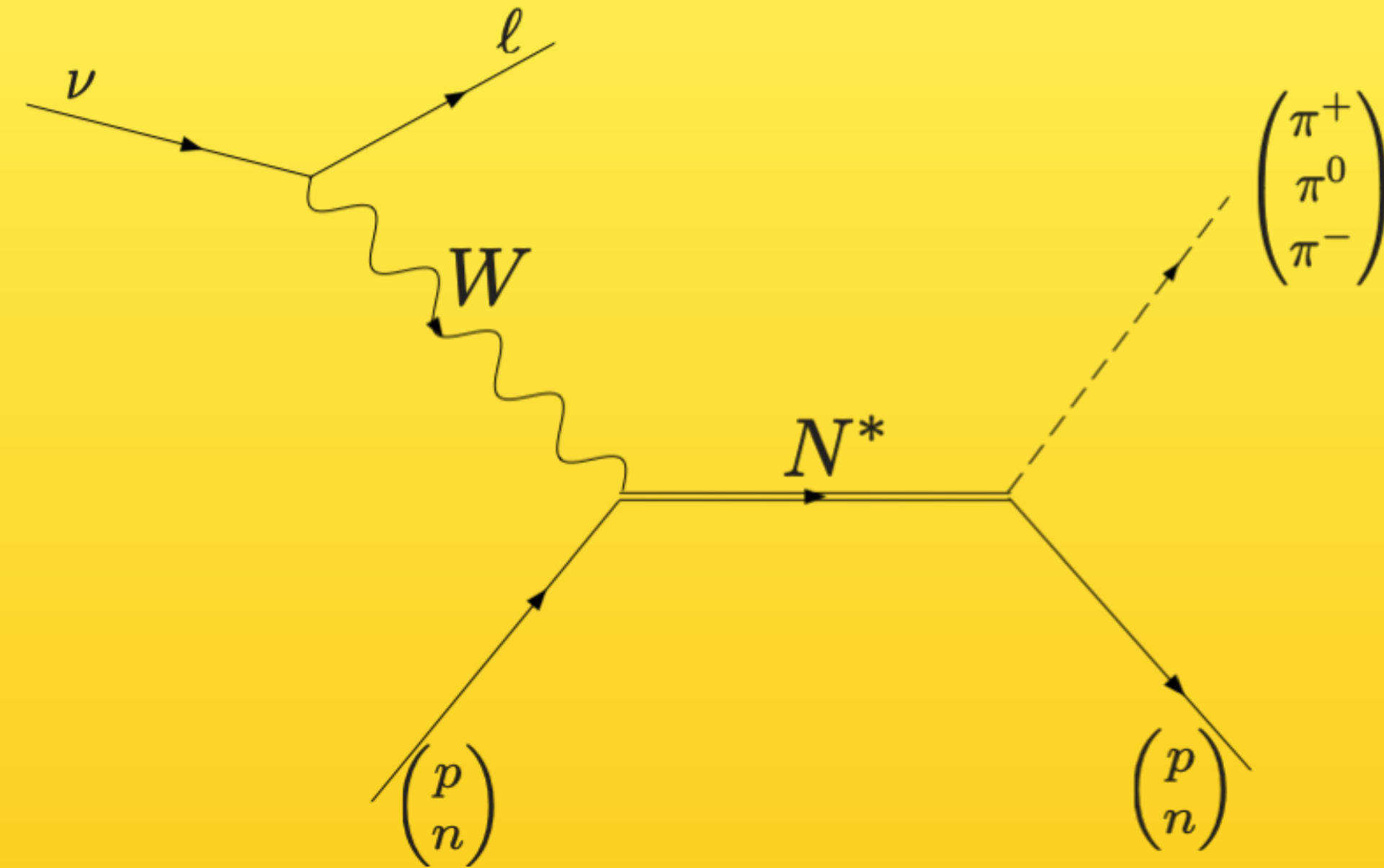


# Neutrino-induced single pion production and the reanalyzed bubble chamber data



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# Summary

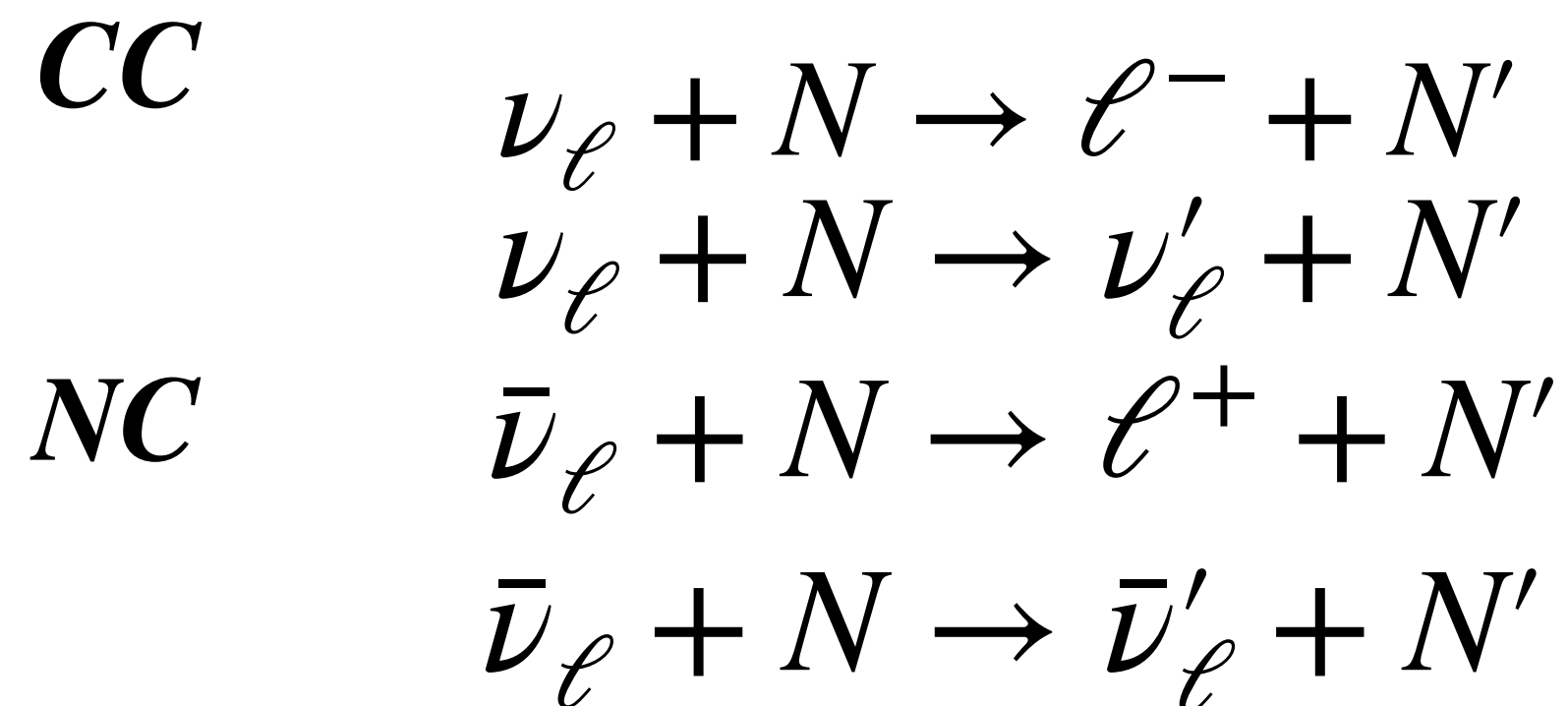
We report the calculation of cross sections for pion production by the scattering of (anti)neutrinos into nucleons incorporating the second resonance region up to 1.6 GeV, in addition to the  $\Delta(1232)$  resonance. Background non resonant contributions are also considered and added coherently. We see that:

- The resonances of the second region included are:  $N^*(1440)(P_{11})$ ,  $N^*(1520)(D_{13})$  y  $N^*(1535)(S_{11})$ . It is shown that they are necessary for the description the cross section up to  $E_\nu \approx 2\text{GeV}$ , and how to handle the behavior and interference amplitude contributions when we grow in energy.
- We work within a framework of chiral effective Lagrangians and discuss approaches that present certain types of inconsistencies or alternative treatments of  $3/2$  resonances.
- We compared with the recent reanalyzed data matching ANL and BNL results and with the antineutrinos CERN PS data, for the sake of consistence.

# Motivation

## From the experimental point of view

- It is known that neutrinos are massive particles that can oscillate (change flavor), it is essential to know precisely the cross sections in the interaction of the neutrino with free, or bounded nucleons in the nucleus detector.
- Quasi-elastic interaction (QE) is the main signal in oscillation experiments or those looking for CP violation



- More than one nucleon could be ejected by (final state interactions) FSI in the detector.

- Inelastic scattering produces additional pions

$$\mathbf{1\pi\ CC} \quad \nu_\ell + N \rightarrow \ell^- + N'\pi$$

$$\bar{\nu}_\ell + N \rightarrow \ell^+ + N'\pi$$

$$\mathbf{1\pi\ NC} \quad \nu_\ell + N \rightarrow \nu'_\ell + N'\pi$$

$$\bar{\nu}_\ell + N \rightarrow \bar{\nu}'_\ell + N'\pi.$$

Can generate false QE events when pions are not detected or are absorbed before emerging, or are an important tool to determine resonances axial weak constants, not obtained from electromagnetic interactions.

- At energies of the order of GeV, the dominant pion production mechanism is through the production and subsequent decay of hadronic resonances.
- The axial form factor (FF) for pion production on free nucleons depends on the old ANL (Argonne National Laboratory) and BNL (Brookhaven National Laboratory) bubble chamber data. These data sets differed in normalization by 30% to 40% for the main channel, creating uncertainties in the parameters predictions.
- A method was more recently presented to eliminate such uncertainties by taking ratios between rates from  $CC1\pi$  and CCQE events, in which normalization is cancelled → [Eur. Phys. J. C 76, 474 \(2016\)](#)



