Coupled cluster theory for neutrino scattering

Joanna Sobczyk

NuInt24, 16 April 2024

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter

JG|U

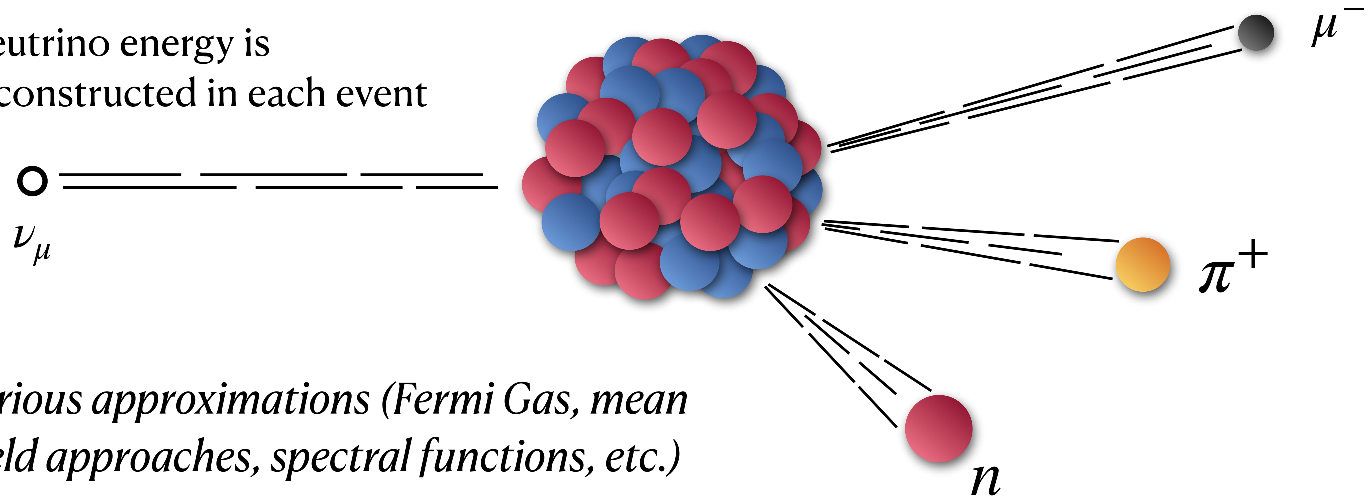
Alexander von Humboldt Stiftung/Foundation

MARIE CURIE ACTIONS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement No. 101026014

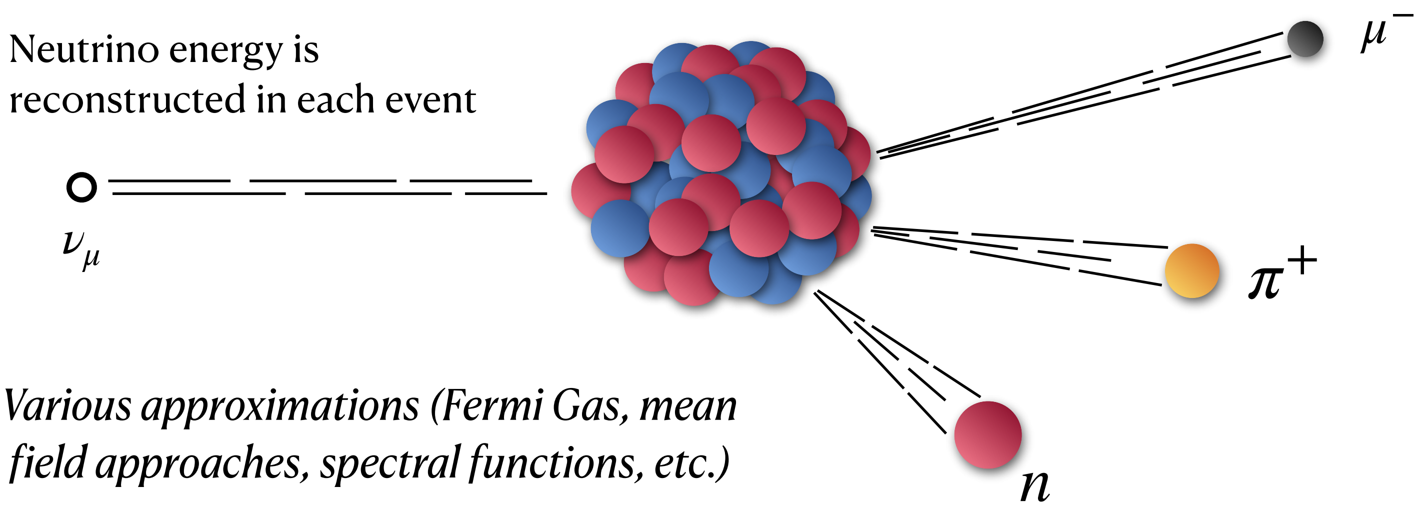
Motivation

Neutrino energy is
reconstructed in each event

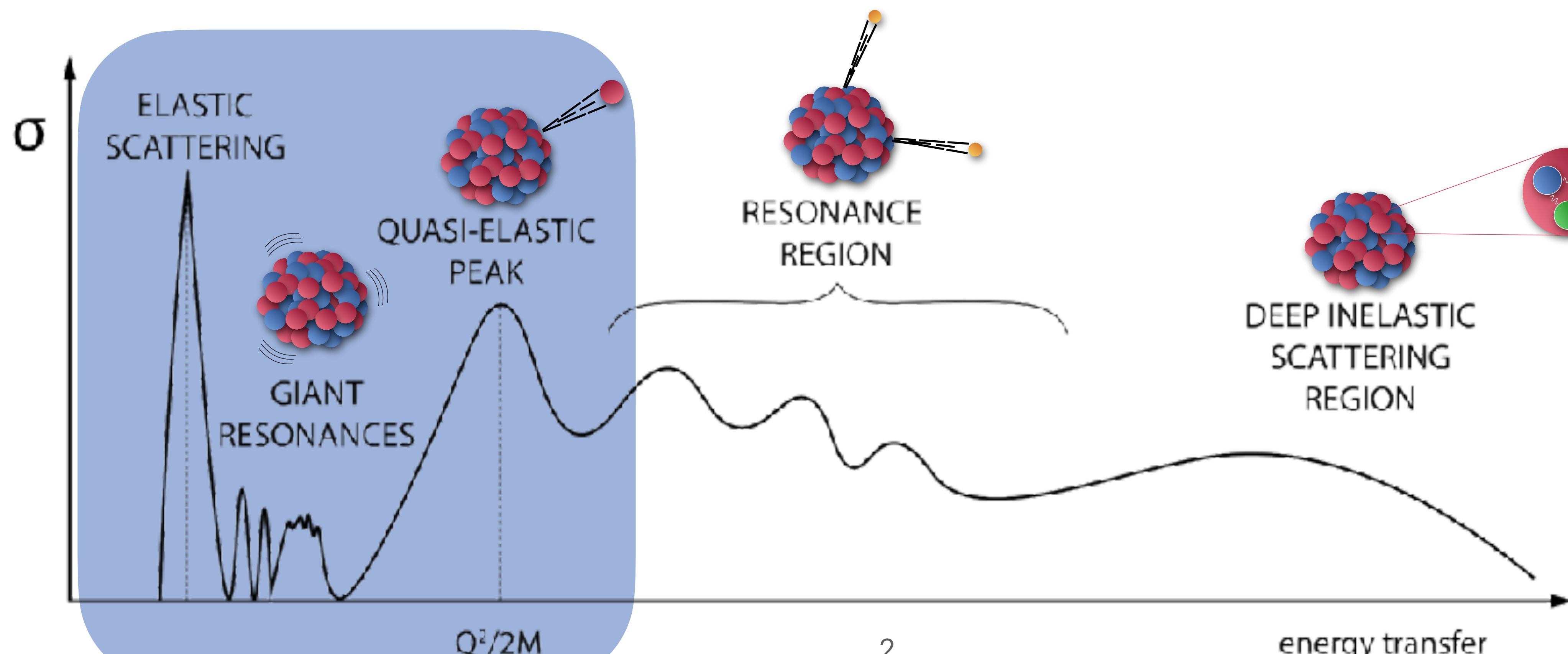


Various approximations (Fermi Gas, mean field approaches, spectral functions, etc.)

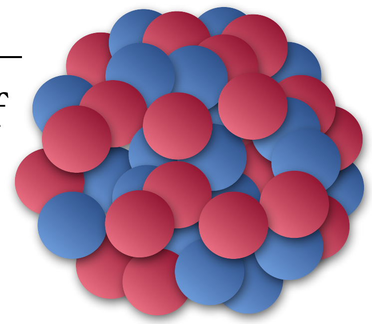
Motivation



WHAT CAN WE LEARN FROM A (MORE) FUNDAMENTAL THEORY?

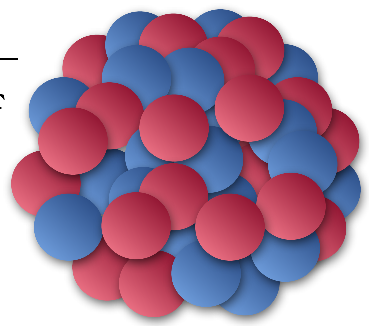


“Ab initio” nuclear theory



$$\mathscr{H}\,|\,\Psi\rangle = E\,|\,\Psi\rangle$$

"Ab initio" nuclear theory



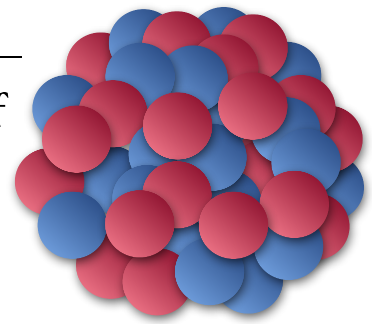
$$\mathscr{H}\,|\,\Psi\rangle = E\,|\,\Psi\rangle$$

What is the dynamics of our system?

$$\mathscr{H} = \sum_{i=1}^{A} t_{kin} + \sum_{i>j=1}^{A} v_{ij} + \sum_{i>j>k=1}^{A} v_{ijk} + \dots$$

“Ab initio” nuclear theory

nucleons — degrees of freedom



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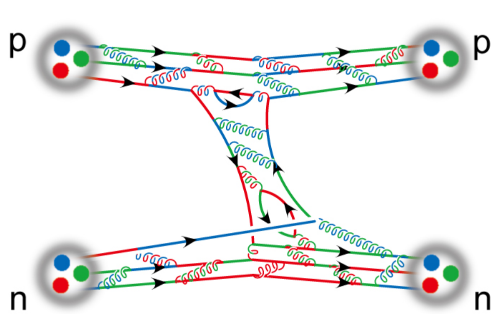
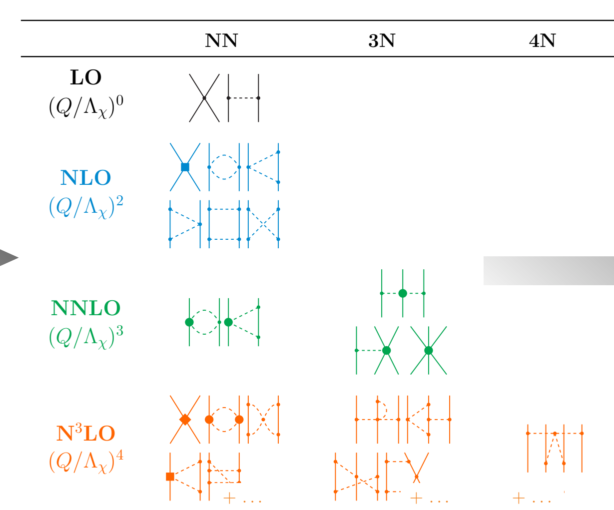
How the nuclear force is rooted in the fundamental theory of QCD?

Quantum Chromodynamics



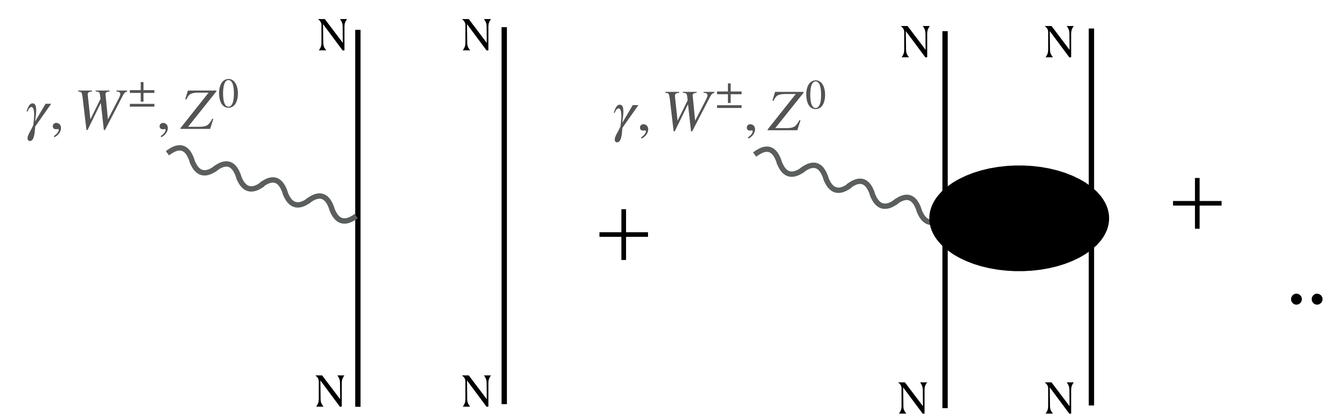


Nuclei & nuclear matter

Electroweak interactions

- Chiral EFT allows to construct electroweak currents consistently with the chiral potential

$$j = \sum_{i=1}^{A} j_i + \sum_{j<i=1}^{A} j_{ij} + \sum_{k<j<i=1}^{A} j_{ijk} + \dots$$



To describe:

➡ Electroweak form-factors

➡ Gamow-Teller ME (β decays)

