Using Bayesian Inference to Study a Network of Atom Interferometers

MAGIS Simulation Meeting

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AION UNIVERSITY OF CAMBRIDGE MAGIS-100

Outline

1. Motivations

2. Bayesian Parameter Estimation in GWs

3. Future Plan

Motivations

1. Localizing gravitational wave (GWs) sources on the sky is crucial for realizing the full potential of GWs in Physics. Sky location of binary black holes (BBHs) can be used to inform LIGO/VIRGO before their observation.
2. Study the potential synergies of using two atomic interferometers (US: MAGIS-100 and UK: AION-100) on exploring mid-frequency GWs and ultra-light dark matter signals.
3. Adapt data analysis and parameter estimation techniques from laser interferometers to AIs, with a long duration GW signal before merger.

Sky Localization

$$\sqrt{\Omega_s} \propto (SNR \cdot \frac{R}{\lambda})^{-1}$$

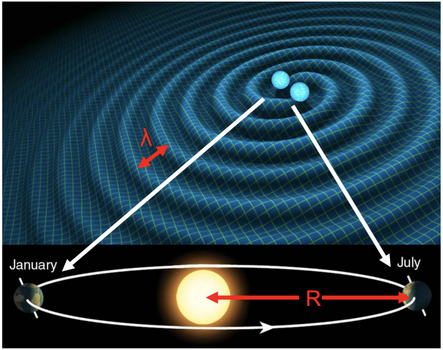


Figure: R. Hurt/Caltech-JPL; 2007 Thomson Higher Education

Outline

Bayesian Inference

A primary aim of Bayesian inference is to evaluate posterior distribution given data, parameters and a model. In the context of Bayesian inference, we are often interested in calculating posterior $p(\Theta|d, M)$ of a set of parameters Θ for a given model M given some data d.

$$p(\Theta|d, M) = \frac{p(d|\Theta, M)p(\Theta|M)}{p(d|M)} = \frac{\mathcal{L}(\Theta)\Pi(\Theta)}{Z},$$

where

$$p(d|\Theta, M) = \mathcal{L}(\Theta)$$

is the likelihood,

$$p(\Theta|M) = \Pi(\Theta)$$

is the prior, and Z is the evidence.

Likelihood

Consider a data set $d = [d_1, d_2, ..., d_n]$ given in frequency domain. In gravitational wave data analysis, we typically assume a Gaussian-noise likelihood in a single frequency bin j given parameters Θ as

$$\mathcal{L}(d_j|\Theta) = \frac{1}{2\pi P_j} \exp(-2\Delta f \frac{|d_j - h_j(f,\Theta)|^2}{P_j}),$$

where Δf is the frequency resolution, $P_j(f)$ is the noise power spectral density (PSD), and $h_j(f,\Theta)$ is the GW waveform in a single frequency bin.

$$\mathcal{L}(d|\Theta) = \prod_j \mathcal{L}(d_j|\Theta).$$

Evidence

The evidence acts as a normalization factor, but also can be used to determine if a real GW signal present by comparing different models.

$$Z = \int_{\Omega_\Theta} \mathcal{L}(\Theta)\Pi(\Theta)d\Theta.$$

Usually computed using sampling method (nested sampling or MCMC).

Waveform

The detector response tensor for AIs with baseline direction unit vector $a_i(t)$ is defined as [3]

$$D_{ij}(t) = a_i(t)a_j(t).$$

The GW strain tensor is given by

$$h_{ij}(t) = h_+(t)e_{ij}^+ + h_\times(t)e_{ij}^\times,$$

and the observed signal is

$$h(t) = D_{ij}h_{ij} = h_+(t)F_+(t) + h_\times(t)F_\times(t)$$

Sky information is encoded in $F_{+,\times}$, and $h_{+,\times}$ depends on luminosity distance, chirp mass and binary orbital inclination angle.

Waveform

The waveform $\tilde{h}$ in the frequency domain is the Fourier-Transform of the time-domain expression, given as [1]

$$\tilde{h}(f) = \sqrt{\frac{5}{96}} \frac{\sqrt{A_+^2 F_+^2 + A_\times^2 F_\times^2}}{D_L} \pi^{-2/3} \mathcal{M}_z^{5/6} f^{-7/6} e^{i\Psi(f)},$$

where the phase is

$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128}(\pi \mathcal{M}_z f)^{-5/3} - \phi_P - \phi_D + ...$$

The polarization phase ϕ_p comes from the reorientation of the detector baseline, which changes the relative angle between the baseline and the source direction.

