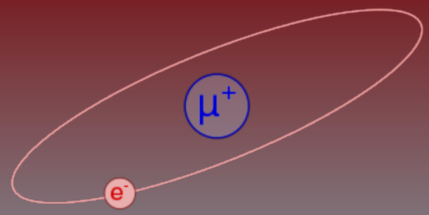
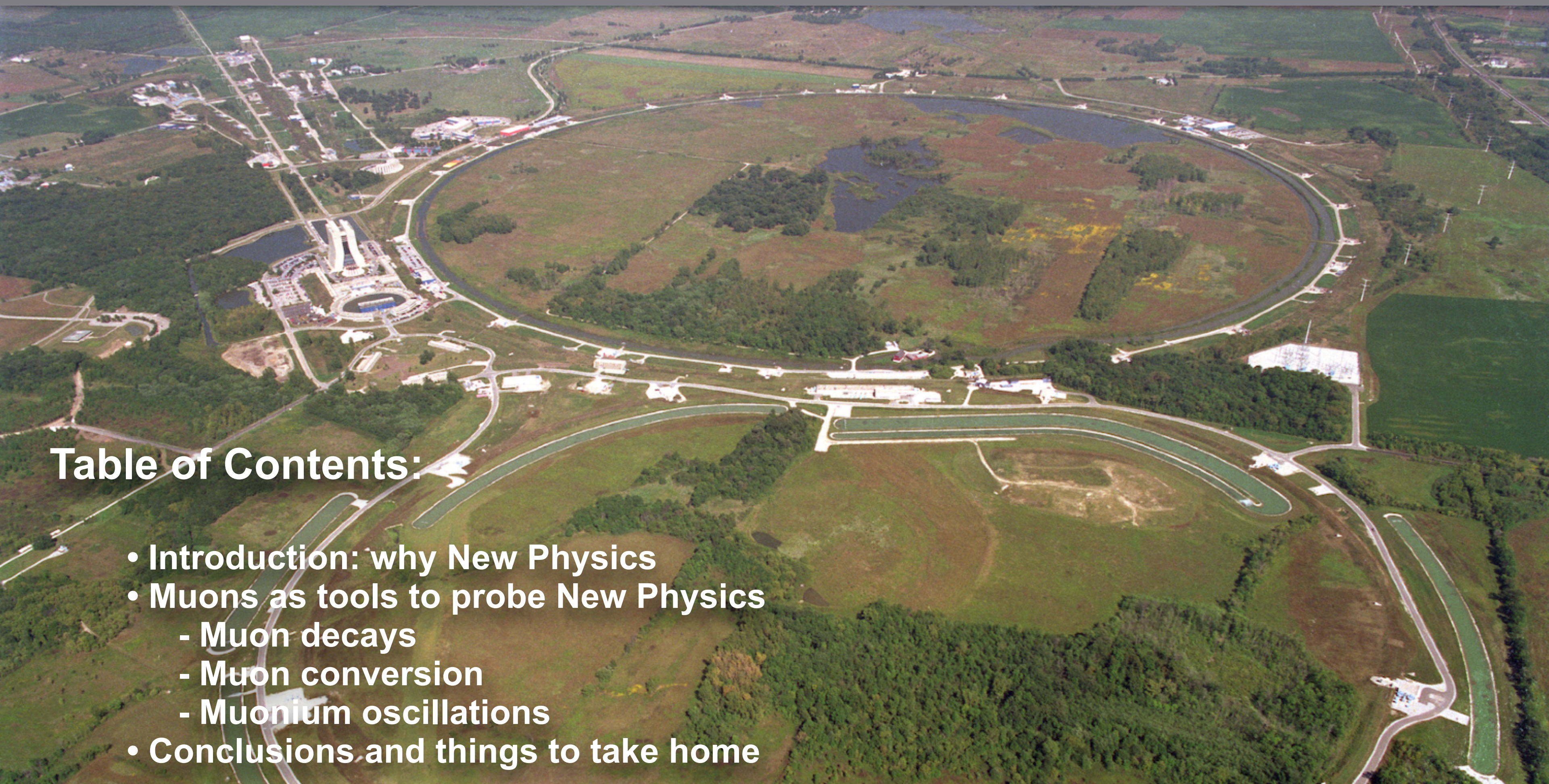


New Ideas for Muon Physics



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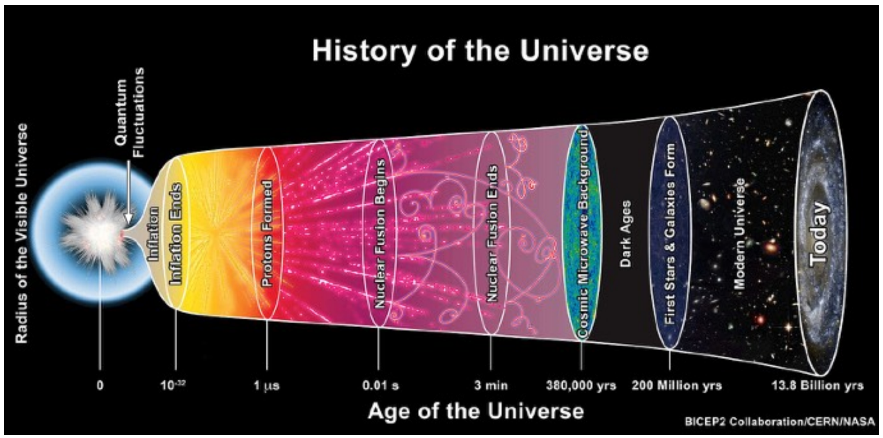
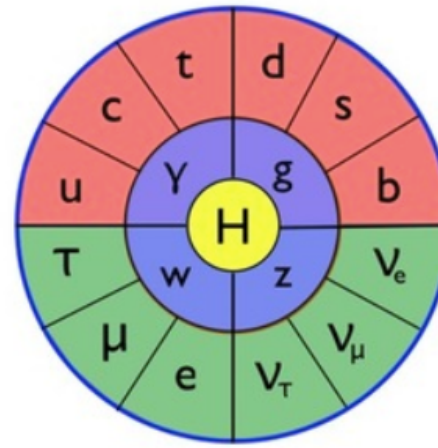
- Introduction: why New Physics
- Muons as tools to probe New Physics
  - Muon decays
  - Muon conversion
  - Muonium oscillations
- Conclusions and things to take home

Alexey A. Petrov
University of South Carolina

★ Is it possible to build the Universe using the Standard Model as a tool?

- no, but maybe it can tell us where to look for new tools





★ The era of "guaranteed discoveries" is over (top quark, electroweak breaking)

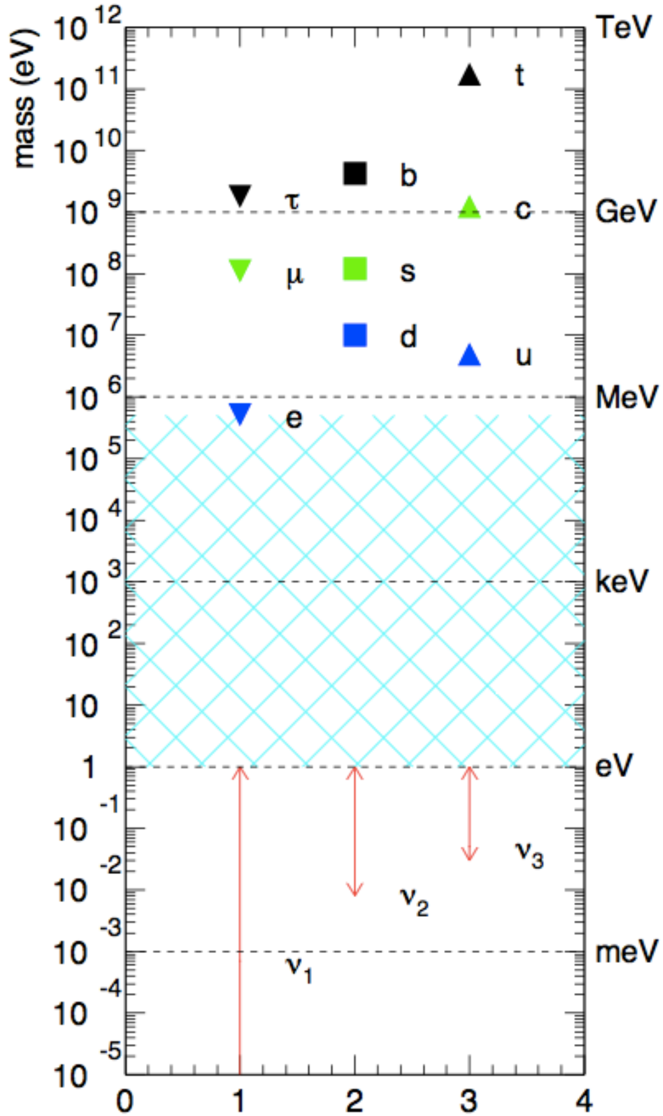
- new experiments designed to study rare decays or perform precision studies of various processes might point us in the right direction

★ What about New Physics?

- no new elementary particles so far at the LHC
- neutrinos oscillations: ν's have mass and so CLFV transitions are guaranteed
- use sphaleron mechanism: baryogenesis via leptogenesis   Fukugita, Yanagida
- new sources of CP-violation in the lepton sector

Should we look for New Physics in the charged lepton sector? Muons?

Fundamental physics with muons: flavor problem



★ SM and BSM Flavor problem

★ Flavor problem: patterns of masses of particles

- quarks

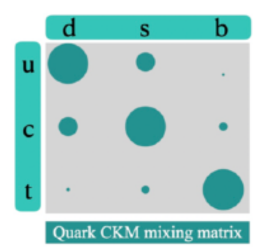
$$\frac{m_d}{m_u} \simeq 2\ , \quad \frac{m_s}{m_d} \simeq 21\ ,$$
$$\frac{m_t}{m_c} \simeq 267\ , \quad \frac{m_c}{m_u} \simeq 431\ , \quad \frac{m_t}{m_u} \simeq 1.2 \times 10^5\ .$$

- leptons

$$\frac{m_\tau}{m_\mu} \simeq 17\ , \quad \frac{m_\mu}{m_e} \simeq 207\ .$$

★ Flavor problem: pattern of fermion mixing

- why is the quark mixing matrix so different from the neutrino mixing matrix?



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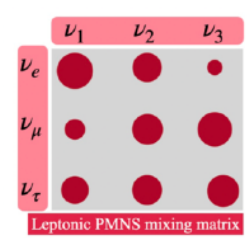


S. Cao, et al.

★ Flavor problem: nature of neutrino mass?

Fundamental physics with muons: flavor problem

★ Flavor problem: flavor-changing neutral currents (FCNC)

- there is no term in the SM Lagrangian that leads to FCNC effects: quantum effects (one loop process)
- quarks: massive quarks and non-zero mixing parameters automatically lead to FCNC processes: $b \to s\gamma, c \to u\ell\bar{\ell}, B^0 - \bar{B}^0$-mixing, etc.
- leptons: massive neutrinos and non-zero mixing parameters automatically lead to FCNC processes: $\mu \to e\gamma, \mu \to eee, \mu A \to eA$, etc.

