Muon Spin Force

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Based on 2308.01356 with Y.Ema and M.Pospelov

Outline

- Muon spin dynamics
  - Why do we care about a force acting on the muon spin
  - Where could this force come from
- Indirect constraints on muon spin force
  - Experiments on nuclear spin force
  - Nuclear spin force as indirect constraints on muon spin force
- A suggestion for direct detection
- Indirect constraints on muon EDM
- Summary

Spin precession in storage ring experiments

In the muon's rest frame

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t} = \boldsymbol{s} \times (\mu_\mu \boldsymbol{B})$$

In storage rings, the observable is the muon's precession frequency relative to its direction of motion

$$\boldsymbol{\Omega}_s - \boldsymbol{\Omega}_\beta = \frac{e}{m_\mu}\left[a_\mu \boldsymbol{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\boldsymbol{\beta}\cdot\boldsymbol{B})\,\boldsymbol{\beta} + \boldsymbol{\beta} \times \left(\frac{1}{\gamma^2-1} - a_\mu\right)\boldsymbol{E}\right]$$

- a_μ can be easily observed if $\frac{e}{m_\mu} a_\mu \boldsymbol{B}$ is the only contributing term
- $\boldsymbol{\beta}\cdot\boldsymbol{B} \approx 0$ in storage ring setup
- BNL and FNAL choose $\gamma \approx 29.3$ to cancel the last term
- A new force acting on the muon spin would mess things up

Contribution to precession frequency from a muon spin force

A muon spin force $H = \Delta E_{(\mu)} \mathbf{n} \cdot \boldsymbol{s}$ would modify the spin dynamics

$$\left(\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}\right)_{\text{rest frame}} = \boldsymbol{s} \times (\mu \boldsymbol{B} - \Delta E_{(\mu)} \mathbf{n})$$

$$\boldsymbol{\Omega}_s - \boldsymbol{\Omega}_\beta = \frac{e}{m_\mu}\left[a_\mu \boldsymbol{B} - a_\mu \left(\frac{\gamma}{\gamma+1}\right)(\boldsymbol{\beta}\cdot\boldsymbol{B})\boldsymbol{\beta} + \boldsymbol{\beta} \times \left(\frac{1}{\gamma^2-1} - a_\mu\right)\boldsymbol{E} - \frac{m_\mu}{\gamma e}\Delta E_{(\mu)} \mathbf{n}\right]$$

The observed a_μ would be shifted by $|\Delta a_\mu| = m_\mu \Delta E_{(\mu)}/(\gamma e B)$ if $\mathbf{n}$ is parallel to $\boldsymbol{B}$

In the FNAL magnetic field $B = 1.45$T,

$$\boxed{\Delta a_\mu \simeq 2.5 \times 10^{-9} \quad \Longleftrightarrow \quad |\Delta E_{(\mu)}| = 6 \times 10^{-14}\text{eV}}$$

Monopole-dipole interaction as origin of the muon spin force

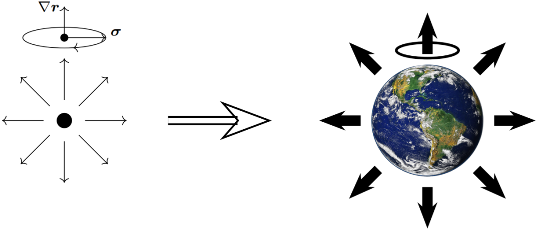
A muon spin force can be generated by a light scalar field which has scalar coupling with nucleons and axial vector/pesudoscalar coupling with the muon

$$\mathcal{L}_{\text{eff}} = \ldots + \frac{1}{2}(\partial_\nu \phi)^2 - \frac{1}{2} m_\phi^2 \phi^2 - g_S \phi(\bar{n}n + \bar{p}p) + \frac{g_A^{(\mu)}}{2m_\mu} \partial^\alpha \phi \bar{\mu} \gamma_\alpha \gamma_5 \mu - g_P^{(\mu)} \phi \bar{\mu} i \gamma_5 \mu .$$

