A Dedicated Muon EDM Experiment in the `g-2' Storage Ring

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Thomas-BMT Equation

The total precession frequency $\vec{\Omega}_S$ of the spin in the presence of $\vec{B}$ and $\vec{E}$ (both $\perp \vec{p}$) would be the net sum of MDM precession and the EDM precession, given by the Thomas-BMT equation:

$$\vec{\Omega}_S = \underbrace{-\frac{q}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]}_{MDM} + \underbrace{\frac{\eta q}{2mc}\left[\vec{E} + c\vec{\beta} \times \vec{B}\right]}_{EDM}$$

If the magnetic field is purely vertical and the electric field is purely radial,

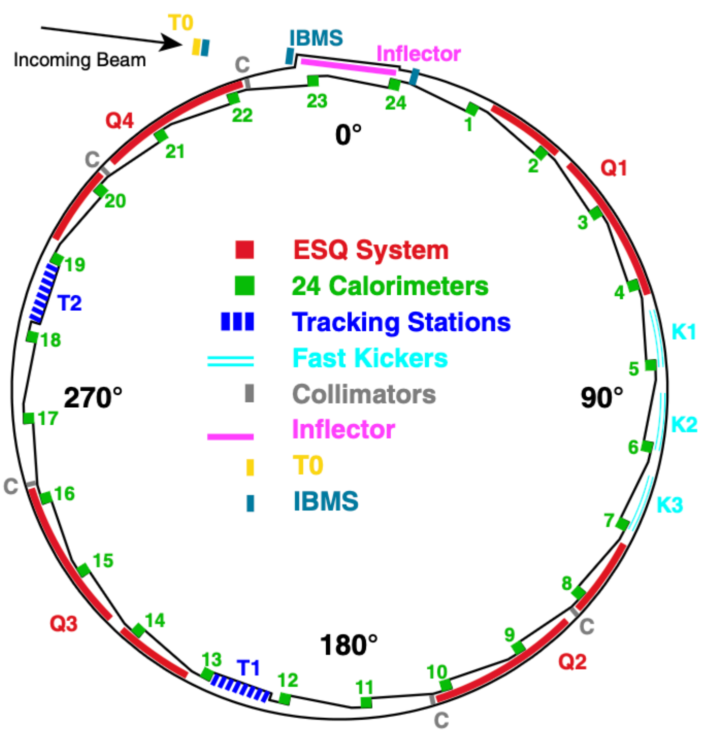
- MDM spin precession would be about the vertical axis in the plane of the ring
- EDM spin precession would be about the radius tipping vertically out of the plane of the ring

The `g-2' Storage Ring (at present)

- The current `g-2' storage ring is being operated for measuring the magnetic dipole moment (MDM) of the muon primarily caused by the vertical magnetic field $\vec{B}$.

The precession frequency Ω_{MDM} is given by:

$$\vec{\Omega}_{MDM} = -\frac{q}{m}\left[G\vec{B} - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\vec{\beta} \times \vec{E}}{c}\right]$$

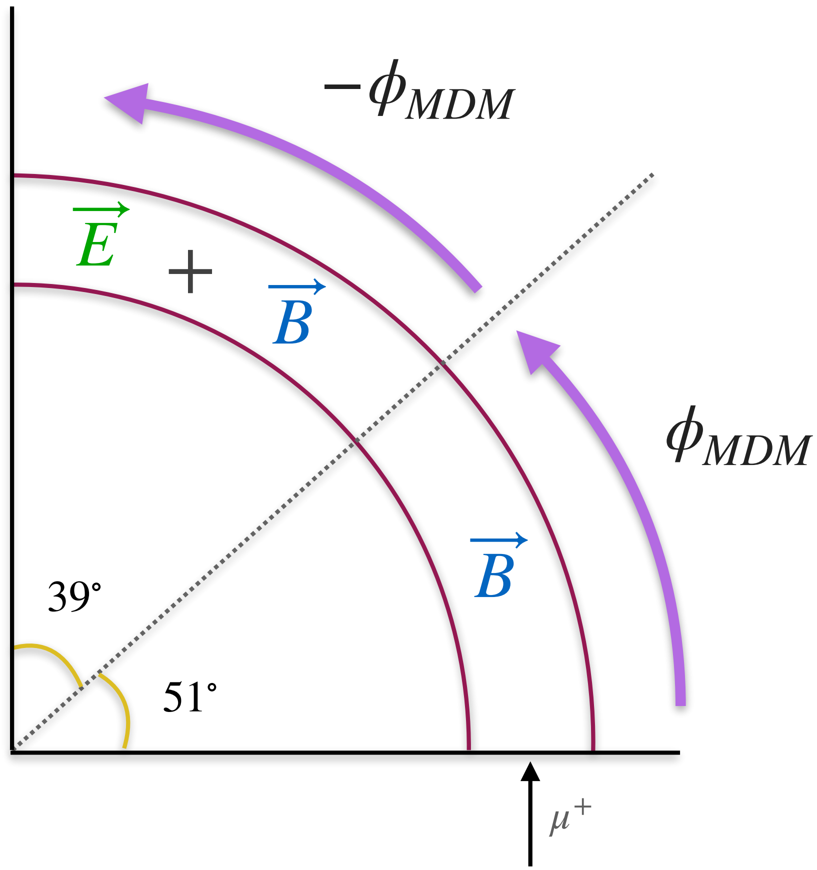


(Figure: Anna Driutti)

- The relativistic γ is chosen such that the second term is zero, operating at muon's 'magic momentum' of 3.09 GeV/c.
- The ring has a radius of 7.112 meters and is four-fold symmetric.
- It has a highly purified constant vertical dipole magnetic field throughout and four isometrically placed electrostatic quadrupoles for vertical focussing.
- Each 90 degree section consists of:
  - 51 degrees of dipole $\vec{B}$-field only region (~ 57%)
  - 39 degrees of (dipole $\vec{B}$ + quadrupole $\vec{E}$) region (~ 43%)
- There are no dipole electric fields in the `g-2' storage ring at present.

Proposed Scheme - A Hybrid Storage Ring

We propose an idea of freezing the MDM precession and enhancing the EDM signal by introducing a dipole electric field in the electrostatic quadrupole sections.



The idea:

1. The μ^+ traverses through 51° $\vec{B}$-only section.
2. The MDM component of the spin precession increases by an amount ϕ_{MDM} due to the $\vec{B}$-field.
3. The μ^+ then enters the 39° section $\vec{E} + \vec{B}$ section.
4. The dipole $\vec{E}$ field (along with $\vec{B}$) in the 39° section is chosen such that the MDM precesses the spin in the opposite direction by the same amount $-\phi_{MDM}$

