# The Axial Form Factor Extracted from Elementary Targets

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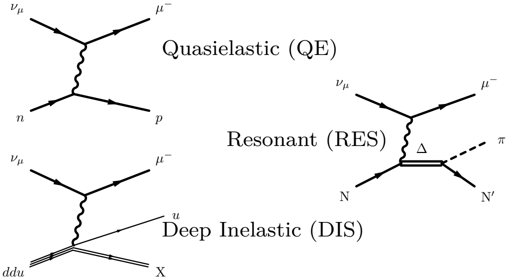
# Outline

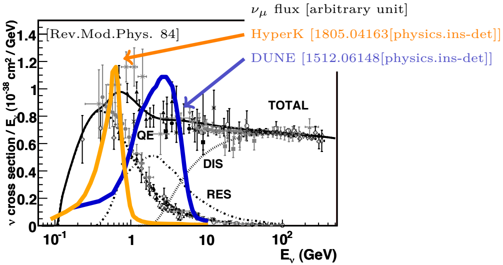
- Introduction
- Deuterium Fits
- LQCD Axial Form Factor Survey
- Combined Hydrogen+Deuterium Fitting
- Conclusions

Note: all references in online slides are hyperlinked

# Introduction

# Neutrino Cross Sections




Energy range spans several *nucleon* interaction topologies

*Nucleon* amplitudes used to build *nuclear* cross sections

$\implies$ inputs to Monte Carlo simulations, $E_\nu$ reconstruction

Goal: isolate, quantify, improve *nucleon* amplitudes

Precise, theoretically robust *nucleon* inputs → definitive statements about *nuclear* uncertainties

# Neutrino Event Topologies

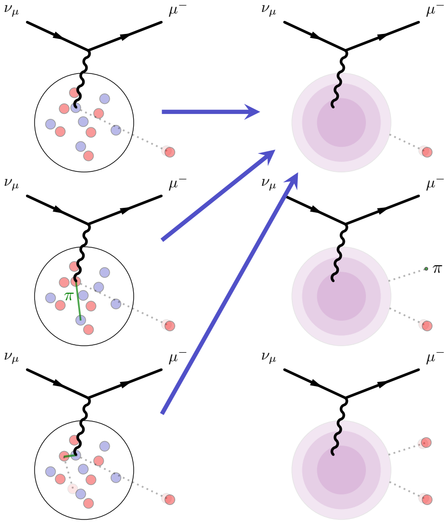
Larger nucleus
  $\Longrightarrow$ more nucleons to interact with
  $\Longrightarrow$ larger cross sections

Nuclear environment complicates measurements:

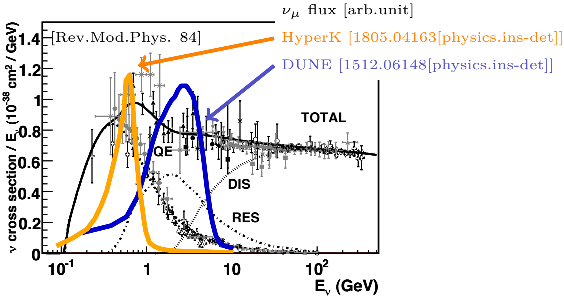
- Many allowed kinematic channels
- Reinteractions within nucleus
- Only final state particles are observable

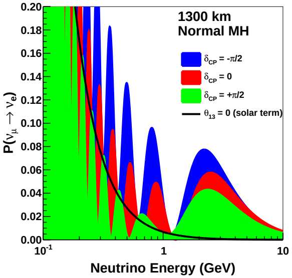
Precise cross sections need precise nucleon amplitudes

Nucleon amplitudes assumed to be precisely known



# Neutrino Cross Sections from Elementary Targets




Quasielastic is lowest $E_\nu$, simplest $\implies$ most important

Question:

How well do we know free nucleon quasielastic cross section from elementary target sources?

Three(!) main sources:

- Hydrogen scattering (new!)
- Deuterium scattering
- Lattice QCD

# Deuterium Fits

# Form Factor Parameterizations

Dipole ansatz — $$F_A(Q^2) = g_A \left(1 + \frac{Q^2}{m_A^2}\right)^{-2}$$

- Overconstrained by both experimental and LQCD data
- Inconsistent with QCD, requirements from unitarity bounds
- Motivated by $Q^2 \to \infty$ limit, data restricted to low $Q^2$

Model independent alternative: $z$ expansion [Phys.Rev.D 84 (2011)] —

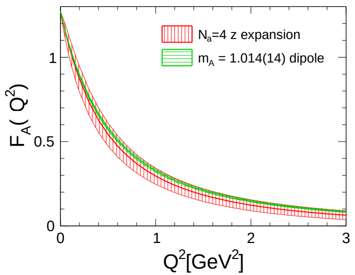
$$F_A(z) = \sum_{k=0}^{\infty} a_k z^k \qquad z(Q^2; t_0, t_{\text{cut}}) = \frac{\sqrt{t_{\text{cut}} + Q^2} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} + Q^2} + \sqrt{t_{\text{cut}} - t_0}} \qquad t_{\text{cut}} \leq (3M_\pi)^2$$

- Rapidly converging expansion
- Controlled procedure for introducing new parameters
- Sum rule constraints to regulate large-$Q^2$ behavior

# Deuterium Constraints on $F_A$

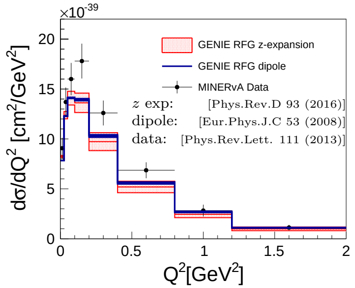
Fits: [Phys.Rev.D 93 (2016)]

