The Physics of Shallow Inelastic Scattering with Neutrinos

(Non-resonant meson production – Resonance Presentations)

NuSTEC Workshop on Pion Production - https://indico.fnal.gov/event/20793/

Target Mass Corrections and Higher-Twist

SIS / DIS Review – M.Sajjad Athar. and JGM - arXiv:2006.08603 **[hep-ph]**

NuSTEC Workshop on SIS and DIS - https://indico.cern.ch/event/727283/

nCTEQ Contributions

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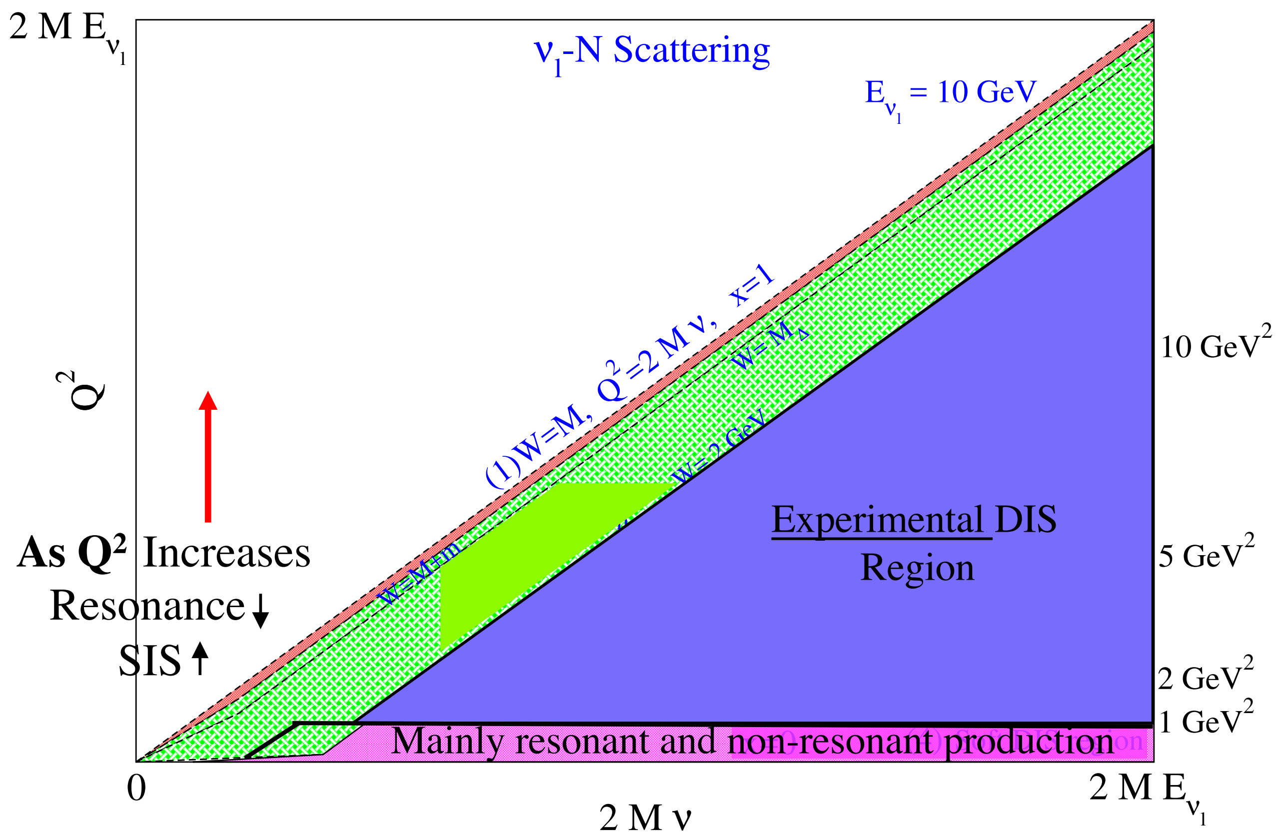
Expanding the Definition of Shallow Inelastic Scattering

Q^2 the most important variable



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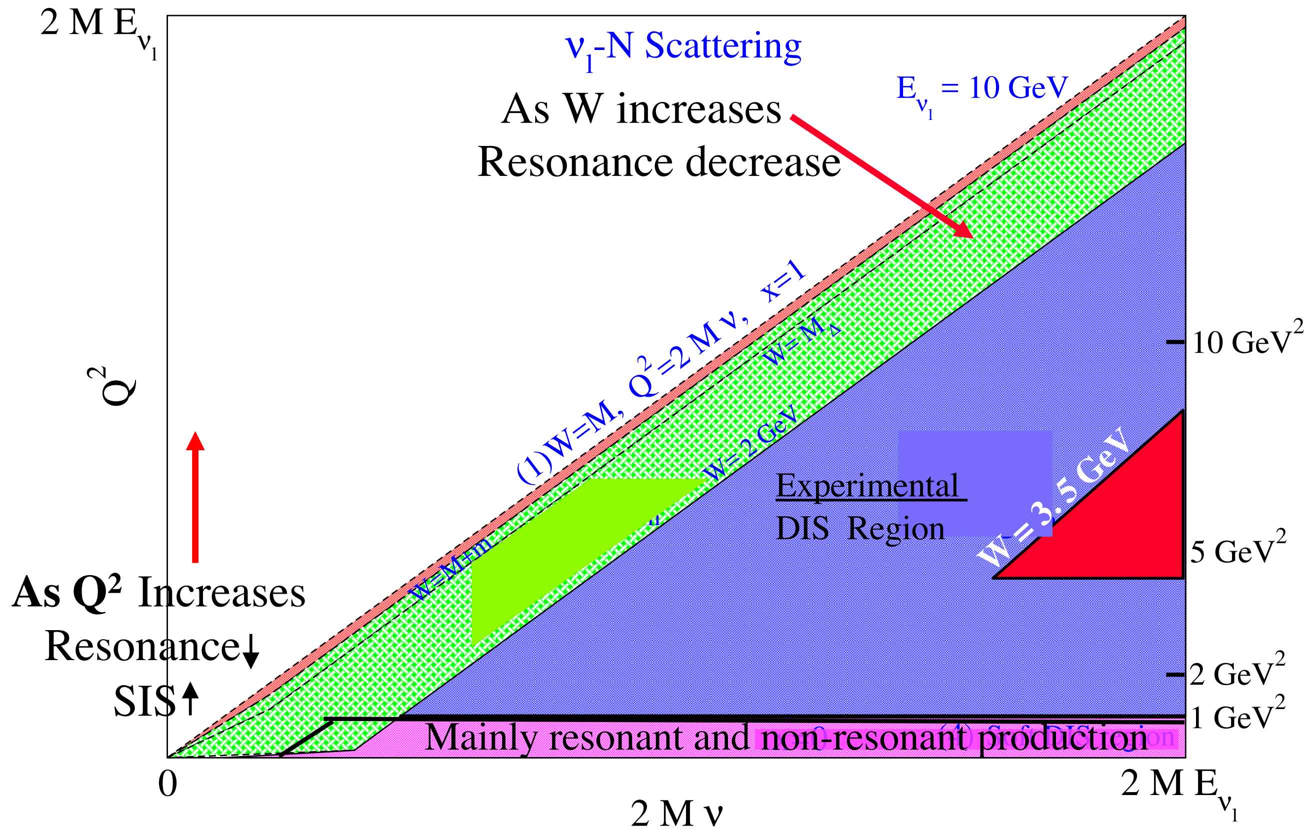
Expanding the Definition of Shallow Inelastic Scattering

Introduce W



- Use W cuts to limit resonant production and, with Q^2 cuts, give a range of x.
- Corresponding range of x gives probability of finding single quark for scattering

