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Neutrino-nucleon interactions

Alexis Nikolakopoulos NuINT school, Sao Paolo, Brazil 11 April 2024

Neutrino-nucleon interactions

A very birds-eye view

Some examples based on introductory works: E.A. Paschos, Electroweak theory M. Thomson, Modern particle physics

Recent research

And lectures from N. Rocco at the last NuSTEC school at Fermilab

For more in-depth overviews:

References in this presentation & NuSTEC White Paper : arxiv.org/abs/1706.03621 & Snowmass 2021 white paper : arxiv.org/abs/2203.09030





Interaction cross section

$$N_{\text{interaction}} = \text{flux} \int_{\Delta\Omega} \mathrm{d}\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}$$



[Taylor, ISBN:0486450139]

 $2 \rightarrow$ N-2 exclusive scattering

$$p_1 + p_2 \rightarrow p_3 + p_4 + \dots + p_n$$

flux Phase space Energy-momentum conservation $d\sigma = \frac{1}{F} \prod_{i}^{n} \frac{d\mathbf{p}_{i}}{(2\pi)^{3} E_{i}} \delta^{4}(p_{1} + p_{2} - \sum_{i}^{n} p_{i}) (2\pi)^{4} |\mathcal{M}|^{2}$ $F^{2} = 16 \left((p_{1} \cdot p_{2})^{2} - m_{1}^{2} m_{2}^{2} \right)$



Interaction cross section: degrees of freedom

 $2 \rightarrow$ N-2 exclusive scattering

 $p_1 + p_2 \rightarrow p_3 + p_4 + \dots + p_n$

$$N_{dof} = 4N - N$$
 (mass-shell conditions)

- 3 (choice of reference system)
- 3 (choice of coordinates)
- 4 (Energy-momentum conservation)



Matrix element for lepton-hadron scattering

 $\overset{\mathbf{k}}{\searrow} j_l^{\mu}(k,k') \overset{\mathbf{k}'}{\checkmark}$ Feynman rules for one-boson exchange: $\mathcal{M} = j_l^{\mu}(k, k') S_{\mu\nu}(q = k - k') \mathcal{J}_H^{\nu}(q, \ldots)$ $S_{\mu\nu}^{W/Z} = -i \frac{g_{\mu\nu} - q_{\mu}q_{\nu}/M_{W/Z}^2}{q^2 - M_{W/Z}^2} \approx -i \frac{g_{\mu\nu}}{M_{W/Z}^2}$ $S^{\gamma}_{\mu\nu} = \frac{-\imath g_{\mu\nu}}{a^2},$

$$\overline{\sum}_{\lambda_l,\lambda_h} |\mathcal{M}|^2 = F_X^2 \overline{\sum} |j_l^{\mu} \mathcal{J}_{\mu}|^2 = F_X^2 \overline{\sum}_{\lambda_l} [j_l^{\mu}]^* j_l^{\nu} \sum_{\lambda_h} [\mathcal{J}_{\mu}]^* \mathcal{J}_{\nu} \equiv F_X^2 L^{\mu\nu} H_{\mu\nu}$$

Matrix element for lepton-hadron scattering

For one-boson exchange: $j_l^\mu = \overline{u}(k')\Gamma^\mu u(k)$ $\Gamma^\mu_\gamma = e\gamma^\mu$

$$\Gamma_Z^{\mu} = \frac{g}{4\cos\theta_W} \gamma^{\mu} \left(1 - \gamma^5\right)$$

$$\Gamma_W^{\mu} = \frac{g}{2\sqrt{2}} \gamma^{\mu} \left(1 - \gamma^5\right)$$

If you calculate the lepton tensor with Γ_w once $\rightarrow\,$ get all the other cases for free

Matrix element for lepton-hadron scattering

For one-boson exchange:

$$\sum_{\lambda_l,\lambda_h} |\mathcal{M}|^2 = F_X^2 \ L^{\mu\nu} H_{\mu\nu}$$
$$\mathcal{L}^{\mu\nu}(s,s') = \left[\overline{u}(k',s')\gamma^{\mu}(1\pm\gamma^5)u(k,s)\right] \left[\overline{u}(k',s')\gamma^{\nu}(1\pm\gamma^5)u(k,s)\right]^{\dagger}$$
$$\sum_{s,s'} \mathcal{L}^{\mu\nu}_{s,s'} \equiv L^{\mu\nu}(k,k',h) = 4 \left[k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} + g^{\mu\nu}(mm'-k\cdot k')\mp ih\epsilon^{\mu\nu\alpha\beta}k_{\alpha}k'_{\beta}\right]$$