- Outdated bubble chamber experiments:
  - Total $O(10^3)$ $\nu_\mu$QE events
  - Digitized event distributions only
  - Unknown corrections to data
  - Deficient deuterium correction




- Dipole overconstrained by data
  underestimated uncertainty ×10

- Prediction discrepancies could be from
  nucleon and/or nuclear origins




Coming up:

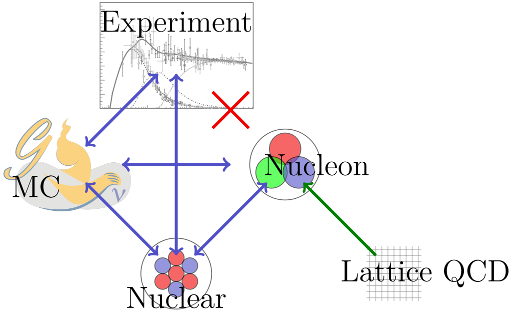
Combined fit with
MINER$\nu$A $\bar{\nu}_\mu p \rightarrow \mu^+ n$ dataset

# LQCD Survey and Implications

# LQCD as Disruptive Technology

LQCD is a complement to experiment

- ✓ No nuclear effects
- ✓ Realistic uncertainty estimates
- ✓ Systematically improvable
- ✓ Computers are (relatively) inexpensive




Build from the ground up:

Nucleon amplitudes from first principles

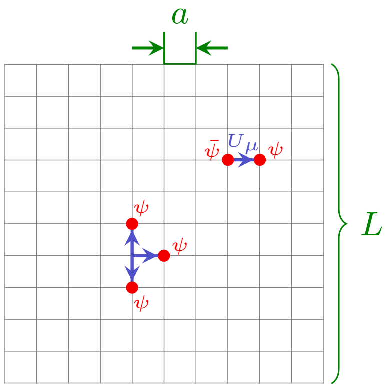
Robust uncertainty quantification

Well motivated theory inputs to nuclear models/EFTs

# Lattice QCD Formalism

Numerical evaluation of path integral
Quark, gluon DOFs —

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}\psi\, \mathcal{D}\overline{\psi}\, \mathcal{D}U \, \exp(-S)\, \mathcal{O}_\psi\,[U]$$




Parameters: $am_{(u,d),\text{bare}}$
$am_{s,\text{bare}}$
$\beta = 6/g^2_{\text{bare}}$

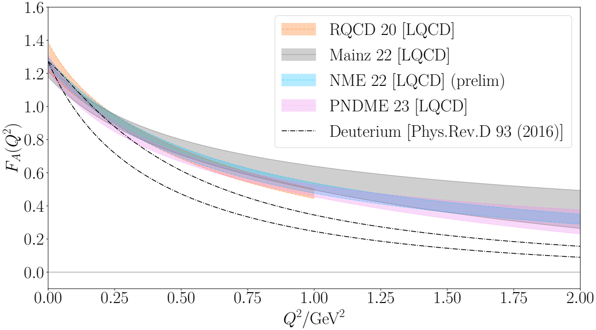
Matching: e.g. $\frac{M_\pi}{M_\Omega}$, $\frac{M_K}{M_\Omega}$, $M_\Omega$
1 per parameter

Results — first principles predictions from QCD,
gluons to all orders

"Complete" error budget $\implies$ extrapolation in $a$, $L$, $M_\pi$ guided by EFT, FV$\chi$PT

- $a \to 0$ (continuum limit)
- $L \to \infty$ (infinite volume limit)
- $M_\pi \to M_\pi^{\text{phys}}$ (chiral limit)

# LQCD Axial Form Factor Summary



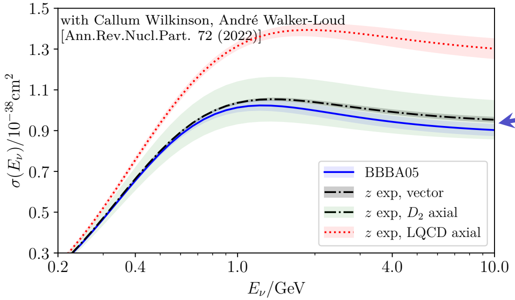

LQCD results maturing:

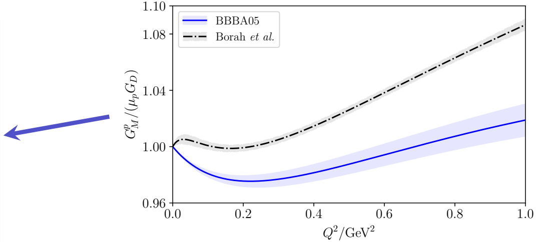
- Many results, all physical $M_\pi$: *independent data & different methods*
- Small systematic effects observed (expectation: largest at $Q^2 \to 0$)
- Nontrivial consistency checks from PCAC

Evidence of slow $Q^2$ falloff, situation unlikely to change drastically

LQCD averages – FLAG (Flavor Lattice Averaging Group) average (upcoming?)

