, 1999
J. B. Rosenzweig, et. al. Physical Review Letters, 95, 195002, 2005
C. Benedetti, et. al. Physical Review Accelerators and Beams, 20:111301, 2017

W. An, et. al., Physical Review Letters, 118:244801, 2017,
Y Zhao, et. al. Physical Review Accelerators and Beams 26 (12), 121301, 2023

Quasi-adiabatic ramps have been shown to preserve the beam emittance in the presence of ion motion

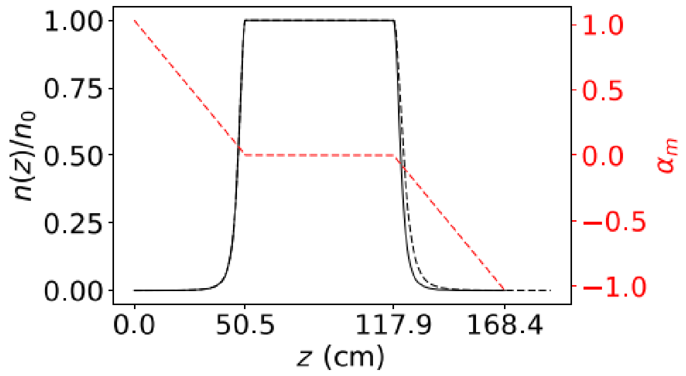
One stage of acceleration of 25 GeV trailing beam

Wide drive beam and $n_0 = 10^{17} cm^{-3}$

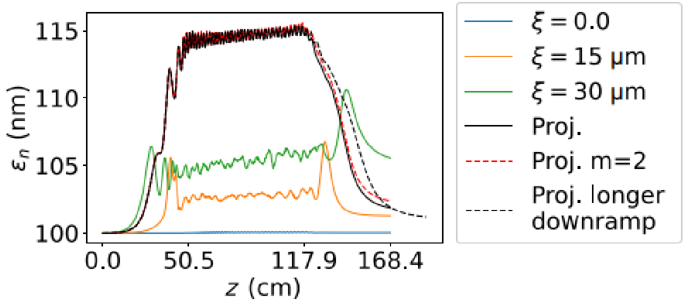
Plasma density ramp $\frac{n(z)}{n_0} = \left(1 + \frac{\alpha_{mi}(z-L)^2}{\beta_{m0} L}\right)^{-2}$

$$\beta_m(z) = \sqrt{2\gamma} c/\omega_p(z), \qquad \alpha_m(z) = -\frac{1}{2}\beta_m'(z).$$

$$\frac{\Omega_b \sigma_z}{c} = 2.2577 \sqrt{\frac{Z N_b(10^9)\sigma_z(um)}{m_i(1836 m_e)}} \left(\frac{n_p(10^{19} cm^{-3})\, E(500\ GeV)}{\epsilon_{Nx}\ (100nm)\epsilon_{Ny}\ (100nm)}\right)^{\frac{1}{4}}$$



(a)



(b)

Y. Zhao, et. al. Physical Review Accelerators and Beams 26 (12), 121301, 2023

L. Hilderbrand, et. al. Submitted to Physical review letters

We investigated plasma lens for the electron arm according to possible linear collider parameters

Technology	PWFA	PWFA	PWFA	PWFA
Beam Aspect Ratio	Flat	Flat	Flat	Round
Center-of-Mass Energy	1	3	15	15
E_{beam} (TeV)	0.5	1.5	7.5	7.5
γ	9.78E5	2.94E6	1.47E7	1.47E7
ε_x (mm mrad)	0.66	0.66	0.66	0.1
ε_y (mm mrad)	0.02	0.02	0.02	0.1
β_x^* (mm)	5	5	5	0.15
β_y^* (mm)	0.1	0.1	0.1	0.15
σ_x^* (nm)	58.07	33.53	15	1.01
σ_y^* (nm)	1.43	0.83	0.4	1.01
N_{bunch} ($\times 10^9$)	5	5	5	5
f (kHz)	4.2	14	13.12	7.73
σ_z (um)	5	5	5	5
Υ	15	78	867	6590
n_γ	1.5	1.5	1.5	5.7
P_{beam} (MW)	1.7	16.8	78.8	55.0
$2P_{\mathrm{beam}}$ (MW)	3.4	33.6	157.6	110.0
$\mathcal{L}_{\mathrm{geo}}$ ($\times 10^{34}$ cm^{-2} s^{-1})	1.01	10.1	47.1	150

We can also use beams accelerated by conventional accelerator or other technology.

