# Eta production from nucleons and nuclei

**Atika Fatima**

Collaborators

M. Sajjad Athar and S. K. Singh

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# Outline

1. *Introduction*
2. *Eta production from free nucleon*
3. *Eta production from bound nucleon*
4. *Conclusion*

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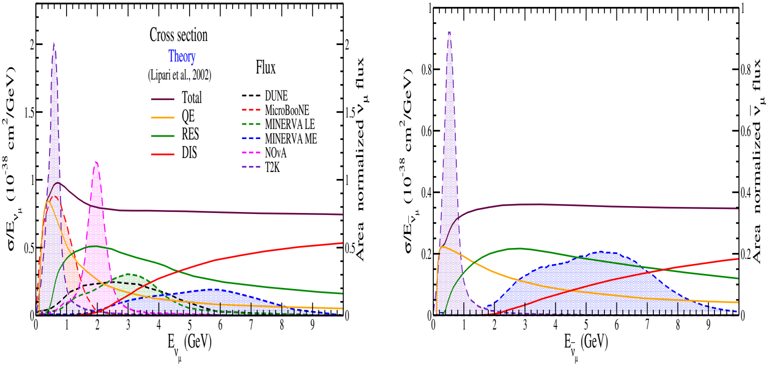
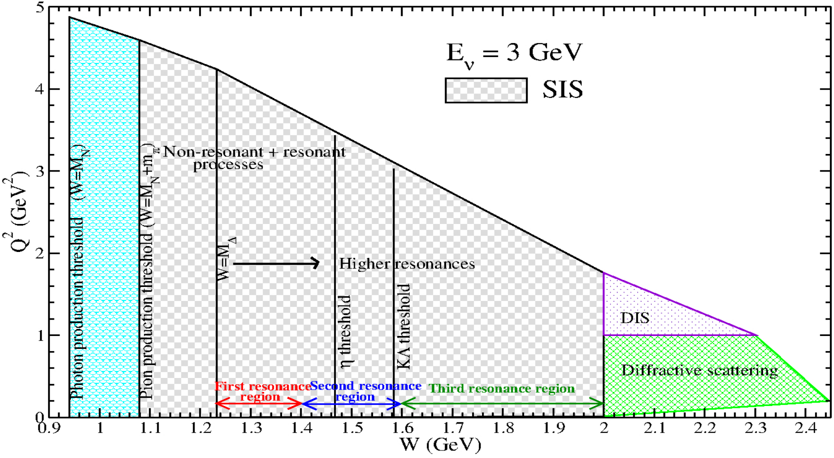
2. *Eta production from free nucleon*
3. *Eta production from bound nucleon*
4. *Conclusion*

# Introduction

- The ongoing accelerator experiments like NOvA, MINERvA and T2K, and the upcoming DUNE experiment have (anti)neutrino peak energy in the few GeV energy region. This energy region is also important for the atmospheric neutrino experiments.



# $Q^2 - W$ plane depicting neutrino-nucleon scattering at $E_\nu = 3$ GeV



# Weak production of eta mesons

## *Theoretical models*

- N. Dombey (1968)
- Nakamura et al. (DCC model)
- Aligarh group

## *Experimental studies*

- BEBC Collaboration (1989)
- ICARUS Collaboration (2015)
- MicroBooNE Collaboration (2023)

MicroBooNE Collaboration has reported the flux averaged cross section for eta production (Phys. Rev. Lett. 132, 151801 (2024)).

## *MC generators*

Most of the MC generators are using Rein-Sehgal model for the resonance model, which was initially developed for the study of single pion production.

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# Weak $\eta$ Production

- $\eta$N couples only to nucleon resonances with isospin I = $\frac{1}{2}$
- A tool to study nucleon resonances especially $N^*(1535)$
- The $\nu/\bar{\nu}$ induced $\eta$ production is interesting because
  - $\eta$ is one of the important probes to search for the strange quark content of the nucleons
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- We have considered
  - Positive and negative parity spin $\frac{1}{2}$ resonances with $M_R < 2$ GeV
  - Contribution from Born diagrams

★ Born terms are calculated using a microscopic model based on the $SU(3)$ chiral Lagrangian.