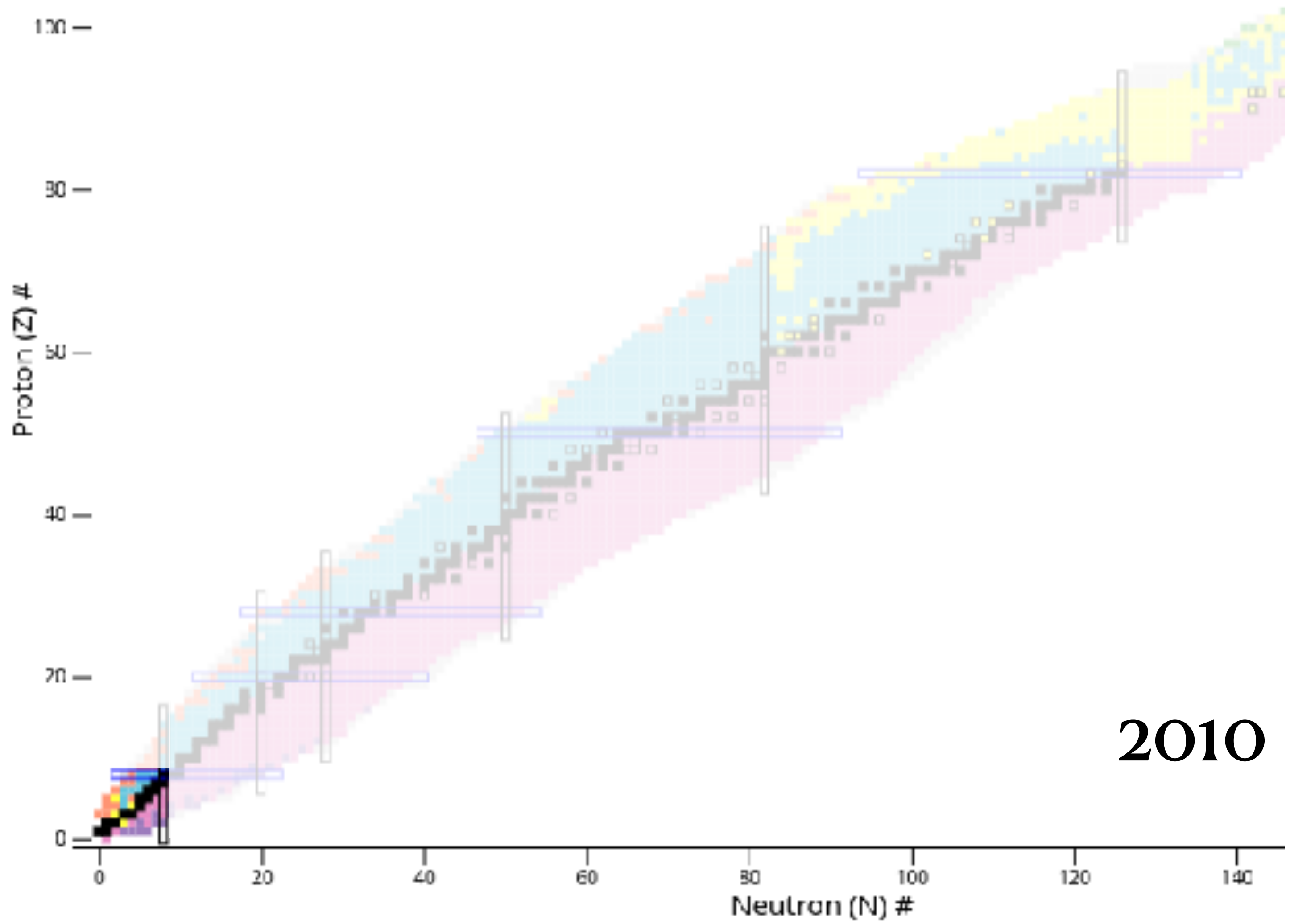
➡ Magnetic moments

➡ Radiative/weak captures

➡ **Electroweak response functions**

Ab initio nuclear theory

$$\mathscr{H}\,|\Psi\rangle = E\,|\Psi\rangle$$



Ab initio nuclear theory

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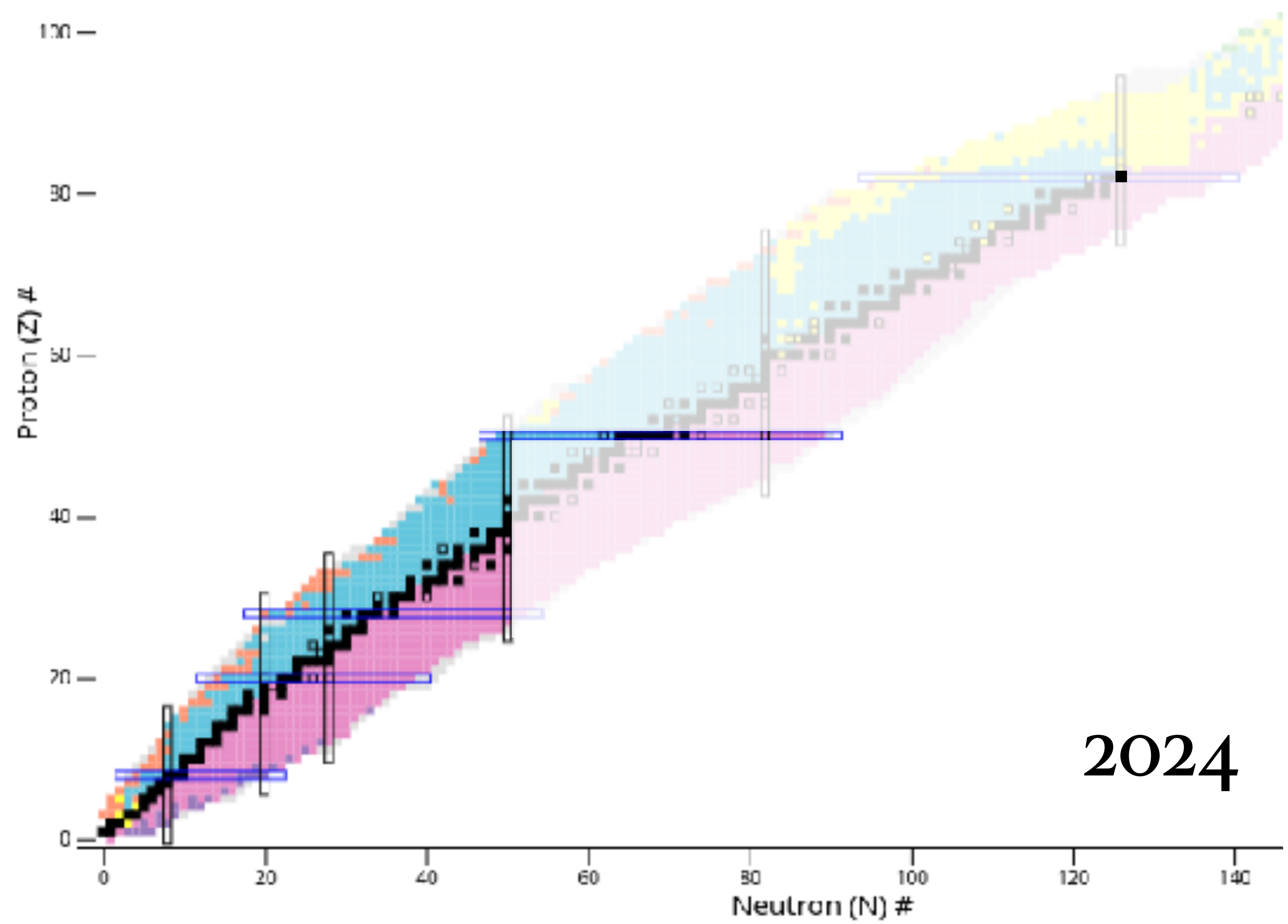

- ✓ Computational power
- ✓ Polynomial scaling with A
- ✓ "Softer" Hamiltonians (better convergence)

Ab initio nuclear theory

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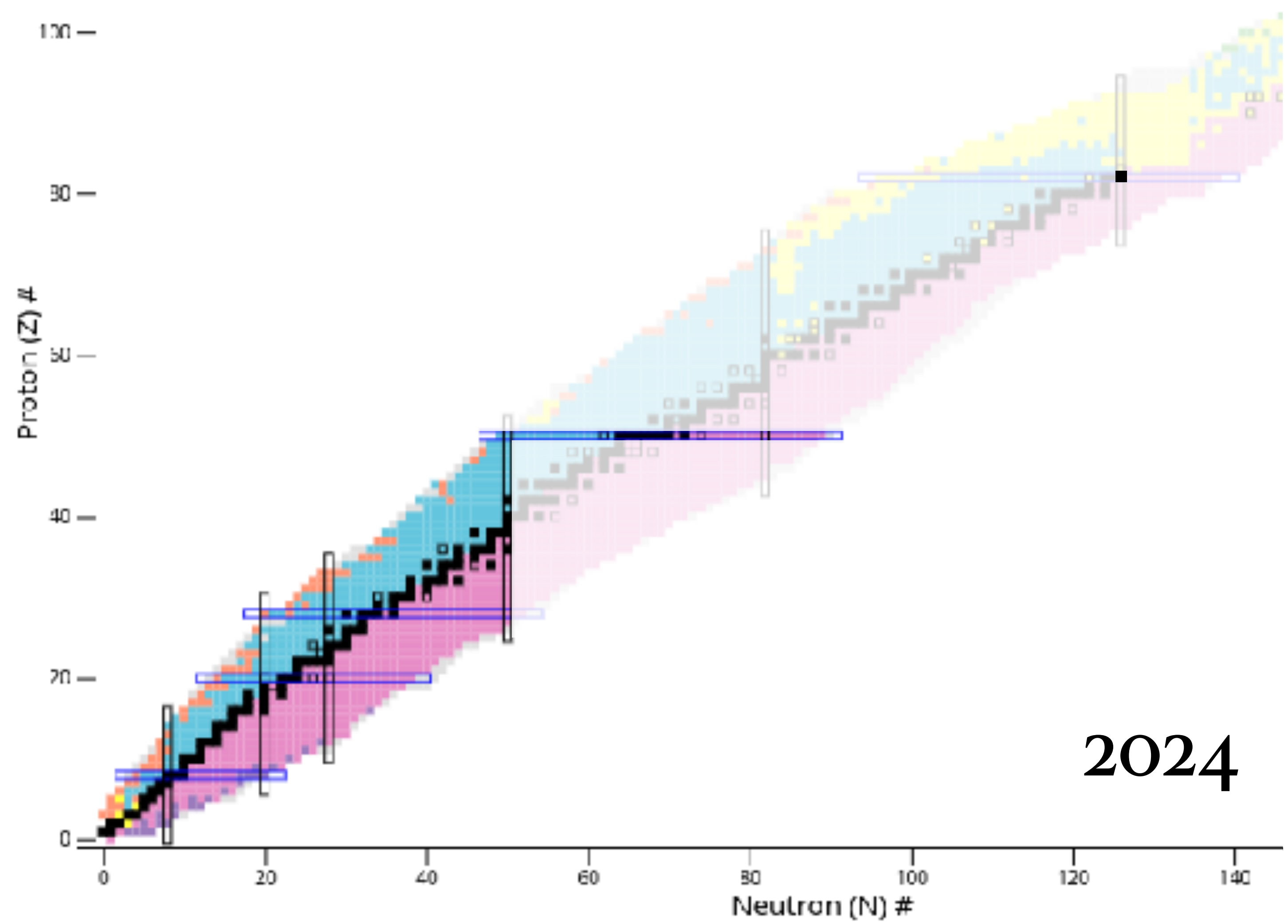

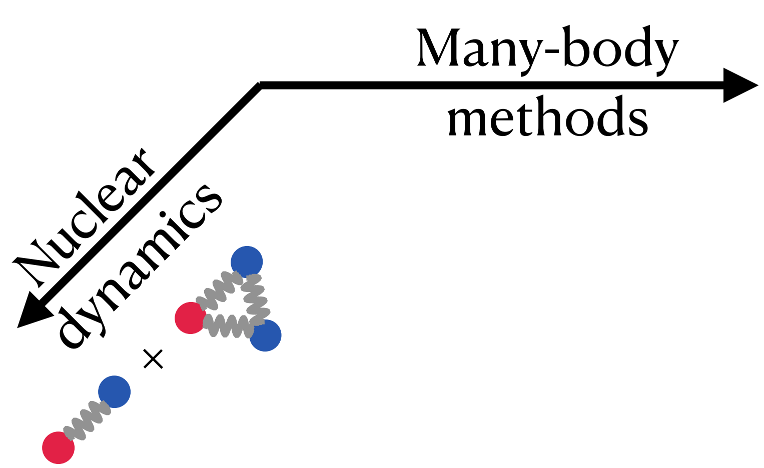

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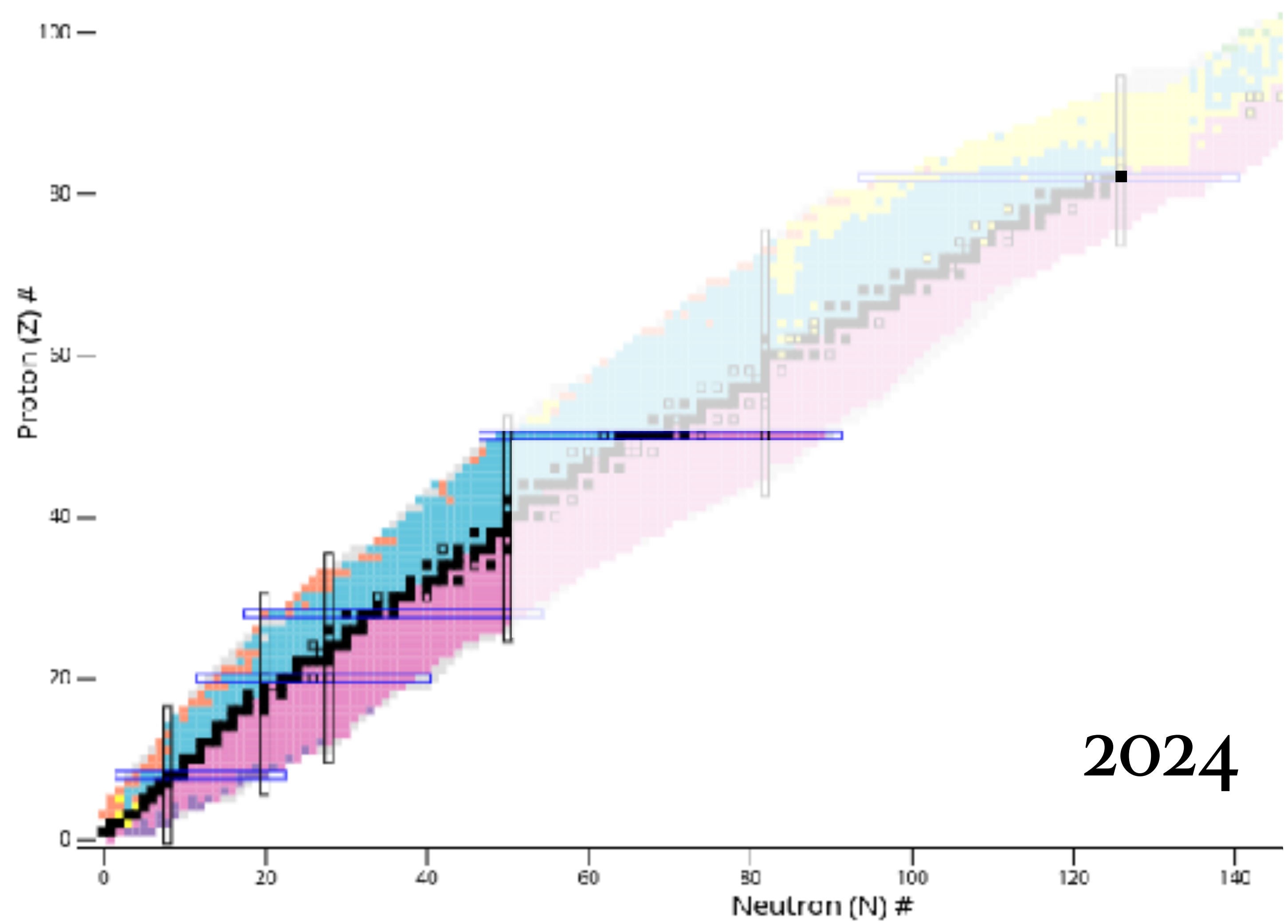

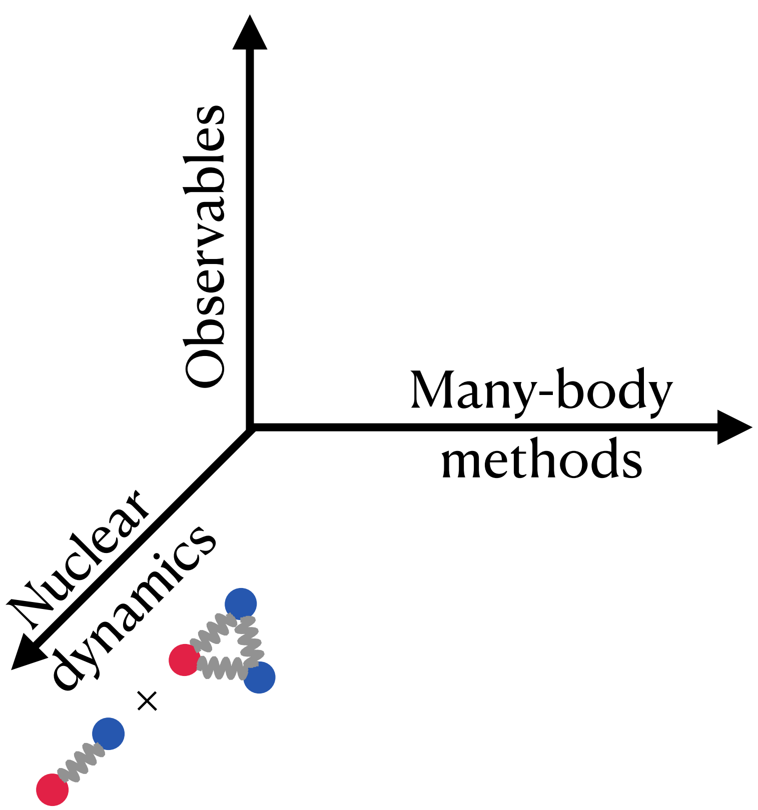

- ✓ Computational power
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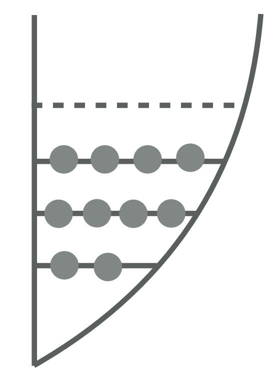
Coupled cluster theory

Reference state (Hartree-Fock): $|\Psi\rangle = a_i^\dagger a_j^\dagger \dots a_k^\dagger |0\rangle$



Include **correlations** through e^T operator

$$\mathcal{H}_N e^T |\Psi\rangle = E e^T |\Psi\rangle$$

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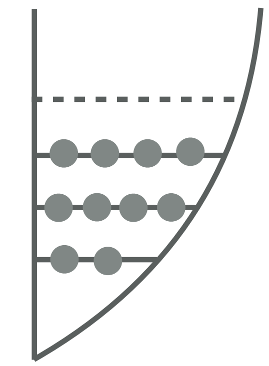