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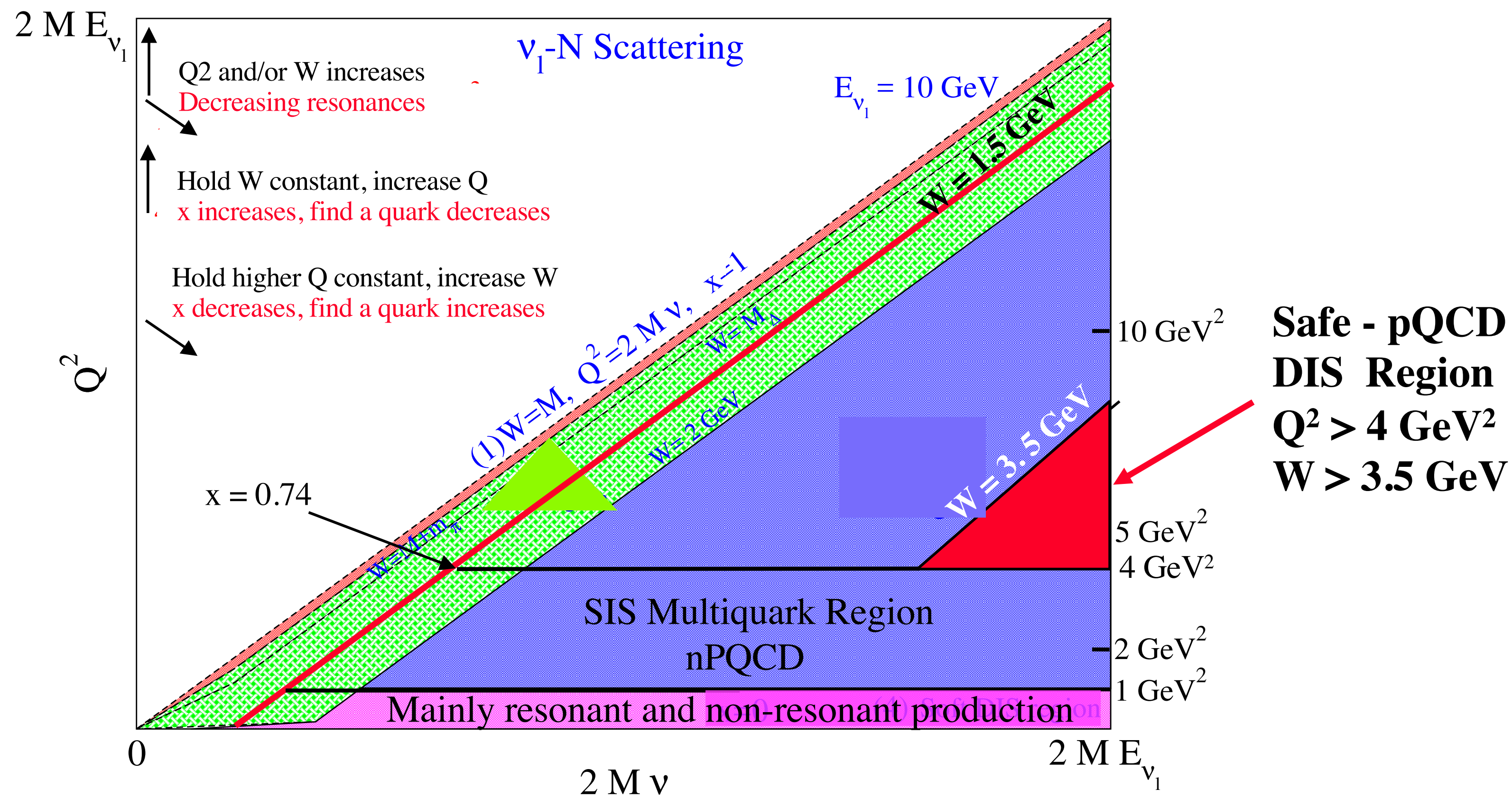
How do we study the SIS region between safe DIS and the resonance region?



- SIS non-perturbative multiquark ($1/Q^2$) effects.
- SAFE DIS (red region) for (nCTEQ) Global pQCD fits for PDFs.
- **≤ 2 % of MINERvA ME $Q^2 > 1$ GeV^2 events are in the SAFE DIS region**

How do we explore the SAFE DIS ⟶ Resonance Transition!
Possibility 1: **Quark – Hadron Duality**

- Quark–hadron duality is a general feature of strongly interacting landscape:
  - How does the physics (language) of quark/gluons from DIS meet the physics of nucleons/mesons (pions) of SIS → **quark-hadron duality.**
- Quark-hadron duality originally studied/confirmed in **e-N scattering.**
- In general, for ν, the resonance structure functions for **proton are much larger than for neutrons** and in the case of DIS structure functions **the situation is opposite.**
- **No general agreement on how to apply duality to ν interactions off nucleons / nuclei. Details in the Backup.**
- **The alternative - consider the physics of the non-perturbative QCD Region**

Language of non-perturbative QCD

For smaller **Q^2** and/or larger **x_{Bj}**, we need to include **M^2x^2/Q^2** corrections to the perturbative theory. Often characterized as "1 / **Q^2** effects"

- **Target Mass Correction** – **kinematic corrections** due to non-negligible mass of targets. Applied to the theory!
  - TMCs were calculated by Nachtmann yielding the "Nachtmann Variable". This is only a first (but significant) step toward the full TMC expression:

$$\xi = \frac{2x}{(1+\sqrt{1+4m_N^2x^2/Q^2})}$$

when $M^2/Q^2 \to 0$, TMCs vanish, $\xi \to x$!

- **Higher Twist** – **Dynamic corrections** to perturbative DIS processes for non-perturbative multiquark/parton interactions (parton-parton correlations) and are mainly extracted experimentally! **NO systematic theoretical approach!**
  - HT effects are extracted experimentally by fitting data to a pQCD + HT:

$$F_2(x, Q^2) = F_2^{pQCD}(x, Q^2)\,[\,1 + C_{HT}(x) / Q^2\,]$$

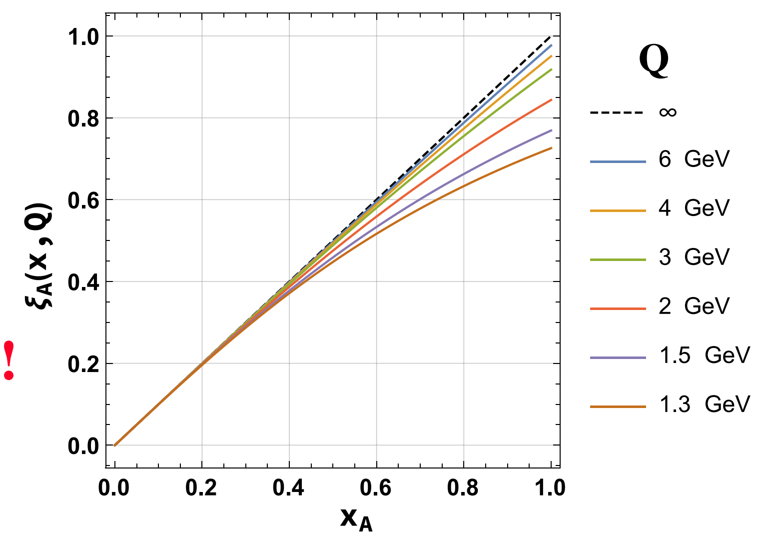
Target Mass Corrections

- Start: familiar $x = Q^2 / 2 M \upsilon$ (target rest frame)
  - this is fraction of the target momentum carried by the interacting parton – right?

Well, only for $Q^2 \to \infty$ limit!

- At finite Q^2, the effects of the target (and quark) masses modify the identification of x with the momentum fraction.
  - The parton momentum fraction (for massless quarks) is then the Nachtmann variable ξ .

$$\xi = \frac{2x}{1+\sqrt{1+4x^2M^2/Q^2}}$$



- **More than a few theorists think the Nachtmann variable, not x, is the natural scaling variable when M/Q is not close to zero!**

- To study TMC:

For nucleons see I. Schienbein et al . 0709.1775 [hep-ph] (2007)

TMC for nucleons and **nuclei- recent nCTEQ publication**

2301.07715 [hep-ph]] (2023)

IFJPAN-IV-2022-18, SMU-HEP-22-12, MS-TP-22-49, ANL-180568

January 20, 2023

arXiv:2301.07715v1 [hep-ph] 18 Jan 2023

Target mass corrections in lepton-nucleus DIS: theory and applications to nuclear PDFs

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Abstract: Motivated by the wide range of kinematics covered by current and planned deep-inelastic scattering (DIS) facilities, we revisit the formalism, practical implementation, and numerical impact of target mass corrections (TMCs) for DIS on unpolarized nuclear targets. An important aspect is that we only use nuclear and later partonic degrees of freedom, carefully avoiding a picture of the nucleus in terms of nucleons. After establishing that formulae used for individual nucleon targets (p, n), derived in the Operator Product Expansion (OPE) formalism, are indeed applicable to nuclear targets, we rewrite expressions for nuclear TMCs in terms of re-scaled (or averaged) kinematic variables. As a consequence, we find a representation for nuclear TMCs that is approximately independent of the nuclear target. We go on to construct a single-parameter fit for all nuclear targets that is in good numerical agreement with full computations of TMCs. We discuss in detail qualitative and quantitative differences between nuclear TMCs built in the OPE and the parton model formalisms, as well as give numerical predictions for current and future facilities.