# Free Nucleon Cross Section



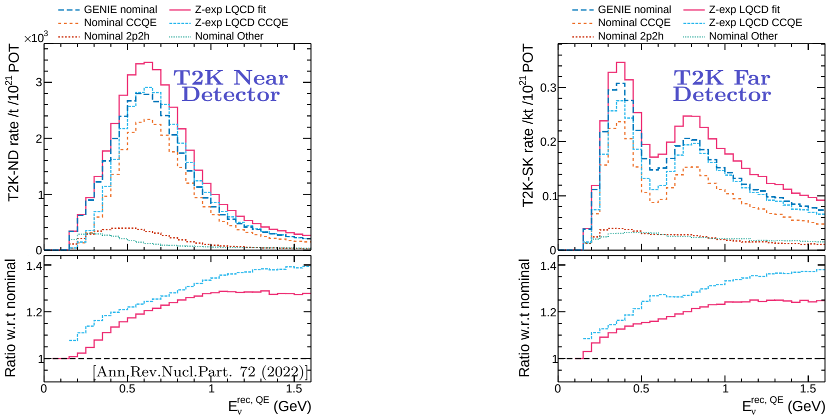




LQCD prefers 30-40% enhancement of $\nu_\mu$ CCQE cross section

recent Monte Carlo tunes require 20% enhancement of QE
[Phys.Rev.D 105 (2022)] [2206.11050 [hep-ph]]

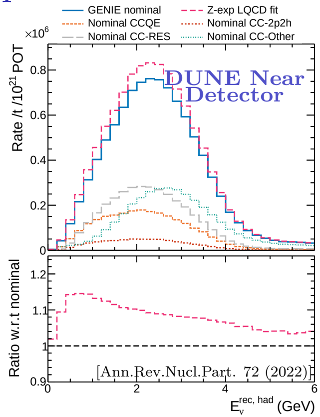
Sensitive to vector form factor tension with improved precision [Phys.Rev.D 102 (2020)] [Nucl.Phys.B Proc.Suppl. 159 (2006)]
(red uncertainty vs black−blue difference)
$\implies$ vector form factors will limit precision in near future

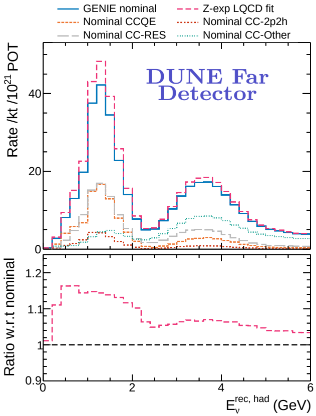
# T2K Implications




- Dashed dark blue (GENIE nominal) vs solid magenta ($z$ exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, $E_\nu$-dependent
- cross section changes at ND $\neq$ *effective* cross section changes at FD:
  insufficient CCQE model freedom $\rightarrow$ bias in FD prediction
- Monte Carlo tuning invalidates more sophisticated comparisons

# DUNE Implications




- Solid dark blue (GENIE nominal) vs dashed magenta ($z$ exp LQCD fit)
- QE enhancements produce 10-20% event rate enhancement, $E_\nu$-dependent
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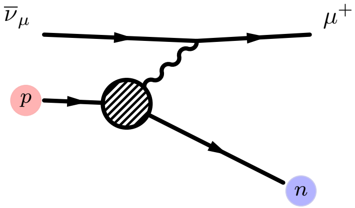
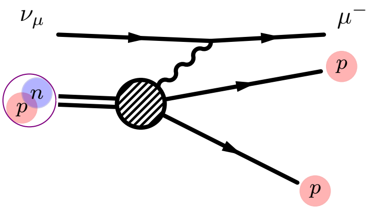
# Combined Hydrogen–Deuterium Fits

# Hydrogen vs Deuterium



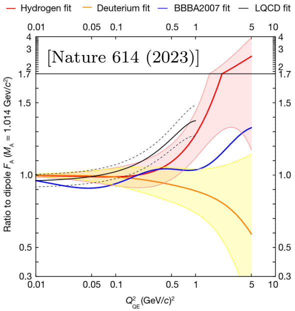




Work done with MINER$\nu$A collaboration on published data
*Special thanks*: Tejin Cai, Kevin McFarland, Miriam Moore

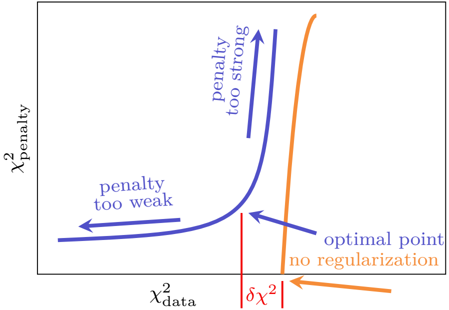
MINER$\nu$A result for $\overline{\nu}$–$p$ scattering in plastic scintillator

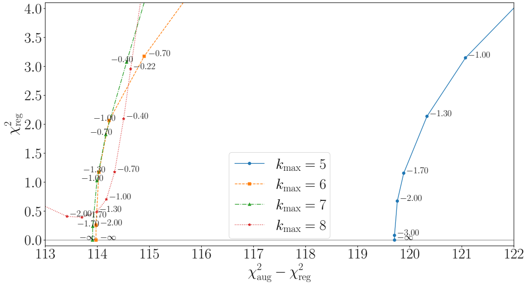
Test consistency between hydrogen, deuterium fit together

Some visible disagreements between hydrogen, deuterium
$\implies$ how does this manifest in combined fit?