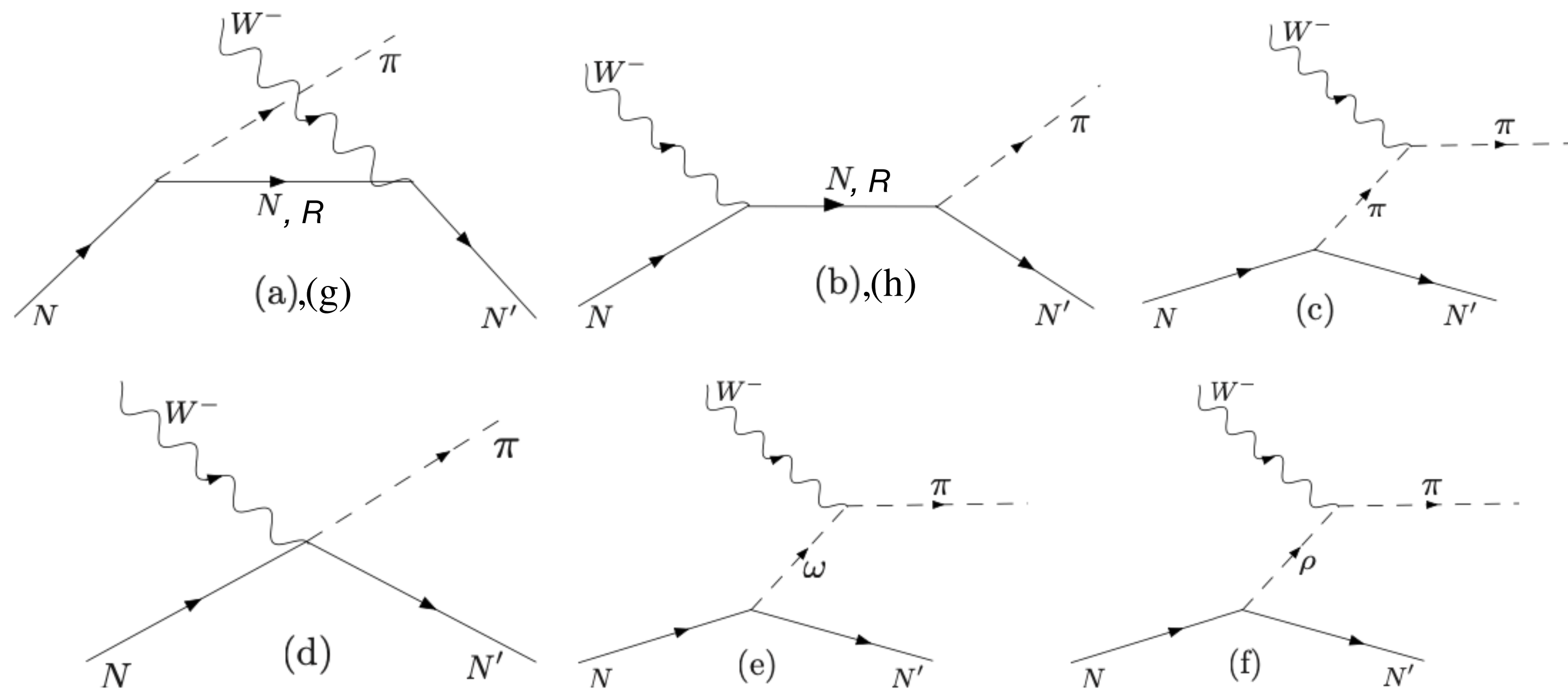
## From the formal point of view

- Several works combine the simplest forms of the free and interaction Lagrangians for spin  $3/2$  resonances without respecting the contact invariance against transformations that change the spurious  $1/2$  component
- In addition to the contribution of resonances, we have a background that comes from crossed resonant terms and non-resonant ones involving nucleons Born and exchange of mesons.
- Many works do not properly consider the interference between both (background + resonant) amplitudes.
- Other separate the production and decay mechanism of a resonance, but resonances are non-perturbative phenomena with poles in the amplitude of the matrix  $S$  and their propagation cannot be omitted.

# Neutrino-Nucleon Scattering

- The total amplitude is constructed from the resonance plus the background terms, both interfering coherently

$$R = \Delta(1232), N^*(1440), N^*(1520), N^*(1535)$$





# Resonance Properties

**$\Delta(1232) 3/2^+$**   $I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$

Re(pole position) = 1209 to 1211 ( $\approx 1210$ ) MeV  
 $-2\text{Im}(\text{pole position}) = 98$  to  $102$  ( $\approx 100$ ) MeV  
 Breit-Wigner mass (mixed charges) = 1230 to 1234 ( $\approx 1232$ ) MeV  
 Breit-Wigner full width (mixed charges) = 114 to 120 ( $\approx 117$ ) MeV

<b><math>\Delta(1232)</math> DECAY MODES</b>	Fraction ( $\Gamma_i/\Gamma$ )	$\rho$ (MeV/c)
$N\pi$	99.4 %	229
$N\gamma$	0.55–0.65 %	259
$N\gamma$ , helicity=1/2	0.11–0.13 %	259
$N\gamma$ , helicity=3/2	0.44–0.52 %	259

**$N(1440) 1/2^+$**   $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

Re(pole position) = 1360 to 1385 ( $\approx 1370$ ) MeV  
 $-2\text{Im}(\text{pole position}) = 160$  to  $195$  ( $\approx 180$ ) MeV  
 Breit-Wigner mass = 1410 to 1450 ( $\approx 1430$ ) MeV  
 Breit-Wigner full width = 250 to 450 ( $\approx 350$ ) MeV

<b><math>N(1440)</math> DECAY MODES</b>	Fraction ( $\Gamma_i/\Gamma$ )	$\rho$ (MeV/c)
$N\pi$	55–75 %	391
$N\eta$	<1 %	†
$N\pi\pi$	25–50 %	338
$\Delta(1232)\pi$	20–30 %	135
$\Delta(1232)\pi$ , <i>P</i> -wave	13–27 %	135
$N\sigma$	11–23 %	–
$p\gamma$ , helicity=1/2	0.035–0.048 %	407
$n\gamma$ , helicity=1/2	0.02–0.04 %	406

**$N(1520) 3/2^-$**   $I(J^P) = \frac{1}{2}(\frac{3}{2}^-)$  Status: \* \* \* \*

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics (generic for all A,B,E,G) **G33** 1 (2006).

**$N(1520)$  DECAY MODES**

The following branching fractions are our estimates, not fits or averages.

Mode	Fraction ( $\Gamma_i/\Gamma$ )
$\Gamma_1$ $N\pi$	55–65 %
$\Gamma_2$ $N\eta$	$(2.3 \pm 0.4) \times 10^{-3}$
$\Gamma_3$ $N\pi\pi$	20–30 %
$\Gamma_4$ $\Delta\pi$	15–25 %
$\Gamma_5$ $\Delta(1232)\pi$ , <i>S</i> -wave	10–20 %
$\Gamma_6$ $\Delta(1232)\pi$ , <i>D</i> -wave	10–15 %
$\Gamma_7$ $N\rho$	15–25 %
$\Gamma_8$ $N\rho$ , <i>S</i> =3/2, <i>S</i> -wave	$(9.0 \pm 1.0)$ %
$\Gamma_9$ $N(\pi\pi)_{S=0}^{J=0}$	<8 %
$\Gamma_{10}$ $p\gamma$	0.31–0.52 %
$\Gamma_{11}$ $p\gamma$ , helicity=1/2	0.01–0.02 %
$\Gamma_{12}$ $p\gamma$ , helicity=3/2	0.30–0.50 %
$\Gamma_{13}$ $n\gamma$	0.30–0.53 %
$\Gamma_{14}$ $n\gamma$ , helicity=1/2	0.04–0.10 %
$\Gamma_{15}$ $n\gamma$ , helicity=3/2	0.25–0.45 %

**$N(1535) 1/2^-$**   $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$  Status: \* \* \* \*

Most of the results published before 1975 were last included in our 1982 edition, Physics Letters **111B** 1 (1982). Some further obsolete results published before 1984 were last included in our 2006 edition, Journal of Physics (generic for all A,B,E,G) **G33** 1 (2006).

**$N(1535)$  DECAY MODES**

The following branching fractions are our estimates, not fits or averages.

Mode	Fraction ( $\Gamma_i/\Gamma$ )
$\Gamma_1$ $N\pi$	35–55 %
$\Gamma_2$ $N\eta$	$(42 \pm 10)$ %
$\Gamma_3$ $N\pi\pi$	1–10 %
$\Gamma_4$ $\Delta\pi$	<1 %
$\Gamma_5$ $\Delta(1232)\pi$ , <i>D</i> -wave	0–4 %
$\Gamma_6$ $N\rho$	<4 %
$\Gamma_7$ $N\rho$ , <i>S</i> =1/2, <i>S</i> -wave	$(2.0 \pm 1.0)$ %
$\Gamma_8$ $N\rho$ , <i>S</i> =3/2, <i>D</i> -wave	$(0.0 \pm 1.0)$ %
$\Gamma_9$ $N(\pi\pi)_{S=0}^{J=0}$	$(2 \pm 1)$ %
$\Gamma_{10}$ $N(1440)\pi$	$(8 \pm 3)$ %
$\Gamma_{11}$ $p\gamma$	0.15–0.30 %
$\Gamma_{12}$ $p\gamma$ , helicity=1/2	0.15–0.30 %
$\Gamma_{13}$ $n\gamma$	0.01–0.25 %
$\Gamma_{14}$ $n\gamma$ , helicity=1/2	0.01–0.25 %

- The CC interaction is given by the weak Lagrangian

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}} \left( J_{lCC}^\mu W_\mu + J_{hCC}^\mu \sqrt{2} (\boldsymbol{\tau} \circ \mathbf{T}^\dagger) \cdot \mathbf{W}^* + \text{h.c.} \right)$$

$$J_{lCC}^\mu = \sum_l \bar{\psi}_l \gamma^\mu (1 - \gamma^5) \psi_{\nu_l}$$

$$J_{hCC}^\mu = \sum_l \bar{\psi}_h (V^\mu - A^\mu) \psi_h$$

- To build the background contribution to hadronic current we

$$\mathcal{L}_{WNN}(x) = -\frac{g}{2\sqrt{2}} \bar{\psi}(x) \left[ \gamma_\mu F_1^V(Q^2) - \frac{F_2^V(Q^2)}{2m_N} \sigma_{\mu\nu} \partial^\nu - F^A(Q^2) \gamma_\mu \gamma_5 \right] \sqrt{2} \mathbf{W}^\mu(x) \cdot \frac{\boldsymbol{\tau}}{2} \psi(x) + \text{H.c.},$$

$$\mathcal{L}_{W\pi\pi}(x) = -\frac{g}{2\sqrt{2}} F_1^V(Q^2) \sqrt{2} [\boldsymbol{\phi}(x) \times \partial_\mu \boldsymbol{\phi}(x)] \cdot \mathbf{W}^\mu(x),$$

$$\mathcal{L}_{W\pi NN}(x) = -\frac{g}{2\sqrt{2}} \frac{f_{\pi NN}}{m_\pi} F_1^V(Q^2) \bar{\psi}(x) \gamma_5 \gamma_\mu \sqrt{2} (\boldsymbol{\tau} \times \boldsymbol{\phi}(x)) \cdot \mathbf{W}^\mu \psi(x),$$

$$\mathcal{L}_{W\pi\rho}(x) = \frac{g}{2\sqrt{2}} f_{\rho\pi A} F^A(Q^2) \sqrt{2} (\boldsymbol{\phi}(x) \times \boldsymbol{\rho}_\mu(x)) \cdot \mathbf{W}(x)^\mu$$

$$\mathcal{L}_{W\pi\omega}(x) = -\frac{g}{2\sqrt{2}} \frac{g_{\omega\pi V}}{m_\omega} F_1^V(Q^2) \epsilon_{\mu\alpha\lambda\nu} (\partial^\lambda \boldsymbol{\phi}(x)) \cdot (\partial^\mu \mathbf{W}^\alpha(x)) \omega^\nu(x).$$

- while strong Lagrangian read

$$\mathcal{L}_{\pi NN}(x) = -\frac{g_{\pi NN}}{2m_N} \bar{\psi}(x) \gamma_5 \gamma_\mu \boldsymbol{\tau} \cdot (\partial^\mu \boldsymbol{\phi}(x)) \psi(x),$$

$$\mathcal{L}_{VNN}(x) = -\frac{g_V}{2} \bar{\psi}(x) \left[ \gamma_\mu \left\{ \begin{array}{c} \boldsymbol{\rho}^\mu(x) \cdot \boldsymbol{\tau} \\ \omega^\mu(x) \end{array} \right\} - \frac{\kappa_V}{2m_N} \sigma_{\mu\nu} \left( \partial^\nu \left\{ \begin{array}{c} \boldsymbol{\rho}^\mu(x) \cdot \boldsymbol{\tau} \\ \omega^\mu(x) \end{array} \right\} \right) \right] \psi(x),$$

- and finally the propagators

$$S(p) = \frac{\not{p} + m_N}{p^2 - m^2}, \text{ nucleon}$$

$$\Delta(p) = \frac{1}{p^2 - m_\pi^2}, \text{ pion}$$

$$D_{\mu\nu}(p) = \frac{-g_{\mu\nu} + \frac{p_\mu p_\nu}{m_V^2}}{p^2 - m_V^2}, \text{ vector-meson}$$

- For  $R(s = 1/2)$  we get from  $\mathcal{L}_{\pi NN}$ ,  $\mathcal{L}_{WNN}$  and  $S(p)$  adding  $\gamma_5$  (parity), and decay width in propagators through  $m \rightarrow m - i\Gamma_R/2$



- $R(s = 3/2)$  are more complex since

$$\psi_\mu \sim A_\mu \otimes \psi$$

$$G(p, A)_{\mu\nu} = R^{-1}(A)_{\mu\alpha} G^{\alpha\beta}(p) R^{-1}(A)_{\beta\nu} \quad R_\sigma^\rho(a) = \delta_\sigma^\rho + a \gamma_\sigma \gamma^\rho \quad a = \left( \frac{1 + 3A}{2} \right)$$

$$G(p) = - \left[ \frac{\not{p} + m}{p^2 - m^2} P^{3/2} + 2/m^2 (\not{p} + m) P_{11}^{1/2} + \sqrt{3}/m (P_{12}^{1/2} + P_{21}^{1/2}) \right]$$

- $\mathcal{L}_{free}$  invariant under contact transformation since change only 1/2 component

$$\psi'^\rho = R(a)^\rho_\sigma \psi^\sigma \quad \psi_{3/2}^\nu \gamma_\nu = 0$$

- Then, any interaction should look, to get A-independence (not respected in some works)

$$\mathcal{L}_{int}(A, Z) = g \bar{\Psi}_\mu R \left( \frac{1}{2} (2Z + (1 + 4Z)A) \right)_{\mu\nu} F^\nu(\psi, \phi, W, \dots) + h.c.,$$

$$R(A)_{\alpha\mu} R^{-1} \left( \frac{1}{2} (1 - 6Z/(1 + 4Z)) \right)_{\beta}^{\mu}$$

- Free parameters Z are set such that in the interaction Lagrange multiplier field  $\Psi_0$  does not acquire dynamics ( $\partial \Psi_0$  absent).