Keywords: DIS, Structure Functions, Target Mass Corrections, OPE, nuclear PDFs

Contents

- Brief outline of one type of derivation using two moments of structure functions, the Cornwall-Norton and Nachtmann moments in backup.
  - Nachtmann moments **already take into account finite M^2/Q^2 corrections**

Example: Full TMC for Structure Functions

- After considerable applied theory/math the "Master Formula" for the target mass corrected structure functions is given by the TMC expansion:

$$\overbrace{F_j^{\rm TMC}(x,Q^2)}^{\textbf{TMC}} = \sum_{i=1}^{5} \underbrace{A_j^i \overbrace{F_i^{(0)}(\xi,Q^2)}^{\textbf{No-TMC}}}_{\textbf{Leading-TMC}} + \underbrace{B_j^i h_i(\xi,Q^2)}_{\textbf{h-term}} + \underbrace{C_j g_2(\xi,Q^2)}_{\textbf{g-term}}, \quad j = 1-5$$

  - **F_i^0** in limit M/Q → 0 = **F_i^0 (x, Q^2) no TMC.**

- The fully TMC corrected structure functions are then

$$F_1^{\rm TMC}(x,Q^2) = \frac{x}{\xi r} F_1^{(0)}(\xi) + \frac{M^2x^2}{Q^2r^2} h_2(\xi) + \frac{2M^4x^3}{Q^4r^3} g_2(\xi) \ ,$$

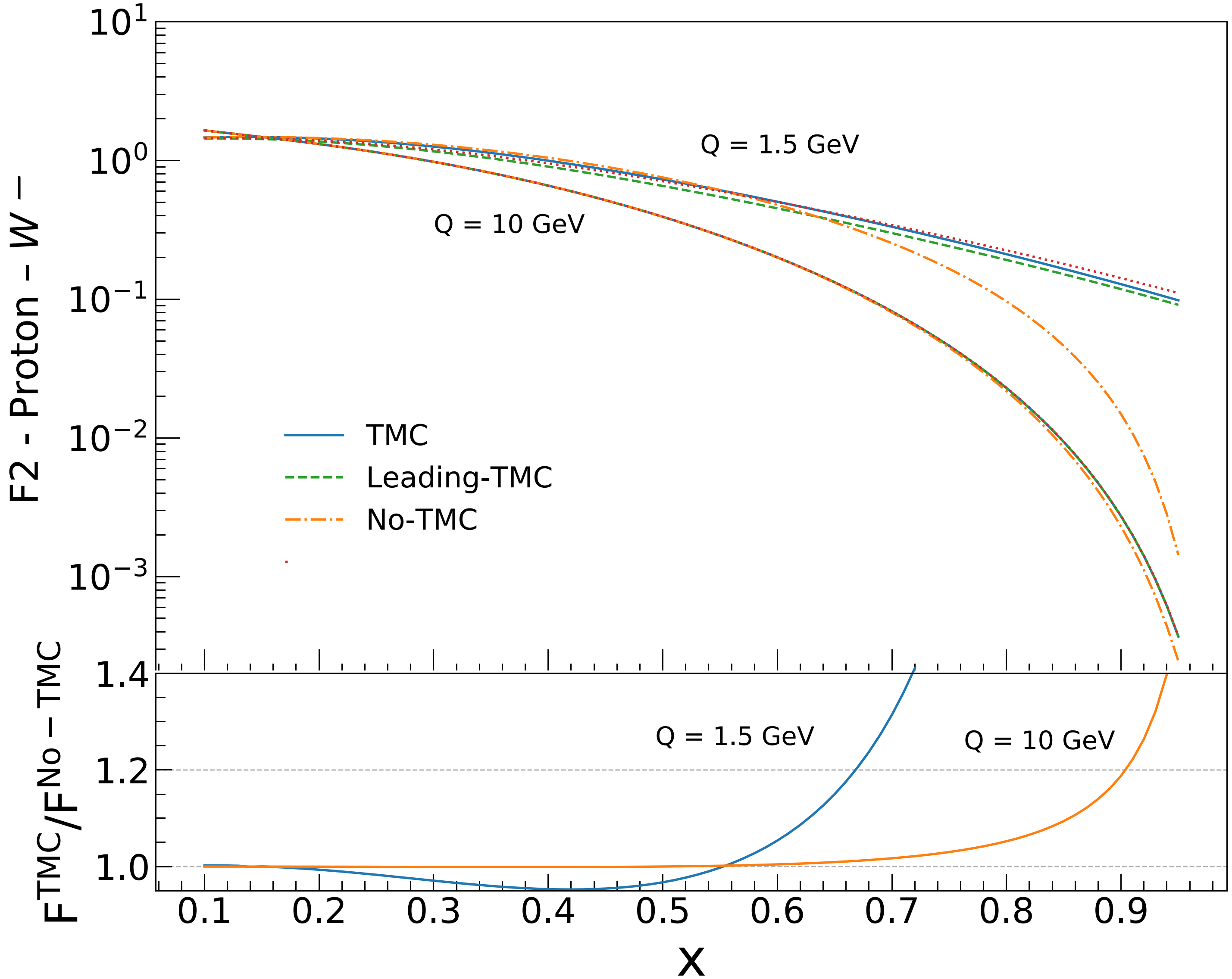
$$F_2^{\rm TMC}(x,Q^2) = \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi) + \frac{6M^2x^3}{Q^2r^4} h_2(\xi) + \frac{12M^4x^4}{Q^4r^5} g_2(\xi)$$

$$r = \sqrt{1+\frac{4x^2M^2}{Q^2}} \equiv \sqrt{1+\frac{Q^2}{\nu^2}}$$

Leading Factor ≈10% | **Scaling Variable ≈ x10!** | **h2 term ≈ 5%** | **g2 term ≈ 1%**

  - Acceptable, much less complicated, approximations in Backup

Size of the TMC



Higher Twist

- The concept was introduced in the early 70's when it was noticed that it was no longer the dimension alone determining the importance of an operator, but rather the difference between the dimension and spin. This became the "twist" on an operator, $\tau = d - s$.

- Today, the name "twist" is used more broadly as a $1/Q^n$ series including the leading term (twist 2) the standard QCD expression.

$$F_{2,T,3}(x,Q^2) = F_{2,T,3}^{\tau=2}(x,Q^2) + \frac{H_{2,T,3}^{\tau=4}(x)}{Q^2} + \frac{H_{2,T,3}^{\tau=6}(x)}{Q^4} + \dots$$

- Note that the leading twist 2 term is also expanded into a series of LO, NLO, NNLO…. perturbative corrections to the t = 2 term,

- DIS process at high Q, the hard interaction time (1/Q) is small compared to a soft interaction time ($1/\Lambda_{QCD}$) - struck quark has "NO TIME" to communicate with the rest of the hadron and is independent of the soft process.

- Higher twist corrections are those where the the struck quark CAN communicate with the hadron remanent (at the expense of a 1/Q factor).

- **HT contributions do not have any simple partonic interpretation.** They are assumed to be generated by parton transverse momentum and multiparton correlation functions, (Insights on quark-gluon correlations and quantum interference effects in hadrons).

Higher Twist

- There is a dearth of theoretically systematic approaches to describing HTs that have been implemented up to now. There are few models trying to answer what is H(x).

$$F_{2,T,3}(x,Q^2) = F_{2,T,3}^{\tau=2}(x,Q^2) + \frac{H_{2,T,3}^{\tau=4}(x)}{Q^2} + \frac{H_{2,T,3}^{\tau=6}(x)}{Q^4} + ...$$

- The most natural choice to maintain a partonic picture is probably a multi-partonic basis with extra gluon fields (or quark-antiquark pairs).

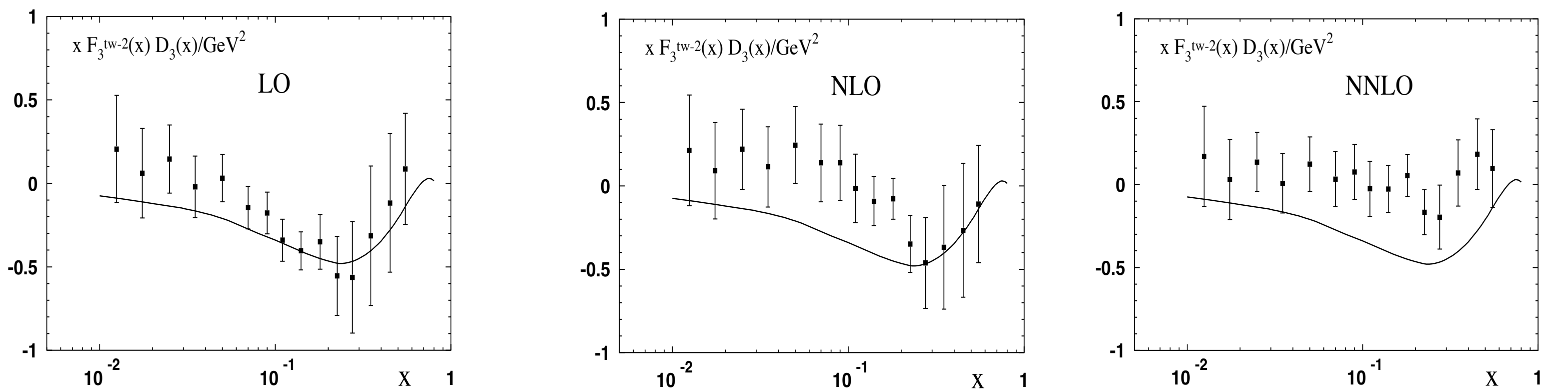
- There are many, many excellent experimental analyses aimed at extracting these higher twist contributions to deep inelastic scattering that generally fit measured structure function data to the form:

$$F_i(x,Q^2) = F_i^{(LT)}(x,Q^2) + \frac{1}{Q^2}h_i(x) = F_i^{(LT)}(x,Q^2)\left(1 + \frac{C_i(x)}{Q^2}\right)$$

- A Higher Twist model occurring in the literature is the Renormalon Model. Renormalons are not real physical things and don't correspond to any physical state. They are simply a construct that allows a more theoretical approach to modeling higher twist and an attempt to understand the H(x):

Experimental Extraction of Higher Twist terms

- Here is a renormalon analysis by Beneke and Braun (hep-ph 0010208) of an CCFR xF_3 HT extraction that serves to show a success of the renormalon model.

