MuonBridge: EFT predictions for $\mu \rightarrow e$ conversion

Jure Zupan
u. of cincinnati

based on Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn

Muons in Minneapolis, UMN, Dec 7 2023

GOALS FOR THIS TALK

- provide an EFT based prediction for $\mu \to e$ conversion
  - assumption: heavy new physics $\Lambda \gg m_\mu$
- open source code **MuonBridge**

OVERVIEW

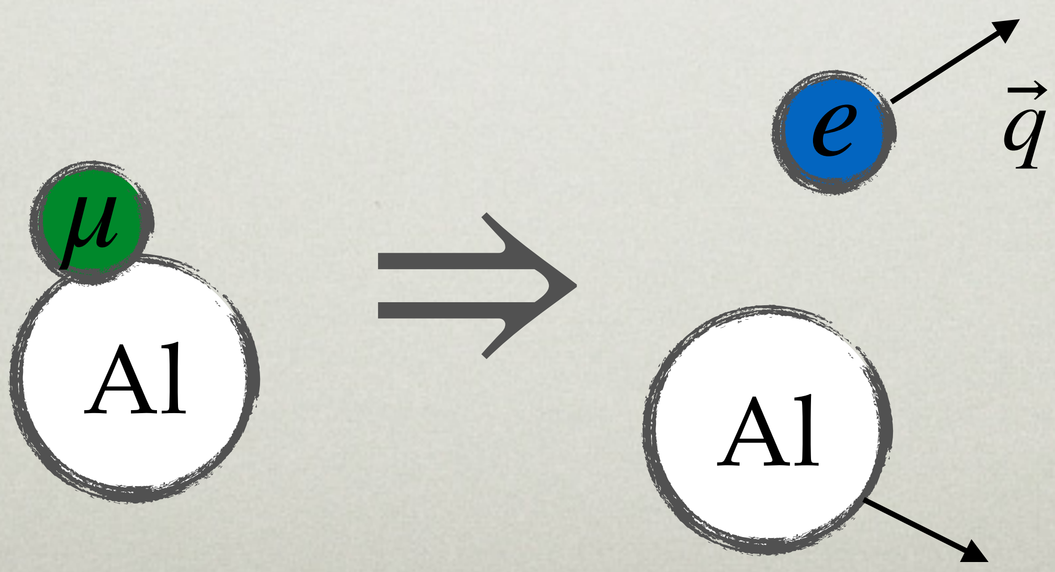
- scales in the problem
  - tower of EFTs, focus on physics below $\mu = 2\,\text{GeV}$
- two NP examples
- some comments about the code

KINEMATICS

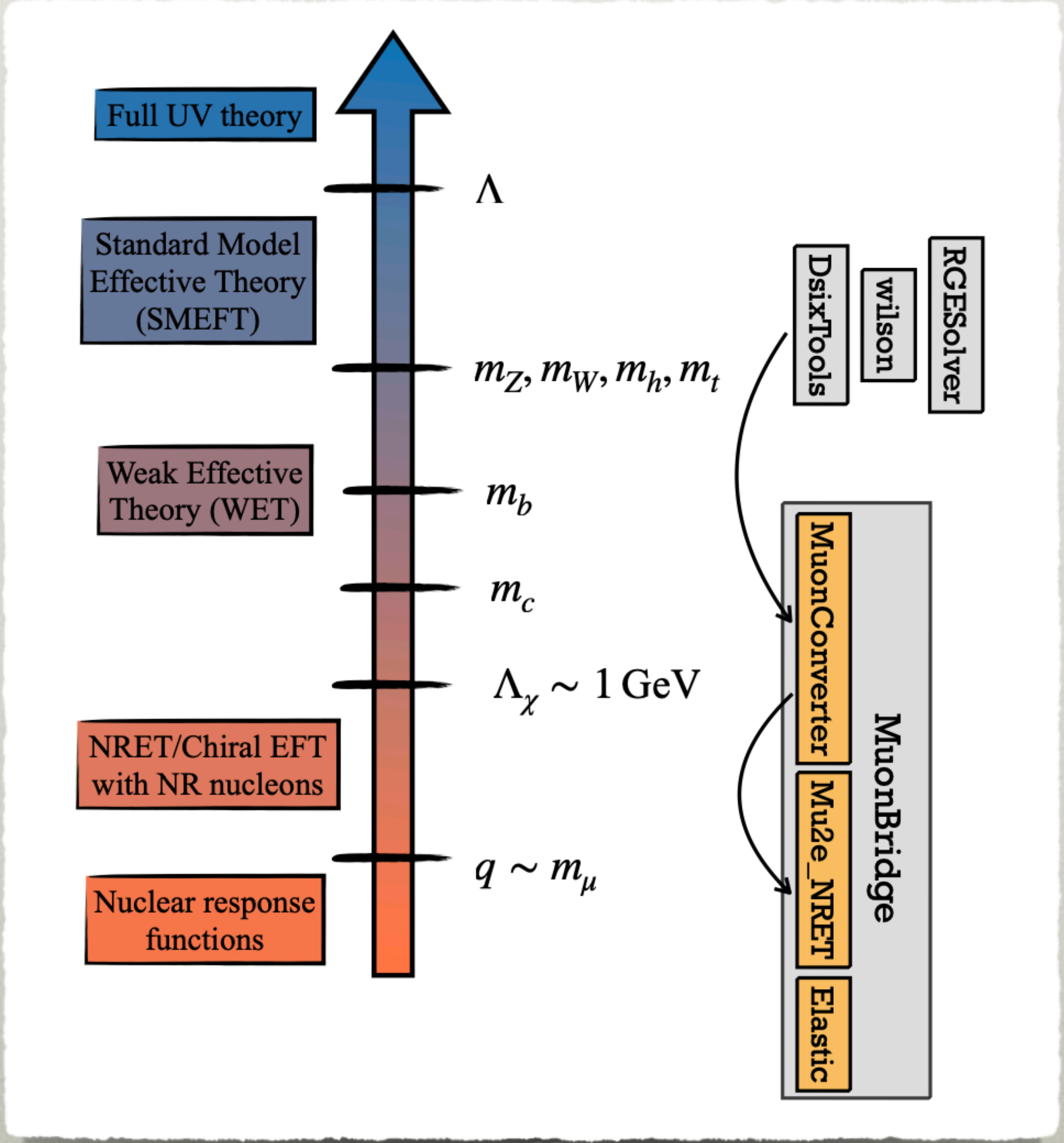
- initial state: μ^- in 1s orbital
- final state: relativistic e^- with three momentum

