Short-range correlations with the generalized contact formalism

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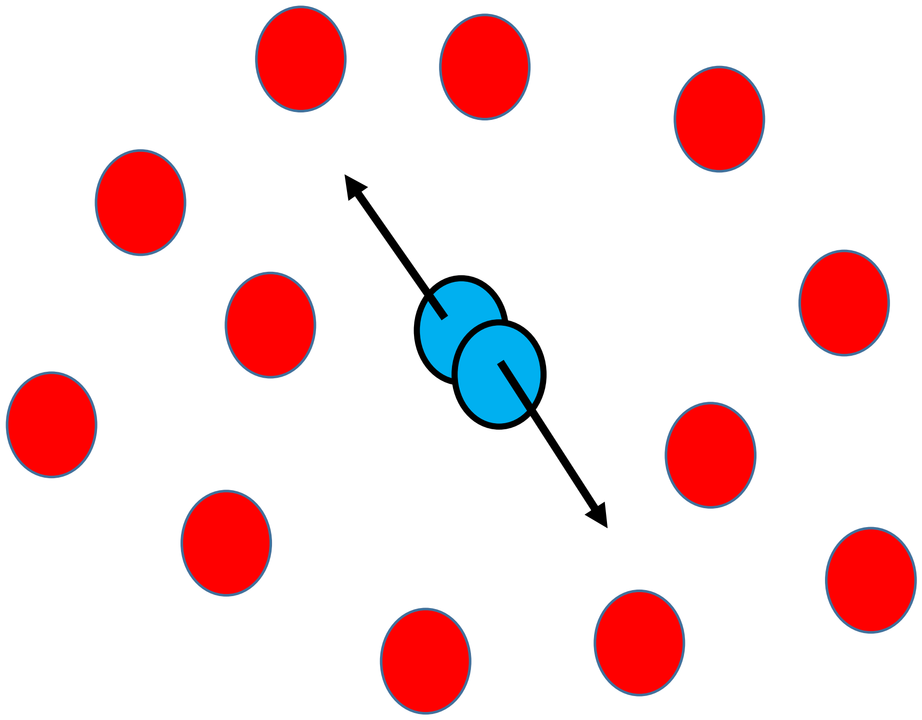
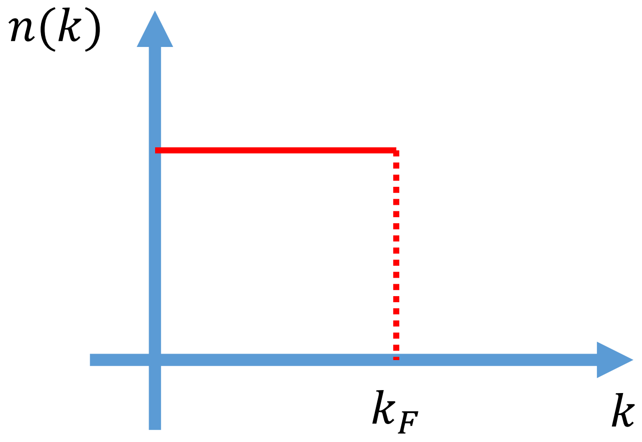
Short-range correlations (SRCs)

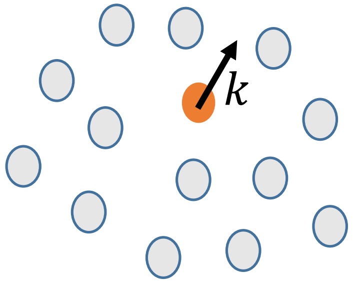
What happens when few particles get close to each other?



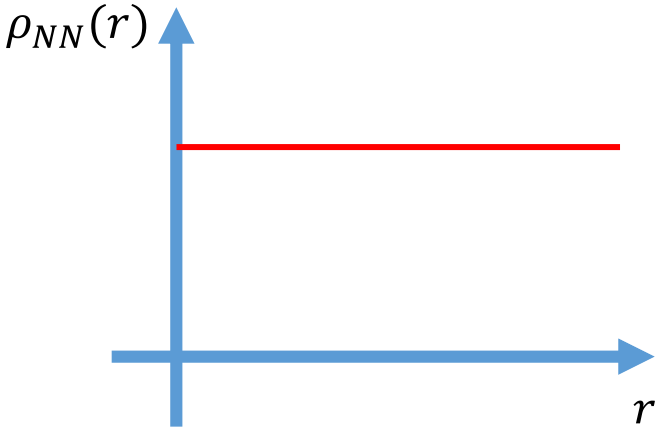
SRCs vs Mean Field (Fermi gas)

One-body Momentum Distribution

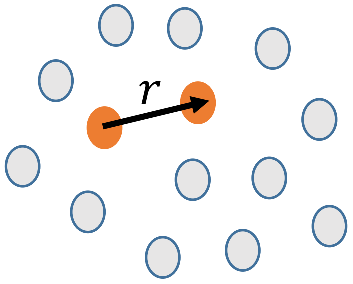




Two-body Relative Density



(Ignoring Fermi correlations)



SRCs vs Mean Field (Fermi gas)

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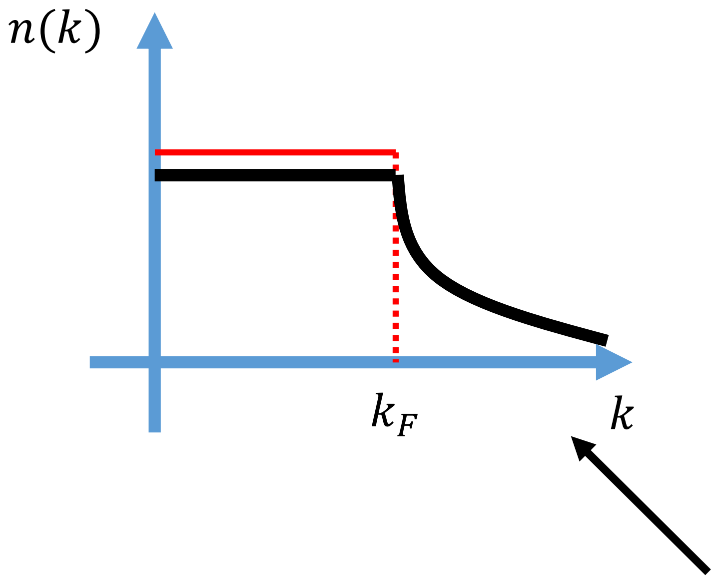

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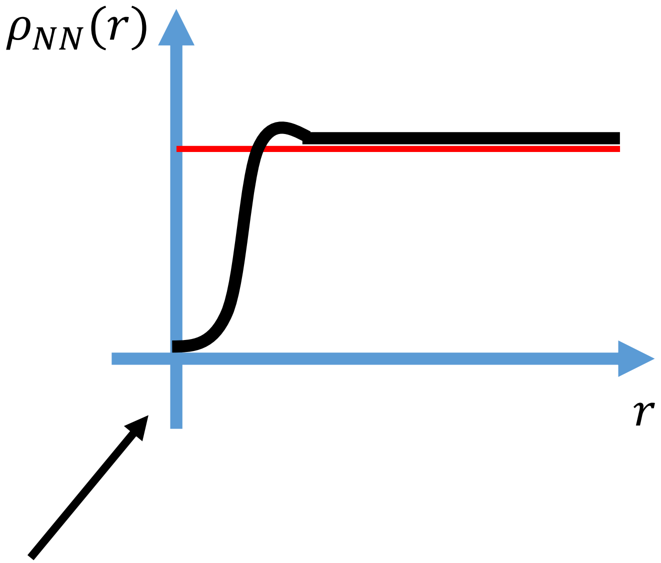
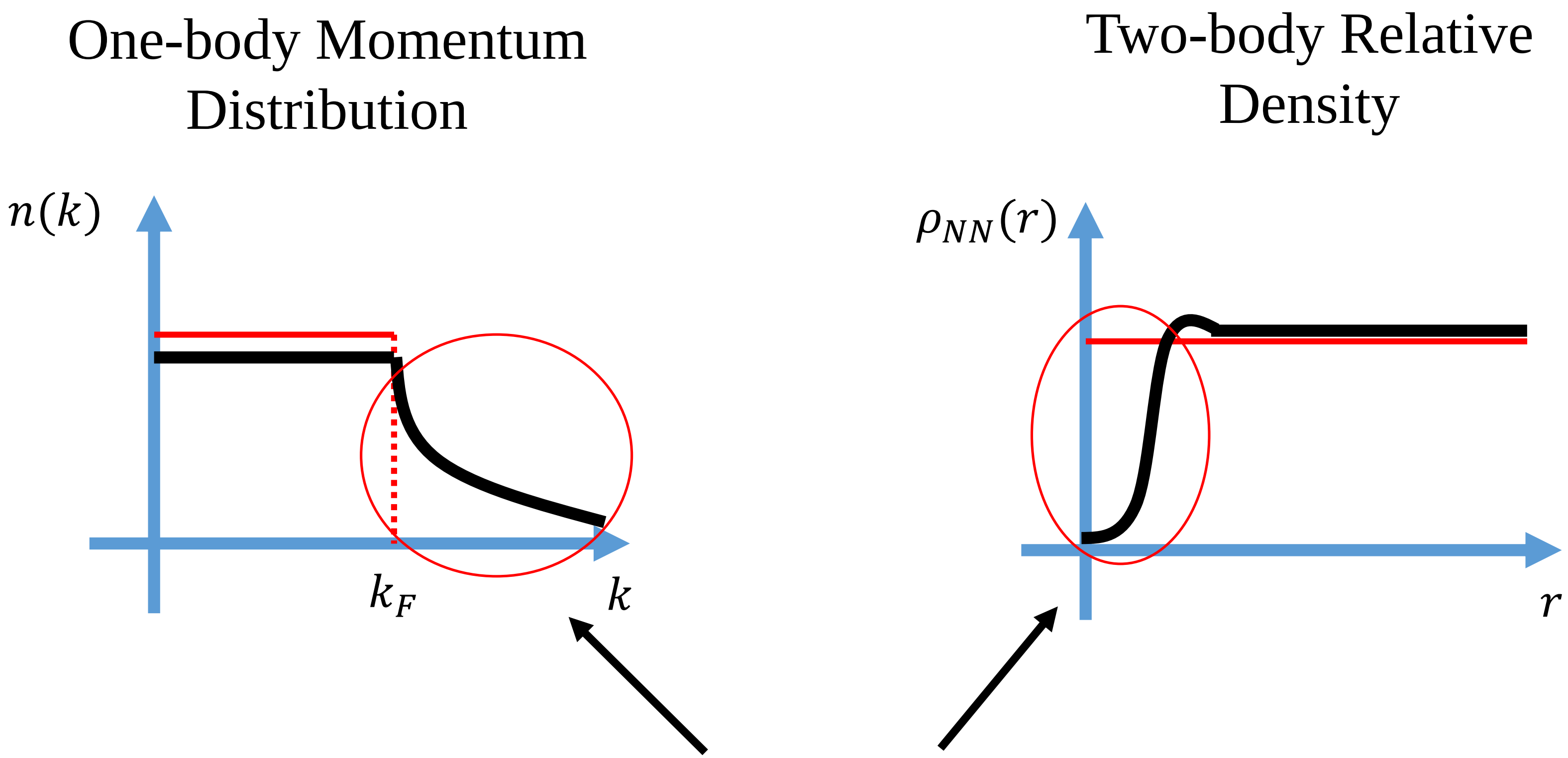


Solving the many-body problem

SRCs vs Mean Field (Fermi gas)

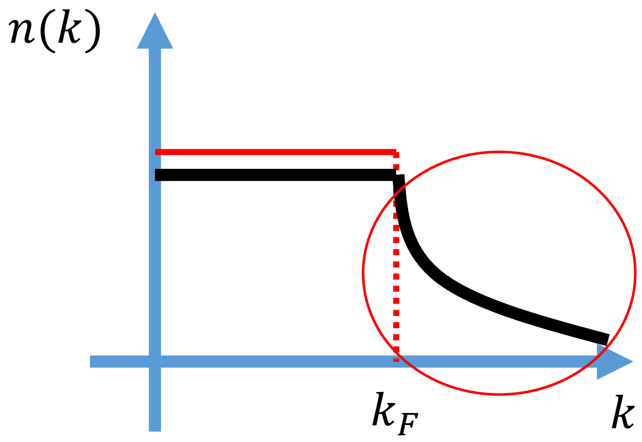


Solving the many-body problem

Effects of short-range correlations (SRCs)

SRCs vs Mean Field (Fermi gas)

One-body Momentum Distribution



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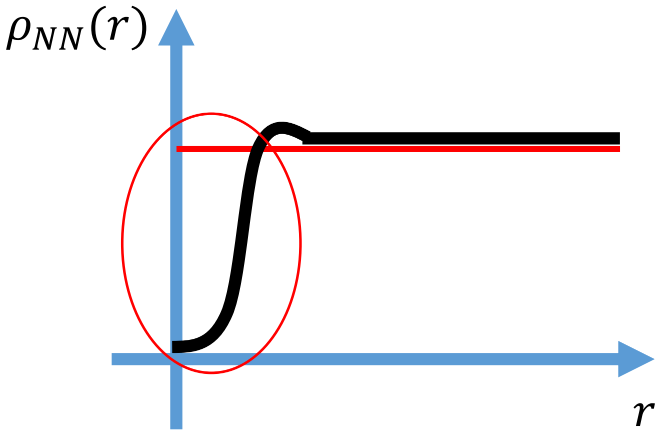
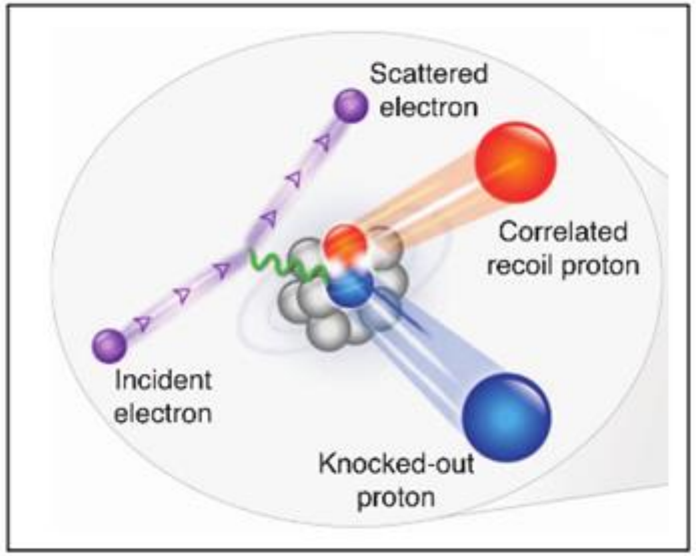


- Significant deviations from mean-field models
- Challenge for the description of quantum systems

Short-range correlations (SRCs)

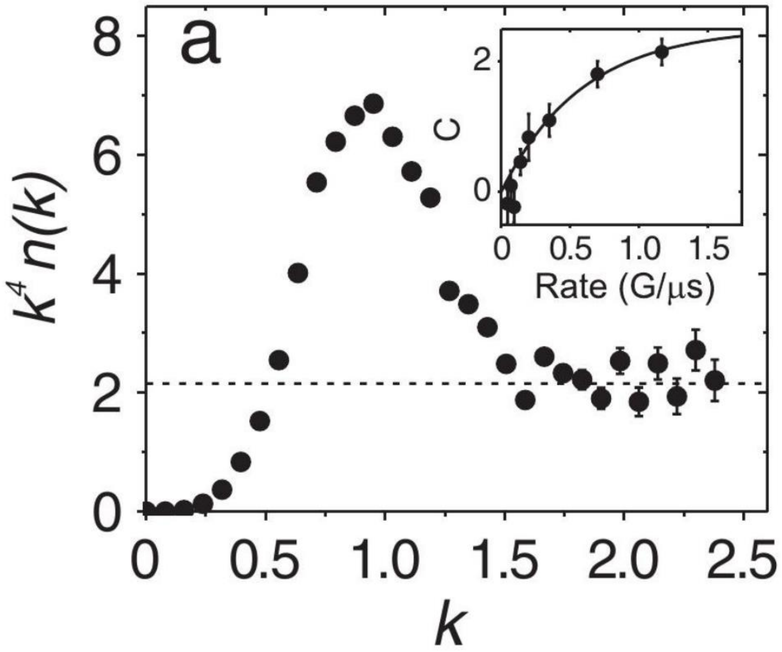
Studied in different systems:

Nuclear systems



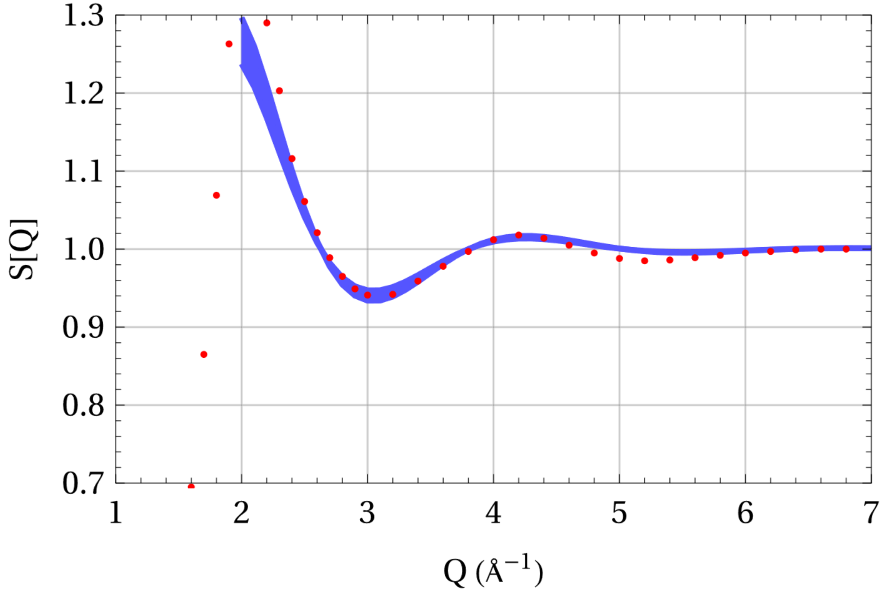
O. Hen et al., Science 346, 614 (2014)

Ultracold atomic systems



J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)

Liquid ^{4}He

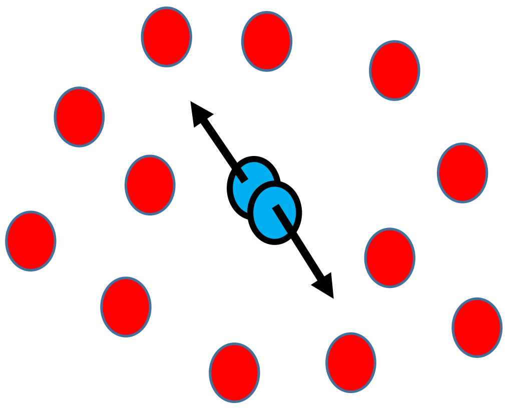


B. Bazak, M. Valiente, N. Barnea, PRA 101, 010501 (2020)

Short-range correlations (SRCs)

Main features:

- High momentum particles with back-to-back configuration
- Universal behavior – "isolated pair"
- Neutron-proton dominance



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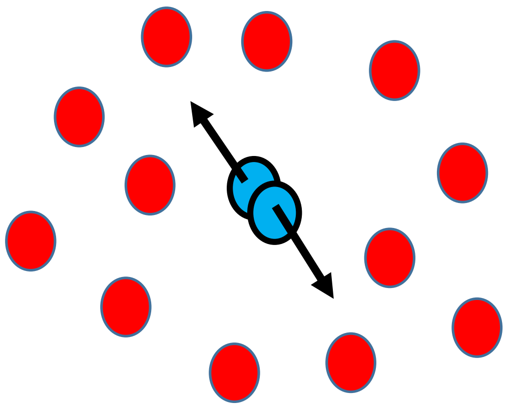


How can we explain these features?