★ Flavor problem: patterns of masses of particles and neutrino mass: new symmetry?

- there could be a mechanism generating mass patterns (Froggatt-Nielsen, etc.)…

A. Blechman, AAP, G.K. Yeghiyan

288 *C.D. Froggatt, H.B. Nielsen / Hierarchy of quark masses*

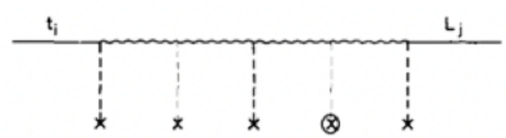
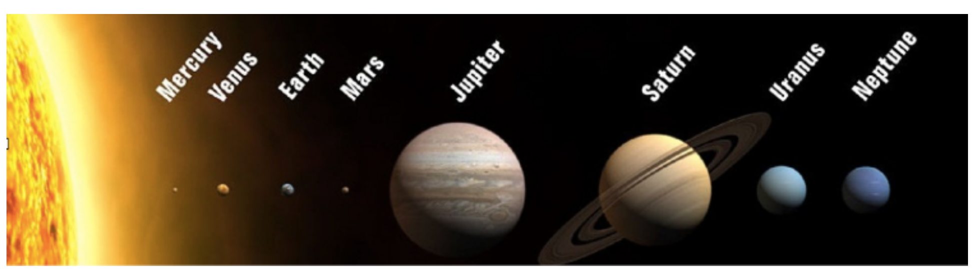
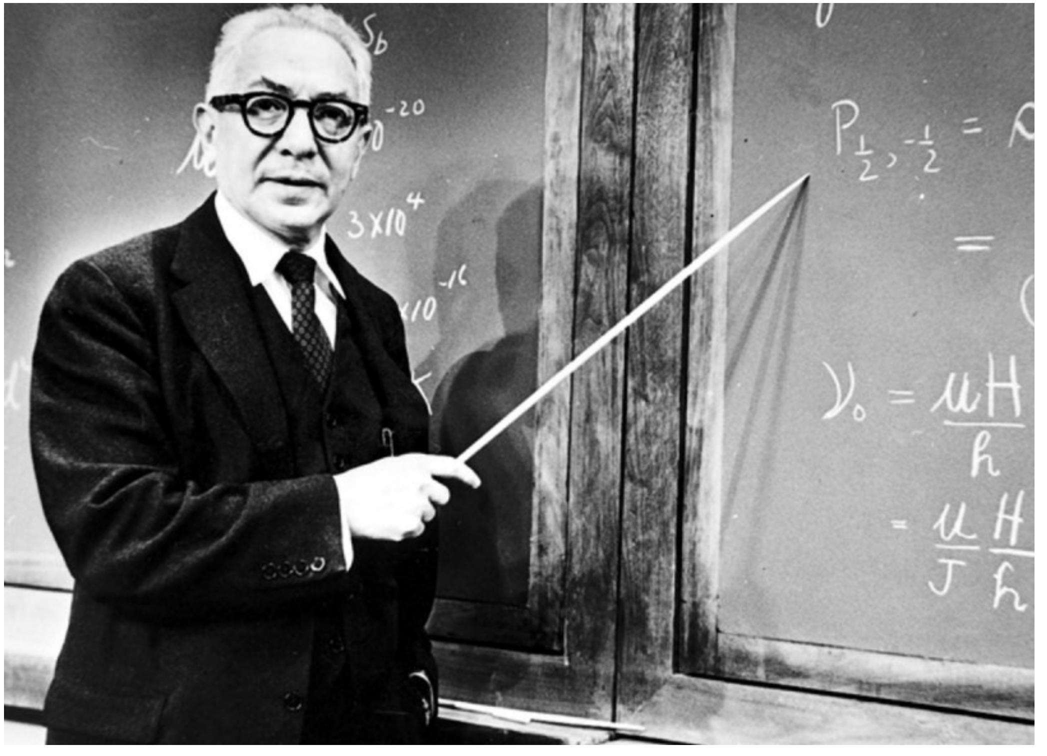


Fig. 1. Feynman diagram which generates the quark mass matrix element $M_{t,ij}$. Full lines represent quarks and wavy lines represent super heavy fermions. The dashed lines represent Higgs tadpoles as follows: $---\times\langle\phi_1\rangle$, and $---\otimes\langle\phi_2\rangle$.

- … or maybe not (a “just so” solution?)



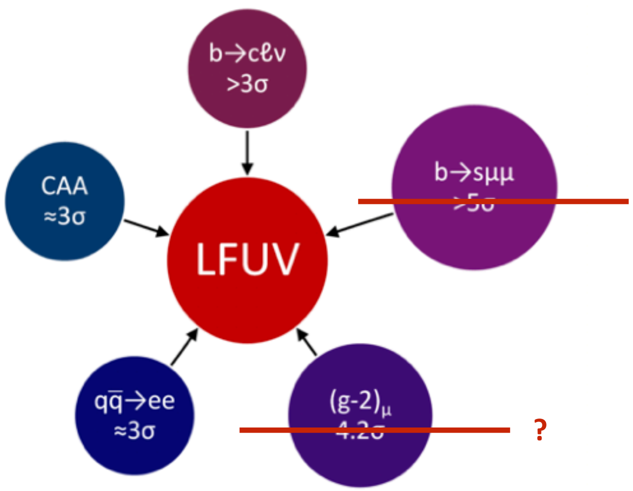
2. Muons as tools to discover New Physics?



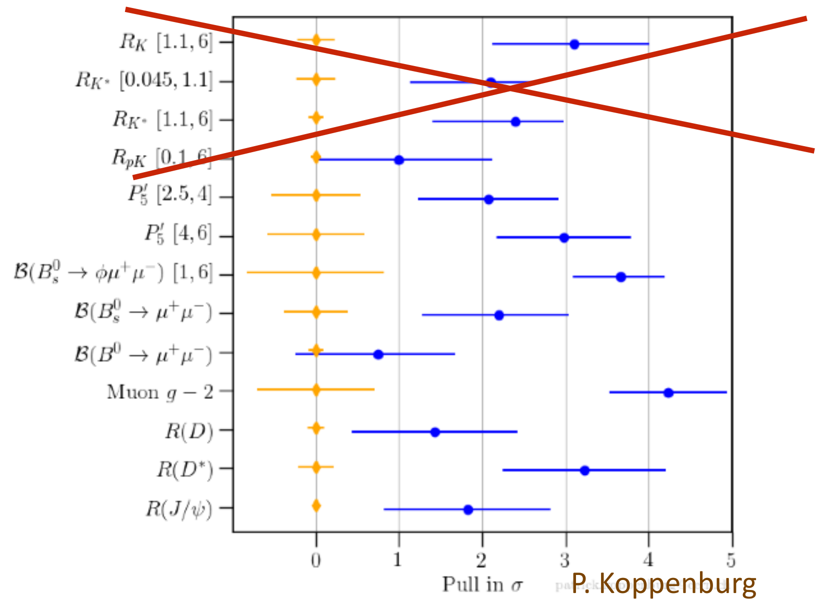
Are there any hints of New Physics with muons?

Muons and recent experimental anomalies

★ Many experimental anomalies involve interactions with muons and taus

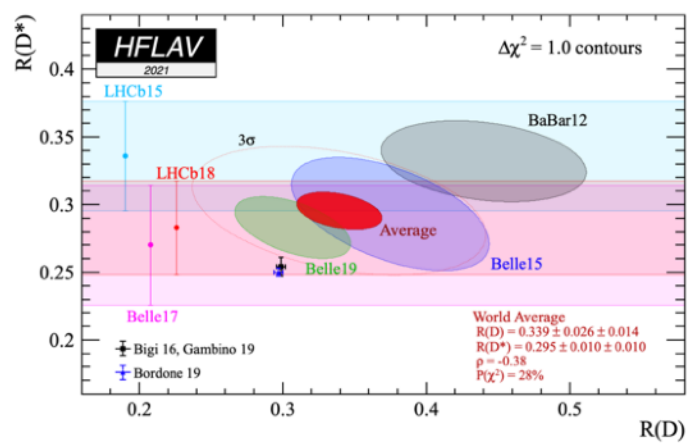


Crivellin, Hoferichter



P. Koppenburg

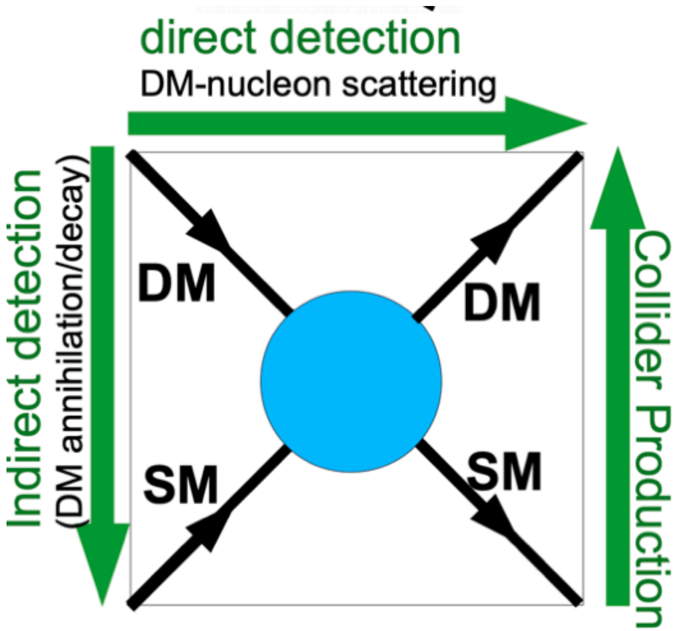
- other lepton-flavor conserving processes
  - magnetic properties: muon g-2
    - currently a discrepancy theory/exp
  - electric properties: muon EDM
    - probes CP-violation in leptons
  - muonic hydrogen
    - proton size/QED/New Physics



How to search for NP with muons?

