Introduction to nuclear effects in neutrino interactions

Artur M. Ankowski
University of Wrocław

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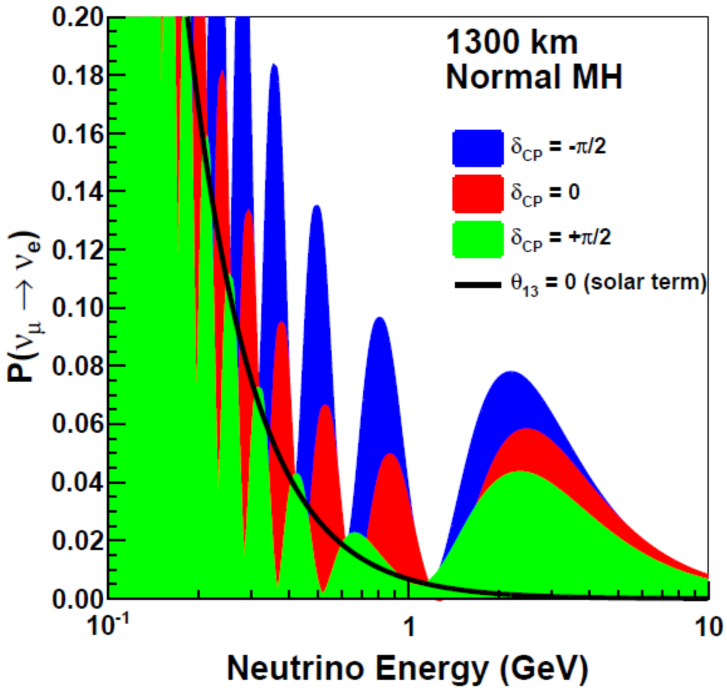
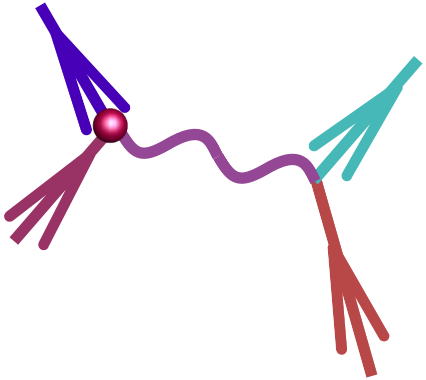
MC Generators in long-baseline neutrino physics

- Main goal: extract the ν & $\bar{\nu}$ oscillation probabilities.
- Polychromatic beams, neutrino energy reconstructed from visible energy deposited by interaction products.
- Energy reconstruction heavily depends on the cross sections implemented in Monte Carlo generators.
- Accuracy of simulations translates into the accuracy of the extracted oscillation parameters.
- **Without reliable cross sections precise oscillation measurements and searches for physics beyond the Standard Model cannot succeed**



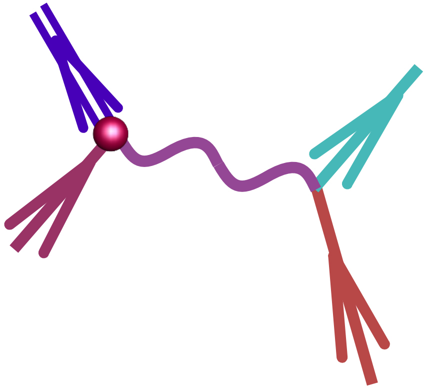
DUNE Collaboration, arXiv:1512.06148

Neutrino-nucleon scattering at GeV energies



quasielastic scattering

$$\nu_l + n \rightarrow l^- + p$$
$$\bar{\nu}_l + p \rightarrow l^+ + n$$



resonance production

$$\nu_l + n \rightarrow l^- + \Delta^+$$
$$\nu_l + p \rightarrow l^- + \Delta^{++}$$
$$\bar{\nu}_l + n \rightarrow l^+ + \Delta^-$$
$$\bar{\nu}_l + p \rightarrow l^+ + \Delta^0$$

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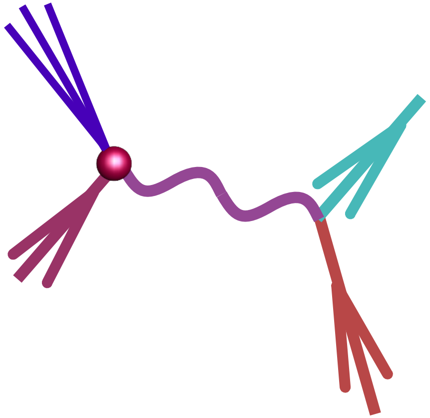
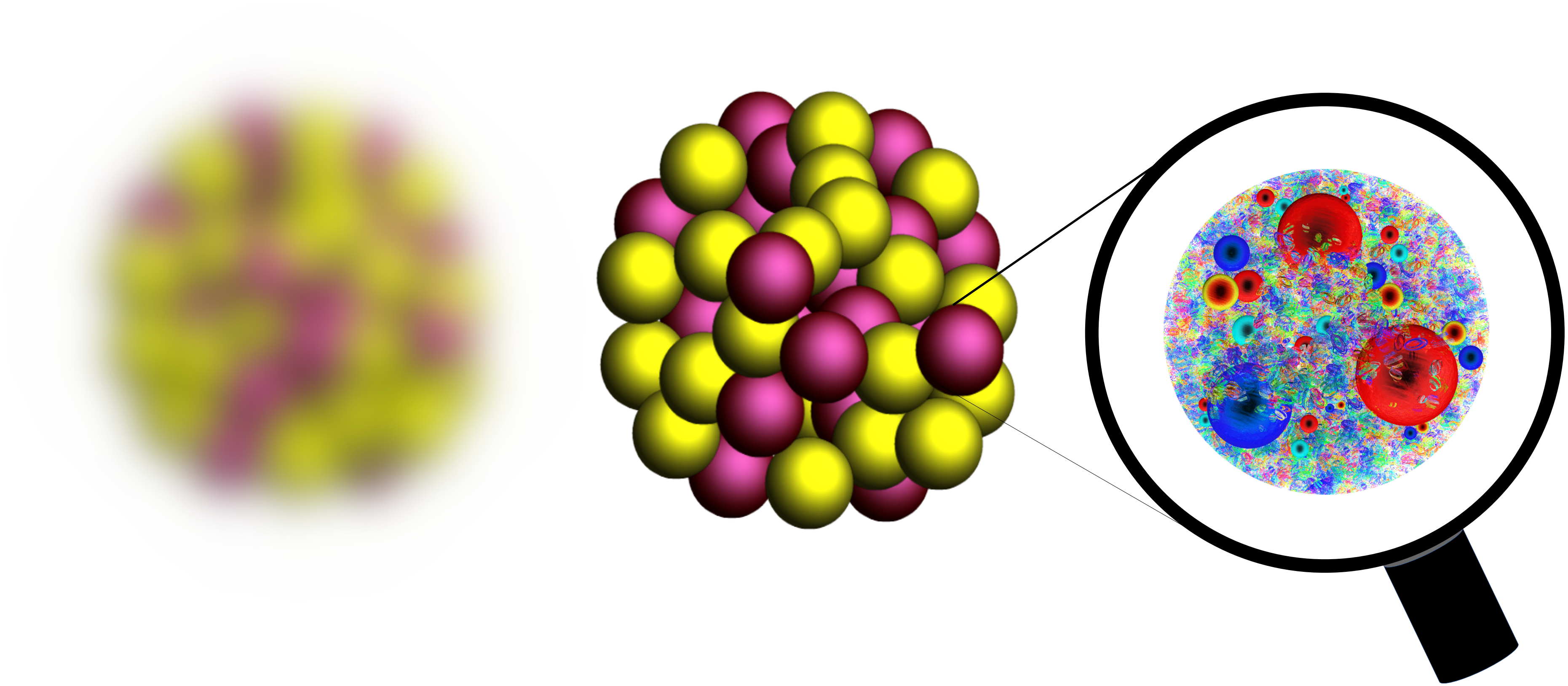


deep-inelastic scattering

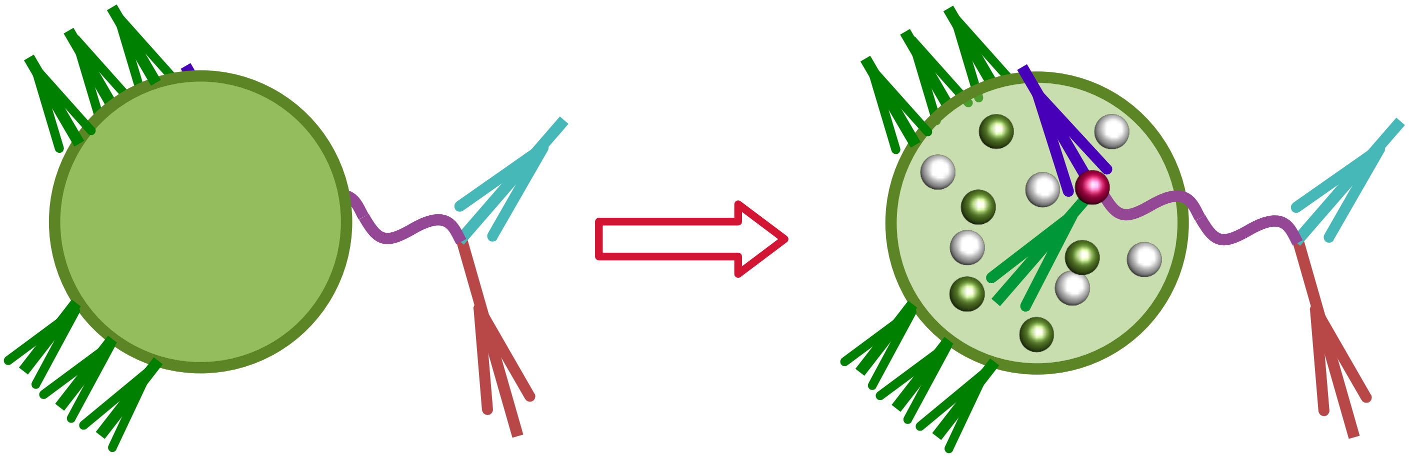
$$\nu_l + N \rightarrow l^- + N' + n\pi$$
$$\bar{\nu}_l + N \rightarrow l^- + N' + n\pi$$



Impulse approximation

At relevant kinematics, the dominant process of neutrino-nucleus interaction is **scattering off a single nucleon**, with the remaining nucleons acting as a spectator system.

This description is valid when the momentum transfer $|\mathbf{q}|$ is high enough ($|\mathbf{q}| \gtrsim 200$ MeV).



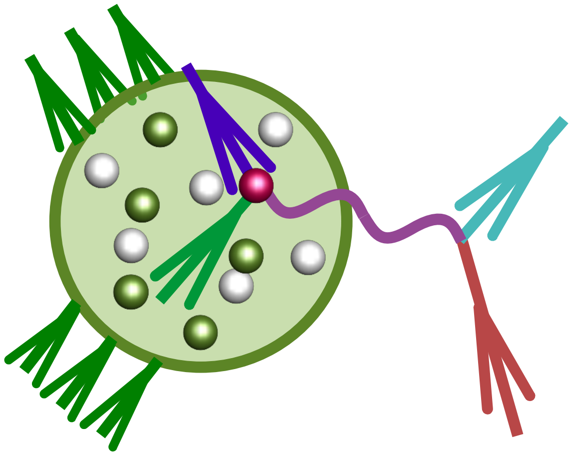
Impulse approximation

$$\frac{d\sigma^{\rm IA}_{\ell A}}{d\omega d\Omega} = \sum_N \int d^3p\, dE\, P^N_{\rm hole}(\mathbf{p}, E)\, \frac{M}{E_{\mathbf{p}}} \frac{d\sigma^{\rm elem}_{\ell N}}{d\omega d\Omega}\, P^N_{\rm part}(\mathbf{p}', \mathcal{T}')$$

average over the initial nucleon state

nucleon cross section

final-state interactions



What data can we use to test our models?

- ν interact weakly: tiny cross sections, probe the whole nuclear volume
- α, d, p, $\pi^{\pm}$ interact strongly: huge cross sections, but only scatter on the nuclear surface
- γ interact electromagnetically: small/large cross sections, probe the whole nuclear volume at $Q^2 = 0$
- e^{-} interact electromagnetically: small/large cross sections, probe the whole nuclear volume at any kinematics

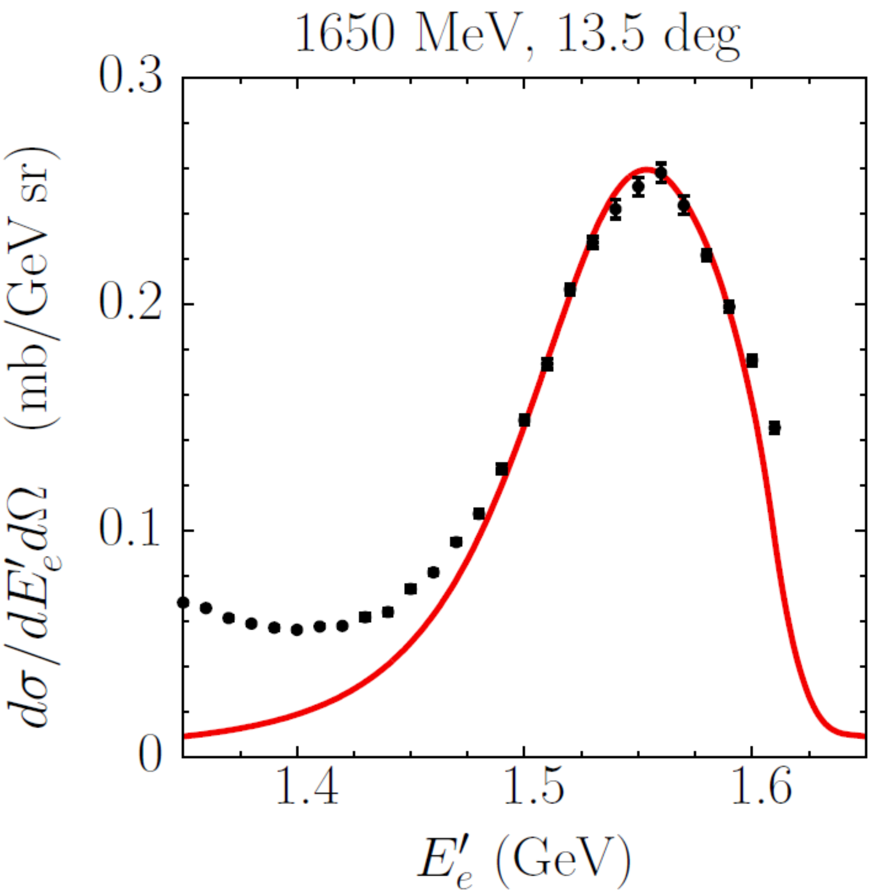
What data can we use to test our models?