Include **correlations** through e^T operator

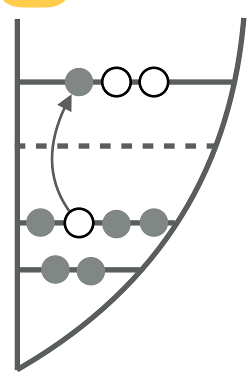
$$\mathscr{H}_N e^T |\Psi\rangle = E e^T |\Psi\rangle$$

1p1h

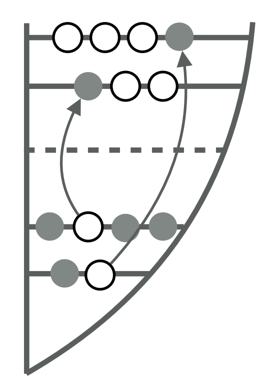
2p2h

Expansion: $T = \sum t_a^i a_a^\dagger a_i + \frac{1}{4} \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \ldots$

singles



doubles



G. Hagen, T. Papenbrock, M. Hjorth-Jensen, D. J. Dean, Rep. Prog. Phys. 77, 096302 (2014).

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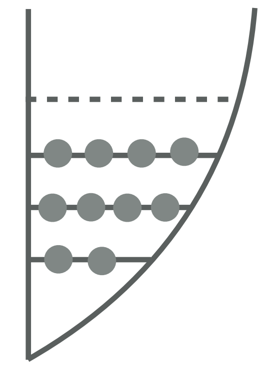

Include **correlations** through e^T operator

$$\mathscr{H}_N e^T|\Psi\rangle = E e^T|\Psi\rangle$$

- ✓ Controlled approximation through truncation in T
- ✓ Polynomial scaling with A (predictions for ^{132}Sn and ^{208}Pb)

1p1h

2p2h

Expansion: $T = \sum t_a^i a_a^\dagger a_i + \frac{1}{4}\sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \ldots$

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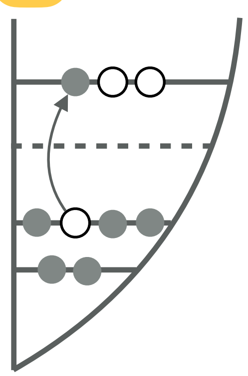

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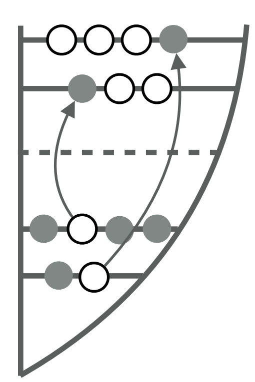

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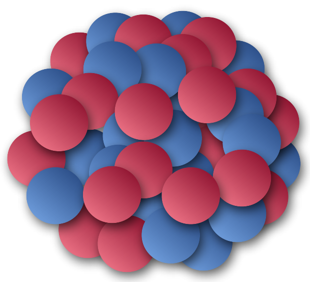
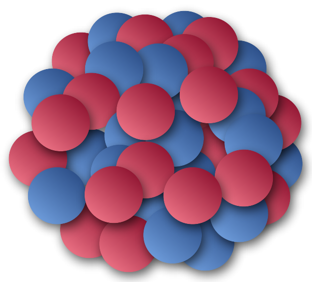
Low/high energies



$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$

Many-body problem

Low/high energies



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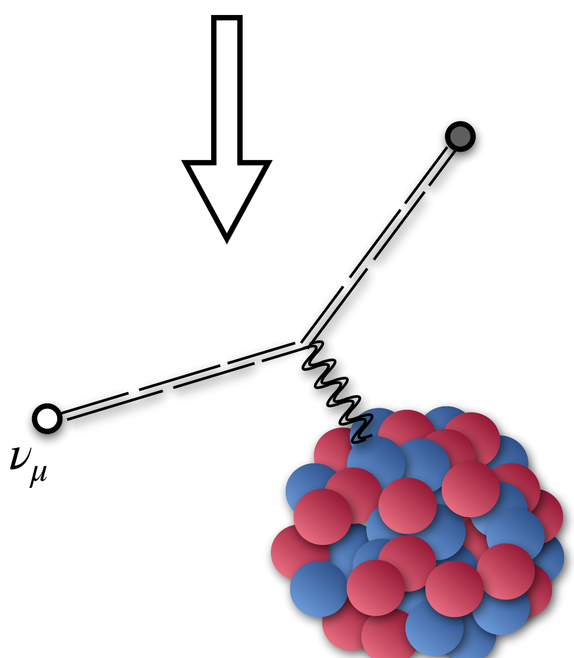
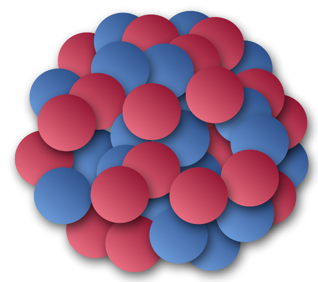


$$\langle\psi_f|\hat{j}|\psi_A\rangle$$

Electroweak responses
consistent treatment of final states

Low/high energies



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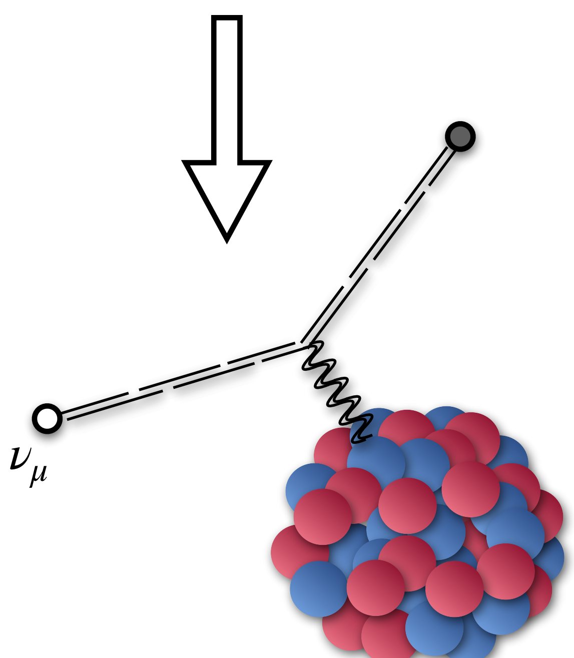

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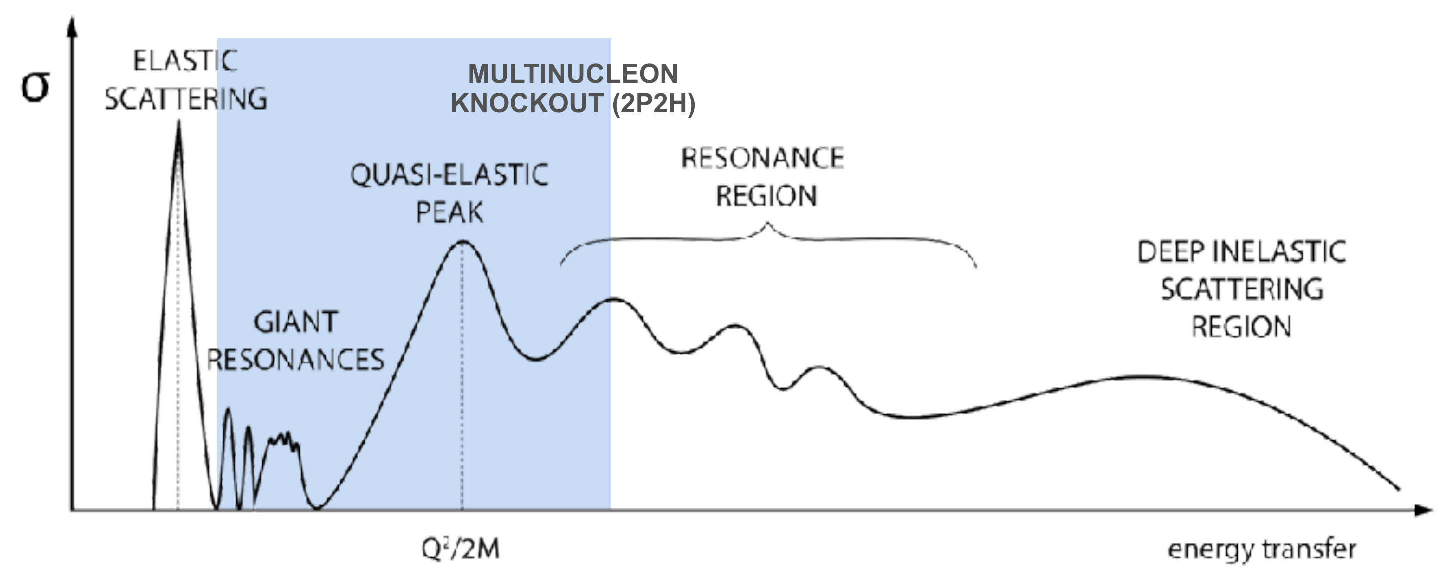

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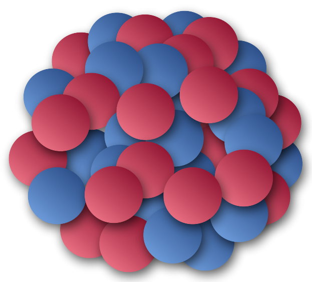

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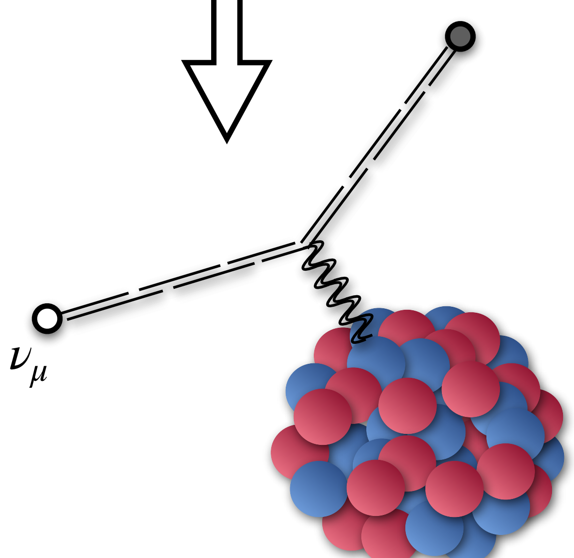

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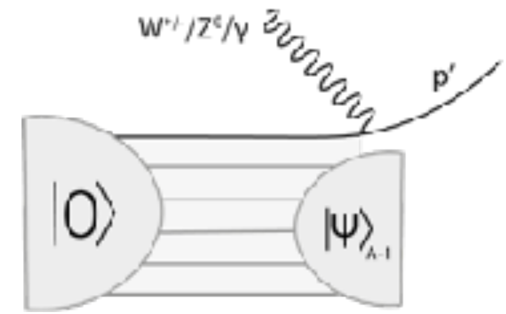
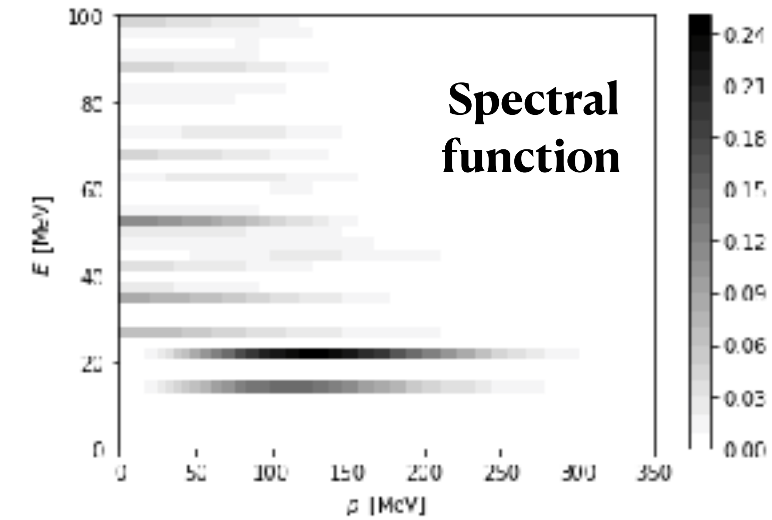
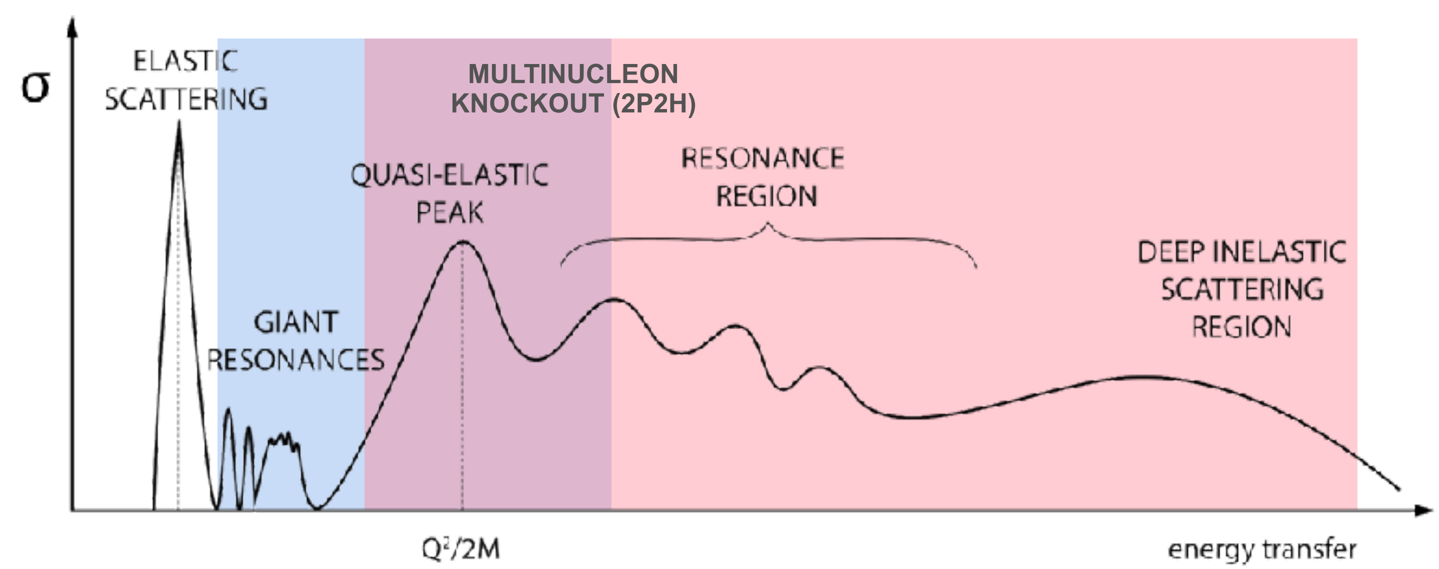


Impulse Approximation



Probability density of finding nucleon $(E, \mathbf{p})$ in ground state nucleus



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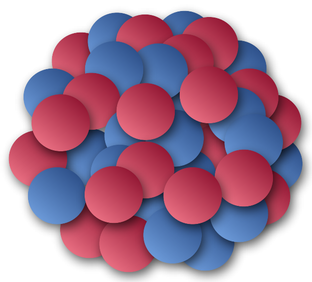

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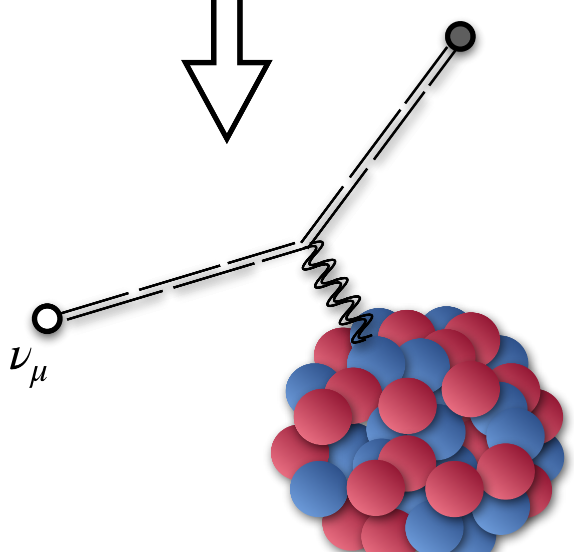

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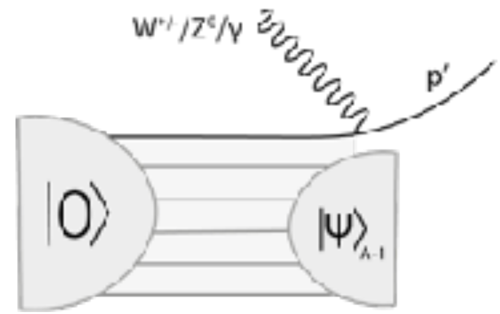