$$\vec{q}^{\,2} = \frac{M_T}{m_\mu + M_T}\left[\left(m_\mu - E_\mu^{\rm bind}\right)^2 - m_e^2\right],$$

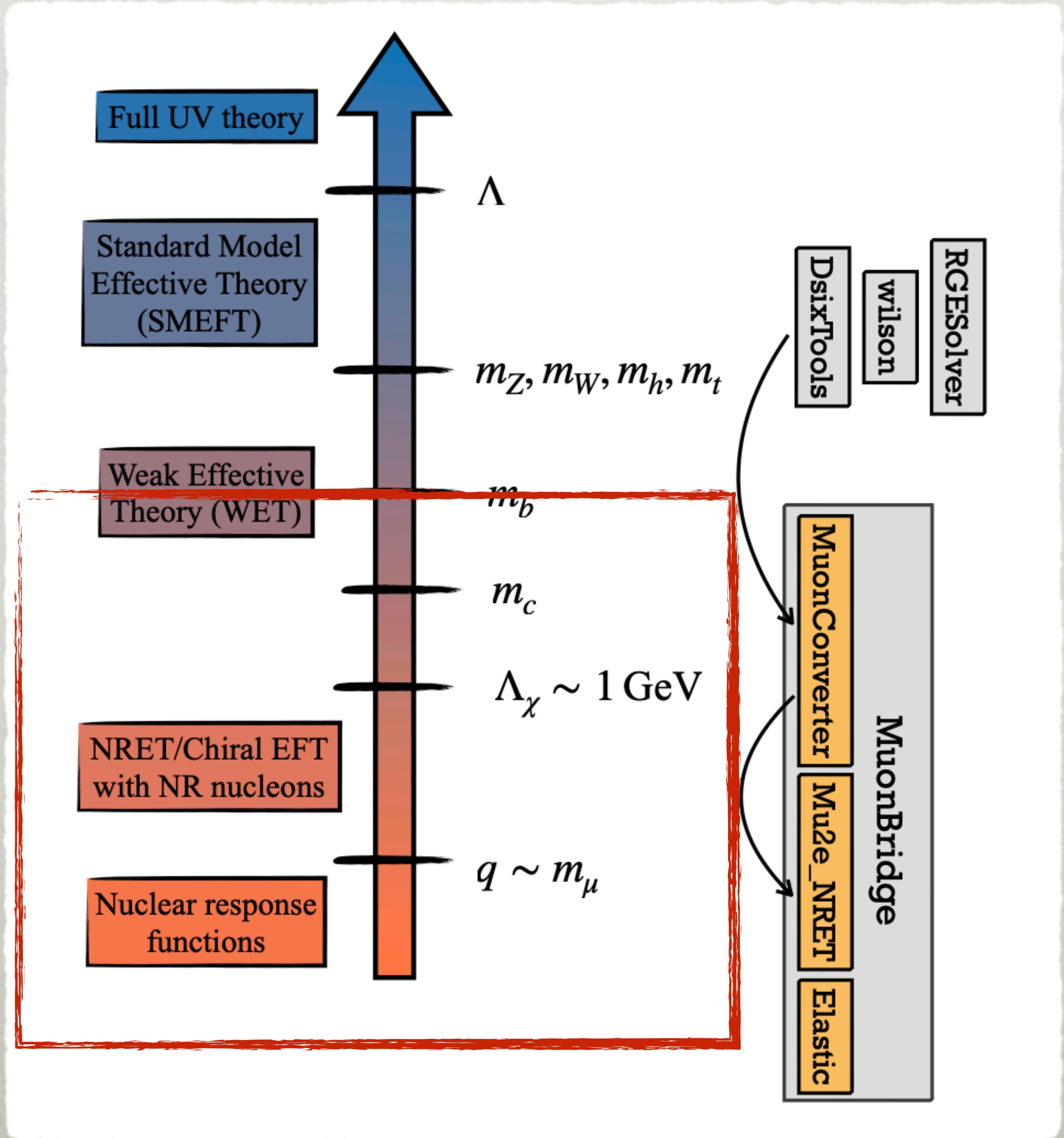
  - $E_\mu^{\rm bind} \ll m_\mu$ (for ^{27}Al $E_\mu^{\rm bind} \simeq 0.463\,\mathrm{MeV}$)
    $\Rightarrow |\vec{q}| \sim \mathcal{O}(100\,\mathrm{MeV})$
- we limit the discussion to processes where nucleus is in ground state