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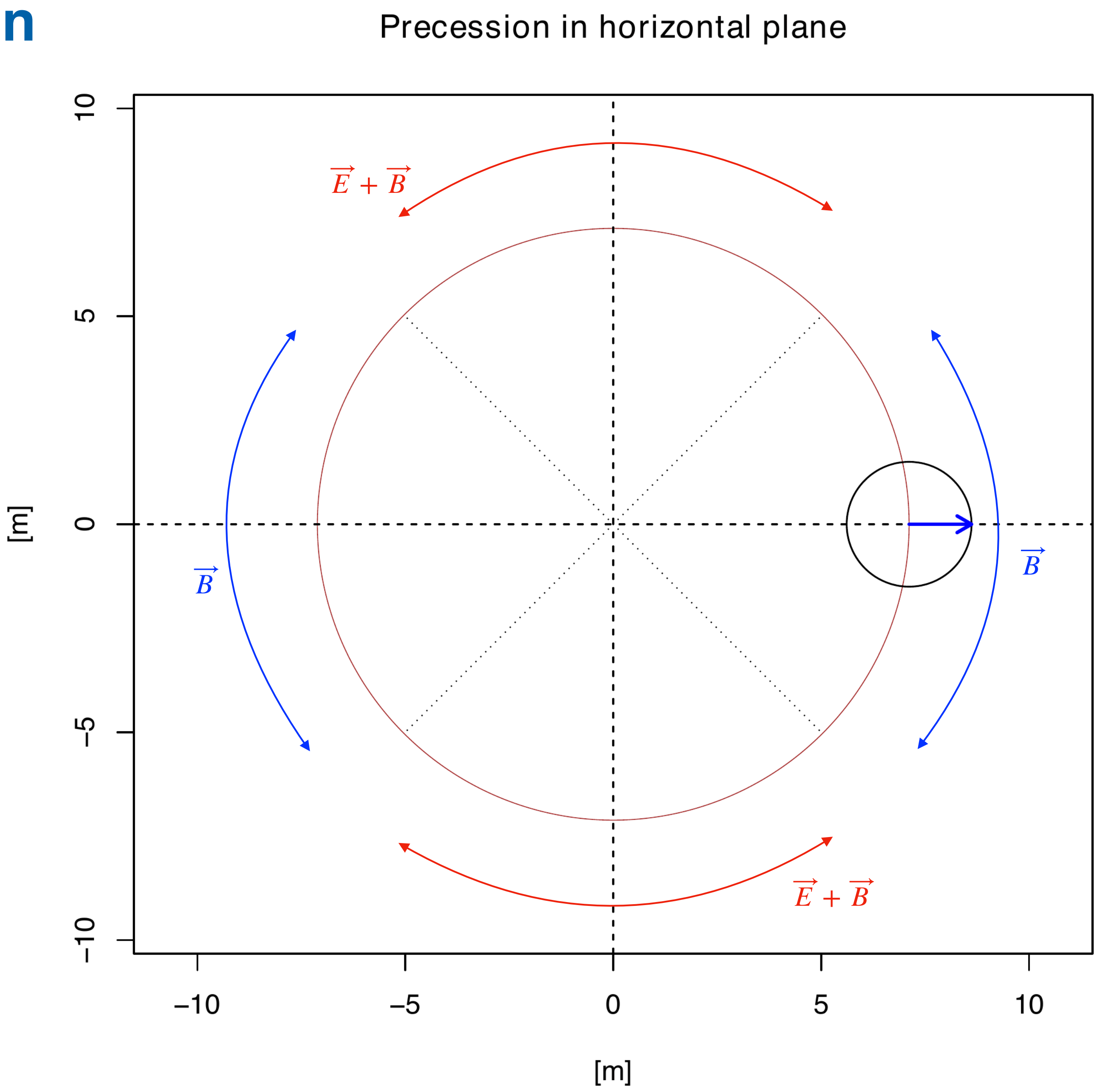
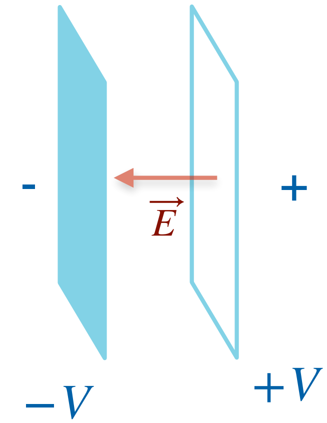
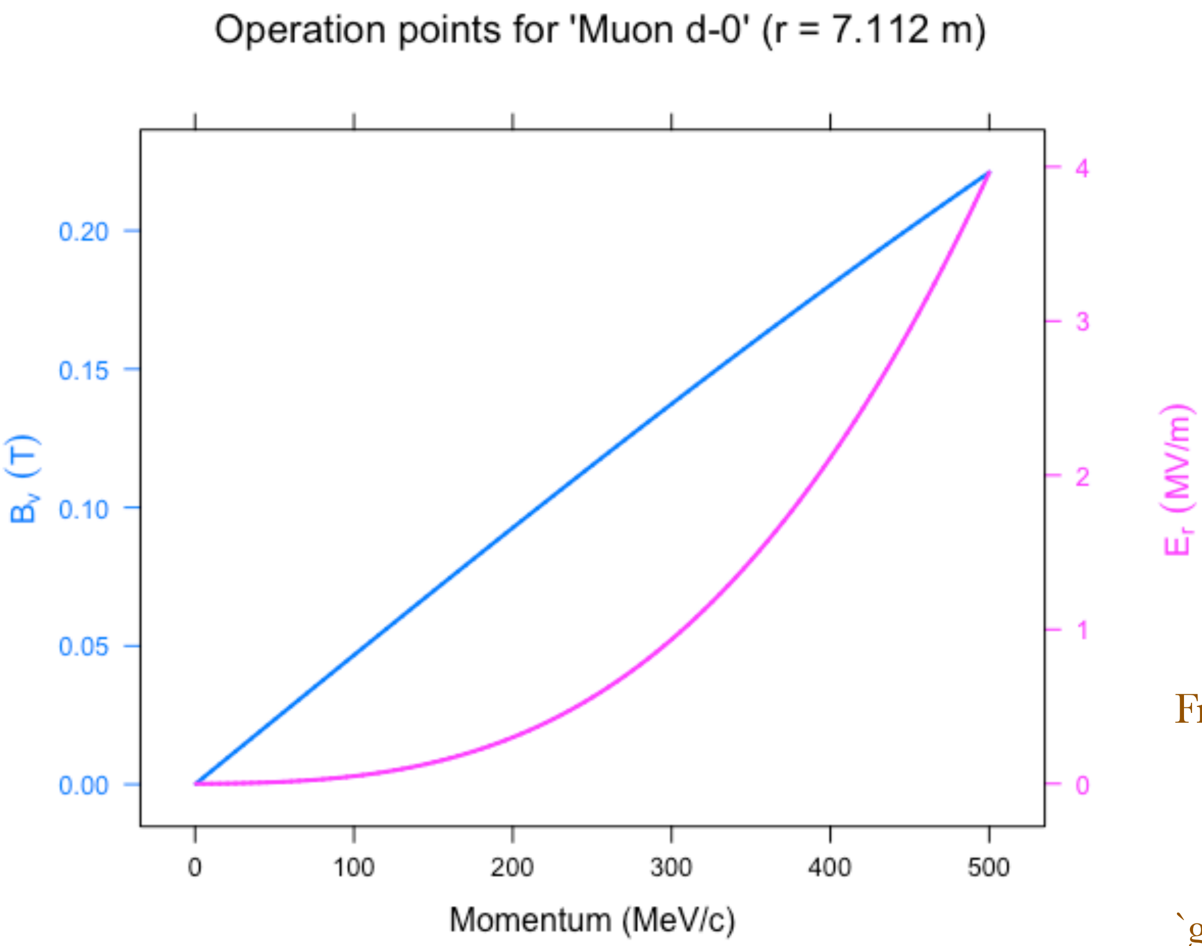
Frozen spin


Precession in horizontal plane
$\vec{E} + \vec{B}$
$\vec{B}$
$\vec{B}$
$\vec{E} + \vec{B}$
10
5
0
−5
−10
[m]
−10
−5
0
5
10
[m]

Which momentum to choose?





Momentum (MeV/c)	Vertical Magnetic Field (Tesla)	Radial Electric Field (MV/m)
100	0.046	0.048
200	0.092	0.300
280	0.131	0.769
400	0.180	2.113
500	0.220	3.963

Frozen spin constraint:

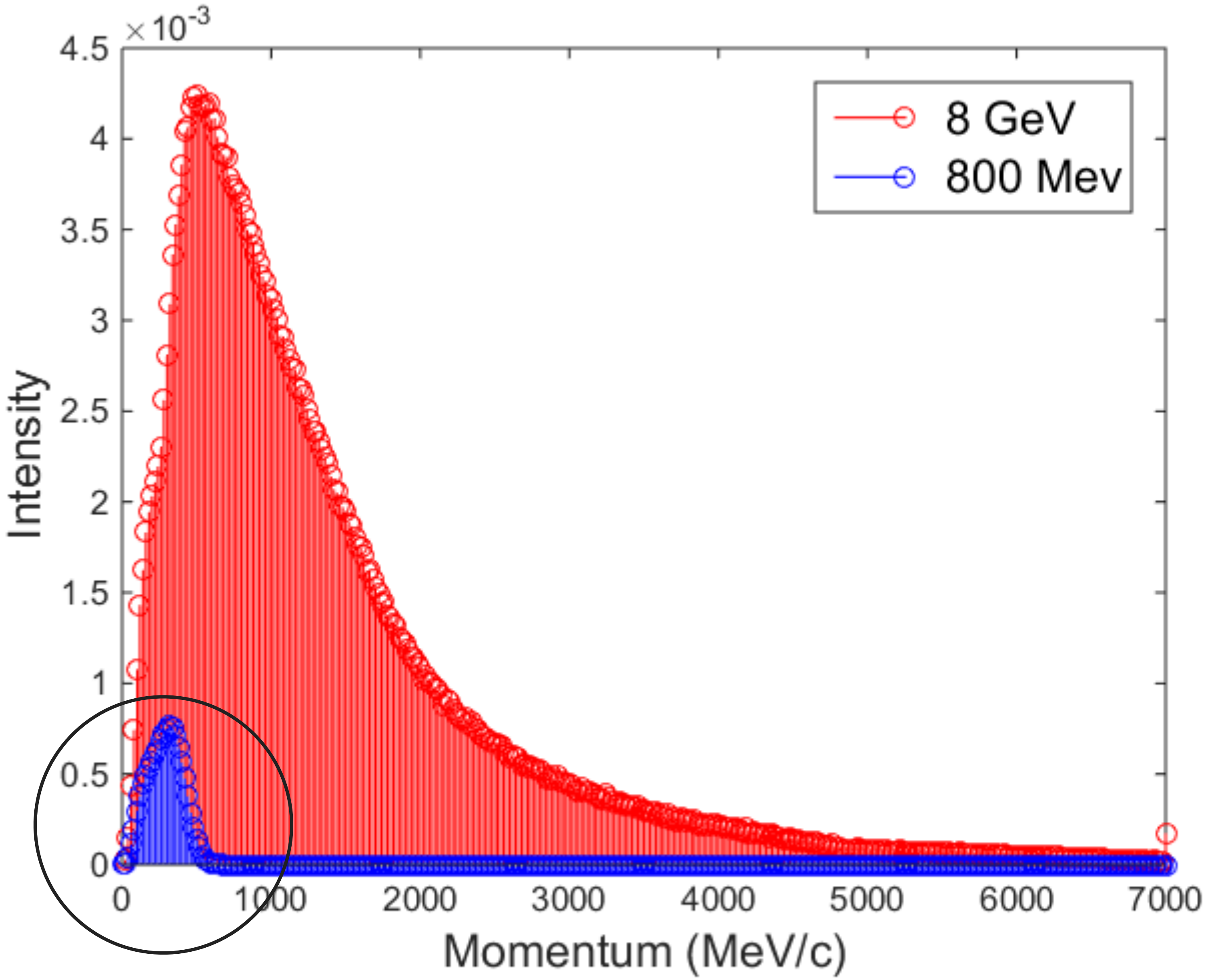
$$\vec{E} - \vec{B}\,.\left[\frac{Gc}{|\beta|}\frac{90^\circ}{39^\circ}\left(G - \frac{1}{\gamma^2 - 1}\right)^{-1}\right] = 0$$

`g-2' ring orbit constraint:

$$\vec{E} + v\vec{B} = \gamma\frac{mv^2}{qR_{ring}}\hat{r}$$

The electric field is a strong decider of the muon central momentum.