★ The vector form factors of the N-$R$ transition are obtained from the helicity amplitudes extracted from

- pion and eta photoproduction data
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★ Properties of the axial N-$R$ transition current are basically unknown

★ Assuming the pion-pole dominance of the pseudoscalar form factor, together with PCAC one can fix the axial coupling using the empirical $N^* \to N\pi$ partial decay width

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# Outline

# Kinematics: $\nu_l/\bar{\nu}_l(k) + N(p) \longrightarrow l^{\mp}(k') + B(p') + m(p_m)$

$$d\sigma = \frac{1}{4ME_\nu(2\pi)^5}\delta^4(k+p-k'-p'-p_m)\frac{d\vec{k}'}{(2E_l)}\frac{d\vec{p}\,'}{(2E_B)}\frac{d\vec{p}_m}{(2E_m)}\overline{\sum}\sum|\mathscr{M}|^2$$

$$\mathscr{M} = \frac{G_F}{\sqrt{2}} j_\mu^{(L)} j^{\mu\,(H)}$$

Fermi coupling constant | Leptonic Current | Hadronic Current

- Leptonic current is

$$j_\mu^{(L)} = \bar{u}(k')\gamma_\mu(1\pm\gamma_5)u(k)$$

- $j^{\mu\,(H)}$ describes hadronic matrix element for

$$W^i + N \rightarrow B' + m$$

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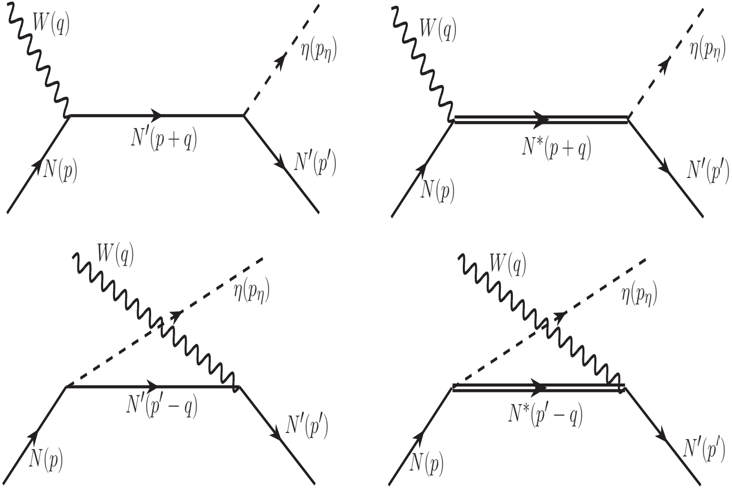
# Eta production: Feynman diagrams




Born terms

Resonance excitations

# Non-linear sigma model

**Also referred to as the Weinberg-Lagrangian**

$$\mathscr{L}_{NLSM} = \bar{\psi}(i\not{\partial} + \gamma^{\mu}V_{\mu} + \gamma^{\mu}\gamma_5 A_{\mu} - M)\psi + \frac{f_{\pi}^2}{4} Tr(\partial_{\mu}U^{\dagger}\partial^{\mu}U)$$

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with

- $V_{\mu} = \frac{1}{2}\left[\xi^{\dagger}\partial_{\mu}\xi + \xi\partial_{\mu}\xi^{\dagger}\right]$
- $A_{\mu} = \frac{i}{2}\left[\xi^{\dagger}\partial_{\mu}\xi - \xi\partial_{\mu}\xi^{\dagger}\right]$
- $\xi = e^{i\frac{\vec{\tau}\cdot\vec{\Phi}(x)}{2f\pi}} \implies U = \xi\xi$
- $\vec{\Phi}$ is identified with the meson field

$$\Phi(x) = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

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**In order to take into account the external fields (gauge bosons) into account, $\partial^\mu$ is replaced by $D^\mu$ as**

$$D^\mu U \equiv \partial^\mu U - ir^\mu U + iUl^\mu,$$
$$D^\mu U^\dagger \equiv \partial^\mu U^\dagger + iU^\dagger r^\mu - il^\mu U^\dagger$$

# Hadronic current for Born terms

$$J^{\mu}_{N(s)} = F_s(s)\,\frac{D-3F}{2\sqrt{3}f_\eta}\bar{u}_N(p')\not{p}_\eta\gamma_5\frac{\not{p}+\not{q}+M}{(p+q)^2-M^2}\mathcal{O}^{\mu}_{V}u_N(p)$$

$$J^{\mu}_{N(u)} = F_u(u)\,\frac{D-3F}{2\sqrt{3}f_\eta}\bar{u}_N(p')\mathcal{O}^{\mu}_{V}\frac{\not{p}-\not{p}_\eta+M}{(p-p_\eta)^2-M^2}\not{p}_\eta\gamma_5u_N(p),$$

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- $D$ and $F$ are the axial-vector couplings of the baryon octet
- $f_\eta$ is the eta decay constant
- $F_s(s)$ and $F_u(u)$ are form factors at the strong vertex, introduced to take into account the hadronic structure
- $\mathcal{O}^{\mu}_{V} = V^{\mu} - A^{\mu}$ is the weak vertex factor, defined in terms of the vector and axial vector form fators