15 TeV colliders can use round beams design

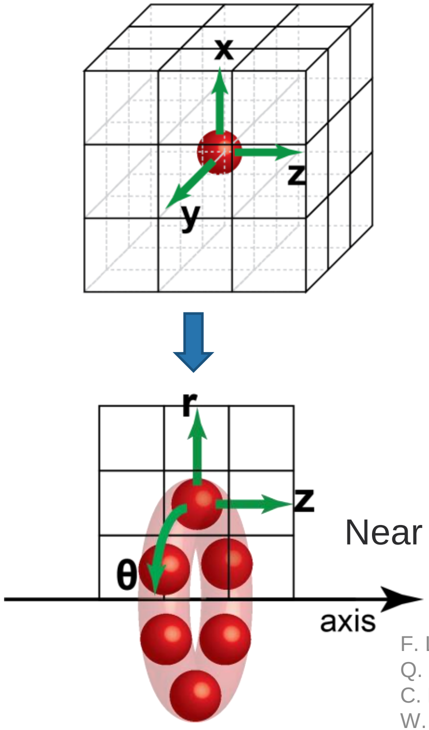
We try to reach the target spot size by choosing the emittance and plasma lens parameter

T. Barklow et. al. Journal of Instrumentation, 18:09, 2023.

Advanced simulation tools opens the possibility to study ion motion in plasma >1000x speed up: QPAD and QuickPIC

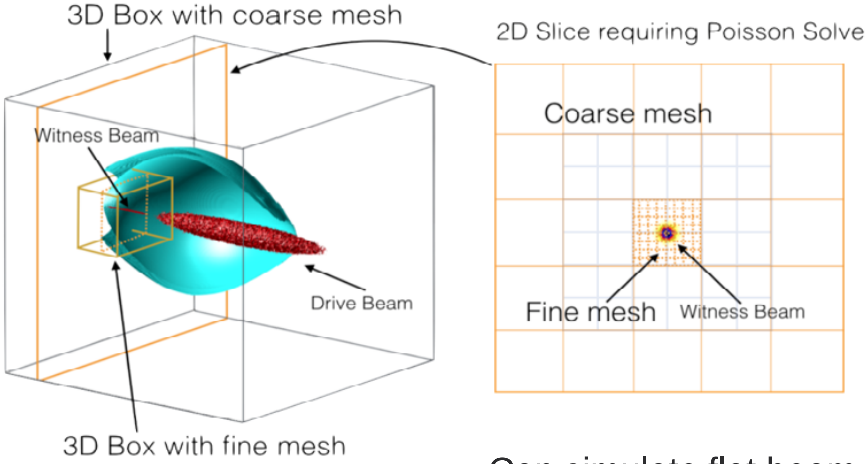
Focused spot size ~ nm (Bubble radius ~ tens of μm for plasma density $10^{17}\sim10^{19} cm^{-3}$**)**

QPAD (Quasi-static code with azimuthal decomposition)