TOWER OF EFTs

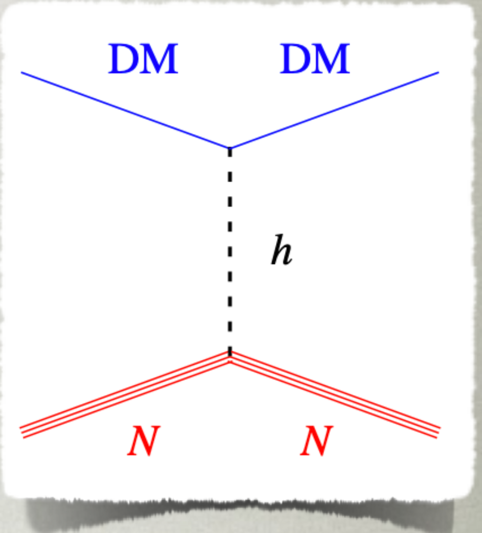


TOWER OF EFTs



SEVERAL COMMENTS

- the tower of EFTs very reminiscent of wimpy DM direct detection
  - instead of nonrelativistic outgoing DM ⇒ relativistic e^-
  - instead of $\vec{q}/m_N$ in NRET we have unit vector $\hat{q}$



- bound muon is approximately nonrelativistic for Al
  - relativistic corrections controlled by $Z\alpha/2$
  - included by including $\vec{v}_\mu$ suppressed terms (lower componenst of Dirac w.f.)
- nucleons bound inside nucleus are roughly nonrel., with $v_{\text{avg}} \approx 0.2$
- outgoing e^- is not a pure plane wave, due to potential from nuclear charge Z
  - at the origin a plane wave with q_{eff} instead of q

$$e^{i\vec{q}\cdot\vec{r}} \to \frac{q_{\text{eff}}}{q} e^{i\vec{q}_{\text{eff}}\cdot\vec{r}}$$

for ^{27}Al: $q = 104.98\,\text{MeV}$
$q_{\text{eff}} = 110.81\,\text{MeV}$

TOWER OF EFTs

- below $\mu = 2\,\mathrm{GeV}$ a series of EFTs
  - Weak Effective Theory (WET): d.o.f.s quarks and gluons

    Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn
  - (Covariant EFT with relativistic nucleons)

    Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945

    Haxton, McElvain, Rule, 2109.13503
  - NRET: d.o.f.s non-rel nucleons
    - (Chiral EFT for nucleus)
    - chiral counting shows that leading effect from $\mu \to e$ on single nucleon currents

WET

Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn

- only need to keep WET operators relevant for $\mu \to e$ conversion
  - work up to and including dimension 7

$$\mathcal{L}_{\rm eff}^{\rm WET} = \sum_{a,d} \hat{\mathcal{C}}_a^{(d)} \mathcal{Q}_a^{(d)},$$

$$\hat{\mathcal{C}}_a^{(d)} = \frac{\mathcal{C}_a^{(d)}}{\Lambda^{d-4}}.$$

- 2 dim 5 operators
  - magnetic (CP-even) and electric (CP-odd) dipole op.

$$\mathcal{Q}_1^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}\mu)F_{\alpha\beta}\,, \qquad \mathcal{Q}_2^{(5)} = \frac{e}{8\pi^2}(\bar{e}\sigma^{\alpha\beta}i\gamma_5\mu)F_{\alpha\beta}\,,$$

WET

Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn

- 10 dimension 6 ops

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$
$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$
$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$
$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta} q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta} q)\,.$$

- note: we use operator basis with well defined parity
  - more convenient for
    - taking nonrel. limit for muons
    - writing down hadronic matrix elements of quark and gluon currents
- related to other WET bases used in the literature by linear transf.
- note: tensor currents appear already at dimension 6

WET

Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn

- 16 operators at dimension 7
  - 8 couple leptonic currents to gauge bosons

$$\mathcal{Q}_1^{(7)} = \frac{\alpha_s}{12\pi}(\bar{e}\mu)G^{a\alpha\beta}G^a_{\alpha\beta}\,, \qquad \mathcal{Q}_2^{(7)} = \frac{\alpha_s}{12\pi}(\bar{e}i\gamma_5\mu)G^{a\alpha\beta}G^a_{\alpha\beta}\,,$$
$$\mathcal{Q}_3^{(7)} = \frac{\alpha_s}{8\pi}(\bar{e}\mu)G^{a\alpha\beta}\widetilde{G}^a_{\alpha\beta}\,, \qquad \mathcal{Q}_4^{(7)} = \frac{\alpha_s}{8\pi}(\bar{e}i\gamma_5\mu)G^{a\alpha\beta}\widetilde{G}^a_{\alpha\beta}\,,$$
$$\mathcal{Q}_5^{(7)} = \frac{\alpha}{12\pi}(\bar{e}\mu)\,F^{\alpha\beta}F_{\alpha\beta}\,, \qquad \mathcal{Q}_6^{(7)} = \frac{\alpha}{12\pi}(\bar{e}i\gamma_5\mu)\,F^{\alpha\beta}F_{\alpha\beta}\,,$$
$$\mathcal{Q}_7^{(7)} = \frac{\alpha}{8\pi}(\bar{e}\mu)\,F^{\alpha\beta}\tilde{F}_{\alpha\beta}\,, \qquad \mathcal{Q}_8^{(7)} = \frac{\alpha}{8\pi}(\bar{e}i\gamma_5\mu)\,F^{\alpha\beta}\tilde{F}_{\alpha\beta}\,,$$