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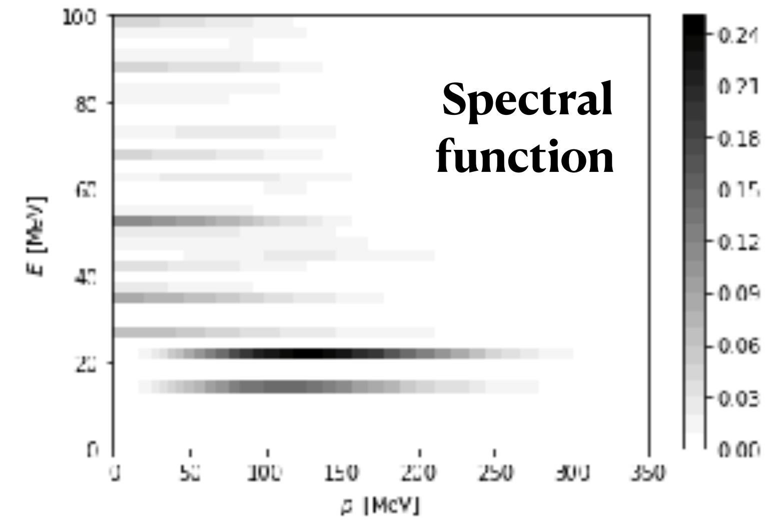

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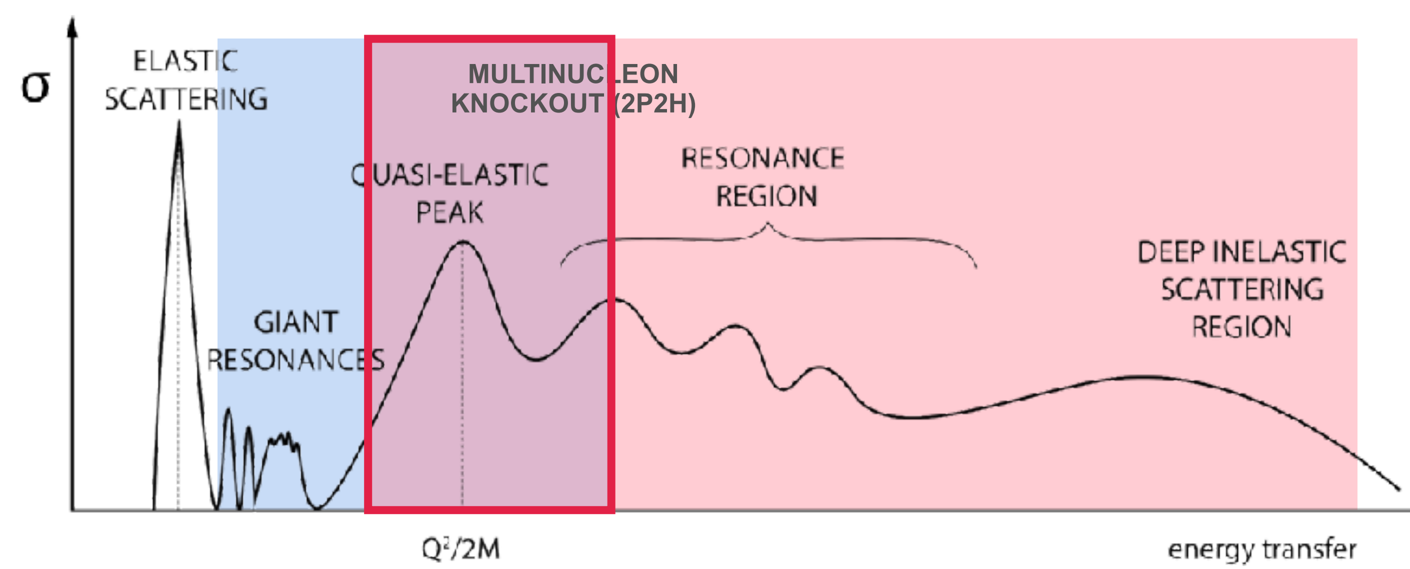
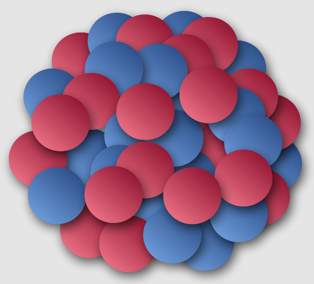


Possible comparison within the same framework

Low/high energies



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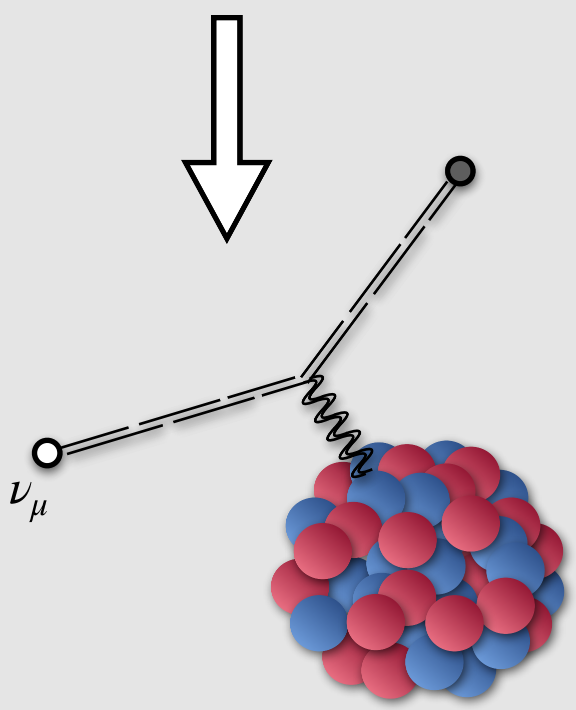
Many-body problem



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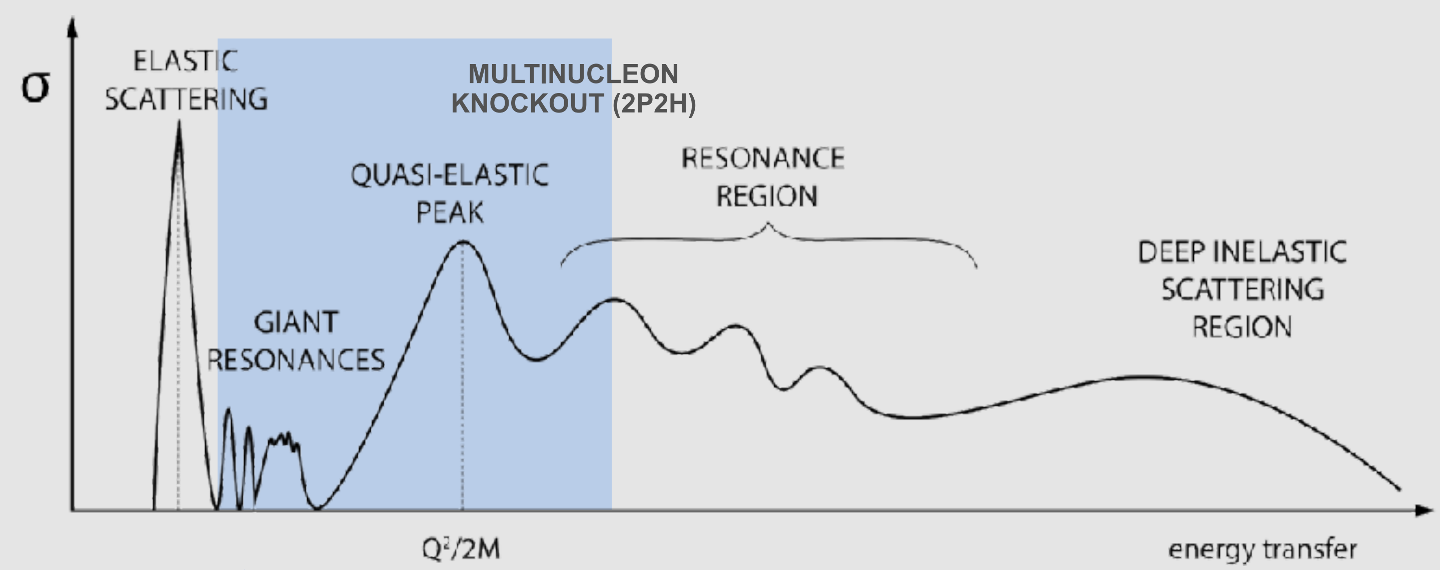
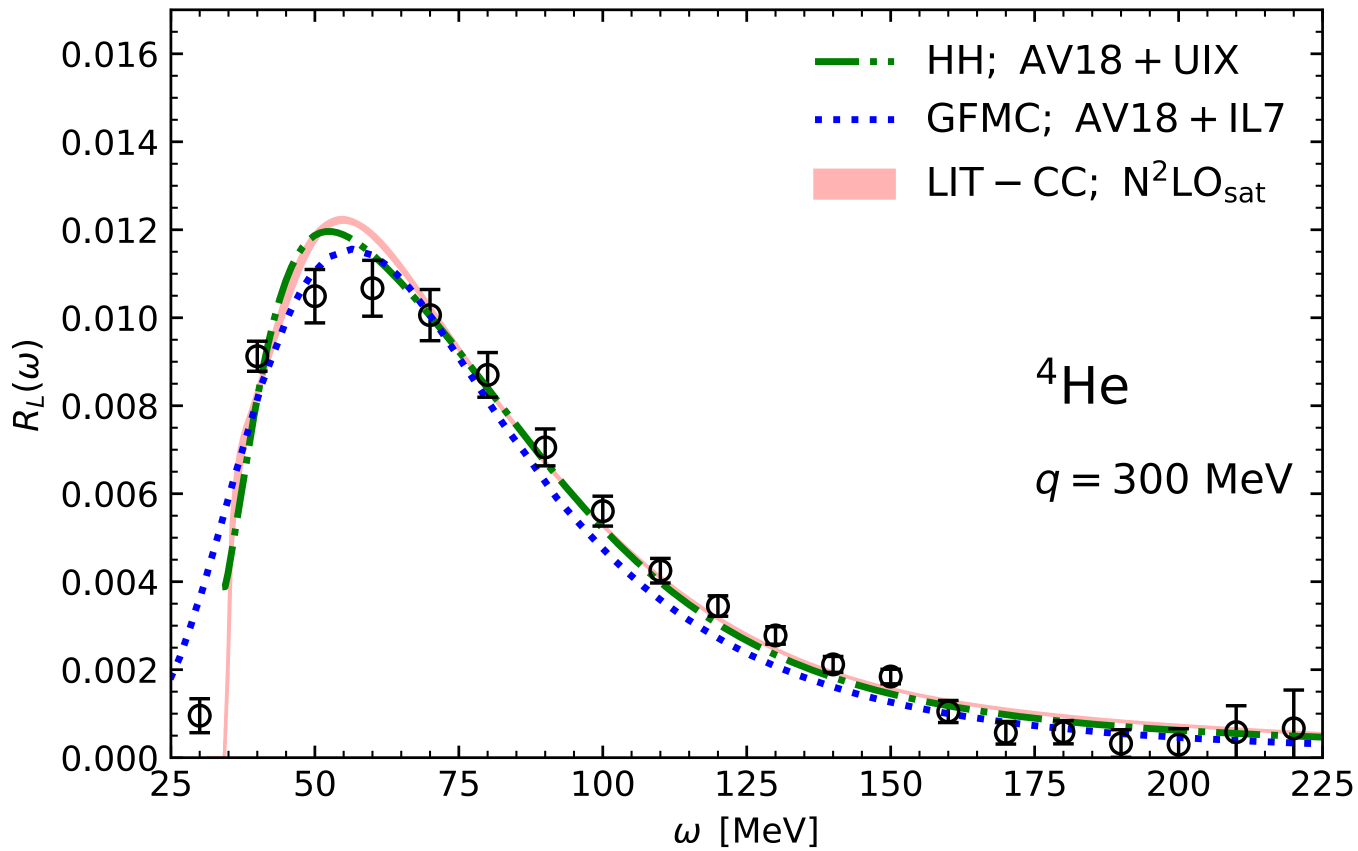
Electroweak responses



Longitudinal response

Lorentz Integral Transform + Coupled Cluster **(LIT-CC)**

$$\left.\frac{d\sigma}{d\omega dq}\right|_e = \sigma_M\left(v_L R_L + v_T R_T\right)$$



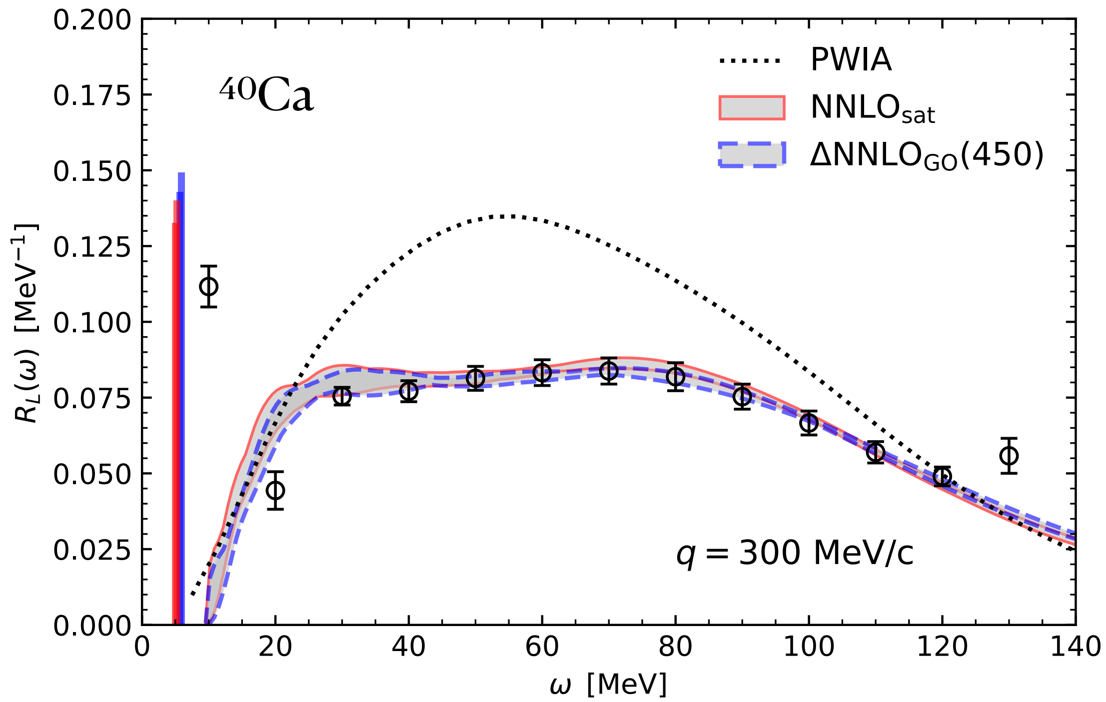
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

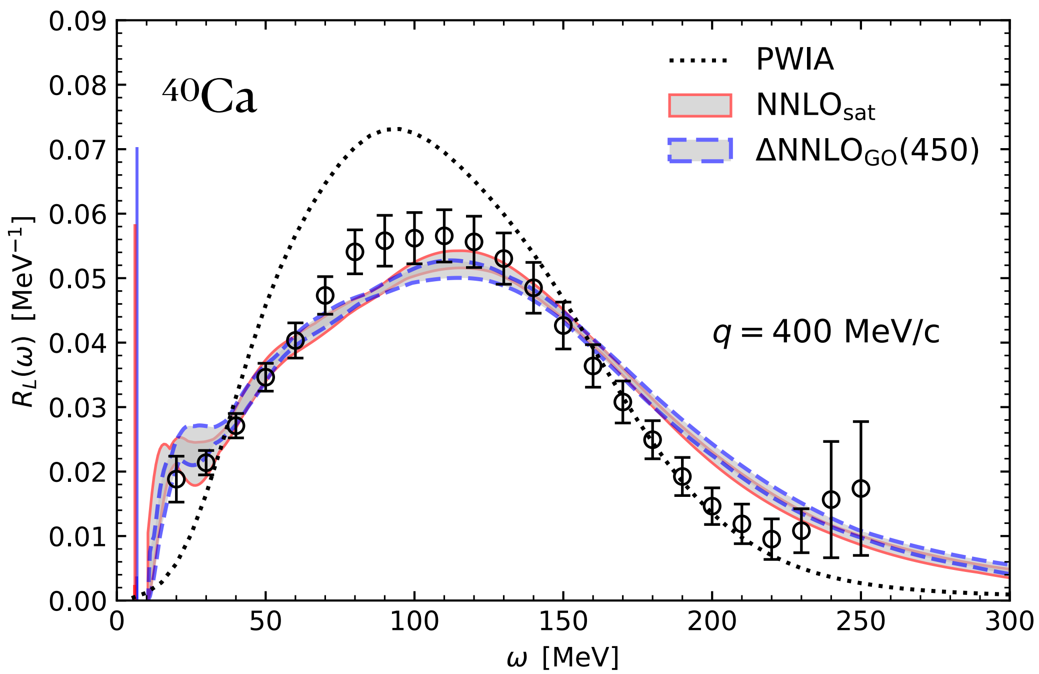
$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle\Psi\,|\,J_\mu^\dagger\,|\,\Psi_f\rangle\langle\Psi_f\,|\,J_\nu\,|\,\Psi\rangle\delta(E_0 + \omega - E_f)$$

Consistent treatment of final state interactions.

