Magnetometry for Precision Muon Physics

Dave Kawall, University of Massachusetts Amherst

With thanks to muon g-2 and KEK collaborators

Muons in Minneapolis Workshop, Dec 6, 2023

U.S. DEPARTMENT OF
ENERGY
Office of
Science

Magnetometry for Precision Muon Physics: Muon g-2

- How will we do 10x better on muon g-2, or muon mass, or muon magnetic moment?
- Consider case of Fermilab muon g-2

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c \quad = \quad \frac{e}{m_\mu c}\left[a_\mu \vec{B} - \left(a_\mu - \left[\frac{m_\mu c}{p}\right]^2\right)\vec{\beta}\times\vec{E} - a_\mu\left(\frac{\gamma}{\gamma+1}\right)\left(\vec{\beta}\cdot\vec{B}\right)\vec{\beta}\right]$$

$$\Rightarrow a_\mu \quad \approx \quad \frac{\omega_a}{B}\left[\frac{m_\mu c}{e}\right]$$

- Maybe determine a_μ to 130 ppb, of which at least 50 ppb will be from B
- How will we ever get a_μ to 15 ppb?
- **Will need significant improvements in absolute magnetometry – few ppb level**

Magnetometry for Precision Muon Physics: Muonium

- Ground-state hyperfine interval $\Delta\nu$ good test of bound state QED
- Size of $\Delta\nu$ depends on of electron/muon mass ratio:

$$\Delta\nu(\text{theory}) \approx \frac{8\pi}{3} g'_e \mu_B g'_\mu \mu_B^\mu \frac{1}{\pi a_{\mu e}^3} \approx 4\ 463\ \text{MHz}$$

$$= \underbrace{\frac{16}{3}\,\alpha^2\, c\, R_\infty\, \frac{m_e}{m_\mu}\left[1+\frac{m_e}{m_\mu}\right]^{-3}}_{\mathrm{E_{Fermi}}} \underbrace{(1+\Delta)}_{\text{Higher Order}}$$

- Can measure $\Delta\nu$, solve for m_e/m_μ
- Gives precise result, but depends on theory

- Muonium $1^2S_{1/2}$ ground-state in B field described by Breit-Rabi Hamiltonian

$$H = h\Delta\nu \vec{I}\cdot\vec{J} + g_j\mu_B^e\vec{J}\cdot\vec{B} - g'_\mu\mu_B^\mu\vec{I}\cdot\vec{B}$$

- Substate energy eigenvalues well-known

$$E_{F=\frac{1}{2}\pm\frac{1}{2},M_F} = -\frac{1}{4}h\Delta\nu - g'_\mu\mu_B^\mu M_F B \pm \frac{1}{2}h\Delta\nu\sqrt{1+2M_F x+x^2}$$

where $x = \frac{\left(g_j\mu_B^e + g'_\mu\mu_B^\mu\right)B}{h\Delta\nu}$ measure of Zeeman splittings compared to hyperfine

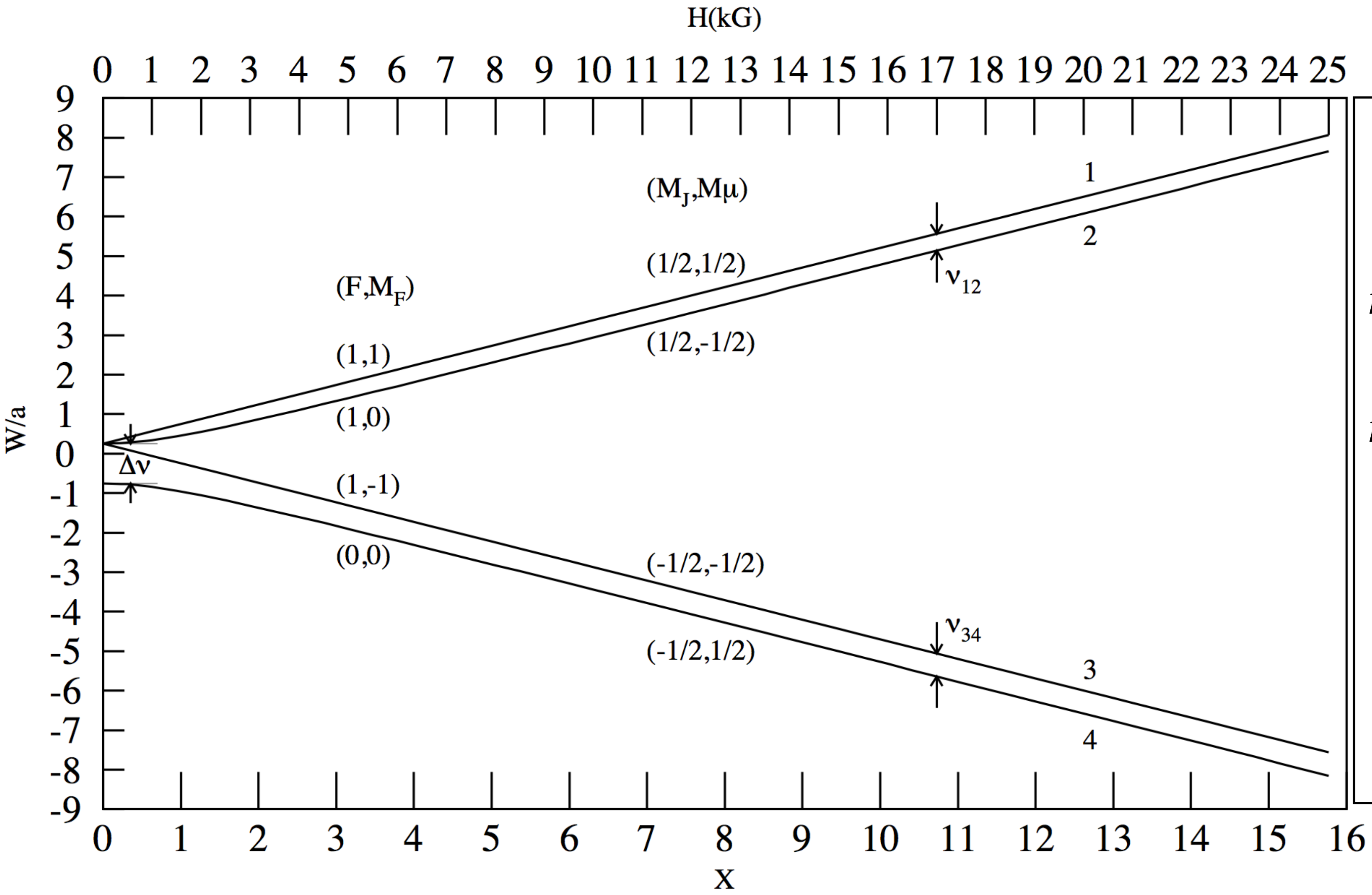
---

$$\nu_{12} = \frac{E_{1,1}-E_{1,0}}{h} = -\frac{g'_\mu\mu_B^\mu B}{h} + \frac{\Delta\nu}{2}\left[(1+x)-\sqrt{1+x^2}\right] \approx \frac{\Delta\nu}{2} - \frac{g'_\mu\mu_B^\mu B}{h}$$

$$\nu_{34} = \frac{E_{1,-1}-E_{0,0}}{h} = +\frac{g'_\mu\mu_B^\mu B}{h} + \frac{\Delta\nu}{2}\left[(1-x)+\sqrt{1+x^2}\right] \approx \frac{\Delta\nu}{2} + \frac{g'_\mu\mu_B^\mu B}{h}$$

- In high field ν_{12} and ν_{34} look like muon spin-flip transitions

- Extract $\Delta\nu$ from Zeeman splittings in high B field of ground-state muonium, **need *relative* B**
  (just need to measure ν_{12} and ν_{34} at same B)
- Splittings directly sensitive to muon magnetic moment (theory independent), **but need *absolute* B**
  (must measure ($\nu_{34}-\nu_{12}$) and B to extract $g_\mu\mu_B$)