How can we utilize information regarding SRCs for the description of the whole system?

The Generalized Contact Formalism (GCF)

RW, B. Bazak, N. Barnea

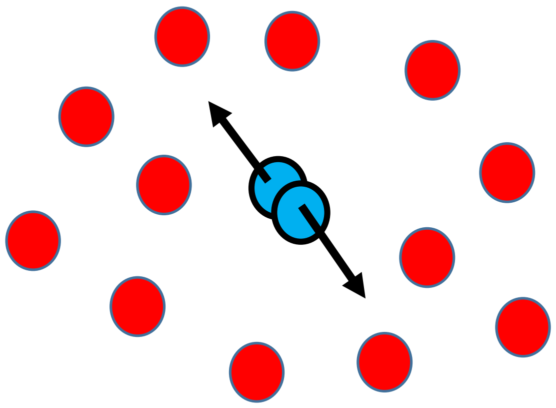
Generalized Contact Formalism

- Generalizing Tan's work for dilute systems

S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008); Ann. Phys. (N.Y.) 323, 2971 (2008); Ann. Phys. (N.Y.) 323, 2987 (2008)

- Starting point – Short-range factorization

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



Universal function
(but depends on the potential)

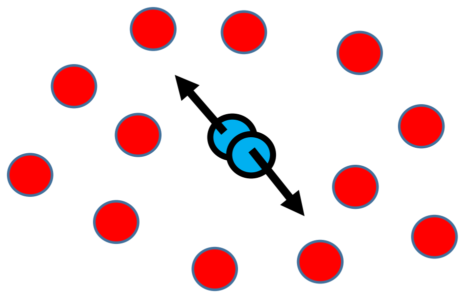
Nucleus-dependent function

$\varphi(r) \equiv$ Zero-energy solution of the **two-body** Schrodinger Eq.

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



universal function

For any **short-range** two-body operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$

$$C \propto \langle A | A \rangle$$

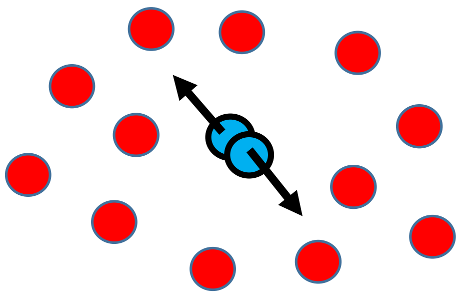
- Two-body dynamics
- Universal for all nuclei
- Simply calculated

- The “contact”
- Number of correlated pairs
- Depends on the nucleus
- Independent of the operator

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



universal function

For any **short-range** two-body operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C \qquad C \propto \langle A | A \rangle$$

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

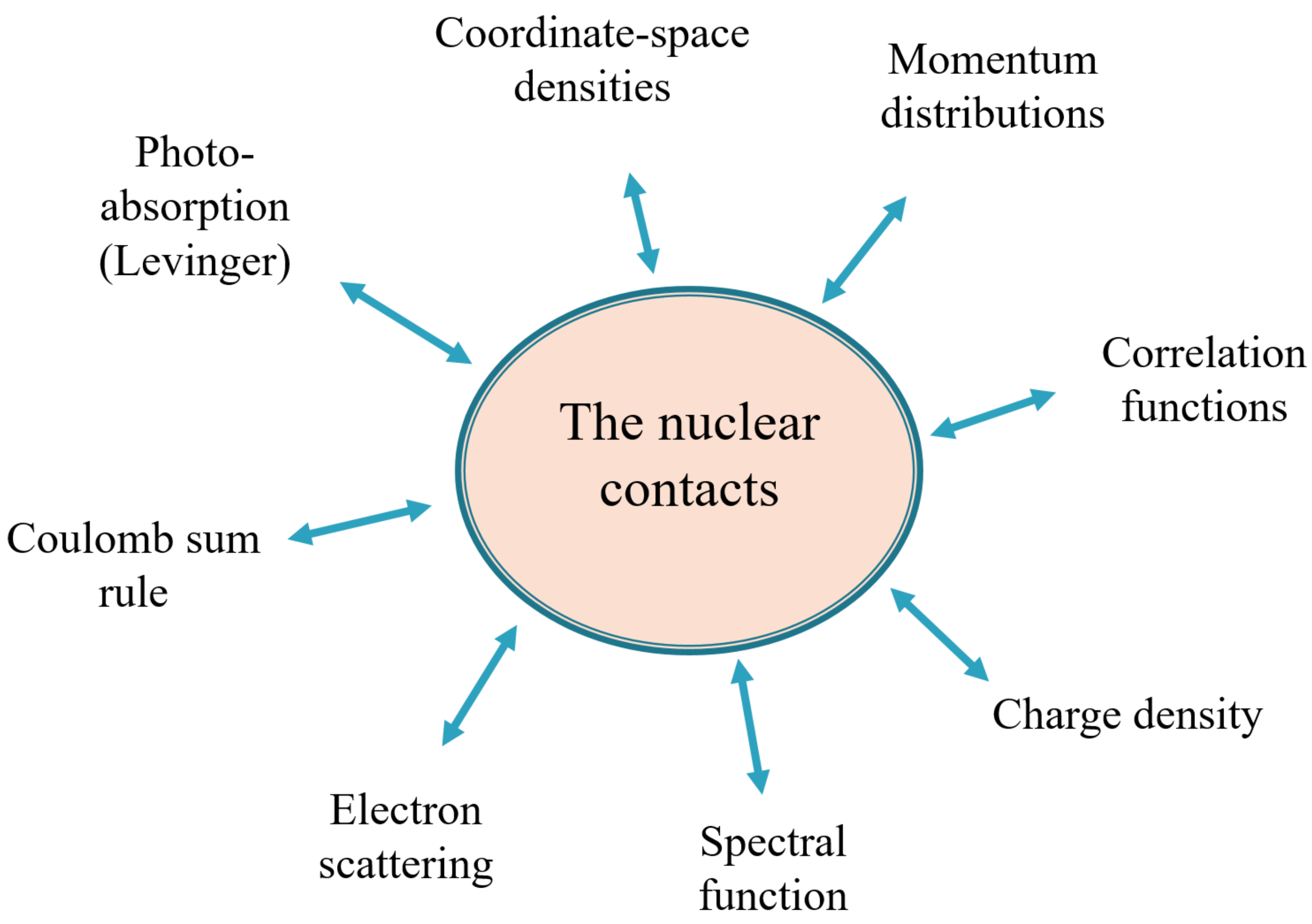
Channels $\alpha = (\ell_2 S_2) j_2 m_2$

Universal functions

The pair kind $ij \in \{pp, nn, pn\}$

3 matrices of Nuclear Contacts

The nuclear contact relations



RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

RW, B. Bazak, N. Barnea, PRL 114, 012501 (2015)

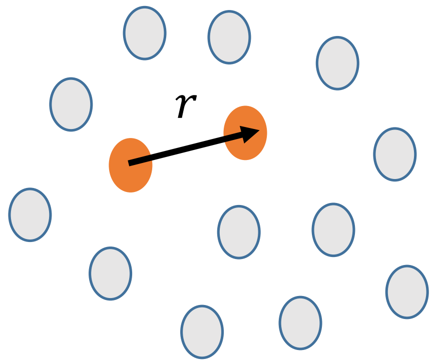
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

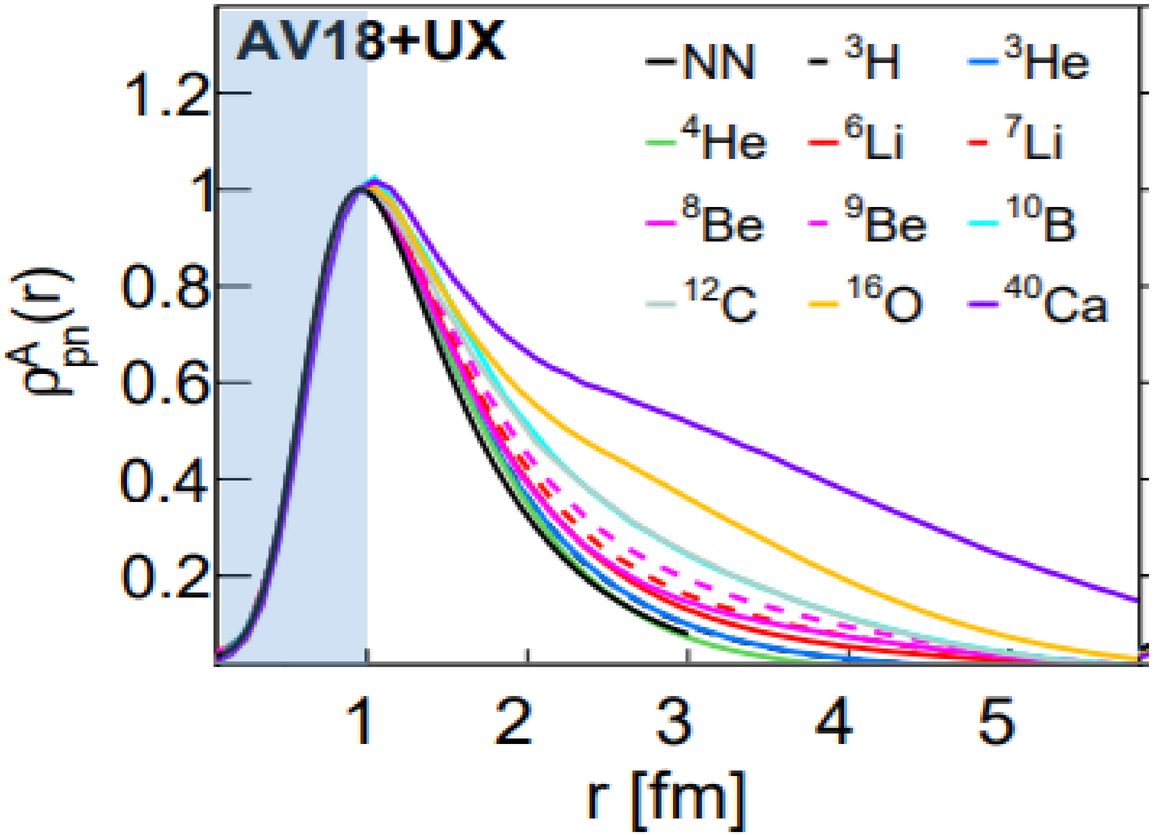
A. Schmidt, J.R. Pybus, RW, et al., Nature 578, 540 (2020)

Two-body density



$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$

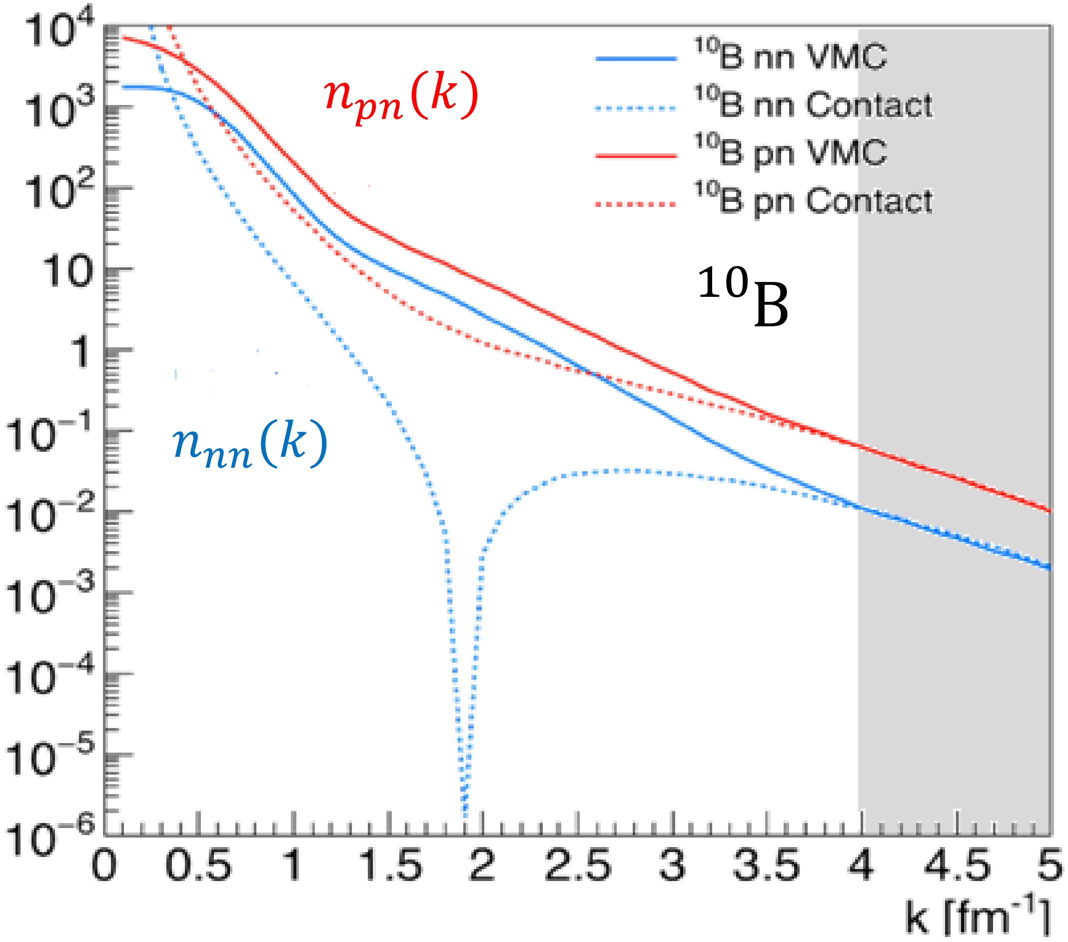
$$\rho_{NN}(\boldsymbol{r}) \xrightarrow{r \to 0} C |\varphi(\boldsymbol{r})|^2$$



Shows the validity of the factorization

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

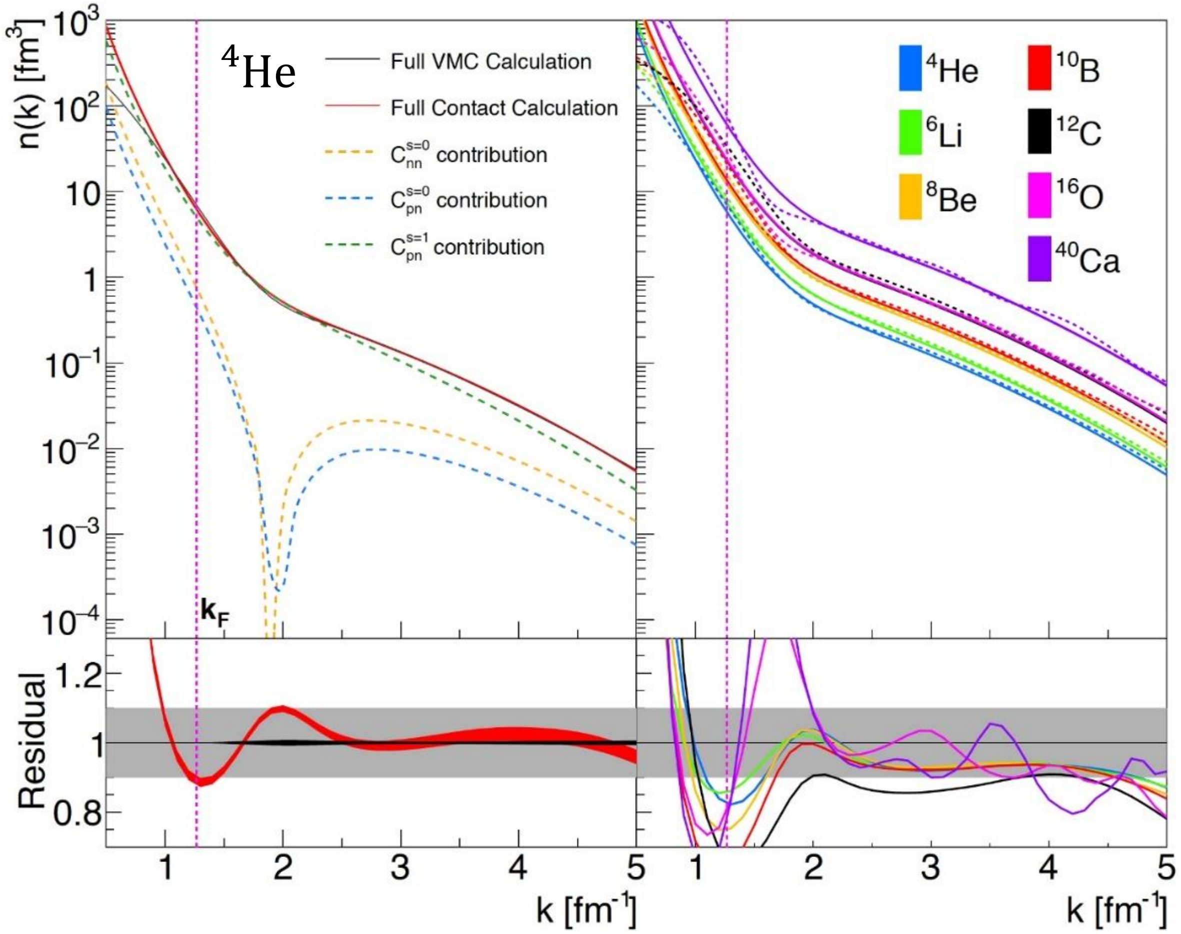
Two-body momentum distribution



One-body momentum distribution

$$n_p(k) \xrightarrow[k\to\infty]{} C_{pn}^{1}\left|\varphi_{pn}^{d}(k)\right|^2 + C_{pn}^{0}\left|\varphi_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\varphi_{pp}^{0}(k)\right|^2$$

$n_p(k)$



- s-wave contributions
- All NN pairs

No fitting parameters!