Near Symmetric beam

3D QuickPIC with mesh refinement

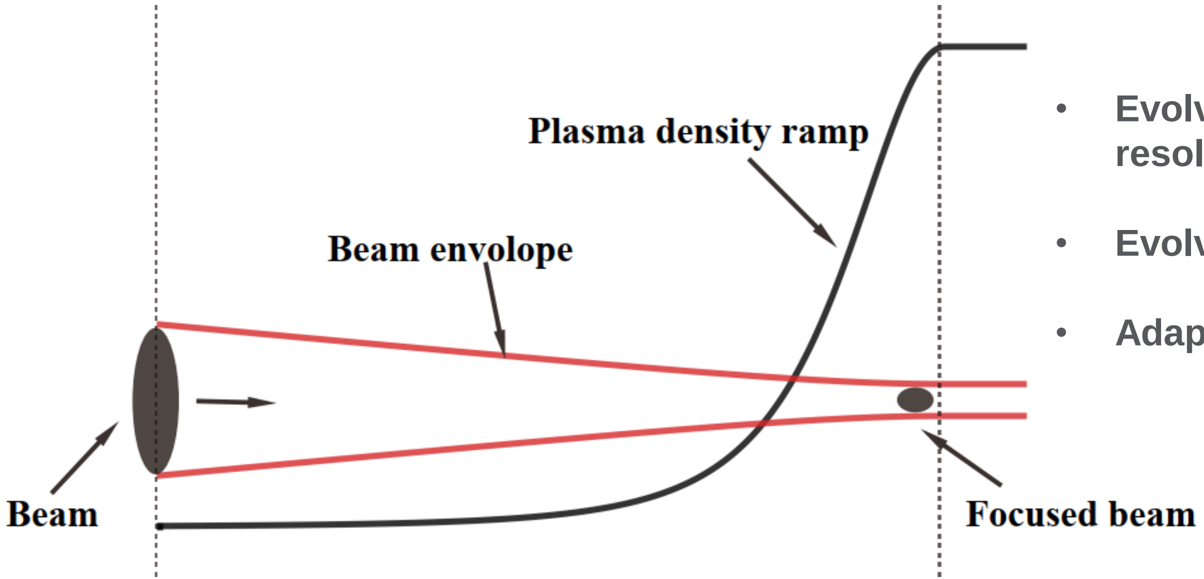


Can simulate flat beam

Simulations shown in this talk use mesh refinement

F. Li, et. al., Computer Physics Communications, 261:107784, 2021
Q. Su, et. al., arXiv:2405.00886
C. Huang et. al. Journal of Computational Physics, 217(2):658–679, 2006
W. An, et. al. Journal of Computational Physics, 250:165–177, 2013.

Adaptive mesh grid setting for beam propagation in a plasma ramp



- Evolving simulation box size and resolution
- Evolving time step
- Adaptive mesh refinement

Large simulation window
Lower refinement ratio
Large time step

→

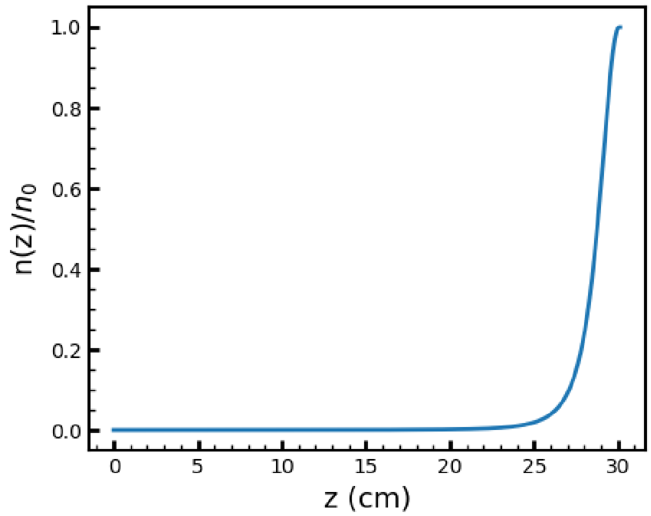
Smaller simulation window
Higher refinement ratio
Small time step

Adiabatic plasma ramp for 15 TeV CM LC – QuickPIC with mesh refinement

Plasma density $\boldsymbol{n_0} = \mathbf{5} \times \mathbf{10^{19}} \boldsymbol{cm^{-3}}$ for normalization, $k_p^{-1} = \frac{c}{\omega} = 0.75$ um,

Drive beam energy 250 GeV, trailing beam energy 7.5 TeV

Matched beam parameters calculated without considering ion motion : found to provide optimal CS parameters at the entrance



$$\frac{n(z)}{n_0} = \frac{1}{\left(1+\frac{\alpha_{mi} z^2}{\beta_{m0} L}\right)^2}$$

In normalized unit

Initial plasma density ratio $n_i/n_0 = 2.02 \times 10^{-5}$

Trailing beam initial α at the entrance $\alpha_{mi} = 3.0$, matched β at uniform density $\beta_{m0} = 5418$

$$\beta = \frac{\langle x^2 \rangle}{\epsilon}, \alpha = -\frac{\langle x x' \rangle}{\epsilon}$$