Example: Dark Matter physics

- How to discover Dark Matter: shake it, break it, or make it
  - sensitive not only to DM, but also to the NP (mediators)

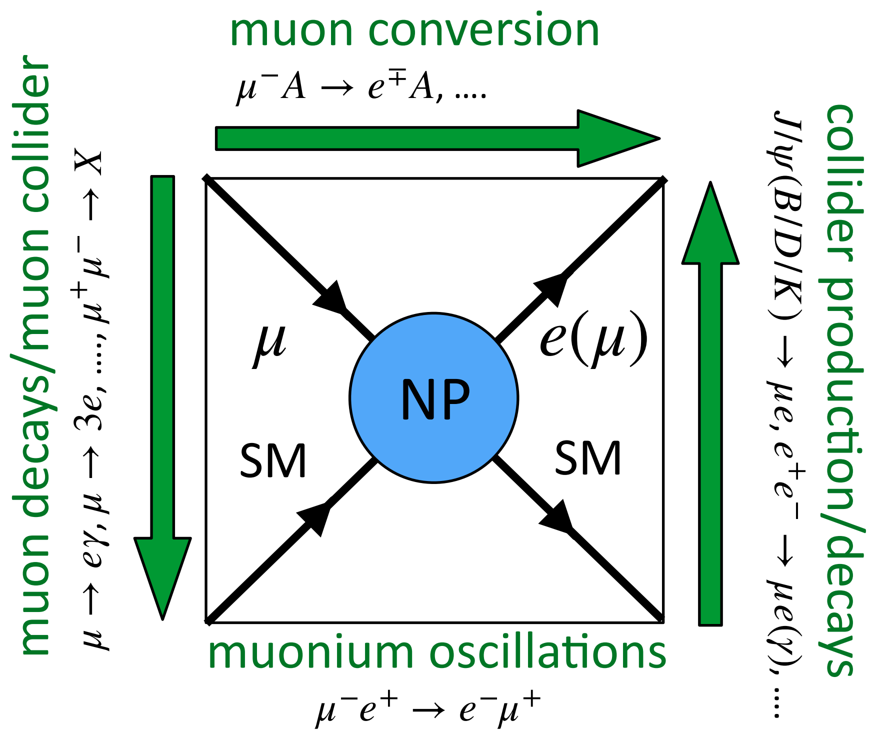


A. Korn

- Studies of DM properties: a comprehensive program of experiments
  - collider studies, “table-top+” experiments, and astrophysics observations

A similar view of muon physics

- How to discover New Physics with muons: multichannel approach
  - note that light NP particles (ALPs, dark photons) can also be probed!



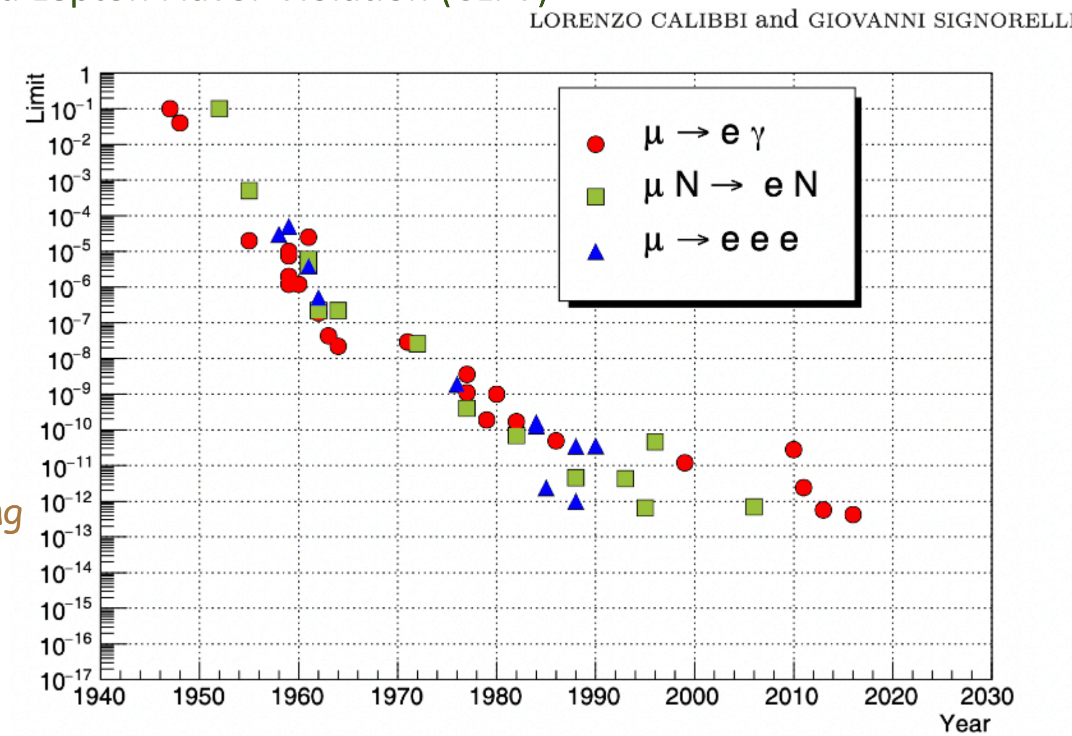
- Studies of NP with muons: a comprehensive program of experiments
  - collider studies, “table-top” and general flavor experiments

FNAL’s Advanced Muon Facility (AMF)?

Fundamental physics with muons: flavor violation

★ Possible experimental searches for Charged Lepton Flavor Violation (CLFV)

- lepton-flavor violating processes
  - $\mu \to e\gamma$, $\tau \to e\gamma$, etc.
  - $\mu \to eee$, $\tau \to \mu ee$, etc.
  - $\mu^+e^- \to e^-\mu^+$ (muonium oscillations)
  - $Z^0 \to \mu e$, τe, etc.
  - $H \to \mu e$, τe, etc.
  - K^0 (B^0, D^0, ...) $\to \mu e$, τe, etc.
  - $\mu^- + (A, Z) \to e^- + (A, Z)$
- lepton number and lepton-flavor violating processes
  - $(A, Z) \to (A, Z\pm2) + e^{\mp}e^{\mp}$
  - $\mu^- + (A, Z) \to e^+ + (A, Z\text{-}2)$



★ Decays are highly suppressed in the Standard Model: $Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i}U_{ei}\frac{m^2_{\nu_i}}{M^2_W}\right|^2 < 10^{-54}$

★ But: no trivial FCNC vertices in the Standard Model: sensitive tests of New Physics!

What kind of New Physics?

NP models and high energy processes

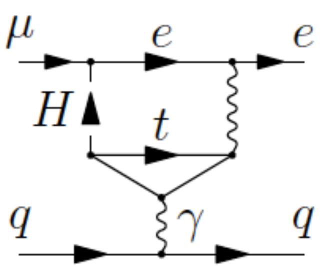
★ Leptonic FCNC could be generated by New Physics

✦ Ex.1 FCNC Higgs decays H → μe, τe, etc.: $Y_{ij} = \frac{m_i}{v}\delta_{ij} + \frac{v^2}{\sqrt{2}\Lambda^2}\hat{\lambda}_{ij}$ Harnik, Kopp, Zupan

Muon collider?

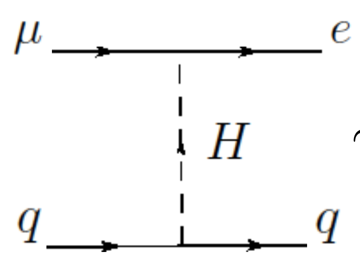
‣ FCNC Higgs model & muon conversion/quarkonium decays

e.g.



$\sim \mathcal{O}(\alpha^2)$

Barr-Zee type



$\sim \mathcal{O}(m_u \text{ or } m_d)$

tree level
(note suppression of light quark couplings)

✦ Ex.2 Exceptional couplings of (flavor-diagonal) NP to third generation $\mathcal{H}_{\rm NP} = G\bar{b}'_L\gamma^\lambda b'_L\bar{\tau}'_L\gamma_\lambda\tau'_L$ flavor "anomalies" Glashow, Guadagnoli, Lane

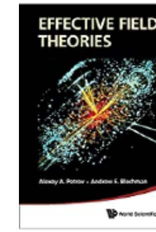
✦ Ex.3 Leptoquarks -> flavor "anomalies"

Muon collider?

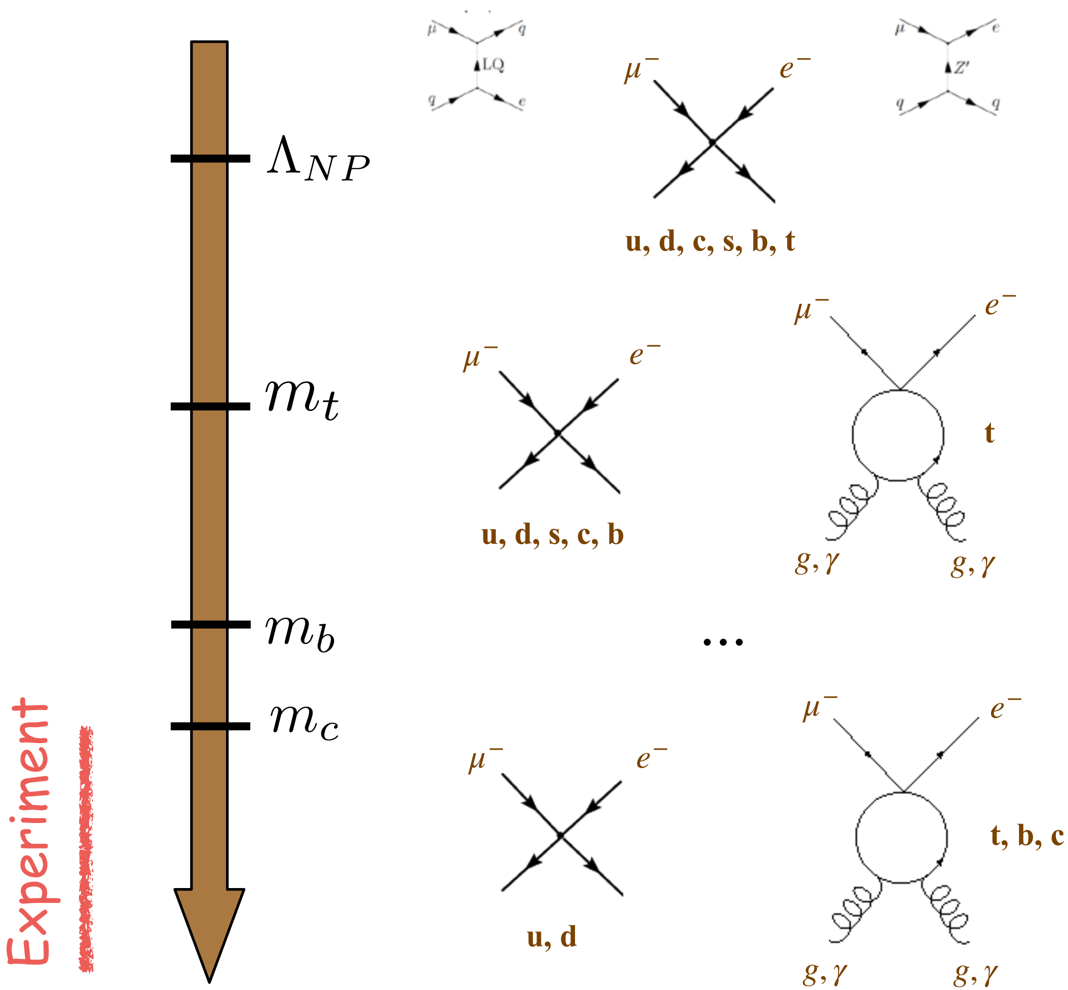
(Number of possible models) > (number of model builders). How do we proceed?

Models and effective Lagrangians

★ Modern approach to flavor physics calculations: effective field theories



★ It is important to understand ALL relevant energy scales for the problem at hand



New Physics generates lepton FCNC

Scales associated with heavy SM particles (quarks, leptons)

. . .

heavy quarks decouple

. . .