In the non-relativistic limit, this turns out to be a force acting on the muon spin ($g_A^{(\mu)}$ and $g_P^{(\mu)}$ have the same effect at tree level)

$$H^{(\mu)} = \frac{g_S (g_A^{(\mu)} + g_P^{(\mu)})}{4\pi \times 2m_\mu} (\boldsymbol{\sigma}^{(\mu)} \cdot \boldsymbol{\nabla}) \frac{\exp(-m_\phi r)}{r} \Longrightarrow \Delta E_{(\mu)} (\mathbf{s} \cdot \mathbf{n})$$

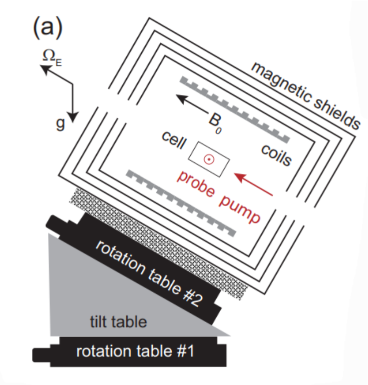
Small for individual nucleon-muon pair, but can be enhanced by the large number of nucleons in the Earth to produce an observable effect.



(Ryan Janish & Harikrishnan Ramani 2006.10069, Prateek Agrawal et al 2210.17547, Hooman Davoudiasl & Robert Szafron 2210.14959)

Indirect constraints from nuclear spin force experiments

Isotope experiments: the design



(S.-B. Zhang et al 2303.10352)

- The comagnetometer cell is filled with ^{129}Xe and ^{131}Xe
- A magnetic field $B_0 \sim 3.5\mu$T is generated along the Earth's rotation direction
- The ratio of the precession frequency $R_+ = \omega_{129}/\omega_{131}$ is measured
- Reversing the direction of the magnetic field gives a new ratio R_-
- The size of the spin force can be obtained from the difference $R_+ - R_-$

Isotope experiments: the observable

The interaction is given by

$$H_i = -\mu_N g_i (\mathbf{I}\cdot\mathbf{B}) + \Delta E_i (\mathbf{I}\cdot\mathbf{n})$$

For Xe isotope,

$$R_+ - R_- = \frac{2}{\omega_{131}} \left(\Delta E_{129} - \Delta E_{131} \frac{g_{129}}{g_{131}} \right) \cos\phi \quad (+\Omega_E \text{ terms})$$

If ΔE_i and g_i are exactly proportional among isotopes, there is no sensitivity.

The observable is the difference in $\Delta E_i / g_i$ between isotopes, current best limits are

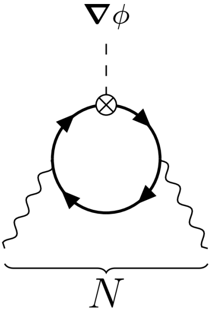
$$|\Delta E_{\rm Hg}| = \left| \Delta E_{201} - \Delta E_{199} \frac{g_{201}}{g_{199}} \right| < 3.0 \times 10^{-21}\,\text{eV} \quad \text{(UW experiment)}$$

$$|\Delta E_{\rm Xe}| = \left| \Delta E_{129} - \Delta E_{131} \frac{g_{129}}{g_{131}} \right| < 1.7 \times 10^{-22}\,\text{eV} \quad \text{(USTC experiment)}$$

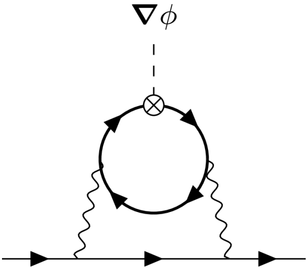
Many orders of magnitude better compared to $|\Delta E_{(\mu)}| = 6 \times 10^{-14}$eV, could possibly afford a loop penalty to consider the spin force transfer.