General expression for the (inclusive) hadron tensor

General expression for the inclusive hadron tensor

$$L_{\mu\nu}(h = -1)H^{\mu\nu} = 2E_i E_f \left[2W_1 \sin^2 \frac{\chi}{2} + W_2 \cos^2 \frac{\chi}{2} - h \frac{W_3}{M_N} \left((E_i + E_f) \sin^2 \frac{\chi}{2} - \frac{m_f^2}{2E_f} \right) + \frac{W_4}{M_N^2} m_f^2 \sin^2 \frac{\chi}{2} + \frac{W_5}{M_N} \frac{m_f^2}{E_f} \right]$$

Completely general and well-defined, but structure functions W_i depend on interaction

 \rightarrow Constraints come at level of the current

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$$\cos \chi = \frac{p_i p_f}{E_i E_f} \cos \theta_f$$
 W (q², p_µq^µ)

Elastic lepton-nucleon scattering: amplitude level

quarks nucleons

$$\overline{u}(p')\gamma^{\mu}(1-\gamma^{5})u(p) \rightarrow \overline{u}(p')\Gamma_{V}^{\mu} - \Gamma_{A}^{\mu}u(p)$$
Structure of vector and axial current

$$\Gamma_V^{\mu} = F_1(Q^2)\gamma^{\mu} - \frac{F_2(Q^2)}{2m_N}(\gamma^{\mu}Q - Q\gamma^{\mu})$$
$$\Gamma_A^{\mu} = F_A(Q^2)\gamma^{\mu}\gamma^5 + \frac{F_P(Q^2)}{2m_N}Q^{\mu}\gamma^5$$

Elastic lepton-nucleon scattering

$$\begin{aligned} & \text{quarks} & \text{nucleons} \\ & \overline{u}(p')\gamma^{\mu}(1-\gamma^{5})u(p) \rightarrow \overline{u}(p')\Gamma_{V}^{\mu} - \Gamma_{A}^{\mu}u(p) \\ & \text{Structure of vector and axial current} \\ & \Gamma_{V}^{\mu} = F_{1}(Q^{2})\gamma^{\mu} - \frac{F_{2}(Q^{2})}{2m_{N}}\left(\gamma^{\mu}\mathcal{Q} - \mathcal{Q}\gamma^{\mu}\right) \\ & \Gamma_{A}^{\mu} = F_{A}(Q^{2})\gamma^{\mu}\gamma^{5} + \frac{F_{P}(Q^{2})}{2m_{N}}Q^{\mu}\gamma^{5} \end{aligned}$$

$$W_{1} = -\tau (F_{1} + F_{2})^{2} - F_{A}^{2}(\tau + 1),$$

$$W_{2} = F_{1}^{2} + \tau F_{2}^{2} + G_{A}^{2},$$

$$W_{3} = G_{A} (F_{1} + F_{2}),$$

$$W_{4} = \frac{\tau - 1}{4} F_{2}^{2} - \frac{F_{1}F_{2}}{2} - F_{P}G_{A}M_{N} + \tau M_{N}^{2}F_{P}^{2},$$

$$W_{5} = \frac{W_{2}}{2},$$

$$W_{5} = \frac{W_{2}}{2},$$

$$W_{5} = F_{1}^{2} + \tau F_{2}^{2}$$

Relativistic electron scattering with Dirac fermion

$$\begin{aligned} \mathcal{J}^{\mu} &= e\overline{u}(k',s')\gamma^{\mu}u(k,s) \\ &= \text{same as the lepton-current calculated previously} \\ H^{\mu\nu}(k,k') &= e^{2}4\left[k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} + g^{\mu\nu}(mm'-k\cdot k')\right] \\ &\rightarrow \text{W1} = Q^{2}/2 \text{ , } \text{W}_{2} = 2 \text{ M}_{\text{p}}^{2} \\ \text{The differential cross section is:} \\ &\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{e'}} = \left(\frac{\alpha}{Q^{2}}\right)^{2} \left(\frac{E'}{E}\right)^{2} \left[\cos^{2}\frac{\theta}{2} + \frac{Q^{2}}{2m_{p}^{2}}\sin^{2}\frac{\theta}{2}\right] \text{ f } \text{ f } \end{aligned}$$

Relativistic electron scattering with Dirac fermion

$$\frac{d\sigma}{d\Omega_{e'}} = \left(\frac{\alpha}{Q^2}\right)^2 \left(\frac{E'}{E}\right)^2 \left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$
Compare to the Mott cross section
= scattering off a spinless point-charge
$$\frac{d\sigma_{Mott}}{d\Omega} = \frac{\alpha^2}{(Q^2)^2} \left(\frac{E'}{E}\right)^2 \cos^2\frac{\theta}{2}$$
Dirac particle includes magnetic