In strong field x >> 1

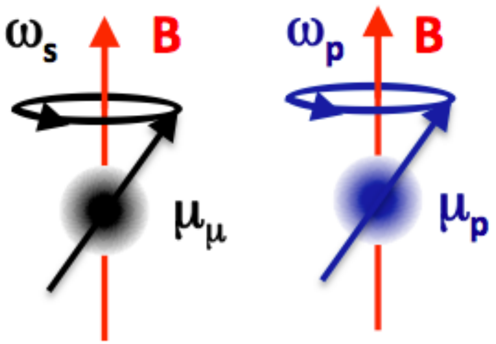
$$(M_J, M_\mu) \Longleftrightarrow (M'_J, M'_\mu)$$

$$\nu_{12} \approx \left(\frac{1}{2}, \frac{1}{2}\right) \Longleftrightarrow \left(\frac{1}{2}, -\frac{1}{2}\right)$$

$$\nu_{34} \approx \left(-\frac{1}{2}, -\frac{1}{2}\right) \Longleftrightarrow \left(-\frac{1}{2}, \frac{1}{2}\right)$$

$$\begin{aligned} \nu_{12} &\approx \Delta\nu/2 - g'_\mu \mu^\mu_B B/h \\ \nu_{34} &\approx \Delta\nu/2 + g'_\mu \mu^\mu_B B/h \\ \Delta\nu &= \nu_{12} + \nu_{34} \\ \nu_{34} - \nu_{12} &\approx 2g'_\mu \mu^\mu_B B/h \end{aligned}$$

Magnetometry for Precision Muon Physics: Absolute Field Measurement with Pulsed NMR



$$
\begin{aligned}
\omega_p &= \gamma_p B \\
\gamma_p &= 42.577\,478\,518(18)\ \mathrm{MHz/T}\ (0.42\ \mathrm{ppb}) \\
\omega_e &= \gamma_e B \\
\gamma_e &= 28\,024.951\,4242(85)\ \mathrm{MHz/T}\ (0.30\ \mathrm{ppb}) \\
\omega_n &= \gamma_n B \\
\gamma_n &= 29.164\,6931(69)\ \mathrm{MHz/T}\ (240\ \mathrm{ppb})
\end{aligned}
$$

Shielded ^{3}He

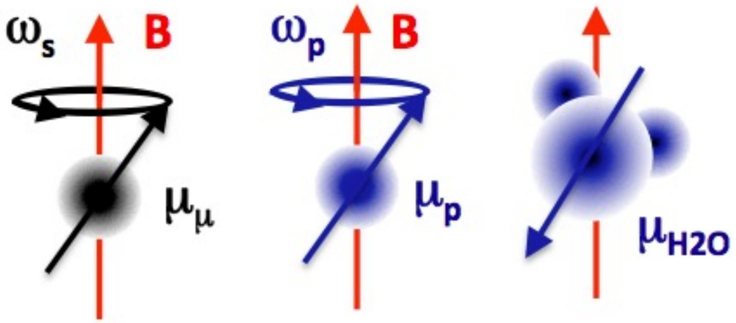
$$
\begin{aligned}
\omega'_h &= \gamma'_h B \\
\gamma'_h &= 32.434\,099\,42(38)\ \mathrm{MHz/T}\ (12\ \mathrm{ppb})
\end{aligned}
$$

Shielded p in H_2O

$$
\begin{aligned}
\omega'_p &= \gamma'_p B \\
\gamma'_p &= 42.576\,384\,74(46)\ \mathrm{MHz/T}\ (10.5\ \mathrm{ppb})
\end{aligned}
$$

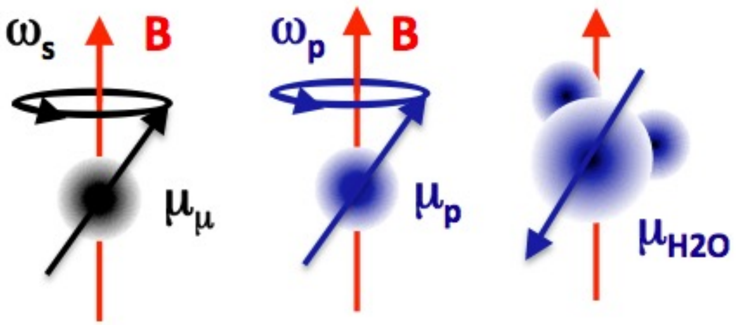
Proton: G. Schneider et al., Science **358**, 1081 (2017)

Absolute Field Measurement with Pulsed NMR



$$\begin{aligned}\omega_p &= \gamma_p B \quad (< 1\,\text{ppb}) \\ \omega_p' &= \gamma_p' B \quad (11\,\text{ppb})\end{aligned}$$

Absolute Field Measurement with Pulsed NMR

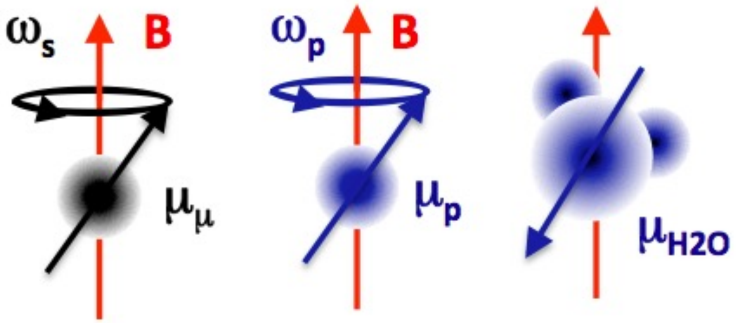


$$
\begin{aligned}
\omega_p &= \gamma_p B \quad (< 1\,\text{ppb}) \\
\omega'_p &= \gamma'_p B \quad (11\,\text{ppb})
\end{aligned}
$$

$$
\omega'_p(T_R) = \omega_p^{\text{probe}} \left[1 + \qquad\qquad\qquad\qquad \right]
$$

⇒ Determine B seen by muons from measurement of ω'_p of protons in spherical H_2O sample

Absolute Field Measurement with Pulsed NMR



$$\begin{aligned} \omega_p &= \gamma_p B \quad (< 1\,\text{ppb}) \\ \omega'_p &= \gamma'_p B \quad (11\,\text{ppb}) \end{aligned}$$

$$\omega'_p(T_R) = \omega_p^{\text{probe}} \left[1 + \frac{1}{\gamma'_p} \frac{d\gamma'_p}{dT} (T - T_R) \right]$$

⇒ Determine B seen by muons from measurement of ω'_p of protons in spherical H_2O sample

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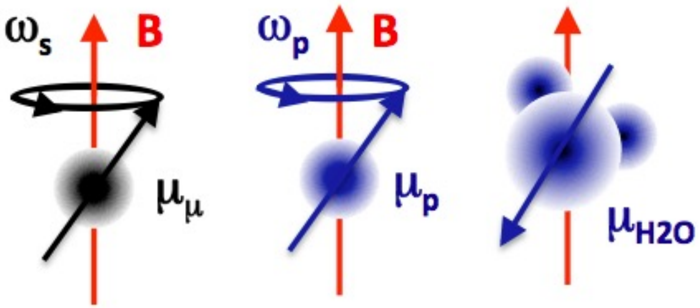
- Complication: Diamagnetic shielding of electrons screens protons, *changes local B*, is T dependent, 10.36 ppb/C

Absolute Field Measurement with Pulsed NMR



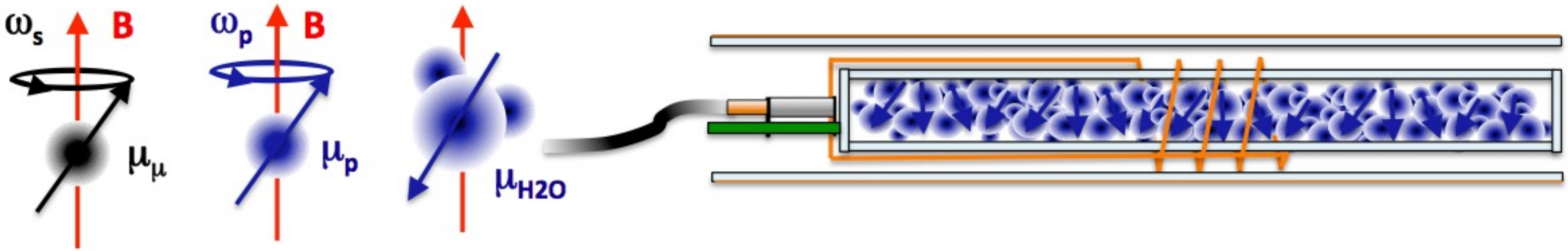
$$\omega_p'(T_R) \;=\; \omega_p^{\mathrm{probe}}\left[1+\frac{1}{\gamma_p'}\frac{d\gamma_p'}{dT}(T-T_R)+\left(\epsilon-\frac{4\pi}{3}\right)\chi_{\mathrm{H_20}}(T)\qquad\qquad\right]$$