Scales associated with experiment

Effective Lagrangians: probing all NP models

★ Systematic approach: Standard Model Effective Field Theory (SMEFT)

- effective Lagrangian

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{C^{(5)}}{\Lambda} Q^{(5)} + \sum_i \frac{C_i^{(6)}}{\Lambda^2} Q_i^{(6)} + \ldots$$

with the Weinberg operator $Q^{(5)}$

$$Q^{(5)} = \epsilon_{jk}\epsilon_{mn} H^j H^m \left(L_p^k\right)^T \mathcal{C} L_r^n$$

and lots (59+5) of $Q_i^{(6)}$ operators

- the strategy of identifying an NP model involves fitting C_i from experimental data and/or matching of $\mathscr{L}$ to UV-completed NP models

TABLE 2.3 Operators with H^n, sets X^3, H^6, H^4D^2, and $\psi^2 H^3$.

X^3		H^6 and H^4D^2		ψ^2H^3+ h.c.	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_H	$\left(H^\dagger H\right)^3$	Q_{eH}	$\left(H^\dagger H\right)\left(\overline{L}_p e_r H\right)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{H\Box}$	$\left(H^\dagger H\right)\Box\left(H^\dagger H\right)$	Q_{uH}	$\left(H^\dagger H\right)\left(\overline{Q}_p u_r \widetilde{H}\right)$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	Q_{HD}	$\left(H^\dagger D^\mu H\right)^*\left(H^\dagger D_\mu H\right)$	Q_{dH}	$\left(H^\dagger H\right)\left(\overline{Q}_p d_r H\right)$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

TABLE 2.4 Operators with H^n, sets X^2H^2, $\psi^2 XH$, and $\psi^2 H^2 D$.

X^2H^2		ψ^2XH+ h.c.		ψ^2H^2D	
Q_{HG}	$H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right)\tau^I H W_{\mu\nu}^I$	$Q_{Hl}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{L}_p\gamma^\mu L_r\right)$
$Q_{H\widetilde{G}}$	$H^\dagger H \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$\left(\overline{L}_p\sigma^{\mu\nu}e_r\right) H B_{\mu\nu}$	$Q_{Hl}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{L}_p\tau^I\gamma^\mu L_r\right)$
Q_{HW}	$H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A u_r\right)\widetilde{H} G_{\mu\nu}^A$	Q_{He}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{e}_p\gamma^\mu e_r\right)$
$Q_{H\widetilde{W}}$	$H^\dagger H \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\tau^I\widetilde{H} W_{\mu\nu}^I$	$Q_{Hq}^{(1)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{Q}_p\gamma^\mu Q_r\right)$
Q_{HB}	$H^\dagger H B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$\left(\overline{Q}_p\sigma^{\mu\nu}u_r\right)\widetilde{H} B_{\mu\nu}$	$Q_{Hq}^{(3)}$	$\left(H^\dagger i\overleftrightarrow{D}_\mu^I H\right)\left(\overline{Q}_p\tau^I\gamma^\mu Q_r\right)$
$Q_{H\widetilde{B}}$	$H^\dagger H \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$\left(\overline{Q}_p\sigma^{\mu\nu}T^A d_r\right) H G_{\mu\nu}^A$	Q_{Hu}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{u}_p\gamma^\mu u_r\right)$
Q_{HWB}	$H^\dagger\tau^I H W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right)\tau^I H W_{\mu\nu}^I$	Q_{Hd}	$\left(H^\dagger i\overleftrightarrow{D}_\mu H\right)\left(\overline{d}_p\gamma^\mu d_r\right)$
$Q_{H\widetilde{W}B}$	$H^\dagger\tau^I H \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$\left(\overline{Q}_p\sigma^{\mu\nu}d_r\right) H B_{\mu\nu}$	Q_{Hud}	$i\left(\widetilde{H}^\dagger D_\mu H\right)\left(\overline{u}_p\gamma^\mu d_r\right)$

TABLE 2.5 Four-fermion operators, classes $(\overline{L}L)(\overline{L}L)$, $(\overline{R}R)(\overline{R}R)$, and $(\overline{L}L)(\overline{R}R)$.

$(\overline{L}L)(\overline{L}L)$		$(\overline{R}R)(\overline{R}R)$		$(\overline{L}L)(\overline{R}R)$	
Q_{ll}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{L}_s\gamma^\mu L_t\right)$	Q_{ee}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$	Q_{le}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{qq}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{uu}	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{lu}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
$Q_{qq}^{(3)}$	$\left(\overline{Q}_p\gamma^\mu\tau^I Q_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{dd}	$\left(\overline{d}_p\gamma^\mu d_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	Q_{ld}	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
$Q_{lq}^{(1)}$	$\left(\overline{L}_p\gamma^\mu L_r\right)\left(\overline{Q}_s\gamma^\mu Q_t\right)$	Q_{eu}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$	Q_{qe}	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{e}_s\gamma^\mu e_t\right)$
$Q_{lq}^{(3)}$	$\left(\overline{L}_p\gamma^\mu\tau^I L_r\right)\left(\overline{Q}_s\gamma^\mu\tau^I Q_t\right)$	Q_{ed}	$\left(\overline{e}_p\gamma^\mu e_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(1)}$	$\left(\overline{Q}_p\gamma^\mu Q_r\right)\left(\overline{u}_s\gamma^\mu u_t\right)$
		$Q_{ud}^{(1)}$	$\left(\overline{u}_p\gamma^\mu u_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$	$Q_{qu}^{(8)}$	$\left(\overline{q}_p\gamma^\mu T^A q_r\right)\left(\overline{u}_s\gamma^\mu T^A u_t\right)$
		$Q_{ud}^{(8)}$	$\left(\overline{u}_p\gamma^\mu T^A u_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$	$Q_{qd}^{(1)}$	$\left(\overline{q}_p\gamma^\mu q_r\right)\left(\overline{d}_s\gamma^\mu d_t\right)$
				$Q_{qd}^{(8)}$	$\left(\overline{Q}_p\gamma^\mu T^A Q_r\right)\left(\overline{d}_s\gamma^\mu T^A d_t\right)$

TABLE 2.6 Four-fermion operators, classes $(LR)(RL)$, and B (baryon-number) violating.

$(\overline{L}R)(\overline{R}L)$		B-violating	
Q_{ledq}	$\left(\left(\overline{L}_p^j e_r\right)\left(\overline{d}_s Q_t^j\right)\right.$	Q_{duq}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[\left(Q_s^{\gamma j}\right)^T C L_t^k\right]$
$Q_{quqd}^{(1)}$	$\left(\left(\overline{Q}_p^j u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k d_t\right)\right.$	Q_{qqu}	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$\left(\left(\overline{Q}_p^j T^A u_r\right)\epsilon_{jk}\left(\overline{Q}_s^k T^A d_t\right)\right.$	$Q_{qqq}^{(1)}$	$\epsilon^{\alpha\beta\gamma}\epsilon_{jk}\epsilon_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(1)}$	$\left(\left(\overline{L}_p^j e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k u_t\right)\right.$	$Q_{qqq}^{(3)}$	$\epsilon^{\alpha\beta\gamma}\left(\tau^I\epsilon\right)_{jk}\left(\tau^I\epsilon\right)_{mn}\left[\left(Q_p^{\alpha j}\right)^T C Q_r^{\beta k}\right]\left[(Q_s^{\gamma m})^T C L_t^n\right]$
$Q_{lequ}^{(3)}$	$\left(\left(\overline{L}_p^j\sigma_{\mu\nu} e_r\right)\epsilon_{jk}\left(\overline{Q}_s^k\sigma^{\mu\nu} u_t\right)\right.$	Q_{duu}	$\epsilon^{\alpha\beta\gamma}\left[\left(d_p^\alpha\right)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

★ Radiative FCNC decays of leptons $\ell_1 \to \ell_2 + \gamma$

- the most general amplitude is

$$A_{\ell_1\to\ell_2\gamma}(p,p') = \frac{i}{m_{\ell_1}}\overline{u}_{\ell_2}(p')\left[A_L P_L + A_R P_R\right]\sigma_{\mu\nu}q^\nu u_{\ell_1}(p)\epsilon^{*\mu},$$

- which leads to the decay rate

$$\Gamma(\ell_1 \to \ell_2\gamma) = \frac{m_{\ell_1}}{16\pi}\left(|A_L|^2 + |A_R|^2\right)$$

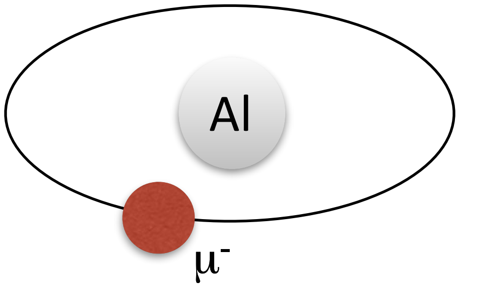
with $A_R = A_L^* = \sqrt{2}\,\frac{vm_i^2}{\Lambda^2}\left(c_W C_{eB}^{fi} - s_W C_{eW}^{fi}\right) \equiv \sqrt{2}\,\frac{vm_i^2}{\Lambda^2}C_\gamma^{fi}$

Effective coupling (example)	Bounds on Λ (TeV) (for $\lvert C_{ij}^6\rvert = 1$)	Bounds on $\lvert C_{ij}^6\rvert$ (for $\Lambda = 1$ TeV)	Observable
$C_{e\gamma}^{\mu e}$	6.3×10^4	2.5×10^{-10}	$\mu \to e\gamma$
$C_{e\gamma}^{\tau e}$	6.5×10^2	2.4×10^{-6}	$\tau \to e\gamma$
$C_{e\gamma}^{\tau\mu}$	6.1×10^2	2.7×10^{-6}	$\tau \to \mu\gamma$
$C_{\ell\ell,ee}^{\mu eee}$	207	2.3×10^{-5}	$\mu \to 3e$
$C_{\ell\ell,ee}^{e\tau ee}$	10.4	9.2×10^{-5}	$\tau \to 3e$
$C_{\ell\ell,ee}^{\mu\tau\mu\mu}$	11.3	7.8×10^{-5}	$\tau \to 3\mu$
$C_{(1,3)H\ell}^{\mu e}$, $C_{He}^{\mu e}$	160	4×10^{-5}	$\mu \to 3e$
$C_{(1,3)H\ell}^{\tau e}$, $C_{He}^{\tau e}$	≈ 8	1.5×10^{-2}	$\tau \to 3e$
$C_{(1,3)H\ell}^{\tau\mu}$, $C_{He}^{\tau\mu}$	≈ 9	$\approx 10^{-2}$	$\tau \to 3\mu$

Teixeira; Feruglio, Paradisi, Pattori

Other interesting modes that probe similar couplings: $\ell_1 \to \ell_2\gamma\gamma$, $\ell_1 \to 3\ell_2$, and others

★ Basic idea for the muon conversion experiment



- ★ take low energy muons (~ 30 MeV) and stop them in a target A(Z,A-Z): muons cascade to atomic 1s state