Making use of PIP-II Protons



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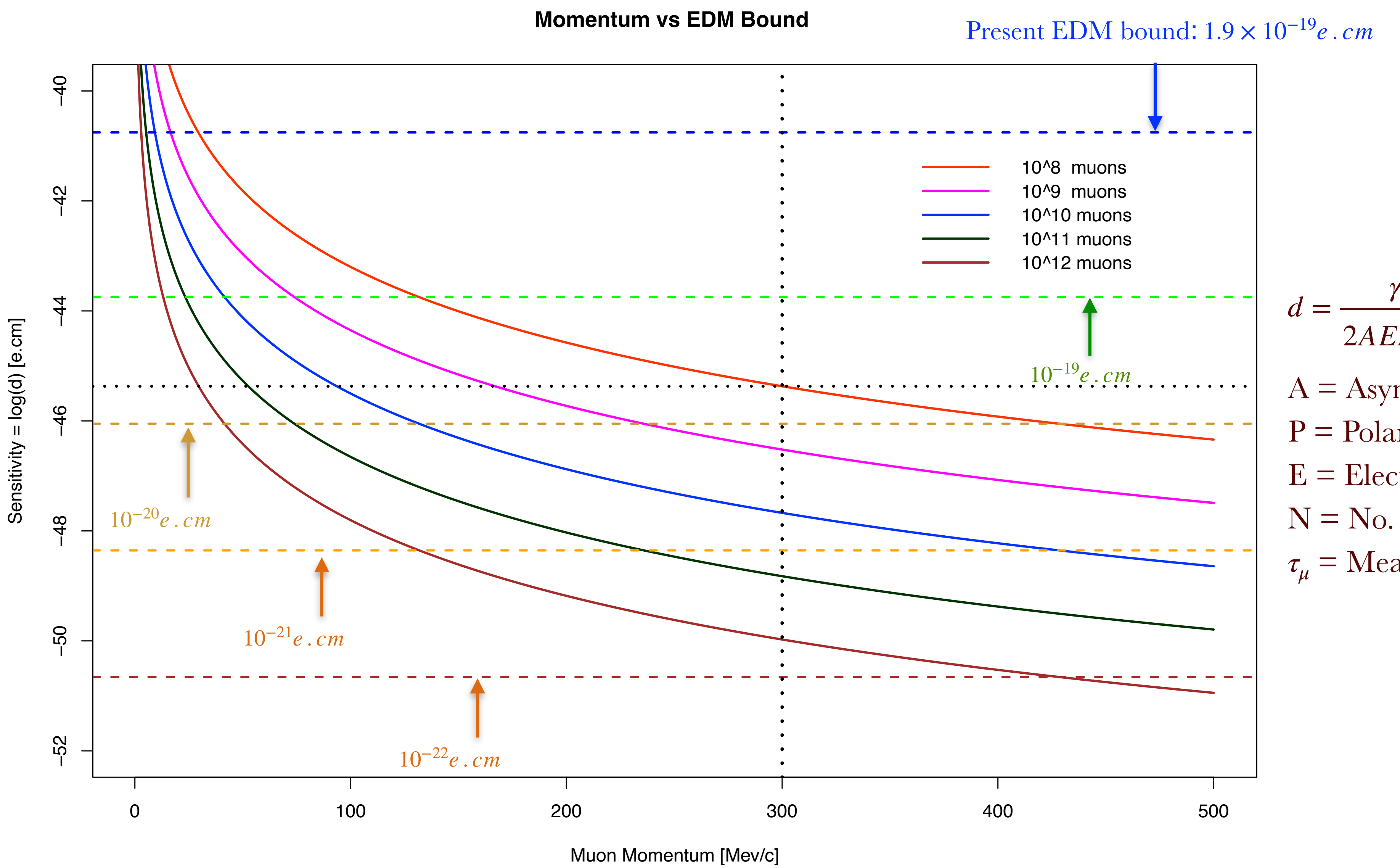
Assumed Particle Rates for PIP-II

RMS Current from PIP-II Linac	0.002 A
Pulse length for `g-2' storage ring	120 ns
No. of PIP-II bunches per EDM pulse	19.5
Protons per EDM pulse	1.5×10^9
Good pions/muons off the target	18000
Muon storage duration	10 lifetimes = 83 microseconds
Muons stored per year	9×10^{11}

Direct EDM Measurement Sensitivity



$$d = \frac{\gamma h G}{2AEP\tau_\mu \sqrt{N}}$$

A = Asymmetry

P = Polarization

E = Electric field

N = No. of μ^+

τ_μ = Mean μ^+ lifetime

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Brendan Casey
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Sam Grant

Hogan Nguyen
Yannis Semertzidis
Dominika Vasilkova

Questions

Q: How does the experiment make use of the ACE beam?
A: Usage of the proton pulses from the PIP-II linac. Higher the PIP-II beam intensity, higher the muon production.

Q: Is the experiment uniquely enabled by The ACE upgrades?
A: The proton intensity given out by the PIP-II linac provides a near-perfect window for creating muons in the interested range of momentum ideal for a frozen-spin EDM experiment.

Q: Can this experiment be performed elsewhere?
A: The `g-2' storage ring offers a unique capability for a spin precession experiment. (In fact, the best present $\mu-$EDM bound comes from BNL `g-2' run.)

Q: What proton energies are needed?
A: 800 MeV protons.

Q: What proton quantities are needed?
A: Greater than 1.5×10^9 protons per EDM pulse.

Q: What time structure is needed? (bunch length, train structure)
A: Greater than 19.5 PIP-II bunches per EDM pulse.

Q: Can the experiment be performed with 800 MeV protons from PIP-II?
A: Absolutely!

Freezing the MDM Precession

The amount of spin's MDM precession in the 51° of $\vec{B}$-only region is given by:

$$\phi_{MDM,\,B} = \frac{q}{m} GB \cdot \frac{51^\circ}{90^\circ} \frac{T_{\text{rev}}}{4}$$

The amount of spin's MDM precession in the $\vec{E} + \vec{B}$-only region is given by:

$$\phi_{MDM,\,E+B} = \frac{q}{m}\left[GB - \left(G - \frac{1}{\gamma^2 - 1}\right)\frac{\beta \cdot E}{c}\right] \cdot \frac{39^\circ}{90^\circ} \frac{T_{\text{rev}}}{4}$$

Equating them both, we can solve for the electric field value needed to cancel the MDM precession accumulated in the $\vec{B}$-only section.

Finding the $\overrightarrow{E}$ and $\overrightarrow{B}$ field values

Simplifying the equation for frozen MDM precession, we have a linear equation in E and B:

$$\overrightarrow{E} - \overrightarrow{B}\,.\left[\frac{Gc}{|\beta|}\frac{90^\circ}{39^\circ}\left(G - \frac{1}{\gamma^2 - 1}\right)^{-1}\right] = 0$$

CONSTRAINT #1

Since we look to re-use the `g-2' storage ring, the radius of the ring imposes a condition via the centripetal Lorentz force required to keep the muons on the 7.112 meter orbit:

$$\overrightarrow{E} + v\overrightarrow{B} = \gamma\frac{mv^2}{qr}\hat{r}$$

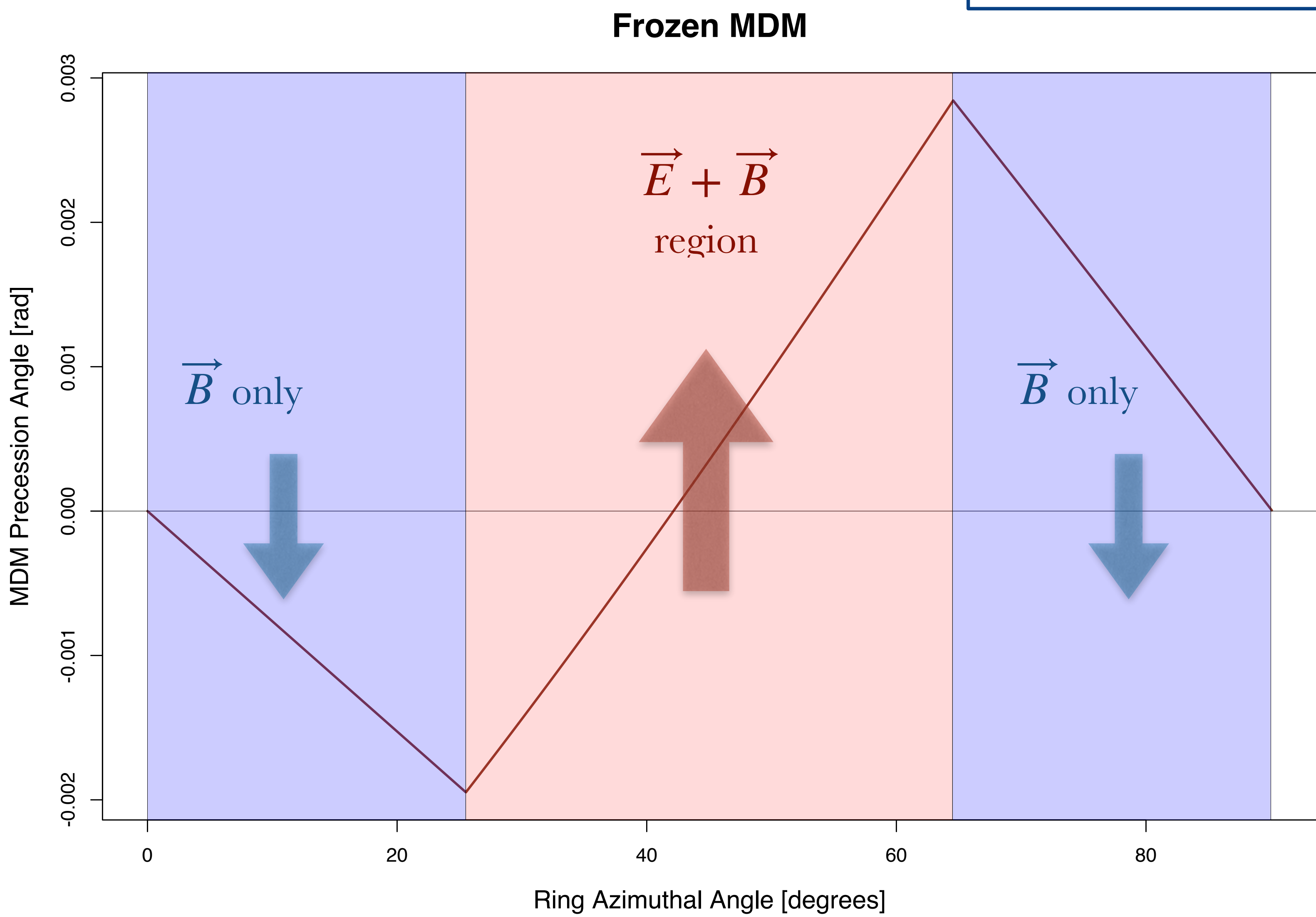
CONSTRAINT #2

(Since we could use both E and B-fields in the ring, there is no constraint to operate on the magic momentum anymore.)

The above two constraints thus give us two linear equations in $\overrightarrow{E}$ and $\overrightarrow{B}$ which we can solve for various values of momentum (γ) for possible operational value of fields.

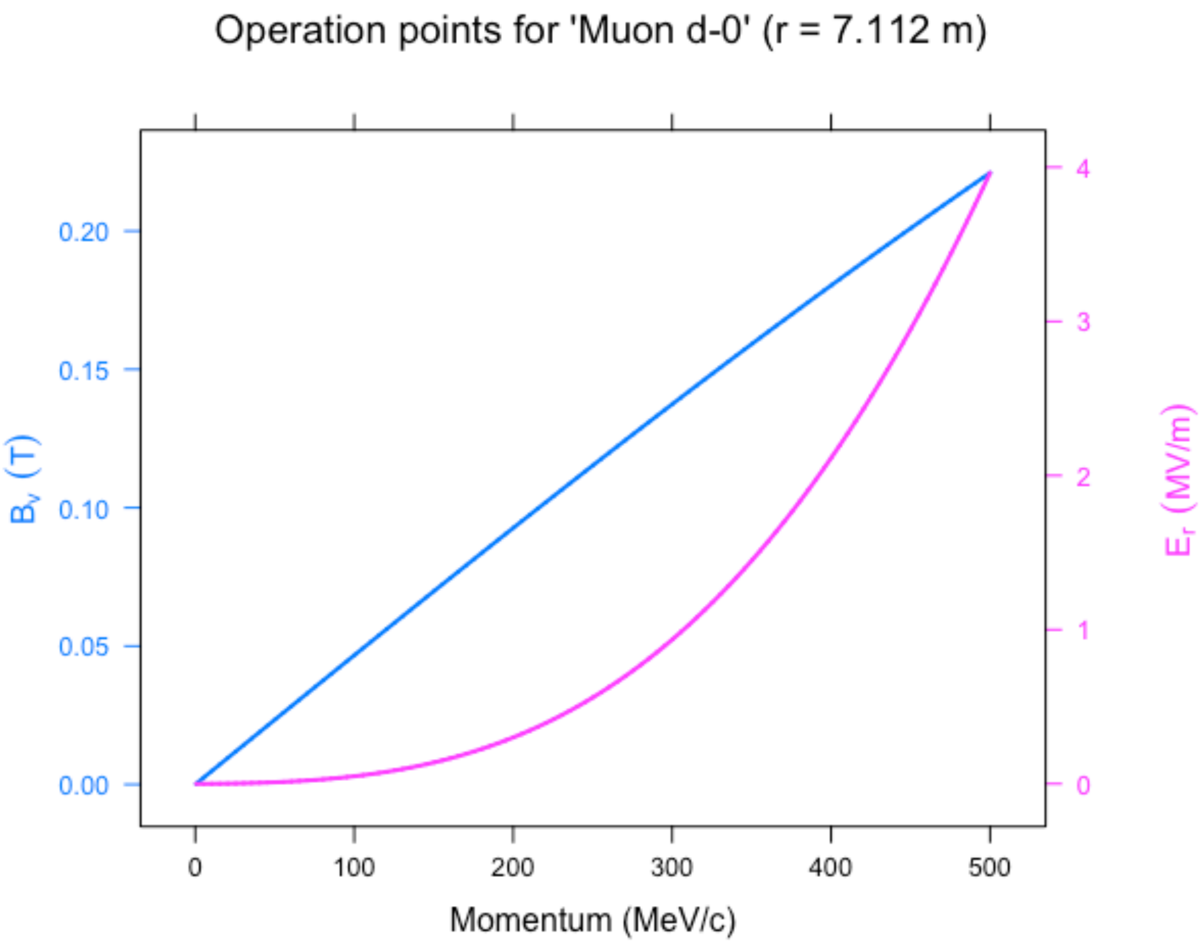
Freezing the MDM Precession

Net MDM precession = 0



If achieved in time, this could be the first ever demonstration of frozen spin technique technique!

Possible field values for frozen MDM condition:



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E-field vs momentum

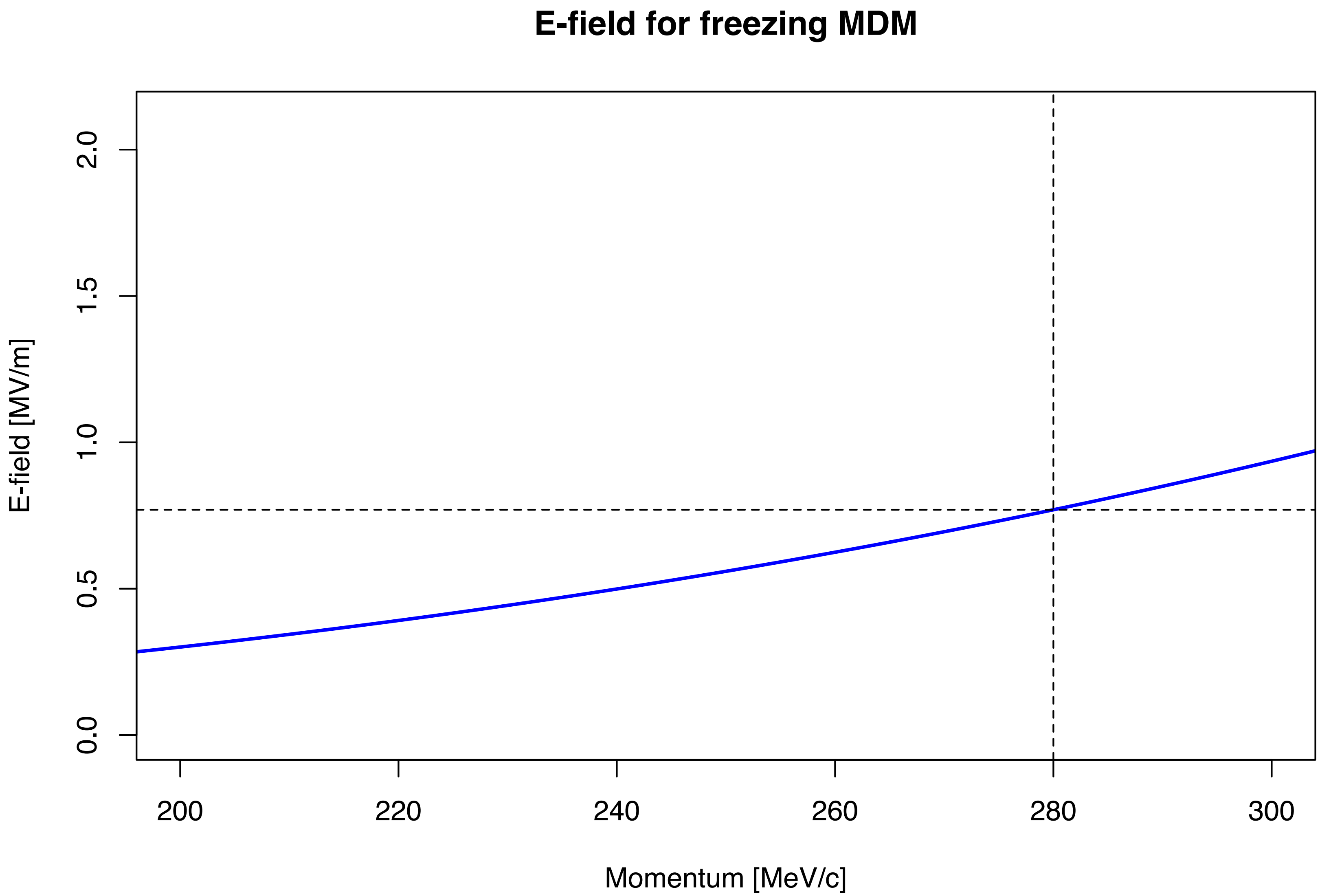
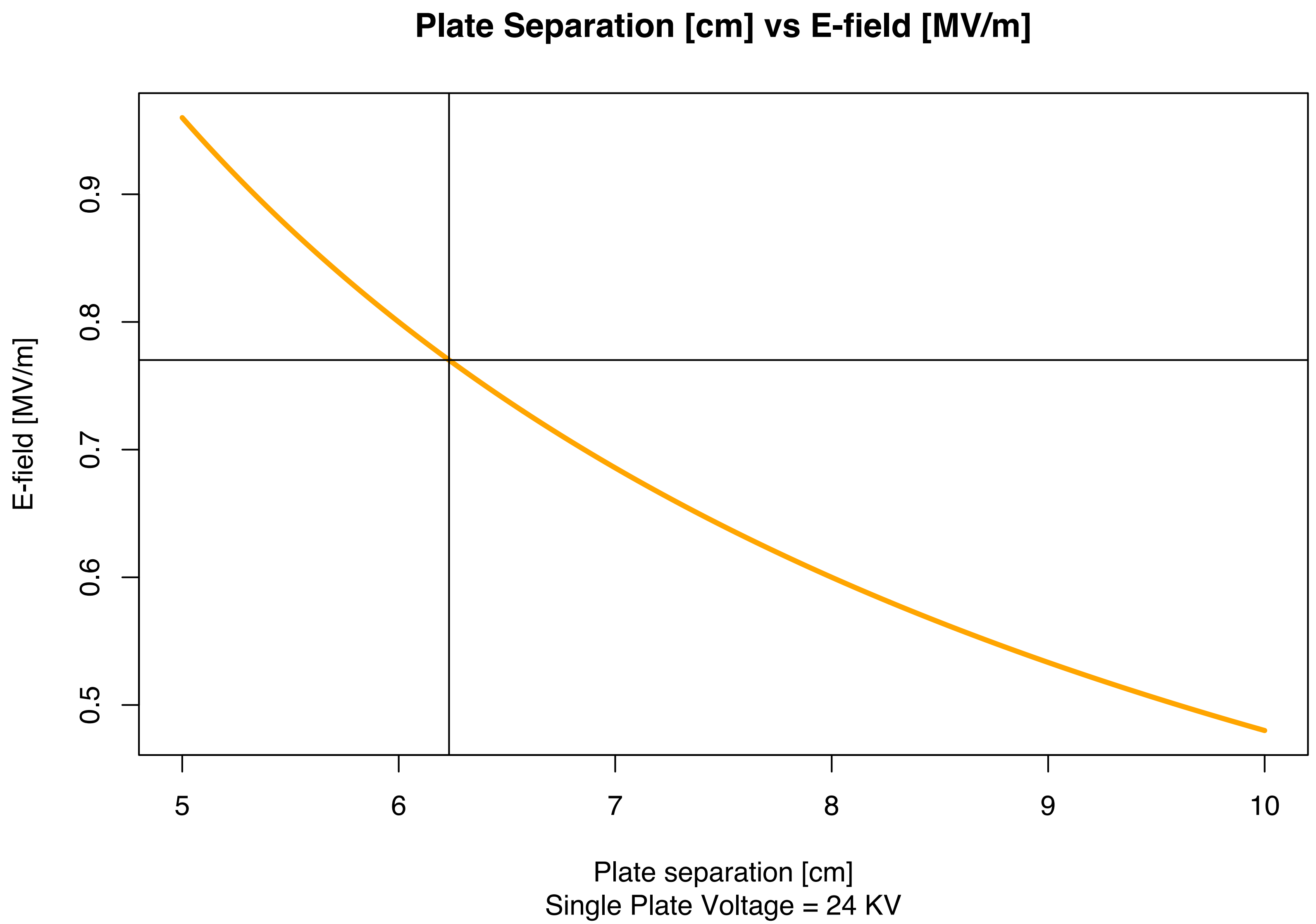


Plate separation vs E-field

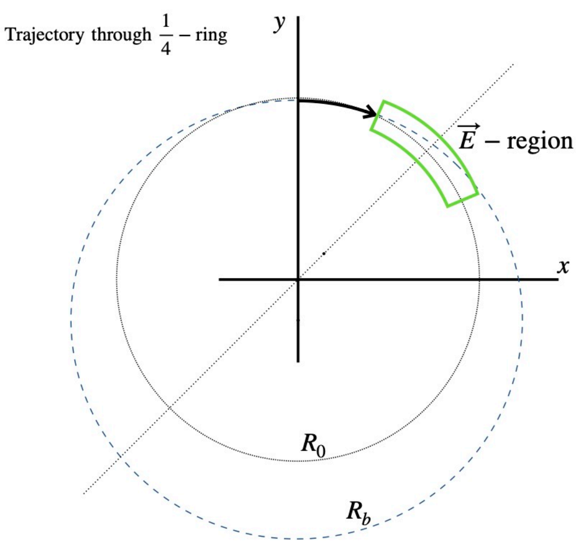


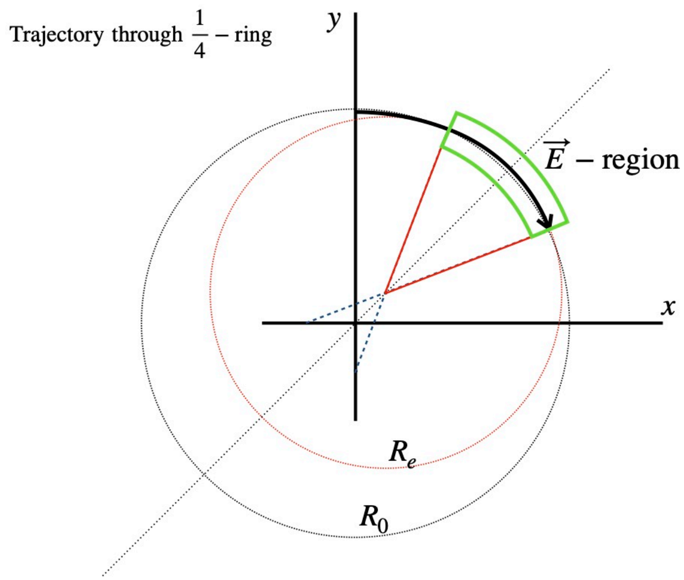
Beam Dynamics

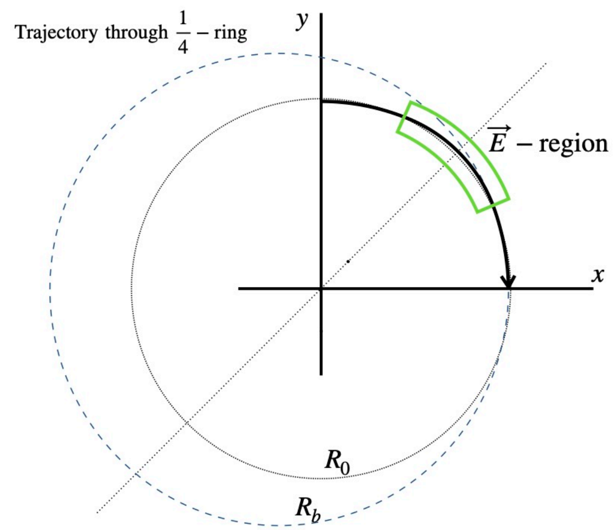
The next question: with both electric and magnetic dipole fields, can we have a stable closed orbit inside the ring with frozen MDM precession conditions?

The answer is: YES!

Only that the stable closed orbit will not be a perfect circle anymore.