# L-curve Basics



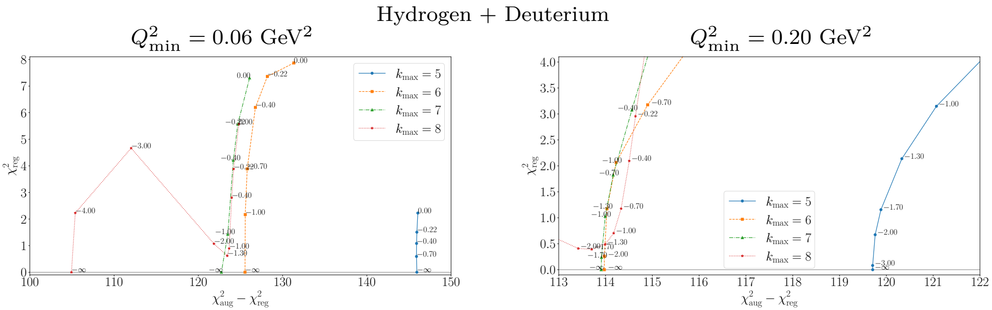




$F_A(z) = \sum_{k=0}^{k_{\max}} a_k z^k$ L-curve heuristic to choose $k_{\max}$, $\lambda$

Optimal $\lambda$ from minimum curvature on L-curve (or $\lambda = 0$), optimal $k_{\max}$ where $\delta\chi^2 < 1$

Regularization term: $\chi^2_{\text{reg}}(\lambda) = \lambda \sum_k \left|\frac{a_k}{\sigma_k}\right|^2$, $\sigma_k = |a_0| \cdot \min[5, 25/k]$ ; $\log_{10}\lambda$ printed on curves

# L-curve Studies




Hydrogen preference for $k_{\rm max} = 5,\ \lambda = 0$

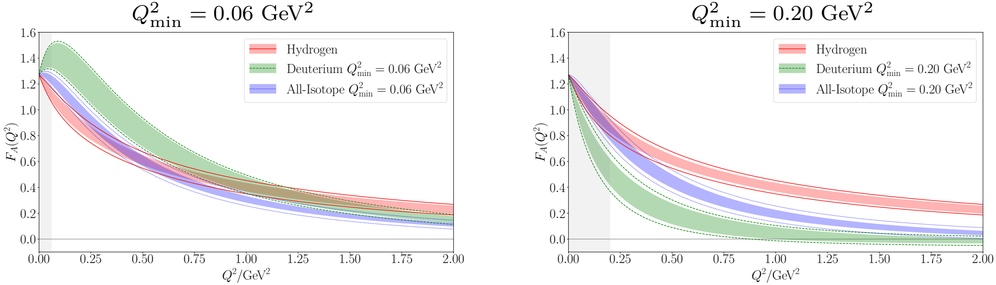
Deuterium preference depends on $Q^2_{\rm cut}$; compromise $k_{\rm max} = 6,\ \lambda = 0$

$t_0 = -0.50\ {\rm GeV}^2,\ k_{\rm max} = 6,\ \lambda = 0$ for nominal studies here

$\implies$ similar quality to $k_{\rm max} \geq 7$, but no regularization

$t_0 = -0.28\ {\rm GeV}^2,\ k_{\rm max} = 8,\ \lambda = 1$ in published deuterium result [Phys.Rev.D 93 (2016)]

# Isotope Fit Comparisons

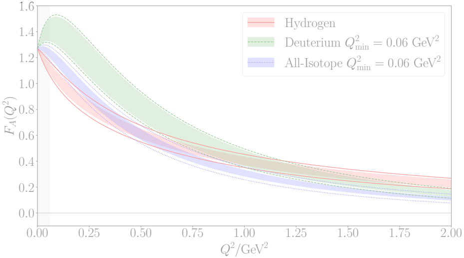


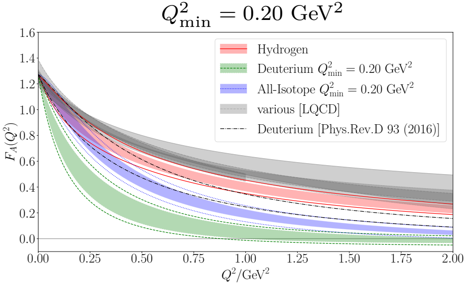

| | | |
|---|---|---|
| Inner band | – | uncertainty from axial only |
| Outer band | – | uncertainty from axial + vector [Phys.Rev.D 102 (2020)] |

Cut low $Q^2$ in deuterium to avoid systematics (nominal $Q^2_{\rm min} = 0.20$ GeV$^2$)

Degeneracy between cross section normalization and axial form factor in deuterium fits
$\Longrightarrow$ strong dependence on $Q^2_{\rm min}$, suppressed by regularization in [Phys.Rev.D 93 (2016)]

# Isotope Fit Comparisons




Tension in fits:

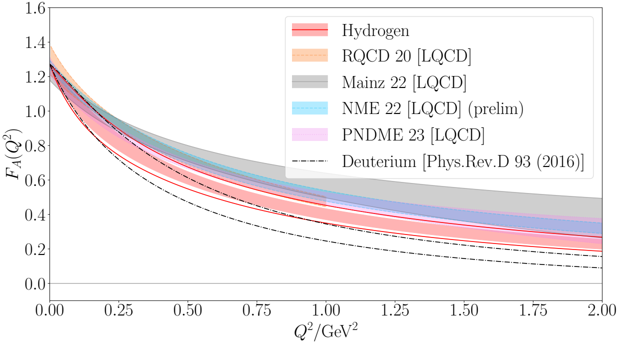
$$\Delta\chi^2 = \chi^2_{\rm H+D} - \chi^2_{\rm D} - \chi^2_{\rm H} \approx 8.8 \quad \Longrightarrow \quad \Delta\chi^2 \;/\; 1 \text{ DoF yields } p\text{-Value} \approx 3.0\times 10^{-3}$$

Test compatibility by fixing axial parameters (marginalize deuterium nuisance parameters):

| | $\{a_k\}_{\rm D}$ | $p_{\rm D}$ | $\{a_k\}_{\rm H}$ | $p_{\rm H}$ |
|---|---|---|---|---|
| $\chi^2_{\rm D}/\mathrm{DoF}_{\rm D}$ | 94.9/94 | 0.45 | 167.7/96 | $8.3\times10^{-6}$ |
| $\chi^2_{\rm H}/\mathrm{DoF}_{\rm H}$ | 23.3/15 | 0.08 | 10.0/13 | 0.69 |