  - 8 couple leptonic currents to quark currents

$$\mathcal{Q}_{9,q}^{(7)} = (\bar{e}\,i\overset{\leftrightarrow}{\partial}_\alpha\,\mu)\,(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{10,q}^{(7)} = (\bar{e}i\gamma_5\,i\overset{\leftrightarrow}{\partial}_\alpha\,\mu)\,(\bar{q}\gamma^\alpha q)\,,$$
$$\mathcal{Q}_{11,q}^{(7)} = (\bar{e}\,i\overset{\leftrightarrow}{\partial}_\alpha\,\mu)\,(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{12,q}^{(7)} = (\bar{e}i\gamma_5\,i\overset{\leftrightarrow}{\partial}_\alpha\,\mu)\,(\bar{q}\gamma^\alpha\gamma_5 q)\,,$$
$$\mathcal{Q}_{13,q}^{(7)} = \partial^\alpha(\bar{e}\gamma^\beta\mu)\,(\bar{q}\sigma_{\alpha\beta}q)\,, \qquad \mathcal{Q}_{14,q}^{(7)} = \partial^\alpha(\bar{e}\gamma^\beta\gamma_5\mu)\,(\bar{q}\sigma_{\alpha\beta}q)\,,$$
$$\mathcal{Q}_{15,q}^{(7)} = \partial^\alpha(\bar{e}\gamma^\beta\mu)\,(\bar{q}i\sigma_{\alpha\beta}\gamma_5 q)\,, \qquad \mathcal{Q}_{16,q}^{(7)} = \partial^\alpha(\bar{e}\gamma^\beta\gamma_5\mu)\,(\bar{q}i\sigma_{\alpha\beta}\gamma_5 q)\,,$$

- note: all derivatives moved to leptonic currents by EOM
  - convenient basis for calculating hadronic matrix elements

Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945
Haxton, McElvain, Rule, 2109.13503

NRET

- a hierarchy of small parameters

$$y \equiv \left(\frac{qb}{2}\right)^2 > |\vec{v}_N| > |\vec{v}_\mu| > |\vec{v}_T|$$

$b \sim$ nuclear size

$\vec{v}_N = (\vec{k}_1 + \vec{k}_2)/2$ average nucleon velocity

bound muon velocity

velocity of outgoing target nucleus

- $y \sim 0.2 - 0.5 \Rightarrow$ nuclear scales are being probed
- Chiral EFT: NP interactions with single nucleon current dominate
- NRET: can expand in v_N and v_μ
    - $v_N > v_\mu \Rightarrow$ leading effect captured from NRET basis with ops. of $\mathcal{O}(v_N, v_\mu^0)$
        - this gives 16 operators (primed operators have structure that differs from NRET for DM)
    - $\mathcal{O}(v_\mu)$ are also included in the predictions in the MuonBridge code: additional 10 operators

NRET

- a

$$
\begin{aligned}
\mathcal{O}_1 &= 1_L\, 1_N, & \mathcal{O}_2' &= 1_L\, i\hat{q}\cdot\vec{v}_N, \\
\mathcal{O}_3 &= 1_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], & \mathcal{O}_4 &= \vec{\sigma}_L\cdot\vec{\sigma}_N, \\
\mathcal{O}_5 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{v}_N), & \mathcal{O}_6 &= i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{\sigma}_N, \\
\mathcal{O}_7 &= 1_L\, \vec{v}_N\cdot\vec{\sigma}_N, & \mathcal{O}_8 &= \vec{\sigma}_L\cdot\vec{v}_N, \\
\mathcal{O}_9 &= \vec{\sigma}_L\cdot(i\hat{q}\times\vec{\sigma}_N), & \mathcal{O}_{10} &= 1_L\, i\hat{q}\cdot\vec{\sigma}_N, \\
\mathcal{O}_{11} &= i\hat{q}\cdot\vec{\sigma}_L\, 1_N, & \mathcal{O}_{12} &= \vec{\sigma}_L\cdot[\vec{v}_N\times\vec{\sigma}_N], \\
\mathcal{O}_{13}' &= \vec{\sigma}_L\cdot(i\hat{q}\times[\vec{v}_N\times\vec{\sigma}_N]), & \mathcal{O}_{14} &= i\hat{q}\cdot\vec{\sigma}_L\, \vec{v}_N\cdot\vec{\sigma}_N, \\
\mathcal{O}_{15} &= i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot[\vec{v}_N\times\vec{\sigma}_N], & \mathcal{O}_{16}' &= i\hat{q}\cdot\vec{\sigma}_L\, i\hat{q}\cdot\vec{v}_N.
\end{aligned}
$$

- y
- C
- N
  - $v_N > v_\mu \Rightarrow$ leading effect captured from NRET basis with ops. of $\mathcal{O}(v_N, v_\mu^0)$
    - this gives 16 operators (primed operators have structure that differs from NRET for DM)
  - $\mathcal{O}(v_\mu)$ are also included in the predictions in the MuonBridge code: additional 10 operators

MATCHING ONTO NRET

- NRET effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{NRET}} = \sum_{N=n,p} \sum_{i=1}^{16} c_i^N \mathcal{O}_i^N + \cdots ,$$