Drive beam parameters obtained from numerical integration

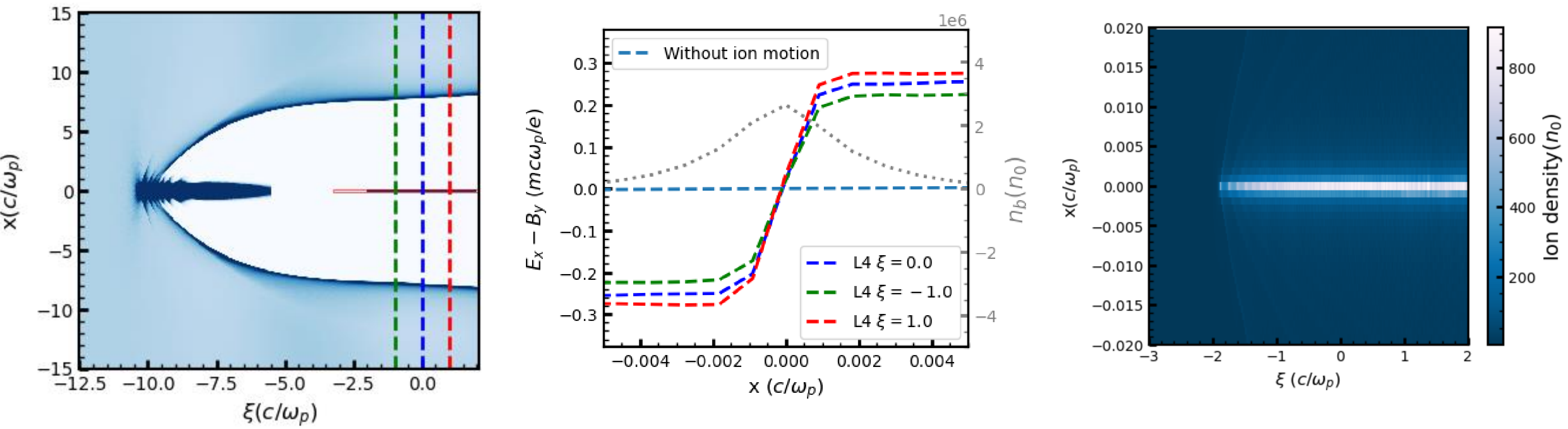
$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + k_\beta^2 \beta^2 = 1 \qquad \alpha = -\frac{1}{2}\beta'$$

Intensely focused beam pull ions inwards and provide a higher focusing force

Drive beam parameter : Normalized emittance $\epsilon_x = \epsilon_y = 15\ um$,$\sigma_{mz} = 0.75\ um$, Charge 0.63 nC, **Energy 250 GeV**

Trailing beam parameter : **Normalized emittance $\boldsymbol{\epsilon_x = \epsilon_y = 100\ nm}$** , Initial spot size $\sigma_{ix} = \sigma_{iy} = 78.5\ nm,$ Bunch length $L_z = 3.0\ um$,**Energy 7.5 TeV**, Charge 0.82 nC， 1% energy spread