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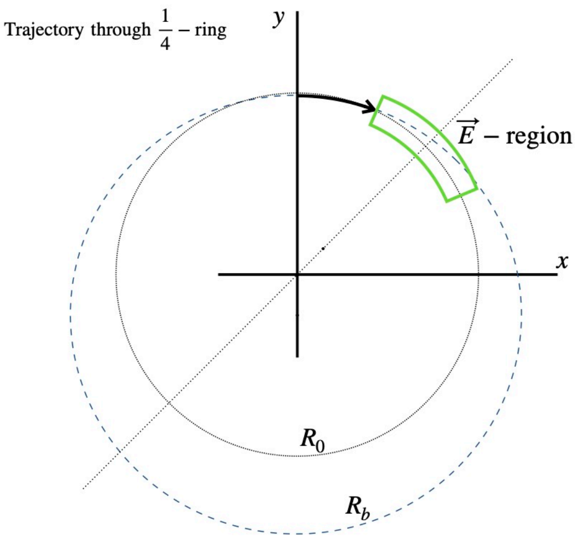
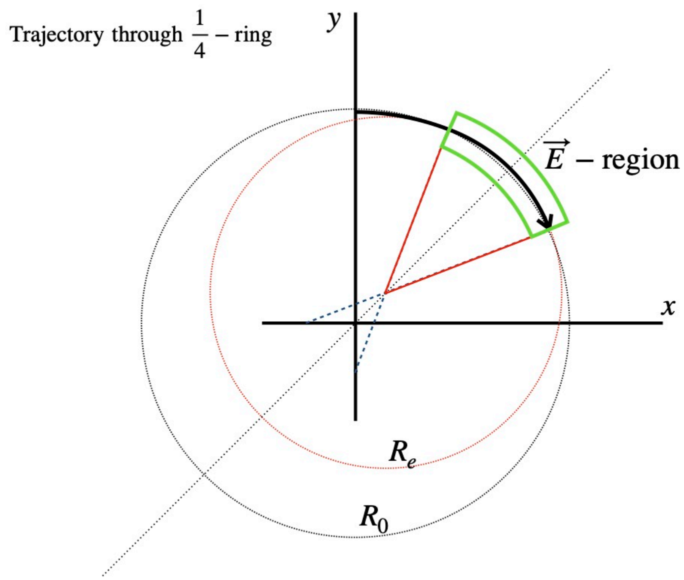
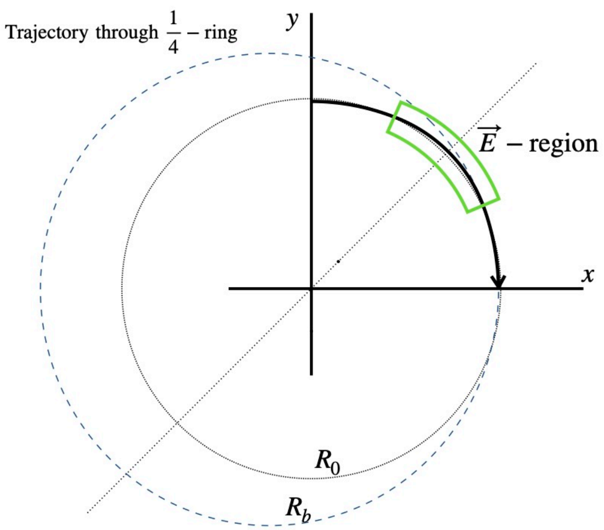
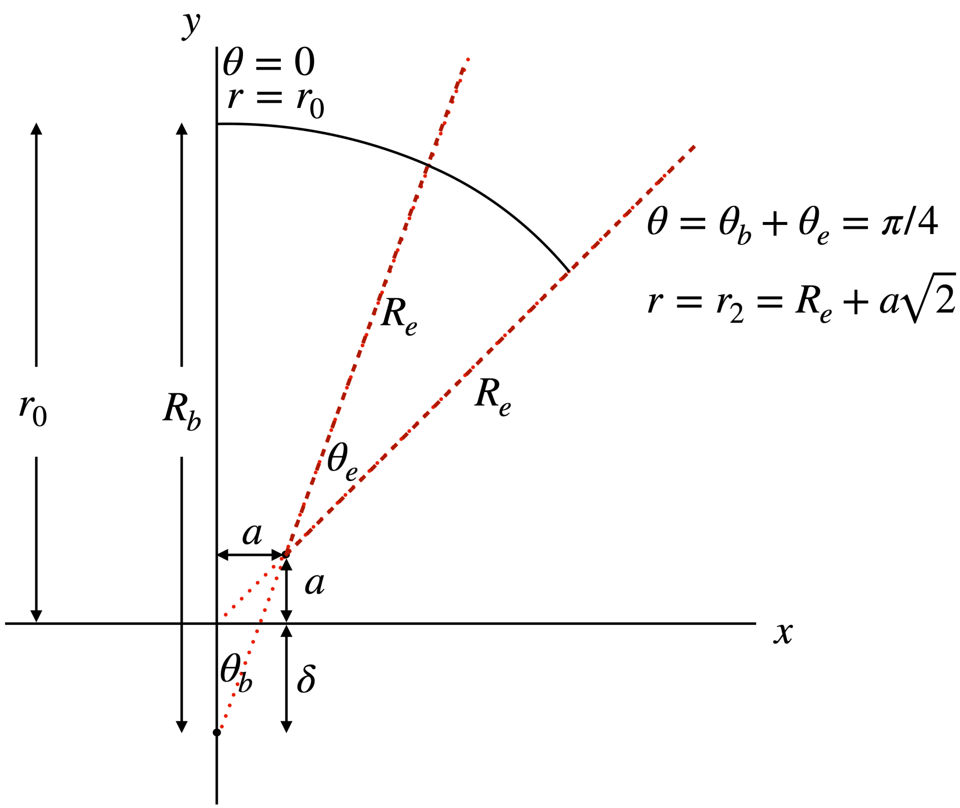


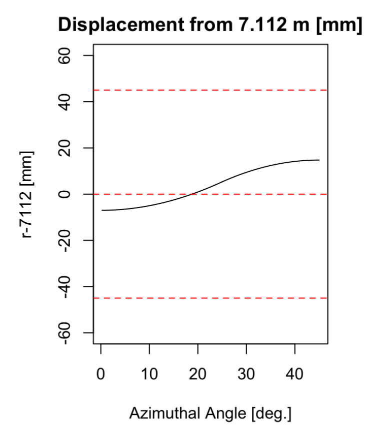


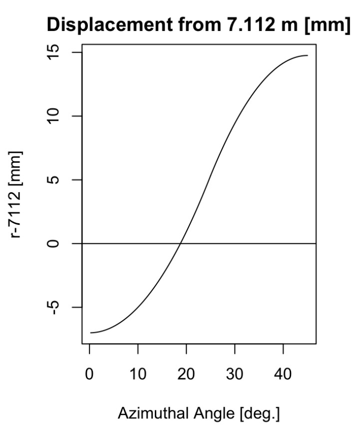


How large are these deviations from the 7.112 meter orbit?

Closed Orbit - Geometric Analysis







With some geometric analysis, we find that,

$$r_0 = R_b[1 - \cos(\theta_b) + \sin(\theta_b)] + R_e[\cos(\theta_b) - \sin(\theta_b)]$$

$$r_2 = \sqrt{2}R_b \sin\theta_b + R_e(1 - \sqrt{2}\sin\theta_b)$$

For example, with the parameters of

$\vec{p} = 387$ MeV/c,
$\vec{E} = 1.98$ MV/m,
$\vec{B} = 0.178$ T,

the maximum radial orbital variations from the 7.112 meters circular orbit are only ± 10.9 mm!

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Closed Orbit - 4th Order Runge-Kutta simulation

One could verify the previous geometric analysis with actual particle tracking to see if we indeed can have a stable closed orbit.

A particle tracking simulation was thus done by solving the coupled differential Lorentz equations in the $\overrightarrow{B}$ only region and $(\overrightarrow{E} + \overrightarrow{B})$-region for various momenta values at a time step of 1 nanosecond.

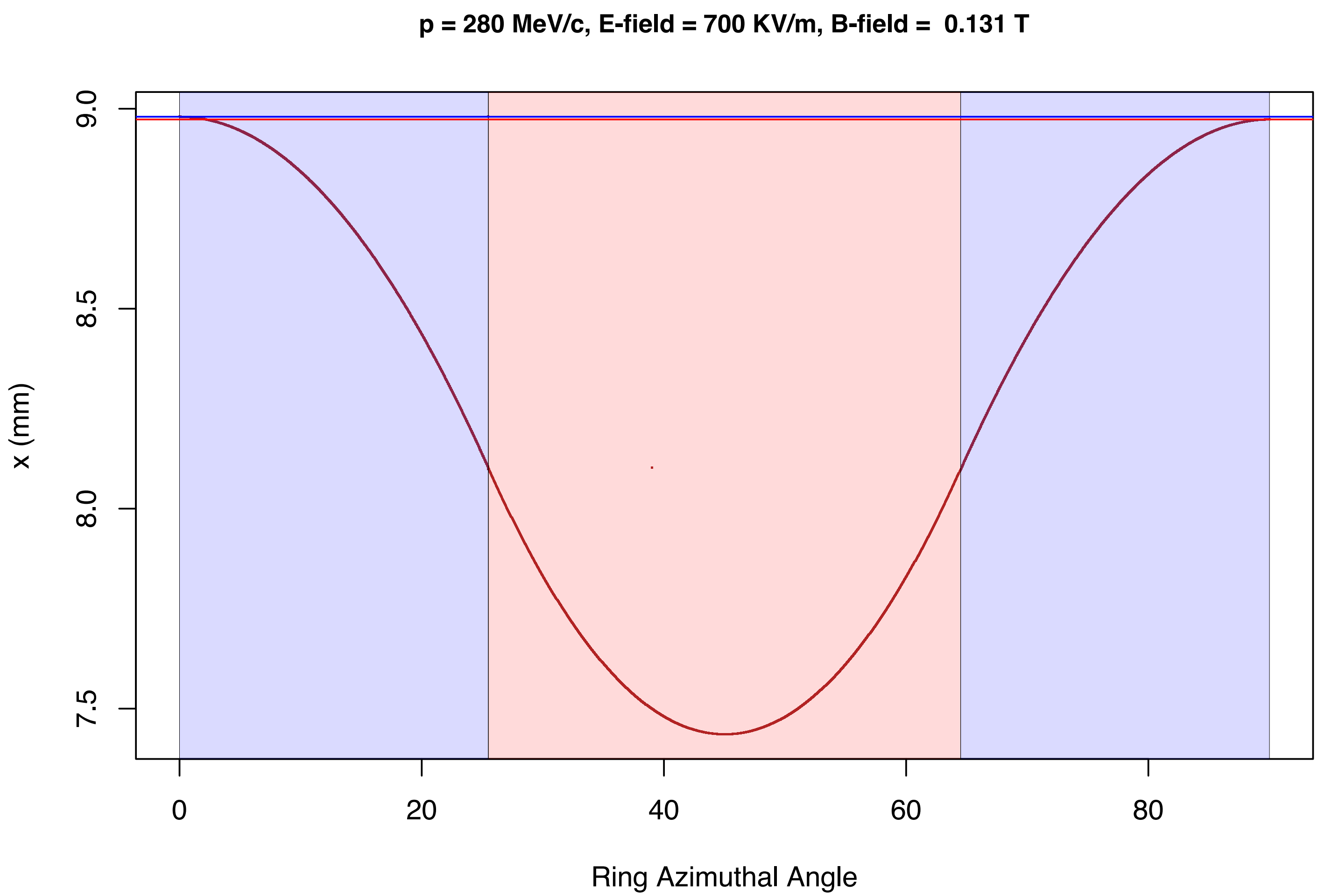
$$\frac{dx_\circ}{dt} = v_{x_\circ} \qquad \frac{dy_\circ}{dt} = v_{y_\circ}$$