- ★ Binding energy and orbit radius for muonic hydrogen-like state

$$E_b = -\frac{Z^2 m e^4}{8n^2} \sim \frac{Z^2 m}{n^2}$$

$$r = \frac{n^2}{Z\pi m e^2} \sim \frac{n^2}{Zm}$$

muonic atom is 200x stronger bound
radius is 200x smaller

- ★ Radial wave function for hydrogen-like system: $R_{nl} \sim r^\ell Z^{3/2}$
  overlap probability: $p \sim r^{2\ell} Z^3$

large overlap for an s-wave and high-Z nucleus

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

see S. Middleton's talk

★ Different nuclei are sensitive to a variety of New Physics scenarios, also

Nucleus	$R_{\mu e}(Z)$ / $R_{\mu e}(Al)$	Bound lifetime	Atomic Bind. Energy(1s)	Conversion Electron Energy	Prob decay >700 ns
Al(13,27)	1.0	.88 μs	0.47 MeV	104.97 MeV	0.45
Ti(22,~48)	1.7	.328 μs	1.36 MeV	104.18 MeV	0.16
Au(79,~197)	~0.8-1.5	.0726 μs	10.08 MeV	95.56 MeV	negligible

J. Miller, 2006

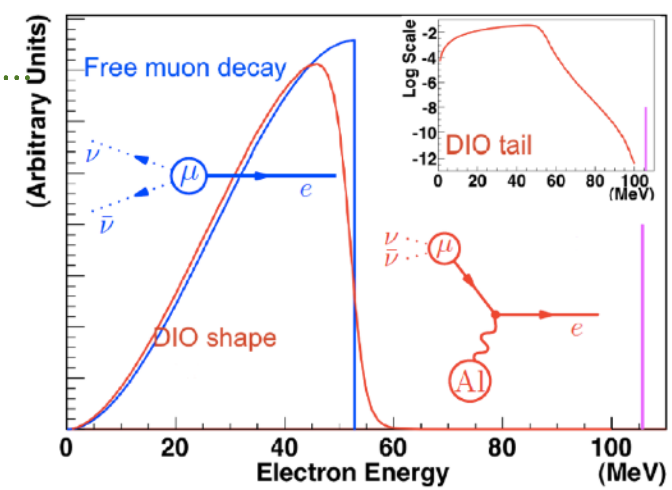
★ The experiment is tricky

✓ Muon conversion gives monoenergetic electrons...
✓ ... yet, there are other sources of electrons as well!

$$\begin{cases} \mu^- \to e^- + \bar{\nu}_e + \nu_\mu & \text{- decay (40\%)} \\ \mu^- + Al \to X + \nu_\mu & \text{- capture (60\%)} \\ \mu^- + Al \to e^- + Al & \text{- conversion} \end{cases}$$

SINDRUM II (PSI), 2006 : $R_{\mu e} < 7 \times 10^{-13}$

M2e goal : $R_{\mu e} < \text{a few} \times 10^{-17}$

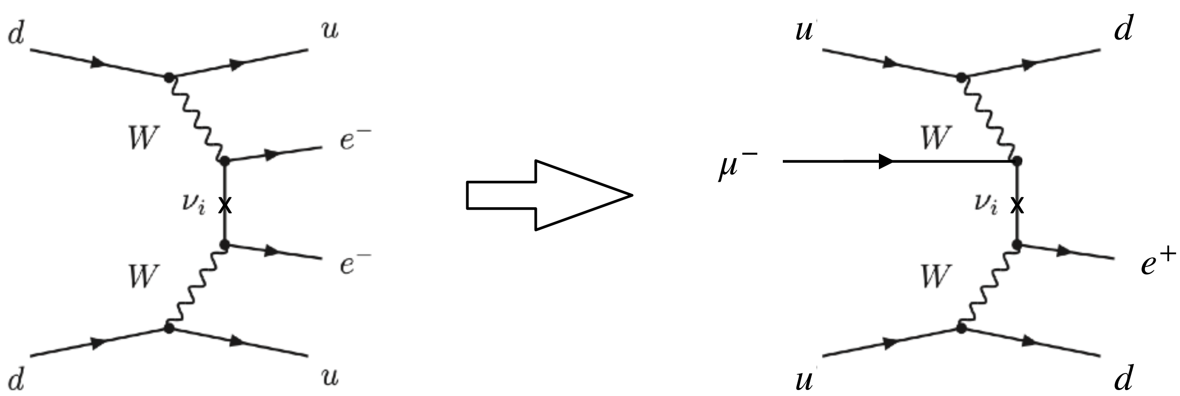


Czarnecki, Marciano, Tormo

Muon conversion: B and L-number violation

★ Muon conversion probes lepton and baryon-number-violating transitions

– ... similar to $0\nu 2\beta$ decays but different!



$0\nu 2\beta$ decay

$\mu^- A \to e^+ A'$ conversion

– ... also probes baryon number violation (similar to $H - \bar{H}$) but different!

P. Fox and AAP

2c. Muonium oscillations

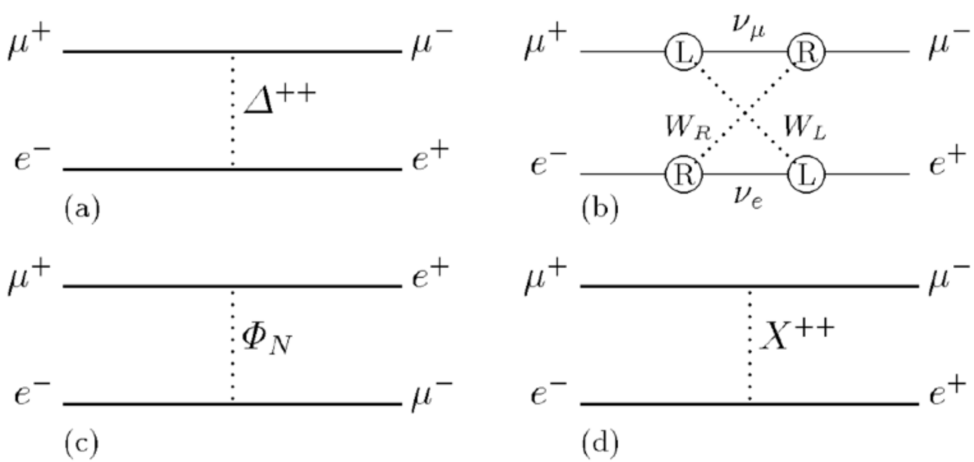
★ Lepton-flavor violating interactions can change $M_\mu \rightarrow \overline{M}_\mu$

- ... just like $B^0\bar{B}^0$ mixing, but simpler!

Pontecorvo (1957)
Feinberg, Weinberg (1961)

- Such transition amplitudes are tiny in the Standard Model
  - ... but there are plenty of New Physics models where it can happen

Clark, Love; Cvetic et al,
Li, Schmidt; Endo, Iguro, Kitahara;
Fukuyama, Mimura, Uesaka; …



$$\sim \left(\bar{\mu}\Gamma e\right)\left(\bar{\mu}\Gamma e\right)$$

effective operator

  - theory: compute transition amplitudes for ALL New Physics models!
  - experiment: produce M_μ but look for the decay products of $\overline{M}_\mu$

Combined evolution = flavor oscillations

- Study oscillations via decays: amplitudes for $M_\mu \to f$ and $\overline{M}_\mu \to \bar{f}$
  - possibility of flavor oscillations ($M_\mu \to \overline{M}_\mu \to \bar{f}$)

$$
\begin{aligned}
|M(t)\rangle &= g_+(t)\,|M_\mu\rangle + g_-(t)\,|\overline{M}_\mu\rangle\,, \\
|\overline{M}(t)\rangle &= g_-(t)\,|M_\mu\rangle + g_+(t)\,|\overline{M}_\mu\rangle\,,
\end{aligned}
$$

with $x = \dfrac{\Delta m}{\Gamma}$, $y = \dfrac{\Delta\Gamma}{2\Gamma}$ and

$$
\begin{aligned}
g_+(t) &= e^{-\Gamma_1 t/2} e^{-i m_1 t}\left[1 + \frac{1}{8}(y - ix)^2 (\Gamma t)^2\right], \\
g_-(t) &= \frac{1}{2} e^{-\Gamma_1 t/2} e^{-i m_1 t}(y - ix)(\Gamma t)\,.
\end{aligned}
$$

  - time-dependent width: $\Gamma(M_\mu \to \bar{f})(t) = \dfrac{1}{2} N_f\,|A_f|^2\, e^{-\Gamma t} (\Gamma t)^2 R_M(x,y)$
  - oscillation probability: $P(M_\mu \to \overline{M}_\mu) = \dfrac{\Gamma(M_\mu \to \bar{f})}{\Gamma(M_\mu \to f)} = R_M(x,y) = \dfrac{1}{2}(x^2 + y^2)$

R. Conlin and AAP

- Mass difference comes from the dispersive part

$$x = \frac{1}{2M_M\Gamma}\mathrm{Re}\left[2\langle\overline{M}_\mu\left|\mathcal{H}_{\rm eff}\right|M_\mu\rangle + \langle\overline{M}_\mu\left|i\int d^4x\ \mathrm{T}\left[\mathcal{H}_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right]\right|M_\mu\rangle\right]$$

  - consider only $\Delta L_\mu = 2$ Lagrangian contributions (largest?)

$$\mathcal{L}_{\rm eff}^{\Delta L_\mu=2} = -\frac{1}{\Lambda^2}\sum_i C_i^{\Delta L=2}(\mu)Q_i(\mu)$$

  - leading order: all heavy New Physics models are encoded in (the Wilson coefficients of) the five dimension-6 operators

$$\begin{aligned} Q_1 &= \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\mu}_L\gamma^\alpha e_L\right), \quad Q_2 = \left(\overline{\mu}_R\gamma_\alpha e_R\right)\left(\overline{\mu}_R\gamma^\alpha e_R\right), \\ Q_3 &= \left(\overline{\mu}_L\gamma_\alpha e_L\right)\left(\overline{\mu}_R\gamma^\alpha e_R\right), \quad Q_4 = \left(\overline{\mu}_L e_R\right)\left(\overline{\mu}_L e_R\right), \\ Q_5 &= \left(\overline{\mu}_R e_L\right)\left(\overline{\mu}_R e_L\right). \end{aligned}$$

  - matrix elements for both singlet and triplet states: easy (QED only)

Effective Lagrangians and particular models

- Effective Lagrangian approach encompasses all models
  - lets look at an example of a model with a doubly charged Higgs Δ^{--}
  - this is common for the left-right models, etc.

$$\mathcal{L}_R = g_{\ell\ell} \overline{\ell}_R \ell^c \Delta + H.c.,$$

  - integrate out Δ^{--} to get

$$\mathcal{H}_\Delta = \frac{g_{ee} g_{\mu\mu}}{2M_\Delta^2} \left(\overline{\mu}_R \gamma_\alpha e_R\right) \left(\overline{\mu}_R \gamma^\alpha e_R\right) + H.c.,$$

  - match to $\mathscr{L}_{\text{eff}}^{\Delta L=2}$ to see that $M_\Delta = \Lambda$ and

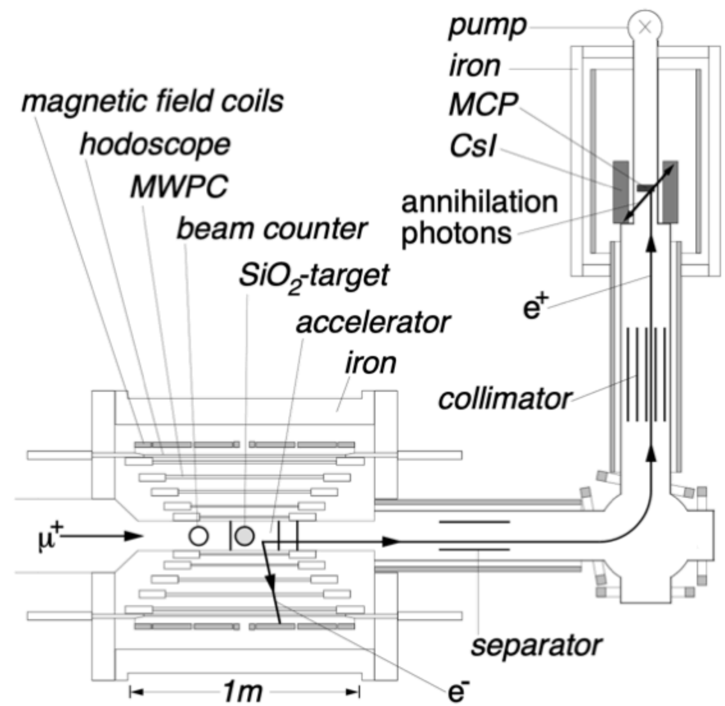
$$C_2^{\Delta L=2} = g_{ee} g_{\mu\mu}/2.$$

Chang, Keung (89);
Schwartz (89);
Han, Tang, Zhang (21)

Is it better than/worse than/complimentary to $\mu \to 3e$? Future: direct production?