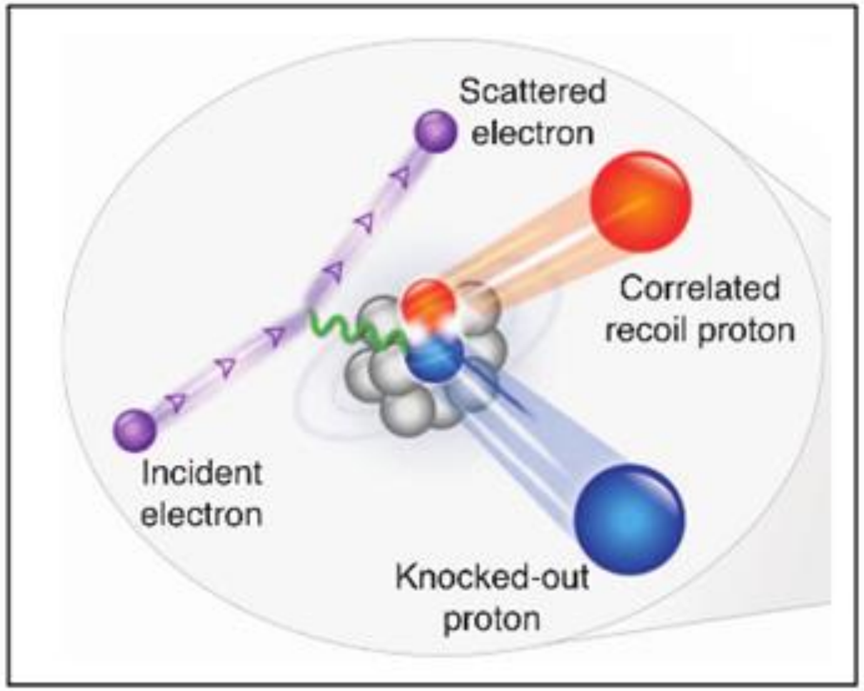
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

Electron-scattering experiments

- $\boldsymbol{A(e,e'N)}$ and $\boldsymbol{A(e,e'NN)}$ cross sections

- $S(\boldsymbol{p_1},\epsilon_1) =$ **spectral function**
  The probability to find nucleon with momentum $\boldsymbol{p}_1$ and energy ϵ_1 in the nucleus

- Using the GCF:



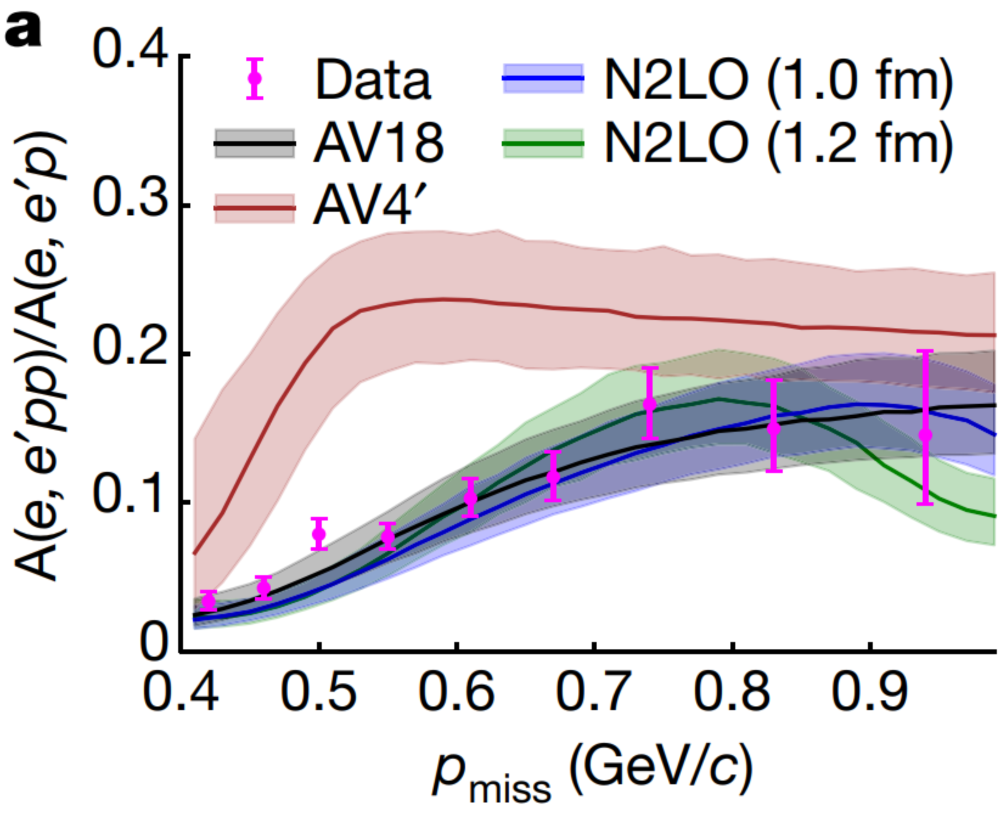
$$S^p(\boldsymbol{p_1},\epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1},\epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1},\epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1},\epsilon_1)$$

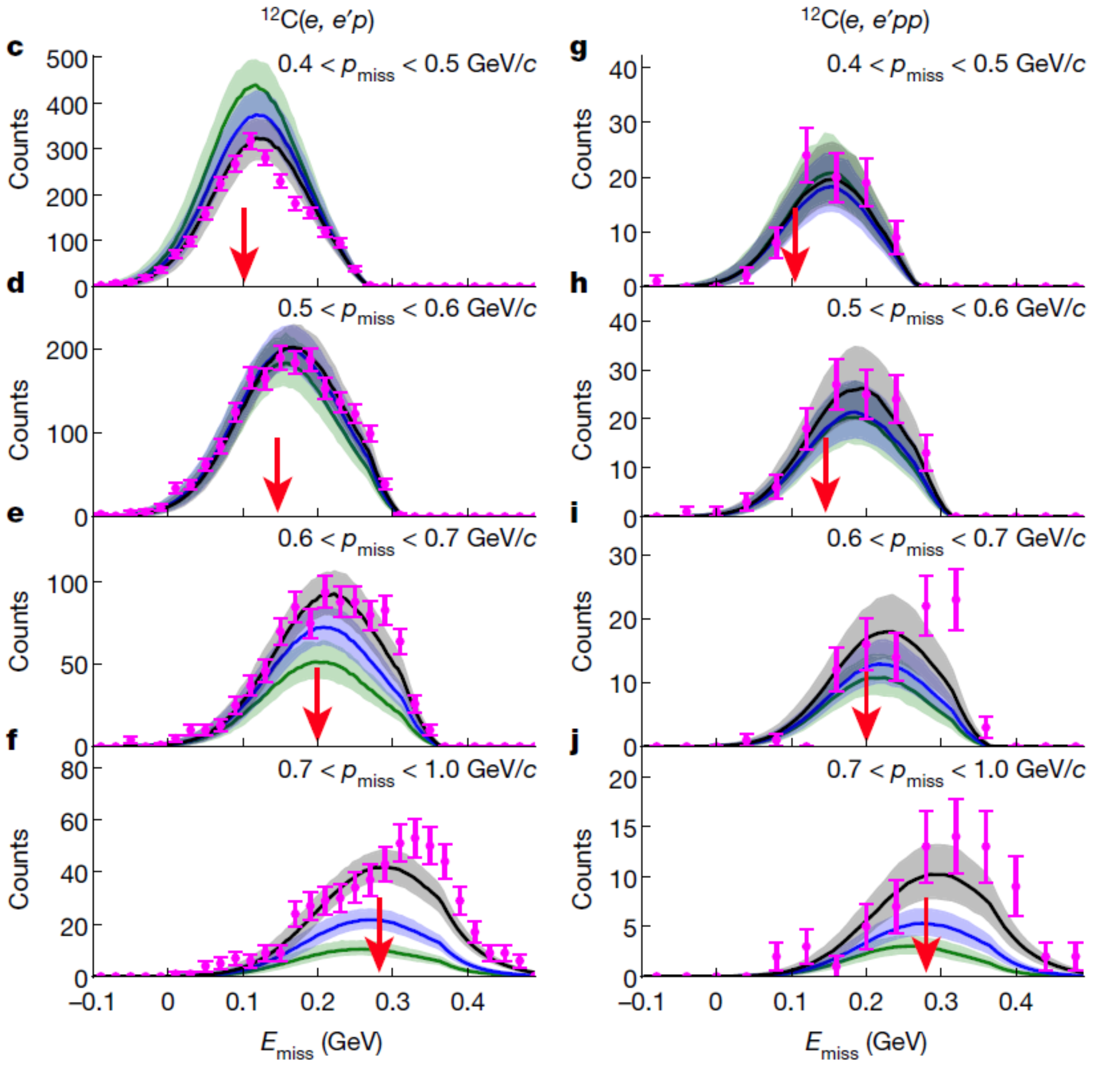
$(p_1 > k_F)$

RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

Electron-scattering experiments

- Good description of experimental data:



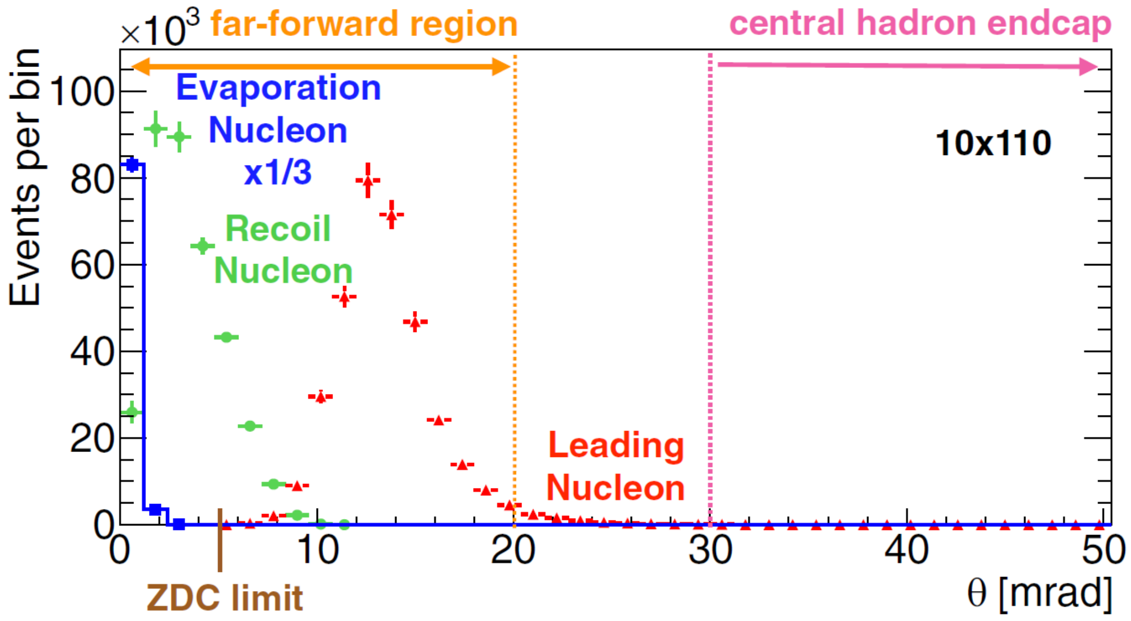


A. Schmidt, J.R. Pybus, RW, E. P. Segarra, A. Hrnjic, A. Denniston, O. Hen, *et al. (CLAS collaboration), Nature 578, 540 (2020)*

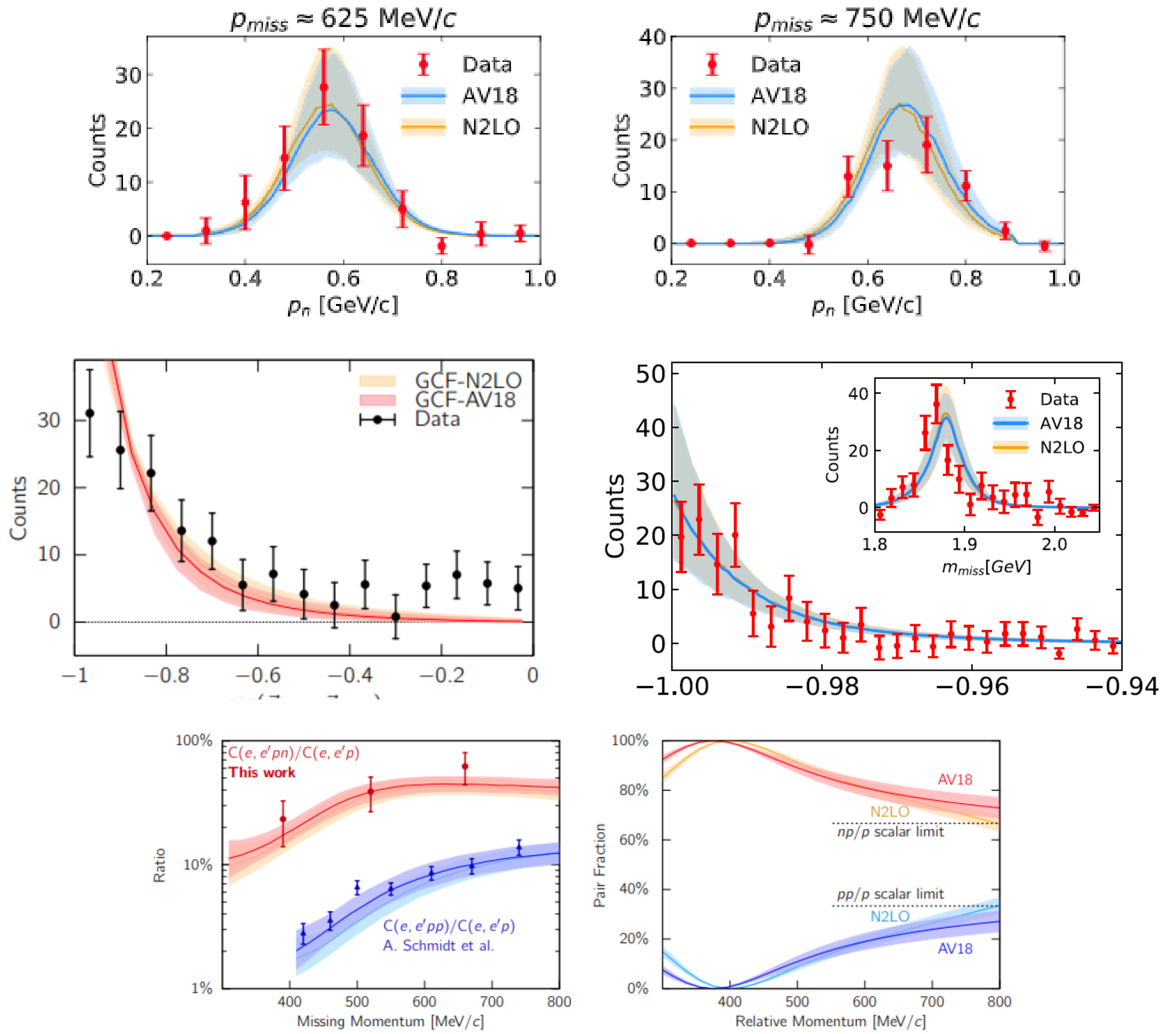
Analyzing and designing experiments

F. Hauenstein, et al., PRC 105, 034001 (2022)

For the EIC



Dedicated event generator



I. Korover et al., PLB 820, 136523 (2021) *J.R. Pybus et al., PLB 805, 135429 (2020)*

Analyzing and designing experiments

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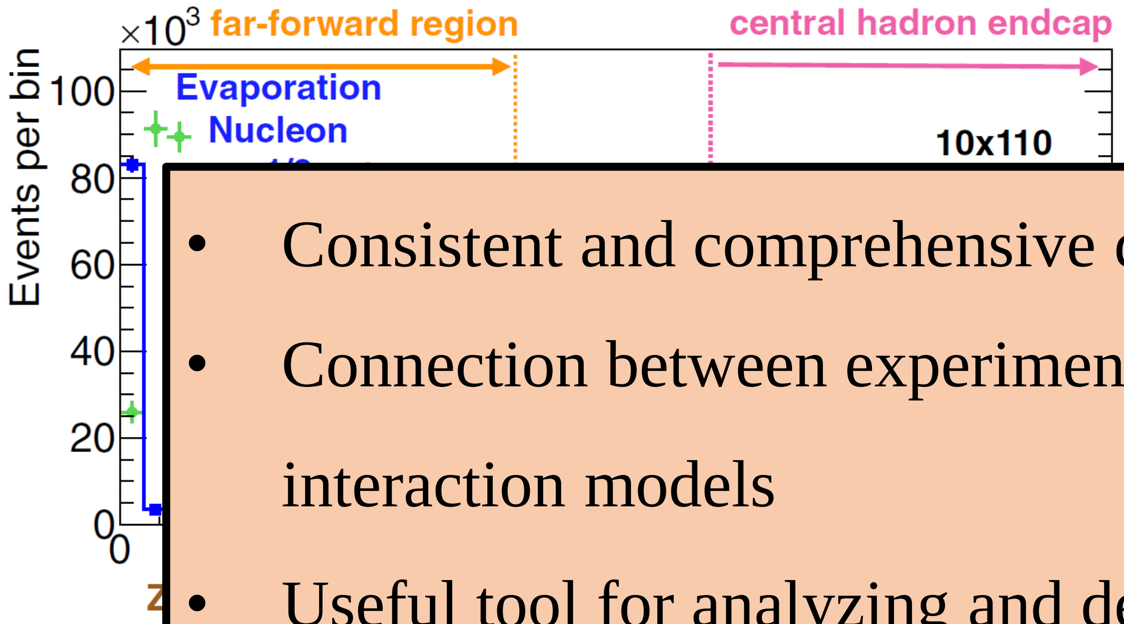
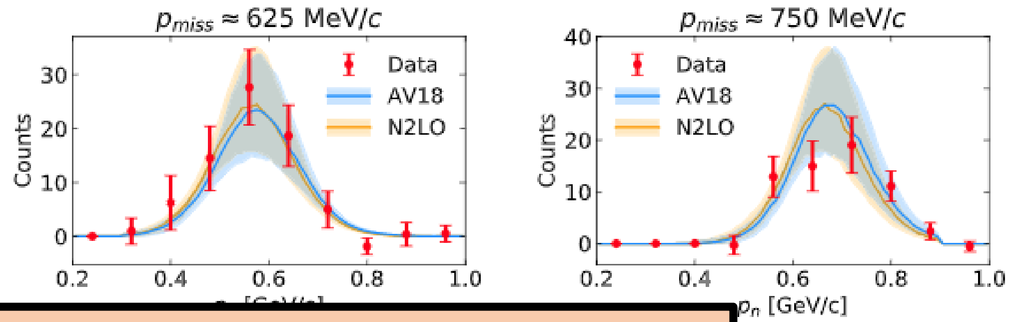
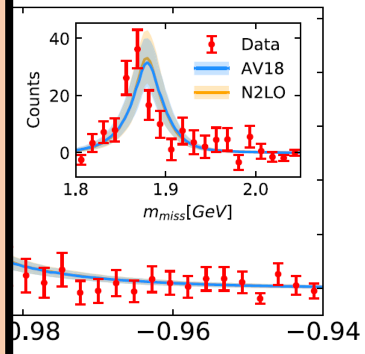
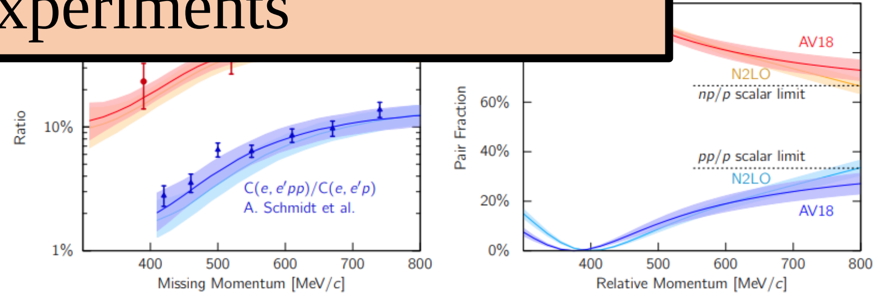






- Consistent and comprehensive description of SRC pairs
- Connection between experiments and nuclear structure and interaction models
- Useful tool for analyzing and designing experiments



Dedicated event generator

I. Korover et al., PLB 820, 136523 (2021) *J.R. Pybus et al., PLB 805, 135429 (2020)*

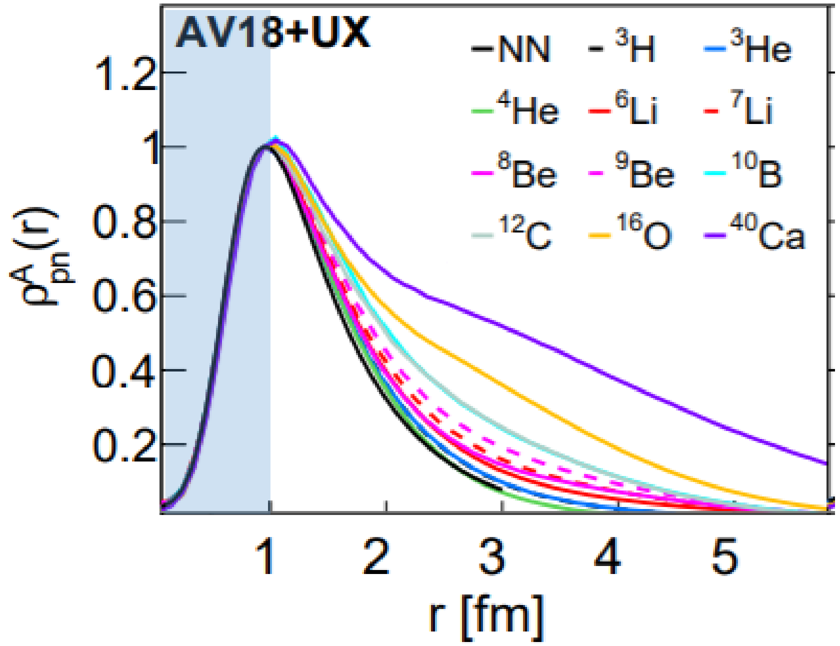
Towards a systematic short-range description

Corrections to the GCF

- GCF is based on the short-range factorization

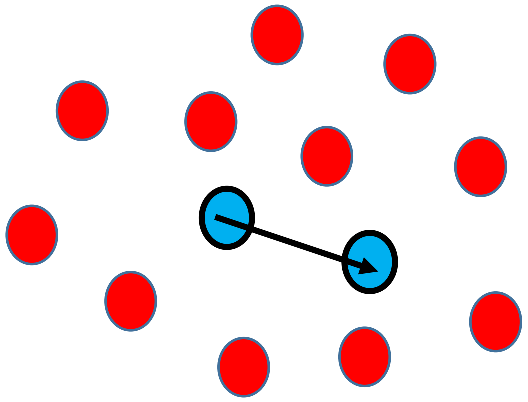
$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

- Begins to fail at larger distances
- Possible corrections:
  - **Three-body correlations**
  - **Pairs at larger distances**



Systematic short-range expansion for SRC pairs:

Beyond factorization

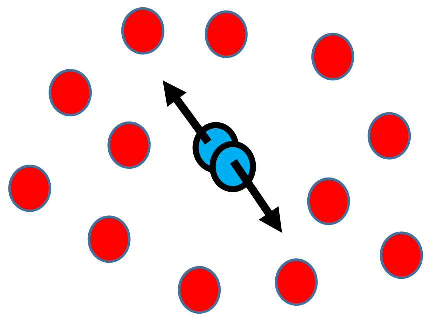


RW, D. Lonardoni, S. Gandolfi, arXiv:2307.05910 [nucl-th] (2023)

Short-range expansion

- Factorization for short distances

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



- $\varphi(r) \equiv$ **Zero-energy** solution of the two-body Schrodinger Eq.