# Resonance contribution

| Resonance → / Parameters ↓ | | $S_{11}(1535)$ | $S_{11}(1650)$ | $P_{11}(1710)$ | $P_{11}(1880)$ | $S_{11}(1895)$ |
|---|---|---|---|---|---|---|
| $M_R$ (GeV) | | $1.510 \pm 0.01$ | $1.655 \pm 0.015$ | $1.700 \pm 0.02$ | $1.860 \pm 0.04$ | $1.910 \pm 0.02$ |
| $\Gamma_R$ (GeV) | | $0.130 \pm 0.02$ | $0.135 \pm 0.035$ | $0.120 \pm 0.04$ | $0.230 \pm 0.05$ | $0.110 \pm 0.03$ |
| $I(J^P)$ | | $\frac{1}{2}(\frac{1}{2}^-)$ | $\frac{1}{2}(\frac{1}{2}^-)$ | $\frac{1}{2}(\frac{1}{2}^+)$ | $\frac{1}{2}(\frac{1}{2}^+)$ | $\frac{1}{2}(\frac{1}{2}^-)$ |
| BR (in %) | $N\pi$ | $32 - 52$ (43) | $50 - 70$ (60) | $5 - 20$ (16) | $3 - 31$ | $2 - 18$ |
| | $N\eta$ | $30 - 55$ (40) | $15 - 35$ (25) | $10 - 50$ (20) | $1 - 55$ (20) | $15 - 45$ (30) |
| | $K\Lambda$ | – | $5 - 15$ (10) | $5 - 25$ (15) | $(1 - 3)$ (2) | $3 - 23$ (13) |
| | $N\pi\pi$ | $4 - 31$ (17) | $20 - 58$ (5) | $14 - 48$ (49) | $(> 32)$ (44) | $(17 - 74)$ (34) |
| $\lvert g_{RN\pi}\rvert$ | | 0.1019 | 0.0915 | 0.0418 | 0.0466 | 0.0229 |
| $\lvert g_{RN\eta}\rvert$ | | 0.3696 | 0.1481 | 0.1567 | 0.1369 | 0.0877 |

# Hadronic current for resonance excitations

$$j^\mu\big|_s = F_s^*(s)\,\frac{g_{RN\eta}}{f_\eta}\bar{u}(p')\not{p}_\eta\gamma_5\Gamma_s\left(\frac{\not{p}+\not{q}+M_R}{s-M_R^2+iM_R\Gamma_R}\right)\Gamma^\mu_{\frac{1}{2}\pm}u(p),$$

$$j^\mu\big|_u = F_u^*(u)\,\frac{g_{RN\eta}}{f_\eta}\bar{u}(p')\Gamma^\mu_{\frac{1}{2}\pm}\left(\frac{\not{p}'-\not{q}+M_R}{u-M_R^2+iM_R\Gamma_R}\right)\not{p}_\eta\gamma_5\Gamma_s u(p)$$

# Spin $\frac{1}{2}$ resonance

$$j^{\mu}_{\frac{1}{2}} = \bar{u}(p')\Gamma^{\mu}_{\frac{1}{2}}u(p)$$

Adjoint Dirac Spinor — N-$R_{\frac{1}{2}}$ transition vertex — Dirac Spinor

*Transition vertex*

- Positive parity state $\Gamma^{\mu}_{\frac{1}{2}^+} = V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}$
- Negative parity state

$$\Gamma^{\mu}_{\frac{1}{2}^-} = \left[V^{\mu}_{\frac{1}{2}} - A^{\mu}_{\frac{1}{2}}\right]\gamma_5$$

$$V^{\mu}_{\frac{1}{2}} = \left[\frac{f_1(Q^2)}{(2M)^2}\left(Q^2\gamma^{\mu} + \not{q}q^{\mu}\right) + \frac{f_2(Q^2)}{2M}i\sigma^{\mu\alpha}q_{\alpha}\right]\gamma_5$$

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# $N - R_{\frac{1}{2}}$ transition vector form factors

★ Isospin symmetry relates weak vector form factors with electromagnetic form factors

$$f_{1,2}^{V}(Q^2) = F_{1,2}^{R+}(Q^2) - F_{1,2}^{R0}(Q^2).$$

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$$f_{1,2}^{V}(Q^2) = F_{1,2}^{R+}(Q^2) - F_{1,2}^{R0}(Q^2).$$