"Neutrino interactions in the energy range of interest to current and near-future experiments (1 to 10 GeV), pose particular problems. In this energy range, bridging the perturbative and nonperturbative pictures of the nucleon, a variety of scattering mechanisms are important.

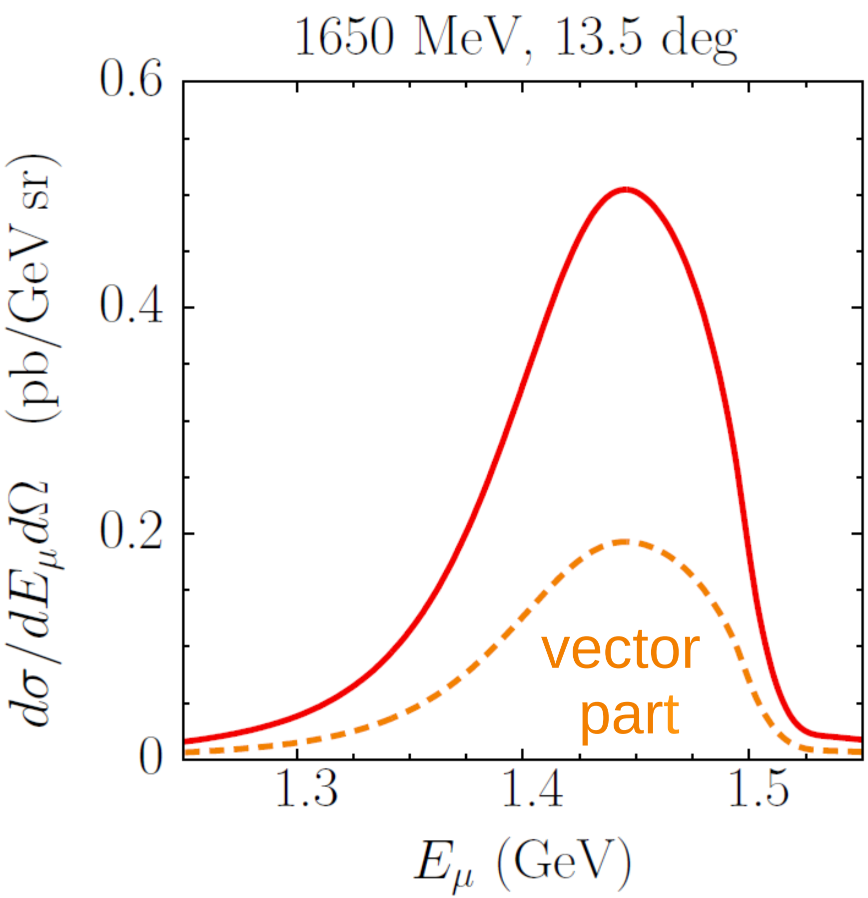
...

The models incorporated into neutrino simulations at these energies have been tuned primarily to this bubble chamber data. This data is not sufficient to completely constrain the models, particularly with regards to the simulation of nuclear effects. **A logical place to turn for guidance are electron scattering experiments.**"

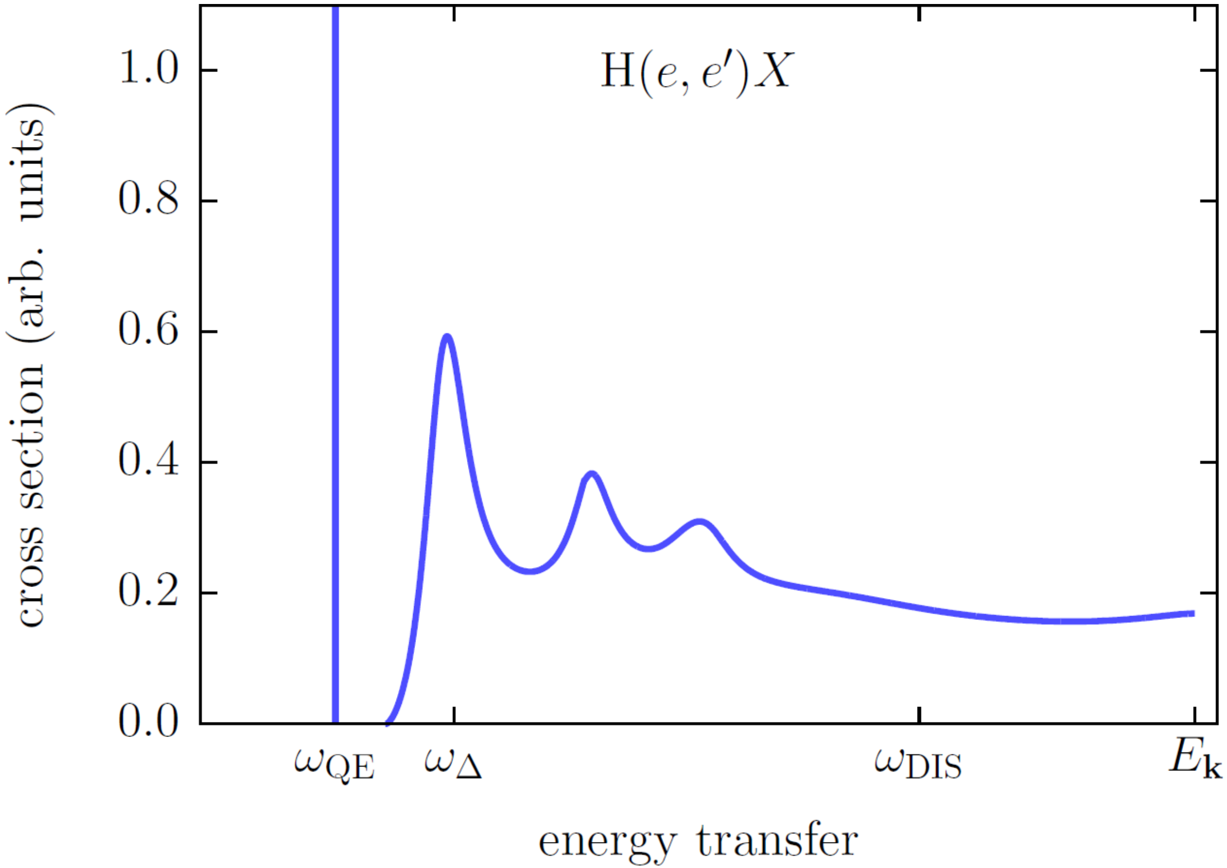
H. Gallagher, AIP Conf. Proc. 698, 153 (2004) electrons