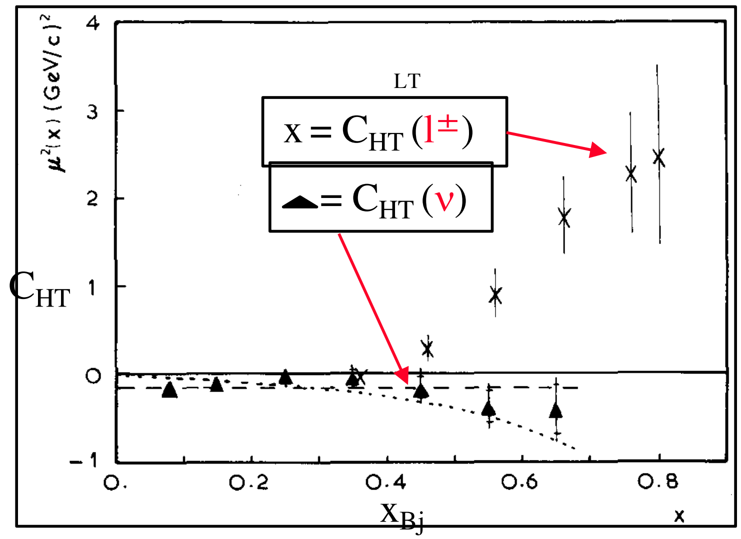


- **Many more recent and accurate experimental extraction of HT terms exist.** Chose this analysis that has the renormalon correction for the $1/Q^2$ (twist 4) term!
- Another important observation is that as the perturbative correction to the leading twist term increases (LO to NLO to NNLO…) the higher twist contribution is absorbed into the correction!
- Note the mainly negative HT term from neutrino scattering for LO x > 0.1

Higher Twist of ν – A compared to e/μ – A

Perhaps HT for ν - A might **NOT** be the same as e/μ -A

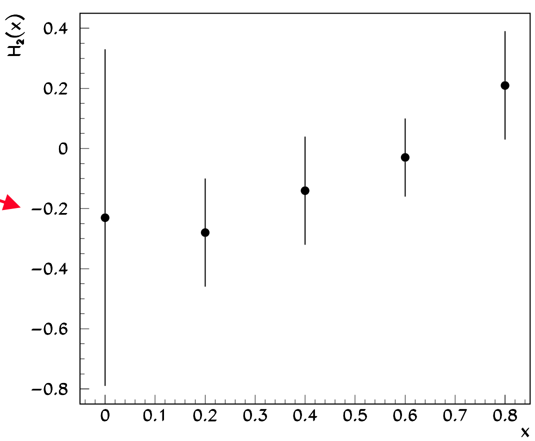
- Gargamelle (CF_3Br) & BEBC (Ne/H) SPS experiments, LO QCD & TMC applied:



- More current: Alekhin and Kataev – HT from CCFR F_2 and xF_3

- That is C_{HT} in neutrino scattering

smaller & mostly negative



Bringing TMC and HT together in a PDF Analysis!

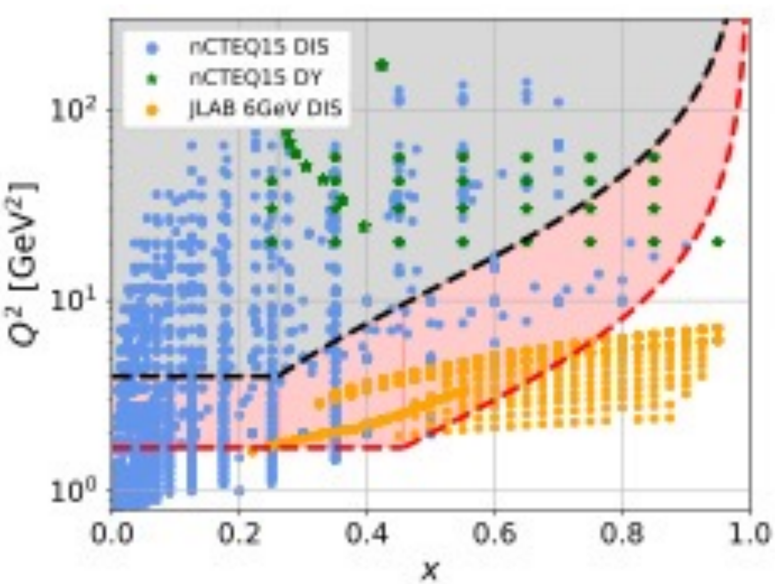
Extrapolating from pQCD to non-pQCD

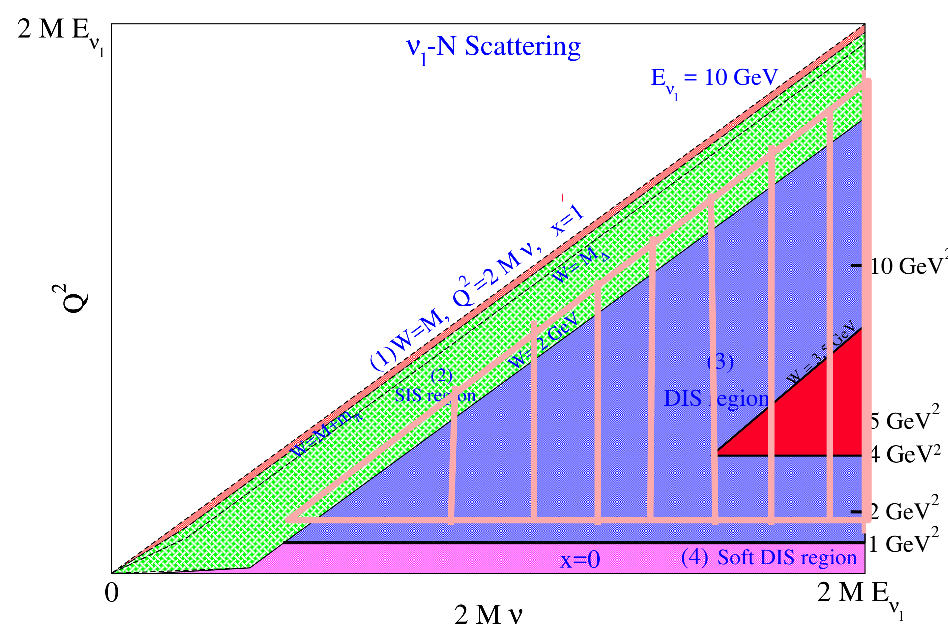
- **Recognize first** the **Bodek-Yang model** keeps **Duality** in mind by extending GRV LO DIS PDFs down in Q^2 and W while including TMC and HT effects!
- **BASED ON ELECTROPRODUCTION EXPERIMENTAL RESULTS!**

- A more rigorous extrapolation to the SIS, non-pQCD transition region is the first **nCTEQ** global fit of **e/μ nuclear ratios** into the **SIS transition region:** e-Print: 2012.11566 [hep-ph]:

- Adding higher–x, lower Q JLab **(eA)** nuclear **ratio** measurements to perform a global fit:

$W > 1.7$ GeV, $Q^2 > 1.69$ GeV^2

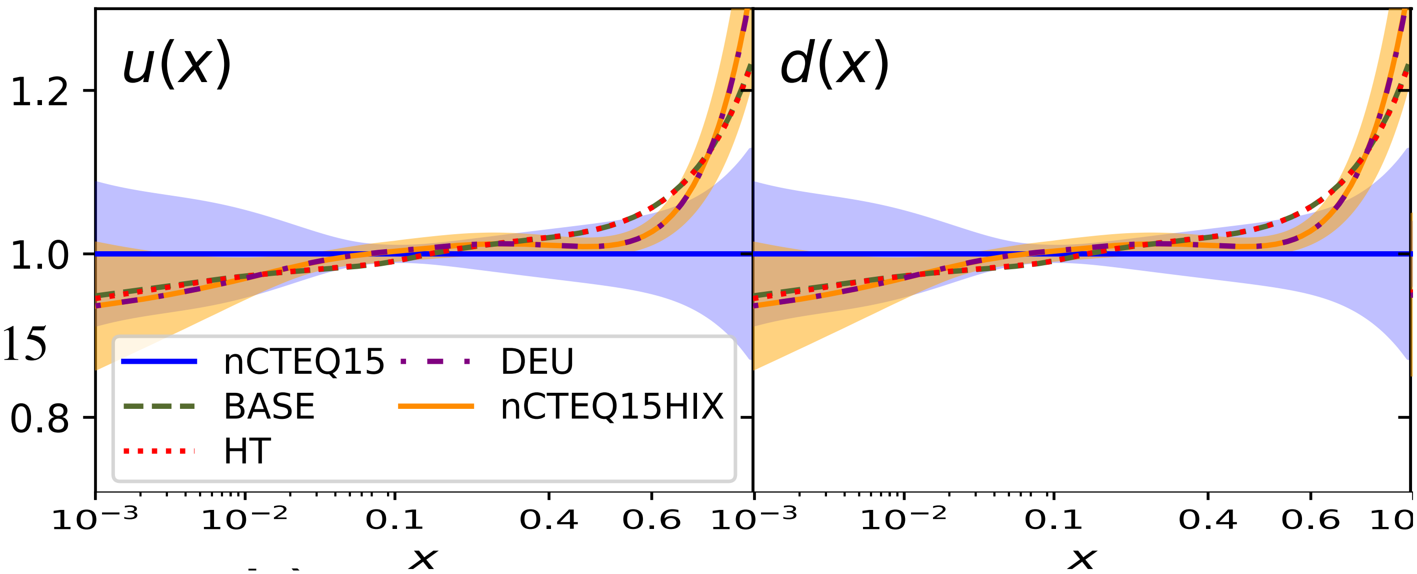




Effective Results for e/μ – A scattering PDFs
pQCD → non-pQCD for electroproduction