Spin force transfer

At the lowest order the muon spin force can be transferred to the nucleus with a muon loop and two photons



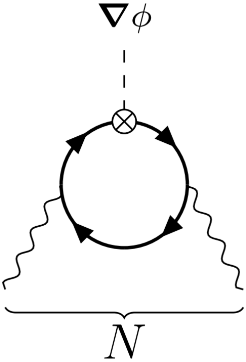
1-loop (on shell photon)



2-loop (off shell photon)

$g_A^{(\mu)}$ and $g_P^{(\mu)}$ have different consequences in loops

Consequences at 1-loop, constraints on pseudoscalar coupling



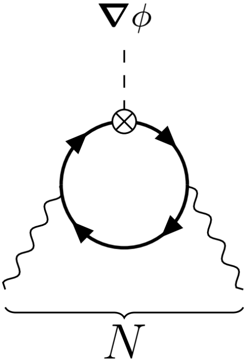
$$\mathcal{L}_{eff} = \frac{e^2 \nabla\phi}{4\pi^2 m_\mu} \cdot \left(g_P^{(\mu)} \mathbf{B} A_0 + \frac{g_A^{(\mu)}}{12 m_\mu^2} \mathbf{B}\, \nabla^2 A_0 \right).$$

- Axial vector contribution suppressed by $1/(R_N m_\mu)^2 \sim 0.1$
- Nuclear charge well approximated by a uniform distribution.
- Magnetic field distribution is uncertain because no nuclear model reliably explains where the nuclear magnetic dipole moment comes from.
- We model the magnetic field to be coming from valance neutron + core polarization

$$\boldsymbol{B} = \boldsymbol{B}_n + \boldsymbol{B}_{\text{core}}$$

- $\boldsymbol{B}_n$ obtained by numerically solving Schrödinger equation in Woods-Saxon potential
- $\boldsymbol{B}_{\text{core}}$ modeled by uniform polarization to reproduce the correct magnetic moment

Consequences at 1-loop, constraints on pseudoscalar coupling



Loop transfer:

$$\left|\frac{\Delta E_{\rm Hg}}{\Delta E_{(\mu)}}\right| = 1\times 10^{-6};\ \left|\frac{\Delta E_{\rm Xe}}{\Delta E_{(\mu)}}\right| = 3\times 10^{-6} \text{ for } g_P^{(\mu)},$$

$$\left|\frac{\Delta E_{\rm Hg}}{\Delta E_{(\mu)}}\right| = 2\times 10^{-8};\ \left|\frac{\Delta E_{\rm Xe}}{\Delta E_{(\mu)}}\right| = 4\times 10^{-9} \text{ for } g_A^{(\mu)}.$$

Combined with the constraint from nuclear experiments:

$$|\Delta E_P^{(\mu)}(\mathrm{Hg})| < 3\times 10^{-15}\,\mathrm{eV},\quad |\Delta E_P^{(\mu)}(\mathrm{Xe})| < 6\times 10^{-17}\,\mathrm{eV}$$

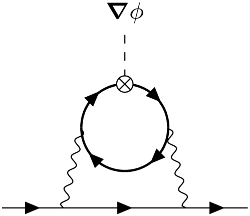
$$|\Delta E_A^{(\mu)}(\mathrm{Hg})| < 2\times 10^{-13}\,\mathrm{eV},\quad |\Delta E_A^{(\mu)}(\mathrm{Xe})| < 4\times 10^{-14}\,\mathrm{eV}$$

To explain g-2:

$$|\Delta E_{(\mu)}| = 6\times 10^{-14}\mathrm{eV}$$

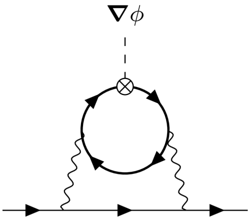
Pseudoscalar coupling is ruled out by a few orders of magnitude, constraint on axial coupling is uncertain due to nuclear uncertainty (Loop transfer to Xe comes from numerical cancellation at 1% level for $g_A^{(\mu)}$).