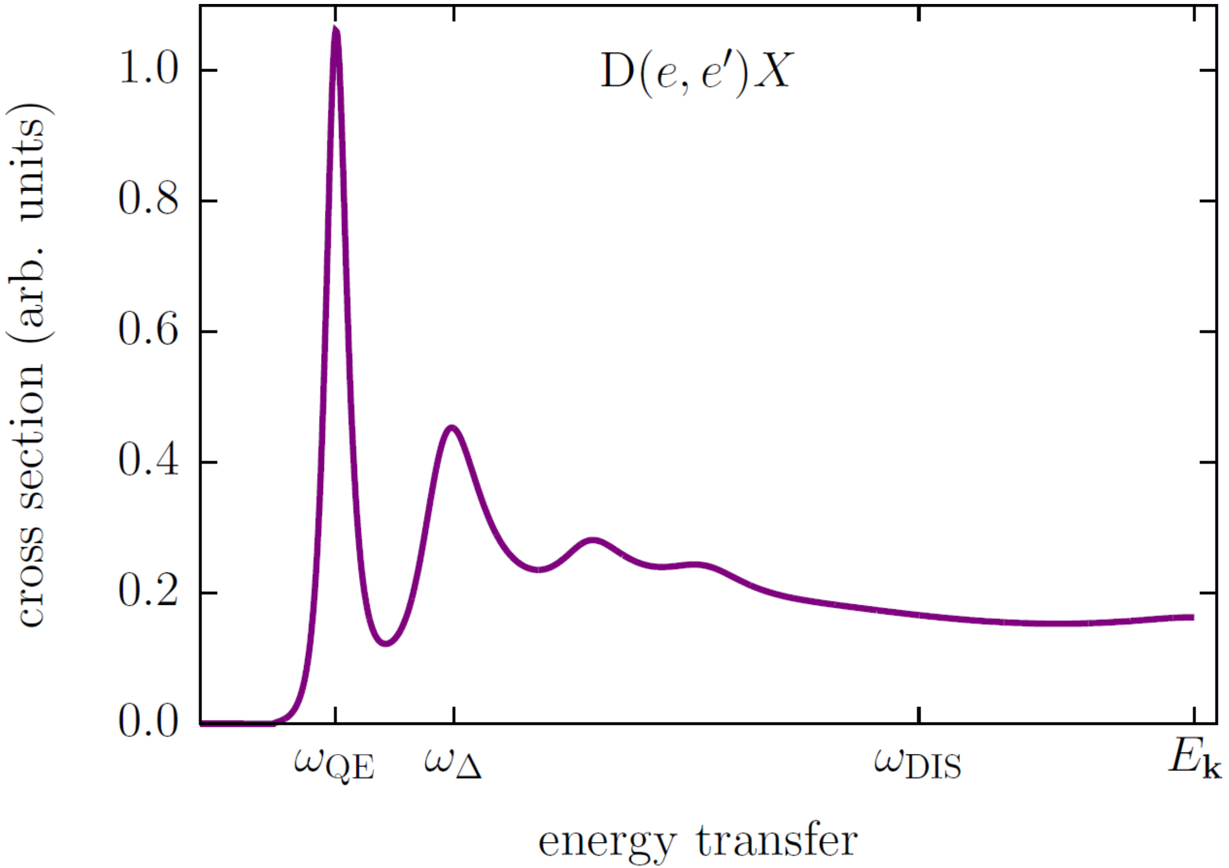
muon neutrinos



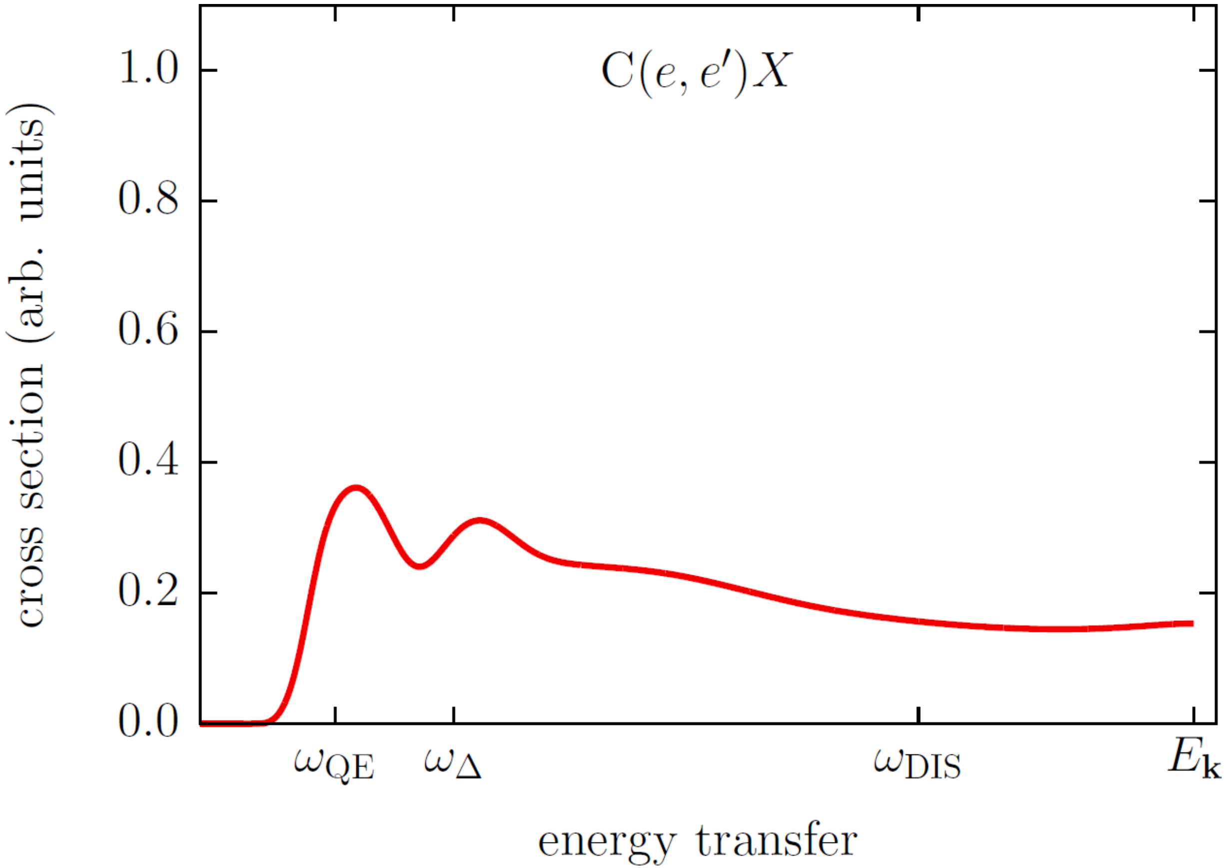
Double differential cross section



Double differential cross section



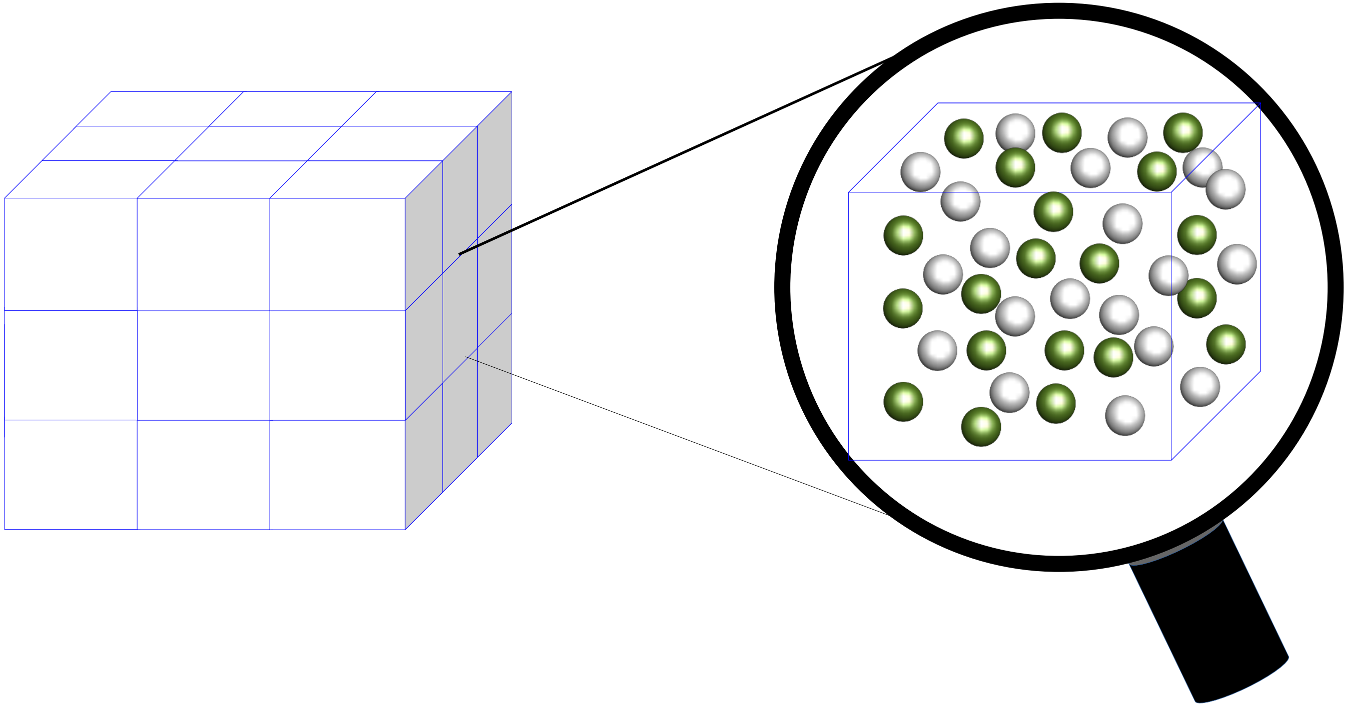
Double differential cross section



Fermi gas model

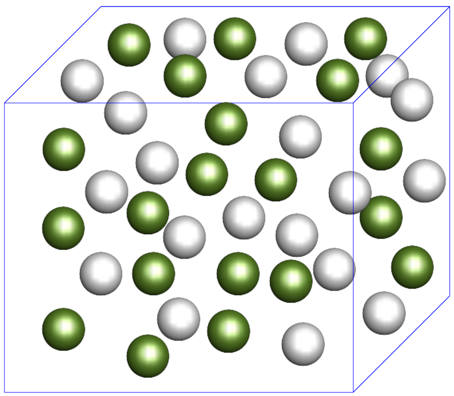
Fermi gas

Imagine an infinite space filled uniformly with nucleons ($N = Z$).
Does *translational invariance* imply that the eigenstates have definite momenta or angular momenta?

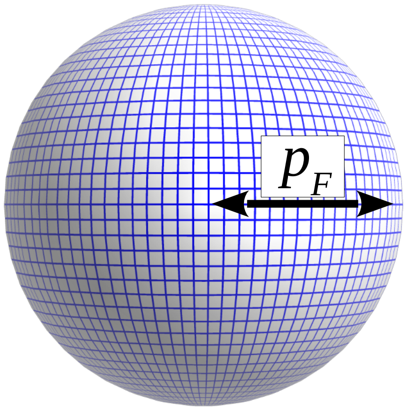


Fermi gas

As fermions, nucleons occupy states up to the maximal momentum ('Fermi momentum'). In a constant potential, their energies are $E_p = \sqrt{M^2 + \boldsymbol{p}^2} - \epsilon$.



Coordinate space



Momentum space

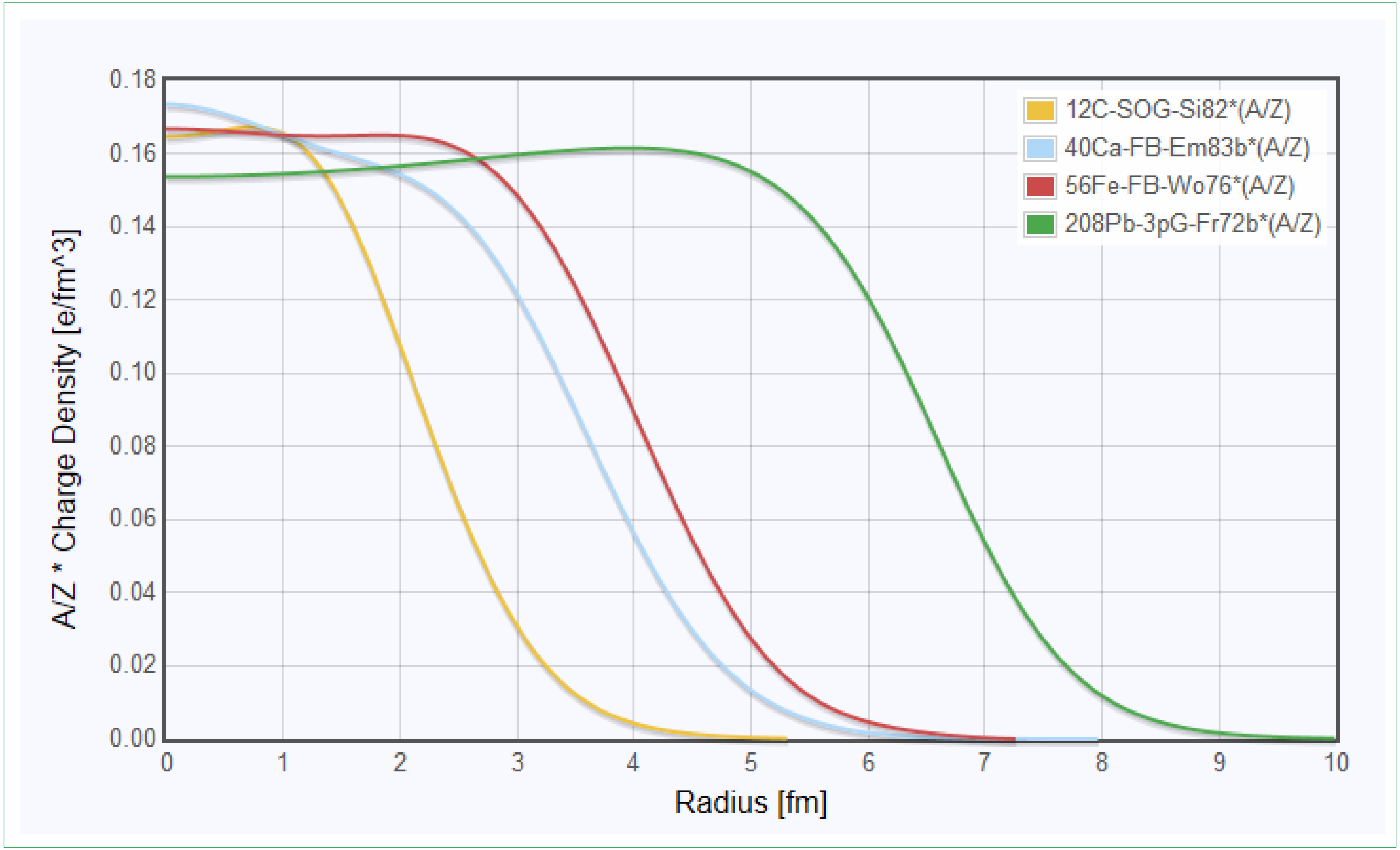
How are p_F and ϱ related?

- Every state of a given momentum can be occupied by: $p\uparrow, p\downarrow, n\uparrow, n\downarrow$
- Its 'volume' is $\frac{2\pi}{L_x}\frac{2\pi}{L_y}\frac{2\pi}{L_z}=\frac{(2\pi)^3}{V}$
- The total phase-space 'volume' is $\frac{A}{4}\frac{(2\pi)^3}{V}=\frac{2\pi^3 A}{V}=2\pi^3\rho$
- It can also be expressed as $\frac{4}{3}\pi\, p_F^3$, so

$$p_F=\sqrt[3]{3\pi^2\rho/2}$$

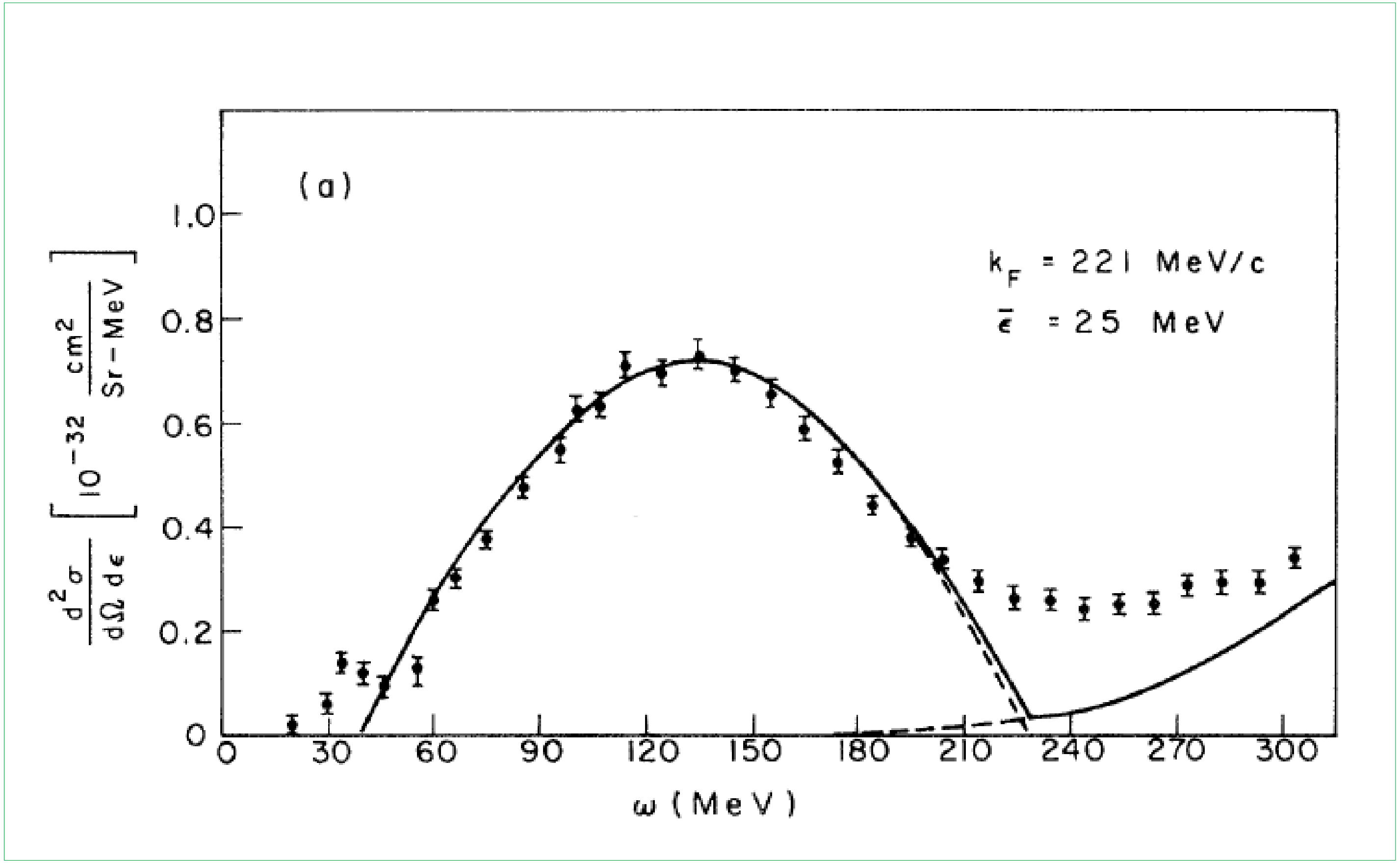
For ϱ = 0.16/fm^3, p_F = 1.33/fm = 263 MeV