★ EM form factors are derived from the helicity amplitudes extracted from the real and/or virtual photon scattering experiments

$$A_{\frac{1}{2}}^{p,n} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\left[\frac{Q^2}{4M^2}F_1^{R^+,R^0} + \frac{M_R \pm M}{2M}F_2^{R^+,R^0}\right]$$

$$S_{\frac{1}{2}}^{p,n} = \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2 + Q^2}{M_R^2 - M^2}}\frac{(M_R \mp M)^2 + Q^2}{4M_R M}\left[\frac{M_R \pm M}{2M}F_1^{R^+,R^0} - F_2^{R^+,R^0}\right]$$

# Parameterization of the helicity amplitudes

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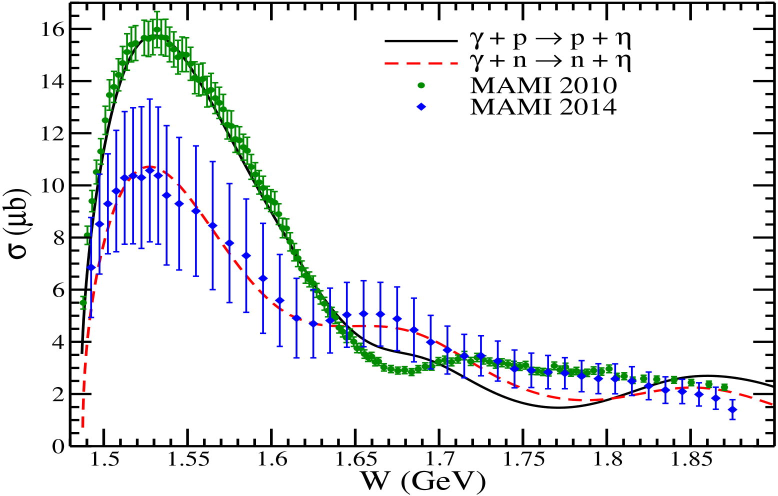
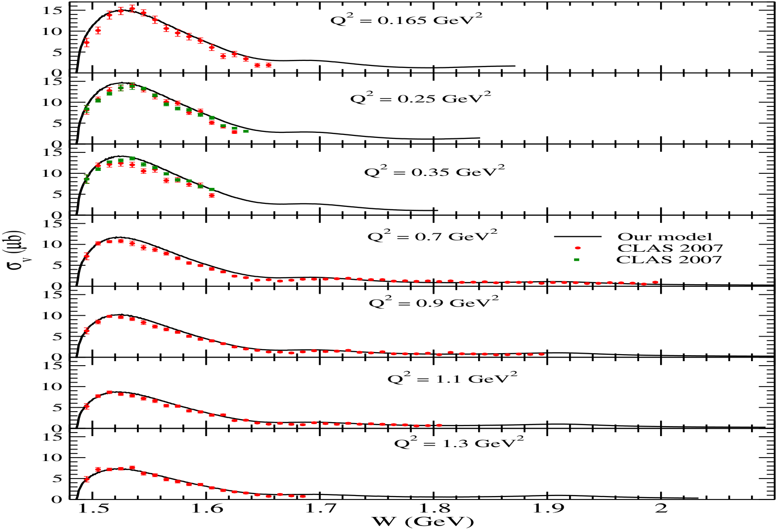
| Resonance | Helicity amplitude | Proton target | | | Neutron target | | |
|---|---|---|---|---|---|---|---|
| | | $\mathscr{A}_\alpha(0)$ | $a_1$ | $b_1$ | $\mathscr{A}_\alpha(0)$ | $a_1$ | $b_1$ |
| $S_{11}(1535)$ | $A_{\frac{1}{2}}$ | 95.0 | 0.85 | 0.85 | −78.0 | 1.75 | 1.75 |
| | $S_{\frac{1}{2}}$ | −2.0 | 1.9 | 0.81 | 32.5 | 0.4 | 1.0 |
| $S_{11}(1650)$ | $A_{\frac{1}{2}}$ | 33.3 | 0.45 | 0.72 | 26.0 | 0.1 | 2.5 |
| | $S_{\frac{1}{2}}$ | 2.5 | 1.88 | 0.96 | 3.8 | 0.4 | 0.71 |
| $P_{11}(1710)$ | $A_{\frac{1}{2}}$ | 55.0 | 1.0 | 1.05 | −45.0 | −0.02 | 0.95 |
| | $S_{\frac{1}{2}}$ | 4.4 | 2.18 | 0.88 | −31.5 | 0.35 | 0.85 |
| $P_{11}(1880)$ | $A_{\frac{1}{2}}$ | −60.0 | 0.4 | 1.0 | −45.0 | −0.02 | 0.95 |
| | $S_{\frac{1}{2}}$ | 0.4 | 0.75 | 0.5 | −31.5 | 0.35 | 0.85 |
| $S_{11}(1895)$ | $A_{\frac{1}{2}}$ | −15.0 | 1.45 | 0.6 | 26.0 | 0.1 | 2.5 |
| | $S_{\frac{1}{2}}$ | −3.5 | 0.88 | 0.6 | 3.8 | 0.4 | 0.71 |