Deuterium is incompatible with hydrogen, LQCD

# Hydrogen–Deuterium Comparison Summary




LQCD "prediction": deuterium fits underestimate axial form factor at high $Q^2$

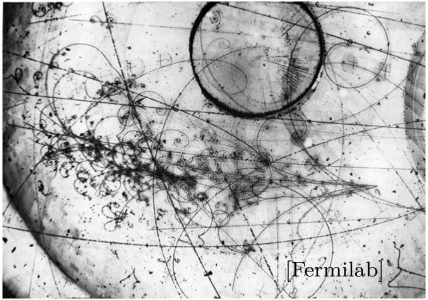
Unphysical deuterium fit degerancy between floating normalization, axial form factor

*Independent of norm degeneracy*, hydrogen & deuterium shapes mutually incompatible

We need more modern hydrogen data!

# Concluding Remarks

# Outlook

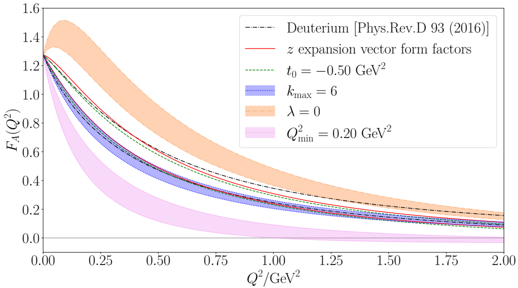



- Nucleon axial form factor uncertainty historically significantly underestimated
- Evidence that QE cross section underestimated,
  beyond published deuterium $1\sigma$ uncertainty band
- LQCD as proxy for (or complementary to) experimental data
- Deuterium fits have degeneracy between normalization, axial form factor scale;
  unphysically modulated by regularization (that was removed)
- Fit deuterium shape inconsistent with hydrogen shape
- Exciting results ahead: hydrogen scattering, LQCD

## Thank you for your attention!

# Backup

# Cumulative Updates to Deuterium



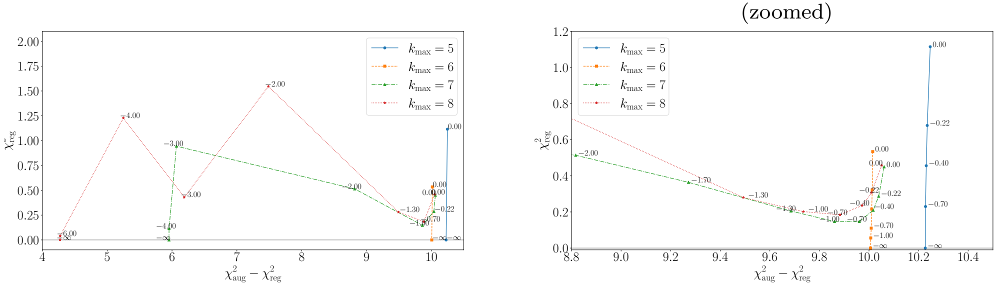

Cumulative changes between fits
$\implies$ moving down legend labels, fits include same modifications as fits above them

Fits all $1\sigma$ consistent until regularization removed

$Q^2$ cut emphasizes axial form factor + normalization degeneracy

# L-curve Studies

Hydrogen

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$\implies$ similar quality to $k_{\max} \geq 7$, but no regularization

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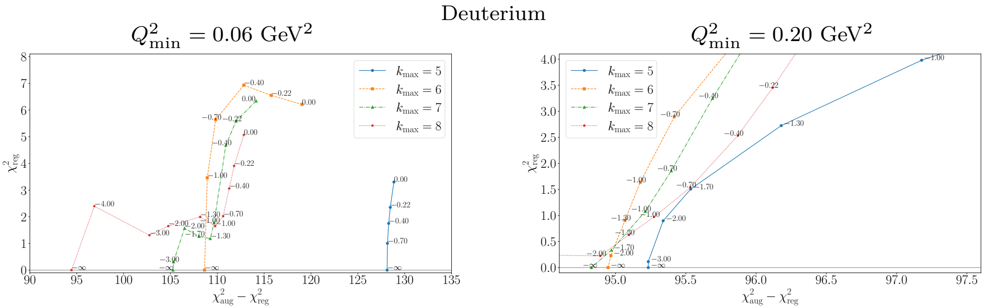


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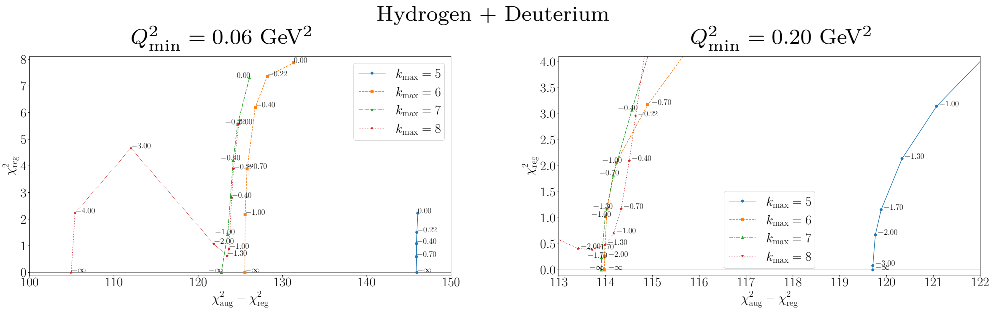