- **The two-body system:**

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = \boldsymbol{E}\varphi^E(r)$$

- For $r \to 0$: The energy becomes negligible

$$E \ll \frac{\hbar^2}{mr^2}$$

Short-range expansion – two-body system

- For $r \to 0$:

$$\varphi^{E}(r) = \varphi^{E=0}(r) + \cdots$$

- Taylor expansion around $E = 0$:

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

Short-range expansion – two-body system

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GCF
leading term

Subleading terms

Short-range expansion – many-body system

- **The many-body case:** Two-body density

Leading order: $\rho_2(r) = \left|\varphi_{\ell=0}^{E=0}(r)\right|^2 C$ (zero energy, s-wave)

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Expansion: (energy derivatives, $\ell > 0$ contributions)

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

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- **Power counting** is needed
  - Can be analyzed analytically for the two-body system

Short-range expansion – many-body system

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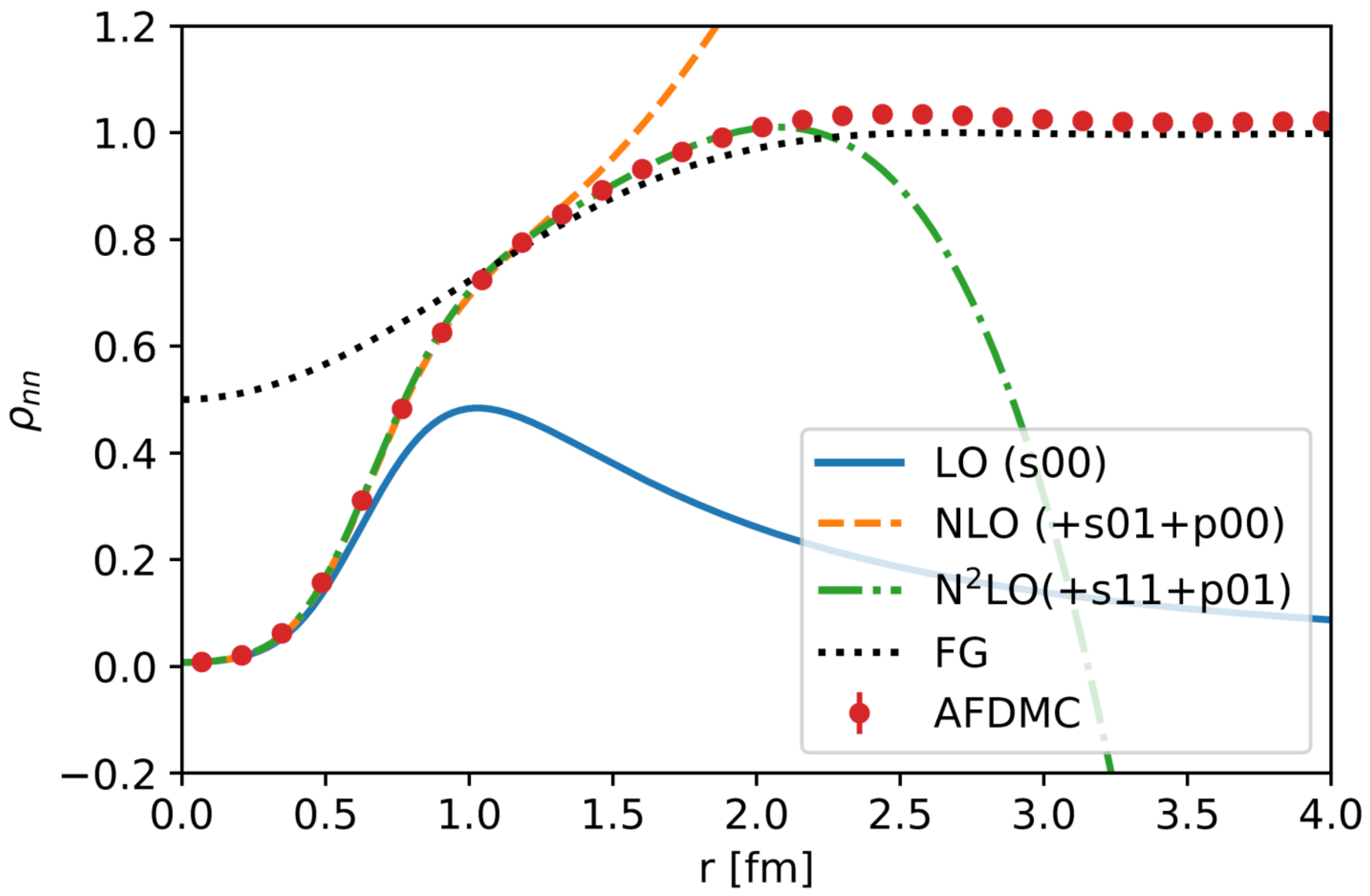
- Neutron matter:

AFDMC by Diego Lonardoni & Stefano Gandolfi:
AV4'+UIX$_C$ n = 0.16 fm^{-3}

5 fitting parameters at N^{2}LO

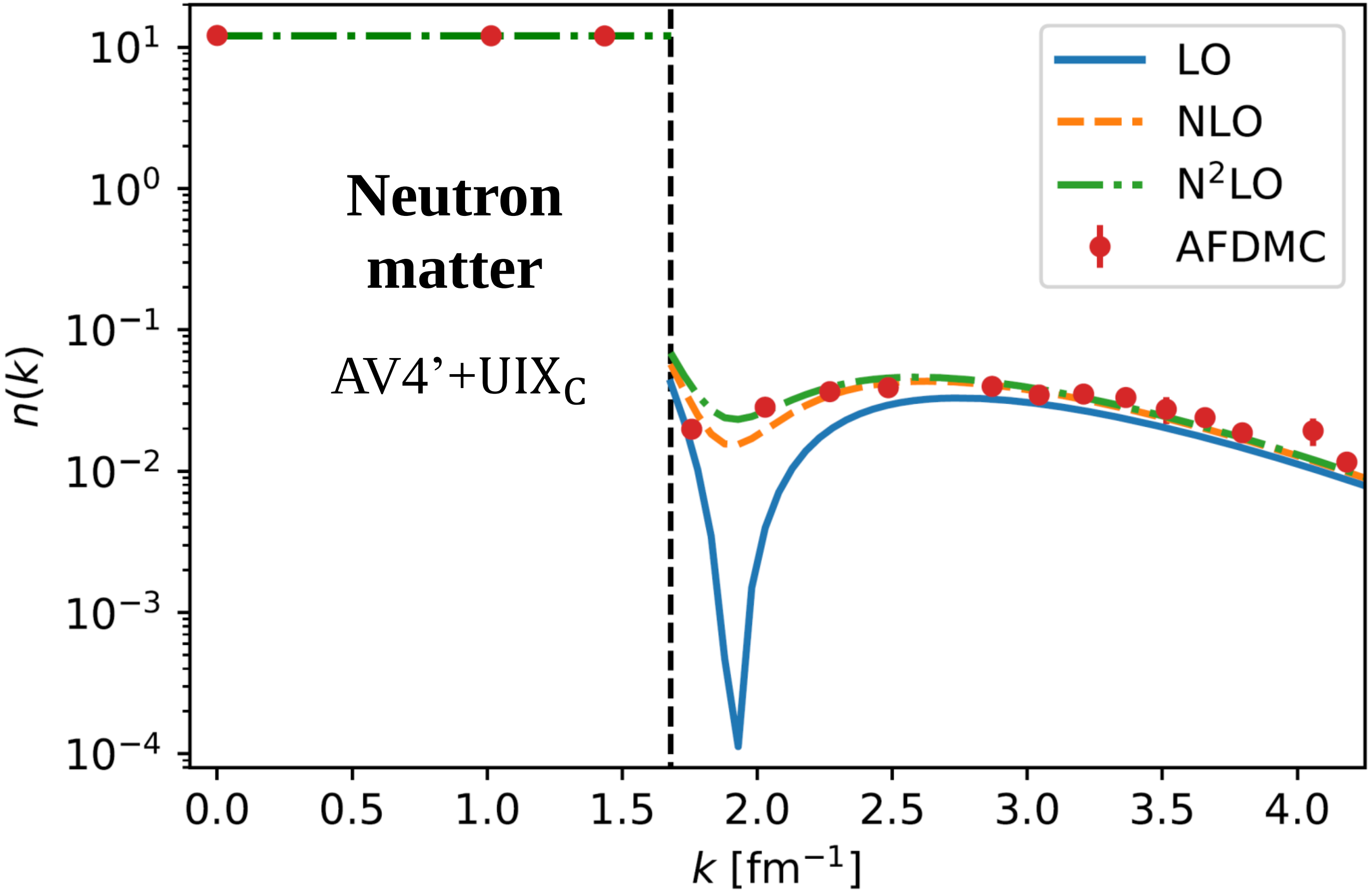
Power counting analyzed analytically for the two-body system



Short-range expansion – many-body system

Momentum distribution

No fitting parameters!

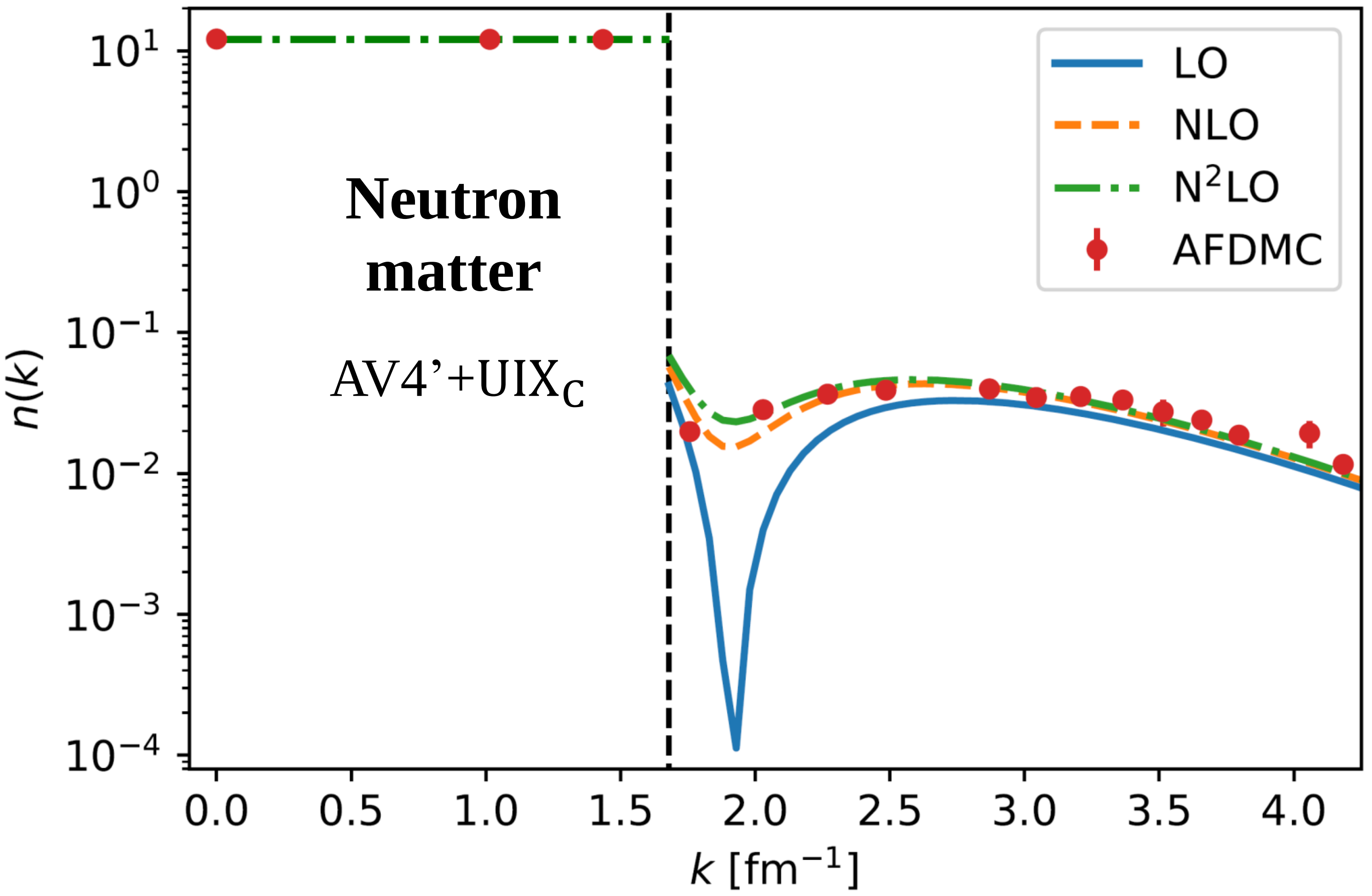


Short-range expansion – many-body system

Momentum distribution

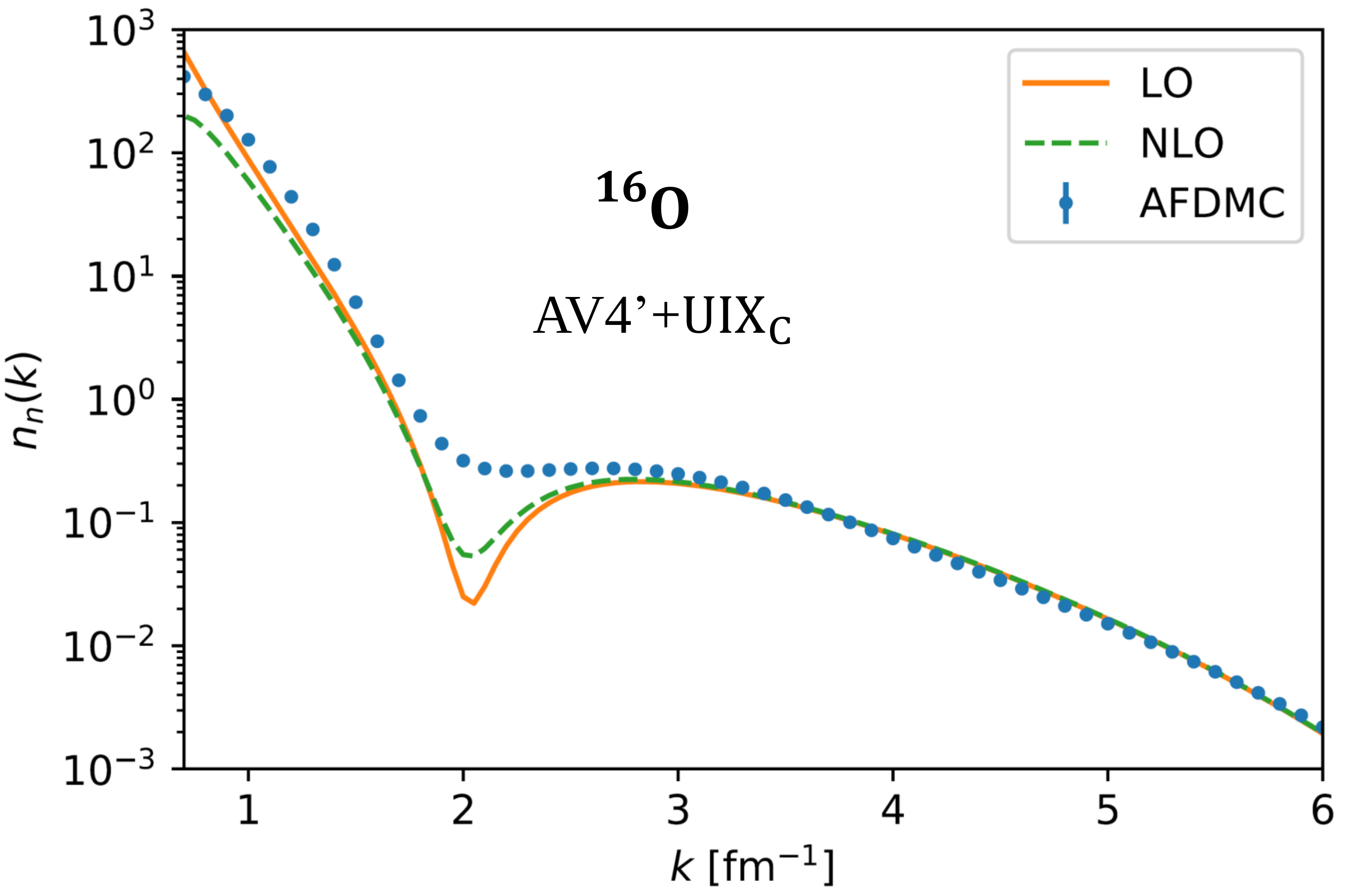
No fitting parameters!

- Systematic description
- Extends to larger distances and smaller momenta
- Future work – analysis of experimental data
- Impact on other observables



Short-range expansion – many-body system

Three-body correlations are needed for finite nuclei



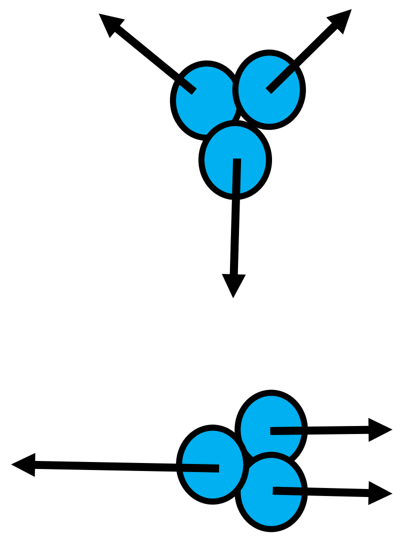
Three-body correlations

RW and S. Gandolfi, Phys. Rev. C 108, L021301 (2023)

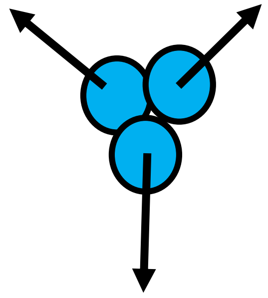
Three-body correlations

Various open questions:

- What are the **dominant configurations**?
- Are 3N SRCs sensitive to the **three-body force**?
- Are they **universal**? What is their **abundance**?
- What is their **contribution to different observables**?