- Once **eliminated the dependence on A and fixed Z,Z'** we have for the strong and weak interactions for  $R = \Delta$

$$\mathcal{L}_{I_1} \equiv \mathcal{L}_{\Delta\pi N}(x) = \frac{f_{\Delta\pi N}}{m_\pi} \bar{\psi} \partial \Phi(x)^{\mu*} \cdot \mathbf{T} \Psi_\mu(x) + \frac{f_{\Delta\pi N}}{m_\pi} \bar{\Psi}_\mu(x) \bar{\psi} \partial \Phi^\mu(x) \cdot \mathbf{T}^\dagger \psi(x)$$

$$\hat{\mathcal{L}}_{WN\Delta}(x) = \bar{\Psi}^\mu(x) (\hat{W}_\mu^V + \hat{W}_\mu^A) (\mathbf{T}^\dagger \cdot \mathbf{W}^*) \psi(x) + \text{h.c}$$

within the Sachs( $G_{M,E}$ ) parametrization in the vector sector

$$\hat{W}_{\mu\nu}^V(p_\Delta, q, p) = \sqrt{2} [(G_M(Q^2) - G_E(Q^2)) K_{\nu\mu}^M + G_E(Q^2) K_{\nu\mu}^E + G_C(Q^2) K_{\nu\mu}^C] \Delta^* \mathbf{W}^* \cdot \mathbf{T}^\dagger \psi$$

$$\hat{W}_{\nu\mu}^A(p_\Delta, q, p) = -i \left[ -D_1(Q^2) g_{\nu\mu} + \frac{D_2(Q^2)}{m_N^2} (p + p_\Delta)^\alpha (g_{\nu\mu} q_\alpha - q_\nu g_{\alpha\mu}) - \right.$$

$$\left. \frac{D_3(Q^2)}{m_N^2} p_\nu q_\mu + i \frac{D_4(Q^2)}{m_N^2} \varepsilon_{\mu\nu\alpha\beta} (p + p_\Delta)^\alpha q^\beta \gamma_5 \right] \Delta^* \mathbf{W}^* \mathbf{T}^\dagger \psi,$$

- If  $R = N^*(1520)$   $\psi_\nu \rightarrow \gamma_5 \psi_\nu$  for the parity change and **usually many works adopt Normal Parity( $C_{i=3,4,5,6}$ ) parametrization**

$$\Gamma_{\nu\mu}^V(p_{D_{13}}, q) = \left[ -\frac{C_3^V(Q^2)}{m_N} H_{3\nu\mu} - \frac{C_4^V(Q^2)}{m_N^2} H_{4\nu\mu} - \frac{C_5^V(Q^2)}{m_N^2} H_{5\nu\mu} + \frac{C_6^V(Q^2)}{m_N^2} H_{6\nu\mu} \right]$$

For consistency and since both parametrization **are not equivalent nor have same behavior**, we make the connection of  $Q^2=0$   $FF$

$$C_3^V(Q^2) = \frac{m_{D_{13}}}{m_N} R_M \left[ G_M(0) - G_E(0) \right] F^V(Q^2)$$

$$C_4^V(Q^2) = -R_M \left[ G_M(0) - \frac{3m_{D_{13}}}{m_{D_{13}} - m_N} G_E(0) \right] F^V(Q^2)$$

$$C_5^V(Q^2) = 0$$

$$C_6^V(Q^2) = -R_M \frac{2m_{D_{13}}}{m_{D_{13}} - m_N} G_E(0) F^V(Q^2),$$

$$R_M = \frac{3}{2} \frac{m_N}{m_{D_{13}} + m_N}$$



## Before those who knows these formal topics jump in their chairs demanding!!!

- We are well aware of the use of the ``spin 3/2'' gauge invariante 2nd order  $\pi N \Delta$  interaction obtained making

$$\Psi_\mu \rightarrow \Psi_\mu - g_2(\partial_\mu \phi)\psi \quad \text{in} \quad \mathcal{L}_{free}(m_\Delta = 0)$$

$$\mathcal{L}_{I_2} = -g_2 \bar{\Psi} \partial_\mu \phi^\dagger \cdot \mathbf{T} \epsilon^{\mu\nu\rho\beta} \gamma_\beta \gamma_5 \partial_\rho \Psi_\nu + h.c.$$

which decouples de spin 1/2 in the propagator, is Z independent, but with problems as  $I_1$  (appearance of negative indefinite norm states when field quantization is achieved). Also 2nd order contribution of the Sachs parametrization also comes from  $\gamma_\nu \Psi_\mu \rightarrow \gamma_\nu \Psi_\mu + i\gamma_5 \partial_\nu W_\mu^Y$

- When  $\Psi_\mu \rightarrow \Psi_\mu - g_2(\partial_\mu \phi)\psi$  is done we get  $\mathcal{L}_{free} + \mathcal{L}_{I_1} \rightarrow \mathcal{L}_{free} + \mathcal{L}_{I_2}(g_2 = -\frac{g_1}{m}) + \mathcal{L}_C(\frac{g_1^2}{m^2})$  and  $I_1$  is shifted (or put under the rug), and absorbed within a contact term in the background.

- Another point of view is, as  $\gamma_\mu G^{\mu\nu}$  and  $G^{\mu\nu} \gamma_\nu$  do not contain pole-contributions, the Z,Z' dependent terms are again shifted to contact background  $\pi N$  ones, with couplings constants to be fitted. This is referred as looking LEC's of CHPT.

- We will analyze the observables:

$$\sigma(E_\nu^{\text{CM}}) = \frac{m_\nu m_N^2}{(2\pi)^4 E_\nu^{\text{CM}} \sqrt{s}} \int_{E_\mu^-}^{E_\mu^+} dE_\mu^{\text{CM}} \int_{E_\mu^-}^{E_\mu^+} dE_\pi^{\text{CM}} \int_{-1}^+ d\cos\theta \int_0^{2\pi} d\eta \frac{1}{16} \sum_{\text{spin}} |\mathcal{M}|^2$$

$$E_\nu^{\text{CM}} = \frac{m_N E_\nu^{\text{Lab}}}{\sqrt{s}} = \frac{m_N E_\nu^{\text{Lab}}}{\sqrt{2E_\nu^{\text{Lab}} m_N + m_N^2}}$$

$$\frac{d\langle\sigma\rangle}{dQ^2} = \frac{\int_{E_\nu^{\text{min}}}^{E_\nu^{\text{max}}} \frac{d\sigma(E_\nu)}{dQ^2} \phi(E_\nu) dE_\nu}{\int_{E_\nu^{\text{min}}}^{E_\nu^{\text{max}}} \phi(E_\nu) dE_\nu}$$

$$\frac{d\sigma}{dQ^2} = \frac{m_\mu m_N^2}{2(2\pi)^4 (E_\nu^{\text{CM}})^2 \sqrt{s}} \int_{E_\mu^-}^{E_\mu^+} \frac{dE_\mu^{\text{CM}}}{\sqrt{(E_\mu^{\text{CM}})^2 - m_\mu^2}} \int_{E_\pi^-}^{E_\pi^+} dE_\pi^{\text{CM}} \int_0^{2\pi} d\eta \frac{1}{16} \sum_{\text{spin}} |\mathcal{M}|^2$$

# Width and Gauge invariance

How to give width without violating electromagnetic Gauge Invariance?

- For all  $R$  (If  $R(1/2) G_{\mu\nu}(p) \rightarrow S(p)$ ) y  $\Gamma_{\alpha\nu\rho} \rightarrow \Gamma_\alpha$ ) Ward's identity must be satisfied

$$i(p - p')_\alpha G_{\mu\nu}(p') \Gamma_{\alpha\nu\rho} G^{\rho\sigma}(p) = G_\mu^\sigma(p) - G_\mu^\sigma(p')$$

- The propagators are obtained solving  $(iG^{-1})^{\mu\nu}(p) = (iG_0^{-1})^{\mu\nu}(p) - \Sigma^{\mu\nu}(p)$  (S. Dyson)

Where the main contribution is

$$\Sigma = \text{Diagram}$$

- $G_R$  structure is changed (Exact propagator), **only if**  $s^{1/2} \in (m_R - \Gamma_R, m_R + \Gamma_R)$

$$G_R, S_R = G_R^0, S_R^0 (m^0 \rightarrow m - i\Gamma(s)/2)$$

$$\Gamma_R(s) \times B_r = \frac{3 \left( \frac{f_{\pi NR}}{m_\pi} \right)^2}{4\pi} \left( \frac{(\sqrt{s} - \pi m_N)^2 - m_\pi^2}{48s^{5/2}} \right) \lambda^{3/2}(s, m_N^2, m_\pi^2)$$

$$\Gamma_R(s) \times B_r = \frac{3}{4\pi} \left( \frac{f_{\pi NR}}{m_\pi} \right)^2 (m_R + \pi m_N)^2 \left( \frac{(\sqrt{s} - \pi m_N)^2 - m_\pi^2}{4s^{3/2}} \right) \lambda^{1/2}(s, m_N^2, m_\pi^2)$$

- If we adopt energy dependent width  $\Gamma(s)$  or we make  $m^0 \rightarrow m - i\Gamma_{exp}/2$  only in  $1/(p^2 - m_R^2)$  Ward identity is violated at order  $\Gamma/m_R$  (at less you introduce vertex corrections)

- Assuming  $\Gamma \approx \Gamma_{CMS} = \text{constant}$  (fitted) in the full propagator it is possible factorize  $(1 + i\Gamma_{CMS}/m_R)$  when we make  $m^0 \rightarrow m - i\Gamma_{CMS}/2$ , keeping the Ward identity.