- low energy coefficients are functions of $\vec{q}_{\text{eff}}^{\,2}$ in general
  - for $\mu \to e$ this is a constant
- their values from nonperturbative matching of WET to NRET
  - follow from nucleon matrix elements
  - for instance

$$c_1^N = \sum_q \frac{1}{m_q} \hat{\mathcal{C}}_{5,q}^{(6)} F_S^{q/N} + \hat{\mathcal{C}}_1^{(7)} F_G^N + \hat{\mathcal{C}}_5^{(7)} F_\gamma^N ,$$

$$+ \sum_q \hat{\mathcal{C}}_{1,q}^{(6)} F_1^{q/N} + m_+ \sum_q \hat{\mathcal{C}}_{9,q}^{(7)} F_1^{q/N} - \sum_q \frac{q^2}{2 m_N m_q} \hat{\mathcal{C}}_{13,q}^{(7)} \left(F_{T,1}^{q/N} - 4 F_{T,2}^{q/N} \right)$$

$$
\begin{aligned}
\langle N'|\bar{q}\gamma^\mu q|N\rangle &= \bar{u}'_N\Big[F_1^{q/N}(q^2)\gamma^\mu - \frac{i}{2m_N}F_2^{q/N}(q^2)\sigma^{\mu\nu}q_\nu\Big]u_N\,,\\
\langle N'|\bar{q}\gamma^\mu\gamma_5 q|N\rangle &= \bar{u}'_N\Big[F_A^{q/N}(q^2)\gamma^\mu\gamma_5 - \frac{1}{2m_N}F_{P'}^{q/N}(q^2)\gamma_5 q^\mu\Big]u_N\,,\\
\langle N'|m_q\bar{q}q|N\rangle &= F_S^{q/N}(q^2)\,\bar{u}'_N u_N\,,\\
\langle N'|m_q\bar{q}i\gamma_5 q|N\rangle &= F_P^{q/N}(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,\\
\langle N'|\frac{\alpha_s}{12\pi}G^{a\mu\nu}G^a_{\mu\nu}|N\rangle &= F_G^N(q^2)\,\bar{u}'_N u_N\,,\\
\langle N'|\frac{\alpha_s}{8\pi}G^{a\mu\nu}\tilde{G}^a_{\mu\nu}|N\rangle &= F_{\tilde{G}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,,\\
\langle N'|m_q\bar{q}\sigma^{\mu\nu}q|N\rangle &= \bar{u}'_N\Big[F_{T,0}^{q/N}(q^2)\,\sigma^{\mu\nu} - \frac{i}{2m_N}\gamma^{[\mu}q^{\nu]}F_{T,1}^{q/N}(q^2)\\
&\qquad - \frac{i}{m_N^2}q^{[\mu}k_{12}^{\nu]}F_{T,2}^{q/N}(q^2)\Big]u_N\,,\\
\langle N'|\frac{\alpha}{12\pi}F^{\mu\nu}F_{\mu\nu}|N\rangle &= F_\gamma^N(q^2)\,\bar{u}'_N u_N\,,\\
\langle N'|\frac{\alpha}{8\pi}F^{\mu\nu}\tilde{F}_{\mu\nu}|N\rangle &= F_{\tilde{\gamma}}^N(q^2)\,\bar{u}'_N i\gamma_5 u_N\,.
\end{aligned}
$$

- N
- lo
  -
- th
  N
  - follow from nucleon matrix elements
  - for instance

$$
\begin{aligned}
c_1^N = &\sum_q \frac{1}{m_q}\hat{\mathcal{C}}_{5,q}^{(6)}F_S^{q/N} + \hat{\mathcal{C}}_1^{(7)}F_G^N + \hat{\mathcal{C}}_5^{(7)}F_\gamma^N,\\
&+\sum_q \hat{\mathcal{C}}_{1,q}^{(6)}F_1^{q/N} + m_+\sum_q \hat{\mathcal{C}}_{9,q}^{(7)}F_1^{q/N} - \sum_q \frac{q^2}{2m_N m_q}\hat{\mathcal{C}}_{13,q}^{(7)}\Big(F_{T,1}^{q/N} - 4F_{T,2}^{q/N}\Big)
\end{aligned}
$$

CONVERSION RATE

- matrix elements squared of NRET operators leads to nuclear response functions W_i
- the $\mu \to e$ conversion rate thus

$$\Gamma(\mu \to e) = \frac{G_F^2}{\pi} \frac{q_{\rm eff}^2}{1+\frac{q}{M_T}} |\phi_{1s}^{Z_{\rm eff}}(\vec{0})|^2 \sum_{\tau=0,1} \sum_{\tau'=0,1} \Big\{ \Big[\tilde{R}_{MM}^{\tau\tau'} W_{MM}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} W_{\Sigma''\Sigma''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} W_{\Sigma'\Sigma'}^{\tau\tau'}(q_{\rm eff}) \Big]$$
$$+ \frac{q_{\rm eff}^2}{m_N^2} \Big[\tilde{R}_{\Phi''\Phi''}^{\tau\tau'} W_{\Phi''\Phi''}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'} W_{\tilde{\Phi}'\tilde{\Phi}'}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Delta}^{\tau\tau'} W_{\Delta\Delta}^{\tau\tau'}(q_{\rm eff}) \Big]$$
$$- \frac{2q_{\rm eff}}{m_N} \Big[\tilde{R}_{\Phi''M}^{\tau\tau'} W_{\Phi''M}^{\tau\tau'}(q_{\rm eff}) + \tilde{R}_{\Delta\Sigma'}^{\tau\tau'} W_{\Delta\Sigma'}^{\tau\tau'}(q_{\rm eff}) \Big] \Big\}$$