Longitudinal response ^{40}Ca

Lorentz Integral Transform + Coupled Cluster **(LIT-CC)**



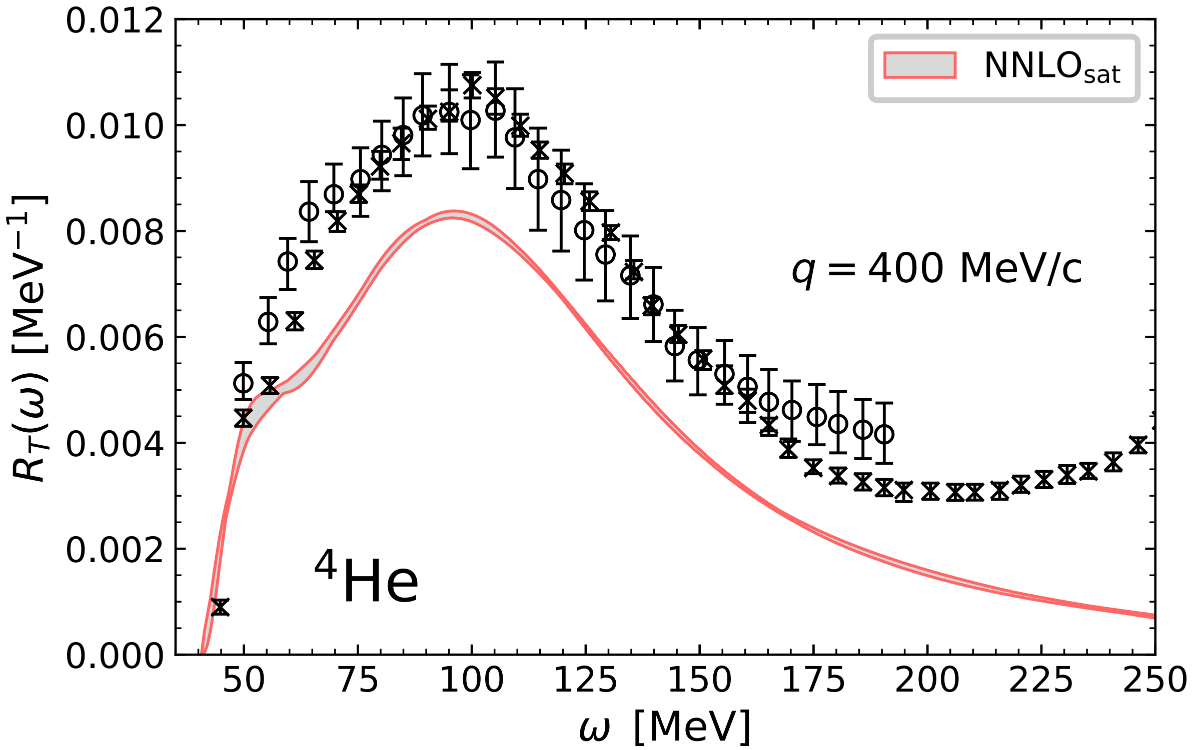


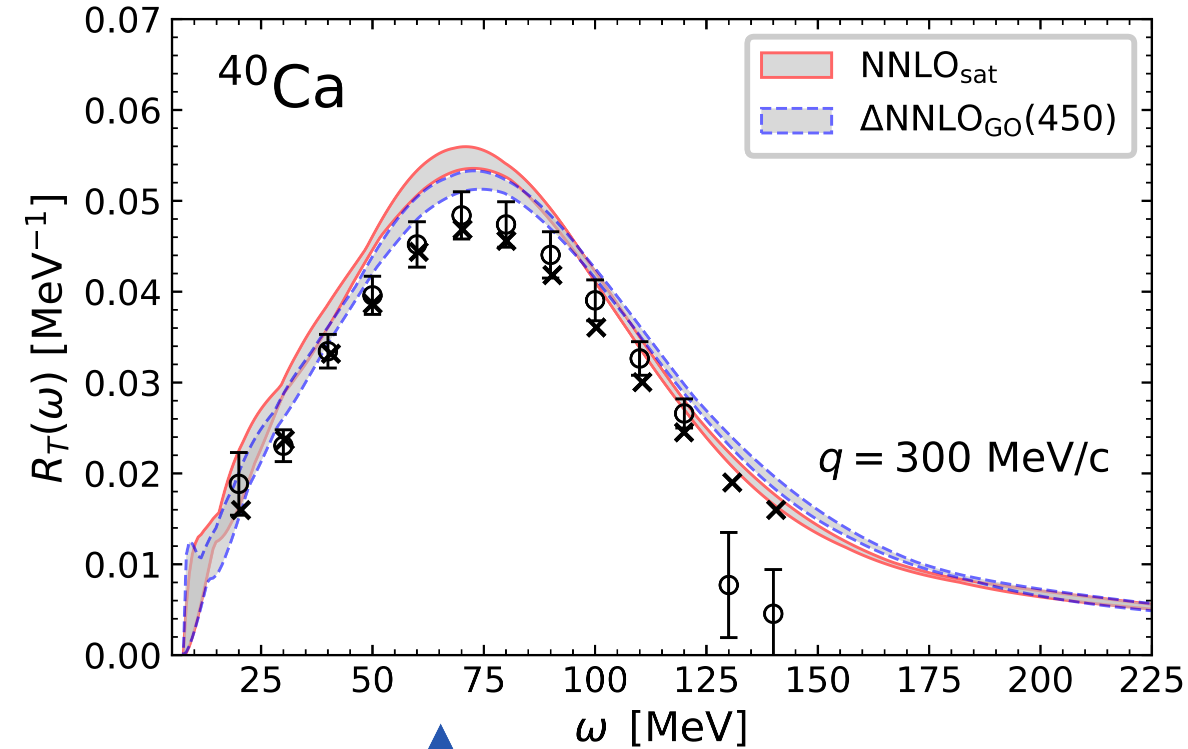
JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501

- ✓ Coupled cluster singles & doubles
- ✓ Two different chiral Hamiltonians
- ✓ Uncertainty from LIT inversion

First ab-initio results for many-body system of 40 nucleons

Transverse response





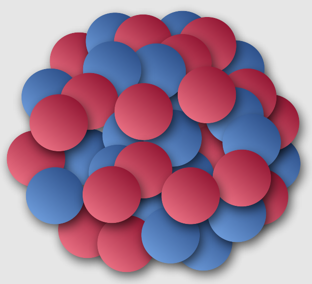
JES, B. Acharya, S. Bacca, G. Hagen;
PRC 109 (2024) 2, 025502

$$\left.\frac{d\sigma}{d\omega dq}\right|_e = \sigma_M\left(v_L R_L + v_T R_T\right)$$

- ➡ This allows to predict electron-nucleus cross-section
- ➡ Currently only 1-body current

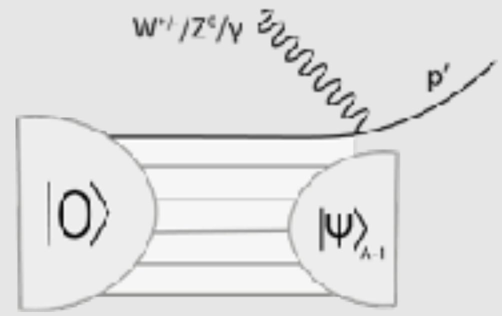
2-body currents important for ^{4}He
→ more correlations needed?
→ 2-body currents strength depends on nucleus?

Low/high energies

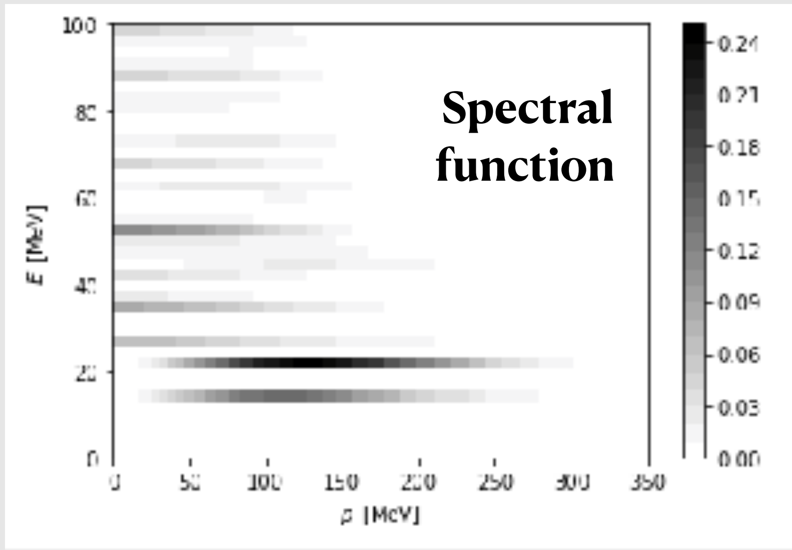


$$\hat{H}\,|\psi_A\rangle = E\,|\psi_A\rangle$$

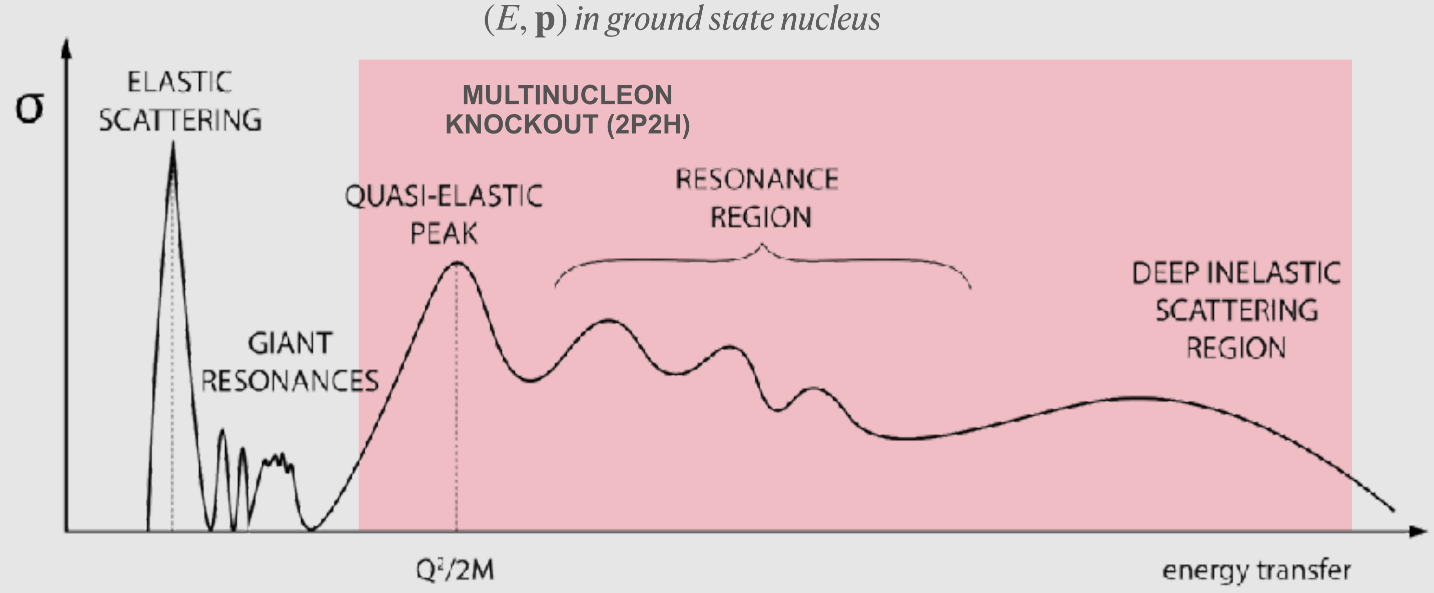
Many-body problem



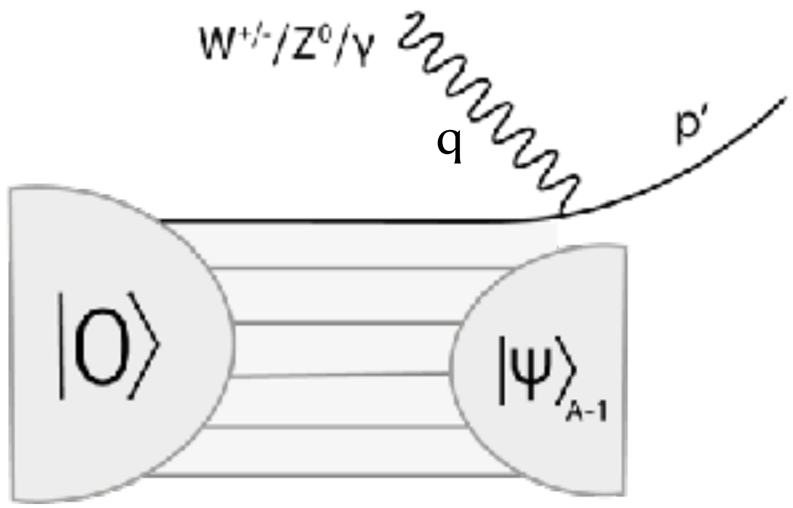
Impulse Approximation



Probability density of finding nucleon
$(E, \mathbf{p})$ *in ground state nucleus*



^{4}He spectral function



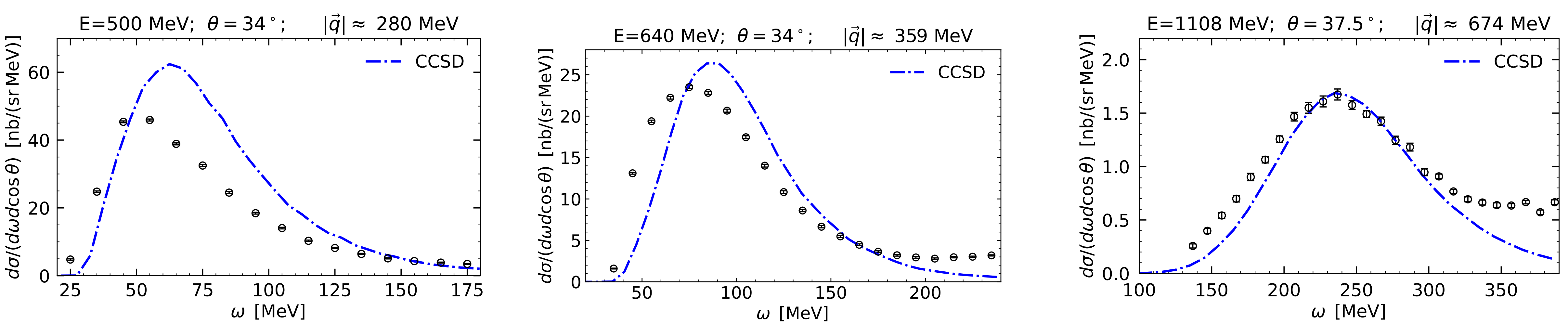
$$\sigma \propto |\mathscr{M}|^2 S(E,p)$$

Factorized interaction vertex (relativistic, pion production...)