The Doppler phase ϕ_D comes from the Doppler shift of the GW phase due to the non-linear motion of the detector.

Multiple Detectors

Suppose there are M frequency bins and N detectors, we have the following likelihood function

$$\mathcal{L}(d|\Theta) = \prod_j^M \prod_k^N \mathcal{L}(d_{j,k}|\Theta),$$

where $\mathcal{L}(d_{j,k}|\Theta)$ is the single likelihood in frequency bin j for an individual detector k.

Different initial location of baselines $\rightarrow$ different amplitudes and phases in the waveform $h_{j,k}$.

Strain Data Construction

$$\tilde{d}(f) = \tilde{h}(f) + \tilde{n}(f)$$

$$P(\tilde{n}_j) \propto e^{-\frac{|\tilde{n}_j|^2}{2\sigma_j^2}}, \sigma_j^2 \propto \frac{1}{2\Delta f} P_n(f)[2]$$

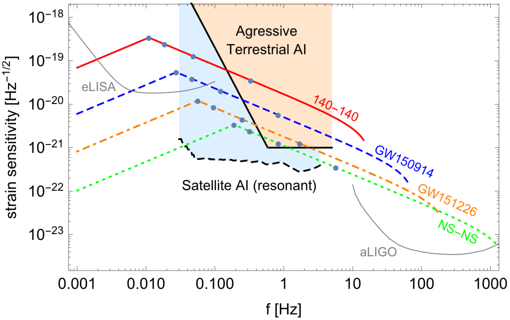
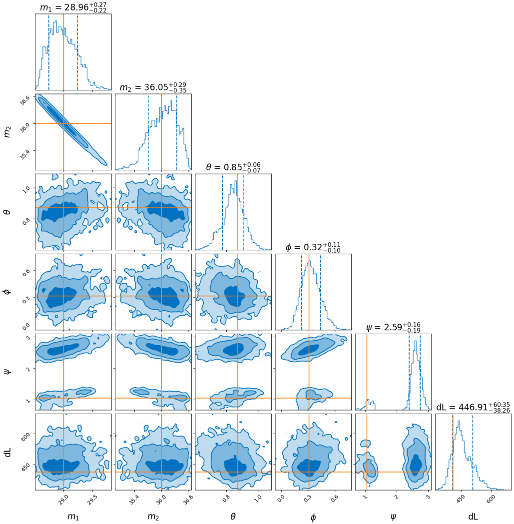


Figure: A simple strain sensitivity from [3]

Corner Plots and Posterior

Parameters: binary masses (m_1, m_2, $M_\odot$), source direction ($\mathbf{n} = \theta$, ϕ, rad), GW polarization (ψ, rad), and luminosity distance (dL, Mpc).



Outline

Future plan

1. $e^{+,\times}$ for each detector (preferred basis).
2. Time delay.
3. Use MAGIS-100 and AION-100 locations.
4. Use sensitivity curve from MAGIS-100 and AION-100.
5. Strain data simulation with stochastic GW background (later story).

References I

[1] Curt Cutler. "Angular Resolution of the LISA Gravitational Wave Detector". en. In: *Physical Review D* 57.12 (June 1998). arXiv:gr-qc/9703068, pp. 7089–7102. ISSN: 0556-2821, 1089-4918. DOI: 10.1103/PhysRevD.57.7089. URL: http://arxiv.org/abs/gr-qc/9703068 (visited on 11/07/2022).

[2] Curt Cutler et al. "The Last Three Minutes: Issues in Gravitational Wave Measurements of Coalescing Compact Binaries". en. In: *Physical Review Letters* 70.20 (May 1993). arXiv:astro-ph/9208005, pp. 2984–2987. ISSN: 0031-9007. DOI: 10.1103/PhysRevLett.70.2984. URL: http://arxiv.org/abs/astro-ph/9208005 (visited on 11/07/2022).

[3] Peter W. Graham and Sunghoon Jung. "Localizing Gravitational Wave Sources with Single-Baseline Atom Interferometers". en. In: *Physical Review D* 97.2 (Jan. 2018). arXiv:1710.03269 [astro-ph, physics:gr-qc, physics:physics], p. 024052. ISSN: 2470-0010, 2470-0029. DOI: `10.1103/PhysRevD.97.024052`. URL: `http://arxiv.org/abs/1710.03269` (visited on 10/14/2022).