Charge density of nuclei



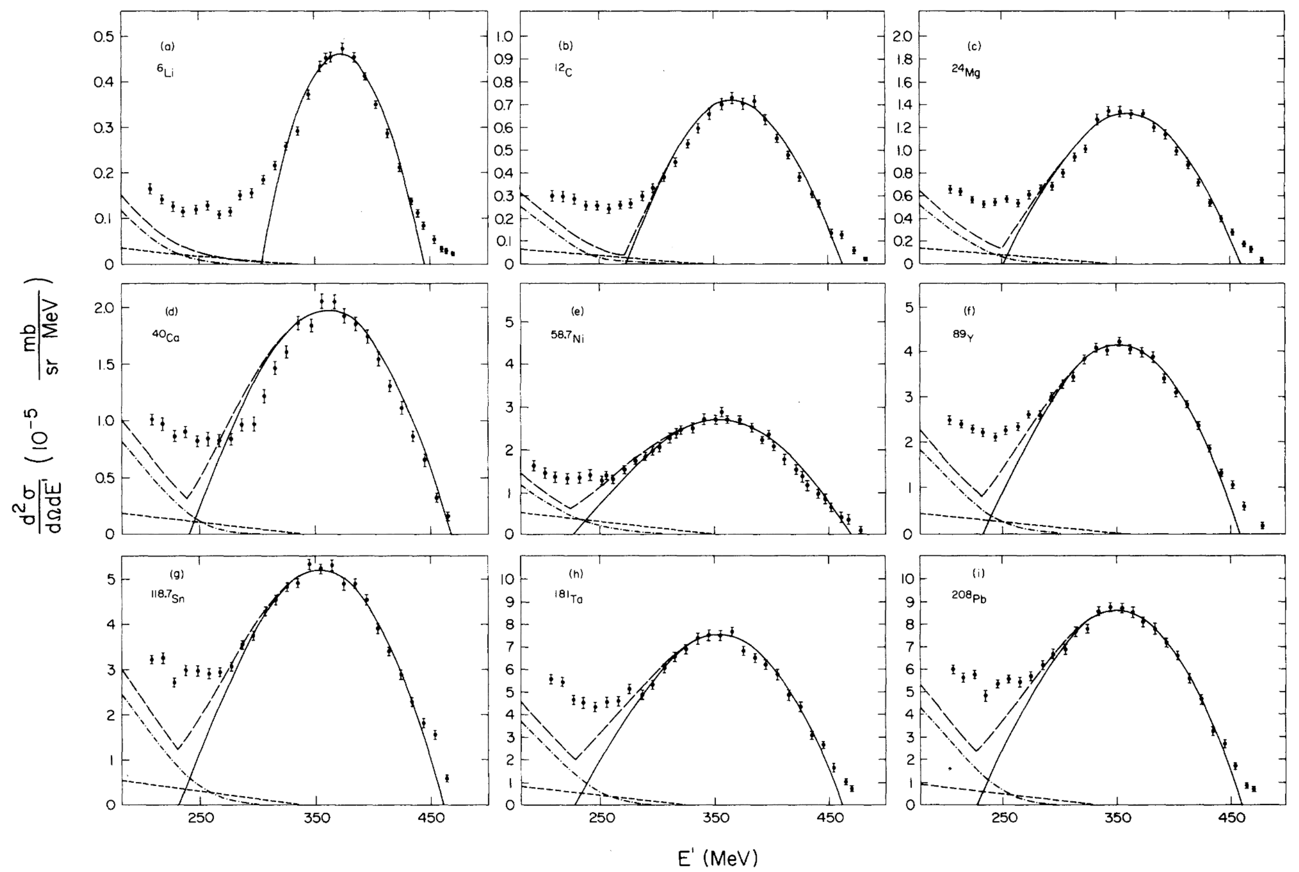
https://discovery.phys.virginia.edu/research/groups/ncd/index.html

Electron scattering on carbon



Moniz *et al.*, PRL 26, 445 (1971)

How about other targets?



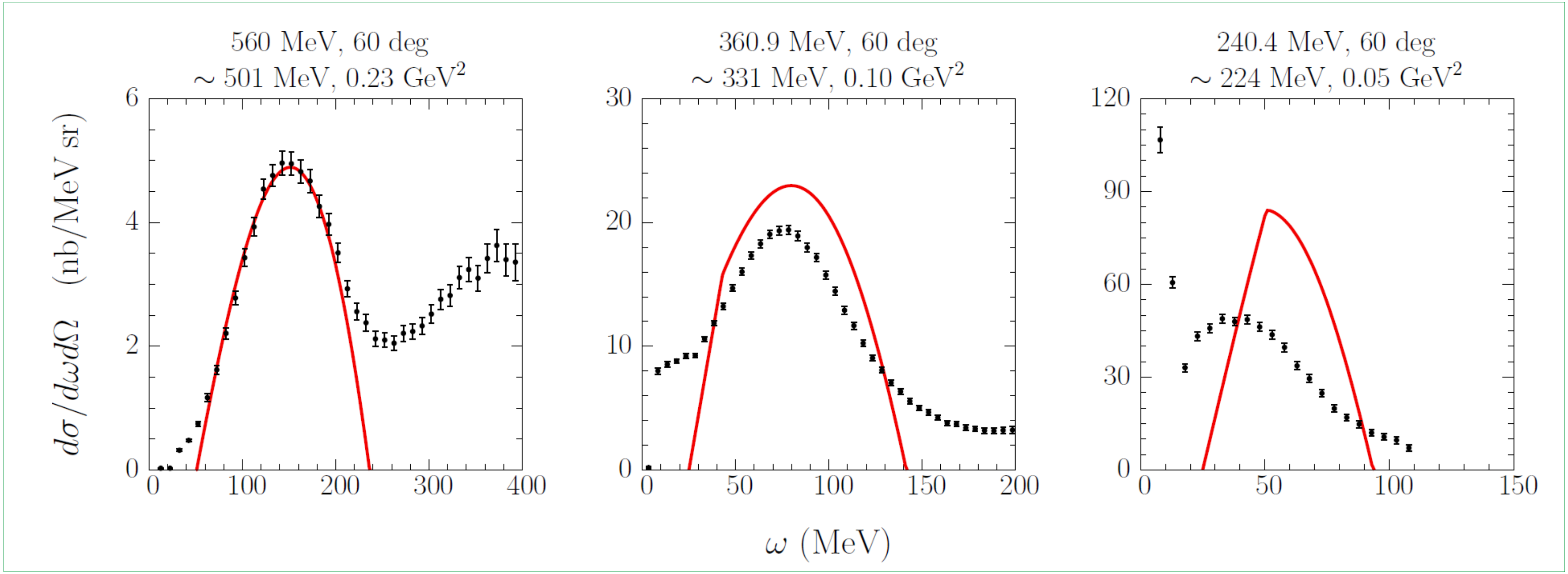
Whitney *et al.*, PRC 9, 2230 (1974)

Fermi gas parameters

target	A	p_F	ϵ
lithium	6	169	17
carbon	12	221	25
magnesium	24	235	32
calcium	40	249	33
nickel	~59	260	36
yttrium	89	254	39
tin	~119	260	42
tantalum	181	265	42
lead	208	265	44

Whitney *et al.*, PRC 9, 2230 (1974)

How about other *kinematics*?



data: Barreau *et al.*, NPA 402, 515 (1983)

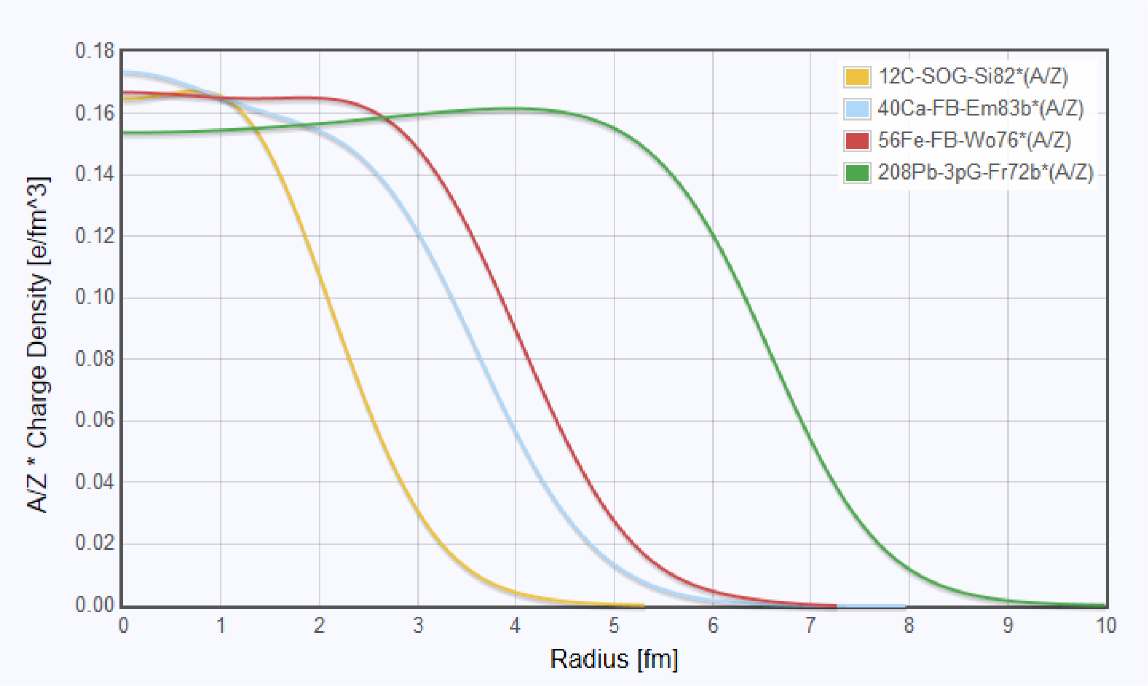
Shortcomings of the Fermi gas model

According to Whitney *et al.* (1974),

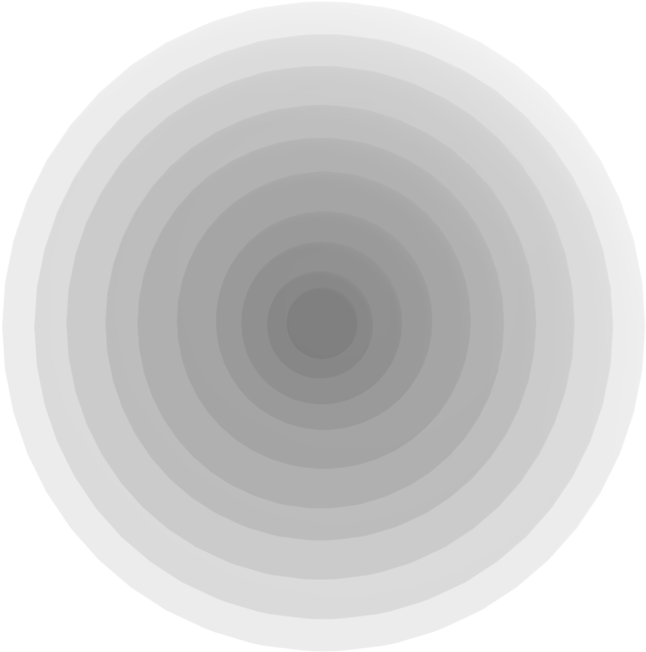
- nucleon-nucleon correlations are ignored
- final-state interactions are not accounted for
- oversimplified momentum distribution
- disregards finite-size effects (‘nuclear surface’)

Local Fermi gas

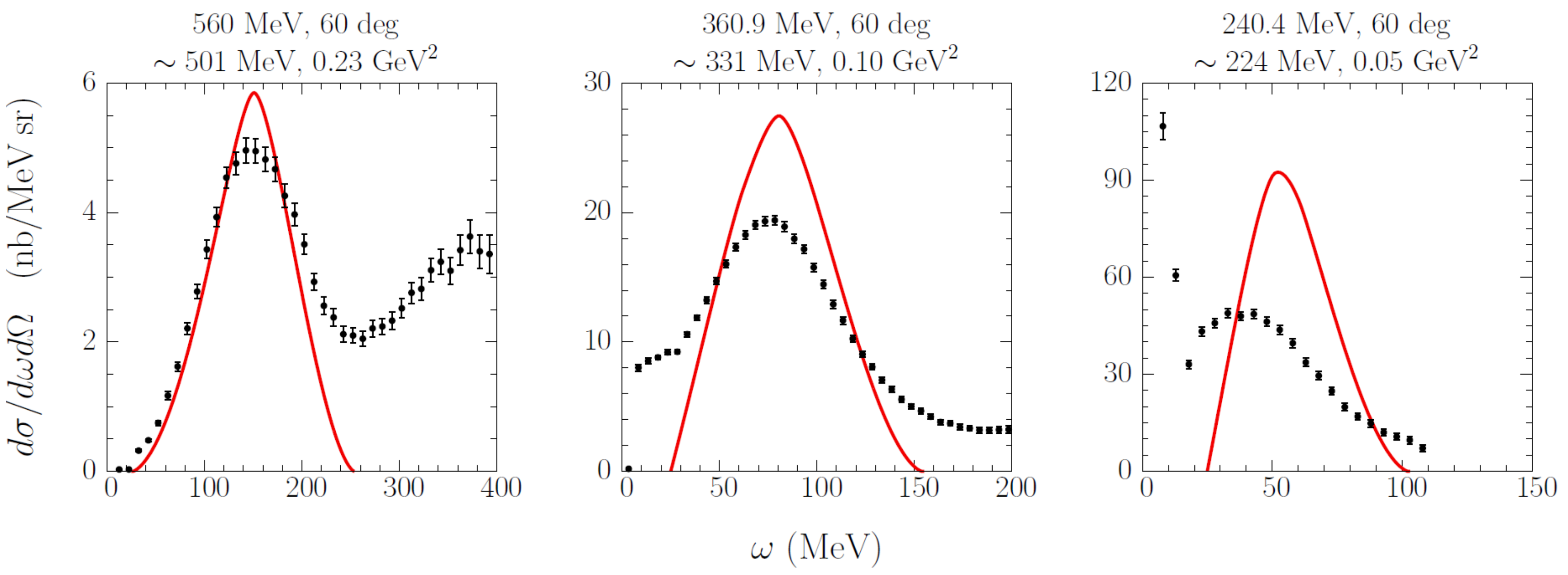
Perhaps we can approximate a spherically symmetric nucleus by a set of concentric spheres of a constant density?