Spectral function - nuclear information

growing **q** momentum transfer → final state interactions play minor role

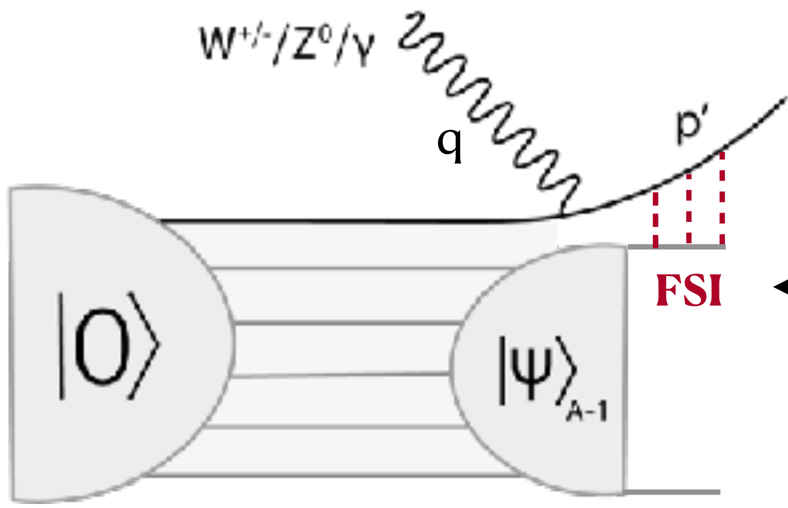
Scattering off **^{4}He**



JES, S. Bacca, G. Hagen, T. Papenbrock *Phys.Rev.C* 106 (2022) 3, 034310

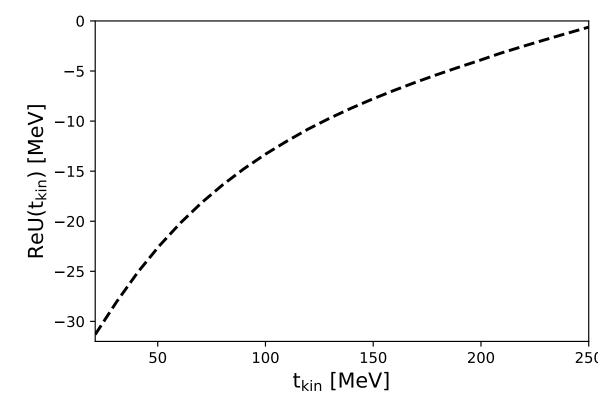
^{16}O spectral function

Error propagation to cross sections



Phenomenological optical potential

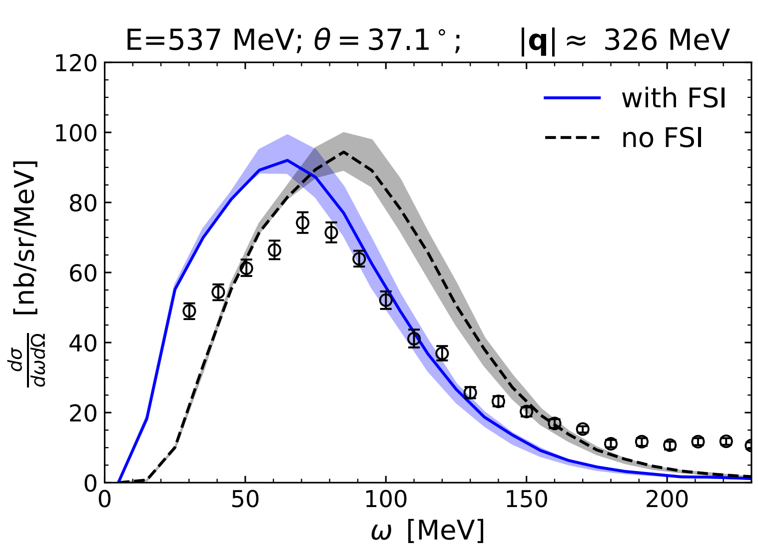
$E_{p+q} \to E_{p+q} + \mathrm{Re}U(t_{kin})$

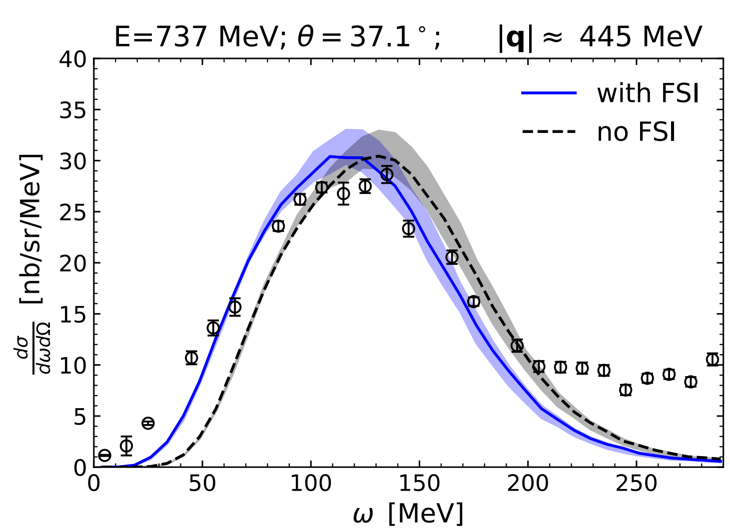


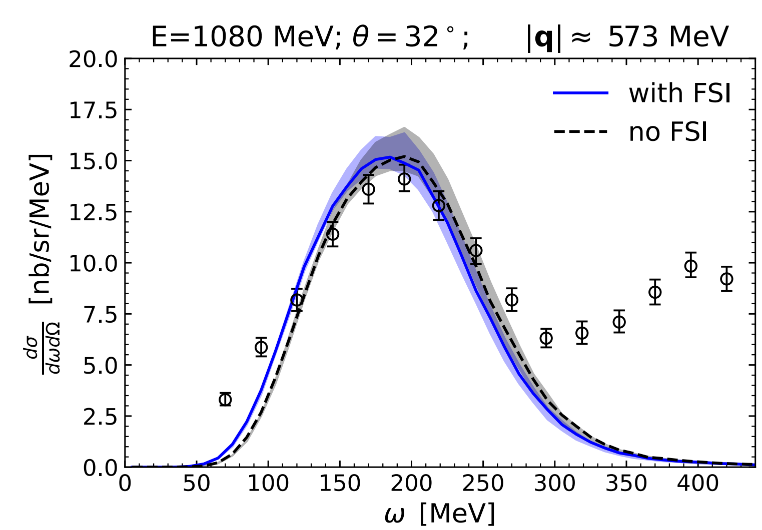
E. D. Cooper et al. *Phys.Rev.C 47, 297–311*

growing **q** momentum transfer → final state interactions play minor role

Scattering off ^{16}O







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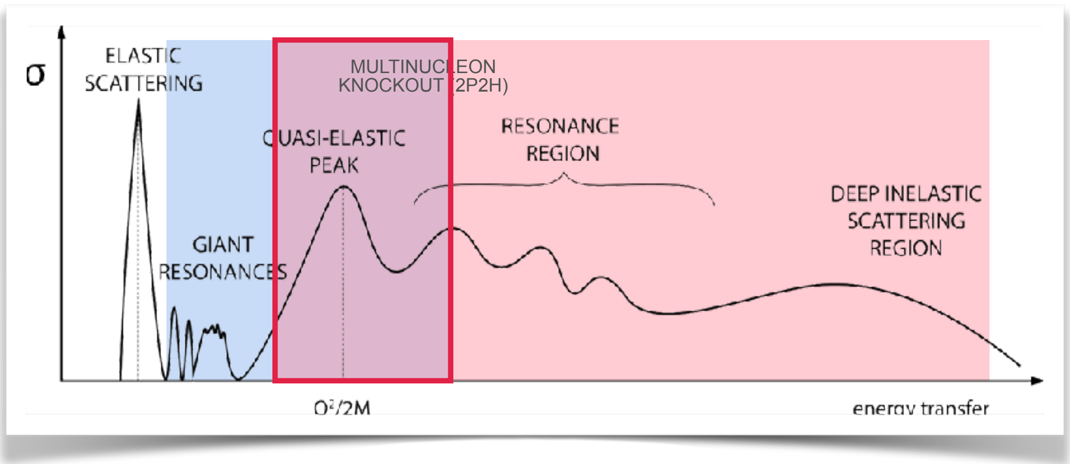
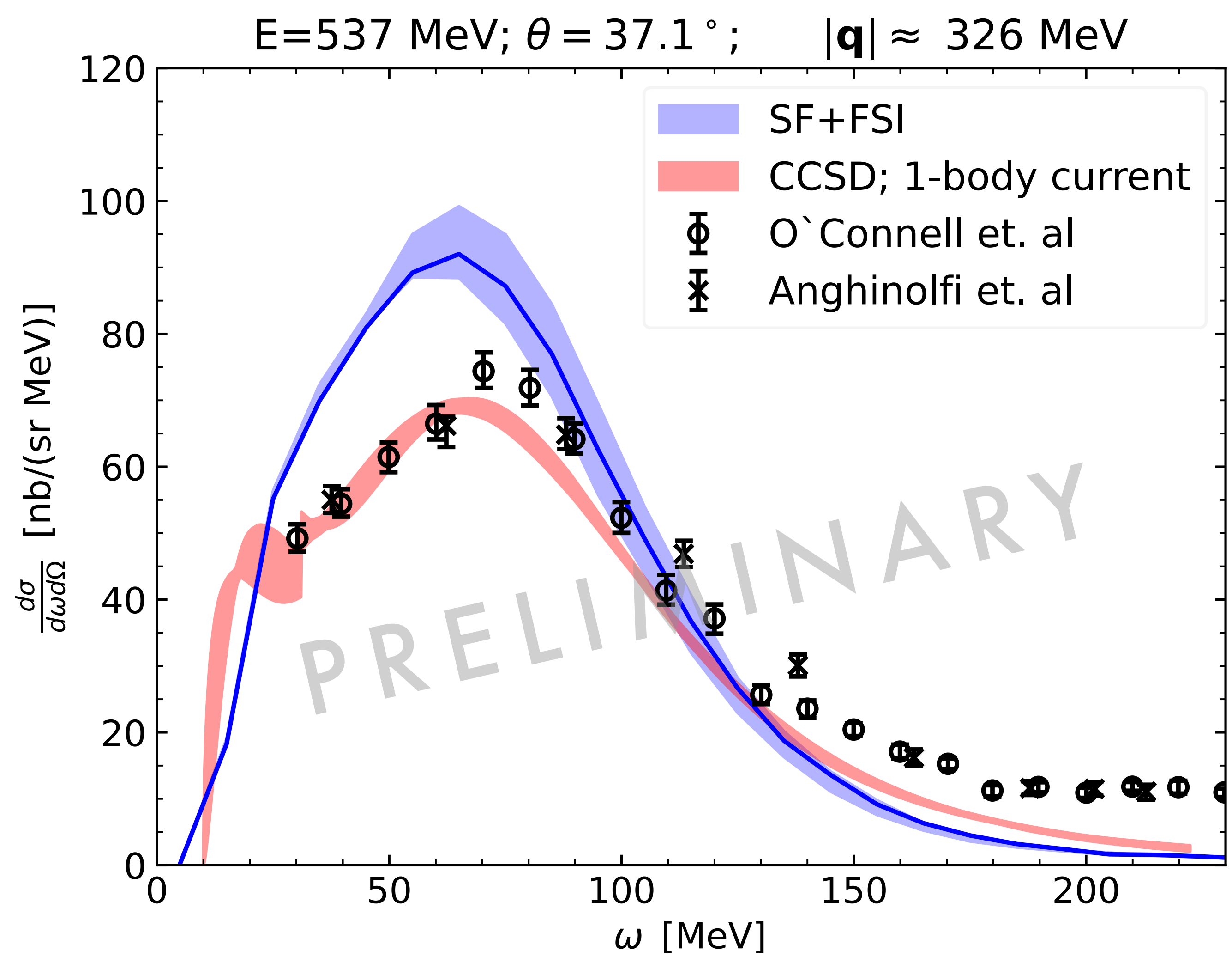
^{16}O cross section

LIT-CC vs SF+FSI

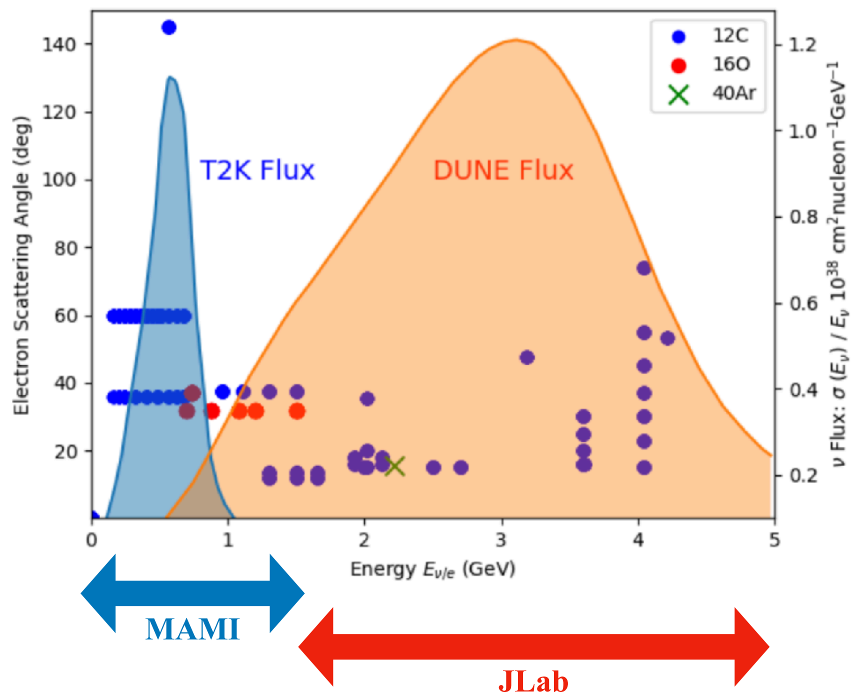


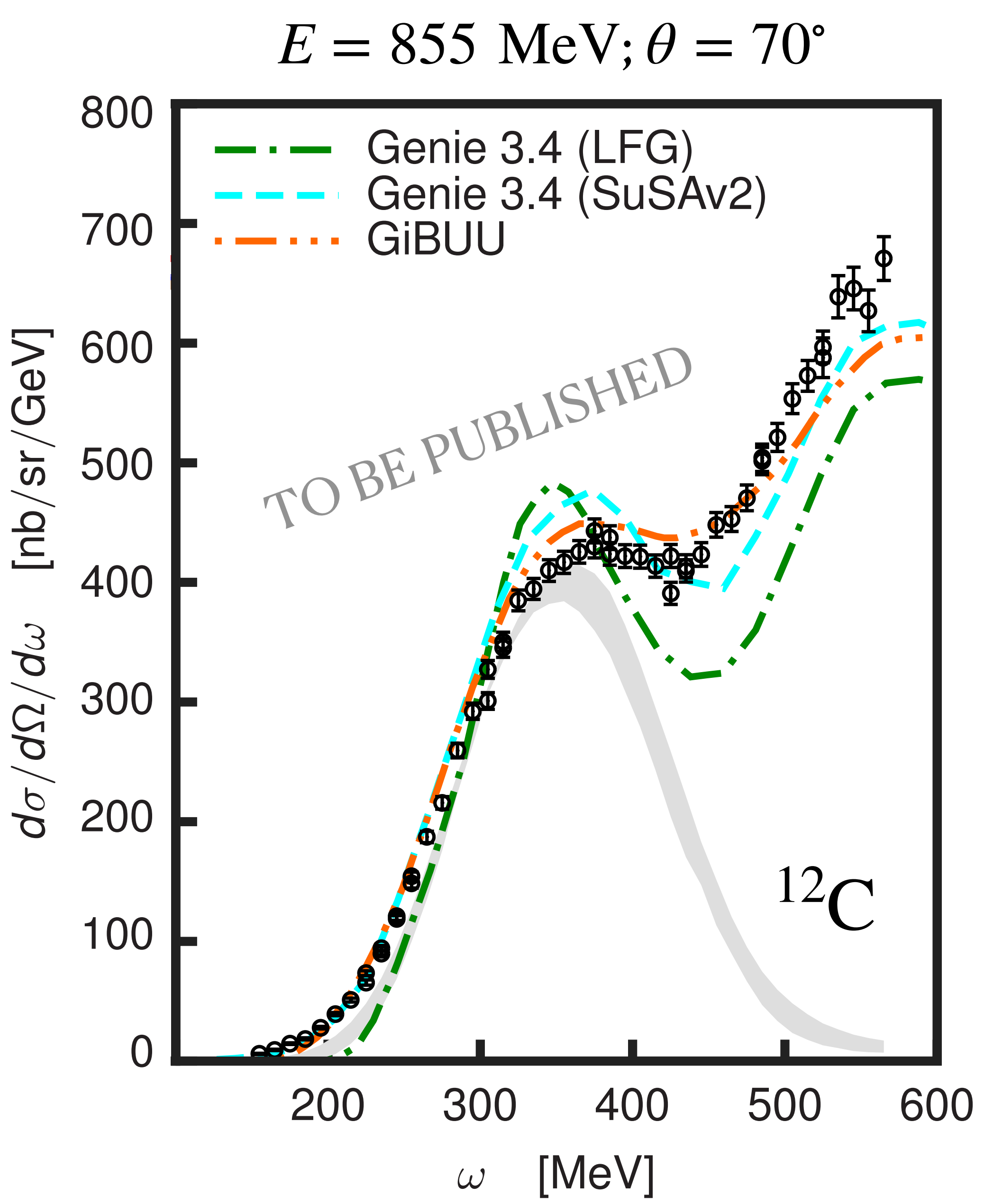
- Only a single low-momentum ^{16}O dataset available
- Dominated by R_L (2-body currents would affect the region above QE peak)