⇒ Determine B seen by muons from measurement of ω_p' of protons in spherical H_2O sample

---

- Complication: Diamagnetic shielding of electrons screens protons, *changes local* B, is T dependent, 10.36 ppb/C
- Complication: magnetic susceptibility $\chi_{H_2O} \approx -0.721(2) \times 10^{-6}$ of water sample gives shape-dependent field perturbation: $\epsilon = 4\pi/3$ for a sphere, $\epsilon = 2\pi$ for cylinder $\perp \vec{B}$, 1.5 ppm shift !!!
- Complication: Very hard to make a spherical water sample
- Complication: Best measurement of $\chi_{\mathrm{H_2O}}$ comes from 1933! Working on update

Absolute Field Measurement with Pulsed NMR



$$\omega'_p(T_R) \;=\; \omega_p^{\text{probe}}\left[1+\frac{1}{\gamma'_p}\frac{d\gamma'_p}{dT}(T-T_R)+\left(\epsilon-\frac{4\pi}{3}\right)\chi_{\mathrm{H_20}}(T)\;+\;\delta_{\text{materials}}\right]$$

⇒ Determine B seen by muons from measurement of ω'_p of protons in spherical H_2O sample

---

- Complication: Diamagnetic shielding of electrons screens protons, *changes local* B, is T dependent, 10.36 ppb/C
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- Complication: Best measurement of $\chi_{\mathrm{H_2O}}$ comes from 1933! Working on update
- Complication: Magnetization of probe materials further perturbs field at site of protons

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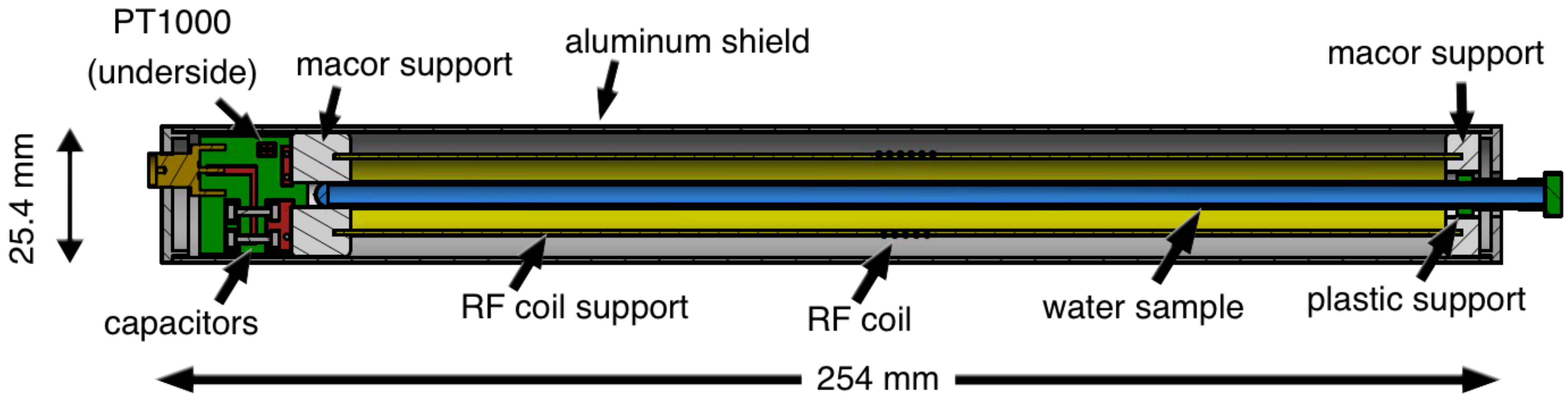
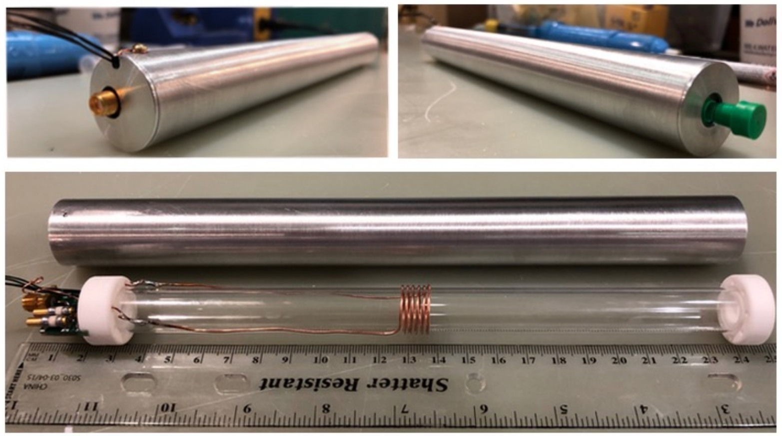
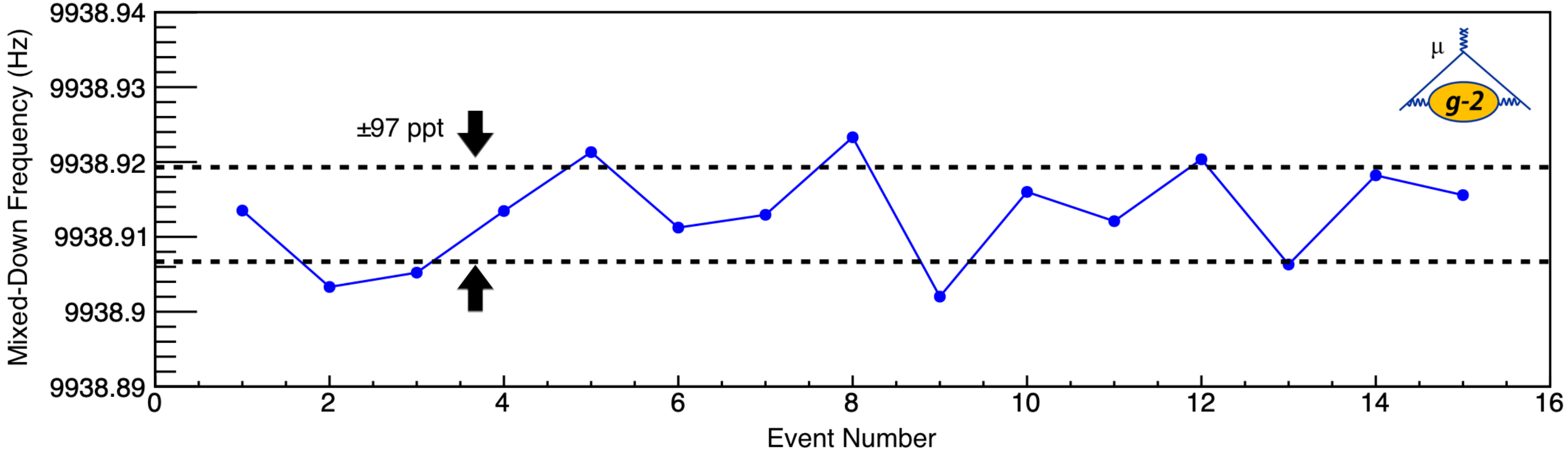
⇒ Determine total correction to 15 ppb accuracy or better using special calibration probes