$$\frac{dv_{x_\circ}}{dt} = \frac{q}{m}E\cos\theta + \frac{q}{m}v_{y_\circ}B_z \qquad \frac{dv_{y_\circ}}{dt} = \frac{q}{m}E\sin\theta - \frac{q}{m}v_{x_\circ}B_z$$

where $x_\circ$ and $y_\circ$ are the coordinates in the horizontal plane of the ring with (0,0) being the centre of the ring..

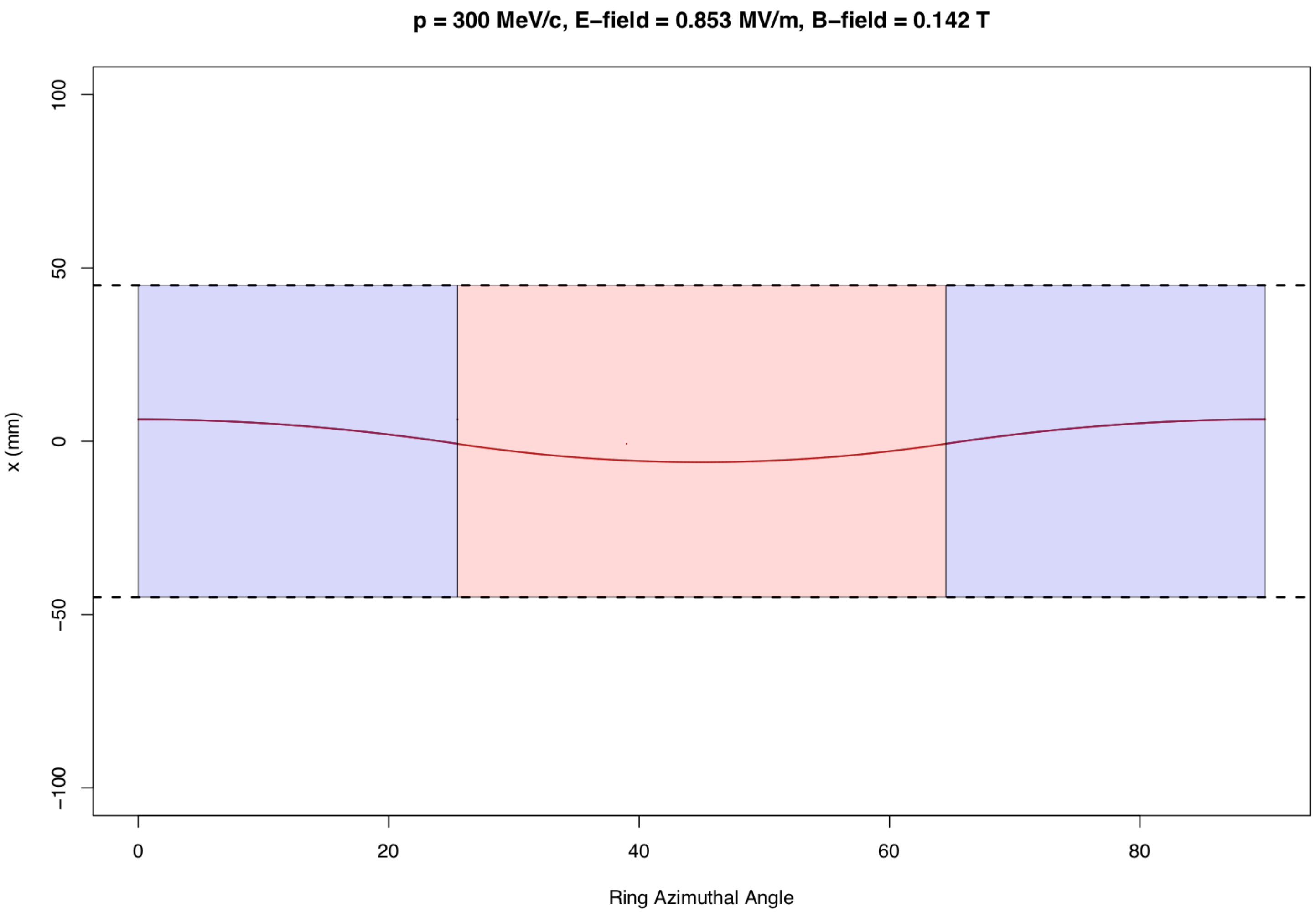
Closed Orbit for 280 MeV/c muon

4th Order Runge-Kutta simulation



Closed Orbit - 4th Order Runge-Kutta simulation

With a ±50 mm aperture, the scale of a typical closed orbit would look like:



EDM Precession Estimates

Unlike the MDM, the precession due to EDM at a given point in the ring is going to keep constantly building up until the muon decays.

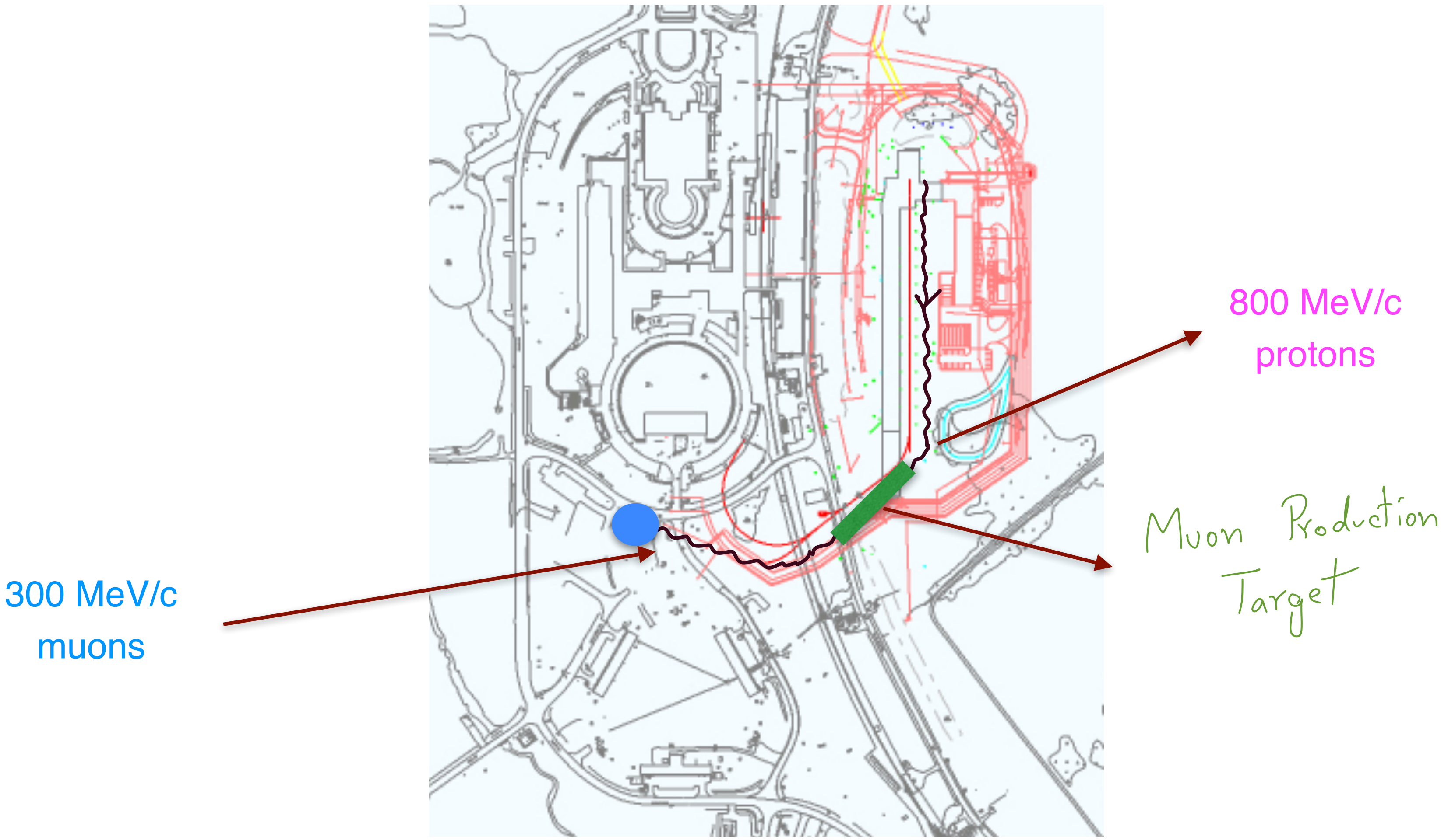
Since we have two distinct regions within a quarter section, the rate of precession will *slightly* vary within the $\overrightarrow{B}$-only section and the $(\overrightarrow{E} + \overrightarrow{B})$-section, albeit by a small factor.