nCTEQ15HIX (https://ncteq.hepforge.org)

Nuclear PDFs for C at $Q^2 = 4$ GeV^2 normalized to nCTEQ15



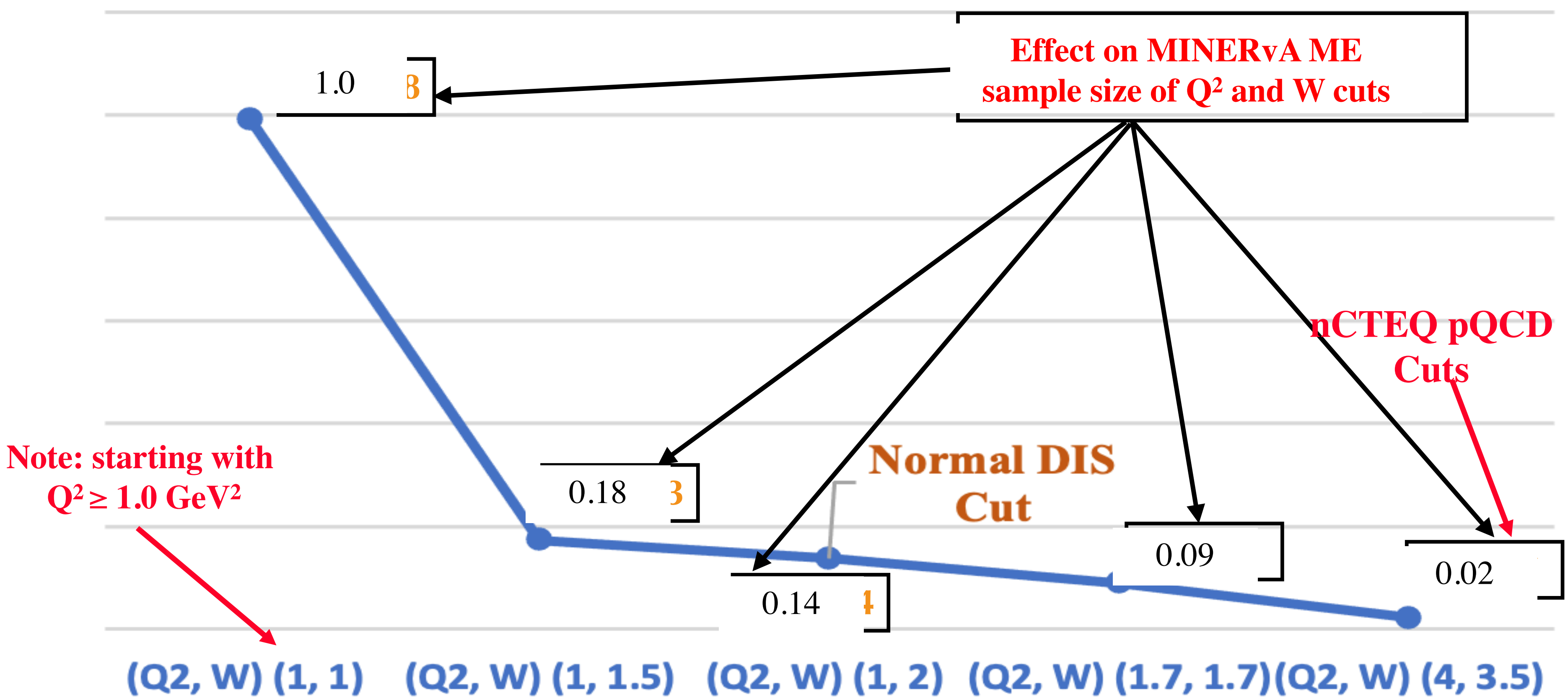
- Comparing the nCTEQ15 (safe DIS) and nCTEQ15hix (lower W and Q) fits to the same expanded data set shows an improvement of 15% in χ^2/N_{dof} for HIX fit.
- **3% of the 15% improvement is coming from the inclusion of HT term for electroproduction!**
- **Need to do the same thing with neutrino data and push down to** $Q^2 = 1.0$ GeV^2

Summary
Understanding the SIS nPQCD Transition Region

- **Kinematic Target Mass Corrections** are quite well understood
  - applied directly to the theory/generators in the relevant low - Q^2 regions.
- **Dynamic Higher Twist Effects**
  - have only a few models and HT are mainly extracted experimentally.
  - **Need better understanding of HT in neutrino scattering**
  - As the perturbative correction to the leading twist term increases (LO to NLO to NNLO…) the higher twist eventually seems to be absorbed in the correction!
  - For electroproduction as long as TMC is applied, the contribution of HT for $x < 0.7$ and $Q^2 > 1.7$ GeV^2 is minimal!
  - **Better understanding of HT in neutrino scattering with the help of completed MINERvA SIS and DIS analyses would be welcome!**
- **Extrapolating from Resonance to DIS is also an important direction!**
  - **Theoretically - work of Natalie Jachowicz et al and Minoo and …**
  - **Experimentally – MINERvA perhaps higher W single and multi-pion results.**

Backup

Effect of Safe DIS cuts $Q^2 > 4$ GeV^2, W> 3.5 GeV on MINERvA ME Sample



Only ≈ 2 % of MINERvA ME events with $Q^2 > 1$ GeV^2 are in the SAFE DIS region

Understanding / Predicting the SIS Region

Quark – Hadron Duality

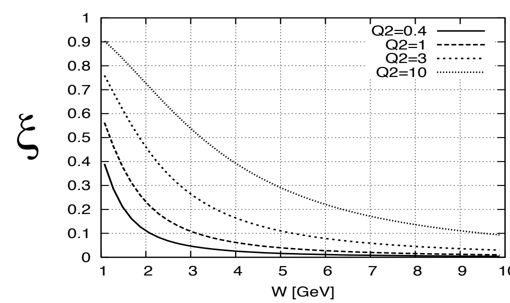
- **Quark–hadron duality** is a general feature of strongly interacting landscape
  - How does the physics (language) of quark/gluons from DIS meet the physics of nucleons/mesons (pions) of SIS → **quark-hadron duality**

ξ

- Ratio of the strength of the SIS to DIS region. Ideal Duality I = 1.0 .

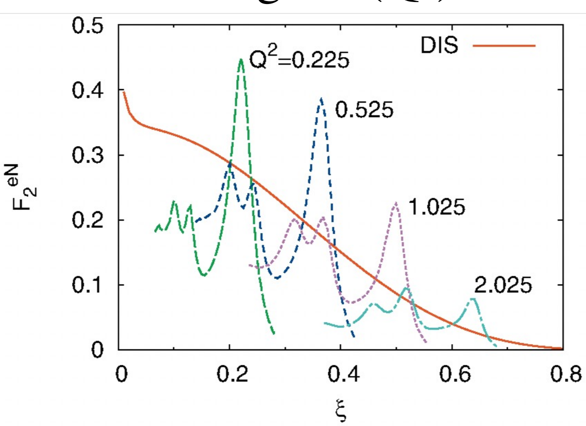
$$\mathcal{I}_{|}(Q^2, Q^2_{DIS}) = \frac{\int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{RES}(\xi, Q^2)}{\int_{\xi_{min}}^{\xi_{max}} d\xi F_j^{DIS}(\xi, Q^2_{DIS})}$$

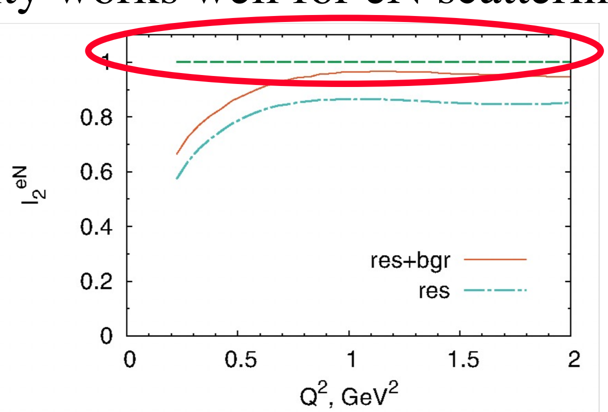
$$\xi = \frac{2x}{(1 + \sqrt{1 + 4m_N^2 x^2/Q^2})}$$



- $F_2^{eN}(\xi)$ for values of Q^2 indicated on spectra compared to LO DIS QCD fit at Q^2 = 10 GeV^2. Value of integral $I(Q^2)$. Duality works well for eN scattering!

e-Nucleon





Stress the importance of including the **non-resonant** pion production!