# σ for eta photoproduction processes




**AF, MSA, SKS, Phys. Rev. D 107, 033002 (2023)**

# σ for eta electroproduction processes

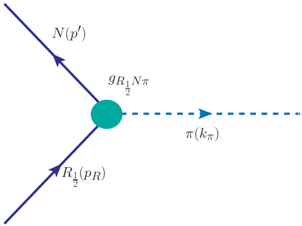



**AF, MSA, SKS, Phys. Rev. D 108, 053009 (2023)**

# $N - R_{\frac{1}{2}}$ transition axial-vector form factors

- Experimentally, the information regarding the axial vector form factors is scarce
- PCAC and PDDAC relates $g_1$ with $g_3$
- Generalized GT relation gives $g_1(0)$ in terms of $g_{RN\pi}$
- $g_{RN\pi}$ is obtained using partial decay width of the $R \to N\pi$

# Axial vector form factors and strong coupling $g_{R_{\frac{1}{2}}N\pi}$




$$\mathscr{L}_{R_{\frac{1}{2}}N\pi} = \frac{g_{R\frac{1}{2}N\pi}}{m_\pi}\bar{\Psi}_{R_{\frac{1}{2}}}\Gamma^{\mu}_{\frac{1}{2}}\partial_\mu\phi^i T_i \Psi$$

$\mathscr{R}N\pi$ coupling strength

Field for spin $\frac{1}{2}$ resonances

Nucleon field

$$\Gamma_{R_{\frac{1}{2}}\to\pi N} = \frac{\mathscr{C}}{4\pi}\left(\frac{g_{R\frac{1}{2}N\pi}}{m_\pi}\right)^2 (M_R \pm M)^2 \frac{E_N \mp M}{M_R}|\vec{q}_{\rm cm}|$$

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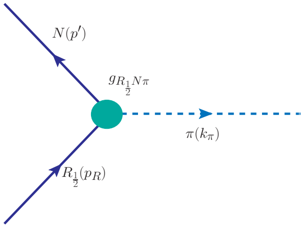


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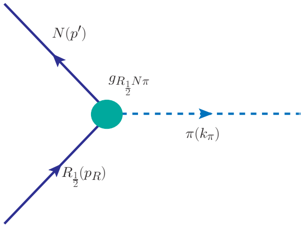


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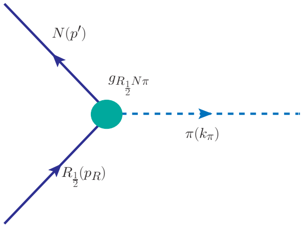


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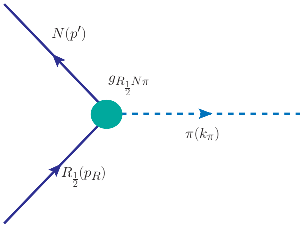


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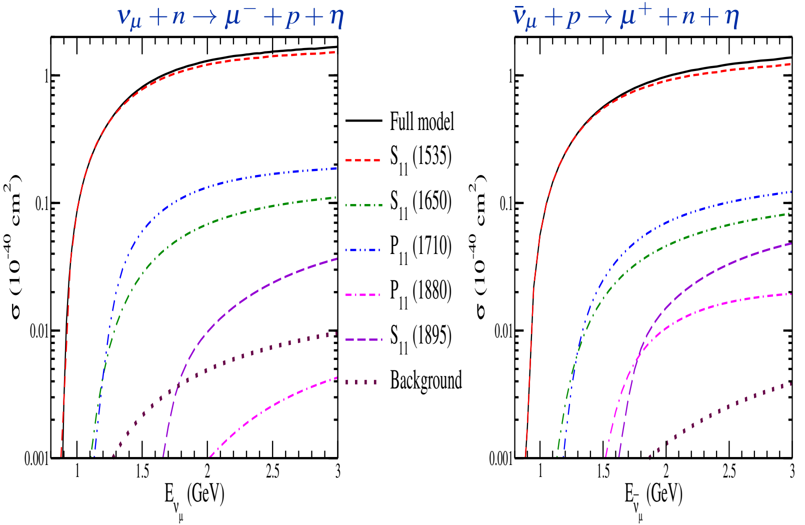
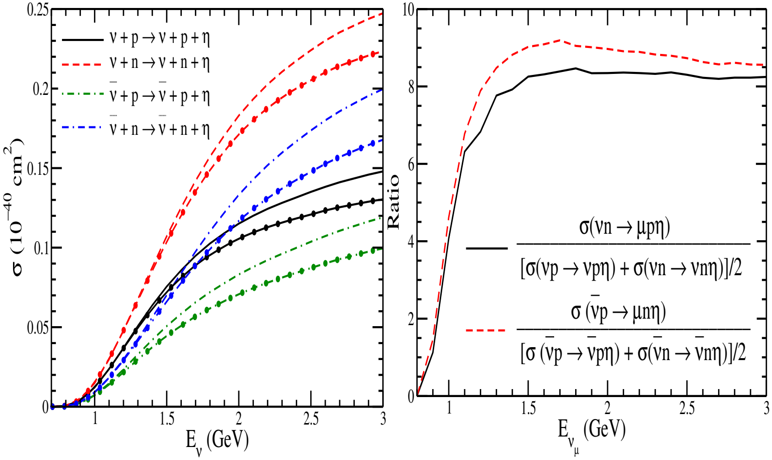
# $\sigma$ for CC induced eta production processes