# Form factors

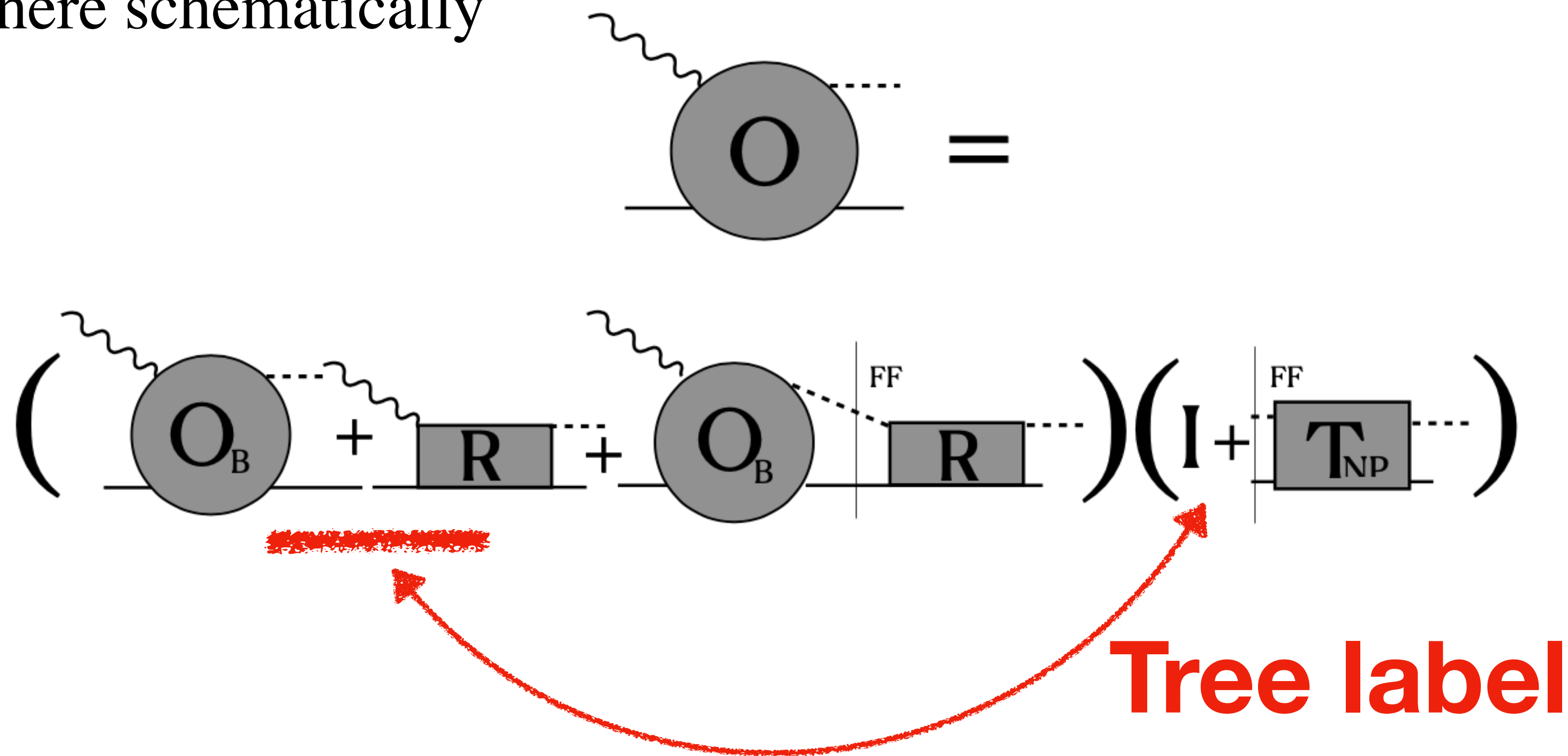
¿How to take into account the structure of the hadrons involved?

- Amplitudes come from effective Lagrangians where hadrons are treated as elementary particles.
- Strong interaction vertices do not consider particles with structure but points. Description would be appropriate for a certain energy range, but will certainly lose its validity for higher energies, requiring FF that account particle sizes.
- Such FF could simulate the inclusion of more energetic resonances not explicitly included in the model.
- Weak vertices already have FF.

- The hadronic amplitude looks like

$$\mathcal{M}_h^\lambda \equiv \bar{u}\phi^* O^\lambda u$$

where schematically



- $S_R, G_R$  are dressed by the  $\pi N$  loop ( $\Gamma_R$ )
- $T_{NP}$  introduces rescattering



- The intermediate momenta integrals are normally divergent, to regularize them, FF must be included in  $O_{B,R}$
- The *non-pole* ( $T_{NP}$ ) iterates to all orders the  $\pi N$  potential that encloses Born, Meson exchange and Crossed(u) resonance terms.
- FF introduce deviations from point couplings due to the quark structure in the nucleons and resonances, play the same role as the electromagnetic FF, which reflect the extension of hadrons, and must be calculated from the underlying theory or from the quarks.
- Different individual FF at each vertex of a graph would require vertex corrections to satisfy electromagnetic gauge invariance in radiative processes.

- Guided by previous calculations in  $\pi N$  scattering, pion photoproduction, and the description of the NC1 $\pi$  data obtained by the CERN Gargamelle experiment without applying cuts in the neutrino energies, where  $(O_B^\lambda + O_R^\lambda)$  is multiplied by a **global** regularizing FF at the vertices  $RN\pi$  y  $NN'\pi$

$$F(k, W_{\pi N}) = \frac{(\Lambda_{\text{eff}})^2}{(\Lambda_{\text{eff}})^2 + k(W_{\pi N})^2}, \Lambda_{\text{eff}} = \Lambda \frac{\Lambda}{W_{\pi N} - W_{\pi N}^{\text{th}}}.$$

- This monopolar FF that takes into account the hadronic structure, with a decreasing effective cut with  $W_{\pi N} - W_{\pi N}^{\text{th}}$ , making that certain term of the amplitude  $\pi N$  “disappears” or contributes less when  $W_{\pi N}$  grows because other resonances, not considered, could be excited.

# Results

## Channels to describe

$$\nu p \rightarrow \mu^- p \pi^+ ,$$

$$\nu n \rightarrow \mu^- n \pi^+ ,$$

$$\nu n \rightarrow \mu^- n \pi^0$$

$$\bar{\nu} n \rightarrow \mu^+ n \pi^- ,$$

$$\bar{\nu} n \rightarrow \mu^+ N \pi^+$$

We will discuss formal aspects and compare the results of our model for  $\sigma(E_\nu)$  and  $d\sigma/dQ^2$  and with the reanalyzed experimental data from ANL, BNL.

## A previous comment regards other calculations

- In the free RS Lagrangian of RS,  $\dot{\Psi}_0$  does not appear, so the equation of motion for  $\Psi_0$  it is a restriction, and has no dynamics. It is then necessary that the interactions do not change this, and it is true for  $Z = 1/2$  and  $A=-1$  (simplest form of G) in

$$\mathcal{L}_{\text{int}}(A, Z) = g_{\text{int}} \bar{\Psi}^\mu R \left( \frac{1}{2} (2Z + (1 + 4Z)A) \right)_{\mu\nu} F^\nu(\psi, \phi, W, \dots) + \text{H.c.}$$

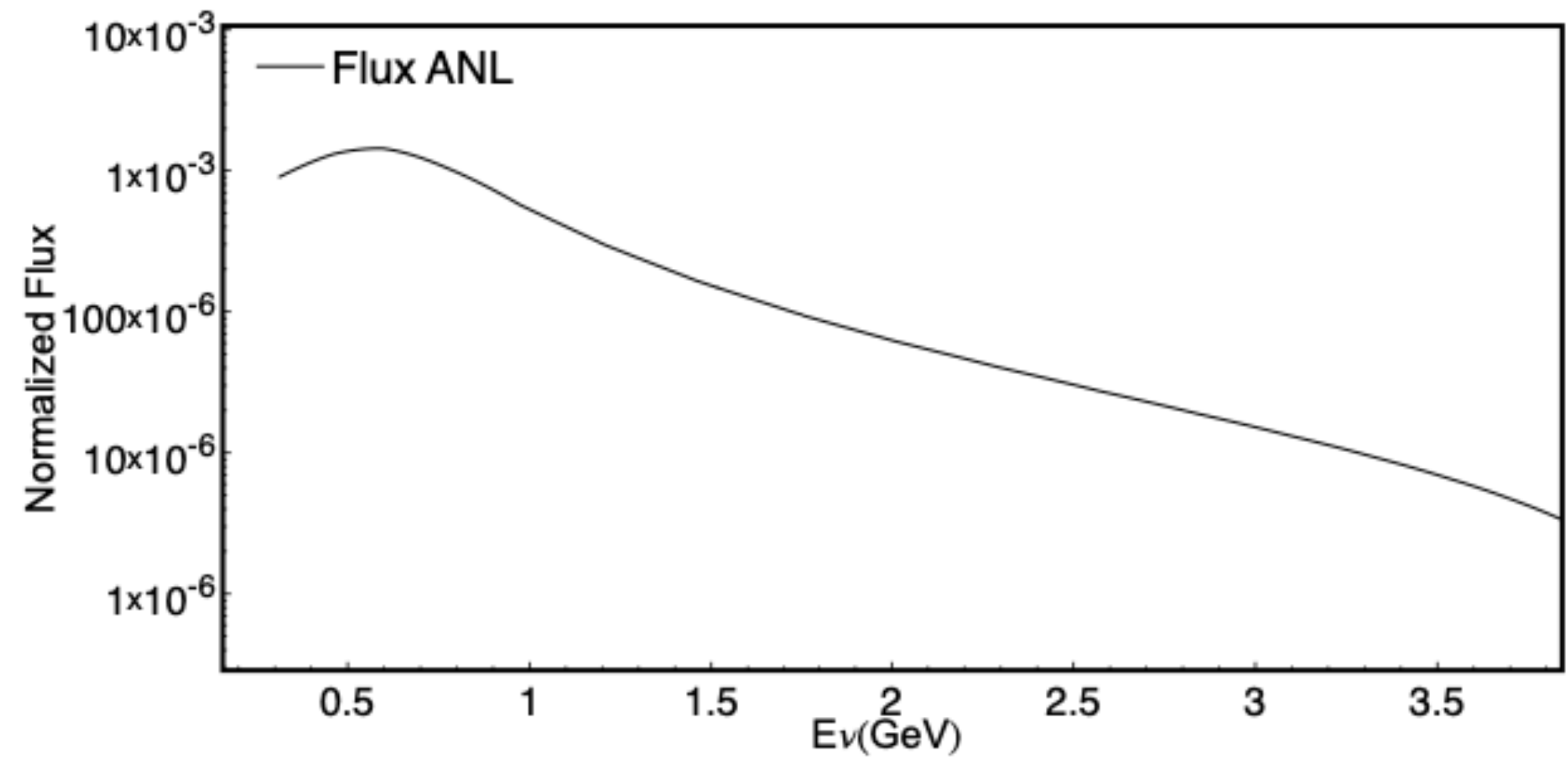
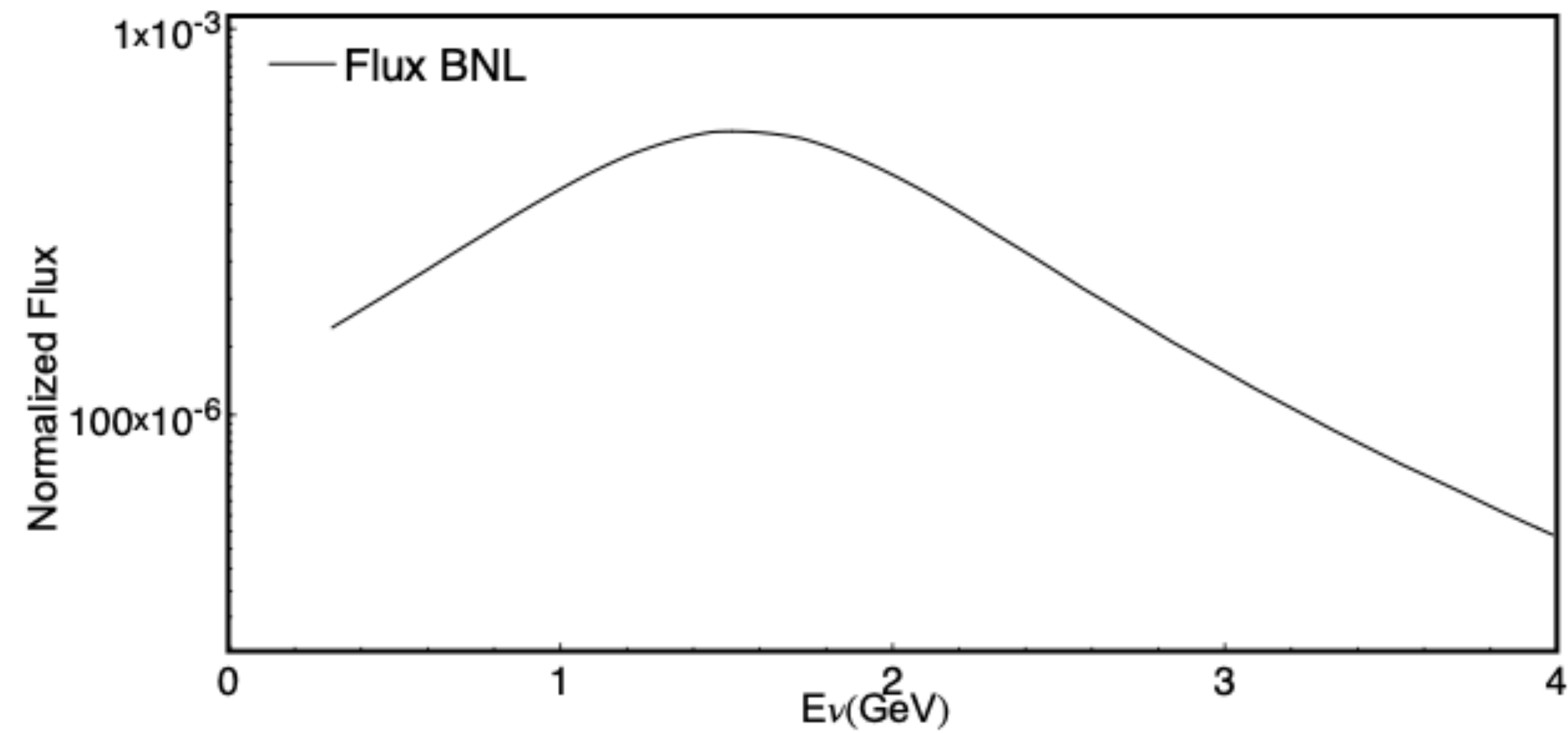
- Some authors adopting  $A=-1$  try to get the simplest vertices and we read it in two ways:
  - i) doing  $Z + \frac{1}{2}(1 + 4Z)A = 0 \rightarrow Z=-1/2$ . This value keeps  $\Psi_0$  dynamics.
  - ii)  $Z=1/2$  is adopted  $Z + \frac{1}{2}(1 + 4Z)A = 0 \rightarrow A=-1/3$  inconsistent with  $A=-1 \rightarrow$  amplitude with  $A$  dependence.
- Results presented without regarding these formal (annoying) issues are only qualified by their proximity to the experimental data.