Plasma and beam profile at 28.5 cm, $n = 2\ \times 10^{19} cm^{-3}$

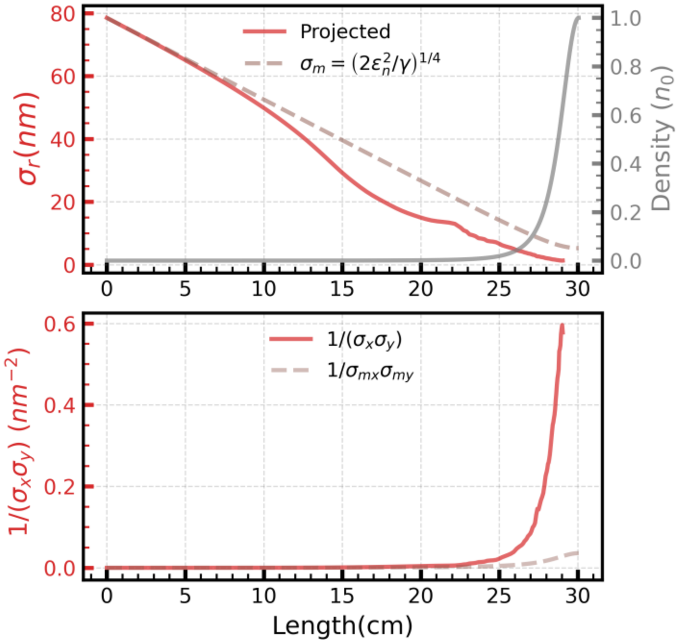


The trailing beam is focused down to nm scale

Trailing beam energy 7.5 TeV, Charge 0.82 nC， 1% energy spread at the end of the plasma lens

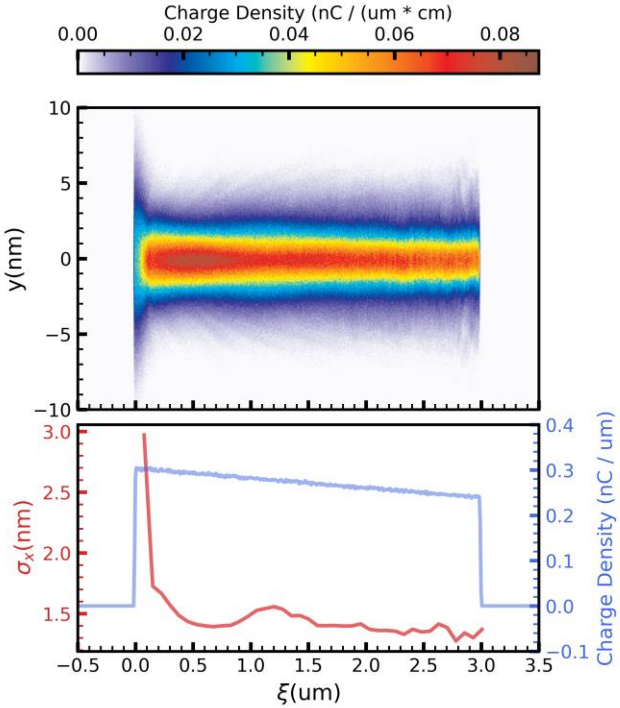
Initial spot size $\sigma_{ix} = \sigma_{iy} = 78.5\,nm$ focused down to **1.3 nm**

Spot size without ion motion **5.9 nm**



20.6 times higher luminosity with ion motion

At end of plasma lens : $n = 2\ \times 10^{19} cm^{-3}$



End plasma lens at 28.5 cm

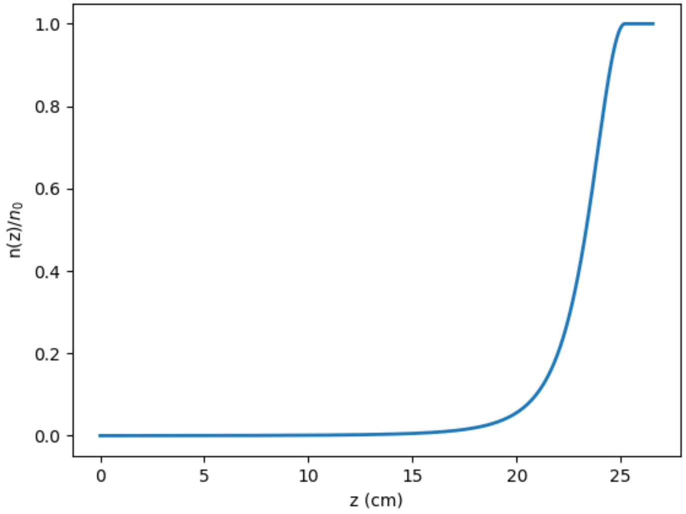
Adiabatic plasma ramp for 1TeV asymmetric beam – QuickPIC with mesh refinement

Maximum plasma density $n_0 = 1 \times 10^{19} cm^{-3}$, $k_p^{-1} = \frac{c}{\omega} = 1.68$ um