- particle physics is in products of Wilson coefficients R_i
  - for instance the terms not vanishing in $v_N \to 0$ limit

$$\tilde{R}_{MM}^{\tau\tau'} = \tilde{c}_1^\tau \tilde{c}_1^{\tau'*} + \tilde{c}_{11}^\tau \tilde{c}_{11}^{\tau'*},$$

$$\tilde{R}_{\Sigma'\Sigma'}^{\tau\tau'} = \tilde{c}_4^\tau \tilde{c}_4^{\tau'*} + \tilde{c}_9^\tau \tilde{c}_9^{\tau'*}$$

$$\tilde{R}_{\Sigma''\Sigma''}^{\tau\tau'} = (\tilde{c}_4^\tau - \tilde{c}_6^\tau)(\tilde{c}_4^{\tau'*} - \tilde{c}_6^{\tau'*}) + \tilde{c}_{10}^\tau \tilde{c}_{10}^{\tau'*}.$$

$$c_i \equiv \tilde{c}_i / v^2 = \sqrt{2} G_F \tilde{c}_i,$$

NUCLEAR RESPONSE FUNCTIONS

- $W_M(q)$: from vector operator
  - in $q \to 0$ limit counts nucleons ⇒ spin-indep. (coherent) scattering
- $W_{\Sigma''}$ and $W_{\Sigma'}$: longit. and transverse axial ops.
  - related to conventional spin form factors

$$S_{00,11} = \frac{1}{4\pi} \sum_{\text{spins}} |\langle \vec{S}_p \pm \vec{S}_n \rangle|^2,$$

$$S_{01} = \frac{1}{2\pi} \sum_{\text{spins}} |\langle \vec{S}_p \rangle|^2 - |\langle \vec{S}_n \rangle|^2,$$

$$W_{\Sigma'}^{\tau\tau'} + W_{\Sigma''}^{\tau\tau'} = S_{\tau\tau'}, \qquad \tau, \tau' = 0, 1.$$

  - measure the nucleon spin content of the nucleus
- W_Δ , $W_{\tilde{\Phi}'}$, $W_{\tilde{\Phi}''}$: come from couplings to velocities of nucleons
  - reflect the composite structure of the nucleus
  - coherence over half-filled shells for $W_{\tilde{\Phi}''}$
- rough scaling for Al (isocalars):

$$W_M \sim \mathcal{O}(A^2) \gg \left\{ W_{\Sigma'}, W_{\Sigma''}, \frac{q_{\text{eff}}^2}{m_N^2} W_{\tilde{\Phi}''} \right\} \gg \left\{ \frac{q_{\text{eff}}^2}{m_N^2} W_\Delta, \frac{q_{\text{eff}}^2}{m_N^2} W_{\tilde{\Phi}'} \right\}$$

- six more response functions + 2 interf. terms at $\mathcal{O}(v_\mu)$

COMMENTS

- since q_{eff} changes by only ~5% from C to W
  - $c_i^N(q_{\text{eff}})$ are basically constants
- $\Rightarrow$ from $\mu \to e$ can measure only a few linear combinations of Wilson coeffs
  - 3 combinations at $\mathcal{O}(v_N^0, v_\mu^0)$
    - vector/scalar, axial, psedoscalar currents
  - + 5 combinations at $\mathcal{O}(v_N, v_\mu^0)$
  - this is for isoscalar-isocalar W's, since also isovector-isoscalar, isovector-isovector, 3x those nos. in total
- to understand UV physics important to measure both $\mu \to e$ on different targets and $\mu \to e\gamma$ ($\mu \to 3e$)

Haxton, McElvain, Menzo, Rule, JZ, 2312.nnnn

MUONBRIDGE

- the code/repository **MuonBridge** consists of three modules
  - **MuonConverter:** matches WET to NRET
    - can interface with RG running codes
  - **Mu2e_NRET:** calculates the $\mu \to e$ rate

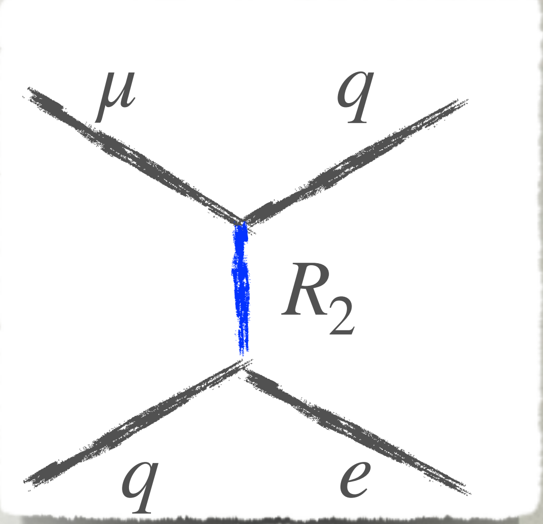
$$B(\mu^- \to e^-) = \frac{\Gamma\left[\mu^- + (A,Z) \to e^- + (A,Z)\right]}{\Gamma\left[\mu^- + (A,Z) \to \nu_\mu + (A,Z-1)\right]},$$

    - particle physics input from **MuonConverter**, i.e., WET Wilson coeffs. C_i
  - **Elastic:** a database of shell model density matrices for calculating nuclear form factors
- comes in both **Python** and **Mathematica** versions