Experimental setup and constraints

- Similar experimental set ups for different experiments
  - example: MACS at PSI
  - idea: form M_μ by scattering muon (μ^+) beam on SiO_2 target
- A couple of "little inconveniences":
  - ➡ how to tell f apart from $\bar{f}$?
    - $M_\mu \to f$ decay: $M_\mu \to e^+ e^- \bar{\nu}_\mu \nu_e$
    - $\overline{M}_\mu \to \bar{f}$ decay: $\overline{M}_\mu \to e^+ e^- \bar{\nu}_e \nu_\mu$
    - $\bar{f}$: fast e^- (~53 MeV), slow e^+ (13.5 eV)
  - ➡ oscillations happen in magnetic field
    - … which selects M_μ vs. $\overline{M}_\mu$



Muonium-Antimuonium Conversion Spectrometer (MACS)

L. Willmann, et al. PRL 82 (1999) 49

The most recent experimental data comes from 1999! Time is ripe for an update!

Experimental constraints

- We can put constraints on the Wilson coefficients of effective operators from the 1999 MACS data (assume single operator dominance)
  - presence of the magnetic field

$$P(M_\mu \to \overline{M}_\mu) \leq 8.3 \times 10^{-11}/S_B(B_0)$$

  - no separation of spin states: average

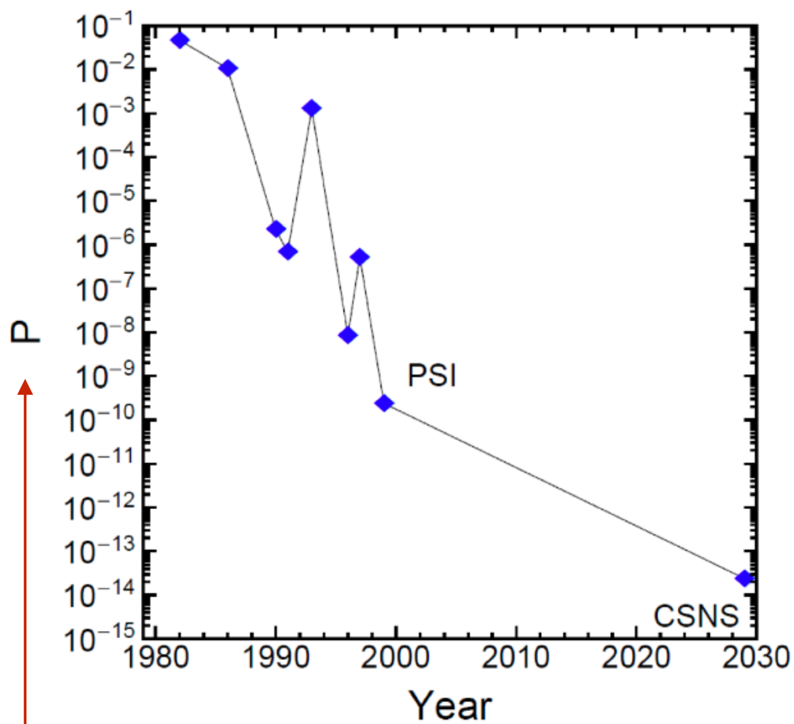
$$P(M_\mu \to \overline{M}_\mu)_{\rm exp} = \sum_{i=P,V} \frac{1}{2S_i+1} P({M_\mu}^i \to {\overline{M}_\mu}^i)$$

  - set Wilson coefficients to one, set constraints on the scale probed

Operator	Interaction type	$S_B(B_0)$ (from [9])	Constraints on the scale Λ, TeV
Q_1	$(V-A)\times(V-A)$	0.75	5.4
Q_2	$(V+A)\times(V+A)$	0.75	5.4
Q_3	$(V-A)\times(V+A)$	0.95	5.4
Q_4	$(S+P)\times(S+P)$	0.75	2.7
Q_5	$(S-P)\times(S-P)$	0.75	2.7
Q_6	$(V-A)\times(V-A)$	0.75	0.58×10^{-3}
Q_7	$(V+A)\times(V-A)$	0.95	0.38×10^{-3}

R. Conlin and AAP, Phys.Rev.D 102 (2020) 9, 095001

Fundamental science with EMuS (China)



$$P(M_\mu \to \overline{M}_\mu) = \frac{\Gamma(M_\mu \to \overline{f})}{\Gamma(M_\mu \to f)} = R_M(x,y)$$

$$R_M(x,y) = \frac{1}{2}\left(x^2+y^2\right)$$

- The latest bound was done at PSI more than 20 years ago with a muon intensity $8 \times 10^6 \mu^+/s$ and high-precision magnetic spectrometer.
- Timing resolution in detector: ~ ns
- Position resolution in detector: ~ mm
- EMuS plan to offer $10^9 \mu^+/s$
- Current timing resolution in detector: ~ ps
- Current position resolution in detector:~μs
- Expect to be improved by > O(10^2)?

MACE experiment at EMuS (Chinese SNS)
Jian Tang, talk at RPPM meeting (Snowmass 2021)

Snowmass2021 Whitepaper: Muonium to antimuonium conversion
A.-Y. Bai, …, AAP, …, arXiv:2203.11406 [hep-ph]

Conclusions and things to take home

- There is no indication from high energy studies where the NP show up
  - this makes indirect searches the most valuable source of information
- Muonium is the simplest atom: atomic physics
  - level splitting (Lamb shift): probe NP w/out QCD complications

MuSEUM experiment (J-PARC)

- Muons are ideal tools to probe fundamental physics
  - flavor-conserving quantities (g-2, EDM)
  - flavor-changing neutral current decays
  - flavor oscillations (muonium-antimuonium conversion)
  - muon transitions already probe the LHC energy domain and can do better!
  - all studies are complimentary to each other
- New experimental facilities are needed (AMF?)

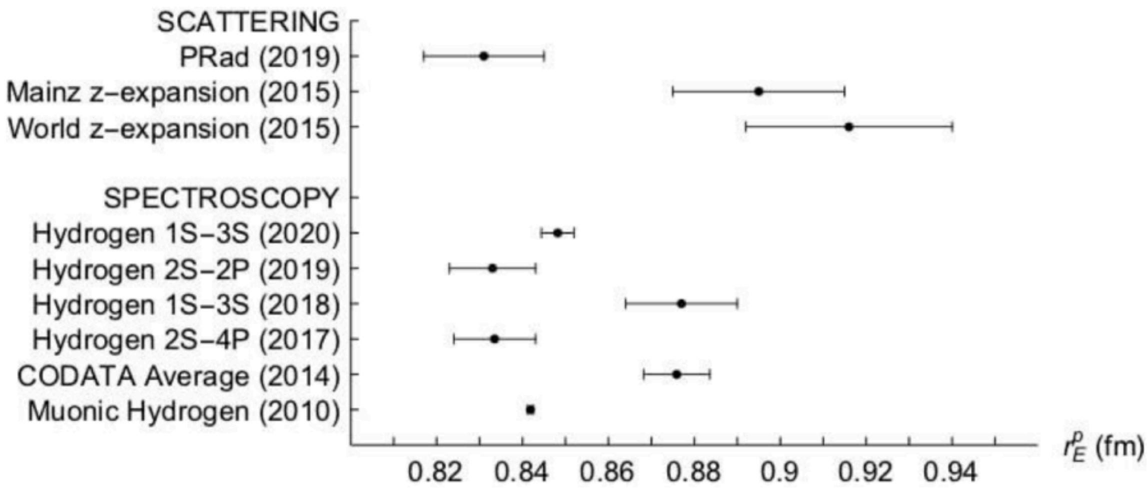
Prospects for precise predictions of a_μ in the Standard Model
G. Colangelo, et. al., arXiv:2203.15810 [hep-ph]

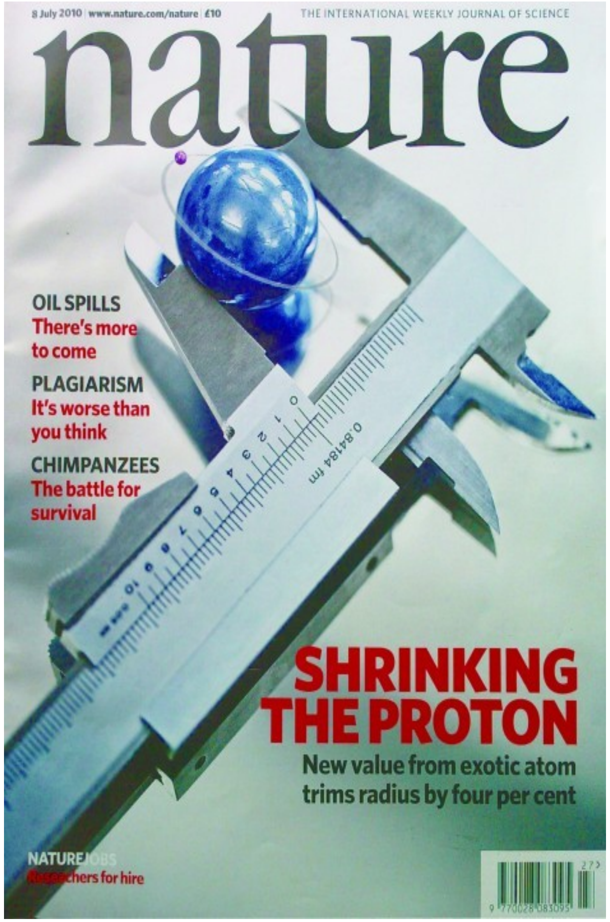
Snowmass2021 Whitepaper: Muonium to antimuonium conversion
A.-Y. Bai, …, AAP, …, arXiv:2203.11406 [hep-ph]


EXTRA!