- To avoid model dependencies by introducing FF at each interaction vertex (violates gauge invariance without vertex corrections), we will introduce a global FF on the amplitude. Since we consider resonances up to  $M_{\pi N} \sim 1.6 \text{ GeV}$ , taking into account the width of the most energetic one  $N^*(1535)$ , we will turn on this FF above its invariant mass. We will adopt  $\Lambda = 600 \text{ MeV}$  as done previously in  $\text{NC}1\pi$

$$F(k) = \frac{\Lambda^4}{\Lambda^4 + k^2 (M_{\pi N} - M_{\pi N}^{th})^2 \theta(M_{\pi N} - 1.6 \text{ GeV})}$$

- This FF is expected to effectively correct the behavior of the distribution  $d\sigma/dM_{\pi N}$  at higher values of  $M_{\pi N}$ , as successfully done in pion photoproduction where was a regularization to include final state interactions.

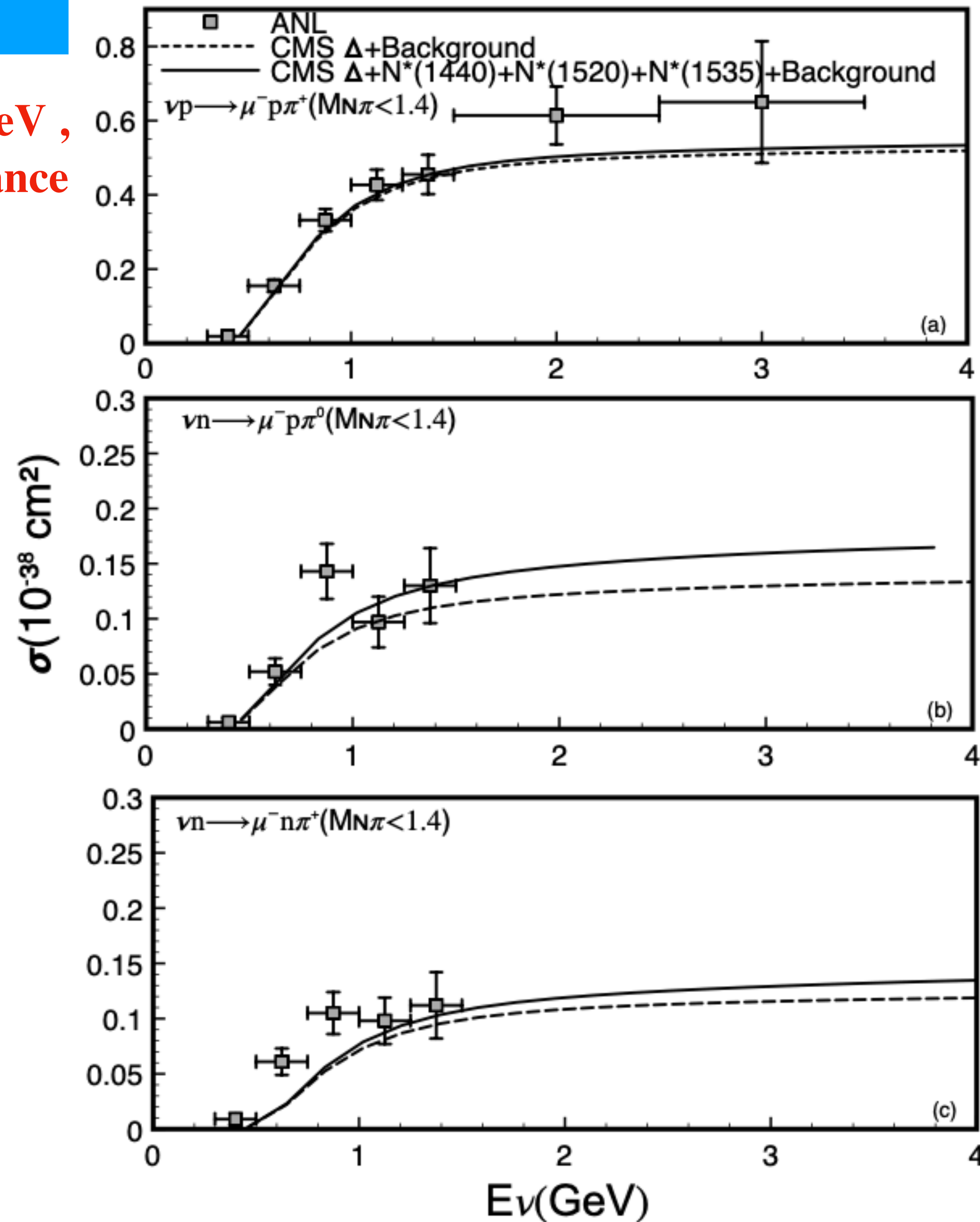
# Neutrino fluxes $\phi(E_\nu)$





# Total cross sections

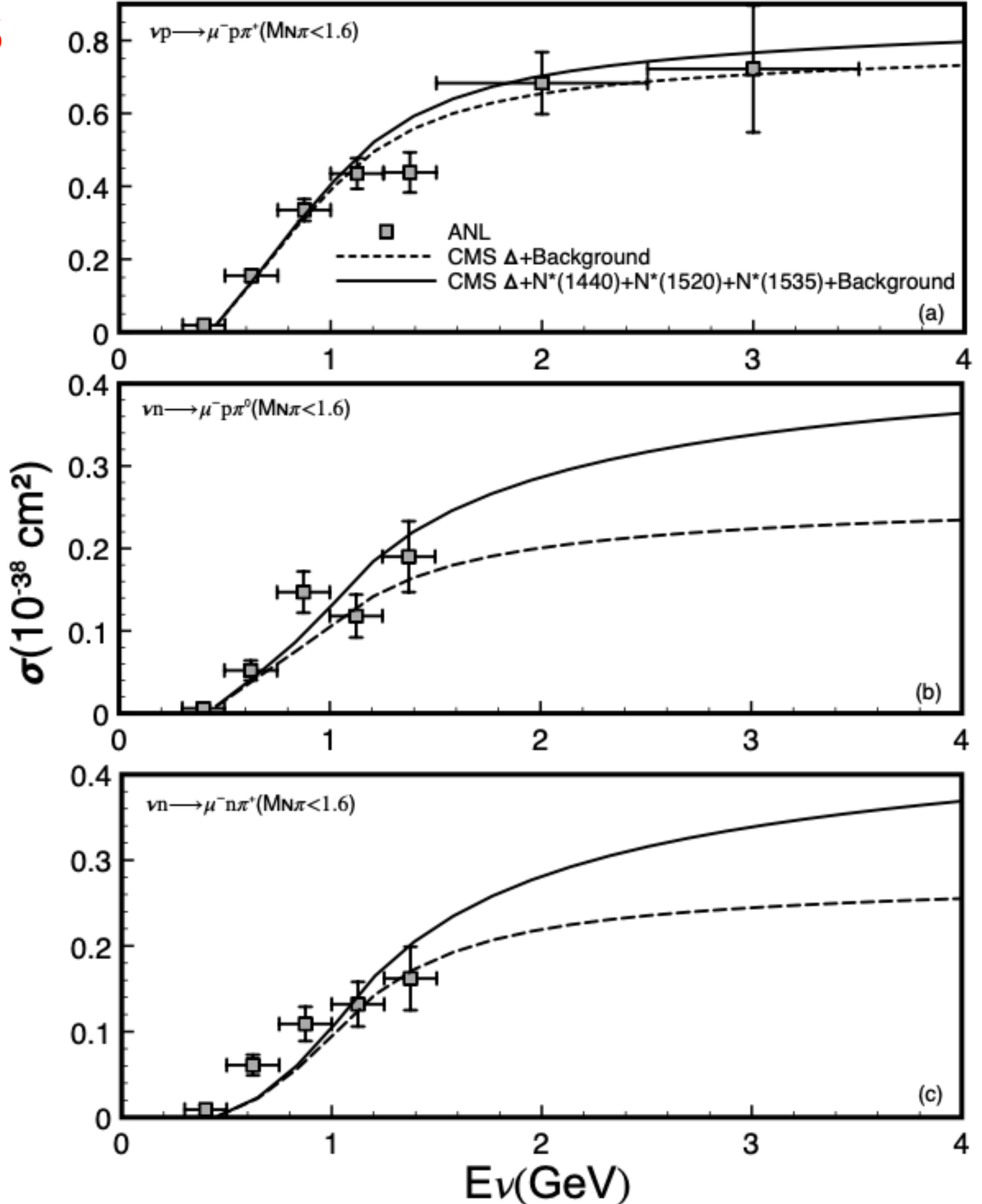
Results with  $M_{\pi N} < 1.4$  GeV,  
 $\Delta$  and  $\Delta$  plus 2nd resonance  
region (old ANL data)



● We use CMS approach previously implemented to get, strong and weak parameters for the  $\Delta$ , with  $M_{\pi N} < 1.4$  GeV.