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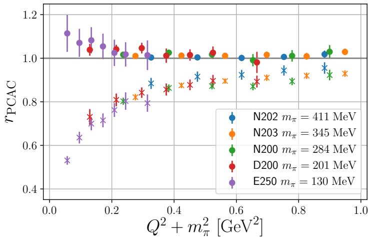
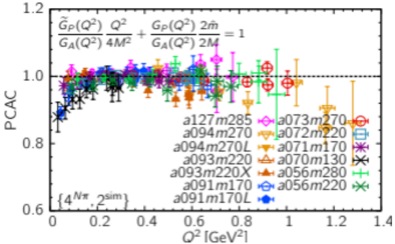
# PCAC Checks

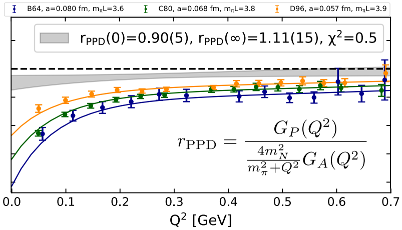
**RQCD** [JHEP 05 (2020)]




**NME** [prelim]

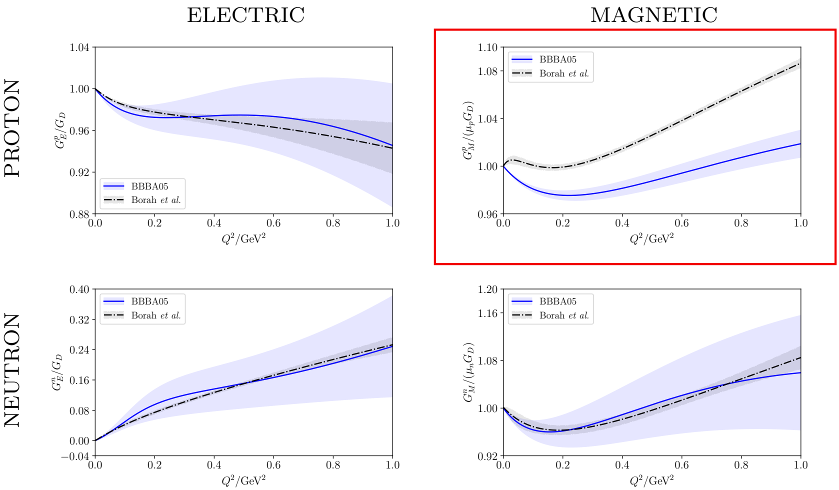



**ETMC** [prelim]



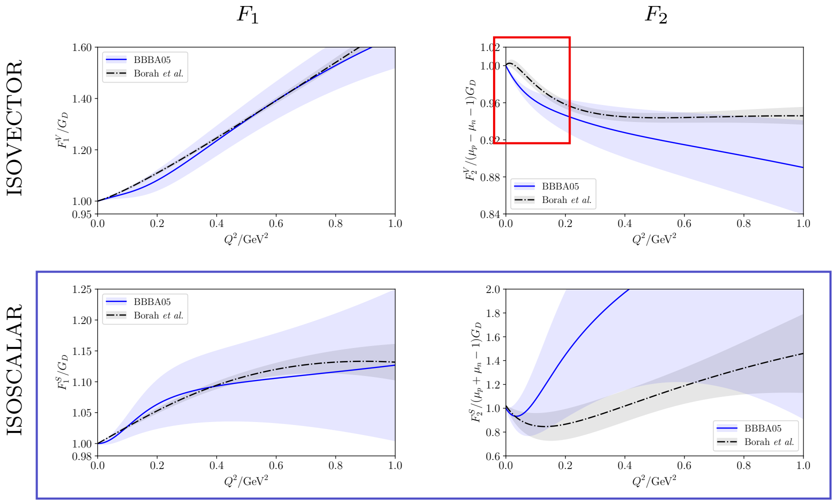

- Relation btw $F_A$, $F_P$, $\tilde{F}_P$ via PCAC
- Contamination in $F_A$ and $\tilde{F}_P$, $F_P$ very different
  $\implies$ nontrivial consistency check
  [Phys.Rev.D 99 (2019)]

# Vector Form Factors - Proton/Neutron




Large tension in proton magnetic form factor

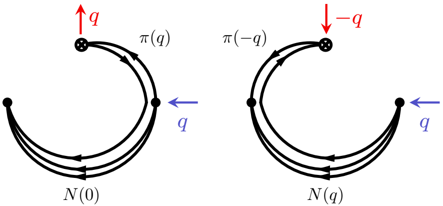
# Vector Form Factors - Isospin Symmetric

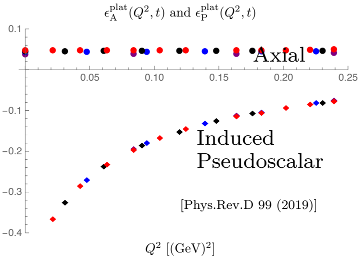



Uncertain slope of $F_2^V$

Large uncertainty on isoscalar form factors

# LQCD Excited States — $\chi$PT and $N\pi$




Contamination in $g_A(Q^2)$ primarily from enhanced $N\pi$, mostly from induced pseudoscalar

Correlator fits without axial current not sensitive to $N\pi$ [Phys.Rev.C 105 (2022)] [Phys.Rev.D 105 (2022)]

Alternate fit strategies:

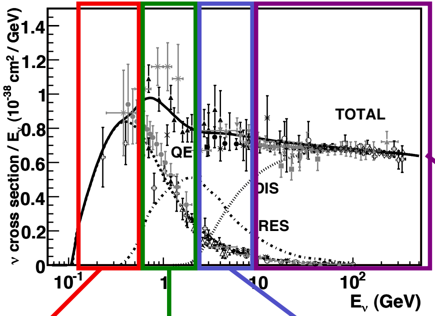
- explicit $N\pi$ operators
- include $\mathcal{A}_4$ (strong $N\pi$ coupling)
- Kinematic constraints ($F_P = 0$)