**AF, MSA, SKS, Phys. Rev. D 108, 053009 (2023)**

# $\sigma$ for NC induced eta production processes

Full line: Full model
Line with circle: Only $S_{11}(1535)$




**AF, MSA, SKS, Phys. Rev. D 108, 053009 (2023)**

# Outline

1. Introduction
2. Eta production from free nucleon
3. Eta production from bound nucleon
4. Conclusion

# Inside the nucleus

### Local Fermi gas model

Cross section is evaluated as a function of local Fermi momentum($p_F(r)$) and integrated over the size of whole nucleus.

Inside the nucleus, the neutrino interacts with a neutron or proton whose local density in the medium is $\rho_n(r)$ or $\rho_p(r)$, respectively. Corresponding local Fermi momenta for neutron and proton are

$$p_{F_n} = [3\pi^2 \rho_n(r)]^{\frac{1}{3}};\ p_{F_p} = [3\pi^2 \rho_p(r)]^{\frac{1}{3}}$$

### Differential scattering cross section

$$\left(\frac{d\sigma}{dE_\eta d\Omega_\eta}\right)_{\nu A} = \int d\vec{r}\, \rho_n(r) \left(\frac{d\sigma}{dE_\eta d\Omega_\eta}\right)_{\nu N}$$

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- Inside the nuclear medium, the properties of the resonances like its mass and decay width gets modified.
- In the literature, other than $\Delta(1232)$ resonance, these modifications have only been studied for $S_{11}(1535)$ resonance by Oset et al. (Phys. Rev. C 44, 738 (1991)).
- The effect of mass modification is almost negligible in the case of $S_{11}(1535)$ resonance.
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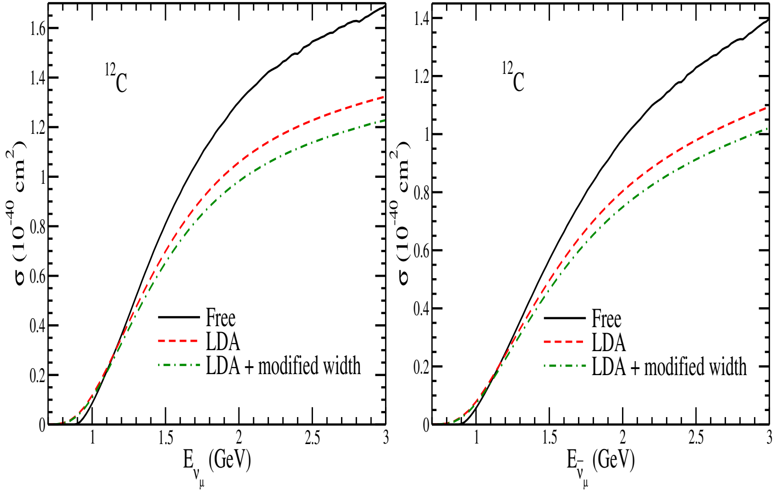
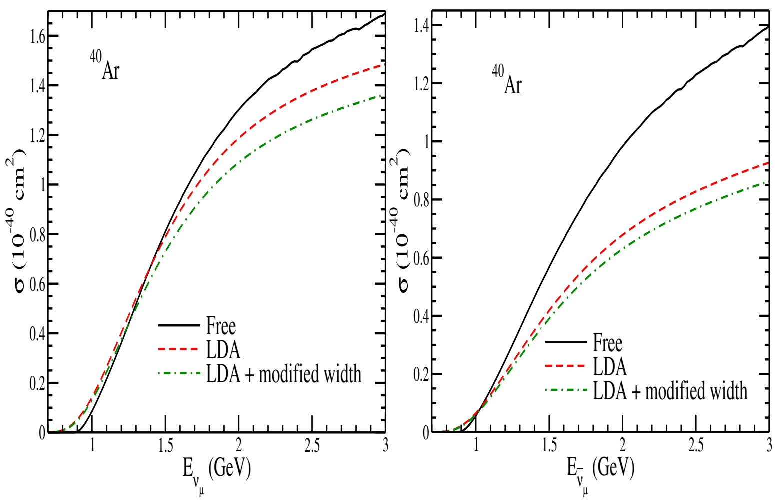
# σ per interacting nucleon for the CC neutrino (left) and antineutrino (right) from $^{12}C$ nuclear target