★ Proton's radius from muonic hydrogen: possible New Physics?

- ★ Level splittings (e.g. Lamb shift) are sensitive to the charge radius of the proton





- ★ They are also sensitive to QED radiative corrections
- ★ Are there possible light New Physics particles that are responsible for this difference?

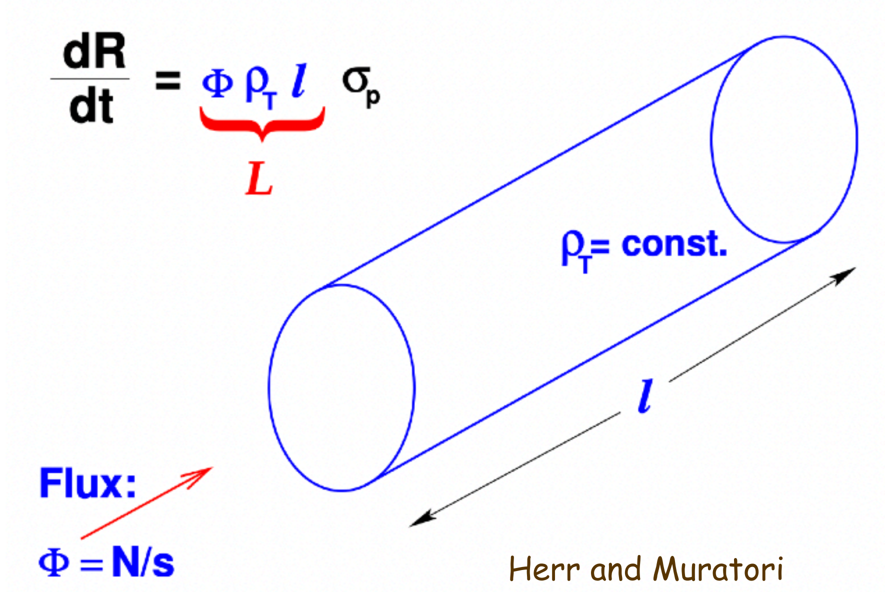
Barger et al, PRL 106 (2011) 153001

Remove proton radius issue from the problem: atomic physics with muonium?

Experimental studies of rare processes: luminosity

★ Need a lot of muons: high luminosity experiments

– Number of events/second



	Energy (GeV)	$\mathcal{L}$ $cm^{-2}s^{-1}$
SPS ($p\bar{p}$)	315x315	$6\ 10^{30}$
Tevatron ($p\bar{p}$)	1000x1000	$50\ 10^{30}$
HERA (e^+p)	30x920	$40\ 10^{30}$
LHC (pp)	7000x7000	$10000\ 10^{30}$
LEP (e^+e^-)	105x105	$100\ 10^{30}$
PEP (e^+e^-)	9x3	$3000\ 10^{30}$
KEKB (e^+e^-)	8x3.5	$10000\ 10^{30}$
eRHIC		10^{33}-10^{35}

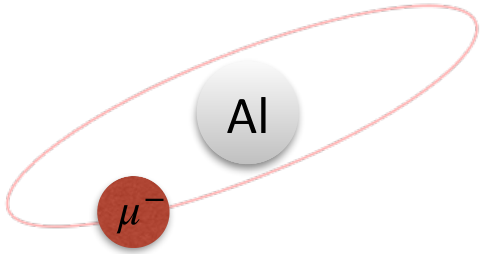
– ... or another way $L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$

What if incident particles formed bound states with target particles?

Bound states: muon conversion

- How effective is this approach compared to scattering?
  - let's compute effective luminosity
  - recall that

$$L = \Phi \rho_T \ell = N \rho_T \frac{\ell}{t} = N \rho_T v$$



  - in this "experiment" the probability density is given by the 1s wave function
  - … and we need to take into account the fact that muon decays
  - Then luminosity = (density)(velocity)(flux of muons)(lifetime)

$$L_{\text{eff}} = |\psi(0)|^2 \times \alpha Z \times \Phi_\mu \times \tau_\mu = \frac{m_\mu^3 Z^4 \alpha^4}{\pi} \Phi_\mu \tau_\mu$$

  - For Al target (Z=13), flux of $\Phi_\mu = 10^{10}$ muons/sec and $\tau_\mu = 2\ \mu$sec

$$L_{\text{eff}} = 10^{48} \text{cm}^{-2}\ \text{sec}^{-1}$$

Bernstein, Czarnecki

Title Text

- A possibility of using muon beams at CMP facilities

Jian Tang, talk at RPPM meeting (Snowmass 2021)

	Proton driver [MW]	Surface muons			Decay muons		
		Intensity [1E6/s]	Polarization [%]	Spread [%]	energy [MeV/c]	Intensity [1E6/s]	Spread [%]
PSI	1.3	420	90	10	85-125	240	3
ISIS	0.16	1.5	95	<15	20-120	0.4	10
RIKEN/RAL	0.16	0.8	95	<15	65-120	1	10
JPARC	1	100	95	15	33-250	10	15
TRIUMF	0.075	1.4	90	7	20-100	0.0014	10
EMuS	0.005	83	50	10	50-450	16	10
Baby EMuS	0.005	1.2	95	10			

×5 CSNS-II upgrade

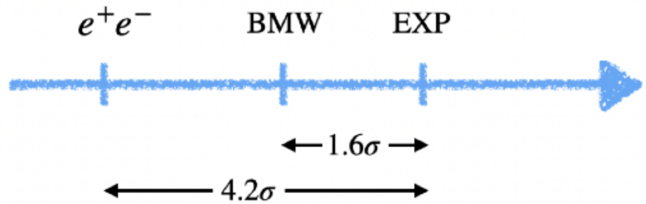
Facility	Source Type	Intensity (μ+/sec)*
ISIS	pulsed	1.5×10^6
J-PARC	continuous	1.8×10^6
PSI	continuous	7.0×10^4
TRIUMF	pulsed	5.0×10^6
SEEMS	pulsed	1.9×10^8

- Muonium Antimuonium Conversion Experiment (MACE) EMuS at CSNS

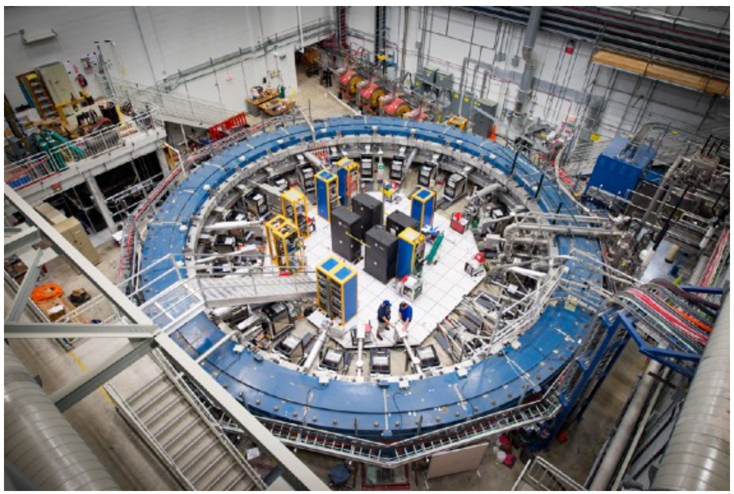
Muons and recent experimental anomalies

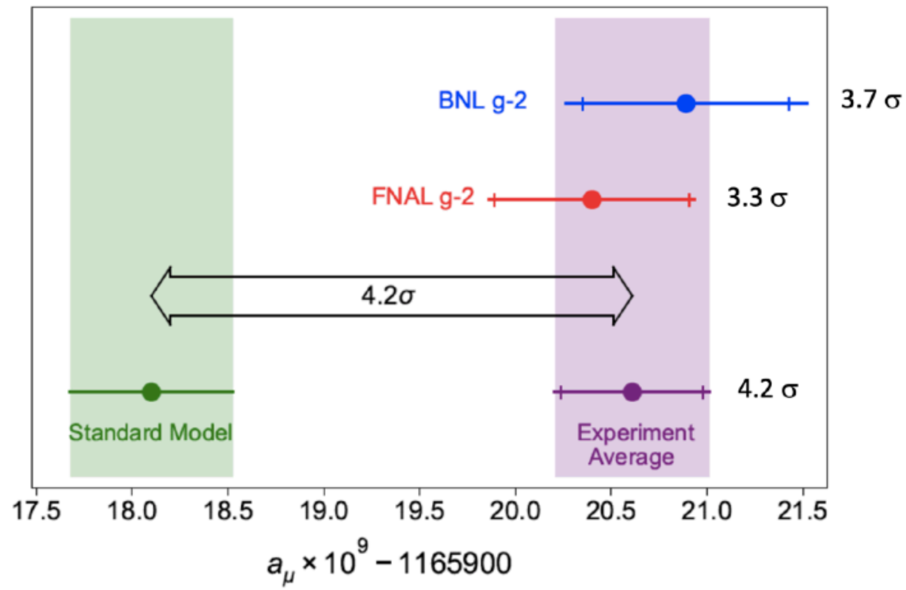
★ Muon's magnetic properties (g-2): $a_\mu = (g-2)/2$ with $\vec{\mu} = g\frac{e}{2m}\vec{s}$

discrepancy



Di Luzio, et al.





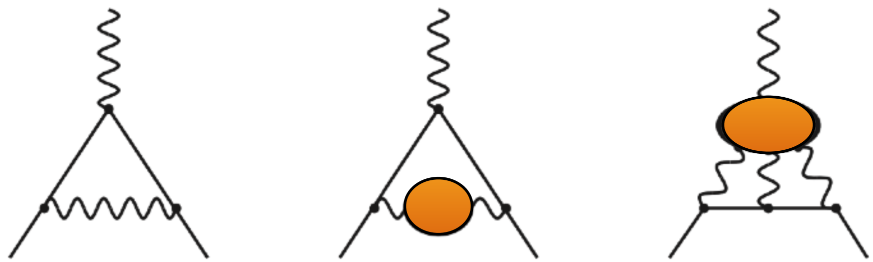
FNAL (g-2): $a_\mu(\text{Exp}) = 116592061(41) \times 10^{-11}$

$a_\mu(\text{Theory}) = 116591810(43) \times 10^{-11}$

$a_\mu(\text{BMW}) = 116591954(55) \times 10^{-11}$

Are there possible New Physics particles that are responsible for this difference?