Prediction from $\chi$PT: [Phys.Rev.D 99 (2019)]

First demonstration of $N\pi$: [Phys.Rev.Lett. 124 (2020)]

$\chi$PT-inspired fit methods for fitting form factor data
[Phys.Rev.D 105 (2022)] [JHEP 05 (2020) 126]

# Energy Regimes



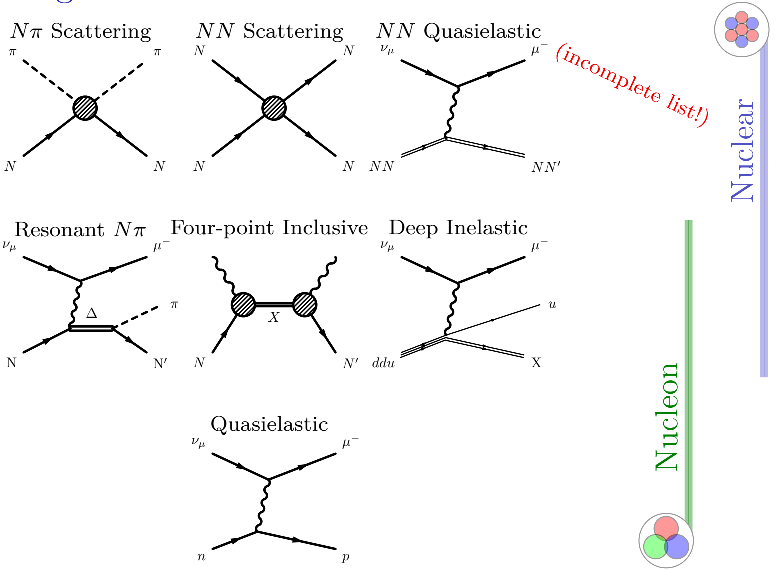

Quasielastic
–Nucleon Form Factors
–Full Error Budgets

$N \to \Delta, N \to N^*$
–Transition Matrix Elements
–Multiparticle Operators

"Shallow Inelastic Scattering" (SIS)
–Hadronic Tensor
–Four Point Functions

Deep Inelastic Scattering
–Axial quasi/pseudo PDF

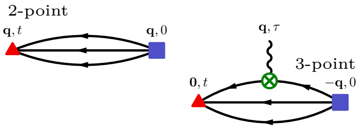
# LQCD Target Calculations



# LQCD Computation Anatomy

Correlation functions in euclidean time:

$\implies e^{-E_n t}$ decay of excited state contribs




2-point function

$\langle \blacktriangle(t) \blacksquare(0) \rangle = \sum_n \langle 0 | \blacktriangle | n \rangle \langle n | \blacksquare | 0 \rangle e^{-E_n t}$

3-point function

$\langle \blacktriangle(t) \otimes(\tau) \blacksquare(0) \rangle = \sum_{mn} \langle 0 | \blacktriangle | n \rangle \langle n | \otimes | m \rangle \langle m | \blacksquare | 0 \rangle e^{-E_n(t-\tau) - E_m \tau}$

Extract masses from 2-point, matrix elements from 3-point

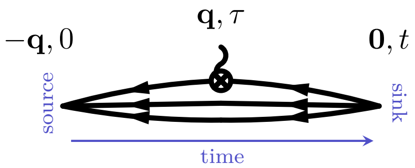
Complications:

- exponentially degrading signal/noise with $t$
- $n > 0$ contaminations from excited states

Use many source/sink operators ($\blacksquare$, $\blacktriangle$) to suppress excited states:

$C_{ij}(t) = \sum_n z_{i,n} z^{\dagger}_{j,n} e^{-E_n t} \quad \implies \quad v^T C(t) v \approx e^{-E_0 t}$ when $\sum_i v_i^T z_{i,n} \approx \delta_{0,n}$

# Fit Setup




Fit exponential dependence of axial "3-point" functions:

$$C^{\rm 3pt}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \langle \mathcal{N}(\mathbf{0},t) \mathcal{A}_z(\mathbf{q},\tau) \overline{\mathcal{N}}(-\mathbf{q},0) \rangle$$

$$\sim \sum_{mn} z_n^{\mathbf{0}} \, A_{nm}^{\mathbf{q}} \, z_m^{\mathbf{q}\dagger} \, e^{-E_n^{\mathbf{0}}(t-\tau)} e^{-E_m^{\mathbf{q}}\tau}$$

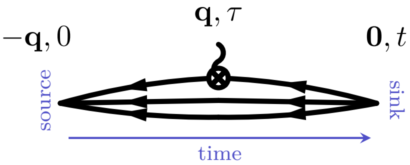
Towers of excited states $m$, $n$ depend on momenta injected

Current $\mathcal{A}_z$ couples to axial, induced pseudoscalar form factors

Overlaps, energies constrained by "2-point" functions

$$C^{\rm 2pt}(t,\mathbf{q}) = \langle \mathcal{N}(\mathbf{q},t) \overline{\mathcal{N}}(-\mathbf{q},0) \rangle \sim \sum_m z_m^{\mathbf{q}} \, z_m^{\mathbf{q}\dagger} \, e^{-E_m^{\mathbf{q}} t}$$