Experimental data

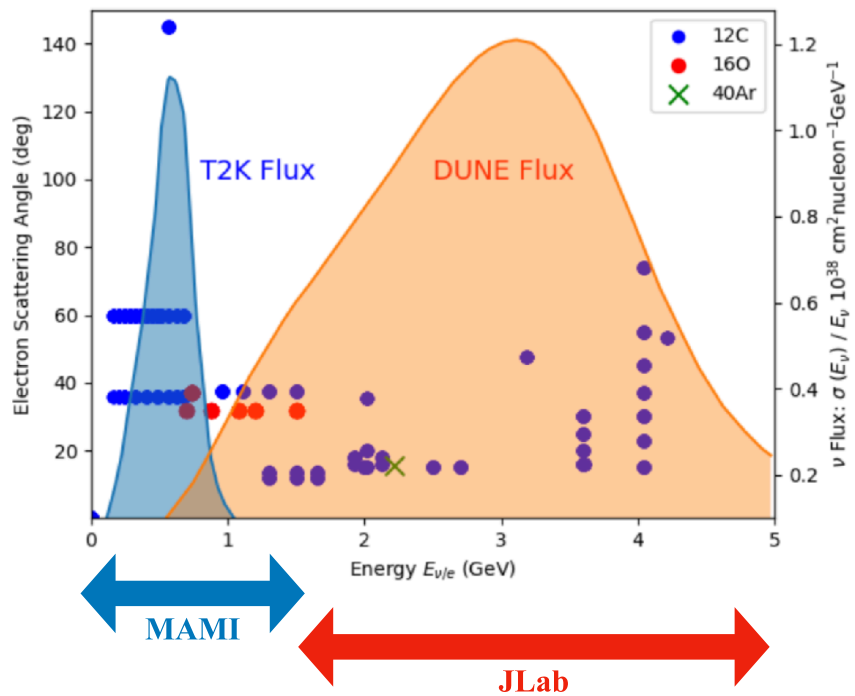
Electron scattering @MAMI

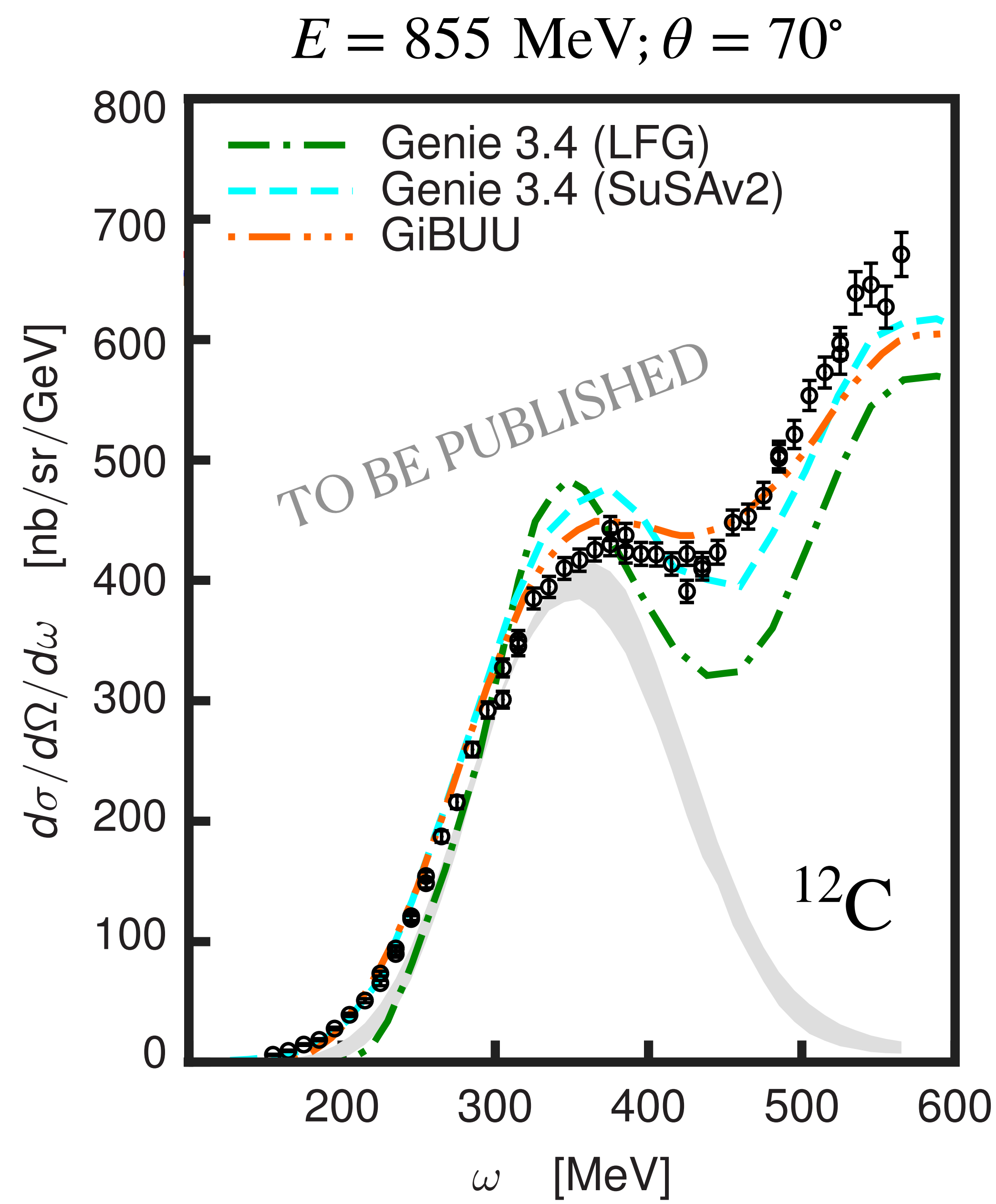




Experimental data

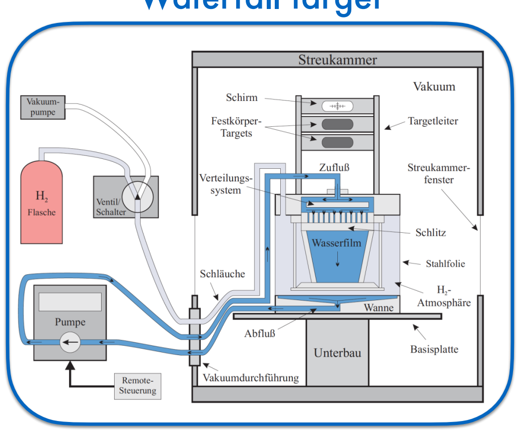
Electron scattering @MAMI



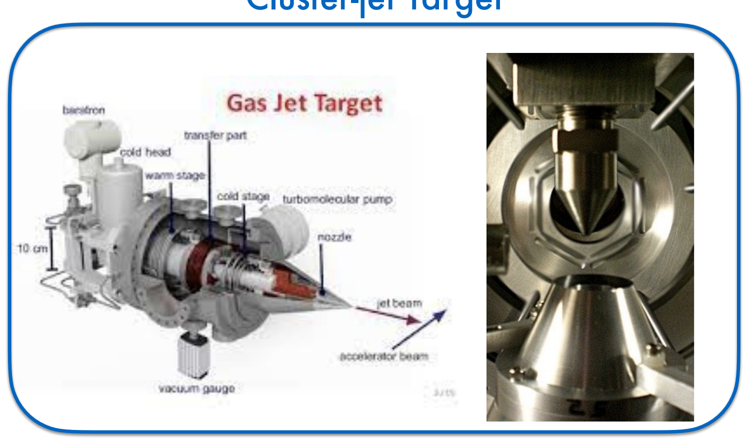


Future targets: ^{16}O, ^{40}Ar

Waterfall target



Cluster-jet Target



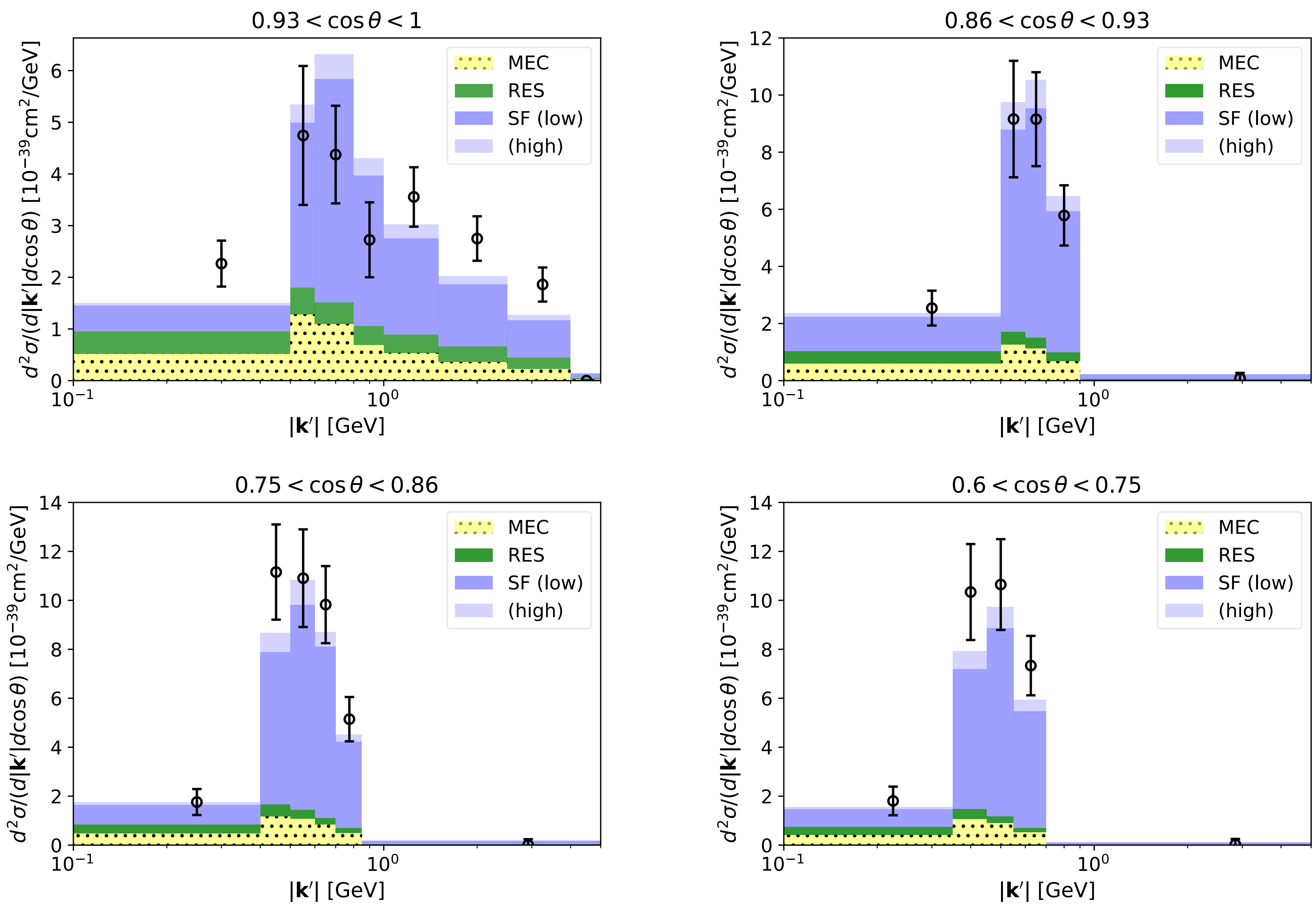
^{16}O spectral function

Error propagation to cross sections

$$\nu_\mu + {}^{16}\text{O} \rightarrow \mu^- + X$$

- Comparison with T2K long baseline ν oscillation experiment
- CC0π events
- Spectral function implemented into NuWro MC generator

Data: Phys. Rev. D 101, 112004 (2020)



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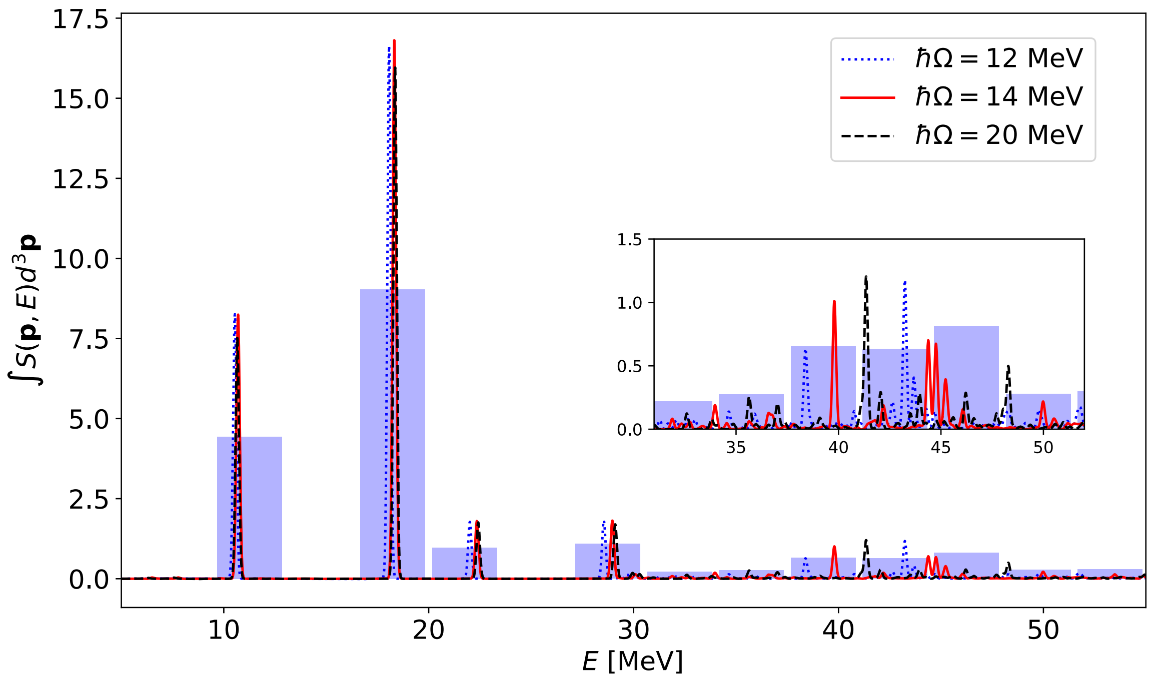
Outlook

- Extension of the formalism to neutrino responses and ^{40}Ar
- Role played by 2-body currents in LIT-CC predictions
- More robust uncertainty quantification: Bayesian analysis of nuclear responses
- Spectral function (accounting for FSI, 2-body currents)

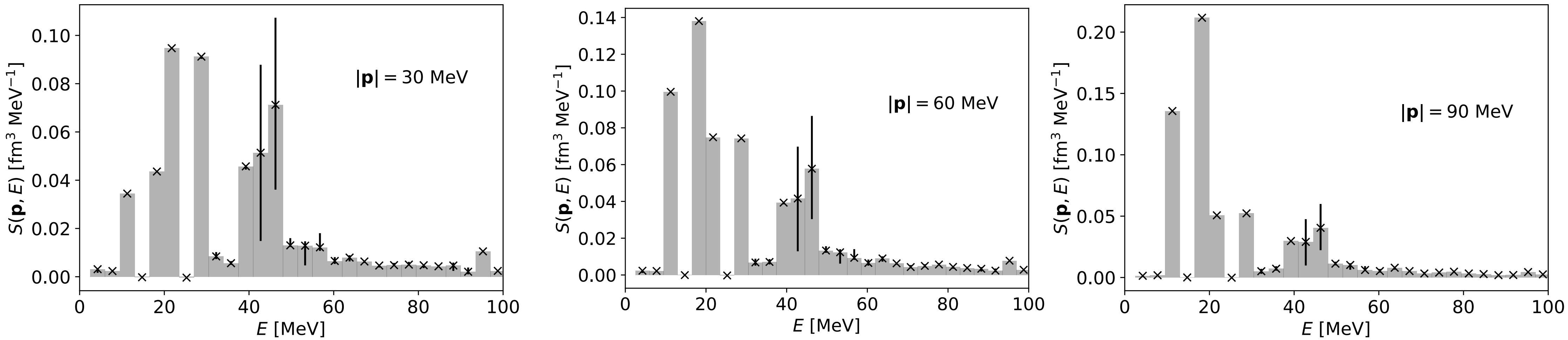
Thank you for attention!