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moment gives sin² term

Electron scattering with nucleons

Spinless point charge:

$$\frac{\mathrm{d}\sigma_{Mott}}{\mathrm{d}\Omega} = \frac{\alpha^2}{\left(Q^2\right)^2} \left(\frac{E'}{E}\right)^2 \cos^2\frac{\theta}{2}$$

Point-like fermion with Dirac magnetic moment

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_{e'}} = \left(\frac{\alpha}{Q^2}\right)^2 \left(\frac{E'}{E}\right)^2 \left[\cos^2\frac{\theta}{2} + \frac{Q^2}{2m_p^2}\sin^2\frac{\theta}{2}\right]$$

Nucleon:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{Mott}}{\mathrm{d}\Omega} \left[G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right] \frac{1}{1+\tau}$$
$$\frac{1}{1+\tau} = 1 + 2(1+\tau) \tan^2 \frac{\theta}{2} \qquad \tau = \frac{Q^2}{4m_p^2}$$
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¹⁵
$$G_E = F_1 - \tau F_2, \quad G_M = F_1 + F_2$$

Nucleon form factors: electron scattering

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\mathrm{d}\sigma_{Mott}}{\mathrm{d}\Omega} \left[G_E^2(Q^2) + \frac{\tau}{\epsilon} G_M^2(Q^2) \right] \frac{1}{1+\tau}$$

Electric and Magnetic form factors of the nucleon

At small Q^2 : $G_E(Q^2) \sim G_E(q) \sim$ 'the Fourier-transform of the charge density' (not strictly true)

Charge and magnetic moment of the nucleon:

$$\begin{split} G^p_E(0) &= 1, \quad G^p_M(0) = \mu/\mu_B = 2.7928 & \longrightarrow \text{Anomalous } \mu\\ G^n_E(0) &= 0, \quad G^n_M(0) = \mu/\mu_B = -1.9130 \end{split}$$

Point-like fermion (no higher-order corrections)

$$G_E(0) = 1, \quad G_M(0) = \mu/\mu_B = 1$$

Nucleon form factors: measurement

Measure the cross section At different combinations of θ , E_i with Q^2 = constant

 $\rightarrow\,$ Can separate electric and magnetic form factors

Nucleon EM form factors

Early analysis found form-factors consistent with dipole-form $G_E^p(Q^2) = G_D(Q^2), \quad G_M^p(Q^2) = \mu^p G_D(Q^2), \quad G_M^n(Q^2) = \mu^n G_D(Q^2)$

 $G_D(Q^2) = (1 + Q^2 / M_V^2)^{-2}$

[Prog.Part.Nucl.Phys. 59 (2007) 694-764]

Nucleon EM form factors

Early analysis found form-factors consistent with dipole-form

Modern analyses: deviation from dipoles

Often used in generators:

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Kelly form [Phys. Rev. C 70, 068202]
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BBBA07 [Eur.Phys.J.C53:349-354,2008]

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Z-expansion [Phys. Lett B 777, 8-15]
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Modern measurements and analyses are of high precision:

[Prog.Part.Nucl.Phys. 59 (2007) 694-764]

[Rev. Mod. Phys. 94, 015002]

Uncertainty is not dominant for neutrino experiments

[Prog.Part.Nucl.Phys. 59 (2007) 694-764]

Nucleon form factors: vector and axial currents

The Dirac structure: 2 vector and 2 axial form factors

The vector current is assumed to be an isovector operator:

Vector form factors determined from e-p and e-n scattering

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$$F_{i}^{p} = F_{i}^{V} + F_{i}^{S}, \quad F_{i}^{n} = F_{i}^{S} - F_{i}^{V}$$
 (modulo factors 2)

Axial form factor

-Neutrino data is less precise → ANL/BNL/FNAL bubble chambers from 80's

-Original fits done with dipole form:

$$G_A(Q^2) = g_A \left(1 + Q^2 / M_A^2\right)^{-2}$$

 $M_A \approx 1 \text{ GeV}$

- Modern analysis with Model-independent parametrization [Meyer et al. PhysRevD.93.113015] 'z-expansion'

Axial form factor

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- Modern analysis with Model-independent parametrization [Meyer et al. PhysRevD.93.113015] 'z-expansion'
- More realistic error budget !