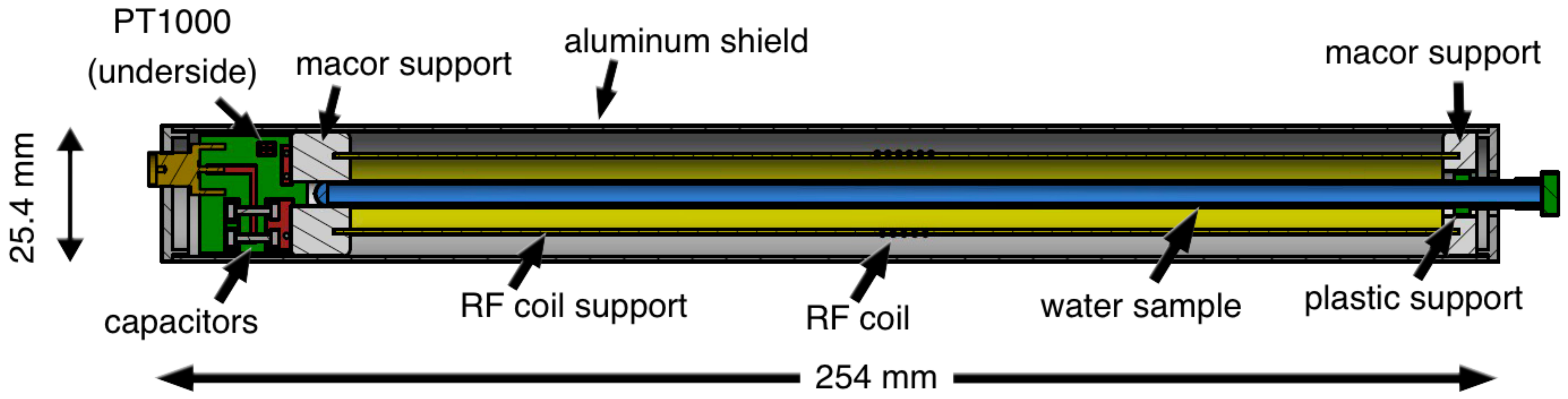
Absolute Field Measurement with Pulsed NMR







Issues with Absolute Field Measurement with Pulsed NMR



- RF coil is asymmetric, close, and source of field perturbation, radiation-damping currents perturb field
- Replace "zero"-susceptibility wire with precision copper/aluminum foils for less material, larger surface area, smaller R, smaller net-χ
- Cu and Al have opposite sign magnetic susceptibilities, combine to reduce image effects
- Replace aluminum shield with Al+Teflon combination for smaller net χ (too heavy for muon g-2)
- Reduce mass, move circuit board and other components farther away, make water sample longer, ...
- $\Delta f_{\text{rad}} = \frac{(f_0 - f_{LC})}{f_0} \frac{Q}{\pi\tau} \frac{M_z(t)}{M_0}$, $\tau \propto 1/$filling factor
- Can measure corrections at ppb level, try to keep sources of perturbation at few ppb level

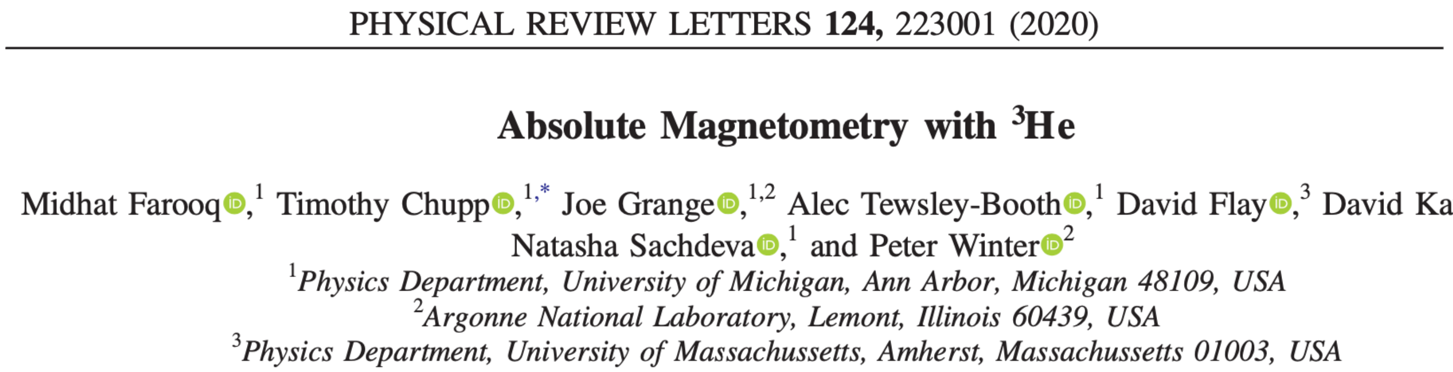
Issues with Absolute Field Measurement with Pulsed NMR

$$\omega_p = \gamma_p B = \omega_{p,\mathsf{H_2O}}^{\text{probe}} \left[1 + \sigma_{\mathsf{H_2O}}(T_{\mathsf{ref}}) + \frac{d\sigma}{dT}(T - T_{\mathsf{ref}}) + \left(\epsilon - \frac{4\pi}{3}\right)\chi_{\mathrm{H_20}}(T) + \delta_{\mathrm{materials}}\right]$$

where $\omega_p' = \omega_p \left[1 - \sigma_{\mathsf{H_2O}}(T)\right]$ defines the diamagnetic shielding σ

- Improve measurement of diamagnetic shielding σ below 10 ppb, always measure at $T_{ref} = 34.7$
- Use different atom/molecule for which σ is known much better 10 ppb: ^{3}He
- Keep probe materials far from sample, long cylinders, combinations of $\pm\chi$ to reduce $\delta_{\mathsf{materials}}$
- Use spherical sample $\epsilon = 4\pi/3$ to eliminate dependence on χ
- For few ppb uncertainty, need sphericity to few 10s of microns on cm scale object

Absolute Field Measurement with Pulsed NMR: Comparison of ^{3}He and Water

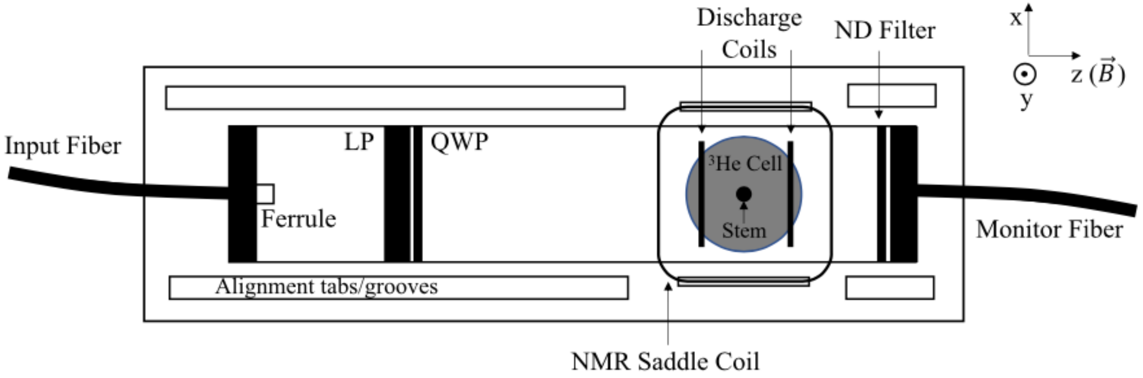


PHYSICAL REVIEW LETTERS **124,** 223001 (2020)

Absolute Magnetometry with ^{3}He

Midhat Farooq,[1] Timothy Chupp,[1,*] Joe Grange,[1,2] Alec Tewsley-Booth,[1] David Flay,[3] David Kawall,[3]
Natasha Sachdeva,[1] and Peter Winter[2]
[1]*Physics Department, University of Michigan, Ann Arbor, Michigan 48109, USA*
[2]*Argonne National Laboratory, Lemont, Illinois 60439, USA*
[3]*Physics Department, University of Massachussetts, Amherst, Massachussetts 01003, USA*