Backup

^{16}O spectral function



- Spectral reconstruction using expansion in Chebyshev polynomials + building histograms
- Uncertainty sources:
  - ✓ $K(\omega, \sigma) = \sum_{k=0}^{N} c_k(\sigma) T_k(\omega)$
  - ✓ Kernel's width Λ



Lorentz Integral Transform (LIT)

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Integral transform

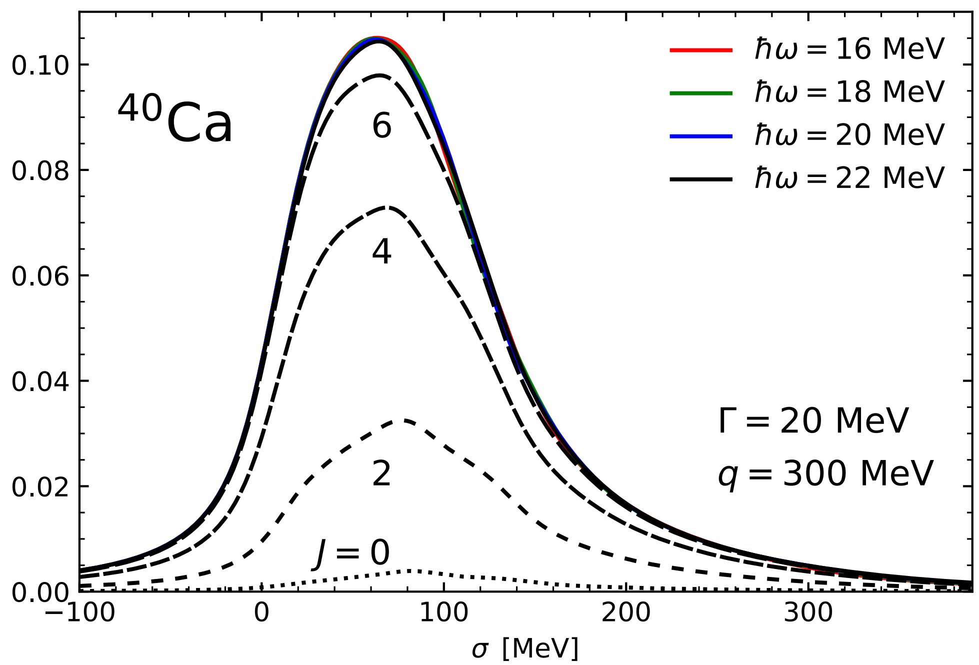
$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger \, K(\mathscr{H} - E_0, \sigma) \, J_\nu | \Psi \rangle$$

Lorentzian kernel:

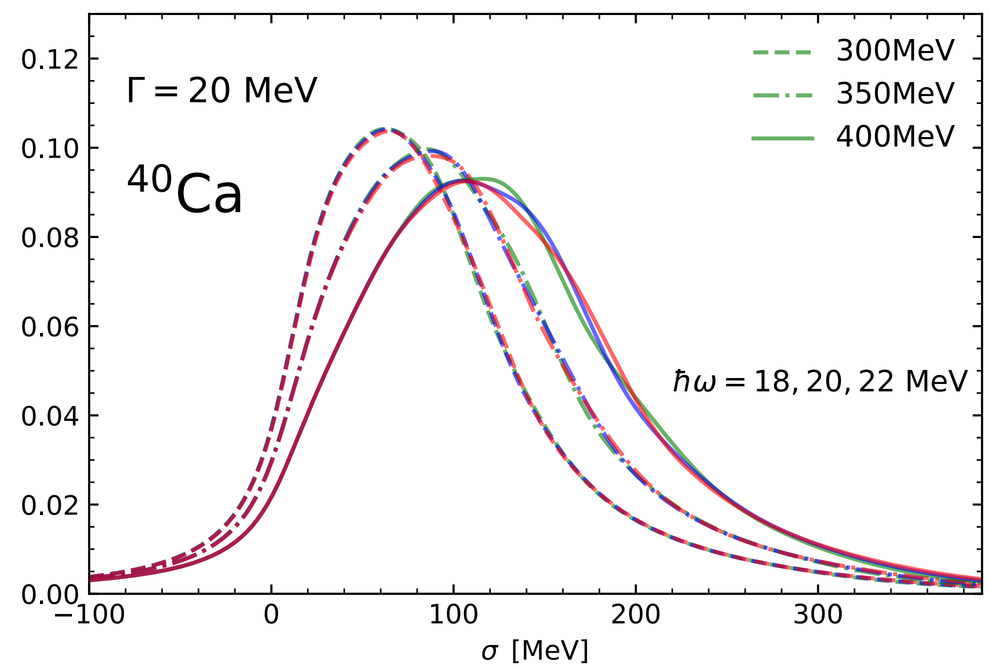
$$K_\Lambda(\omega, \sigma) = \frac{1}{\pi} \frac{\Lambda}{\Lambda^2 + (\omega - \sigma)^2}$$

$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

Longitudinal response ^{40}Ca

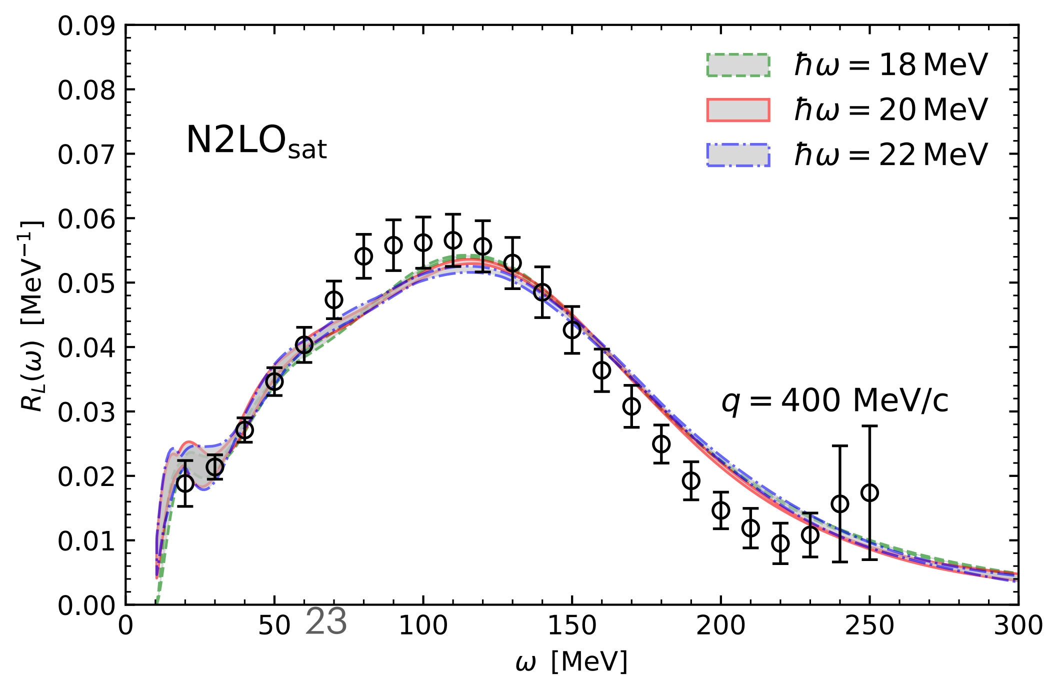


Sum over multipoles



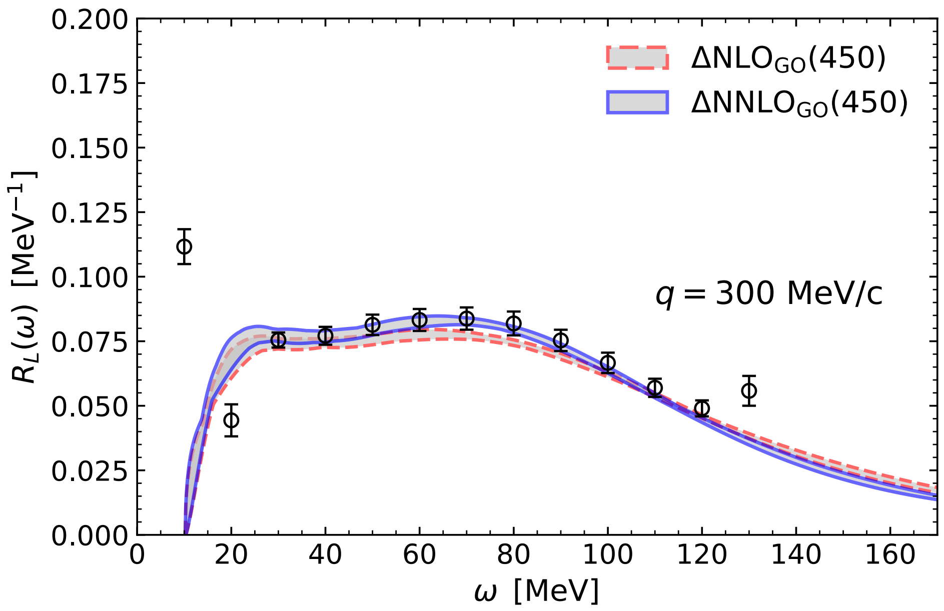
Underlying oscillator frequency

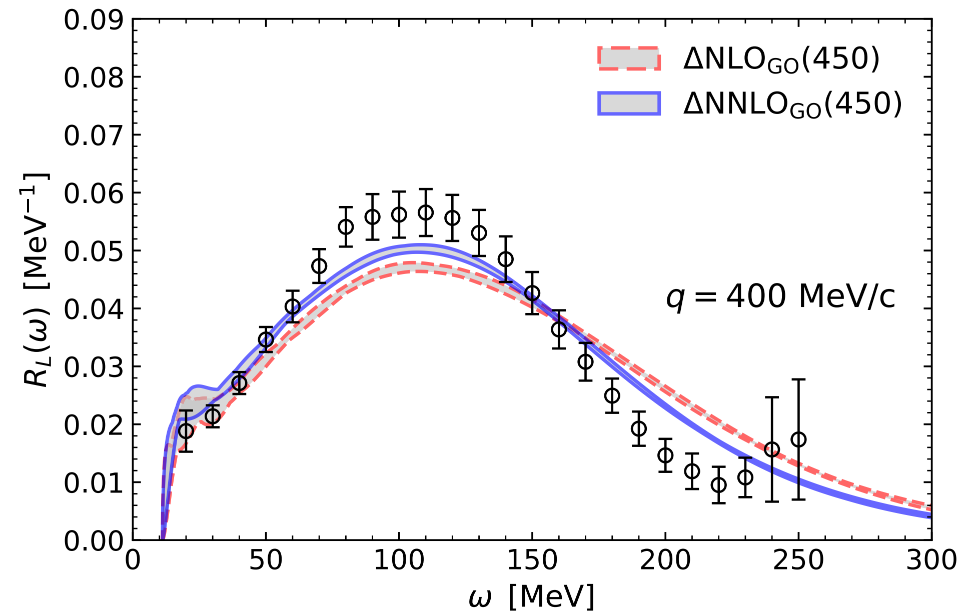
Inversion

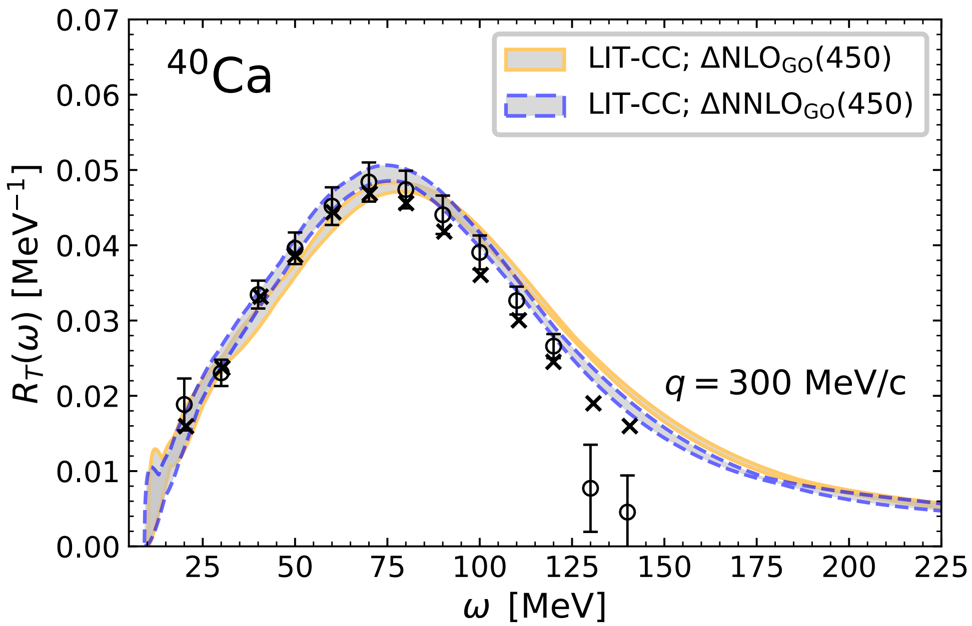


Chiral expansion for ^{40}Ca

(Electromagnetic responses)







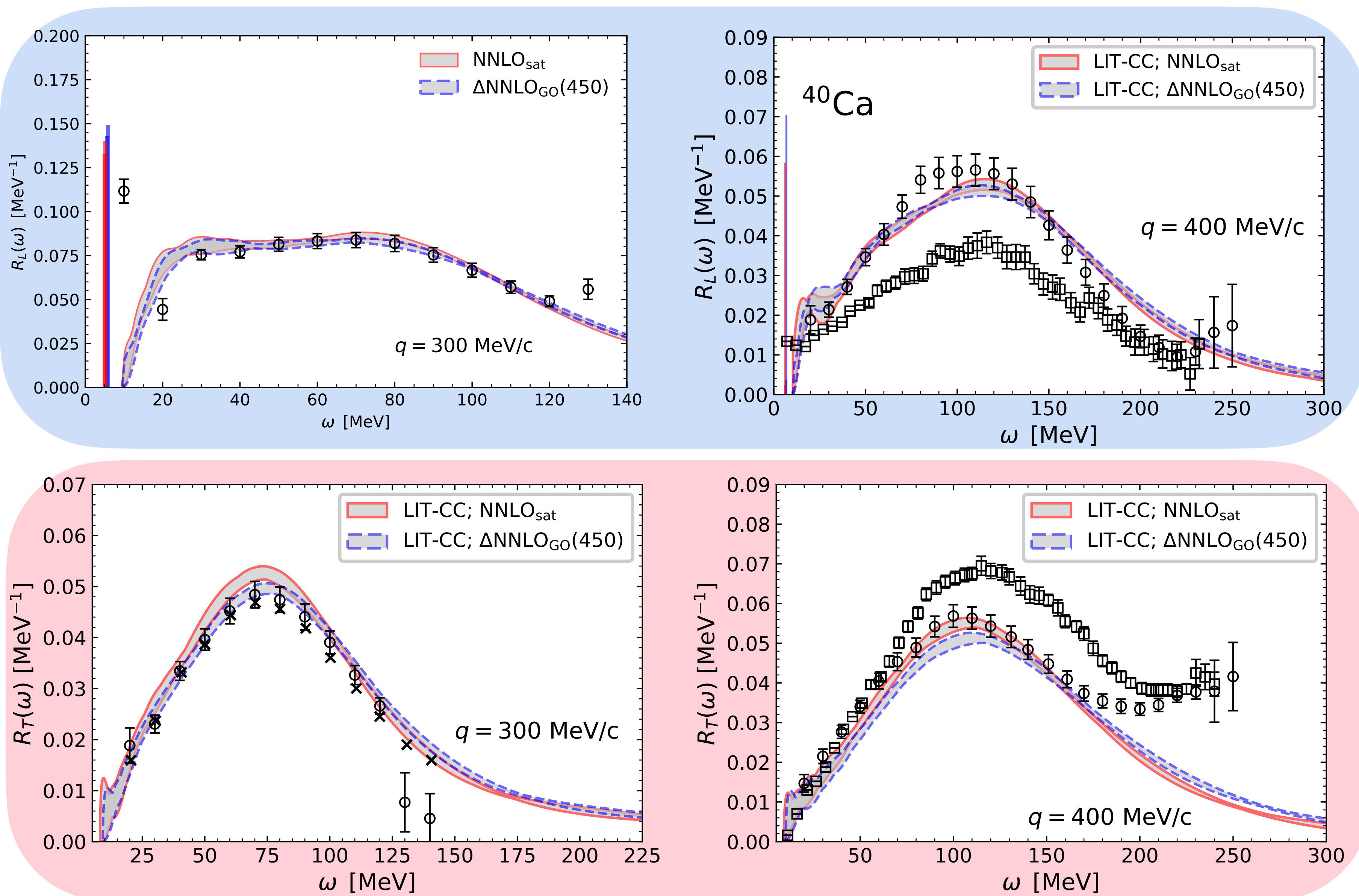
- ✓ Two orders of chiral expansion
- ✓ Convergence better for lower q (as expected)
- ✓ Higher order brings results closer to the data

B. Acharya, S. Bacca, JES et al. Front. Phys. 1066035(2022)

JES, B. Acharya, S. Bacca, G. Hagen; arXiV: 2310.03109

Electromagnetic responses ^{40}Ca

$$\left.\frac{d\sigma}{d\omega dq}\right|_e = \sigma_M\left(\upsilon_L R_L + \upsilon_T R_T\right)$$



- ✓ CC singles & doubles
- ✓ two different chiral Hamiltonians
- ✓ *inversion procedure*

JES, B. Acharya, S. Bacca, G. Hagen; *PRL* 127 (2021) 7, 072501
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