Axial form factor: Lattice QCD

- Recent Lattice QCD calculations for the axial form factor

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Axial form factor: Lattice QCD

- Recent Lattice QCD calculations for the axial form factor
- \rightarrow Significant increase for the total elastic cross section
- \rightarrow Status currently unclear

$$\begin{split} H^{\mu\nu} &= -g^{\mu\nu}W_1 + \frac{P^{\mu}P^{\nu}}{M_N^2}W_2 + i\frac{\epsilon^{\mu\nu\alpha\beta}P_{\alpha}Q_{\beta}}{2M_N^2}W_3 + \frac{Q^{\mu}Q^{\nu}}{M_N^2}W_4 + \frac{P^{\mu}Q^{\nu} + P^{\nu}Q^{\mu}}{M_N^2}W_5 \\ W_i^{elastic}(q^2) &\rightarrow W_i^{inelastic}(q^2, p \cdot q) = W_i(Q^2, x) \\ \frac{\mathrm{d}\sigma(E)}{\mathrm{d}Q^2} &\rightarrow \frac{\mathrm{d}\sigma(E)}{\mathrm{d}Q^2\mathrm{d}x} \end{split}$$

ZEUS

Bjorken scaling In the limit $\omega \to \infty$, $Q^2 \to \infty$, $0 < x = \frac{Q^2}{2M_N\omega} < 1$ Structure functions F_i become independent of Q^2 $\omega W_{2,3}(x, Q^2) \equiv F_{2,3}(x, Q^2) \to F_{2,3}(x)$

$$MW_1(x,Q^2) \equiv F_1(x,Q^2) \to F_1(x)$$

[Phys. Rev. D 67, 012007]

Simple explanation of **Bjorken scaling** is the parton picture: "Scattering of point-like fermions with momentum fraction x"

$$\frac{\mathrm{d}\sigma^{\gamma}}{\mathrm{d}x\mathrm{d}Q^2} = 4\pi \left(\frac{\alpha}{Q^2}\right)^2 \left[(1-y) + y^2/2\right] n_p(x)$$
$$\frac{\mathrm{d}\sigma^{\gamma}}{\mathrm{d}x\mathrm{d}Q^2} = 4\pi \left(\frac{\alpha}{Q^2}\right)^2 \left[(1-y)\frac{F_2(x,Q^2)}{x} + y^2F_1(x,Q^2)\right] n_p(x)$$

$$- F_2 = 2xF_1 = xn_p(x)$$

Callan-Gross relation

See e.g. [E.A. Paschos, Electroweak theory] [Thomson, Modern particle physics]

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$$y = (k \cdot q)/(p \cdot k)$$

Simple explanation of **Bjorken scaling** is the parton picture: "Scattering of point-like fermions with momentum fraction x"

In practice

Rigorous factorization theorems [Collins et al. Factorization of Hard Processes in QCD]

Higher order QCD + non-perturbative corrections

Relevance of and overlap with neutrino program: [NuSTEC white paper, arXiv:1706.03621]

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Neutrino generators use DIS at low Q^2 and W ~ 1.5 GeV

Often called 'Shallow inelastic scattering' (SIS) 'transition' between hadronic and parton degrees of freedom

Interactions in the nucleon resonance region

• Significant contribution to event rate in ~1 GeV experiments

GENIE estimates for **SBND**:

~ 23 % of the signal has a pion

Of which ~80 % in resonance region

Non-perturbative, no factorization → Hadron d.o.f

Proton F₂ structure functions (e,e') CLAS From [clas.sinp.msu.ru/strfun/]

Electro and photoproduction of pions

Much expertise from electromagnetic interactions with hadrons

Many approaches available in the literature

- MAID07, CLAS analyses ('unitary isobar model')
- Julich-Bonn, ANL-Osaka, ... (Dynamical models)

Electro and photoproduction

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Supported by a large amount of data!

- ...

Electron-induced SPP: high quality proton target data

Figures from M. Kabirnezhad [arxiv:2203.15594]

Differential exclusive cross sections are abundant for large kinematic range

Form factors for resonance excitation extracted from data

Nikolakopoulos et al. [Phys Rev D 107, 05300 (2023)]

Axial couplings are not well constrained

The quality of neutrino-nucleon scattering data is not up to par

Spin 3/2 resonance:

 \rightarrow 4 axial form-factors

 π

N'

Axial couplings are not well constrained

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Spin 3/2 resonance:

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Constraints on $C_5^A(Q^2=0)$ and $C_6^A(Q^2=0)$, can be formulated from PCAC

No constraints on other form factors or Q² - dependence

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Axial couplings are not well constrained

The quality of neutrino-nucleon scattering data is not up to par

Data shows large uncertainties

Quark-hadron duality

[W. Melnitchouk et al. , Phys. Rept. 406, 127–301 (2005), arXiv:hep-ph/0501217]

Quark-hadron duality in neutrino interactions [T. Sato EPJ:ST (2021) 230:4409-4418 (2022)]

F₂ structure function In electron scattering in RES region

Approaches and averages to DIS structure function

For CC ν scattering

- ANL-Osaka DCC model underestimates F₂ from DIS
- Modifying the Q²-dependence of resonance axial form factor to that from electrons :

Improved agreement with DIS

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Questions ? Answers ?