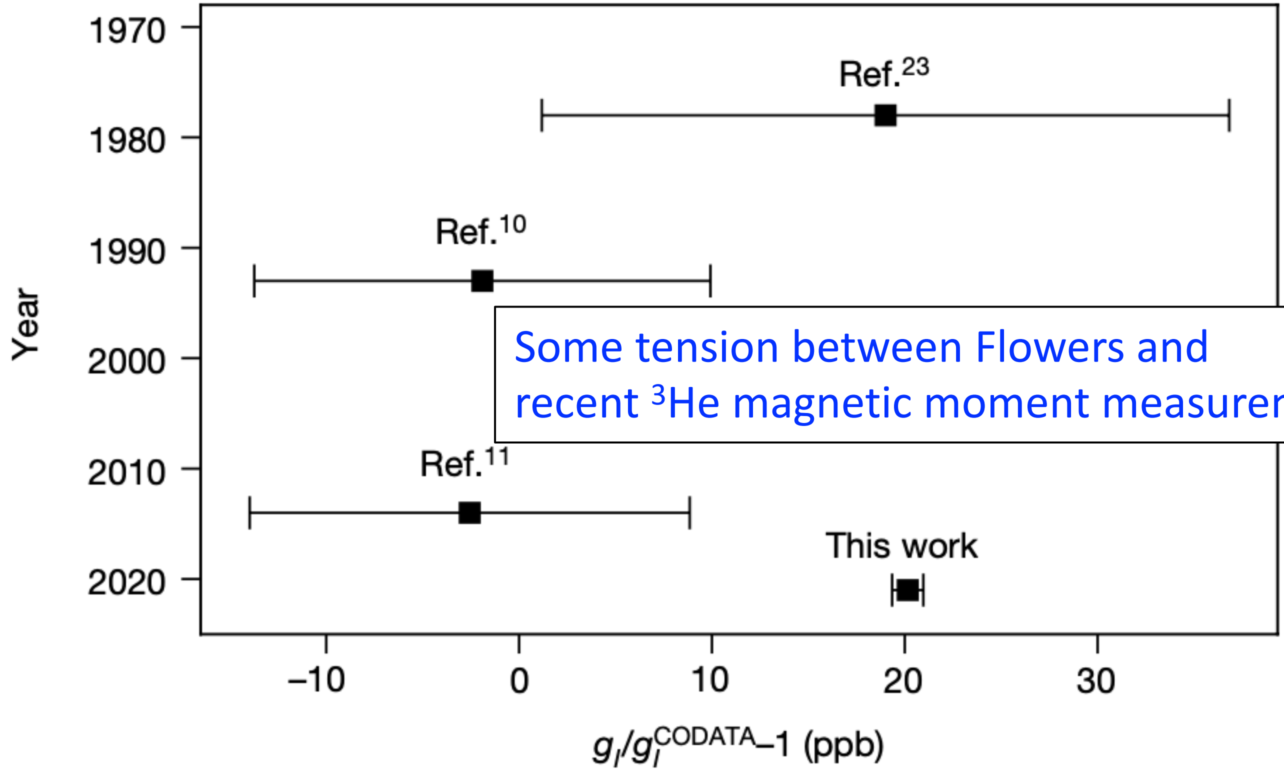
$$
\begin{aligned}
h\nu'_{\mathrm{H_2O}} &= 2g_p\mu_N\left(1-\sigma_{\mathrm{H_2O}}\right)B \\
h\nu'_{^3\mathrm{He}} &= 2g_3\mu_N\left(1-\sigma_{^3\mathrm{He}}\right)B
\end{aligned}
$$



- $\sigma_{\mathrm{H_2O}}(25°C) = 25\,689(11)\times10^{-9}$ (W.D. Phillips, W.E.Cooke, and D. Kleppner, Metrologia **13**, 179 (1977))
- $\sigma_{\mathrm{H_2O}}(25°C) = 25\,680(2.5)\times10^{-9}$ known to 2.5 ppb (Yu.I.Neronov and N.N. Seregin, Metrologia **51**, 54 (2014))
- $\sigma_{^3\mathrm{He}} = 59\,967.43(10)\times10^{-9}$, "perfectly" known, calculated Rudzinski, Puchalski, Pachucki
- $\mu'_h/\mu'_p = -0.761\,786\,1313(33)$ known to 4.3 ppb, J.L. Flowers, B.W. Petley and M.G. Richards, Metrologia **30**, 75 (1993)
- Relative comparison of water and ^{3}He to 4.3 ppb, much different systematics
- New measurement of ^{3}He moment differs from previous. Hope to check at 5-10 ppb level or better

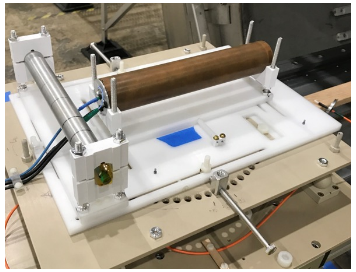
Absolute Field Measurement with Pulsed NMR: Comparison of ^{3}He and Water

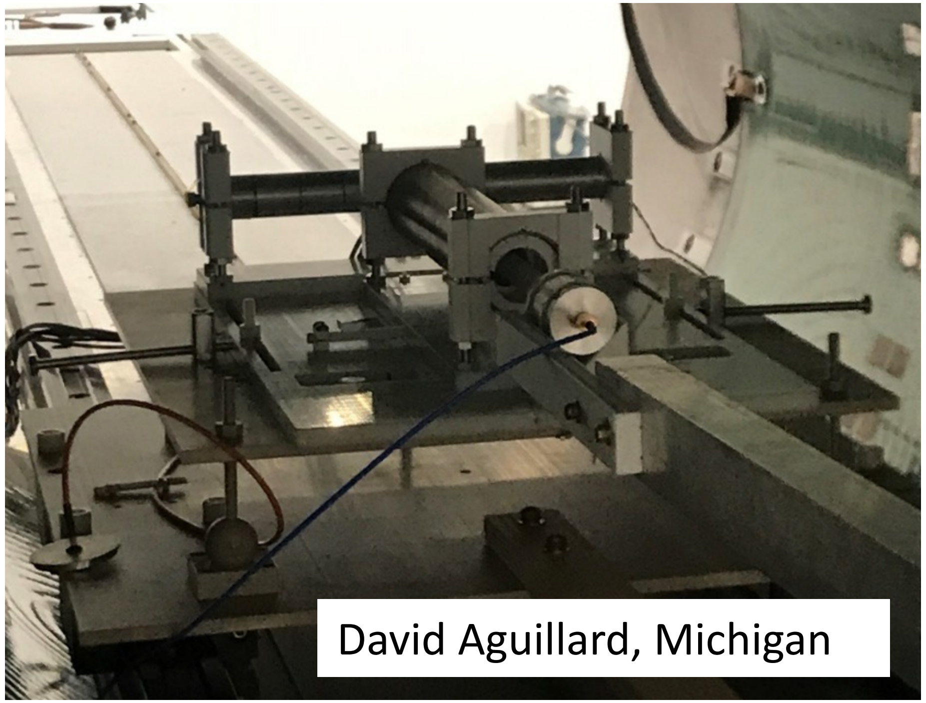
A. Schneider et al., ''Direct Measurement of the ^{3}He$^+$ Magnetic Moments", Nature **606**, 878 (2022)



Some tension between Flowers and recent ^{3}He magnetic moment measurement

Fig. 4 | History of ^{3}He nuclear *g*-factor determinations. Comparison of previous measurements of the bare nuclear *g*-factor g_I of ^{3}He and the value given in this work. All previous results were derived from comparisons of the NMR frequency of ^{3}He to that of water or molecular hydrogen. All error bars correspond to the 1σ confidence interval (68%).