https://discovery.phys.virginia.edu/research/groups/ncd/index.html

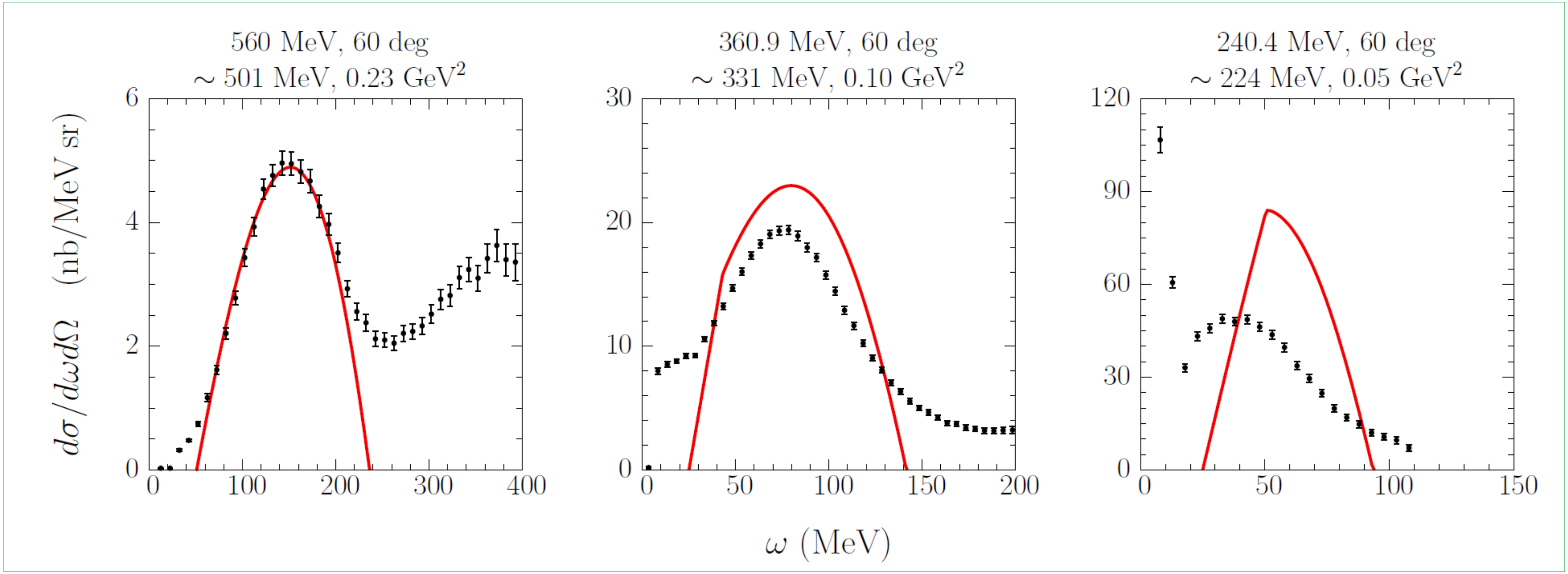


Local Fermi gas



data: Barreau *et al.*, NPA 402, 515 (1983)

Global Fermi gas

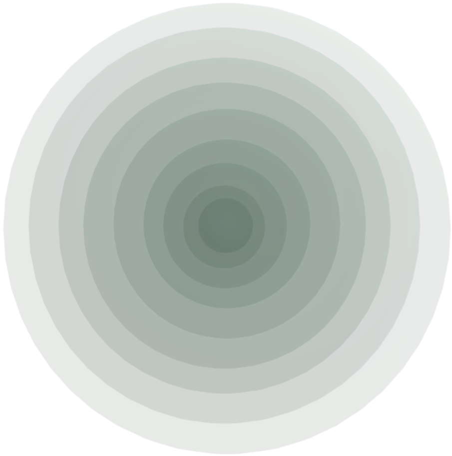


data: Barreau *et al.*, NPA 402, 515 (1983)

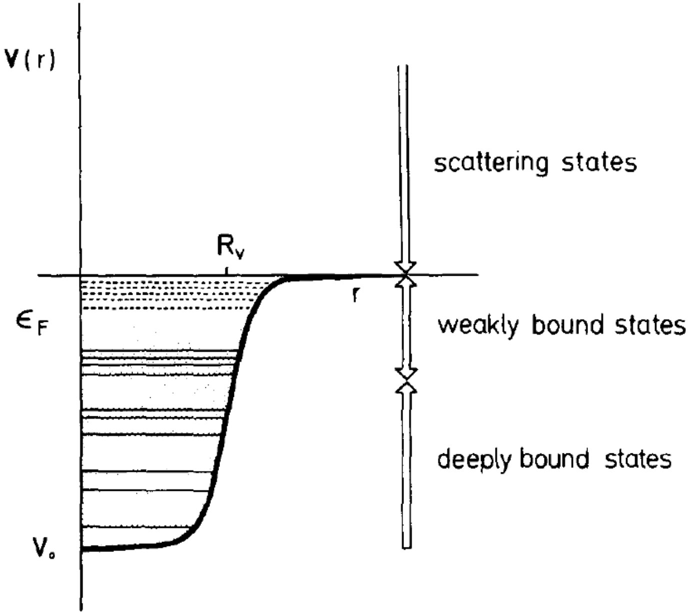
Shell model

Shell model

Imagine a spherically symmetric potential. Does *rotational invariance* imply that the eigenstates of the Hamiltonian have definite momenta or angular momenta?

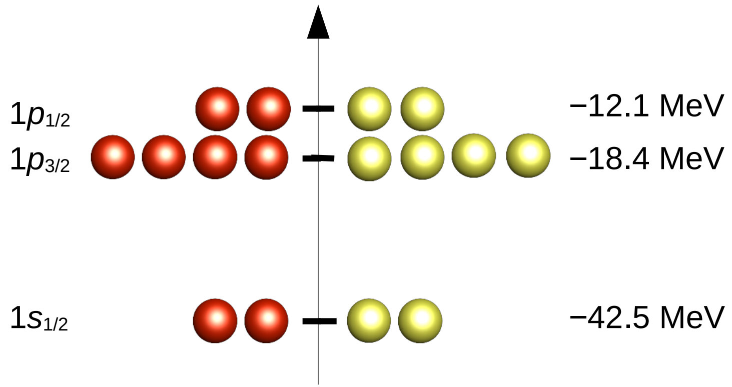


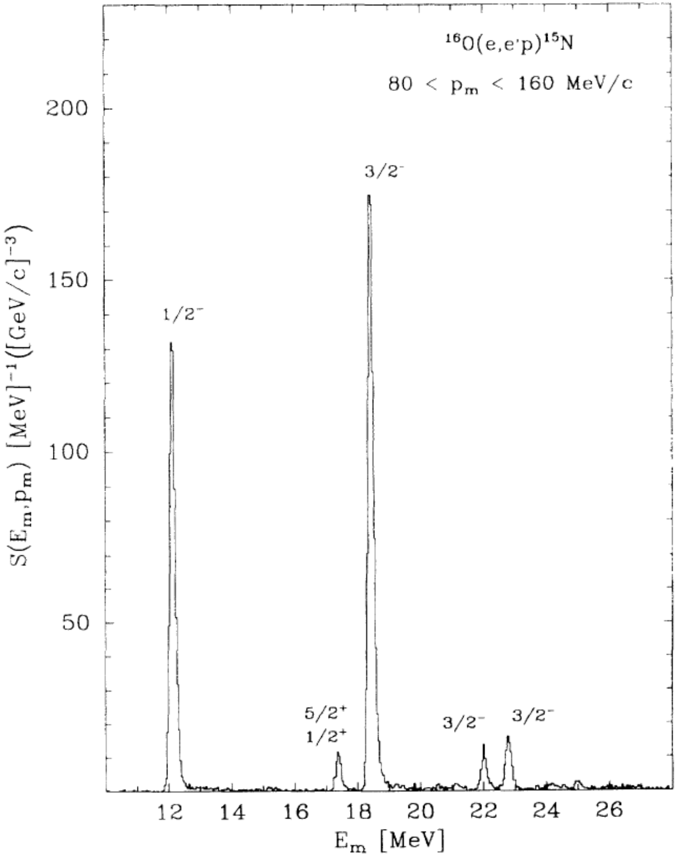
Potential in
the coordinate space



Mahaux *et al.*, Phys. Rep. 120, 1 (1985)

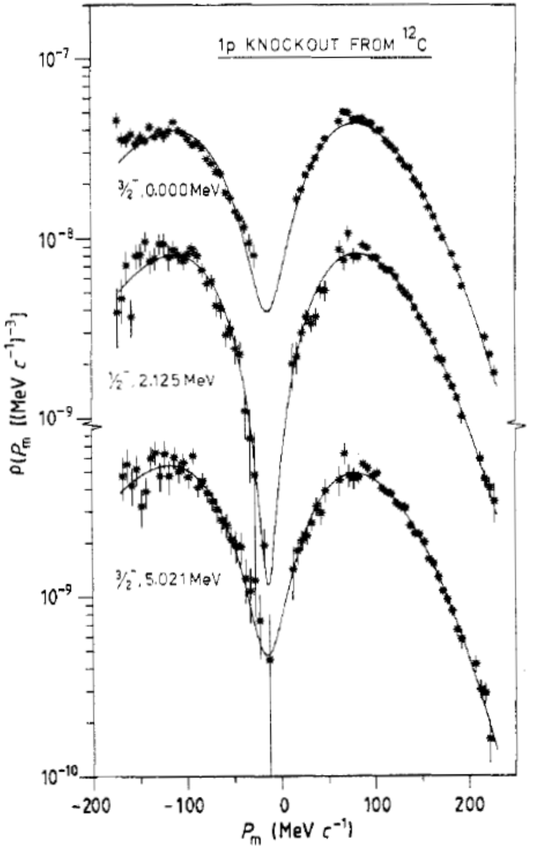
Example: oxygen nucleus

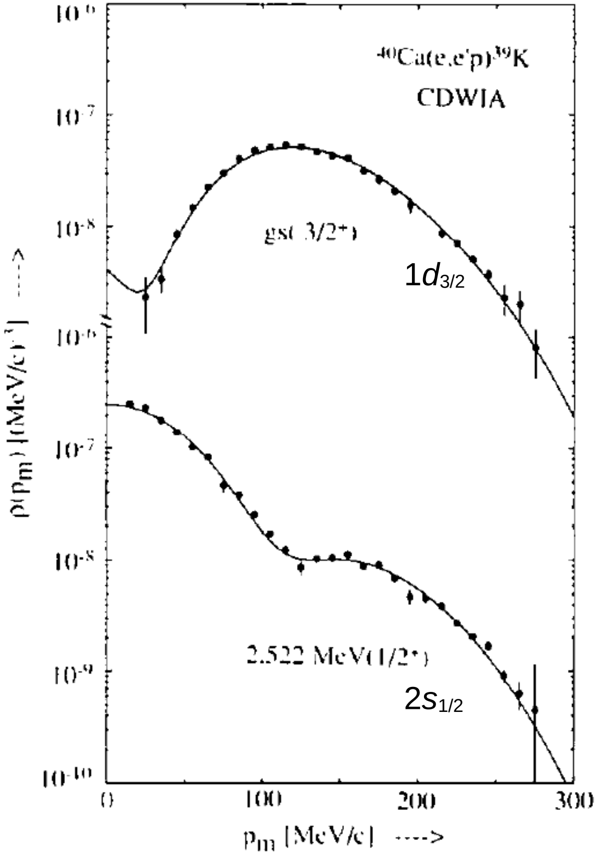


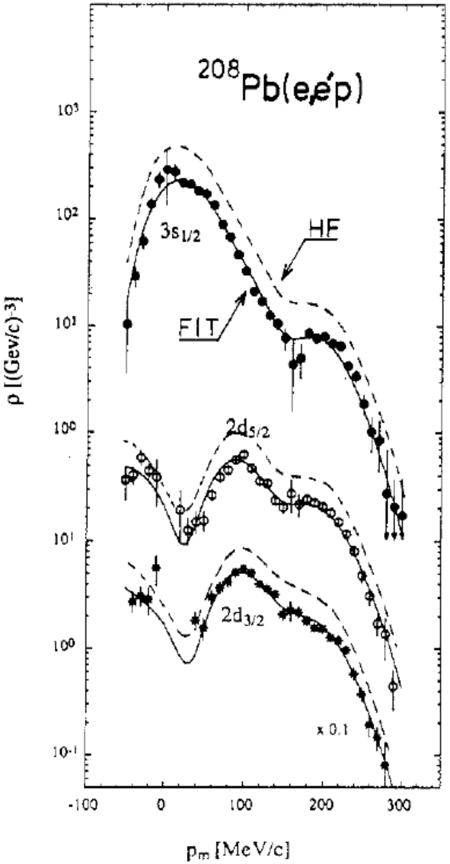


Leuschner *et al.*, PRC 49, 955 (1994)

Momentum distributions





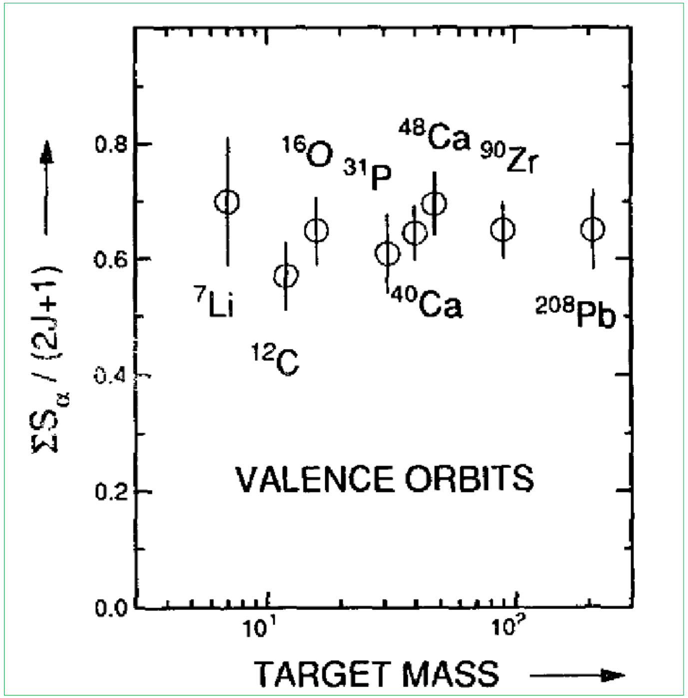


De Witt Huberts, JPG 16, 507 (1990)

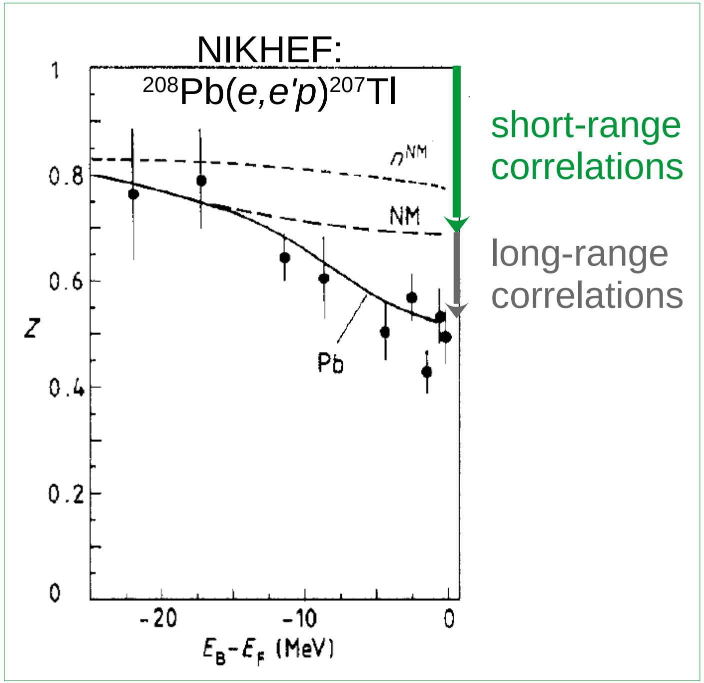
Kramer *et al.*, PLB 227, 199 (1989)

Depletion of the shell-model states

States are occupied to between ~65% (valence shells) and 80% (core shells) of the predictions of the independent-particle shell model.