Three-body correlations



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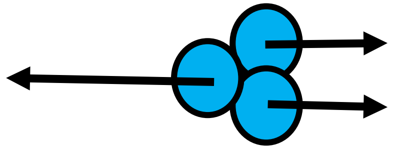
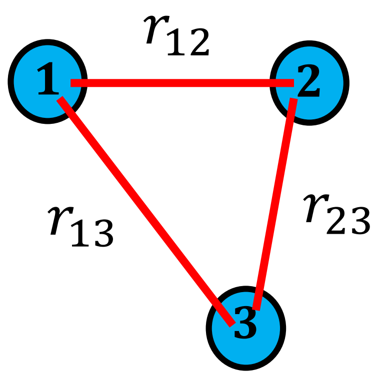


We performed first ab-initio calculations of 3N SRC

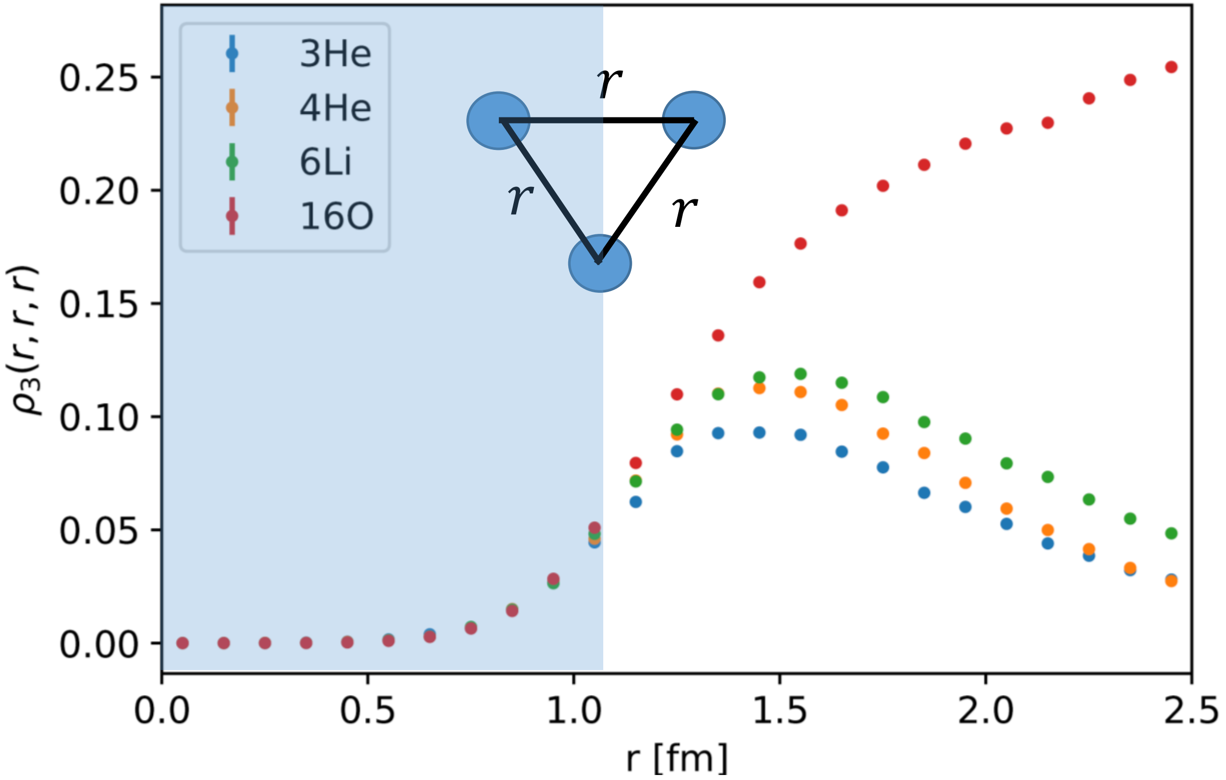
$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle \Psi | \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12}) \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13}) \delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23}) | \Psi \rangle$$

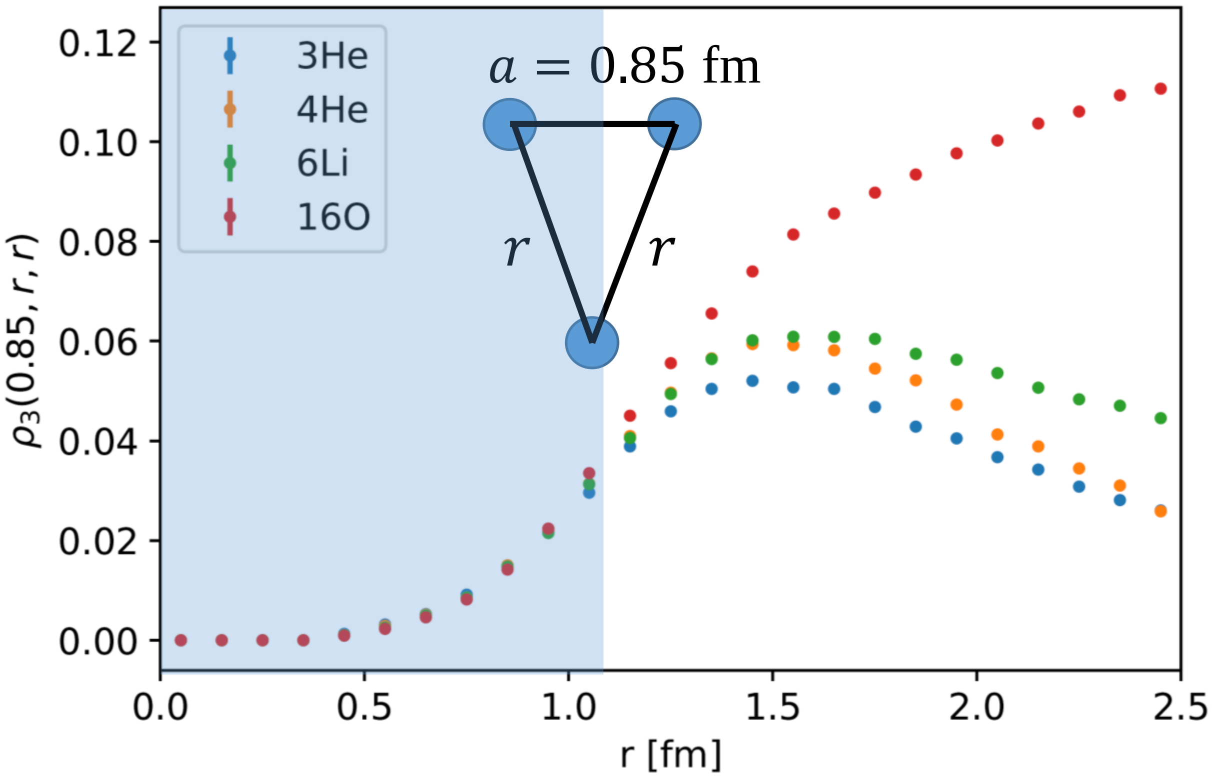


Three-body density

Same scaling factor for all geometries!

Universality





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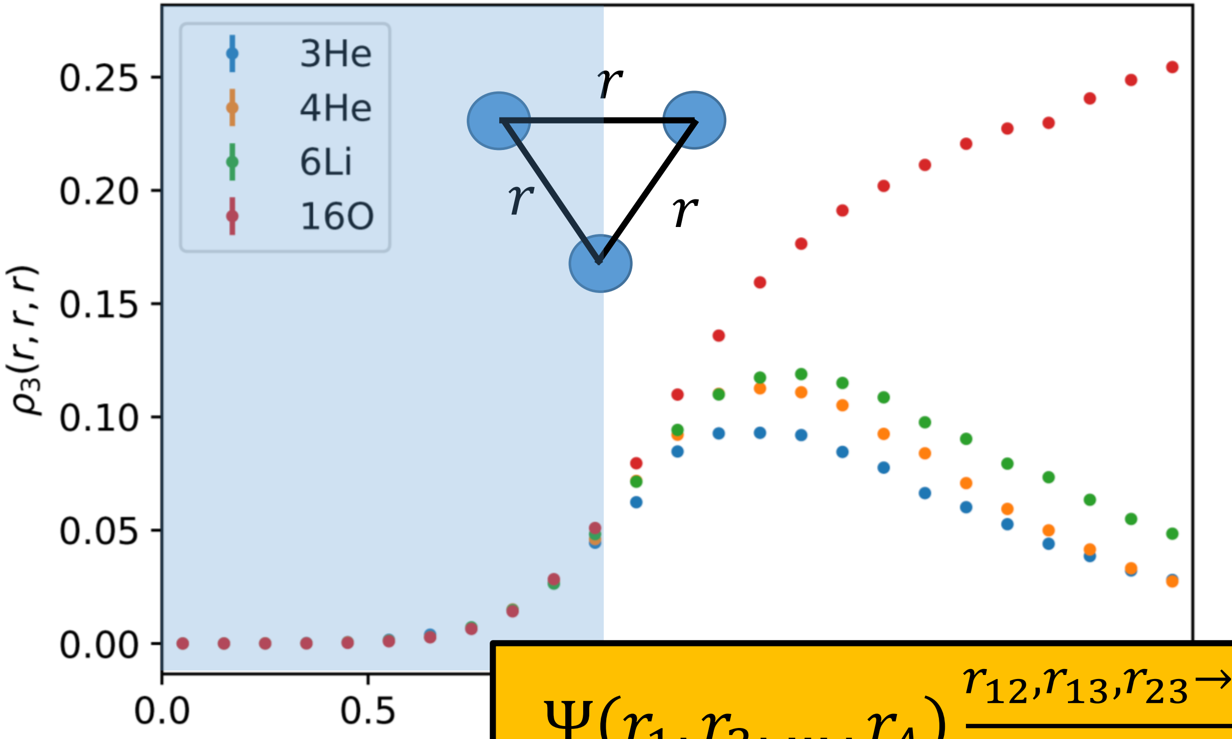
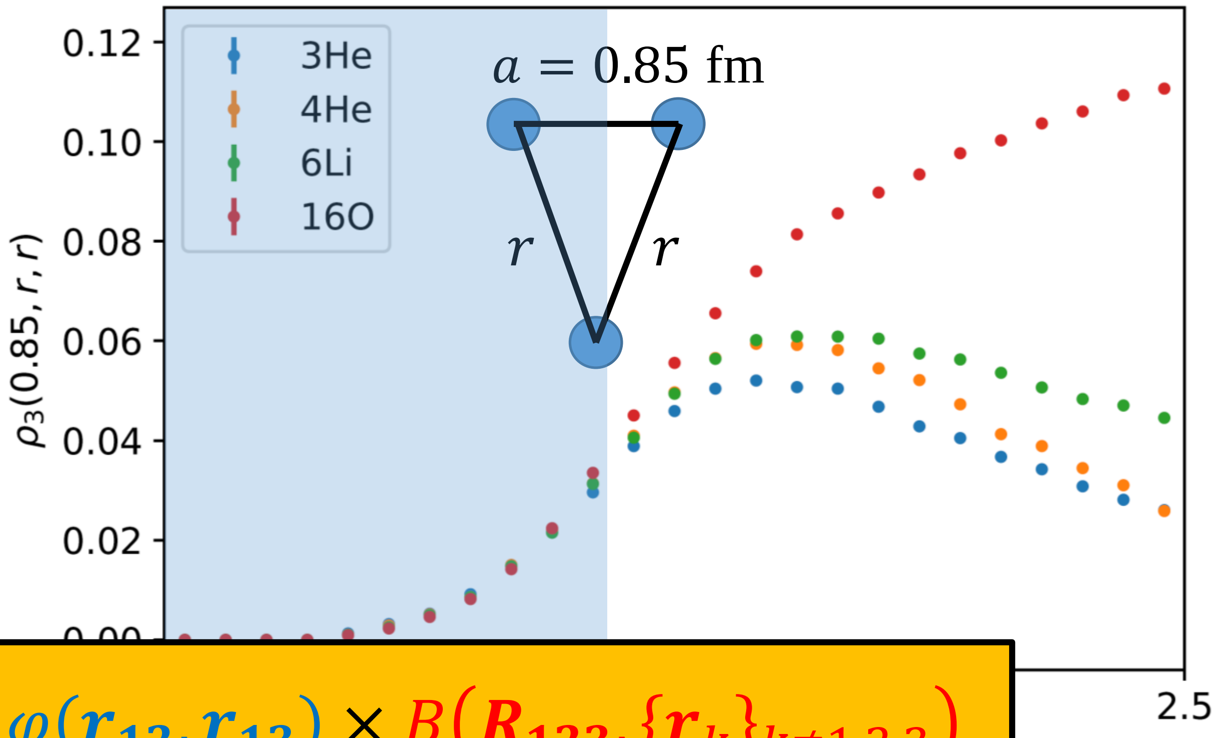




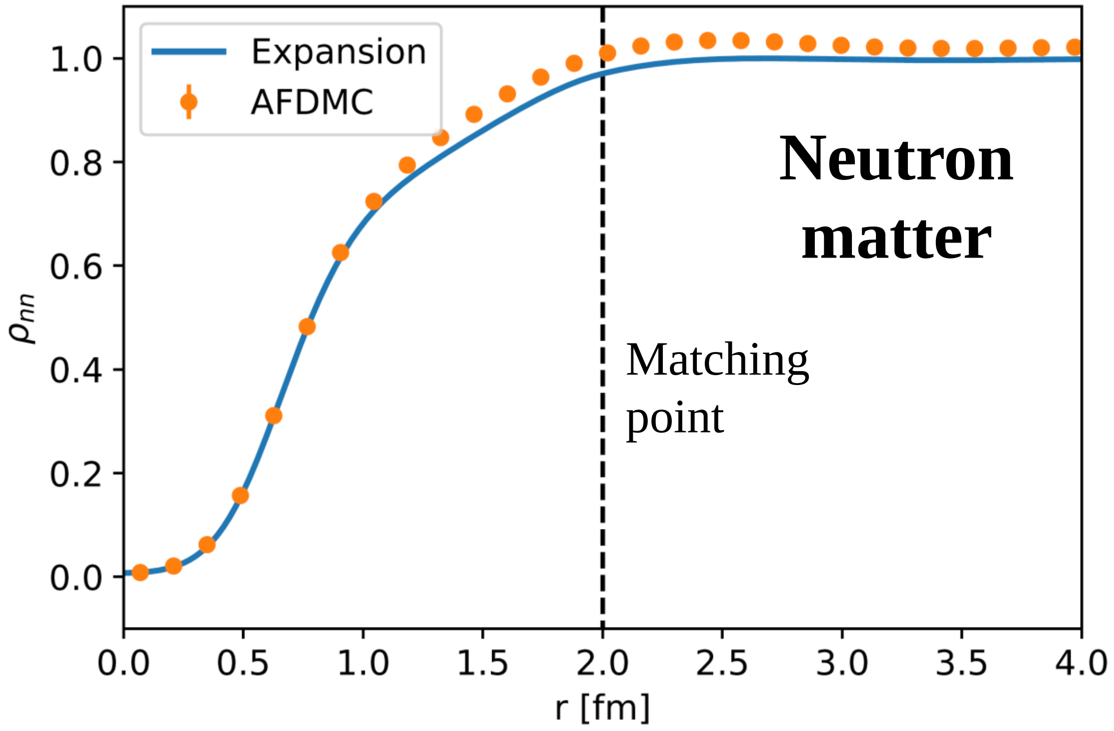
$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{r}_{12}, \boldsymbol{r}_{13}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

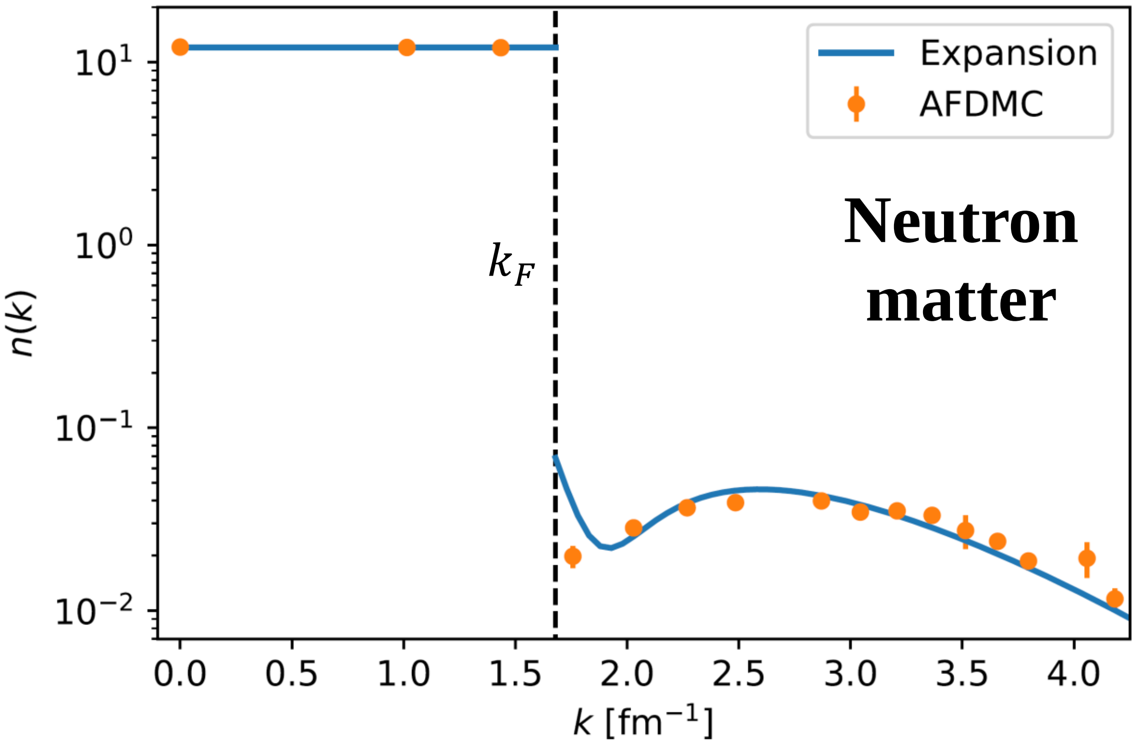
universal function

Describing more general quantities

Matching to long-range model

Fitting only the LO contact and matching to FG:





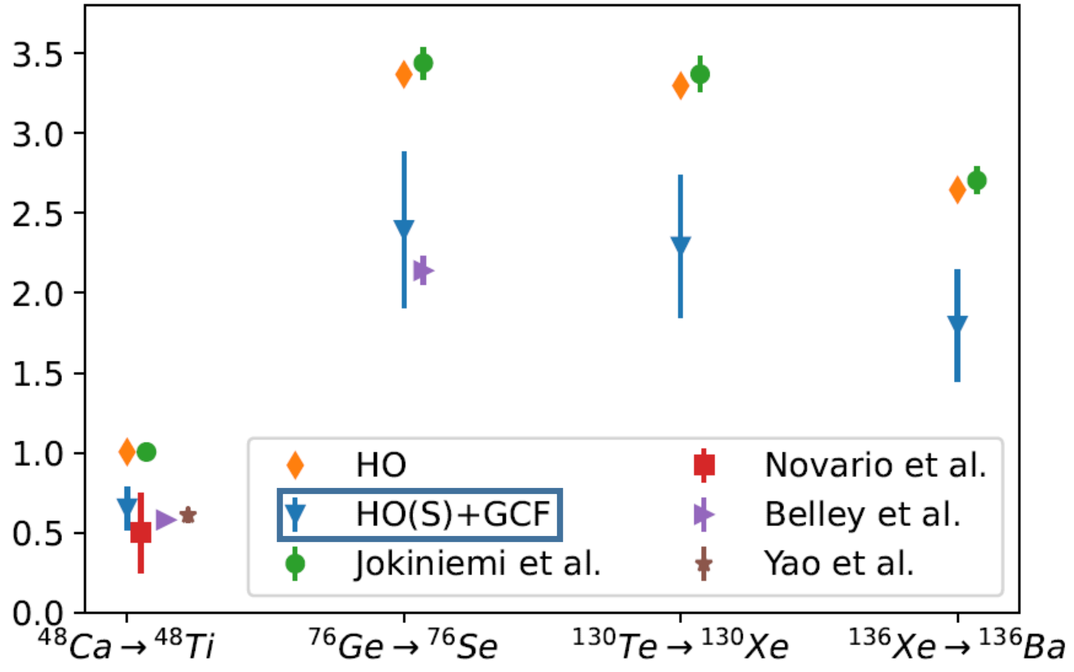
Obtained NN Potential Energy $\langle V_2 \rangle = -29.3$ MeV
Exact NN Potential Energy $\langle V_2 \rangle = -30.1$ MeV