Matched beam parameters calculated without considering ion motion

Drive beam energy same as trailing beam energy

$$\frac{n(z)}{n_0} = \frac{1}{\left(1+\frac{\alpha_{mi} z^2}{\beta_{m0} L}\right)^2}$$



In normalized unit

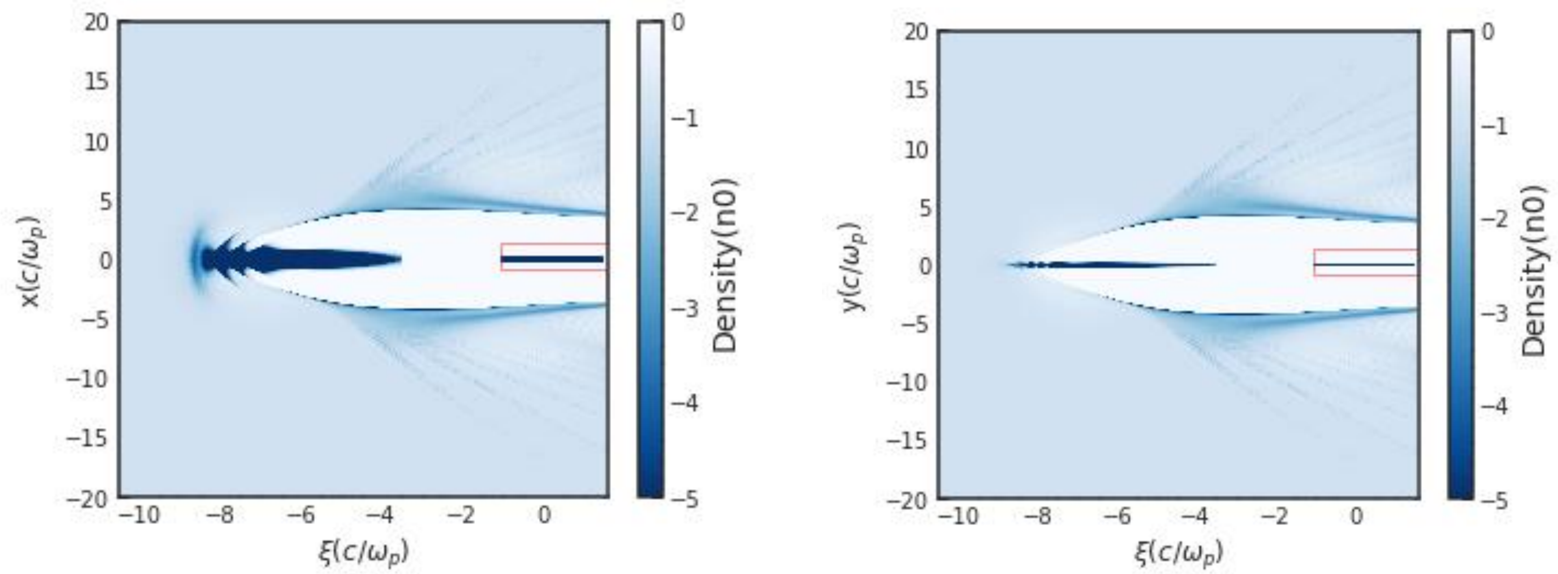
Initial plasma density ratio $n_i/n_0 = 0.00058$
Initial α at the entrance $\alpha_{mi} = 1.0$
Matched β at uniform density $\beta_{m0} = 1978$

$$\beta = \frac{\langle x^2 \rangle}{\epsilon}, \alpha = -\frac{\langle xx' \rangle}{\epsilon}$$

Asymmetric witness beam with asymmetric driver provide asymmetry for plasma lens

Drive beam parameter **: Normalized emittance $\boldsymbol{\epsilon_x = 158\ um, \epsilon_y = 10\ um}$** ,$\sigma_{mz} = 1.68\ um$, Energy 1TeV, Charge 0.63 nC, **aspect ratio $\boldsymbol{\sigma_x/\sigma_y}$ = 3.9**

Trailing beam parameter : **Normalized emittance $\boldsymbol{\epsilon_x = 12.6\ um,\ \epsilon_y = 44\ nm}$**, Bunch length $L_z = 4.2\ um$, Energy 1TeV, Charge 0.58 nC, **aspect ratio $\boldsymbol{\sigma_x/\sigma_y}$ = 16.7**