Lapikás, NPA 553, 297c (1993)

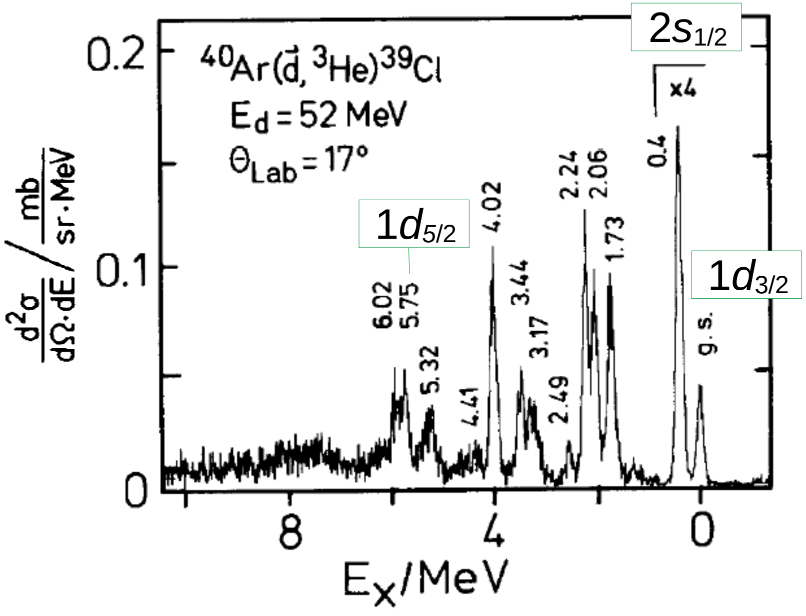


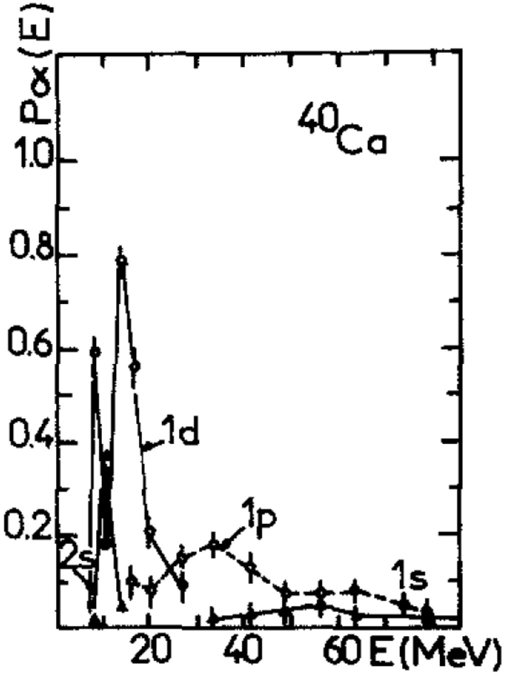
de Witt Huberts, NPA 553, 297c (1993)
Benhar *et al.*, PRC 41, R24 (1990)

Deeply bound states

The state's width increases with increasing distance from the Fermi energy.

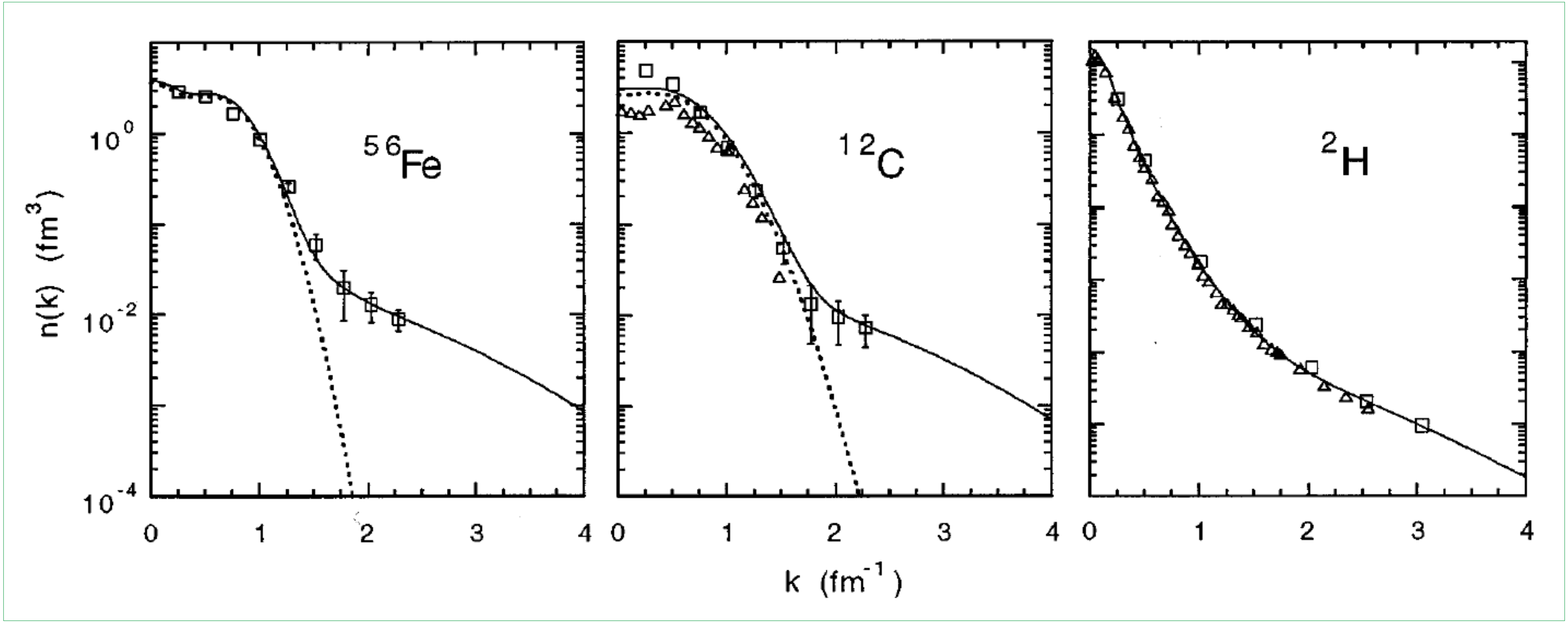
Mairle *et al.,* NPA 565, 543 (1993)





Mougey *et al.*, NPA 262, 461 (1976)

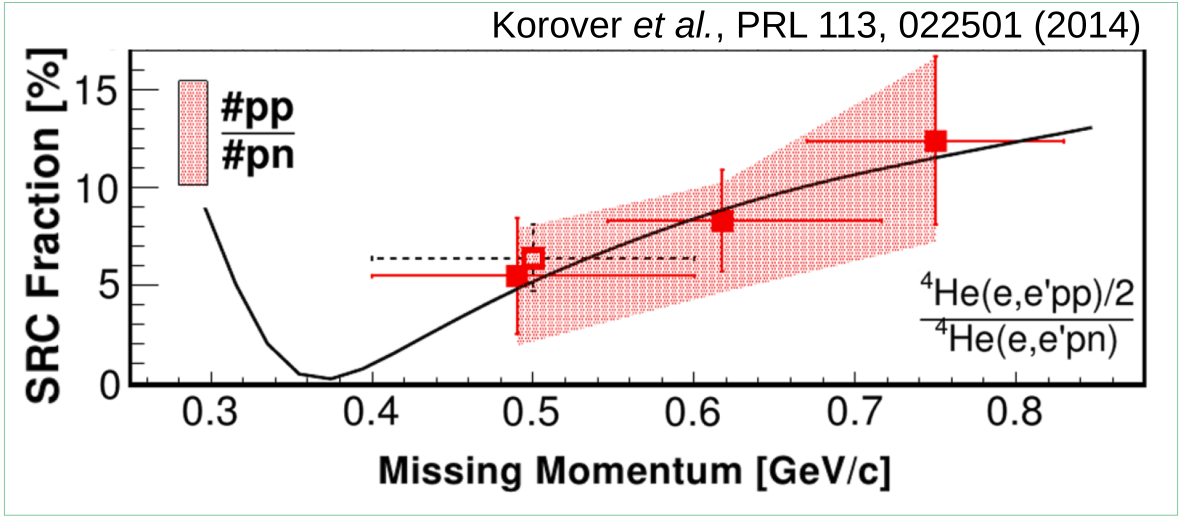
Total momentum distributions

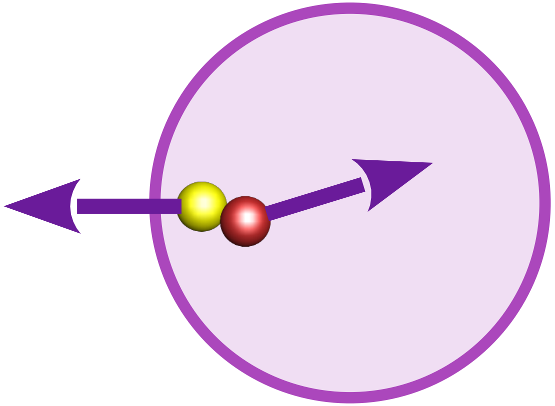


Ciofi degli Atti and Simula, PRC 53, 1689 (1996)

Short-range correlations

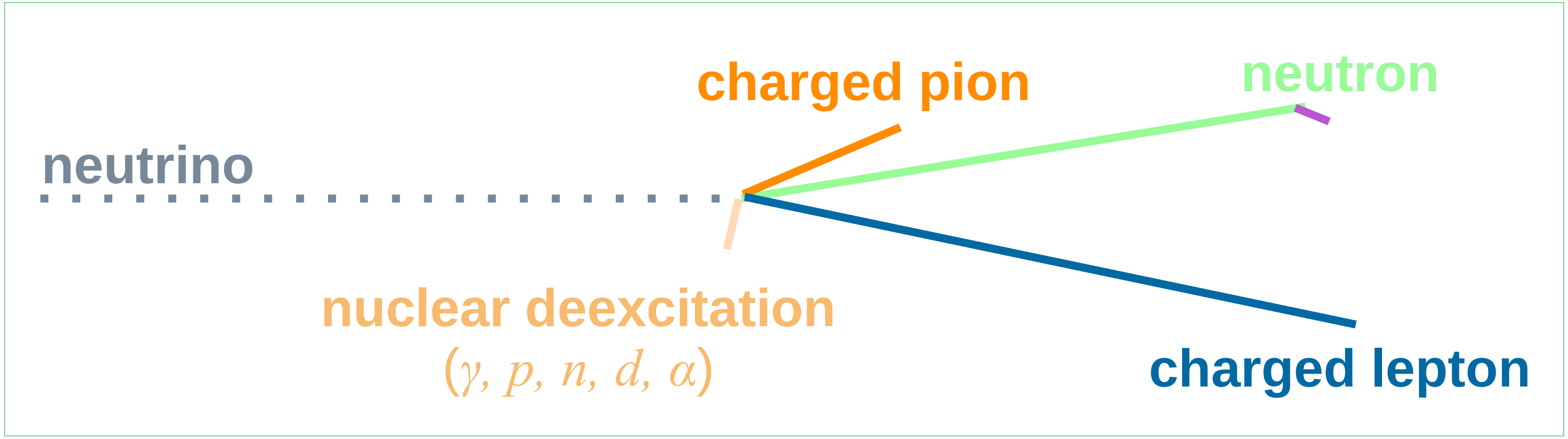
At short distances, repulsive interactions between nucleons create *NN* pairs (typically *pn*) with high relative momenta, which can only be probed at large energy transfers. Their presence depletes shell-model states.