**AF, MSA, SKS, Paper in preparation**

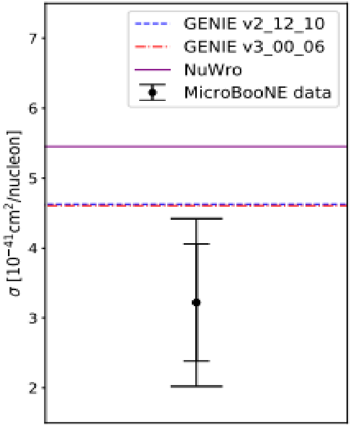
# $\sigma$ per interacting nucleon for the CC neutrino (left) and antineutrino (right) from $^{40}$Ar nuclear target




**AF, MSA, SKS, Paper in preparation**

# MicroBooNE $\eta$ production result

$$\langle\sigma\rangle = (3.22 \pm 0.84 \pm 0.86) \times 10^{-41}\ \text{cm}^2/\text{nucleon}$$




**Phys. Rev. Lett. 132, 151801 (2024)**

- $\langle\sigma\rangle_{\text{free}} = 1.87 \times 10^{-41}\ \text{cm}^2/\text{nucleon}$
- $\langle\sigma\rangle_{^{40}\text{Ar}} = 1.78 \times 10^{-41}\ \text{cm}^2/\text{nucleon}$

- GENIE $v2\_12\_10$: $4.63 \times 10^{-41}\ \text{cm}^2/\text{nucleon}$
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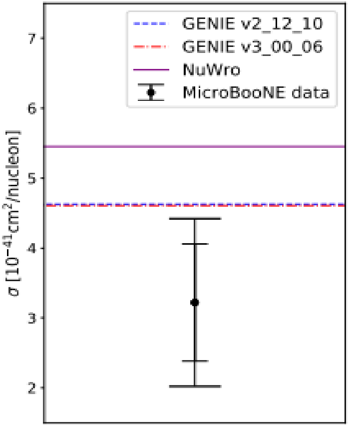


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# Outline

# Conclusion

- The study of $\eta$ production is important:
  - in modelling the neutrino event generators
  - to understand the axial-vector response of the hadronic sector
- The Born terms are determined using the non-linear sigma model based on the chiral SU(3) symmetry.
- The model is first applied to the eta production in the electromagnetic sector and a good agreement with the experimental data is found.
- The effect of nuclear medium and width modification in the case of $S_{11}(1535)$ resonance are studied, and found to be significant in the case of both neutrino and antineutrino induced reactions.

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Thank you!

# BACKUP

# Non-linear sigma model

- This is an effective field theory (EFT).
- EFT is a low energy approximation to some underlying, more fundamental theory. Low is defined with respect to some energy scale.
- The basic idea consists of writing down the most general possible Lagrangian, including all terms consistent with assumed symmetry principles, and then calculating matrix elements with this Lagrangian within some perturbative scheme.

# Meson-Meson Interaction

The lowest order Lagrangian with the minimal number of derivatives describing the interaction of the Goldstone bosons

$$\mathscr{L} = \frac{f_\pi^2}{4} Tr(D_\mu U^\dagger D^\mu U)$$

**U is $SU(3)$ matrix containing the Goldstone boson fields**

$$U(x) = \exp\left(i\frac{\Phi(x)}{f_\pi}\right),$$

**$SU(3)$ representation of pseudoscalar fields**

$$\Phi(x) = \sum_{k=1}^{8} \phi_k(x)\lambda_k = \begin{pmatrix} \pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}$$

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# Interaction of pseudoscalar fields with baryons

We consider the octet of $\frac{1}{2}^+$ baryons. With each member of the octet we associate a complex, four-component Dirac field

$$B(x) = \sum_{k=1}^{8} \frac{1}{\sqrt{2}} b_k(x)\lambda_k = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix},$$

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The lowest-order chiral Lagrangian for the baryon octet in the presence of an external current may be written in terms of the SU(3) matrix B as,