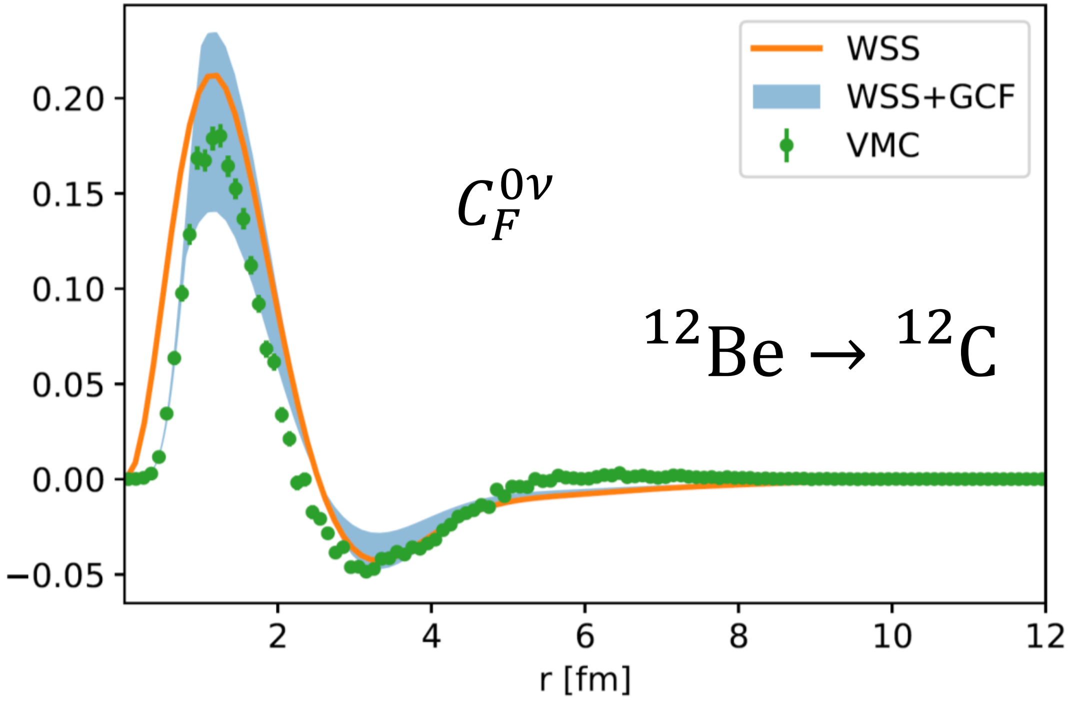
Obtained Kinetic Energy $\langle T \rangle = 43.2$ MeV
Exact Kinetic Energy $\langle T \rangle = 43.3$ MeV

Neutrinoless double beta decay

QMC + Shell-Model + GCF

$M_L^{0\nu}$





$$M_F + M_{GT} + M_T$$

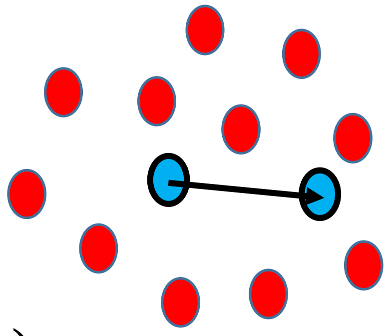
Significant reduction due to SRCs

Similar approach is relevant for more quantities!

Summary and outlook

Summary

- **Nuclear short-range correlations – beyond mean-field effects**
- **Generalized Contact Formalism:** $\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2})$
  - Consistent and comprehensive description of short-range correlated pairs at leading order
  - Accurate description of large-momentum transfer **electron scattering reactions**
- **Short-range expansion:**
  - **Systematic framework** with organized subleading contributions
  - Valid for larger distances / lower momenta
  - Various observables can be described (kinetic and potential energy, $0\nu\beta\beta$,…)



- **3N SRCs**

Future plans

- **Formalism development:**
  - Including **3N SRCs** in systematic expansion
  - Improving methods to **extract contact values**
  - Improving description of **reactions**: final state interactions, relativistic effects…
  - Matching with **long-range models**
  - Imbedding SRC knowledge in **ab-initio methods**

Future plans

- **Formalism development:**
  - Including **3N SRCs** in systematic expansion
  - Improving methods to **extract contact values**
  - Improving description of **reactions**: final state interactions, relativistic effects…
  - Matching with **long-range models**
  - Imbedding SRC knowledge in **ab-initio methods**
- **Applications: Neutrino-nucleus scattering**
  - Comparing to **experimental data**
  - Extraction of relevant contact values
  - Combination with **JLab Ar spectral function** analysis
  - Matching with low-momentum approaches
  - **Event generators?**

Future plans

- **Formalism development:**
  - Including **3N SRCs** in systematic expansion
  - Improving methods to **extract contact values**
  - Improving description of **reactions**: final state interactions, relativistic effects…
  - Matching with **long-range models**
  - Imbedding SRC knowledge in **ab-initio methods**
- **Applications:**
  - **Beta decay** rates, magnetic moments...
  - $\mathbf{0\nu\beta\beta}$ matrix elements
  - Guiding the **detection of 3N SRCs**
  - Testing nuclear **three-body forces**
  - Design and analysis of **exp** (JLab, GSI, EIC)
  - Liquid ^4He structure factor, Dipolar excitons

Questions?

BACKUP

The spectral function

$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_s \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left| \langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle \right|^2$$

The initial wave function

$$\psi_0 \to \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j})$$

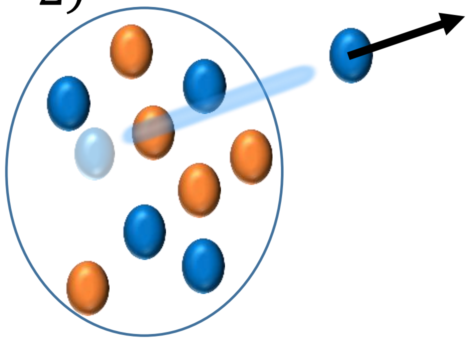
The final wave function

$$|\psi_f^{12}\rangle = a_{\boldsymbol{p_1},s}^\dagger |f_{A-1}\rangle \propto |\Psi_\nu^{A-2}\rangle e^{i\boldsymbol{p_1}\cdot r_1 + i\boldsymbol{p}_2\cdot r_2} \chi_{s_1}\chi_{s_2}$$

Energy conservation:

$$E_f^{A-1} = \epsilon_2 + (A-2)m - B_f^{A-2} + \frac{P_{12}^2}{2m(A-2)}$$

$$B_f^{A-2} \approx \langle B_f^{A-2} \rangle$$



The spectral function

$p_1 > k_F$

$$S^p(\boldsymbol{p_1}, \epsilon_1) = C^1_{pn} S^1_{pn}(\boldsymbol{p_1}, \epsilon_1) + C^0_{pn} S^0_{pn}(\boldsymbol{p_1}, \epsilon_1) + 2C^0_{pp} S^0_{pp}(\boldsymbol{p_1}, \epsilon_1)$$

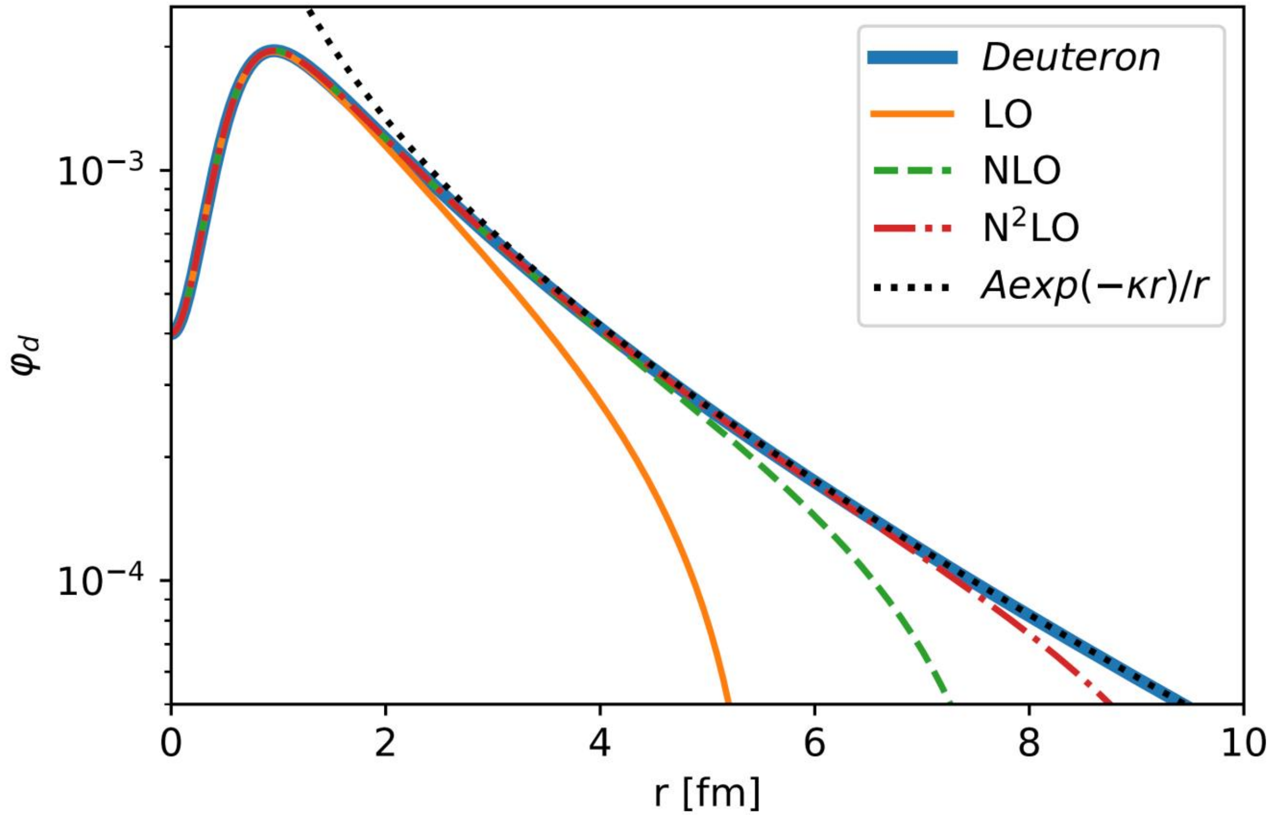
$$S^\alpha_{ab}(\boldsymbol{p_1}, \epsilon_1) = \frac{1}{4\pi} \int \frac{d^3p_2}{(2\pi)^3} \underbrace{\delta(f(\boldsymbol{p}_2))}_{\text{Energy conservation}} \underbrace{n_{CM}(\boldsymbol{p_1} + \boldsymbol{p_2})}_{\text{CM momentum distribution (Gaussian)}} \underbrace{|\tilde{\varphi}^\alpha_{ab}(|\boldsymbol{p_1} - \boldsymbol{p_2}|/2)|^2}_{\text{Two-body function}}$$

Similar to the convolution model

C. Ciofi degli Atti, S. Simula, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. C 44, R7(R) (1991),
C. Ciofi degli Atti and S. Simula PRC ***53,*** *1689 (1996)*

Short-range expansion – two-body system

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4’
Deuteron channel
Bound state

Short-range expansion – many-body system

- **The many-body case:** Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

(α – quantum numbers)

Complete set of
two-body functions

Short-range expansion – many-body system

- **The many-body case:** Exact expansion

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_{E,\alpha}\varphi_\alpha^E(\boldsymbol{r}_{12})A_\alpha^E(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

(α – quantum numbers)

- Taylor expansion around $E=0$:

$$\varphi_\alpha^E(\boldsymbol{r})=\varphi_\alpha^{E=0}(\boldsymbol{r})+\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)E+\frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)E^2+\cdots$$

GCF factorization

Subleading terms

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_\alpha\varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)}+\sum_\alpha\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)}+\sum_\alpha\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(2)}+\cdots$$

Short-range expansion – many-body system

- The many-body case:

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)}+\sum_\alpha\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)}+\sum_\alpha\left(\frac{d^2}{dE^2}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(2)}+\cdots$$

- Two-body density:

$$\rho_2(r)=\sum_\alpha\left(|\varphi_\alpha^{E=0}(r)|^2C_\alpha^{00}+\varphi_\alpha^{E=0}(r)\frac{d}{dE}\varphi_\alpha^{E=0}(r)C_\alpha^{01}+\cdots\right)$$

- Subleading contacts:

$$C_\alpha^{mn}\propto\langle A_\alpha^{(m)}|A_\alpha^{(n)}\rangle$$

Short-range expansion – many-body system

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

- **Power counting** is needed
- Two relevant parameters:
  - Number of energy derivatives
  - Orbital angular momentum $(s, p, d, \dots)$
- Can be analyzed analytically for the two-body system

Short-range expansion: Next order terms

The many-body case:

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{\alpha} \varphi_{\alpha}^{E=0}(\boldsymbol{r}_{12}) A_{\alpha}^{(0)} + \sum_{\alpha} \left(\frac{d}{dE} \varphi_{\alpha}^{E=0}(\boldsymbol{r}) \right) A_{\alpha}^{(1)} + \sum_{\alpha} \left(\frac{d^2}{dE^2} \varphi_{\alpha}^{E=0}(\boldsymbol{r}) \right) A_{\alpha}^{(2)} + \cdots$$

$$A_{\alpha}^{(0)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A) = \sum_{E} A_{\alpha}^{E}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

$$A_{\alpha}^{(1)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A) = \sum_{E} E A_{\alpha}^{E}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

$$A_{\alpha}^{(2)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A) = \frac{1}{2!} \sum_{E} E^2 A_{\alpha}^{E}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

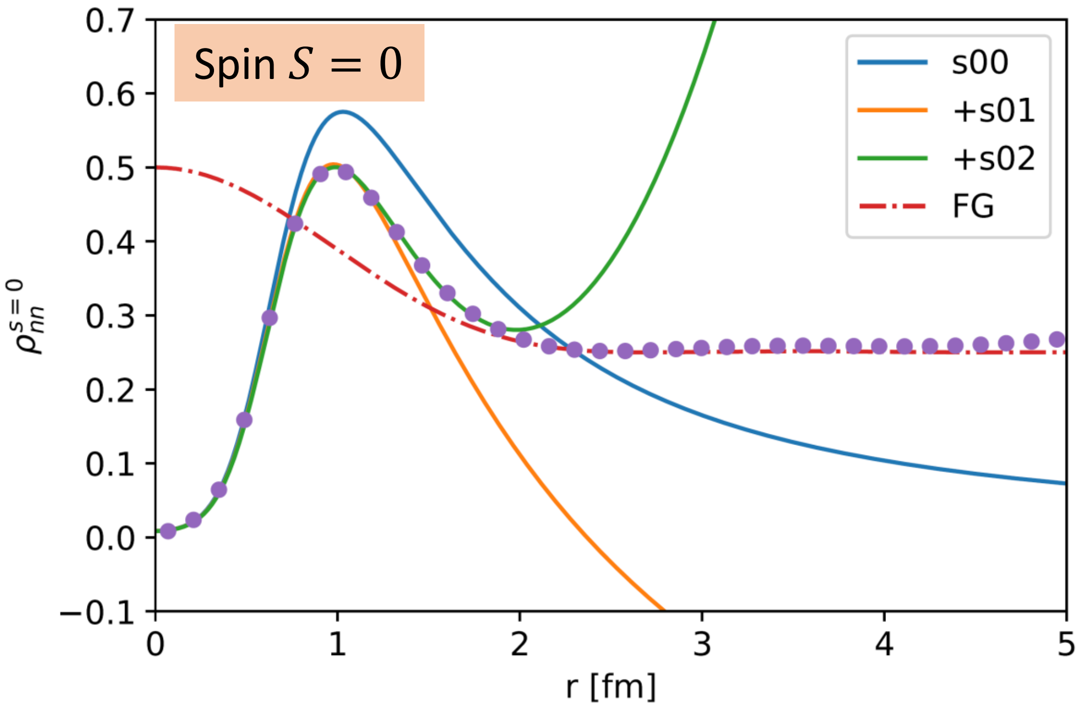
Short-range expansion

$$\rho_2(r) = \sum_\alpha \left(|\varphi_\alpha^{E=0}(r)|^2 C_\alpha^{00} + \varphi_\alpha^{E=0}(r) \frac{d}{dE} \varphi_\alpha^{E=0}(r) C_\alpha^{01} + \cdots \right)$$

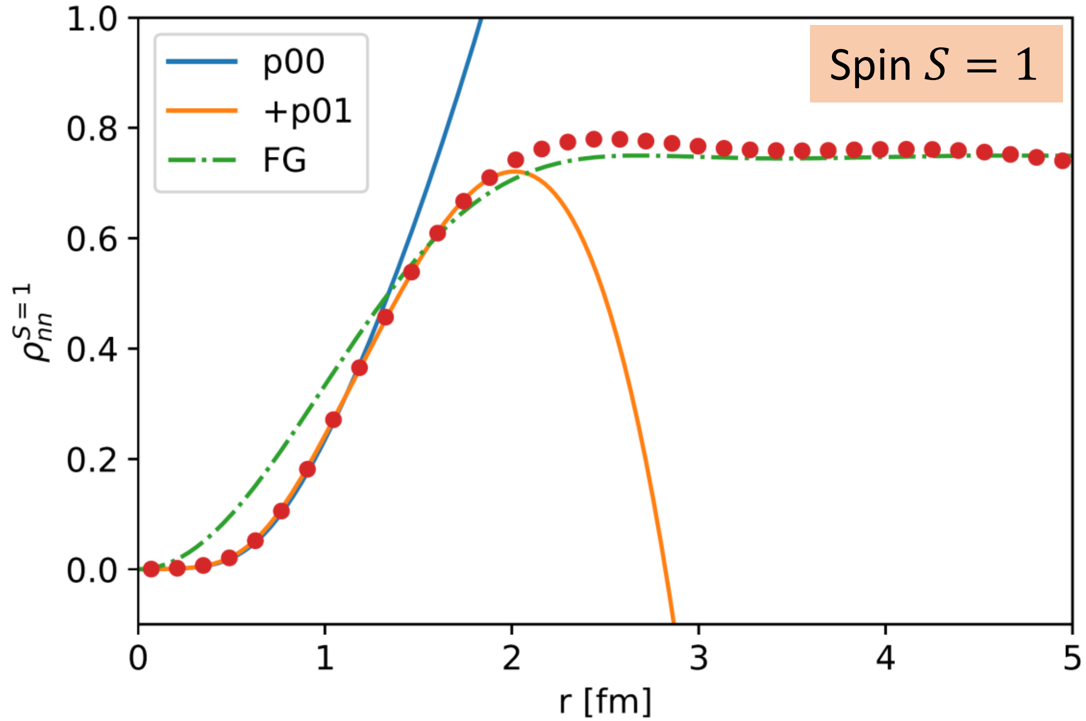
- Neutron matter:

  $(S + \ell = \text{Even})$

AFDMC by Diego Lonardoni & Stefano Gandolfi: AV4' n = 0.16 fm^{-3}



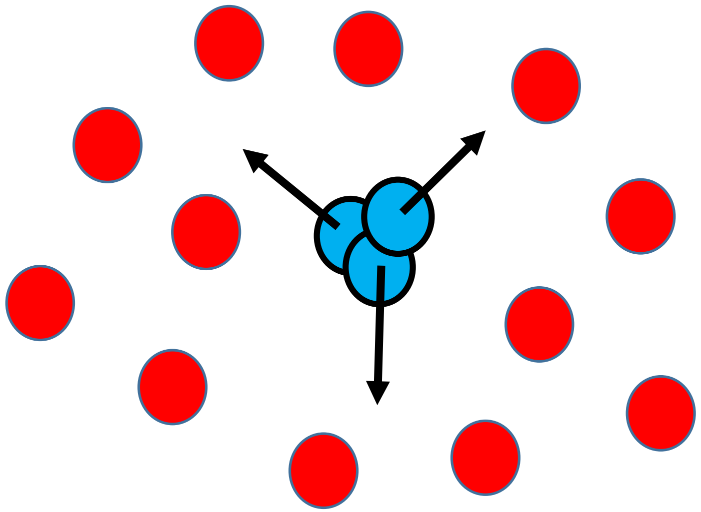
s-wave: $\ell = 0, S = 0, j = 0$

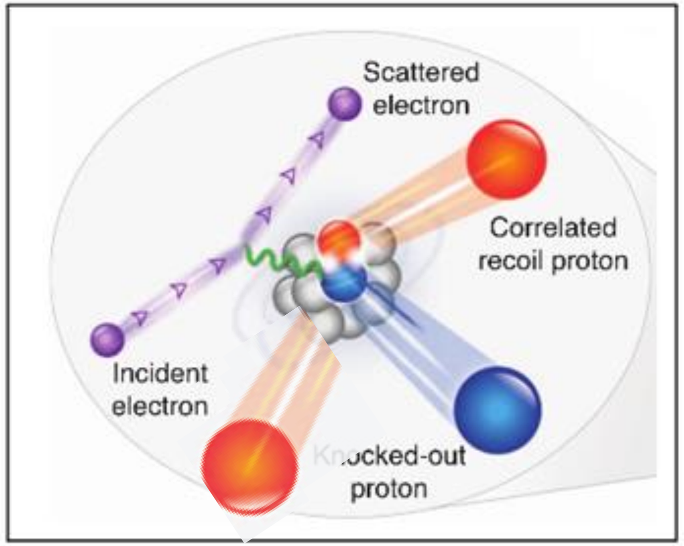


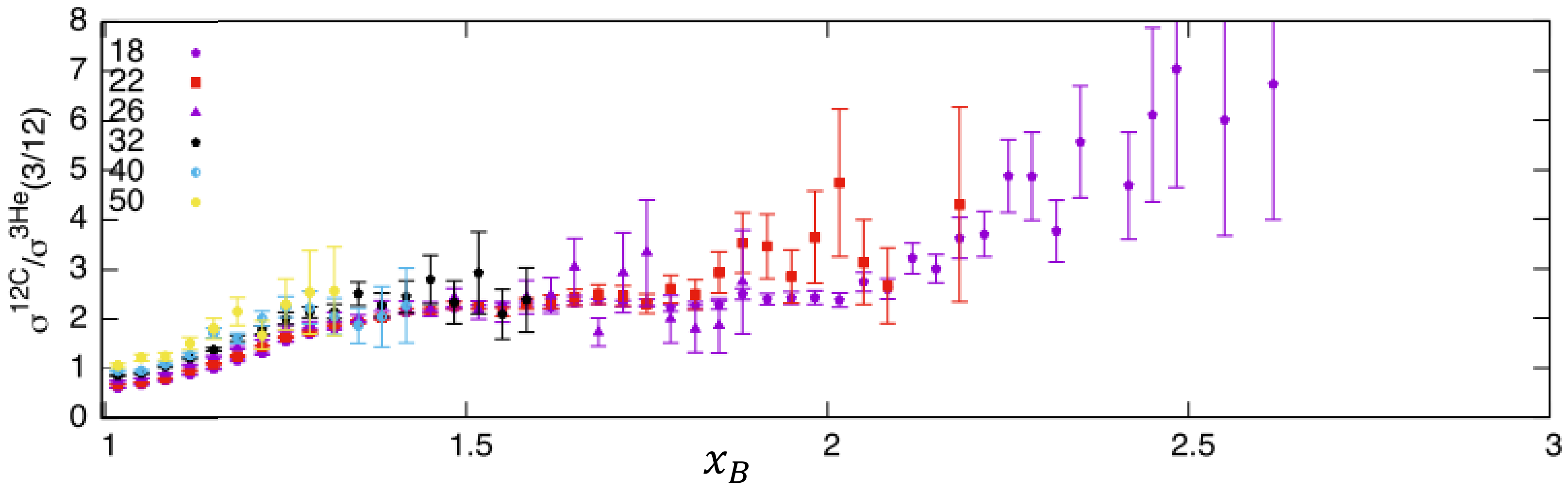
p-wave: $\ell = 1, S = 1, j = 0/1/2$

Three-body correlations

- There is no clear experimental signal of 3N SRCs in nuclear systems.
- But significant experimental efforts



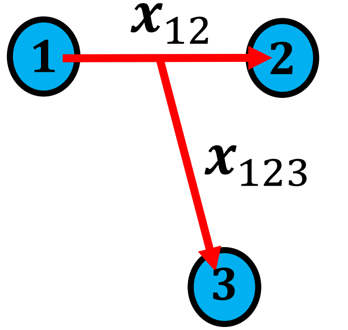




D. B. Day, et al, PRC 107, 014319 (2023)

Three-body correlations

$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{x}_{\boldsymbol{12}}, \boldsymbol{x}_{\boldsymbol{123}}) \times B\left(\boldsymbol{R}_{\boldsymbol{123}}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

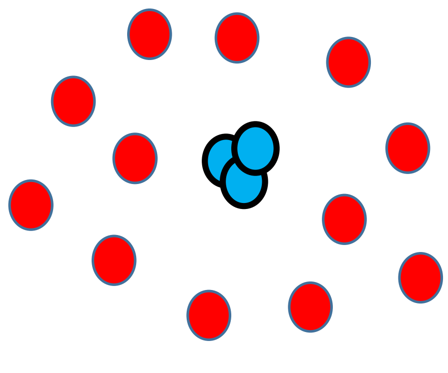


- A **single** leading channel:

$$j^{\pi} = \frac{1}{2}^{+}, t = \frac{1}{2}$$

- The same quantum numbers as ^{3}He

- Therefore, **at short-distances** we expect:
  - **$T = 1/2$ dominance** (over $T = 3/2$)
  - **Universality** - All nuclei should behave like ^{3}He

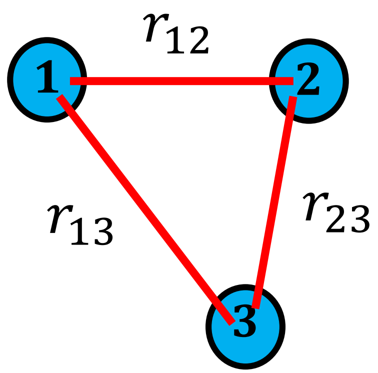


Three-body density

Ab-initio calculations – AFDMC (with Stefano Gandolfi):

$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle \Psi | \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12}) \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13}) \delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23}) | \Psi \rangle$$

- Projections to $T = \frac{1}{2}$ and $T = \frac{3}{2}$
- N2LO($R = 1.0$ fm)E1 local chiral interaction
- Nuclei: ^{3}He, ^{4}He, ^{6}Li, , ^{16}O



Three-body contact values ($T = 1/2$)

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3 \qquad \frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3 \qquad \frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

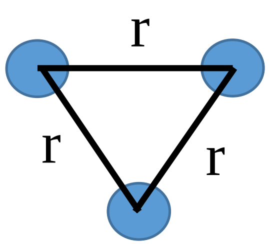
Can be compared to inclusive cross section ratios (in the appropriate kinematics)

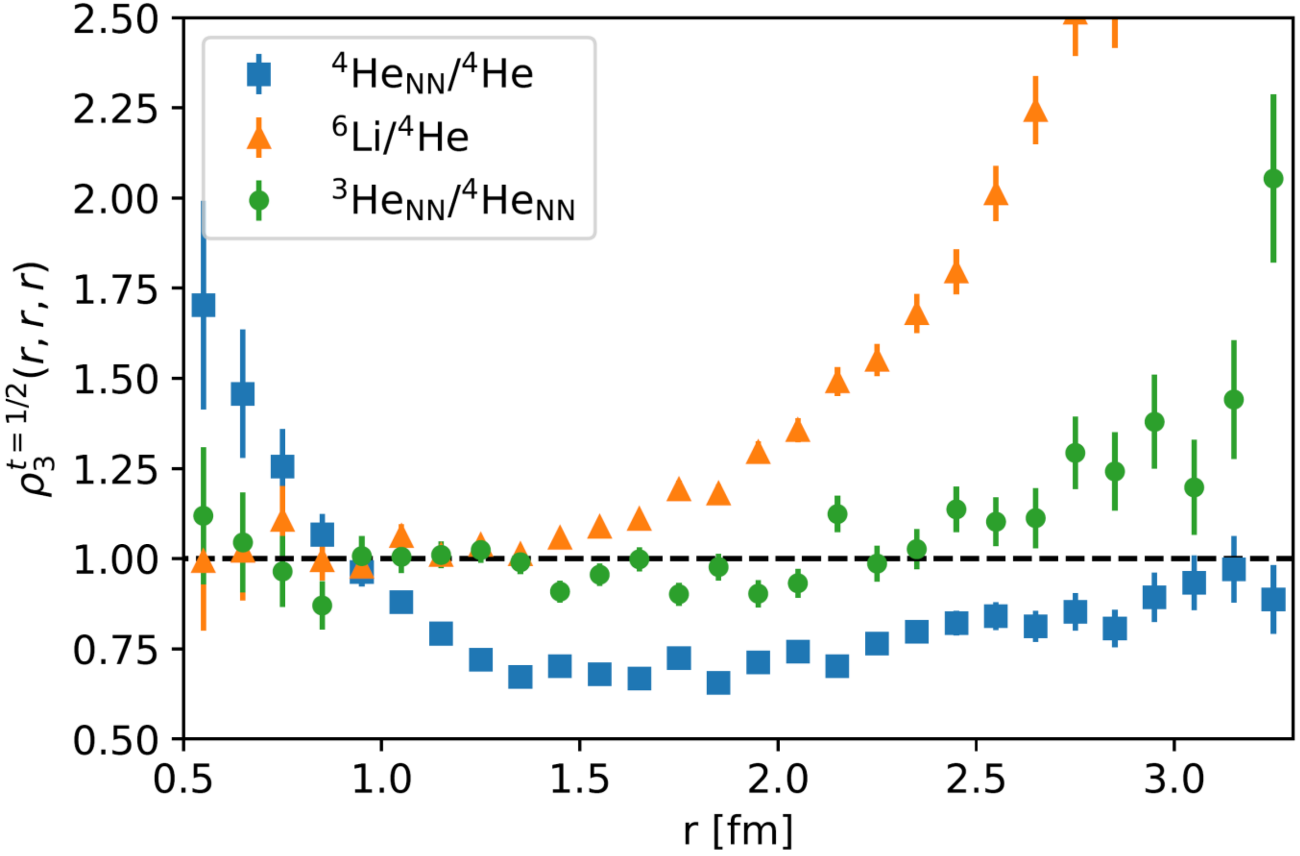
$$a_3(A) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e\,^3\mathrm{He}} + \sigma_{e\,^3\mathrm{H}})/2}$$

For a symmetric nucleus A

$$a_3(A) = \frac{3}{A} \frac{C(A)}{C(^3\mathrm{He})}$$

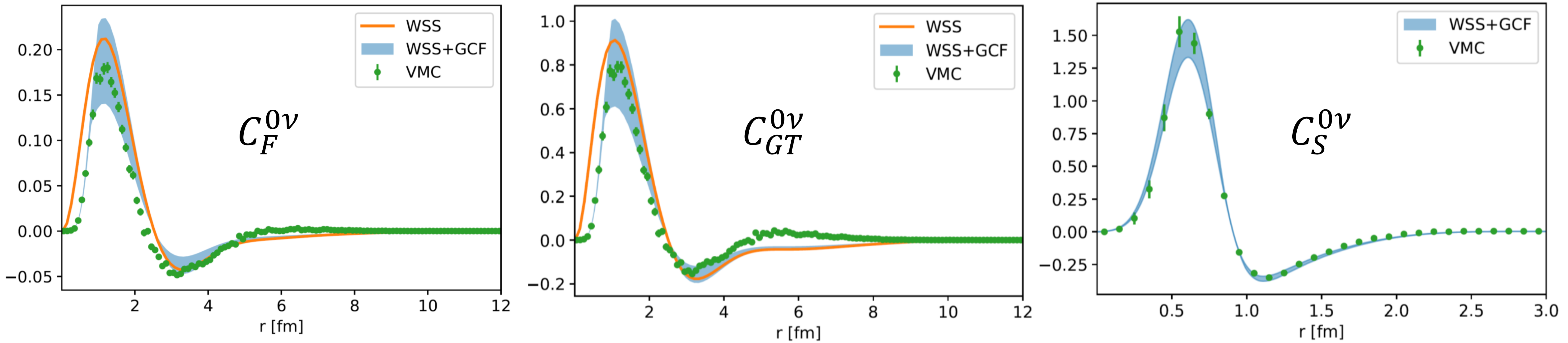
Sensitivity to three-body forces





Results – light nuclei (AV18)

Using $^{6}He \rightarrow {}^{6}Be$ and $^{10}Be \rightarrow {}^{10}C$ to "predict" $^{12}Be \rightarrow {}^{12}C$

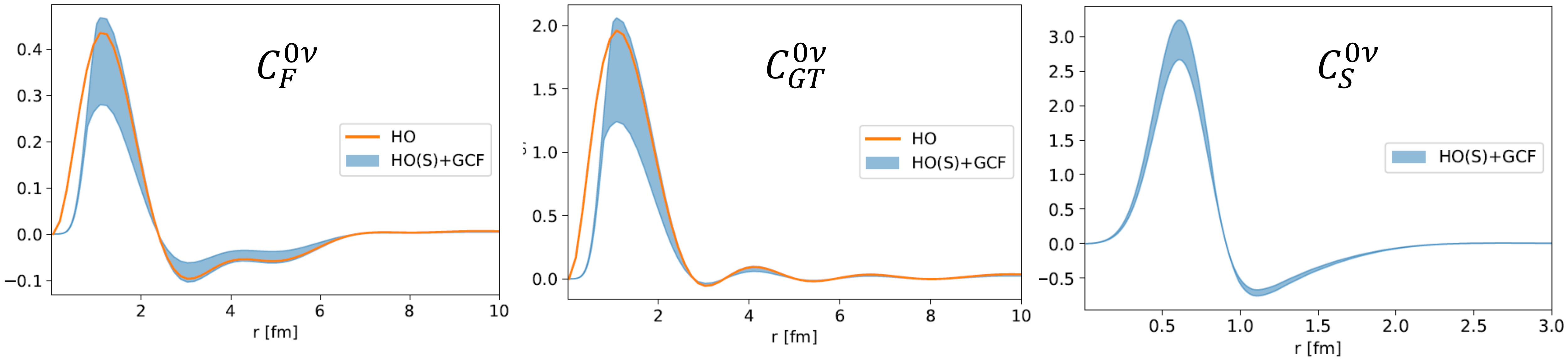


Short distances - GCF

Long distances – Shell model

Results – heavy nuclei (AV18)

- Transition densities of **heavy nuclei**:



$^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}$

Model independence of contact ratios

- For $0\nu 2\beta$:

$$\frac{C^{AV18}(f_1,i_1)}{C^{AV18}(f_2,i_2)} = \frac{C^{SM}(f_1,i_1)}{C^{SM}(f_2,i_2)}$$

$$C^{AV18}(f_1,i_1) = \frac{C^{SM}(f_1,i_1)}{C^{SM}(f_2,i_2)} C^{AV18}(f_2,i_2)$$

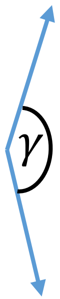
- For example

$$C^{AV18}\left({}^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}\right) = \frac{C^{SM}\left({}^{76}\mathrm{Ge} \rightarrow {}^{76}\mathrm{Se}\right)}{C^{SM}\left({}^{12}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}\right)} C^{AV18}\left({}^{12}\mathrm{Be} \rightarrow {}^{12}\mathrm{C}\right)$$

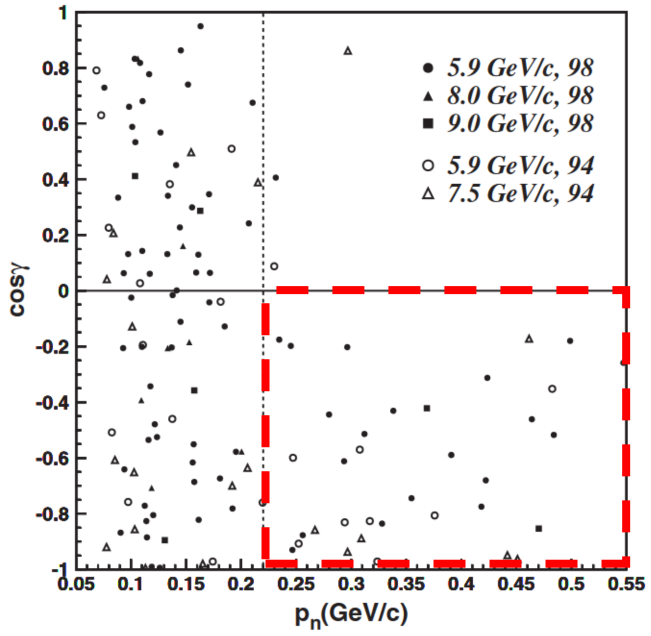
Exact QMC calculations

SRCs in Nuclear Systems

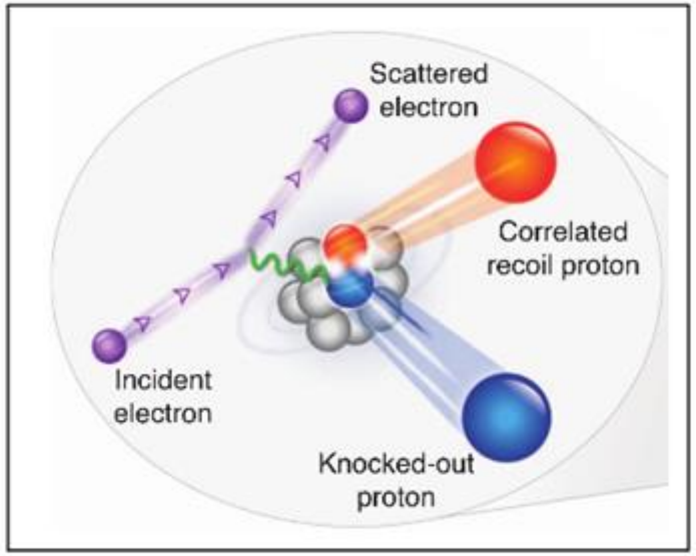
Studied **experimentally** using large momentum transfer quasi-elastic reactions