# Fit Setup




Plot ratio correlator:

$$\mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) = \frac{C^{3\mathrm{pt}}_{\mathcal{A}_z}(t,\tau,\mathbf{q})}{\sqrt{C^{2\mathrm{pt}}(t-\tau,\mathbf{0}) C^{2\mathrm{pt}}(\tau,\mathbf{q})}} \sqrt{\frac{C^{2\mathrm{pt}}(\tau,\mathbf{0})}{C^{2\mathrm{pt}}(t,\mathbf{0})} \frac{C^{2\mathrm{pt}}(t-\tau,\mathbf{q})}{C^{2\mathrm{pt}}(t,\mathbf{q})}}$$

$$\xrightarrow[t-\tau,\tau\to\infty]{} \frac{1}{\sqrt{2E_0^{\mathbf{q}}(E_0^{\mathbf{q}}+M)}} \left[ -\frac{q_z^2}{2M} \mathring{F}_P(Q^2) + (E_0^{\mathbf{q}}+M) \mathring{F}_A(Q^2) \right]$$

$Q^2 = |\mathbf{q}|^2 - (E_0^{\mathbf{q}} - M)^2$

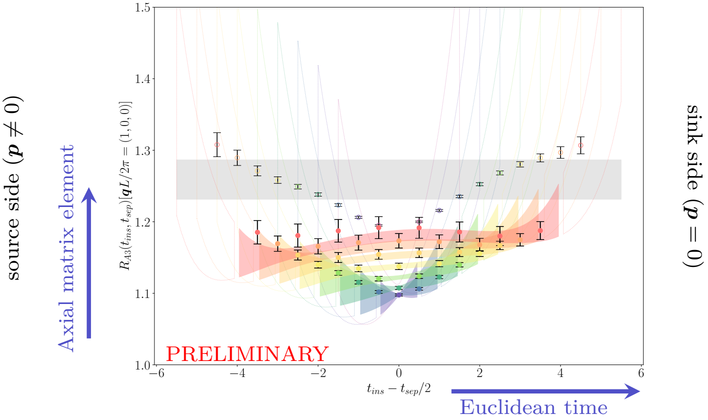
$\mathcal{A}_z$ with $q_z = 0 \quad \Longrightarrow \quad \mathcal{R}_{\mathcal{A}_z}(t,\tau,\mathbf{q}) \to \sqrt{\frac{E_0^{\mathbf{q}}+M}{2E_0^{\mathbf{q}}}} \mathring{g}_A(Q^2)$

$\Longrightarrow$ No induced pseudoscalar

$\Longrightarrow$ Simplified analysis of $\mathring{F}_A(Q^2) = \mathring{g}_A(Q^2)$

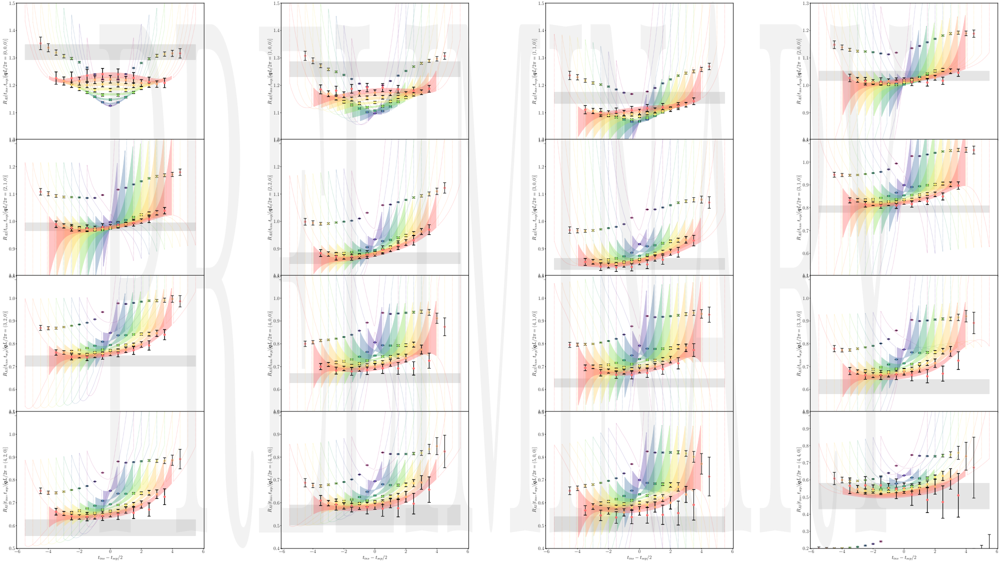
$\Longrightarrow$ `a12m130` ensemble only, $N_{state} = 3$ only

# Correlation Function Ratio

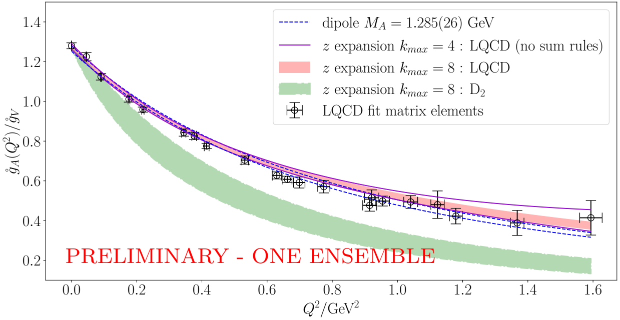



- Color: source-sink separation time
- Colored bands: fit
- Gray band: $\mathring{g}_A$ posterior value
- Curvature: excited state contamination

# $\mathring{g}_A(Q^2)$ Correlators



# Axial Form Factor Fit




Trend of high-$Q^2$ enhancement seen in other LQCD results

2–4% LQCD uncertainty vs 10% uncertainty on $D_2$ result

TODO list:

- $\boldsymbol{q}L/2\pi = (1,0,0)$ matrix element larger than expectation
- Deep dive into excited states systematics, prior dependence
- More momenta, $q_z \neq 0$, full set of ensembles