- Independent lattice computations of HVP
- Data-driven estimates of hadronic vacuum polarization (HVP)
  - discrepancy between KLOE and BaBar data used in HVP

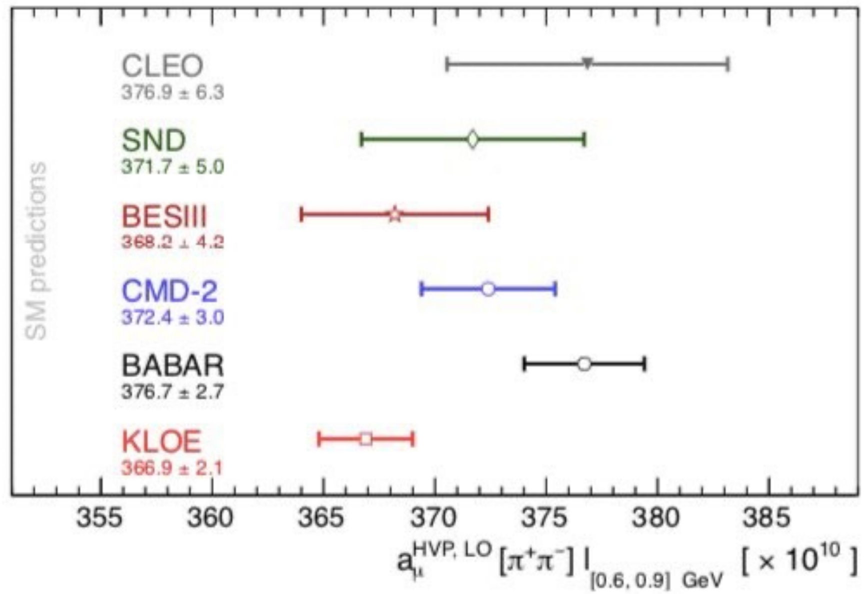


Leading order | Vacuum polarization | Light-by-light scattering

$$a_\mu^{hvp} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s) R(s)$$

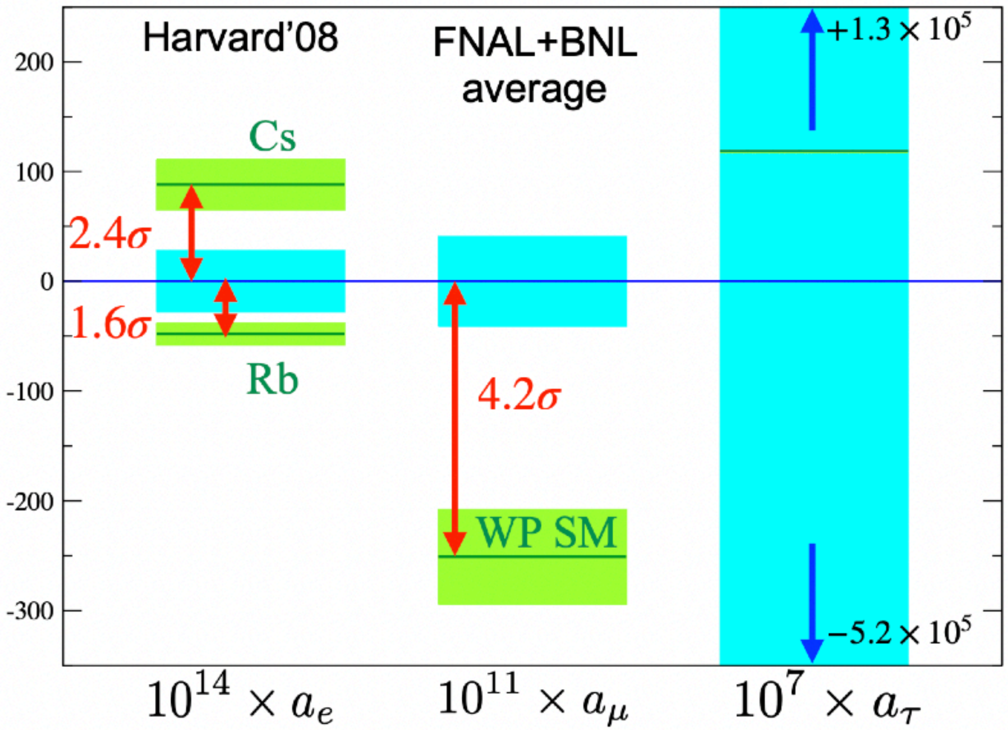
$$\frac{1}{12\pi} R(s) = \frac{1}{12\pi} \frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)};$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)m_\mu^2}{x^2 m_\mu^2 + (1-x)s}$$



  - need radiative return Belle II data to eliminate the discrepancy
  - τ-decay data is not currently used: Belle II + lattice?

$$a_\ell^{\rm SM} - a_\ell^{\rm Exp}$$



Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]

A. El-Khadra (talk at LP21)

- Muon decay $\mu \to 3e$:

$$\Gamma\ (\mu \to 3e) =$$
$$= \frac{\alpha m_\mu^5}{3\Lambda^4(4\pi)^2}\left(|C_{DL}|^2 + |C_{DR}|^2\right)\left(8\log\left[\frac{m_\mu}{m_e}\right] - 11\right)$$
$$+ \frac{4m_\mu^5}{3\Lambda^4(16\pi)^3}\left(m_e^4 G_F^2\left(|C_{SR}^e|^2 + |C_{SL}^e|^2\right)\right.$$
$$+ 2\left(2\left(|C_{VR}^e|^2 + |C_{VL}^e|^2 + |C_{AR}^e|^2 + |C_{AL}^e|^2\right) + |C_{AR}^e + C_{VR}^e|^2 + |C_{AL}^e - C_{VL}^e|^2\right))$$
$$- \frac{\sqrt{4\pi\alpha}m_\mu^5}{3\Lambda^4(4\pi)^3}\left(\Re\left[C_{DL}\left(3C_{VR}^e + C_{AR}^e\right)^*\right] + \Re\left[C_R^D\left(3C_{VL}^e - C_{AL}^e\right)^*\right]\right)$$

- Muonium decay $M_\mu^V \to e^+e^-$:

$$\Gamma\left(M_\mu^V \to e^+e^-\right) = \frac{f_M^2 M_M^3}{48\pi\Lambda^4}\left\{\frac{3}{2}|C_{VR}^e + C_{AR}^e|^2 - \frac{3}{2}|C_{VL}^e + C_{AL}^e|^2\right.$$
$$\left. + |2C_{VL}^e + C_{VR}^e|^2 + |2C_{AL}^e + C_{AR}^e|^2\right\}$$

- Note: different combination of Wilson coefficients!

R. Conlin, J. Osborne, AAP

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics - and QED!

Measure $$R_{\mu e} = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]}$$ to probe NP

★ Lepton wave functions are taken as solutions of Dirac equation

- with usual substitutions $u_1(r) = r\, g(r)$ and $u_2(r) = r\, f(r)$

$$\frac{d}{dr}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V + m_i \\ -(W - V - m_i) & \kappa/r \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\psi = \psi^\mu_\kappa = \begin{pmatrix} g(r)\chi^\mu_\kappa(\theta,\phi) \\ i f(r)\chi^\mu_{-\kappa}(\theta,\phi) \end{pmatrix}$$

★ ... with Dirac equation in a potential $V(r) = -e\int_r^\infty E(r')dr'$

$$E(r) = \frac{Ze}{r^2}\int_0^r r'^2 \rho^{(p)}(r')dr'$$

SINDRUM II (PSI), 2006 :	$R_{\mu e} < 7\times 10^{-13}$
M2e goal :	$R_{\mu e} <$ a few $\times 10^{-17}$

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a general quark operator Q

$$\langle N|Q|N\rangle = \int d^3r \left[Z\rho_p(r)\langle p|Q|p\rangle + (A-Z)\,\rho_n(r)\langle n|Q|n\rangle\right]$$

p(n) densities

$$\rho_{p(n)}(r) = \frac{\rho_0}{1+\exp[(r-c)/z]}, \quad \int d^3\rho_{p(n)}(r) = 1$$

★ Matrix elements of light quark currents are easily computed

- since $(m_\mu - m_e) \ll m_N$ we can neglect space components of the quark current

$$\langle p|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|p\rangle = 2 + c_d$$
$$\langle n|\bar{u}\gamma^0 u + c_d \bar{d}\gamma^0 d|n\rangle = 1 + 2c_d$$

count number of quarks

★ Gluonic contribution can be removed removed using anomaly equation or can be computed

★ Calculation of muon conversion probability involves interesting interplay of particle and nuclear physics

★ Nuclear averages are often done as an approximation. For a gluonic Rayleigh operator

$$\langle N|\frac{\beta_L}{4\alpha_s}G^a_{\mu\nu}G^{a\mu\nu}|N\rangle = -\frac{9}{2}\left[ZG^{(g,p)}\rho^{(p)} + (A-Z)G^{(g,n)}\rho^{(n)}\right],$$

where $G^{(g,\mathcal{N})} = \langle\mathcal{N}|\frac{\alpha_s}{4\pi}G^a_{\mu\nu}G^{a\mu\nu}|\mathcal{N}\rangle \approx -189 \text{ MeV}$

★ The (coherent) conversion rate is

$$\Gamma_{(}\mu^- + (A,Z) \to e^- + (A,Z)) = \frac{4a_N^2}{\Lambda^4}\left(|c_1|^2 + |c_3|^2\right)$$

with $a_N = G^{(g,p)}S^{(p)} + G^{(g,n)}S^{(n)}$

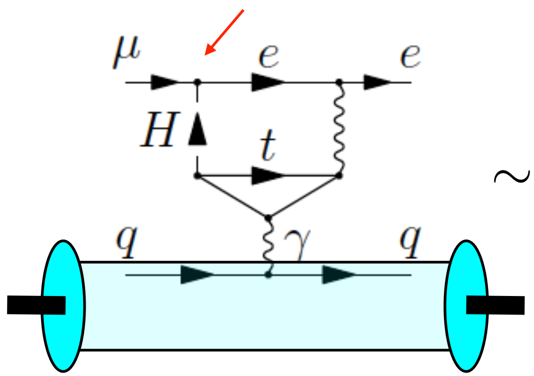
The overlap integrals $S^{(p,n)}$ with muon and electron wave functions are

$$S^{(p)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 Z\rho^{(p)}(g_e^- g_\mu^- - f_e^- f_\mu^-),$$

$$S^{(n)} = \frac{1}{2\sqrt{2}}\int_0^\infty drr^2 (A-Z)\rho^{(n)}(g_e^- g_\mu^- - f_e^- f_\mu^-).$$

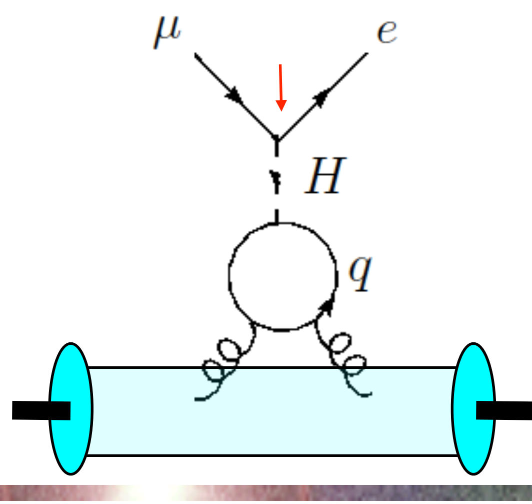
★ Contribution of heavy quarks can, in principle, be large even at low energies

★ Two-loop sensitivity to NP in muon conversion experiment...



$\sim \mathcal{O}(\alpha^2)$

★ ... becomes one-loop!



➡ gluonic couplings to hadrons are not (always) suppressed!
➡ NP couplings to heavy quarks are not well constrained and could be large

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