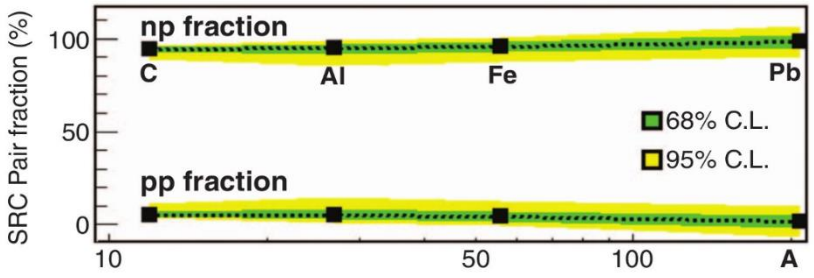


$^{12}\mathrm{C}(p, ppn)X$

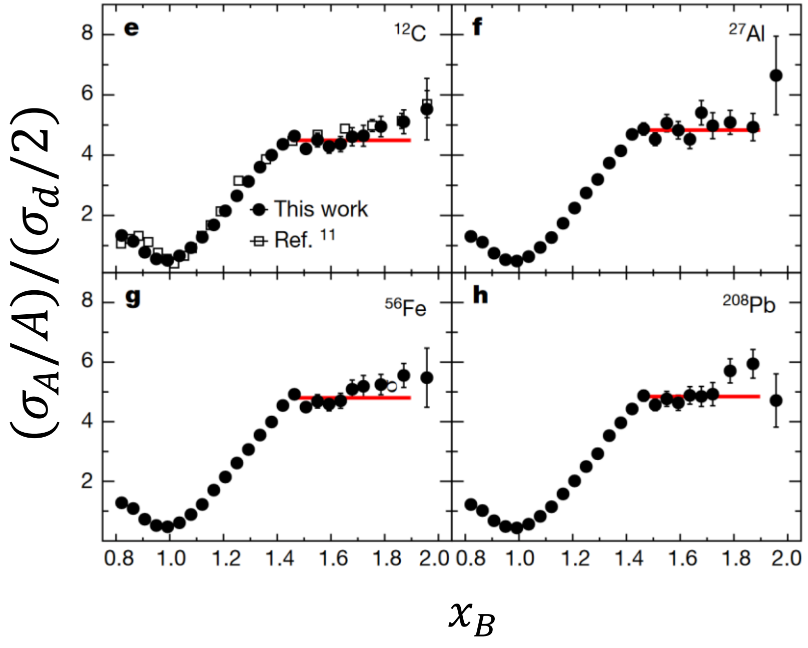


Piasetzky et al., PRL 97, 162504 (2006)





O. Hen et al., Science 346, 614 (2014)



B. Schmookler et. al. (CLAS Collaboration), Nature 566, 354 (2019)

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij},\{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^\alpha | A_{ij}^\beta \rangle$$

Channels α $= (\ell_2 S_2) j_2 m_2$

Universal functions

The pair kind $ij \in \{pp, nn, pn\}$

3 matrices of Nuclear Contacts

Main channels for nuclear systems:

The **spin-one** channel: $\ell_2 = 0,2\,;\, s_2 = 1\,;\, j_2 = 1\,;\, t_2 = 0$ (only **np pairs**)

The **spin-zero** channel: $\ell_2 = 0\,;\, s_2 = 0\,;\, j_2 = 0\,;\, t_2 = 1$ (**All pairs**)

RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij}\rightarrow 0} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{\boldsymbol{ij}}^{\boldsymbol{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

Channels α $= (\ell_2 S_2) j_2 m_2$

Universal functions

The pair kind $ij \in \{pp, nn, pn\}$

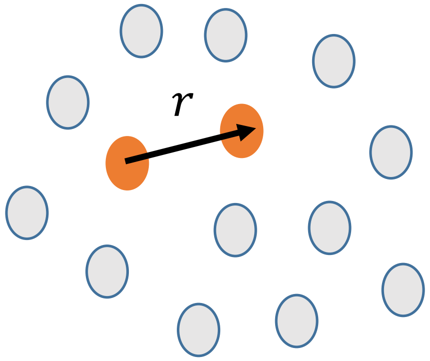
3 matrices of Nuclear Contacts

Some features can be explained using:

- RG arguments — S. K. Bogner and D. Roscher, Phys. Rev. C 86, 064304 (2012). A. J. Tropiano, S. K. Bogner, and R. J. Furnstahl, Phys. Rev. C 104, 034311 (2021)
- Coupled Cluster expansion — S. Beck, RW, N. Barnea, Phys. Rev. C 107, 064306 (2023) S. Beck, RW, N. Barnea, arXiv:2305.17649 [nucl-th] (2023)

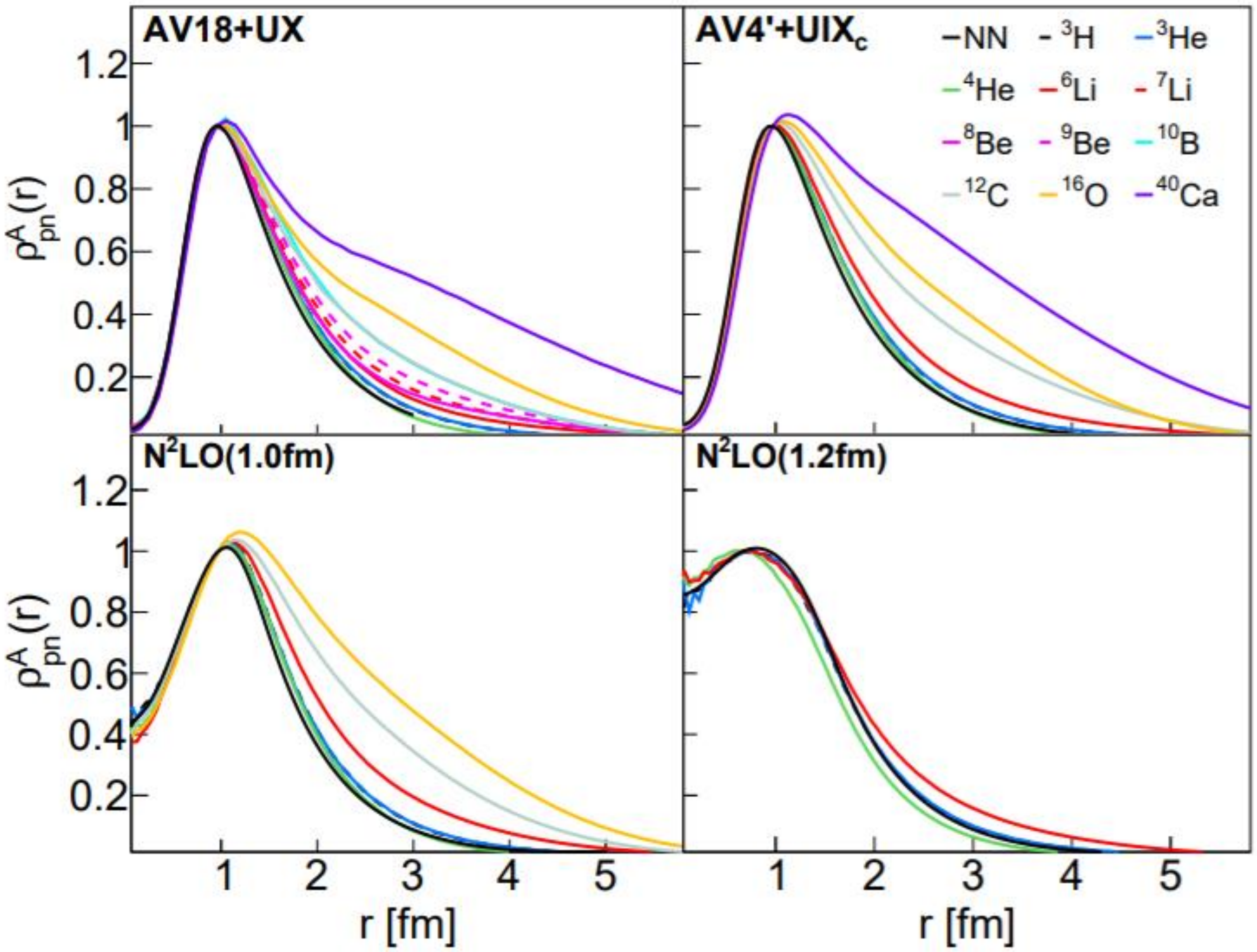
RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Two-body density



$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$

$$\rho_{NN}(\boldsymbol{r}) \xrightarrow{r \to 0} C |\varphi(\boldsymbol{r})|^2$$



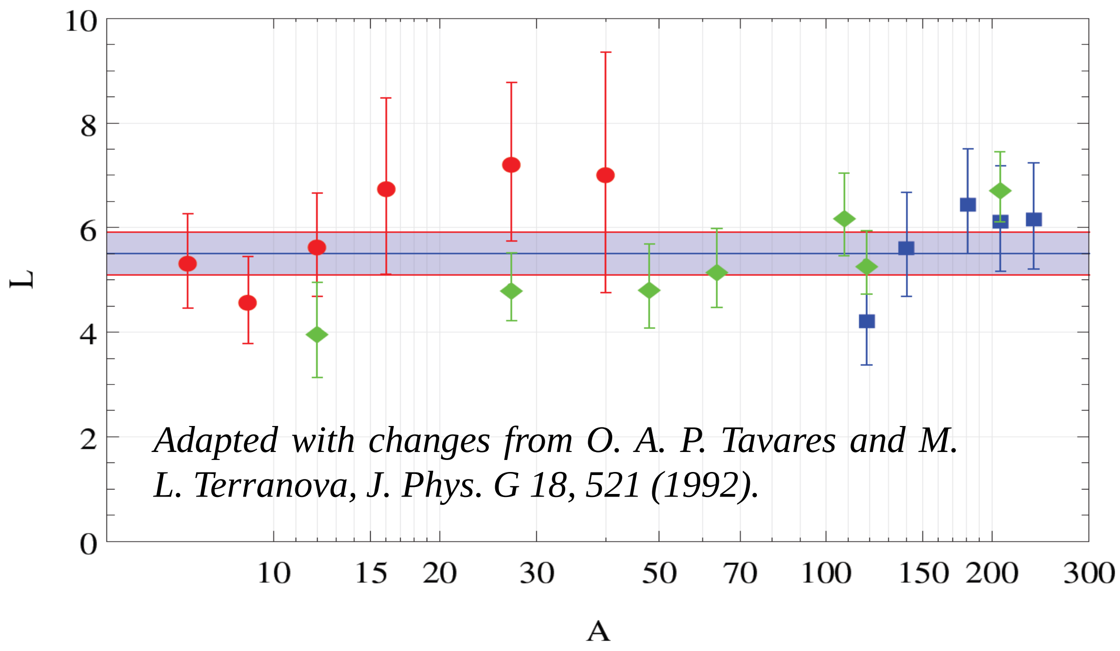
Shows the validity of the factorization

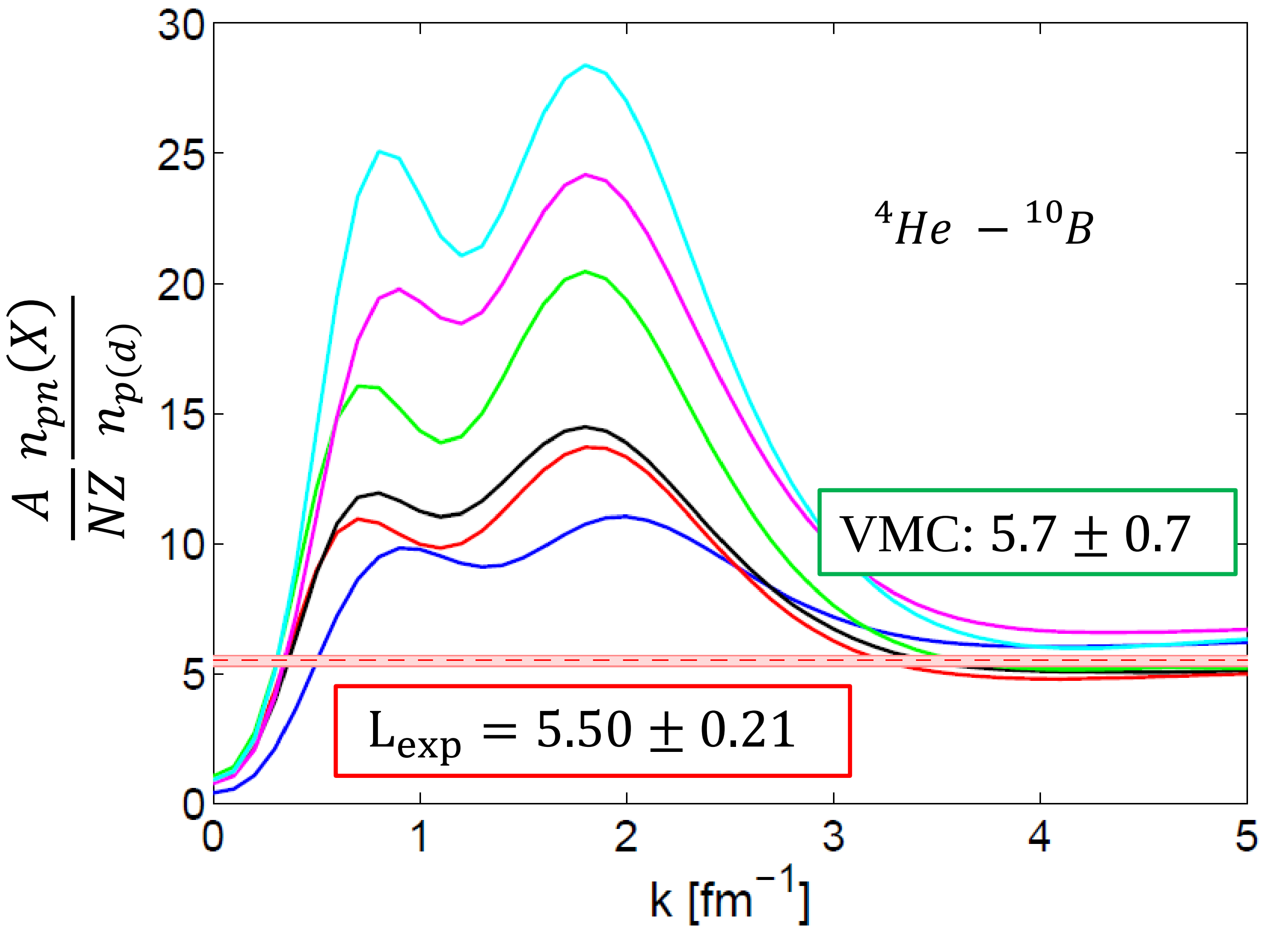
R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

Nuclear photoabsorption

$$\sigma_X(\omega) = L\frac{NZ}{A}\sigma_d(\omega)$$

Levinger Model





RW, B. Bazak, N. Barnea, PRL 114, 012501 (2015) ; PRC 92, 054311 (2015)

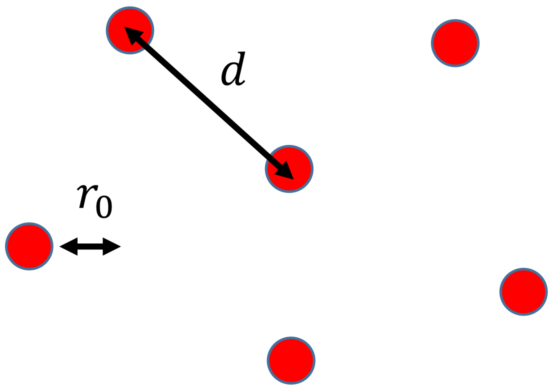
The Contact Theory

- Dilute systems - with **negligible interaction range**

- Zero-range condition:

$$r_0 \ll a, d$$

Interaction range — Scattering length — Average distance between particles



- Zero-range model: Non-interacting particles with boundary condition

Independent of the details of the interaction

$$\Psi(r_1, r_2, \ldots, r_N) \xrightarrow{r_{12}\to 0} \left(\frac{1}{r_{12}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{12}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

The Contact Theory

$$\Psi(r_1, r_2, \dots, r_N) \xrightarrow{r_{12}\to 0} \left(\frac{1}{r_{12}} - \frac{1}{a}\right) \times A\left(\mathbf{R}_{12}, \{\mathbf{r}_k\}_{k\neq 1,2}\right)$$

- A parameter – **the contact** – can be defined:

$$\boldsymbol{C} \propto \langle A|A\rangle$$

- C ≈ **number of SRC pairs** in the system

- Connected to many quantities in the system

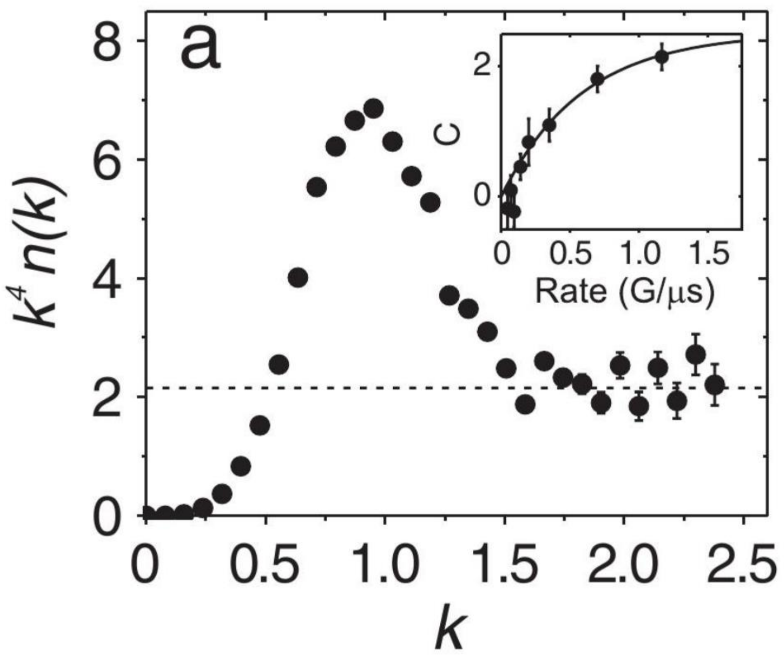
$$n(k) \xrightarrow{k\to\infty} \boldsymbol{C}/k^4$$

$$T + U = \frac{\hbar^2}{4\pi m a}\boldsymbol{C} + \sum_{\sigma} \frac{d^3k}{(2\pi)^3} \frac{\hbar^2 k^2}{2m} \left(n_\sigma(\boldsymbol{k}) - \frac{\boldsymbol{C}}{k^4}\right) \qquad \dots$$

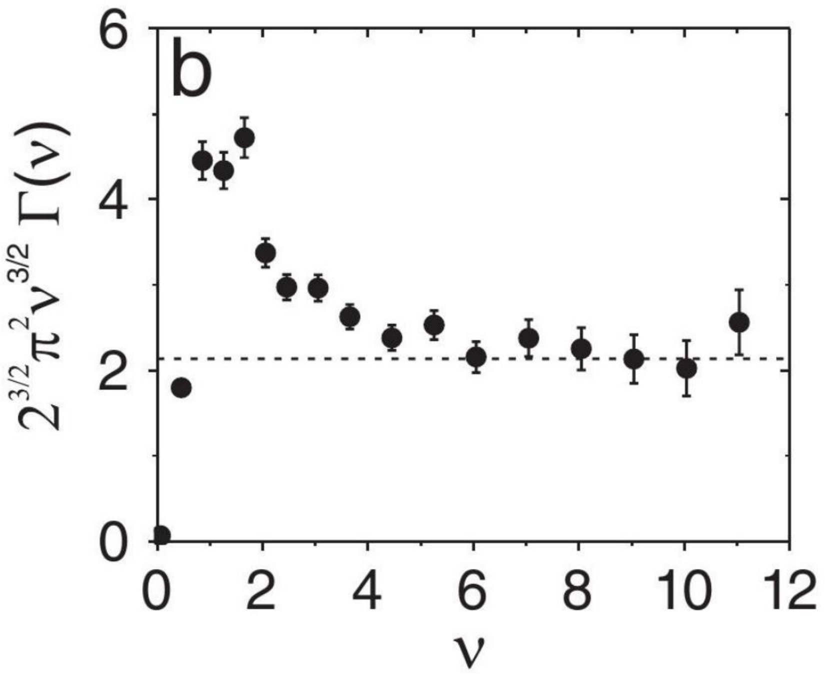
The Contact Theory

- Verified experimentally: (ultra-cold atomic systems)

Momentum distribution



RF line shape



J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)