Consequences at 2-loop, constraints on axial vector coupling



- Pseudoscalar coupling contribution is suppressed at 2-loops
  - Induced electron coupling is small
  - UV finite, cannot see free quarks
  - Neutron contribution suppressed by magnetic form factors
- Axial vector coupling can be important
  - UV scale compensates loop smallness
  - Not suppressed by $1/(R_N^3 m_\mu^2 m_p) \sim 5 \times 10^{-3}$ compared to 1-loop

Consequences at 2-loop, constraints on axial vector coupling



$$\mathcal{L}_{\text{eff}} = \sum_{i=u,d,s,e} \frac{g_A^{(i)}}{2m_\mu} \partial_\alpha \phi \times \bar{\psi}_i \gamma^\alpha \gamma_5 \psi,$$

$$g_A^{(i)} = -g_A^{(\mu)} \times \frac{3}{4} \left(\frac{\alpha}{\pi}\right)^2 Q_i^2 \log(\Lambda_{\text{UV}}^2/\Lambda_{\text{IR}}^2),$$

- Induced axial coupling with quarks can be transferred to the nucleus

Quarks —(lattice QCD / axial charge)→ Nucleon —(nucleon spin / expectation values)→ Nucleus

- Result relatively consistent among various models

$$\Delta E_{\text{Xe}} = (3-7)\times 10^{-8} \times \Delta E_{(\mu)} \times \frac{\log(\Lambda_{\text{UV}}^2/\Lambda_{\text{IR}}^2)}{\log(m_\tau^2/m_p^2)} \quad \to \quad \left|\Delta E_{(\mu)}\right| < 6\times 10^{-15}\text{eV}$$

- Kind of stronger than the muon g-2 requirement, but this result could be undermined by nuclear uncertainty at 1-loop.
- Constraint from Hg isotopes has better 1-loop behavior, but their spin content are poorly known.

Direct detection using stopped muons

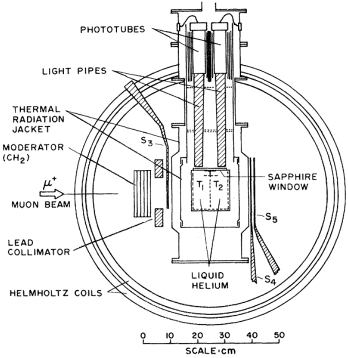
General idea for direct detection

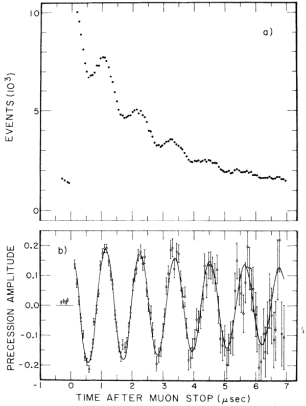
$$\boldsymbol{\Omega}_s = \frac{e}{m_\mu}\left(a_\mu + \frac{1}{\gamma}\right)\boldsymbol{B} - \frac{1}{\gamma}\Delta E_{(\mu)}\mathbf{n} + \cdots$$

- Need a way to distinguish the two effects $\longrightarrow$ the ability to change the size of magnetic field, preferably with a possibility of reversing direction
- Effect is larger for small $\gamma \longrightarrow$ easier to detect with stopped muons
- $|\Delta E_{(\mu)}| = 6 \times 10^{-14}$eV is equivalent to $B^{\text{eq}} = \Delta E_{(\mu)}/\mu_\mu \simeq 1.1$mGs $\longrightarrow$ might be easier to detect at low/medium values of magnetic field

Existing experiments in similar conditions

An experiment using stopped muons was done half century ago using magnetic field $\sim$66Gs to measure the magnetic moment of μ^+. (V. Hughes et al PhysRevLett.33.572)





Their result is $\Omega = 2\pi \times (13.58 \pm 0.02\text{kHz/G}) \times B$, about two orders of magnitude weaker in accuracy than our requirement.

A possible setup

Muon spin resonance in weak magnetic field using stopped μ^+

- $$\Delta\psi = \frac{\Delta E_{(\mu)} t}{\hbar} \implies \Delta\psi(t = \tau_\mu) = 2 \times 10^{-4} \implies N_{(\mu)} > 1/(\Delta\psi)^2 \sim 10^8$$

  10^5 muons/s for continuous beam, even higher intensity with pulsed beam
- Uniform vertical magnetic field in the range $5 - 50$Gs with a possibility of reversing direction
- Co-magnetometer to monitor the magnetic field
- Liquid or gaseous target to stop the muons
- $\Delta E_{(\mu)}$ can be extracted from