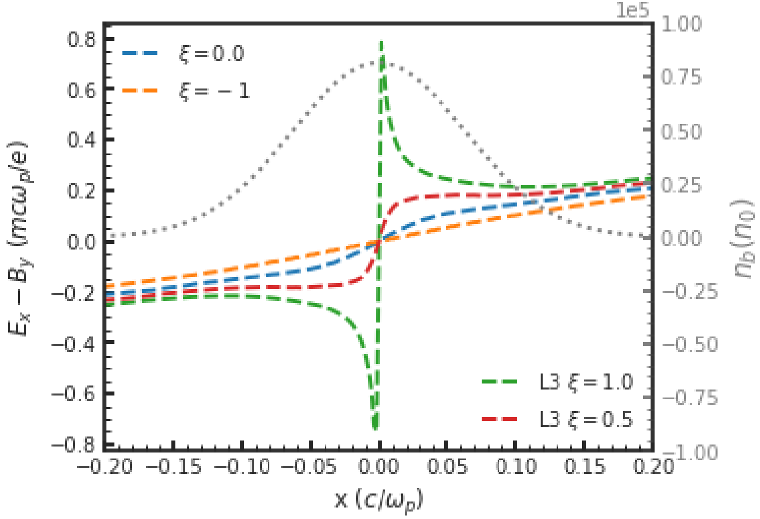


Ion motion increases the focusing field 10-100 times

Drive beam parameter : Normalized emittance $\epsilon_x = 158\ um, \epsilon_y = 10\ um$,$\sigma_{mz} = 1.68\ um$
Charge 0.63 nC, aspect ratio 3.9

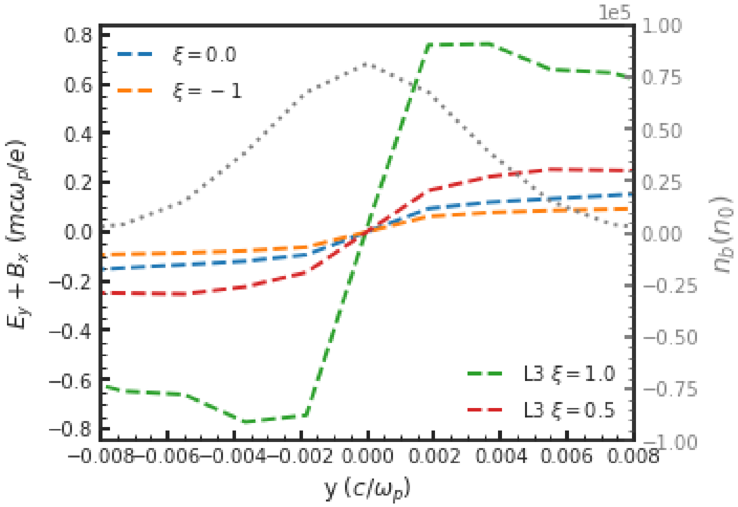
Trailing beam parameter : Normalized emittance $\epsilon_x = 12.6\ um,\ \epsilon_y = 44\ nm,$ Bunch length $L_z = 4.2\ um,$
Energy 1TeV, Charge 0.58 nC, aspect ratio 16.7