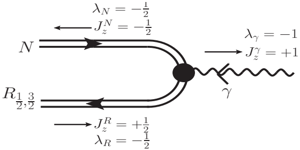
$$\mathcal{L}_{MB}^{(1)} = \mathrm{Tr}\left[\bar{B}\left(i\not{D} - M\right)B\right] - \frac{D}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5\{u_{\mu},B\}\right) - \frac{F}{2}\mathrm{Tr}\left(\bar{B}\gamma^{\mu}\gamma_5[u_{\mu},B]\right),$$

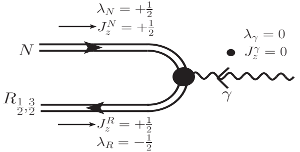
covariant derivative of B:

$$D_{\mu}B = \partial_{\mu}B + [\Gamma_{\mu},B],$$

$$\Gamma^{\mu} = \frac{1}{2}\left[u^{\dagger}(\partial^{\mu} - ir^{\mu})u + u(\partial^{\mu} - il^{\mu})u^{\dagger}\right]$$

# Electromagnetic form factors and helicity amplitudes



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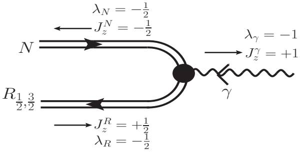


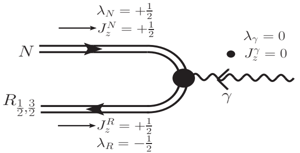


$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{2\pi\alpha}{M}\frac{(M_R \mp M)^2+Q^2}{M_R^2-M^2}}\left[\frac{Q^2}{4M^2}F_1^{R^+,R^0}+\frac{M_R \pm M}{2M}F_2^{R^+,R^0}\right]$$

$$S^{p,n}_{\frac{1}{2}} = \mp\sqrt{\frac{\pi\alpha}{M}\frac{(M \pm M_R)^2+Q^2}{M_R^2-M^2}}\frac{(M_R \mp M)^2+Q^2}{4M_R M}\left[\frac{M_R \pm M}{2M}F_1^{R^+,R^0}-F_2^{R^+,R^0}\right]$$

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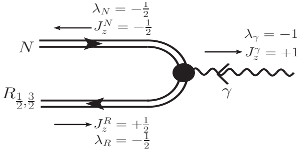
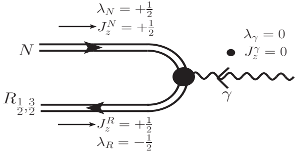
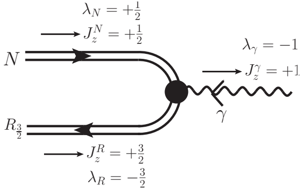



In case of spin $\frac{3}{2}$ resonances



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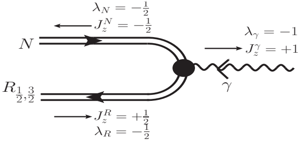


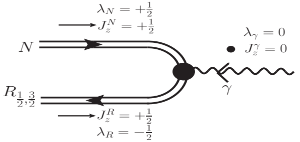


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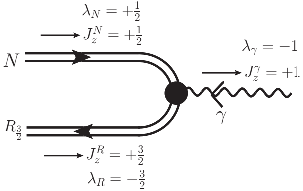


$$A^{p,n}_{\frac{3}{2}} = \sqrt{\frac{\pi\alpha}{M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\left[\frac{C_3^{R+,R0}}{M}(M \pm M_R) \pm \frac{C_4^{R+,R0}}{M^2}\frac{M_R^2 - M^2 - Q^2}{2} \pm \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2 - M^2 + Q^2}{2}\right]$$

$$A^{p,n}_{\frac{1}{2}} = \sqrt{\frac{\pi\alpha}{3M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\left[\frac{C_3^{R+,R0}}{M}\frac{M^2 + MM_R + Q^2}{M_R} - \frac{C_4^{R+,R0}}{M^2}\frac{M_R^2 - M^2 - Q^2}{2} - \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2 - M^2 + Q^2}{2}\right]$$

$$S^{p,n}_{\frac{1}{2}} = \pm\sqrt{\frac{\pi\alpha}{6M}\frac{(M_R \mp M)^2 + Q^2}{M_R^2 - M^2}}\frac{\sqrt{Q^4 + 2Q^2(M_R^2 + M^2) + (M_R^2 - M^2)^2}}{M_R^2}$$

$$\times \left[\frac{C_3^{R+,R0}}{M}M_R + \frac{C_4^{R+,R0}}{M^2}M_R^2 + \frac{C_5^{R+,R0}}{M^2}\frac{M_R^2 + M^2 + Q^2}{2}\right]$$