● The effect of adding 2nd resonance region depends on the channel, for  $E_\nu = 3.0, 1.5, 1.5$  GeV we get a 4%, 17% and 10% of contribution respectively.

Idem with  $M_{\pi N} < 1.6$

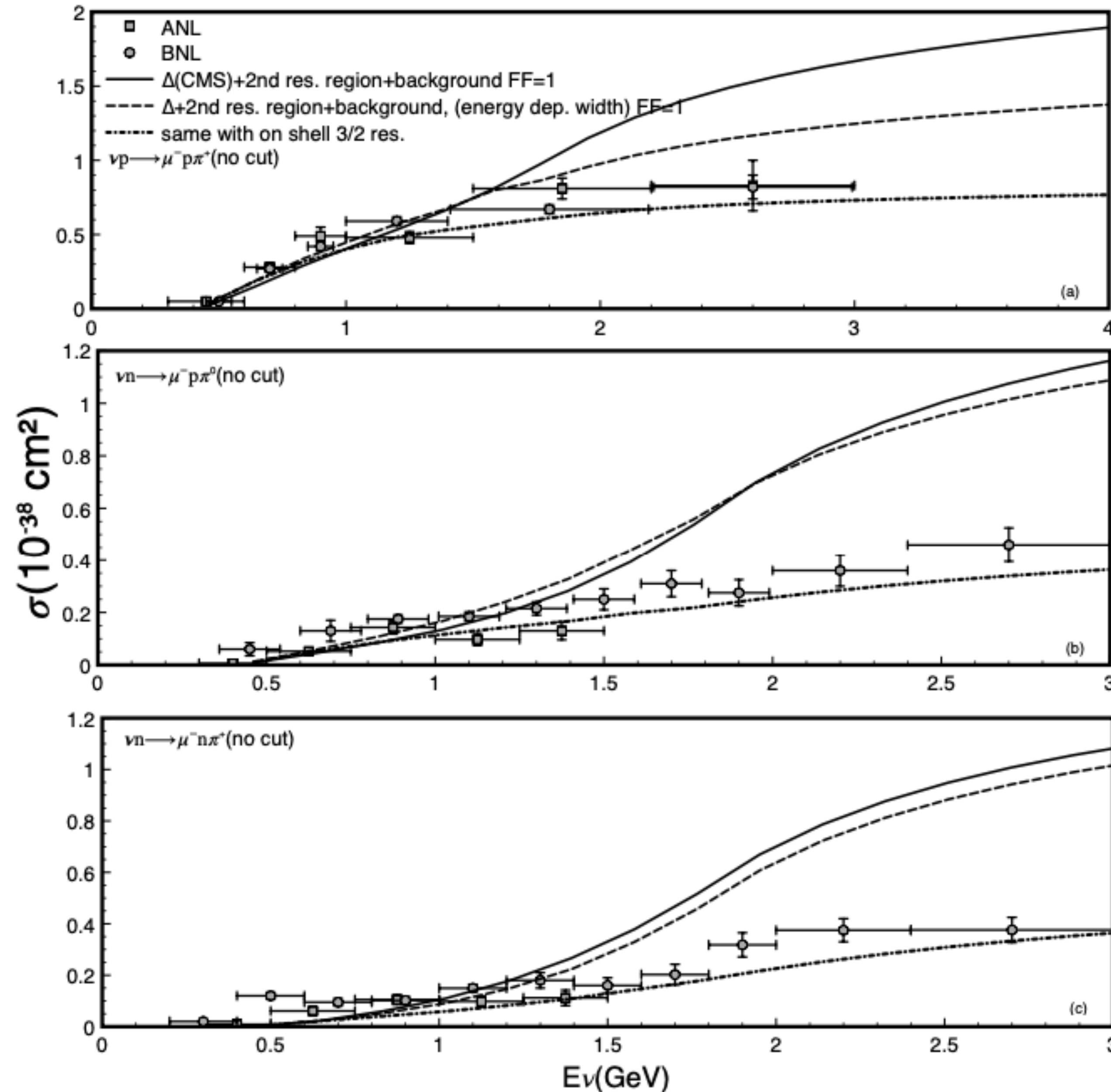


- As expected, contribution of 2nd region resonances is more important and necessary to improve the agreement with the data.
- Although in first channel the difference is small, the improvement is clear for the last two ones where background-resonance interference is most important.

## A pause to take a breath and summarize

- Until this moment we stayed within the CMS approach, the simplest to treat consistently all resonances, and backgrounds are added coherently. However, the data also includes uncut ( $M_{\pi N} < 2.0 \text{ GeV}$ ) results  $\rightarrow$  We need to extend the model to higher energies.
- Before moving on calculations without cutting, we will first analyze the behavior of the total cross section without adding any FF.
- We increase the cut both, within the CMS or introducing energy dependency on the widths, which violates gauge invariance without vertex corrections.

# Results with $M_{\pi N} < 2.0 \text{ GeV}$



- The R-cross terms grow and should be affected by rescattering.
- Using different  $A$  values for  $3/2$  propagators and vertexes, results are undervalued.
- Due to what was discussed before, we include a global FF:

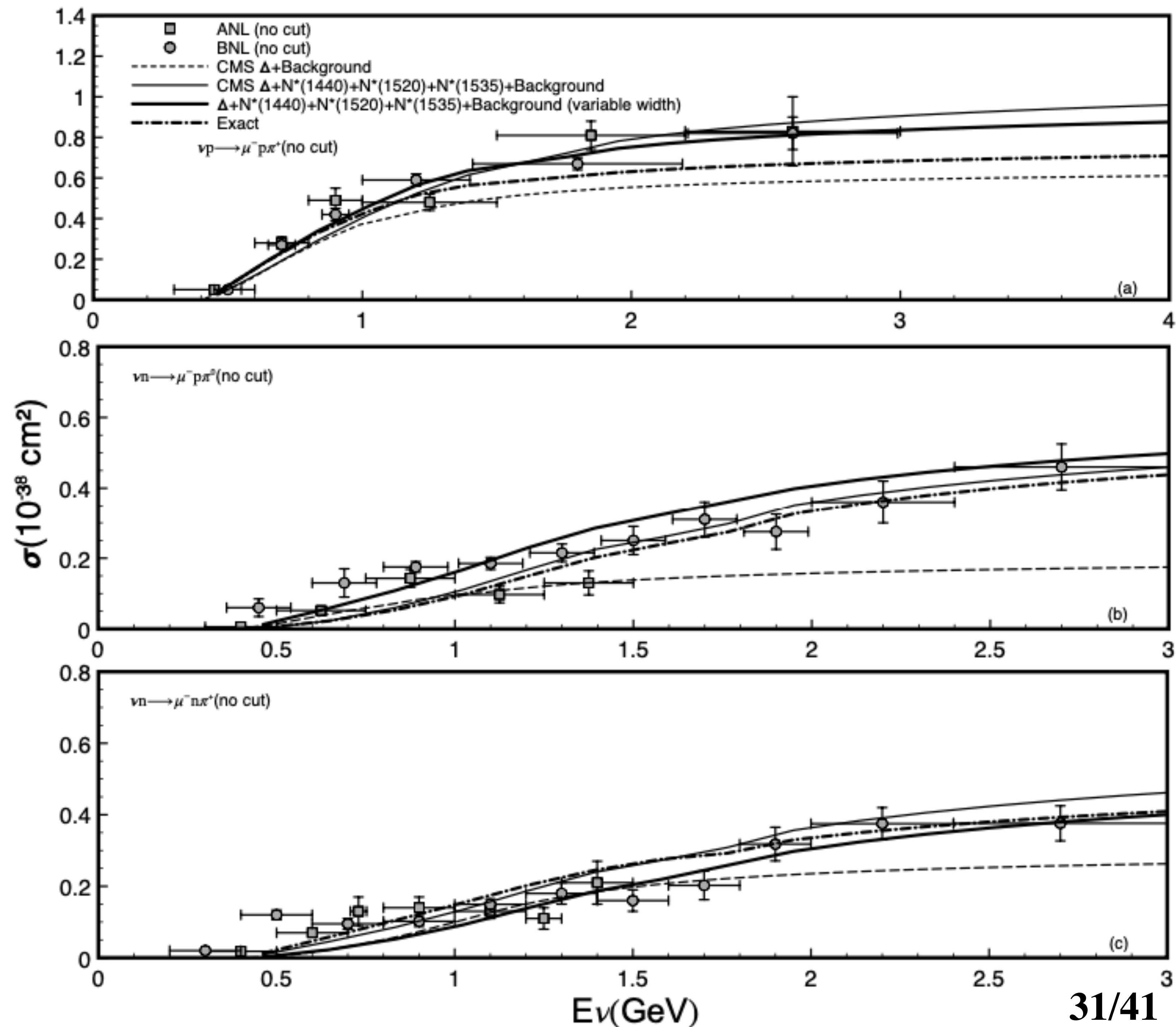
$$O_R + O_B \rightarrow (O_R + O_B) \times \text{FF}$$

$$F(k) = \frac{\Lambda^4}{\Lambda^4 + k^2(M_{\pi N} - M_{\pi N}^{th})^2 \theta(M_{\pi N} - 1.6 \text{ GeV})}$$



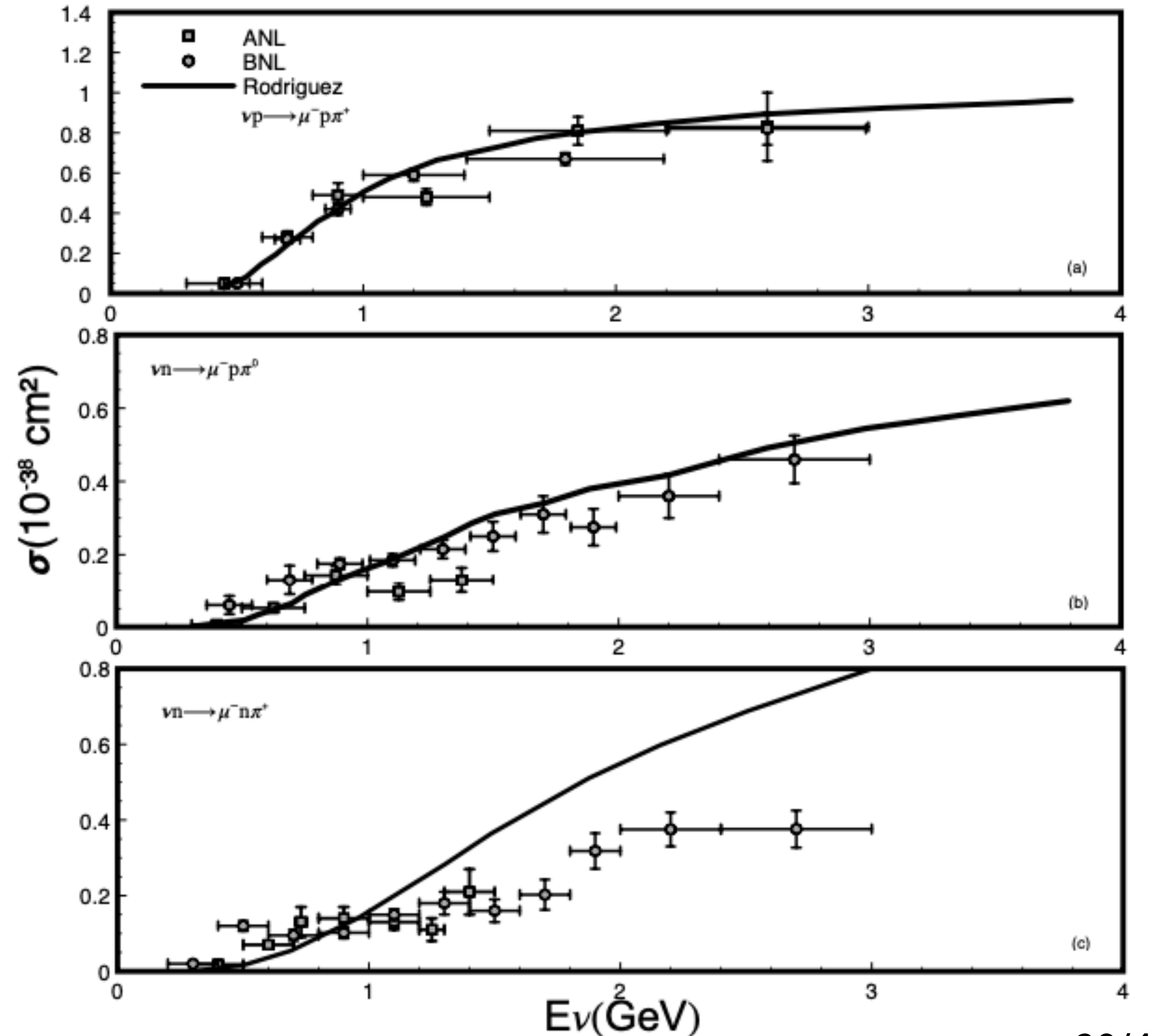
## Comparison with reanalyzed ANL and BNL

- The increase in the cross section due additional resonances is persistent and the best working approach is CMS as before . We will use  $\chi^2/dof$  in spite we are not fitting anything.
- We have also shown results with variable widths, which are not consistent without vertex corrections, since there are works with this approach. Also we show results with the exact propagator, which has a more complex structure and one should consistently include the rescattering in the total amplitude.