Energy reconstruction

Neutrino energy reconstruction



Neutrino energy is converted to

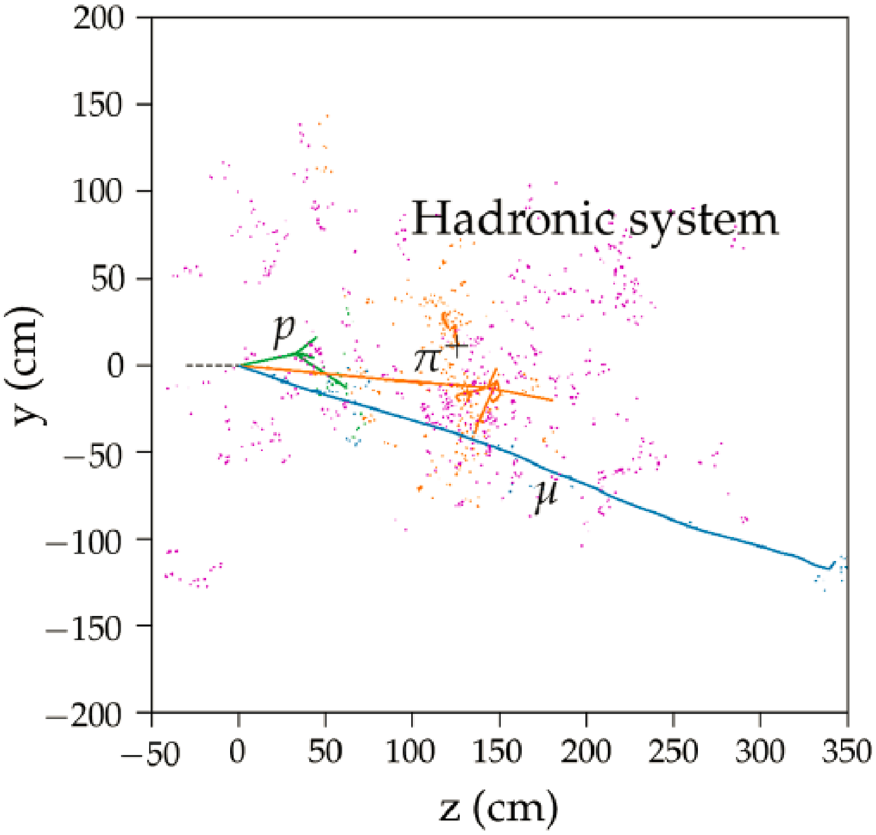
- the kinetic energies of the knocked-out nucleons,
- the total energies of leptons, mesons, and gammas,
- nuclear breakup

Calorimetric energy reconstruction

neutrino energy = Σ (total energies of leptons, mesons, and gammas) + Σ (proton kinetic energies) + …

- Applicable to any final state
- Hadron energies need to be reconstructed
- Requires estimating the missing energy in simulations, based on the kinematics of the observed particles

GENIE+FLUKA simulation of a 4-GeV ν_μAr event



A. Friedland & S.W. Li, PRD 99, 036009 (2019)

Exclusive final states: CC0π

+ quasielastic interactions with any number of nucleons

+ resonance excitation, nonresonant pion production, or deep-inelastic scattering followed by pion absorption

+ resonance excitation, nonresonant pion production, or deep-inelastic scattering with pions below the detection threshold

— quasielastic interactions followed by pion production in final-state interactions

...

Exclusive final states: CC1π

+ (some) resonance excitation or nonresonant pion production

+ deep-inelastic scattering or higher resonance excitation followed by pion absorption

+ quasielastic interaction followed by pion production in final-state interaction

− single pion production followed by pion production in final-state interactions

...

Exclusive final states: CC2π

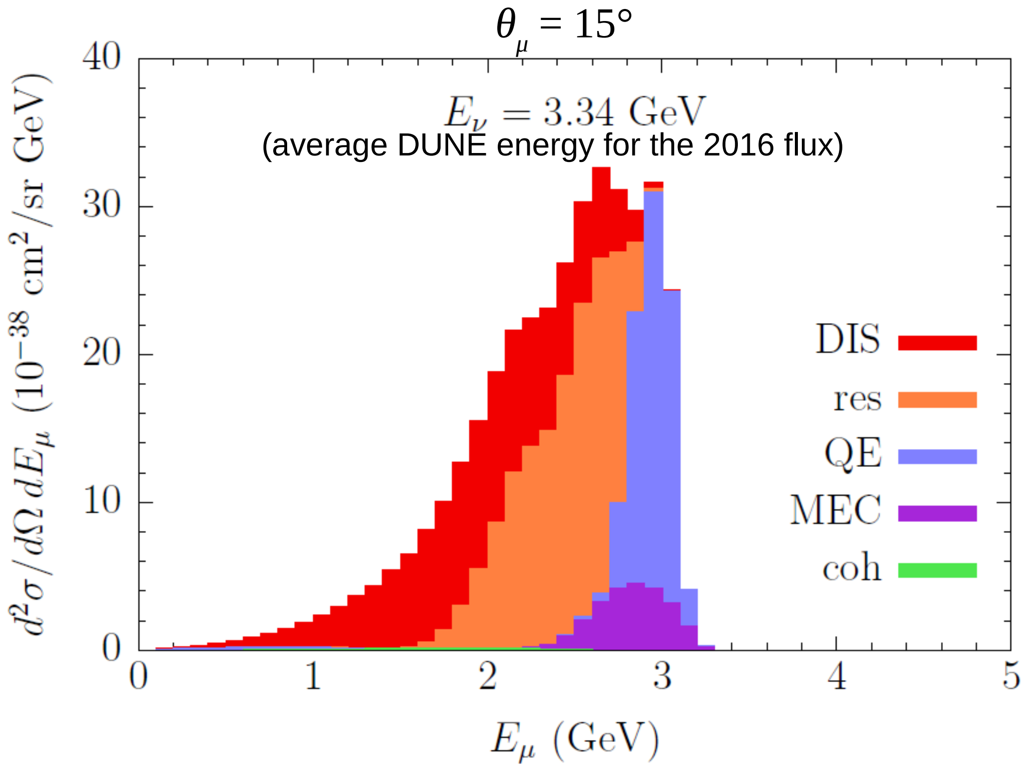
+ higher resonance excitation

+ deep-inelastic scattering

+ single pion production followed by pion production in final-state interactions

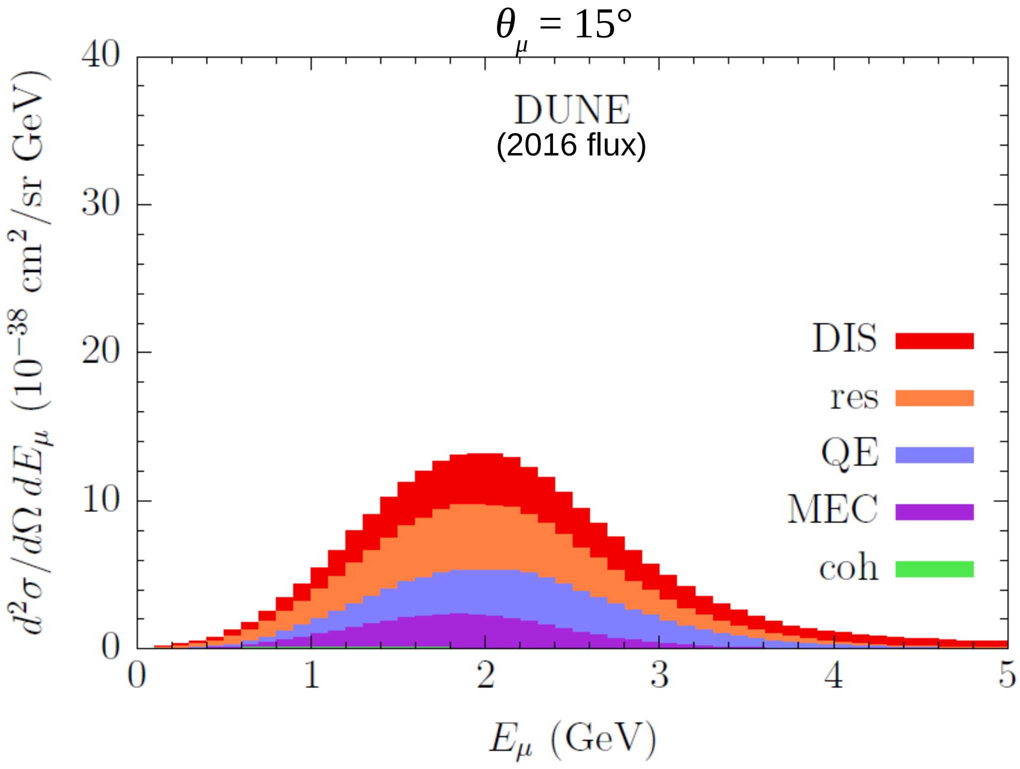
...

Neutrino double differential cross section



A.M.A. & A. Friedland, PRD 102, 053001 (2020)

Neutrino double differential cross section



A.M.A. & A. Friedland, PRD 102, 053001 (2020)

Polychromatic vs. monochromatic beams

Photo credit: delish.com

Kinematic energy reconstruction

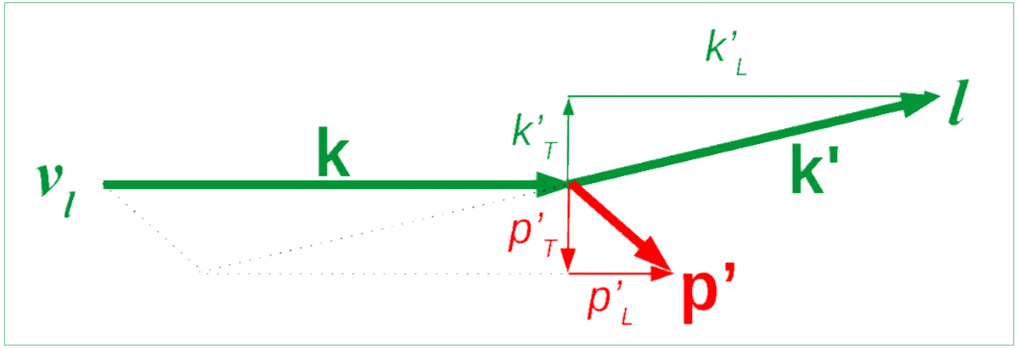
In quasielastic scattering on free nucleons, $\nu + p \rightarrow l + n$ and $\nu + n \rightarrow l + p$, we can infer the neutrino energy from the charged lepton's kinematics.

Energy conservation:

$$E_\nu + M = E_l + \sqrt{M^2 + p'^2_L + p'^2_T}$$

Momentum conservation:

$$E_\nu = |k'|\cos\theta + p_L'$$
$$0 = |k'|\sin\theta + p_T'$$



Therefore

$$E_\nu = \frac{2ME_l + m^2}{2\left(M - E_l + |k'|\cos\theta\right)}$$

Kinematic energy reconstruction

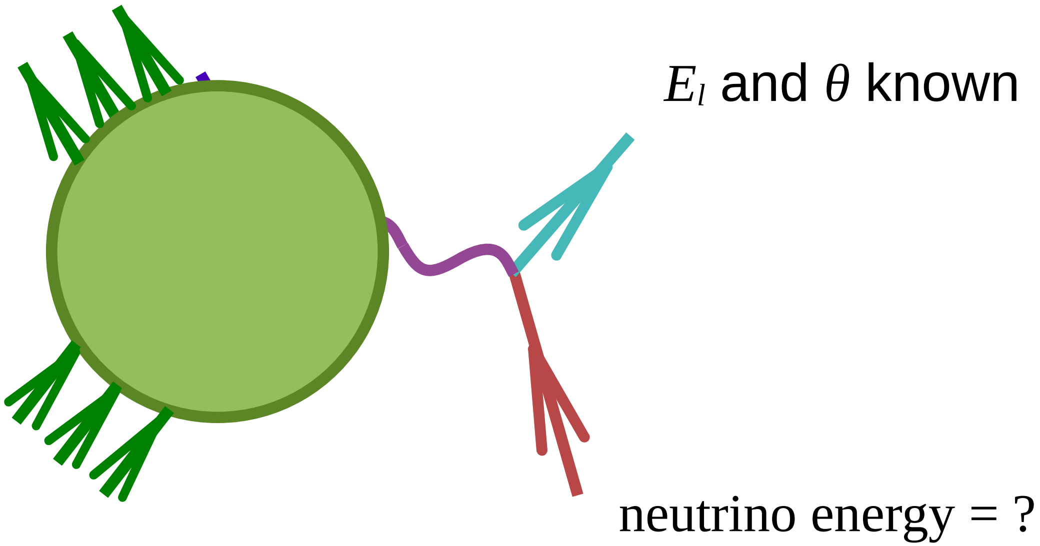
$$E_\nu = \frac{2(M-\epsilon)E_l + m^2 + M^2 - (M-\epsilon)^2}{2[(M-\epsilon) - E_l + |k'|\cos\theta]}$$

- Applicable only to charged-current quasielastic events
- Based on the lepton kinematics only
- No need to reconstruct hadrons
- *What is the binding energy ϵ value?*

Thought experiment: unknown monochromatic beam

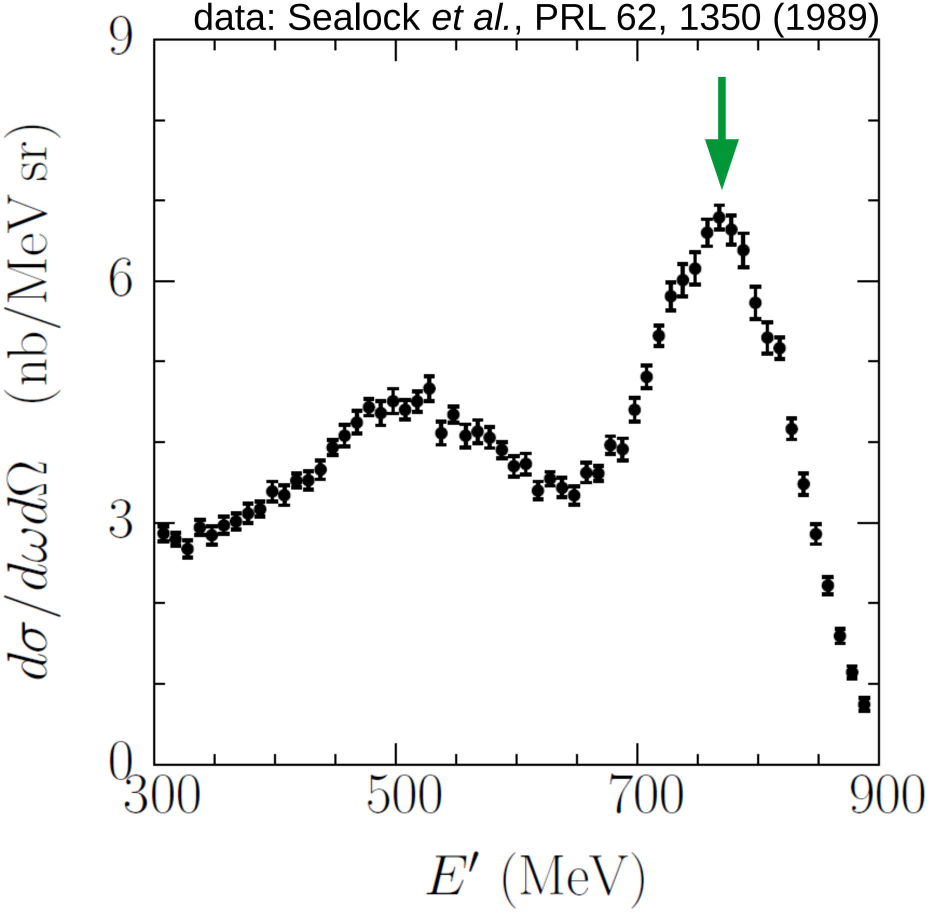
Consider the simplest (unrealistic) case: the beam is monochromatic but its energy is unknown and has to be reconstructed