Focusing field in x direction



$\frac{E_x - B_y}{x}$ from 1 to 50

Focusing field in y direction

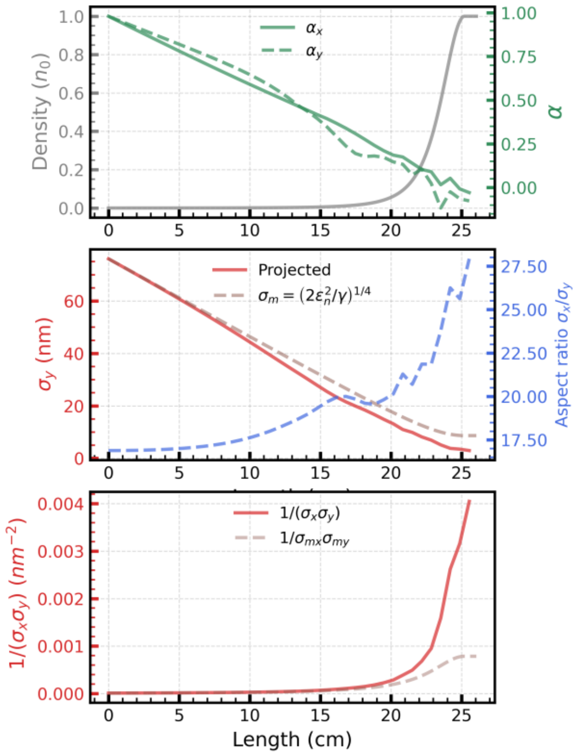


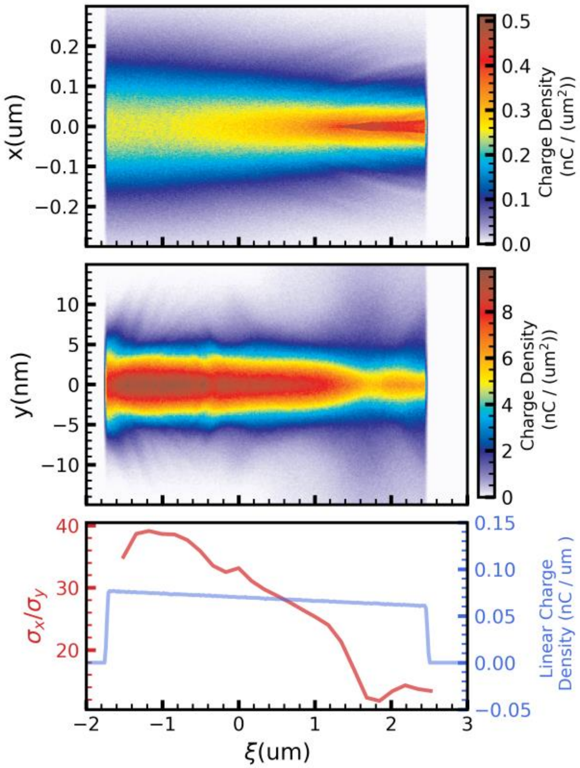
$\frac{E_y + B_x}{y}$ from 20 to 250

The ion motion provide extra asymmetry for the trailing beam

Trailing beam focused to : σ_y = 83 nm, $\sigma_x = 3.0\ nm$

Matched spot size in y direction without ion motion : 8.7 nm , Aspect ratio increased from 16.7 to around 28





With ion motion

Three times smaller spot size in y direction

Aspect ratio doubled at the beginning of the beam

5 times higher luminosity

Achieved beam parameters using maximum plasma density $n_0 \sim 10^{19} cm^{-3}$

Center-of-mass Energy	1-3TeV		15 TeV			
	Target	Plasma lens	Target	Plasma lens	Target	Plasma lens
Ebeam (TeV)	1.5	1	7.5	7.5	7.5	7.5
Charge(nC)	0.8	0.58	0.8	0.82	0.8	0.82
σ_x(nm)	33	83	1.0	1.3	15	40
σ_y(nm)	0.8	3.0	1.0	1.3	0.4	1.4
Designed ϵ_x (nm)	660	12600	100	100	660	10000
Designed ϵ_y (nm)	20	44	100	100	20	25
σ_z or l_z	5.0	4.2	5.0	3.0	5.0	3.0
Aspect ratio	40	From 17 to 28	1	1	37.5	From 20 to 29

Summary

- Plasma lens including ion motion shows potential for the electron-arm final focus of a LC (PBA or conventional)
  - Plasma lens with maximum density $n_0 = 2 \times 10^{19} cm^3$ can provide target spot size for a 15 TeV round beam example
    - Beams focused down to 1.3 nm
    - 20 times higher luminosity with ion motion
    - Synchrotron radiation needs to be self-consistently included in the analysis
  - 1-3 TeV asymmetric examples
    - 5 times higher luminosity compared to without ion motion
    - Asymmetric drive beam improve the trailing beam aspect ratio
    - Aspect ratio of 20-30 was achieved in preliminary simulation
    - The examples can be improved with higher plasma density and lower emittance for the smaller spot size direction.

Thanks for your attention!