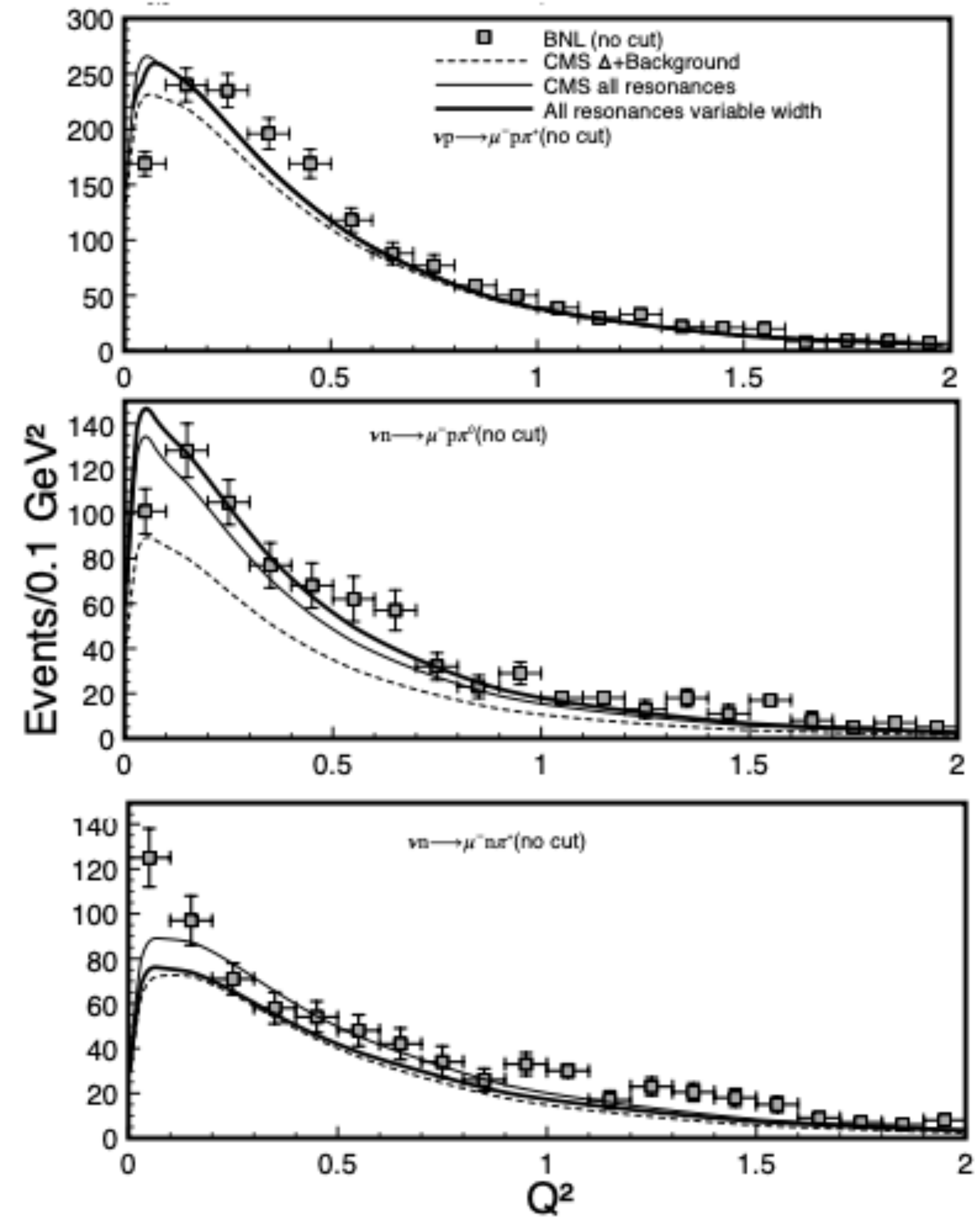
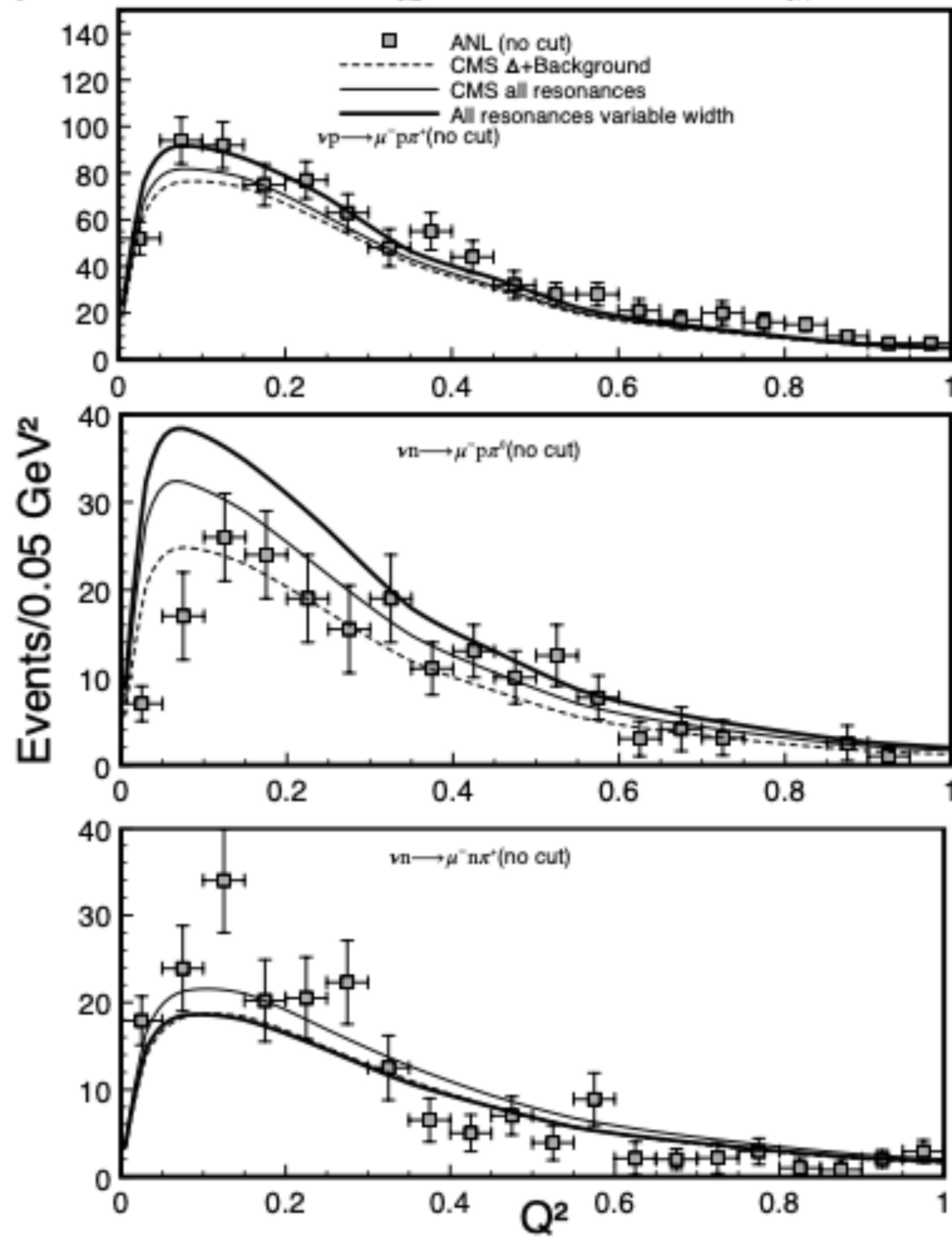
- For comparison, we show the result by GENIE, where seems to be problems in describing well all the channels for the same reanalyzed data sets.

- We can verify that for the total cross sections in the three channels we obtain for  $\chi^2/dof$  in ANL 0.4(1.13), 1.1(0.34), 1.0(2.6) and BNL 0.6(1.9), 0.7(0.8), 0.8(5.2) results within the CMS (GENIE) models.