David Aguillard, Michigan

- -Working at Argonne on a precision ^{3}He vs water probe comparison, with spherical and cylindrical water samples
- -Below 10 ppb would be great
- -W. Heil at Mainz has pushed resolution

Volume Magnetic Susceptibility of Water

$$\omega_p'(T_R) \;=\; \omega_p^{\text{probe}}\left[1+\frac{1}{\gamma_p'}\frac{d\gamma_p'}{dT}(T-T_R)+\left(\epsilon-\frac{4\pi}{3}\right)\chi_{\mathrm{H_20}}(T)\;+\;\delta_{\text{materials}}\right]$$

- For a cylindrical sample, $\epsilon = 1/2$ and $\left(\epsilon-\frac{1}{3}\right)\chi_{\mathrm{H_2O}} \approx 1\,500$ ppb correction

---

P.J. Mohr and B.N. Taylor: CODATA recommended values, Rev. Mod. Phys **72**, 351 (2000)

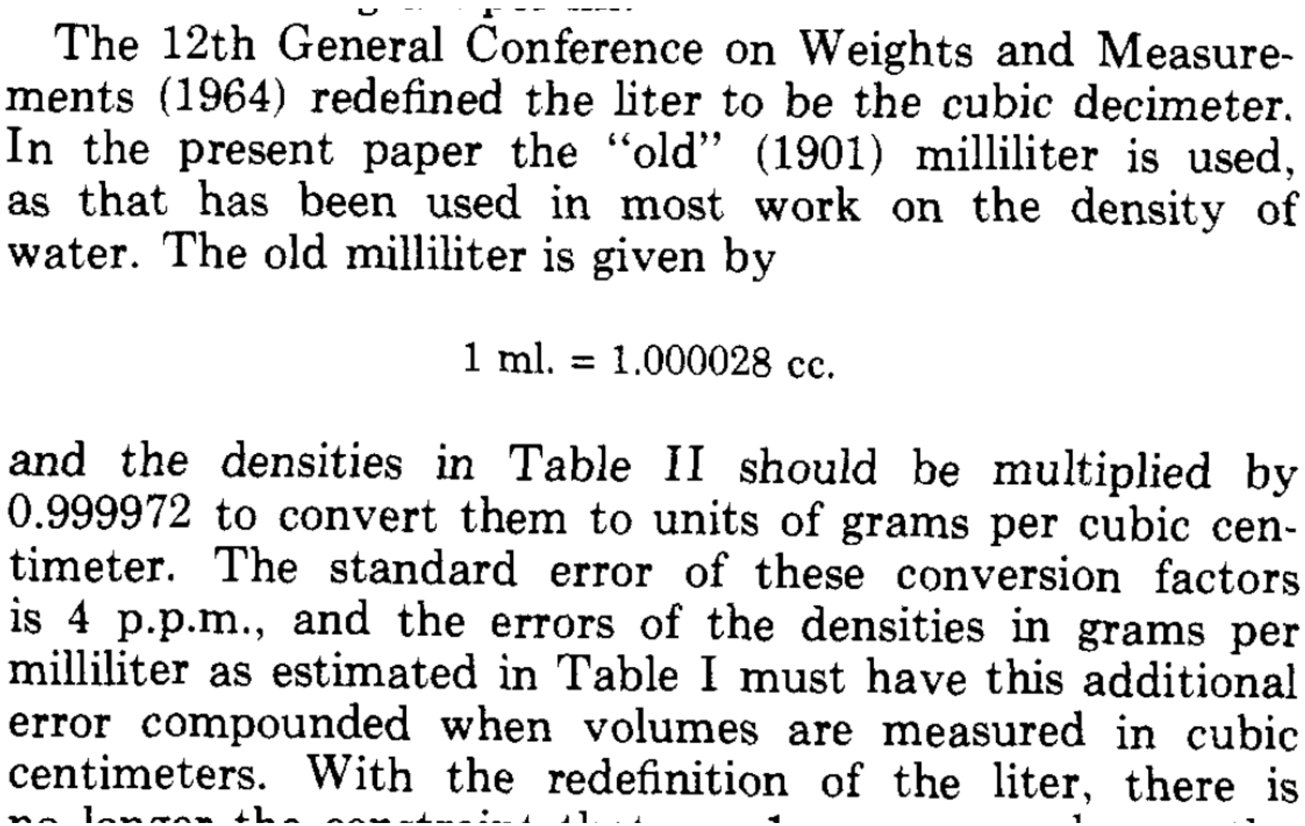
``where χ(22 °C)= -9.0559(61) x 10^{-6} [0.067 %] is the volume magnetic susceptibility of water at 22°C. This value of χ(*t*) is the mass susceptibility result of Auer (1933) corrected to 22°C using the H_2O mass susceptibility versus temperature data of Philo and Fairbank (1980) and converted to a volume susceptibility using the H_2O mass density vs. temperature data of Patterson and Morris (1994). We have also corrected the result of Auer for the accepted difference between the international ampere, which he used in his experiment as a unit to express the values of currents, and the SI ampere (Hamer, 1965). ”

J.S. Philo and W.M. Fairbank, J. Chem. Phys. **72**, 4429 (1980)

``The ratio of the water signal at each temperature to that at 20°C was taken, and this volume susceptibility ratio converted to a mass susceptibility ratio using the p-V-T formulas of Kell[12] and the vapor pressure of water. water.[13] Susceptibility *ratios* are reported because the absolute value for the susceptibility of water is poorly known.[14]”

Volume Magnetic Susceptibility of Water

- International temperature scales revised: ITS-1927, IPTS-1948, IPTS-1968, ITS-1990
- Some uncertainty in water isotopic composition and density depending on preparation method (distillation)
- $\chi_m = \chi/\rho$, definition of cc, ml changed over time
- G. Kell, "Precise Representation of Volume Properties of Water at One Atmosphere", J. Chem. Eng. Data **12**, 66 (1967).



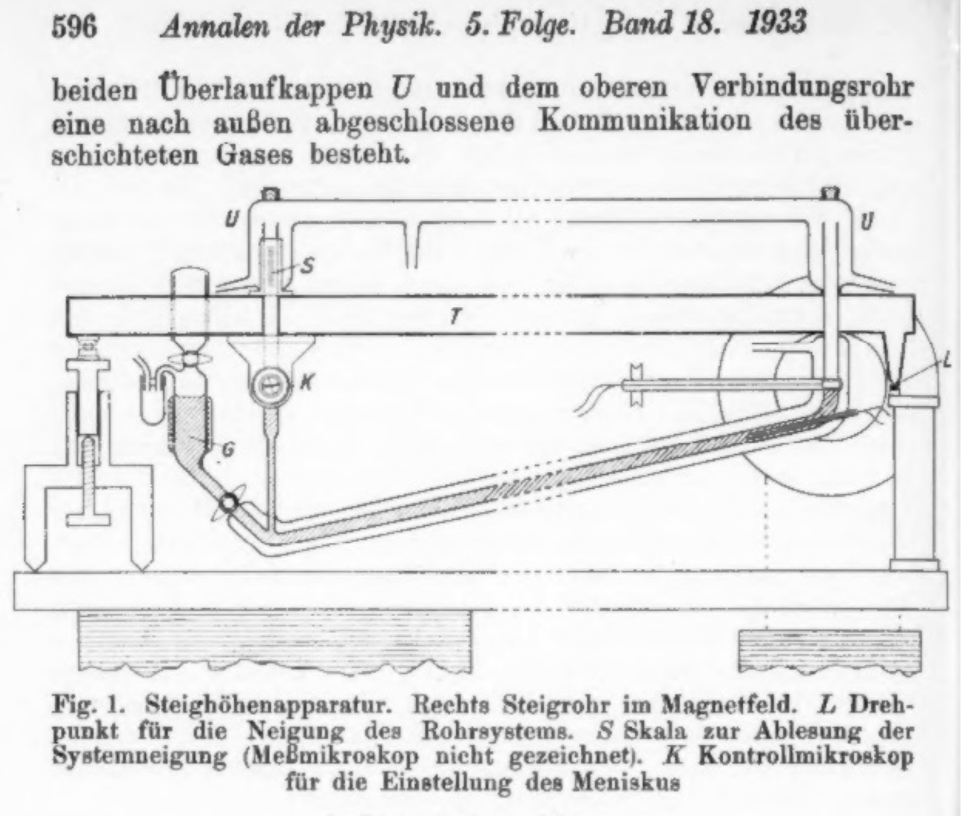
The 12th General Conference on Weights and Measurements (1964) redefined the liter to be the cubic decimeter. In the present paper the "old" (1901) milliliter is used, as that has been used in most work on the density of water. The old milliliter is given by