NEW PHYSICS EXAMPLES

- two examples
  - leptoquarks
  - light ALP

LEPTOQUARK EXAMPLE



- scalar leptoquark R_2 in the $(3, 2, 7/6)$ of the SM gauge group

$$\mathcal{L} \supset y^{RL}_{2\,ij}\, \bar{u}^i_R R_2 L^j_L + y^{LR}_{2\,ij}\, \bar{e}^i_R R^*_2 Q^j_L + \text{h.c.},$$

- integrating out R_2 at $\mu = m_{R_2}$ all 10 of the dim 6 operators in WET basis are generated
- in particular operators with quark tensor currents are generated
  - these have coherently enhanced contribs. at subleading powers in v_N, v_μ $\Rightarrow$ required to be kept in **MuonBridge**

$$\mathcal{Q}_{1,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha q)\,, \qquad \mathcal{Q}_{2,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha q)\,,$$

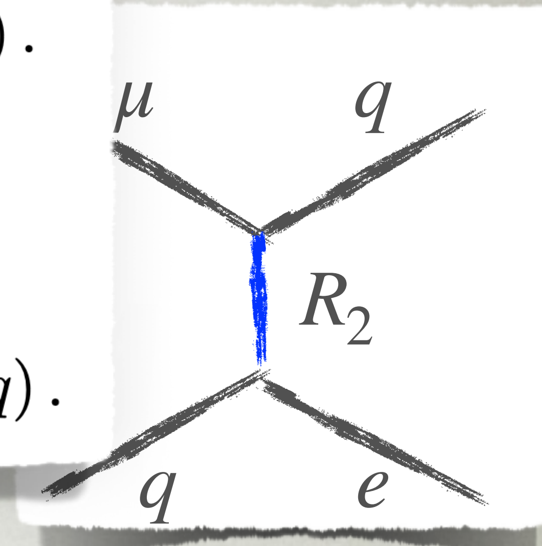
$$\mathcal{Q}_{3,q}^{(6)} = (\bar{e}\gamma_\alpha\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,, \qquad \mathcal{Q}_{4,q}^{(6)} = (\bar{e}\gamma_\alpha\gamma_5\mu)(\bar{q}\gamma^\alpha\gamma_5 q)\,.$$

$$\mathcal{Q}_{5,q}^{(6)} = (\bar{e}\mu)(\bar{q}q)\,, \qquad \mathcal{Q}_{6,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}q)\,,$$

$$\mathcal{Q}_{7,q}^{(6)} = (\bar{e}\mu)(\bar{q}i\gamma_5 q)\,, \qquad \mathcal{Q}_{8,q}^{(6)} = (\bar{e}i\gamma_5\mu)(\bar{q}i\gamma_5 q)\,,$$

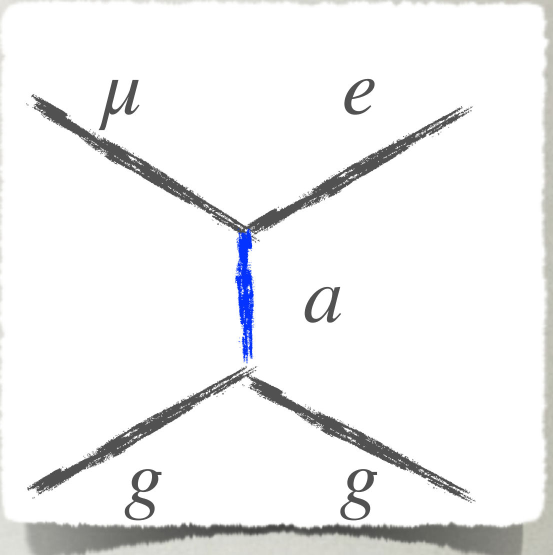
$$\mathcal{Q}_{9,q}^{(6)} = (\bar{e}\sigma^{\alpha\beta}\mu)(\bar{q}\sigma_{\alpha\beta} q)\,, \qquad \mathcal{Q}_{10,q}^{(6)} = (\bar{e}i\sigma^{\alpha\beta}\gamma_5\mu)(\bar{q}\sigma_{\alpha\beta} q)\,.$$

$$\mathcal{L} \supset y_{2\,ij}^{RL}\,\bar{u}_R^i R_2 L_L^j + y_{2\,ij}^{LR}\,\bar{e}_R^i R_2^* Q_L^j + \text{h.c.},$$



- integrating out R_2 at $\mu = m_{R_2}$ all 10 of the dim 6 operators in WET basis are generated
- in particular operators with quark tensor currents are generated
  - these have coherently enhanced contribs. at subleading powers in v_N, v_μ ⇒ required to be kept in **MuonBridge**

LIGHT ALP



- the same formalism trivially extends to light mediators
- example light ALP coupling to μe and gluons
- strictly speaking WET no longer an appropriate EFT
  - but trivial fix, allow Wilson coeffs to be q^2 dependent, $C_i \propto 1/(m_a^2 - q^2)$
  - since a only weakly couples to gluons: corrections to QCD can be neglected, i.e., just an external probe
  - in $\mu \to e$ the q is fixed, so C_i are even constants

CONCLUSIONS

- EFT approach well suited for predicting the $\mu \to e$ conversion rates
- presented results that include up to dim-7 ops in WET, subleading effects in NRET
- results will be available in the form of a public code **MuonBridge**

BACKUP SLIDES

TABLE I. An incomplete survey of elastic $\mu \to e$ conversion studies including the nuclear targets considered, the nuclear multipole operators evaluated, and the form of the lepton wave functions employed. $\mathcal{O}_{J;\tau}$ means that both isospin structures and all allowed J were included. For the Dirac electron, all of the references surveyed restrict attention to the lowest partial waves $\kappa = \pm 1$. Besides the Dirac solution, the remaining forms of the muon wave function are all constant approximations: $\langle|\psi_\mu|^2\rangle_\rho$ is obtained by averaging the probability of the Dirac solution over the nuclear density, $|\phi_{1s}^Z(\vec{0})|^2$ is the probability of the point-like Schrodinger solution evaluated at the origin, $|G(R_N)|^2$ is the upper component of the muon's Dirac wave function evaluated at the nuclear radius. Superscript † indicates that the reference considers the inelastic process as well, although the information in the table reflects only the treatment of the elastic process.