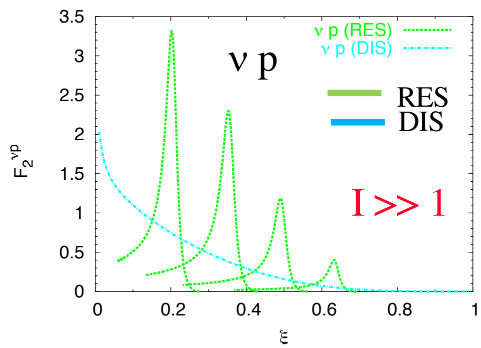
Neutrinos – NO HIGH STATISTIC NUCLEON DATA

must rely on models for ν-n, ν-p and ν-N scattering

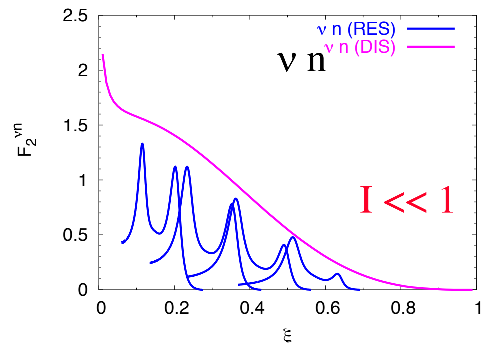
Resonance estimates from Lalakulich, Melnitchouk and Paschos for ν-n and ν-p scattering.

$F_2^{\nu p(res-3/2)} = 3F_2^{\nu n(res-3/2)}$

$F_2^{\nu p(res-1/2)} \equiv 0$



$F_2^{\nu n(res)}$: finite contributions from isospin 3/2 and -1/2 resonances



In general, for neutrinos the **resonance structure functions** for **proton are much larger than for neutrons** however **DIS structure functions the situation is opposite.**

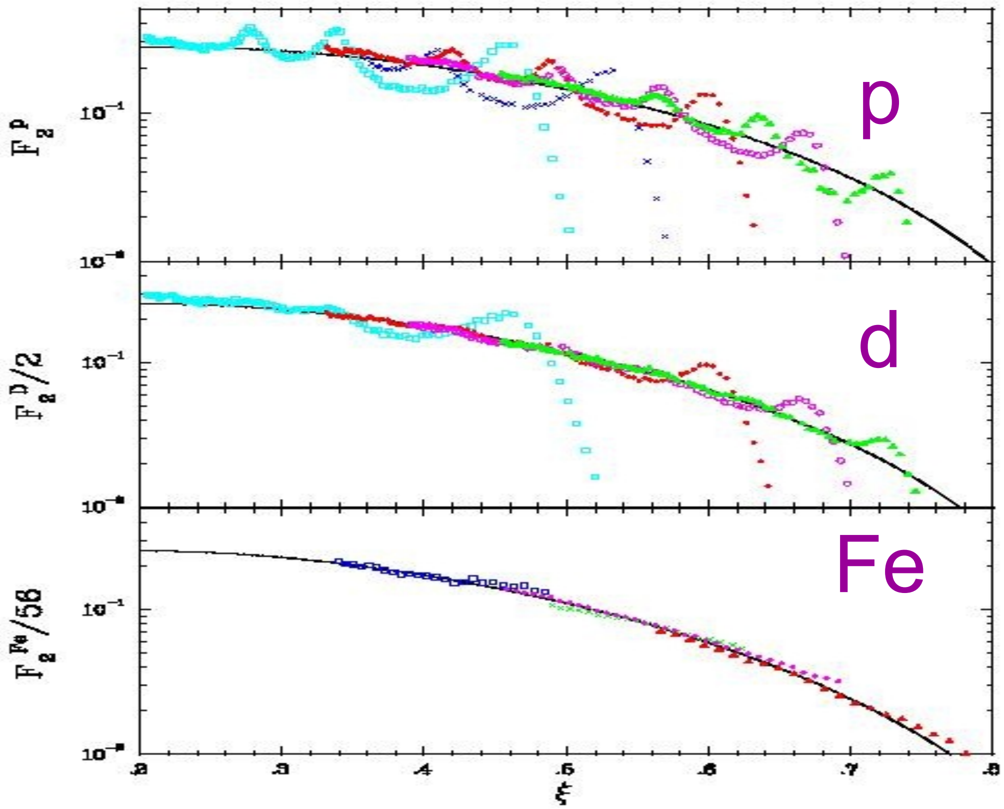
Strong suggestion here that **for neutrinos**:
duality holds for isoscalar nucleon ($F_2^{\nu p} + F_2^{\nu n}$)/2.

What does that imply for **duality for nuclei with large neutron excess??**

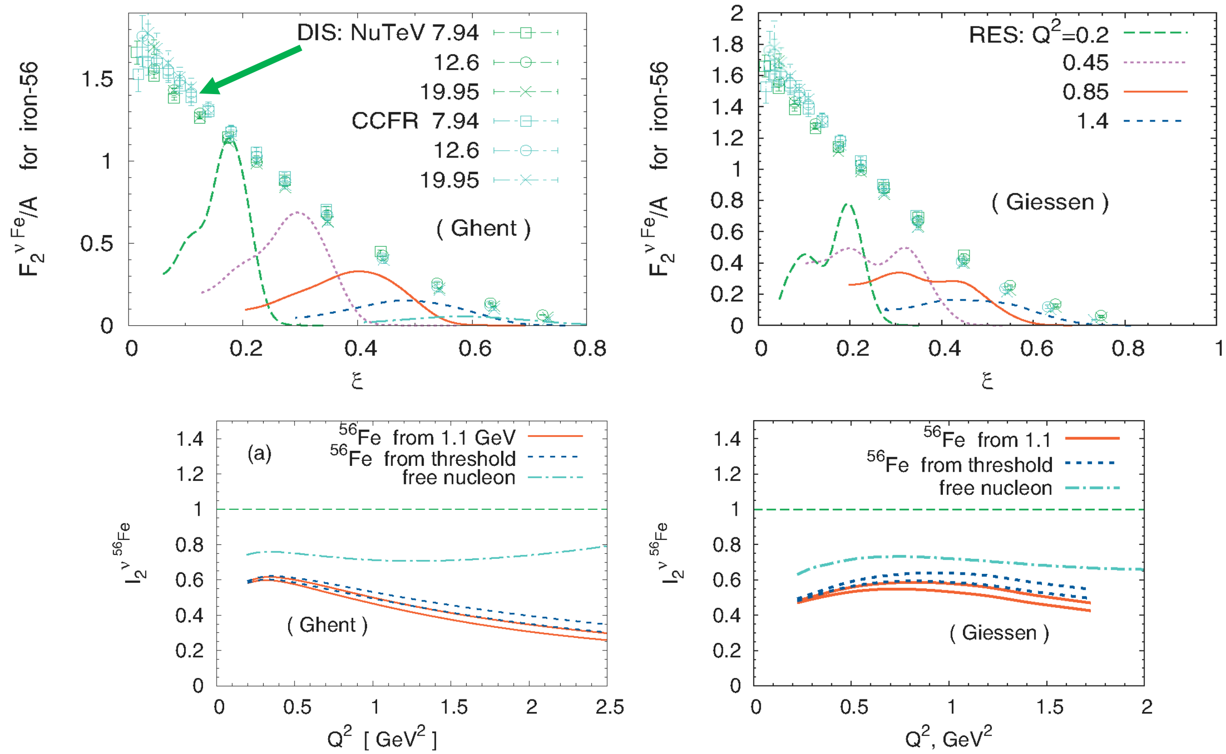
How duality should be applied with neutrinos is still an open question!

Now Nucleus not Nucleon Qualitative look at Q-H Duality: e-A results

- Now **e-nucleus** – individual resonances visible in e-P, somewhat less in e-D and mostly smeared out by e-Fe. Curved line is from MRST global **DIS fits** with **EMC effect** for Fe applied.



Even more uncertain for ν when talking of NUCLEI not NUCLEON- Is the problem for Fe the neutron excess and/or **models for Final State Interactions?**



- In general, for neutrinos the **resonance structure functions** for **proton are much larger than for neutrons** and in the case of **DIS structure functions the situation is opposite.**

- Although to some extent model dependent, a general tendency is that **DIS structure functions** are much larger than the **resonance contribution** at lower W.