1 ml. = 1.000028 cc.

and the densities in Table II should be multiplied by 0.999972 to convert them to units of grams per cubic centimeter. The standard error of these conversion factors is 4 p.p.m., and the errors of the densities in grams per milliliter as estimated in Table I must have this additional error compounded when volumes are measured in cubic centimeters. With the redefinition of the liter, there is

Volume Magnetic Susceptibility of Water

H. Auer, Ann. Phys. Leipzig **18**, 593 (1933)



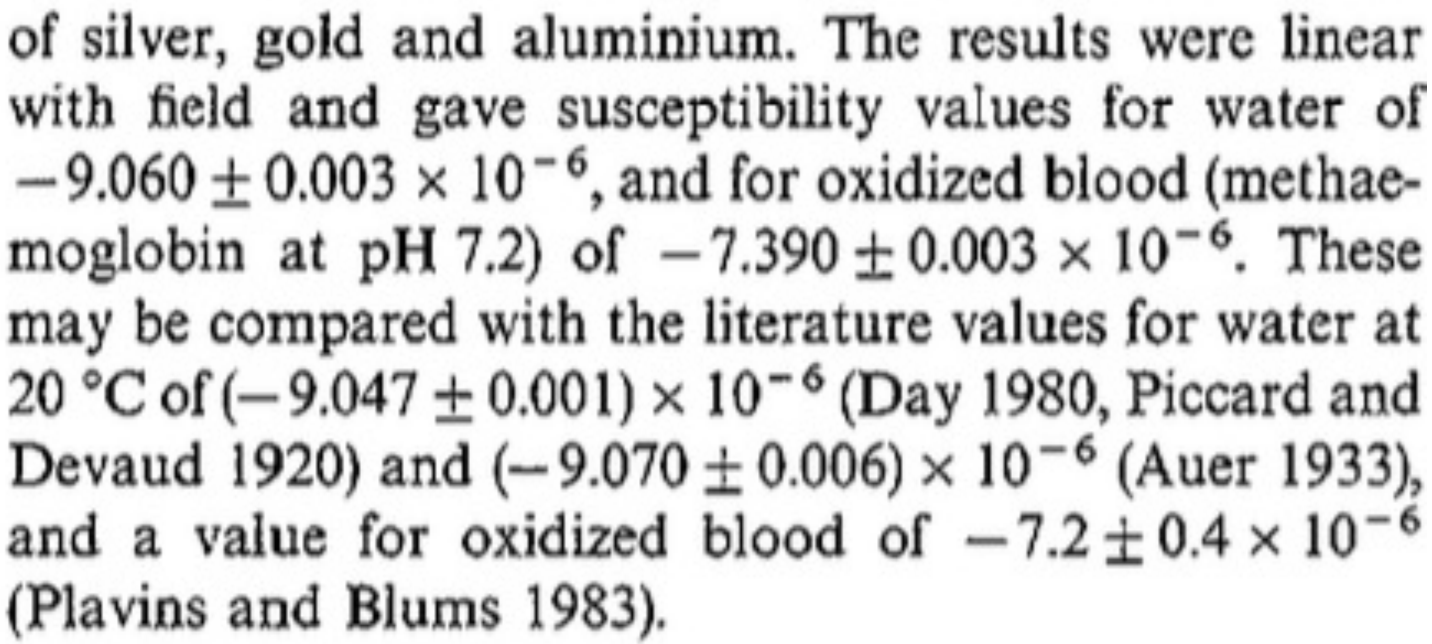
596 *Annalen der Physik. 5. Folge. Band 18. 1933*

beiden Überlaufkappen *U* und dem oberen Verbindungsrohr eine nach außen abgeschlossene Kommunikation des überschichteten Gases besteht.

Fig. 1. Steighöhenapparatur. Rechts Steigrohr im Magnetfeld. *L* Drehpunkt für die Neigung des Rohrsystems. *S* Skala zur Ablesung der Systemneigung (Meßmikroskop nicht gezeichnet). *K* Kontrollmikroskop für die Einstellung des Meniskus

- Looked at distortion of water meniscus from magnetic forces

B H Blott and G J Daniell, ‘‘The determination of magnetic moments of extended samples in a SQUID magnetometer”, 1993 *Meas. Sci. Technol.* **4** 462 (1993)



of silver, gold and aluminium. The results were linear with field and gave susceptibility values for water of $-9.060 \pm 0.003 \times 10^{-6}$, and for oxidized blood (methaemoglobin at pH 7.2) of $-7.390 \pm 0.003 \times 10^{-6}$. These may be compared with the literature values for water at 20 °C of $(-9.047 \pm 0.001) \times 10^{-6}$ (Day 1980, Piccard and Devaud 1920) and $(-9.070 \pm 0.006) \times 10^{-6}$ (Auer 1933), and a value for oxidized blood of $-7.2 \pm 0.4 \times 10^{-6}$ (Plavins and Blums 1983).

- Blott measurement at unknown temperature
- Schenck: $\chi(20^{0}\mathrm{C}) = -9.032 \text{ x } 10^{-6}$,

J.F. Schenck, “The role of magnetic susceptibility in magnetic resonance imaging: MRI magnetic compatibility of the first and second kinds”, Med. Phys. **23**, 815 (1996).

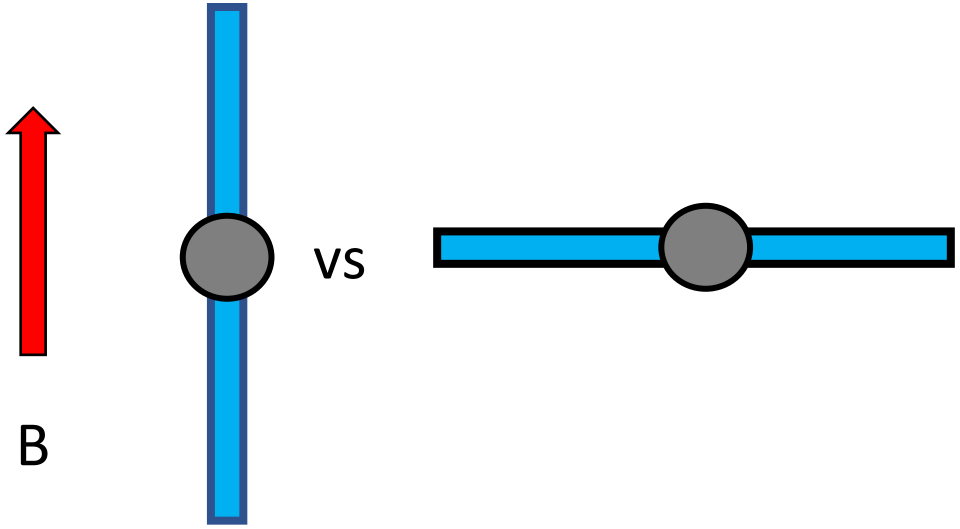
So what is the problem?

- Maybe nothing, but it certainly seems like a lot is based on a 1933 measurement
- Non-trivial corrections required as definitions changed
- Measurements not in great agreement
- Note that Auer found temperature dependence of χ of 2.9 x $10^{-4}/^{0}C$ at $5^{0}C$ to 0.62 x $10^{-4}/C$ at $70^{0}C$. Philo and Fairbank found 1.38810 x $10^{-4}/^{0}C$ at $20^{0}C$, and very small quadratic term, disagrees with Auer
- Using $\chi_{\mathrm{H_2O}} = -9.0559 \times 10^{-6}$, $\left(\frac{1}{2} - \frac{1}{3}\right)\chi_{\mathrm{H_2O}} \;=\; 1\,509$ ppb
- Using $\chi_{\mathrm{H_2O}} = -9.032 \times 10^{-6}$, $\left(\frac{1}{2} - \frac{1}{3}\right)\chi_{\mathrm{H_2O}} \;=\; 1\,505$ ppb
- People will question ***everything*** if we find a result different from Standard Model
- Would be good to have another measurement of χ
- **Required** if we want progress in magnetometry, precision muon physics