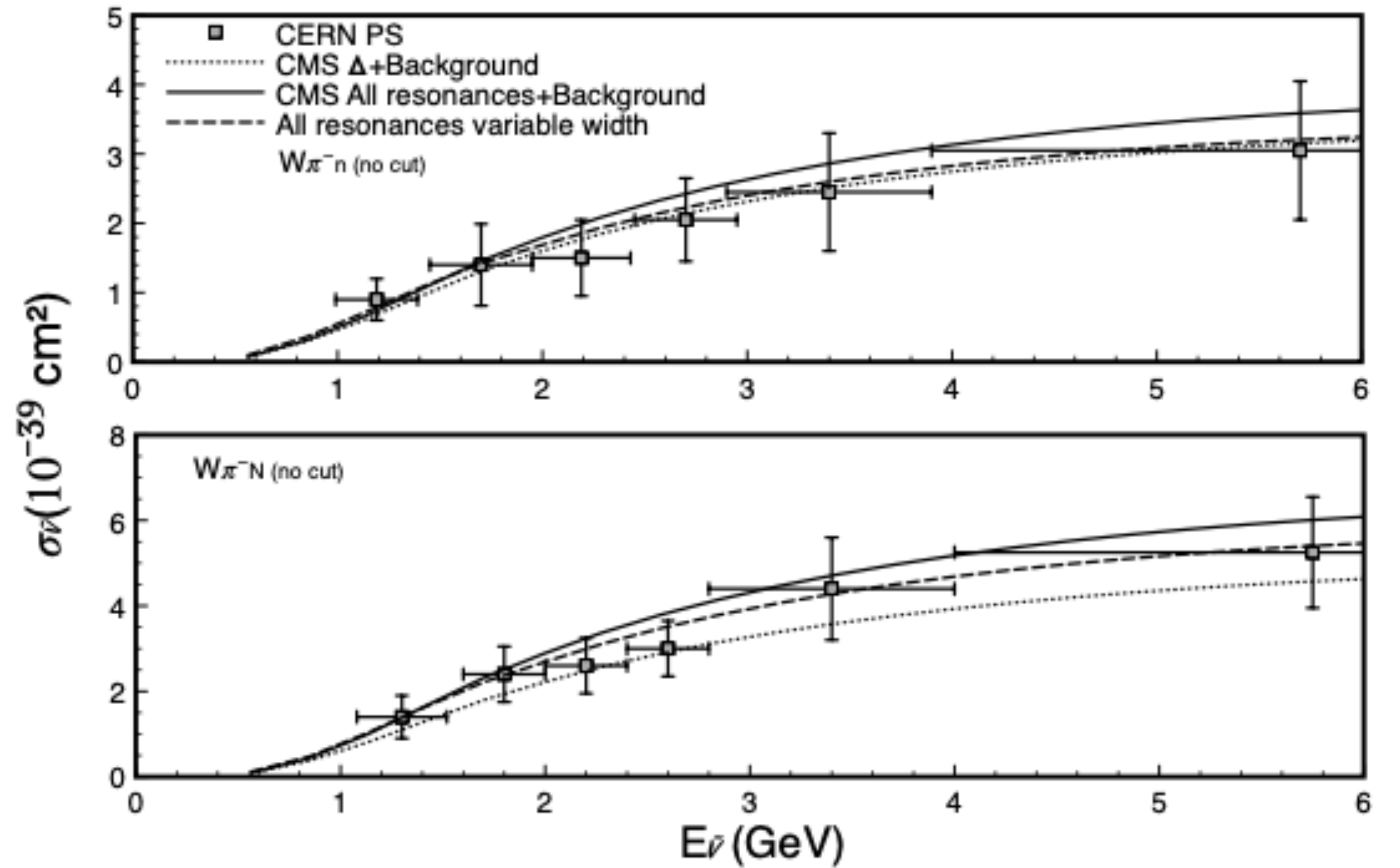
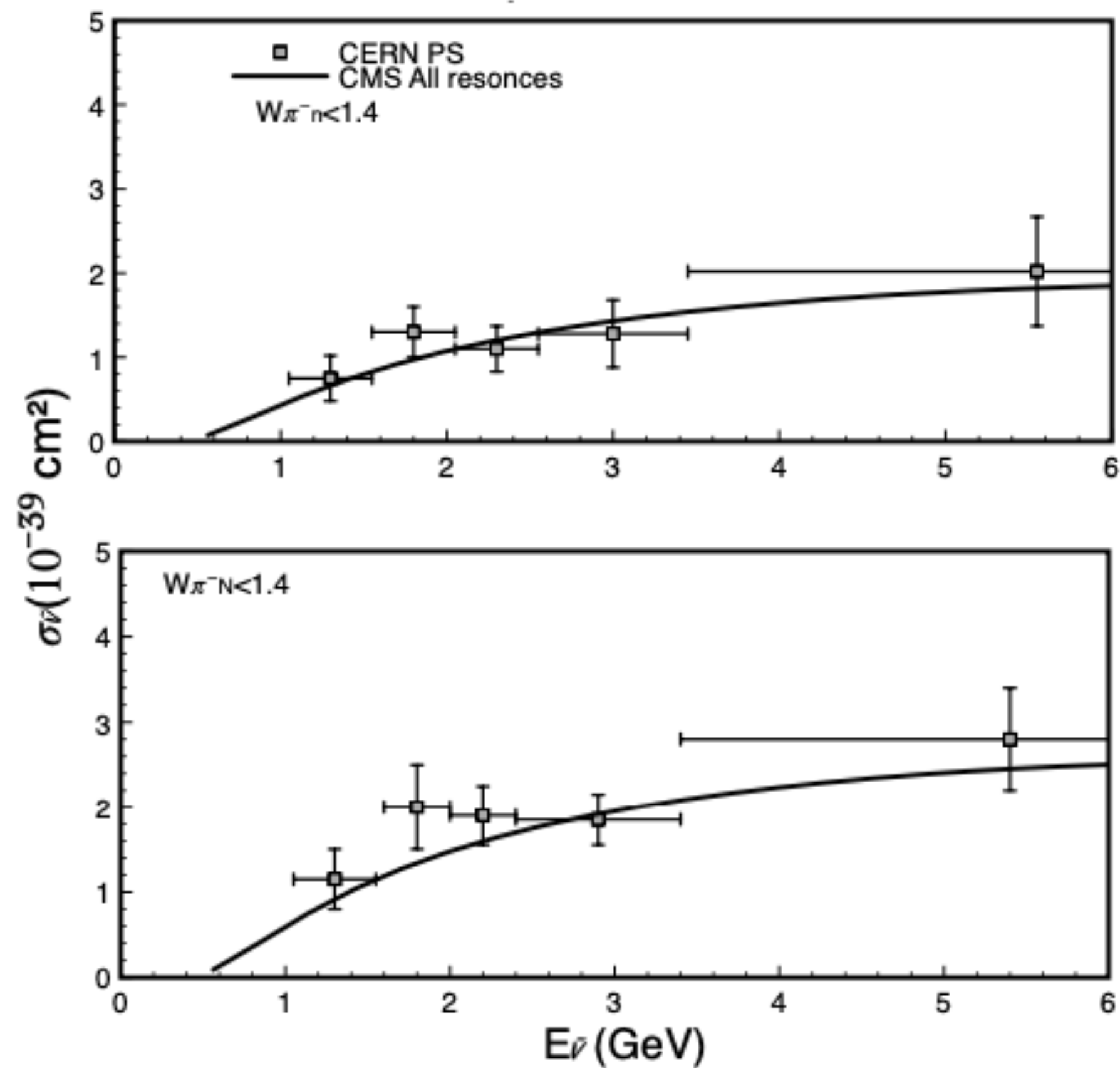
# Flux averaged differential cross sections



- We show the CMS approximation calculations, and omit the exact one since we should readjust the parameters of the  $\Delta$  , while for the energy-dependent width, it should not be crucial since we maintain the same structure of the CMS propagator but only replacing the width in the singular terms.
- The results within the CMS are acceptable in all three channels, considering we are not making any adjustments.
- The use of the energy-dependent width increases the theoretical results of the first and second channels and decreases those of the third, this leads to a worse agreement with data depending on the experiment considered ANL or BNL.
- If we compare with the GENIE results, as before, we obtain a better overall reproduction of the data.

# Antineutrinos

- The interactions of neutrinos with hadrons are not the same as those of antineutrinos. We have a sign of difference in the contraction of the lepton currents that makes a different coupling with the hadronic ones. In the experiment we have the detector of a mixture of heavy freon  $\text{CF}_3\text{Br}$  that was exposed to the CERN PS antineutrino beam (with a peak at  $E_{\bar{\nu}} \sim 1.5 \text{ GeV}$ ). It is reported that we have 0.44 % neutrons and 0.55 % protons, and since our calculations were for free nucleons we weighted the results with these percentages depending on the channel.
- First, we only show results within the CMS approach with all resonances.
- Second, we show results without cuts and with the global FF.



- For the  $W_{\pi N} \equiv M_{\pi N} = 1.4$  GeV cut, we can see that the data is reproduced but just below the center of the error bars. In the second figure we show the CMS with only  $\Delta$  the and the  $\Delta$  + other resonancias, where we see an appreciable difference. By adding the energy-dependent width, we obtain an improvement, but at the expense of a violation of electromagnetic gauge invariance as it is known.

# Conclusions

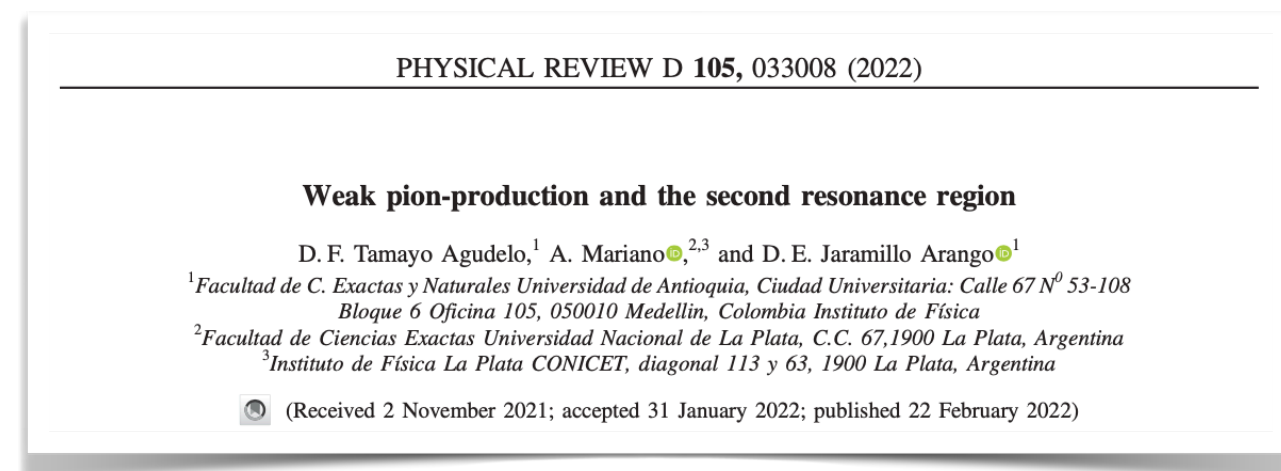


- Throughout this work we have addressed the description of the weak production of pions, within consistent effective models from the point of view of the treatment of spin 3/2 resonances. Additionally, we have discussed how to include resonances in the second region and how to treat the hadron structure as energy grows. We also include indirectly, the effect of more energetic resonances not considered in the model. We have described total(differential) neutrino and antineutrino scattering cross sections.
- We used the complex mass scheme (CMS) that corresponds to the replacement  $m_R \rightarrow m_R - i \frac{\Gamma_R^{\text{CMS}}}{2}$  (with  $m_R, \Gamma_R$  constants) in the full unperturbed propagator, which allows us to comply with the Ward identities that relate propagators and vertices for the radiation of a resonance, and maintain gauge invariance. Finally, use the exact structure of the propagator for the 3/2 resonances, however, in this case a readjustment of the parameters must be made, since the background should also be treated exactly .
- It is important to note that for the first time the Sachs parameterization was used for the  $D_{13}$  resonance, where the values of  $G_E(Q^2 = 0), G_M(Q^2 = 0), C_A^5(Q^2 = 0)$  were obtained by comparison with those usually adopted values for the Parity Conserving parameterization.

- For results without cuttings a global FF was necessary, adopting the same used successfully in pion photoproduction and NC1 $\pi$  production, taking into account the hadronic structure of the final pair and redispersion (in a troglodyte way) into more energetic resonances.
- Also, results for the average flow cross section  $\langle d\sigma/dQ^2 \rangle$  and the total cross section for antineutrinos were made.
- It should be noted that we have not made adjustments to “any” parameters, as they were taken from previous works, in order to be consistent.
- Regarding the quality of our results, seems that consistency leads to better results than other equivalent but inconsistent models of Literature.
- In GENIE, pion production is separated into resonant and non-resonant terms, and the interference terms between them, as well as the interferences between resonances, are neglected. As consequence is not possible to describe together all the channels. The data for channels  $\nu n \rightarrow \mu^- n \pi^+$ ,  $\nu n \rightarrow \mu^- n \pi^0$  are very similar, but there are large differences between GENIE's nominal predictions for them.

# Shortcomings of the model and perspective

- Recall that the axial  $C_A^5(0)$  was set to  $M_{\pi N} < 1.4$  GeV data with only the  $\Delta$  resonance included. Perhaps a new adjustment would be necessary that included the other resonances. When  $M_{\pi N}$  is expanded, new resonances can be added gradually and the corresponding axial parameters would be adjusted as was done for  $\Delta$ .
- Rescattering and unitarity must be introduced by a formalism of T-matrix in a partial waves expansion, that is not a trivial task.
- More details in



**That's all**

**Thanks to organizers for making possible my  
participation**