The total precession through a half-quadrant will be:

$$\Delta\phi_{EDM} = \frac{d}{S} . B_0 \left[\ell_b + \ell_e \left(1 + \frac{E_0}{B_0 \beta c} \right) \right]$$

Plugging in appropriate set of field values for a momentum range of 300 MeV/c and the path lengths, we see that the EDM precesses in the order of 10 mrad for 5 muon lifetimes ($\sim$30 μs).

PIP-II Beam Potential for a ‘d-0’ EDM Experiment



Summary and Outlook

We could have a potential EDM experiment in the `g-2 storage ring by altering the quadrupole system to create a radial electric dipole field pointing radially inward.

The new system would not be significantly different from the current set-up, other than

- the radius of curvature for the orbits would be different,
- the inner/outer plates would be at a higher potential difference than the upper/lower plates in order to create the electric dipole field.

Since the muons' central momentum would be around 300 MeV/c, the requirements for the existing magnetic dipole field, inflector system, and kicker system would all reduce by a factor of ten.

Electric field levels of 0.77 MV/m could be tried to be achieved with potentials of ±24 kV with lateral plate separations on the scale of ~70 mm.

An excellent opportunity to study the systematics for a possible future dedicated run, and could be the first demonstration of the frozen spin technique with the possibility to do some physics EDM measurement in a limited time (few weeks?).

This could prove as an able demonstrator for a dedicated future EDM physics experiment.

Muon path length from target to storage ring

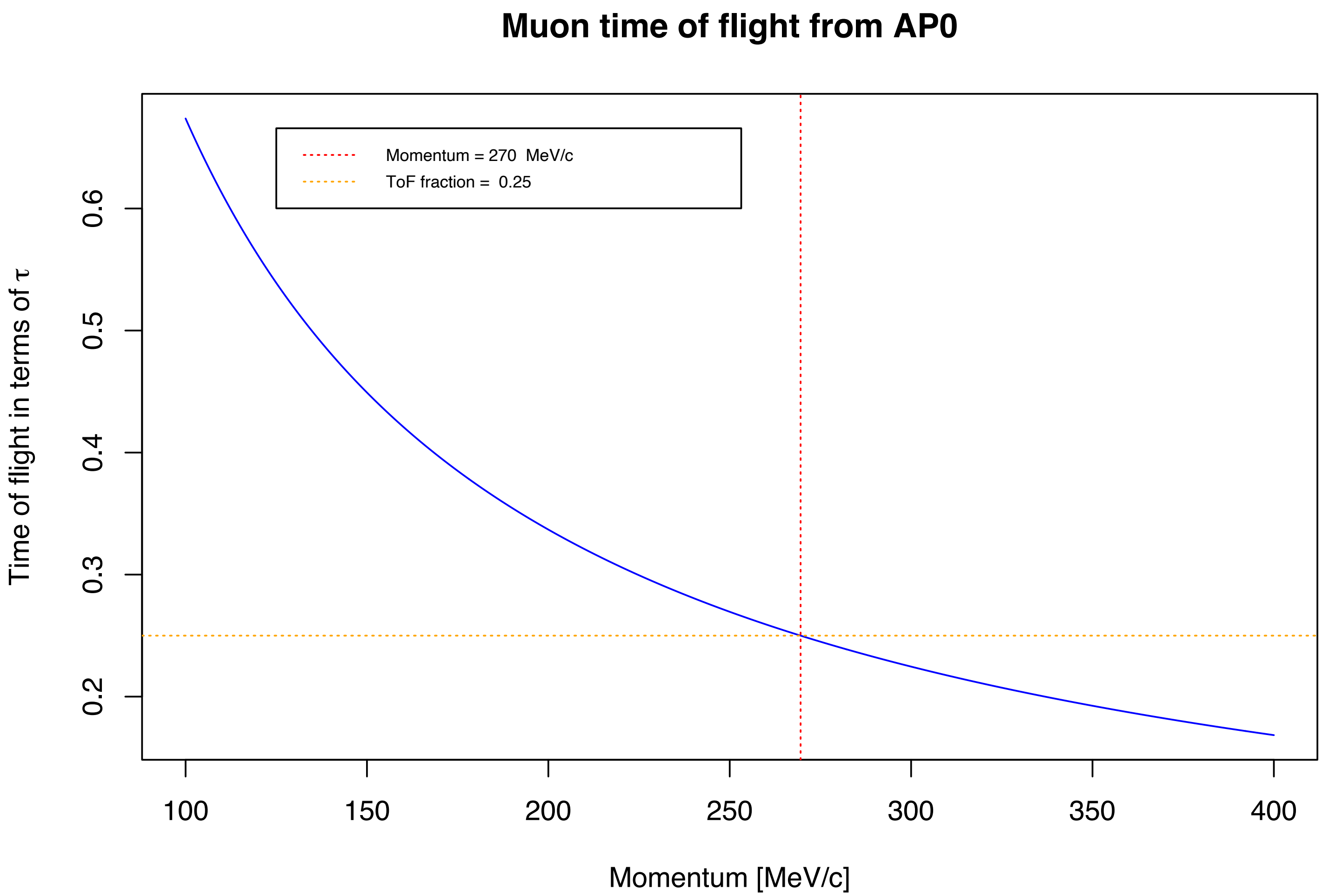
$$\gamma = \sqrt{1 + \frac{p^2}{m^2 + c^2}}$$

$$t = \frac{L}{c\sqrt{1 - 1/\gamma^2}}$$

For L ~ 420 meters and $p \approx 300$ MeV/c, $\gamma \approx 3.01$ and the time of flight is approximately 1.48 μs, which is about 22% of a 300 MeV/c muon lifetime.

How to handle the decay products from AP0 if routed directly to the storage ring?

Time of flight from target



Beam Line Simulations

Generating a Gaussian distribution after the target station (with the Twiss parameters from *g-2*) transmits the same number of particles at 300 MeV/c and at 3100 MeV/c.

- 300 MeV/c: Currents had to be scaled down based on new momentum

Next step: Simulate a realistic distribution coming out from the target

- Not a complete model yet – work is in progress
- Results are encouraging based on the number of particles at 300 MeV/c
- Key items to look at are: optical properties of low momentum distribution, effect of magnet material (Li lens and Be-windows), impact of AIR around the target station, beamline Ti - windows

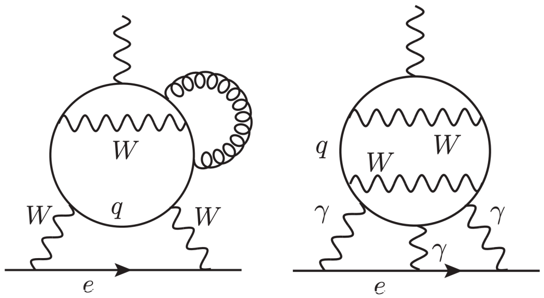
Quick Summary

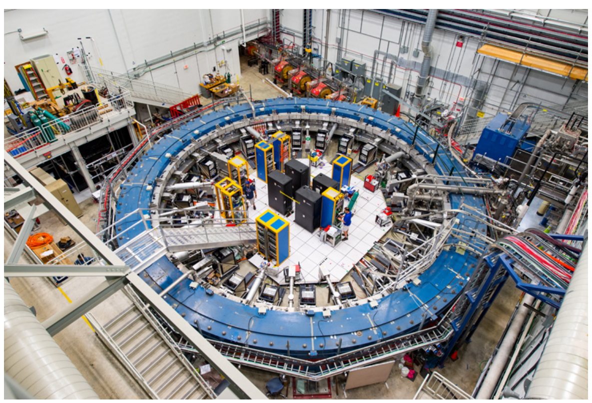
Parameter	Value	Unit
Muon Momentum	387	MeV/c
Magnetic Field	0.178	T
Radial Electric Field	-1.98	MV/m
Plate Separation	$\pm$ 35	mm
Plate Voltage	$\pm$ 69.283	kV
Quadrupole Gradient	TBD	MV/m/m
Central Orbit Radius	7112	mm
Radial Orbit Deviations	$\pm$ 10.9	mm
Ring Admittance (Horiz., central momentum)	153	π mm-mrad

Next up…

- What is the natural first-order focusing due to set of curved 'dipole' plates?
- What quadrupole gradient would we need? What tunes to choose (especially vertical tune)?
- What would be the expansion coefficients of the E-field due to plate distortions and misalignments? What are its effect on EDM measurement?
- How bad can the radial and azimuthal magnetic field be?
- To what accuracy must the E-field be measured in the ring? And how to do it?
- Detector related and other systematics.

Motivation for EDM at Fermilab and in the `g-2’ Ring





- A non-zero EDM value is an indication of combined CPV violation.
- EDM of a muon is heavily suppressed in the Standard Model unlike in few other BSM models - an excellent probe for new physics.
- The current muon EDM limit of $d_\mu < 1.8 \times 10^{-19}$ is the the only EDM of fundamental particle probed directly on the bare particle, that too done using the same `g-2’ storage ring!
- We have the combined wisdom of operating the `g-2’ storage ring for over two decades.

PIP-II Layout