Author(s)	Year [Ref]	Target	Operators	ψ_e	ψ_μ
Weinberg and Feinberg†	1959 [23]	multiple	$M_{0;p}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Marciano and Sanda	1977 [24]	multiple	$M_{0;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Shanker	1979 [25]	multiple	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Kosmas and Vergados†	1990 [26]	multiple	$M_{0;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Chiang et al.†	1993 [27]	multiple	$M_{0;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Kosmas et al.†	1994 [28]	^{48}Ti	$M_{0;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Czarnecki, Marciano, and Melnikov	1998 [29]	^{27}Al, ^{48}Ti, ^{208}Pb	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Siiskonen, Suhonen, and Kosmas†	2000 [30]	^{27}Al, ^{48}Ti	$M_{J;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Kosmas†	2001 [31]	^{48}Ti, ^{208}Pb	$M_{J;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$
Kitano, Koike, and Okada	2002 [32]	multiple	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Kosmas	2003 [33]	multiple	$M_{0;\tau}$	Plane wave	Dirac
Cirigliano et al.	2009 [6]	multiple	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Crivellin et al.	2017 [34]	^{27}Al, ^{197}Au	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Bartolotta and Ramsey-Musolf	2018 [35]	^{27}Al	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Cirigliano, Davidson, and Kuno	2018 [36]	^{27}Al	$M_{0;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$	Plane wave	$\|\phi_{1s}^Z(\vec{0})\|^2$
Davidson, Kuno, and Saporta	2018 [37]	^{27}Al, Ti	$M_{0;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$	Plane wave	$\|\phi_{1s}^Z(\vec{0})\|^2$
Civitarese and Tarutina†	2019 [38]	^{208}Pb	$M_{0;\tau}$	Plane wave	$\|G(R_N)\|^2$
Rule, Haxton, and McElvain	2021 [16]	^{27}Al, Ti	$M_{J;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$ $\Delta_{J;\tau}$ $\tilde{\Phi}'_{J;\tau}$ $\Phi''_{J;\tau}$	Full Dirac (q_{eff})	$\|\phi_{1s}^{Z_{\text{eff}}}(\vec{0})\|^2$
Heeck, Szafron, and Uesaka	2022 [39]	multiple	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Cirigliano et al.	2022 [40]	^{27}Al,^{48}Ti,^{197}Au,^{208}Pb	$M_{0;\tau}$	Dirac, $\|\kappa\| = 1$	Dirac
Hoferichter, Menéndez, and Noël	2022 [41]	^{27}Al,Ti	$M_{J;\tau}$ $\Sigma'_{J;\tau}$ $\Sigma''_{J;\tau}$ $\Phi''_{J;\tau}$	Plane wave	$\langle\|\psi_\mu\|^2\rangle_\rho$

Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945

TABLE XI. Input parameters and output quantities for the muon and electron Dirac solutions discussed in the text.

Nucleus	c (fm)	β (fm)	$\sqrt{\langle r^2 \rangle}$ (fm)	$E_\mu^{\rm bind}$ (MeV)	$\int_0^\infty \lvert F_{1s}\rvert^2 dr$	Z	$Z_{\rm eff}$	R	q (MeV)	$q_{\rm eff}$ (MeV)
^{12}C	2.215	0.491	2.505	0.1000	0.00047	6	5.7030	0.8587	105.07	108.40
^{16}O	2.534	0.514	2.739	0.1775	0.00083	8	7.4210	0.7982	105.11	109.16
^{19}F	2.580	0.567	2.904	0.2242	0.00104	9	8.2298	0.7646	105.12	109.44
^{23}Na	2.760	0.543	2.940	0.3337	0.00154	11	9.8547	0.7190	105.07	110.25
^{27}Al	3.070	0.519	3.062	0.4630	0.00211	13	11.3086	0.6583	104.98	110.81
^{28}Si	3.140	0.537	3.146	0.5346	0.00241	14	12.0009	0.6299	104.91	111.03
^{32}S	3.161	0.578	3.239	0.6924	0.00308	16	13.1839	0.5595	104.78	111.56
^{40}Ca	3.621	0.563	3.499	1.0585	0.00453	20	15.6916	0.4830	104.45	112.28
^{48}Ti	3.843	0.588	3.693	1.2615	0.00527	22	16.6562	0.4340	104.28	112.43
^{56}Fe	4.111	0.558	3.800	1.7182	0.00690	26	18.6028	0.3663	103.84	113.16
^{63}Cu	4.218	0.596	3.947	2.0884	0.00811	29	19.8563	0.3210	103.48	113.50
^{184}W	6.510	0.535	5.421	9.0851	0.01169	74	32.2914	0.0831	96.54	114.95

Haxton, McElvain, Ramsey-Mussolf, Rule, 2208.07945