- **How duality should be applied with neutrinos is still an open question**

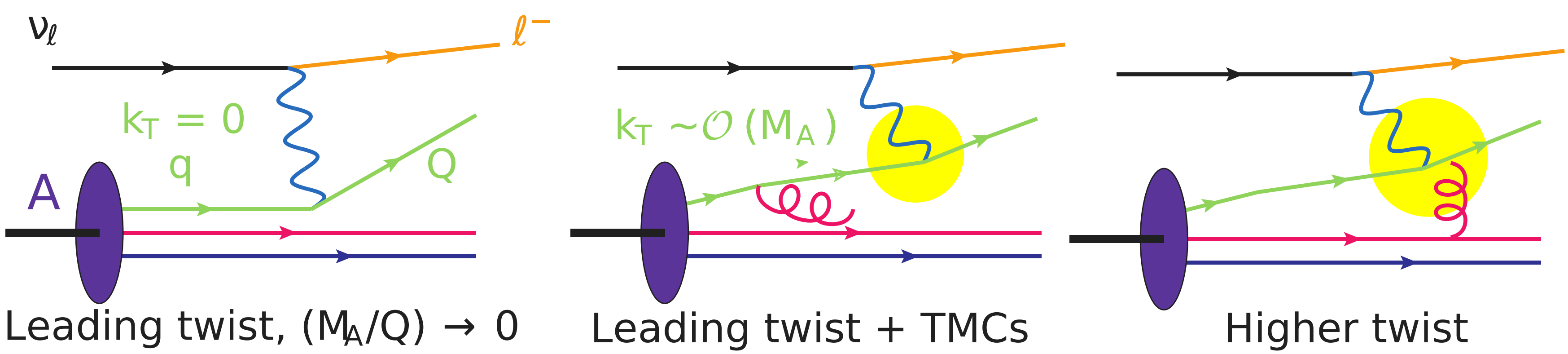
Summary: Quark-Hadron Duality for e-N/A and ν-N/A

- F_2 *ep en*: Qualitative and quantitative duality HOLDS in electron–nucleon scattering.
- F_2 ν*p* : In neutrino–nucleon scattering, duality seems to roughly holds for the **average** nucleon but NOT individually for neutron and proton.
- F_2 *eA* : Different story, looks good but quantitative check in e–A not as good as e–n/p
- ***F_2* νA : Not at all clear how duality works here, particularly in nuclei with an excess number of neutrons.**
- In general, for neutrinos, the resonance structure functions for **proton are much larger than for neutrons** and in the case of DIS structure functions **the situation is opposite.**
- Although to some extent model dependent, a general tendency is that DIS structure functions are **much larger** than the resonance contribution at lower W.
- **For neutrinos: not yet at all clear how duality should be applied!**

Physics of the Lower Q, W (SIS) non-Perturbative QCD Region

- The "Infinite Momentum Frame" or at least "SAFE Deep Inelastic Region" → perturbative QCD region.
  - We agree it certainly, does not describe the environment of our 1 - 10 GeV neutrino beams! How do we know we are there or at least getting close…?

- For smaller Q^2 and/or larger x_{Bj}, we need to include M^2x^2/Q^2 corrections to the perturbative theory. Often characterized as "1 / Q^2 effects"



KINEMATIC

DYNAMIC

TMC - Brief outline of one type of derivation

- The two standard moments of structure functions are the Cornwall-Norton and Nachtmann moments.
  - The Cornwall-Norton moments appropriate for the region $Q^2 \gg M^2$ of F2 are given by:

$$M_2^n(Q^2) \;=\; \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

  - The Nachtmann moments **already take into account finite M^2/Q^2 corrections:** given by:

$$\mu_2^n(Q^2) \;=\; \int_0^1 dx\; \frac{\xi^{n+1}}{x^3}\left[\frac{3+3(n+1)r+n(n+2)r^2}{(n+2)(n+3)}\right] F_2(x, Q^2)$$

- Relate Nachtmann and CN moments by expanding the moments in powers of $1/Q^2$:

$$M_2^n(Q^2) \;=\; \mu_2^n(Q^2) \;+\; \frac{n(n-1)}{n+2}\frac{M^2}{Q^2}\mu_2^{n+2}(Q^2) \;+\; \frac{n(n^2-1)(n+2)}{2(n+3)(n+4)}\frac{M^4}{Q^4}\mu_2^{n+4}(Q^2) + \frac{n(n^2-1)(n+2)(n+3)}{6(n+5)(n+6)}\frac{M^6}{Q^6}\mu_2^{n+6} \;+\; \cdots$$

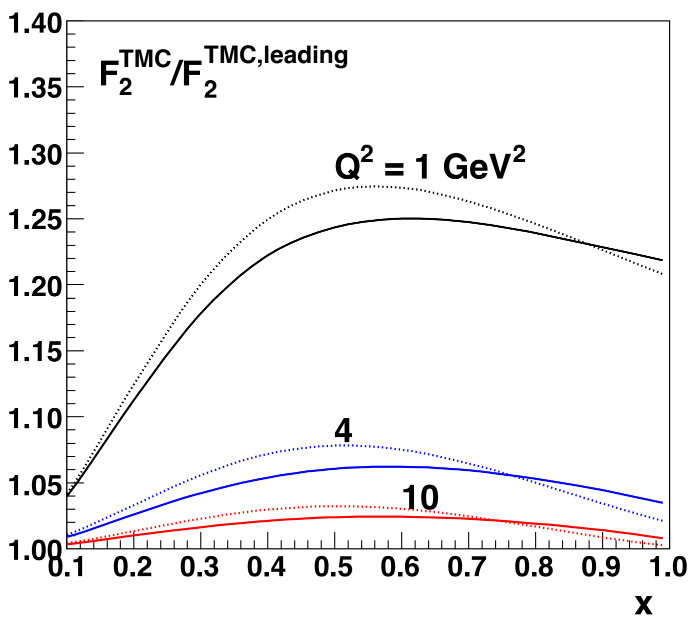
- Since Nachtmann protect the moments of the structure functions from target mass effects the TM effects can be identified directly with the moments of the quark distributions.

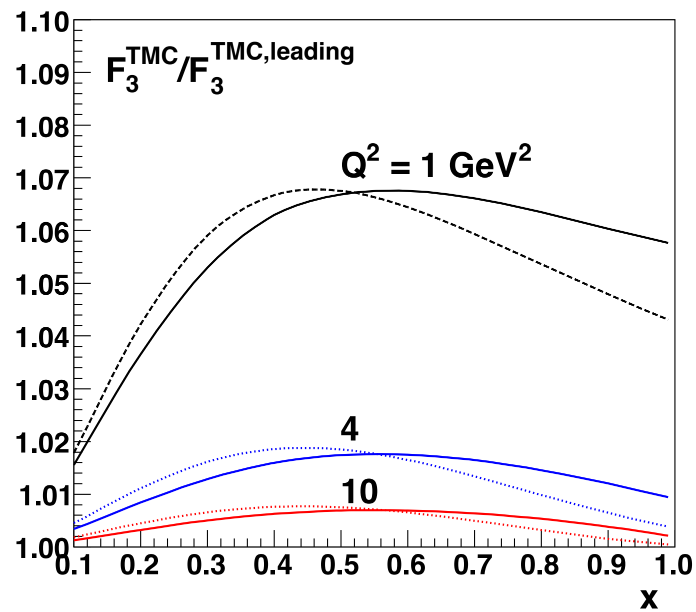
$$F_2(x, Q^2) \;=\; \frac{\xi^2(1-a^2\xi^2)}{(1+a^2\xi^2)^3}F(\xi) \;+\; 6a^2\frac{\xi^3(1-a^2\xi^2)}{(1+a^2\xi^2)^4}H(\xi) \;+\; 12a^4\frac{\xi^4(1-a^2\xi^2)}{(1+a^2\xi^2)^5}G(\xi)$$

Approximations to the full TMC

- Rather than the full expressions for the structure functions good approximations yield quite acceptable results that can easily be placed on-line:

$$F_2^{\rm TMC}(x,Q^2) \simeq \frac{x^2}{\xi^2 r^3} F_2^{(0)}(\xi)\left[1+\frac{6\mu x\xi}{r}(1-\xi)^2\right] \qquad F_3^{\rm TMC}(x,Q^2) \simeq \frac{x}{\xi r^2} F_3^{(0)}(\xi)\left[1-\frac{\mu x\xi}{r}(1-\xi)\ln\xi\right]$$





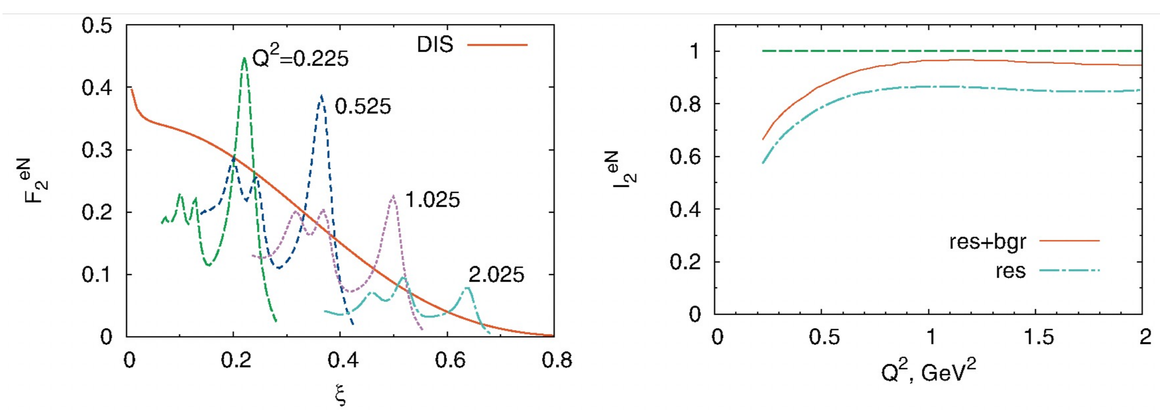
- Ratio of the fully target mass corrected $F^{TMC}(x,Q^2)$ structure functions to the leading contributions at $Q^2 = 1, 4$ and 10 GeV^2 . The solid curves represent the exact results, while the dotted curves use the approximate formulas.