Volume Magnetic Susceptibility of Water

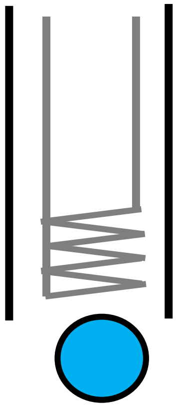
- Possible measurement technique: $\frac{\Delta\nu}{\nu} = \left(\frac{1}{2}\sin^2(\phi) - \frac{1}{3}\right)(\chi_{\mathrm{H_2O}} - \chi_{\mathrm{air}})$ where ϕ=angle between cylinder and B
- Cylinder of water parallel and perpendicular to B yields fractional change in Larmor frequency:

$$\delta\left[\frac{\Delta\nu}{\nu}\right] = \frac{1}{2}\chi_{\mathrm{H_2O}} \approx 4\,500 \text{ ppb effect}$$

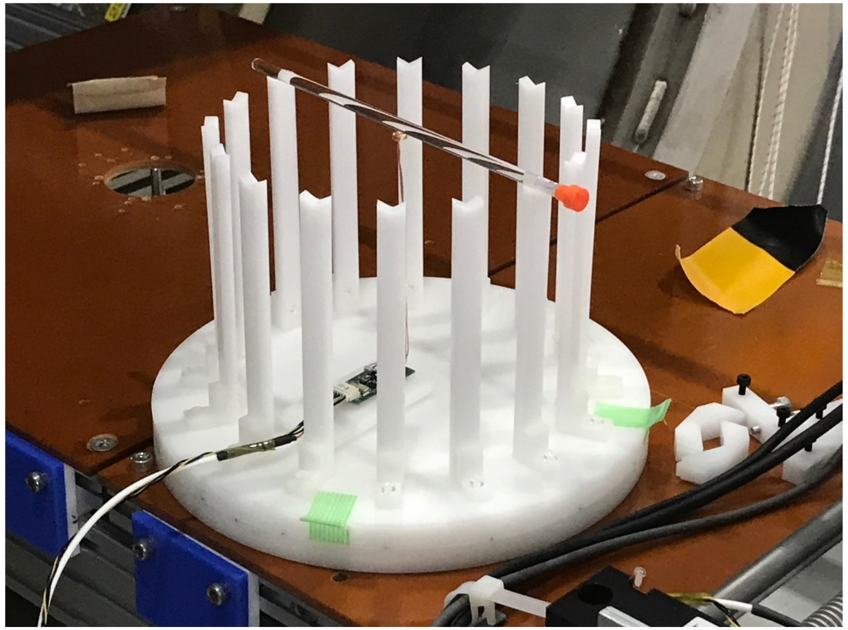


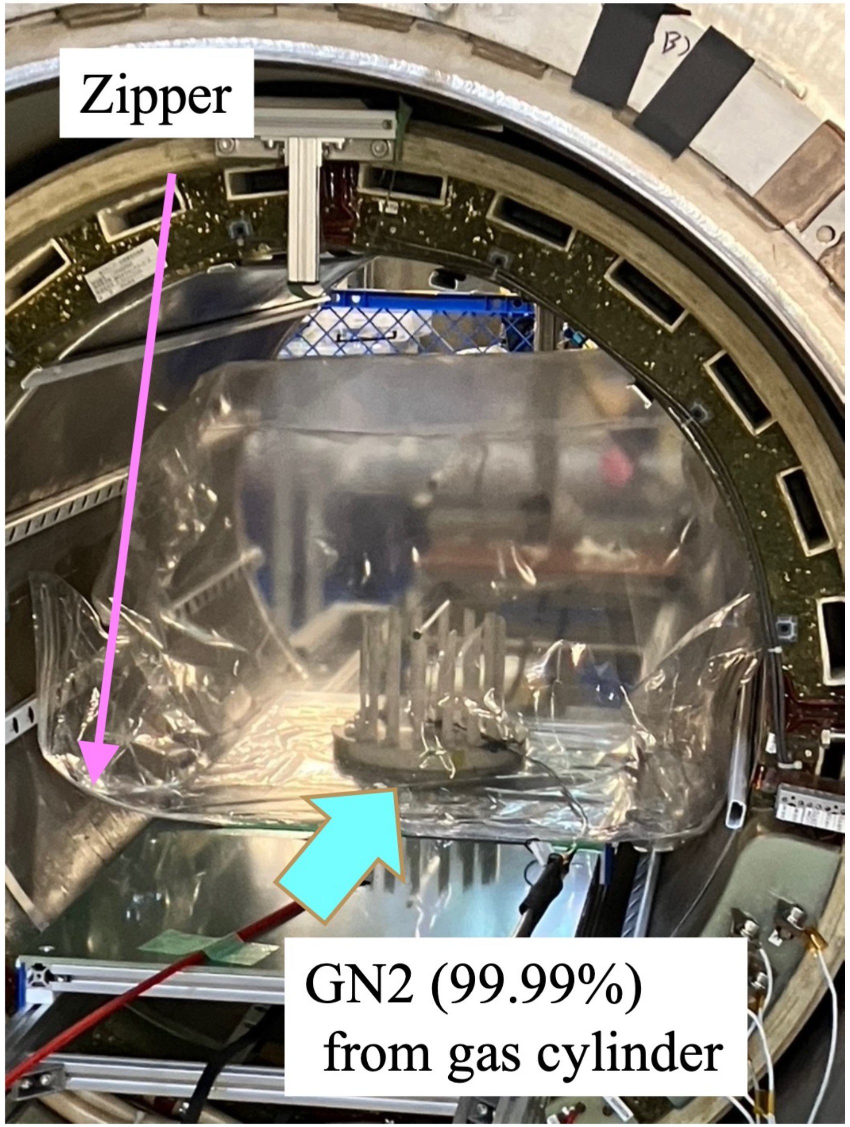
Top View

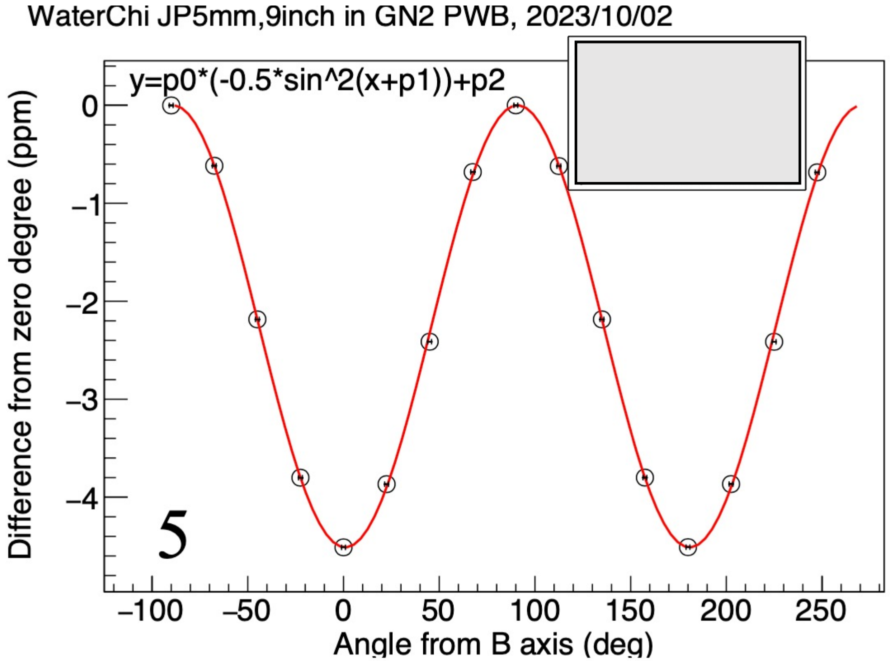
- Need a special NMR probe with open end that sits above rotating cylinder of water



Volume Magnetic Susceptibility of Water: **Ken-ichi Sasaki at KEK/J-PARC**







Technique looks promising

Magnetometry for Precision Muon Physics

- Should look ahead to next generation of experiments
- Target absolute B-field calibration standards at few ppb level
- Many other changes required in experiment design to make B-field measurements at 5-10 ppb level
- No-iron, persistent mode superconducting g-2 magnet?