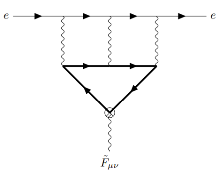
$$-B \frac{d\omega_{(\mu)}/\omega_{(p)}}{dB} = \frac{\Delta E_{(\mu)}}{2\mu_p B} = 3.4 \times 10^{-4} \times \frac{\Delta E_{(\mu)}}{6 \times 10^{-14}\,\text{eV}} \times \frac{10\,\text{Gs}}{B}$$

Indirect constraints on muon EDM

(Based on 2108.05398 and 2207.01679 with Y.Ema and M.Pospelov)

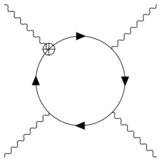
Muon EDM: Pathways to atomic EDMs

Muon EDM



electron EDM

$$d_e = \left(\frac{19}{4}\zeta(3) - \frac{71}{24}\right)\frac{m_e}{m_L}\left(\frac{\alpha}{\pi}\right)^3 d_L$$



CP-odd light-by-light operator

$$\mathcal{L} = -\frac{(d_L/e)}{96\pi^2 m_L^3} e^4 F_{\alpha\beta}F^{\alpha\beta}F_{\gamma\delta}\tilde{F}^{\gamma\delta}$$

C_S operator

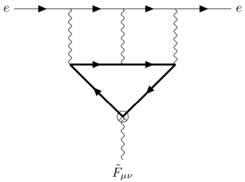
Schiff moment

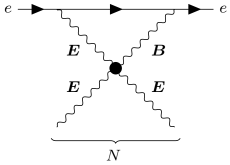
paramagnetic EDM

diamagnetic EDM

Indirect constraints on muon EDM

Paramagnetic EDM





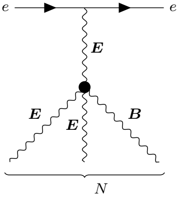
$$|d_e^{eq}(\text{HfF}^+)| < 4.1 \times 10^{-30} e\text{cm}$$

$$|d_e^{eq}(\text{HfF}^+)| \simeq 4.6 \times 10^{-10} d_\mu$$

$$\Longrightarrow |d_\mu| < 8.9 \times 10^{-21} e\text{cm}$$

Factor of $\sim$20 better than previous BNL direct measurement $|d_\mu| < 1.8 \times 10^{-19} e$cm

Schiff moment



$$|S_{\text{Hg}}| < 3.1 \times 10^{-31} e \text{ fm}^3$$

$$S_{^{199}\text{Hg}}/e \simeq (d_\mu/e) \times 4.9 \times 10^{-7} \text{fm}^2$$

$$\Longrightarrow |d_\mu| < 6.4 \times 10^{-20} e\text{cm}$$

Factor of $\sim$3 better than previous BNL direct measurement

Summary

- The existence of a muon spin force of the size $|\Delta E_{(\mu)}| = 6 \times 10^{-14}$eV could explain the muon g-2 anomaly
- The muon spin force can be indirectly constrained by nuclear experiments, the constraint
  - strongly rules out the pseudoscalar type of muon spin force.
  - is uncertain for axial coupling due to significant nuclear uncertainty.
- The muon spin force can be directly probed by μSR experiments using stopped muons
- The same idea of loop transfer improves muon EDM bounds by a factor of 20

Thank you!

Back up slides

lab-based g_S^N and supernova bounds on g_A^μ

$|\Delta E_{(\mu)}| = 6 \times 10^{-14}$eV corresponds to $g_S(g_A^{(\mu)} + g_P^{(\mu)}) \sim 2 \times 10^{-29}$ in the $1/m_\phi \gg R_\oplus$ limit.

g_S^N is constrained by fifth-forth experiments, g_A^μ is constrained by supernova bounds

$$|g_S^N(\text{lab})| < 8 \times 10^{-25} \quad \text{(MICROSCOPE Collaboration)}$$
$$|g_A^\mu(\text{SN})| < 4 \times 10^{-8} \quad \text{(Robert Bollig et al 2005.07141)}$$

The supernova bound could be relaxed when the muon has scalar interaction with ϕ

(Hooman Davoudiasl and Robert Szafron 2210.14959)