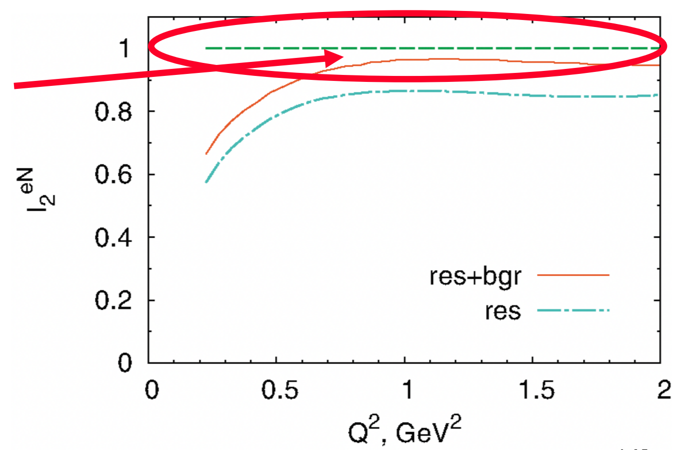
Duality and Higher Twist

- Does the fact that duality holds so well for e N resonance scattering compared to LO, **leading twist** DIS results suggest there is little room for higher twist contributions for $Q^2 > 1$ GeV^2 and $x < 0.65$??

- **Multiple studies of this available in the literature and all seem to agree with the above statement.** A. Fantoni, N. Bianchi, and S. Liuti. Quark-hadron duality and higher twist contributions in structure functions. *AIP Conf. Proc.*, 747(1):126–129, 2005.

- Using Giessen fit to e-N scattering – $F_2^{eN}(\xi)$ for values of Q^2 indicated on spectra compared to LO DIS QCD fit at $Q^2 = 10$ GeV^2





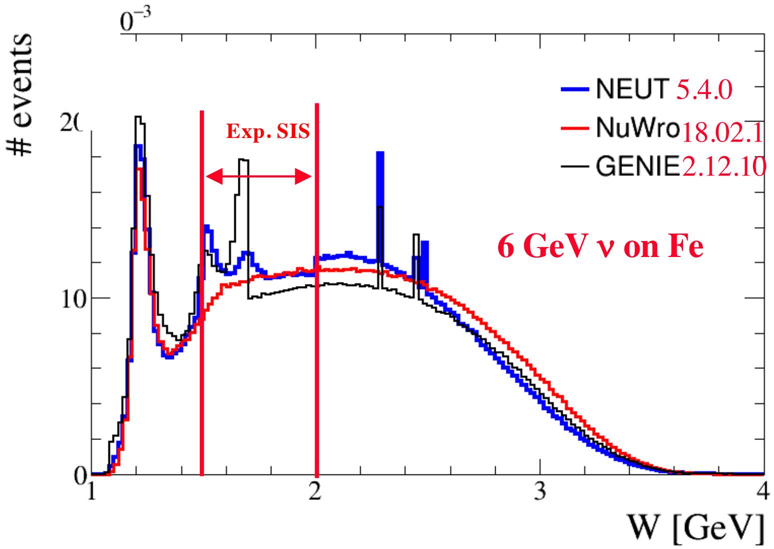
To perform the extrapolated fits for Neutrinos we need **MINERvA (ME) SIS and DIS Analyses**

- **SIS** – $1.5 < W < 2.0$ GeV – First Inclusive Cross sections in this restricted W region $d\sigma/dQ^2$, $d\sigma/dp_\mu^t$ and $d\sigma/dp_\mu^z$ for both ν and $\bar{\nu}$ completed, $d\sigma/dx$ and $d\sigma/dx$ underway.

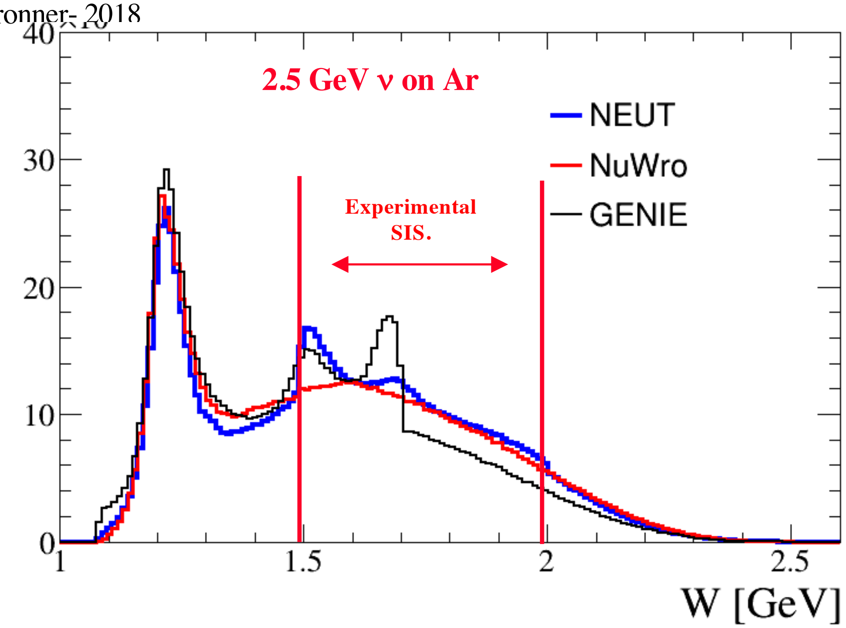
- **DIS** – ($W>2$ GeV and $Q^2 > 1$ GeV^2): $d\sigma/dx$ and $d\sigma/dE_\nu$ in nuclear targets (C, CH, Fe AND Pb) for nuclear ratios with both ν and $\bar{\nu}$

- **DIS** – ($W>2$ GeV and $Q^2 > 1$ GeV^2): $d\sigma/dxdy$ for ν and $\bar{\nu}$. These expressions can be included directly in (nCTEQ) global fits (reduced Q^2 and W cuts) to study **higher-twist** with neutrinos.

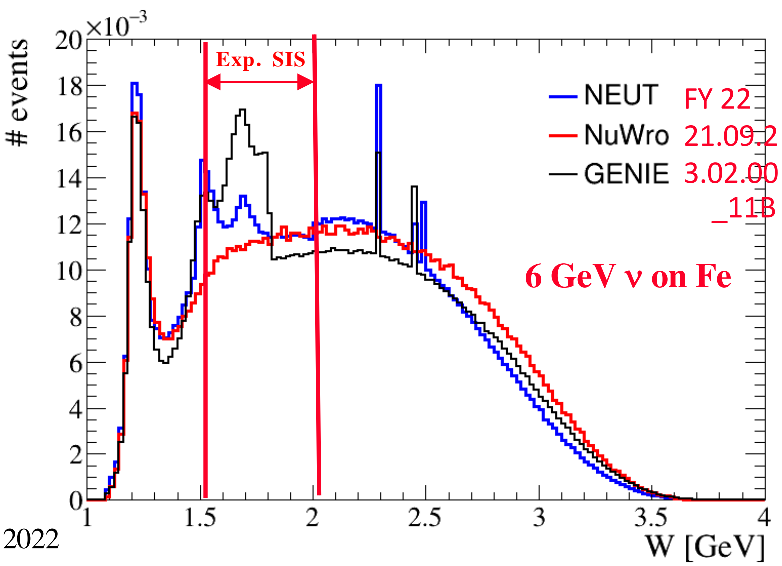
The SIS and Overall Landscape vs W



C. Bronner- 2018



2022 version



C. Bronner- 2022

Evolution from 2018: significant changes only for GENIE resonances & Res-DIS transition

Obvious mix of resonant and SIS meson production in experimental SIS W range.

Significance for DUNE - 45 % of ν_μ CC events have W > 1.5 GeV.