A.M.A., PoS(NuFact2014)004



“Unknown” monochromatic electron beam

data: Sealock *et al.*, PRL 62, 1350 (1989)



quasielastic peak:
θ = 37.5 deg
E' = 768 MeV
$\Delta E'$ = 5 MeV

A.M.A., PoS(NuFact2014)004

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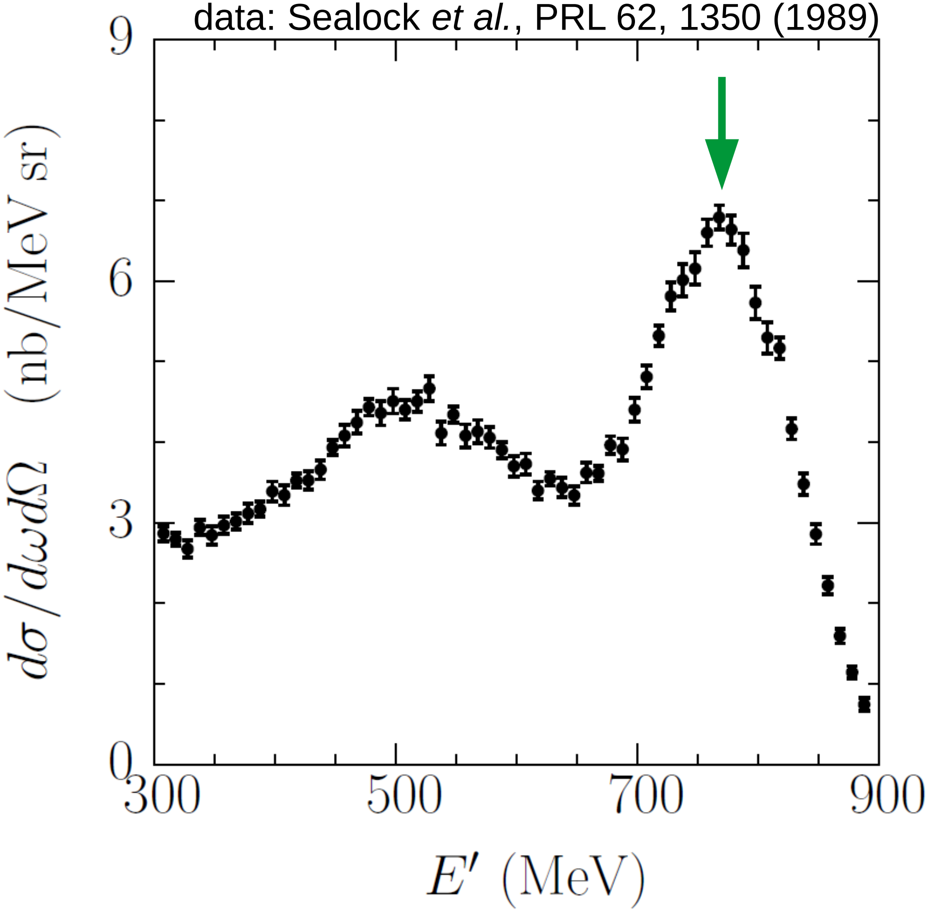

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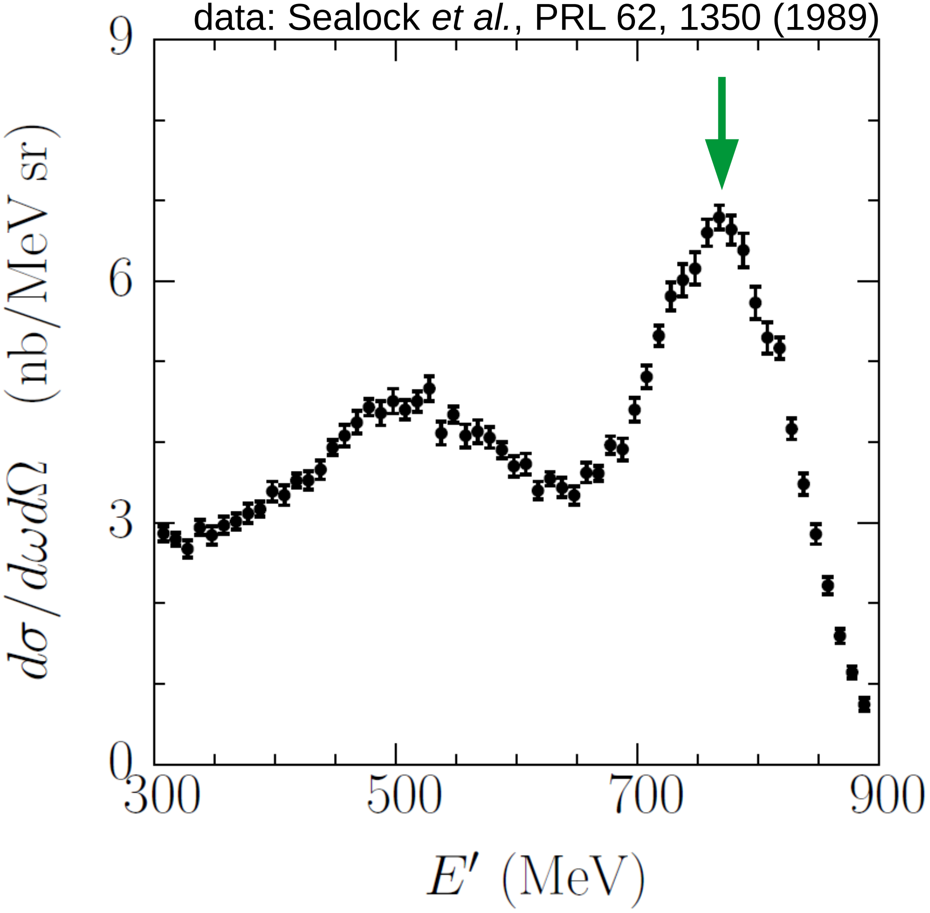

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true value
E = 961 MeV

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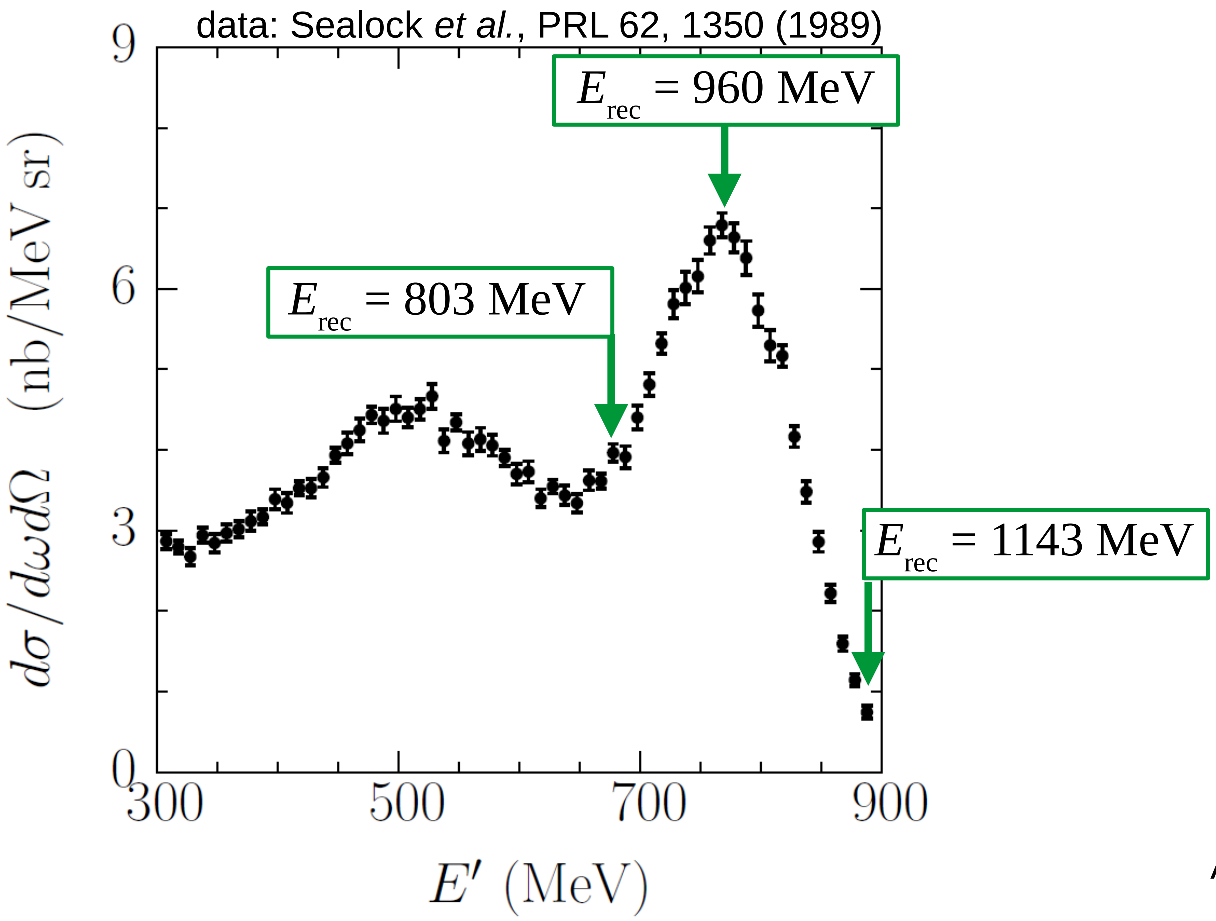

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“Unknown” monochromatic electron beam

θ (deg)	37.5	37.5	37.1	36.0	36.0
E' (MeV)	976	768	615	487.5	287.5
$\Delta E'$ (MeV)	5	5	5	5	2.5

Assuming ϵ = 25 MeV:

rec. E	1285 ± 8	960 ± 7	741 ± 7	571 ± 6	333 ± 3
true E	1299	961	730	560	320

A.M.A., PoS(NuFact2014)004

“Unknown” monochromatic electron beam

θ (deg)	37.5	37.5	37.1	36.0	36.0
E' (MeV)	976	768	615	487.5	287.5
$\Delta E'$ (MeV)	5	5	5	5	2.5

What is the appropriate ϵ value?

true E	1299	961	730	560	320
ϵ	33 ± 5	26 ± 5	16 ± 5	16 ± 3	13 ± 3

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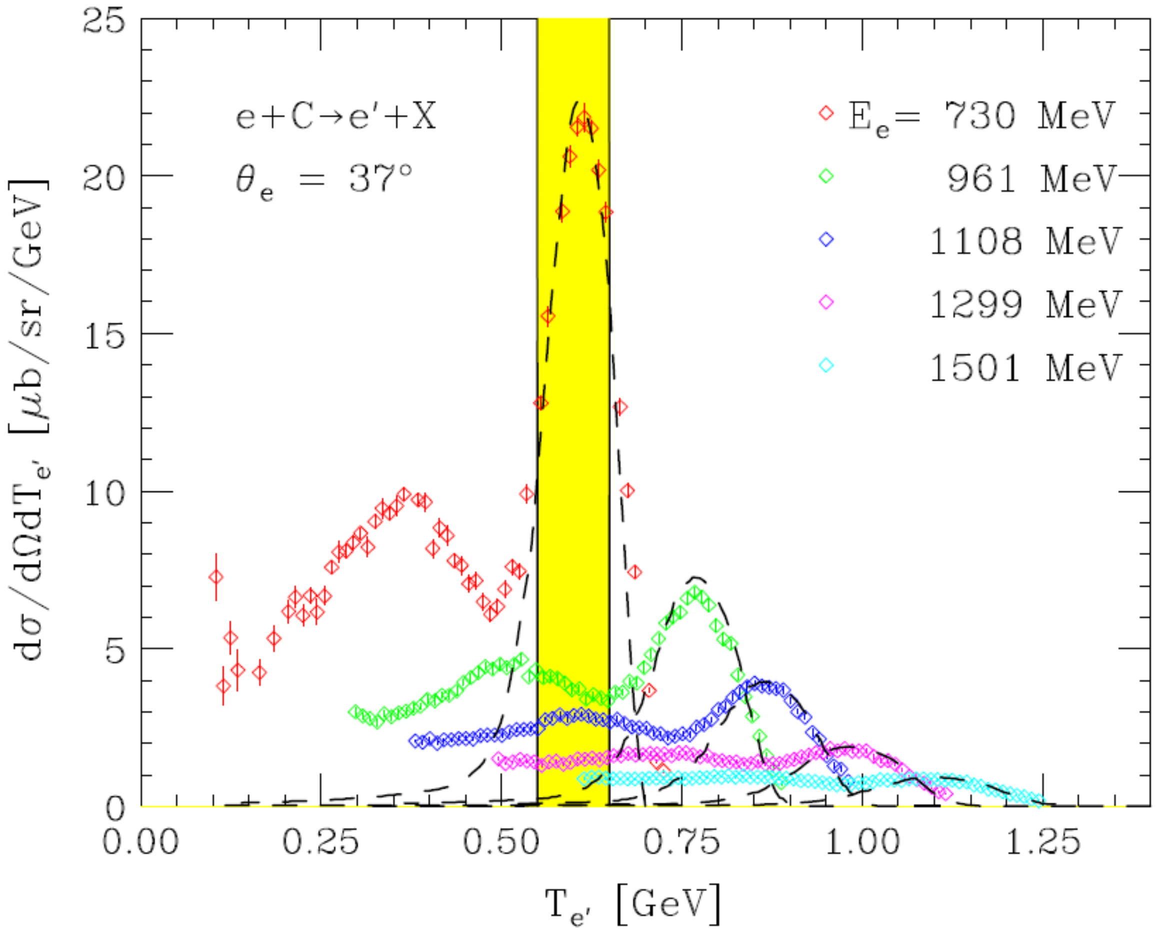
different E or different Q^2 or different θ → **different ϵ**

Backgrounds



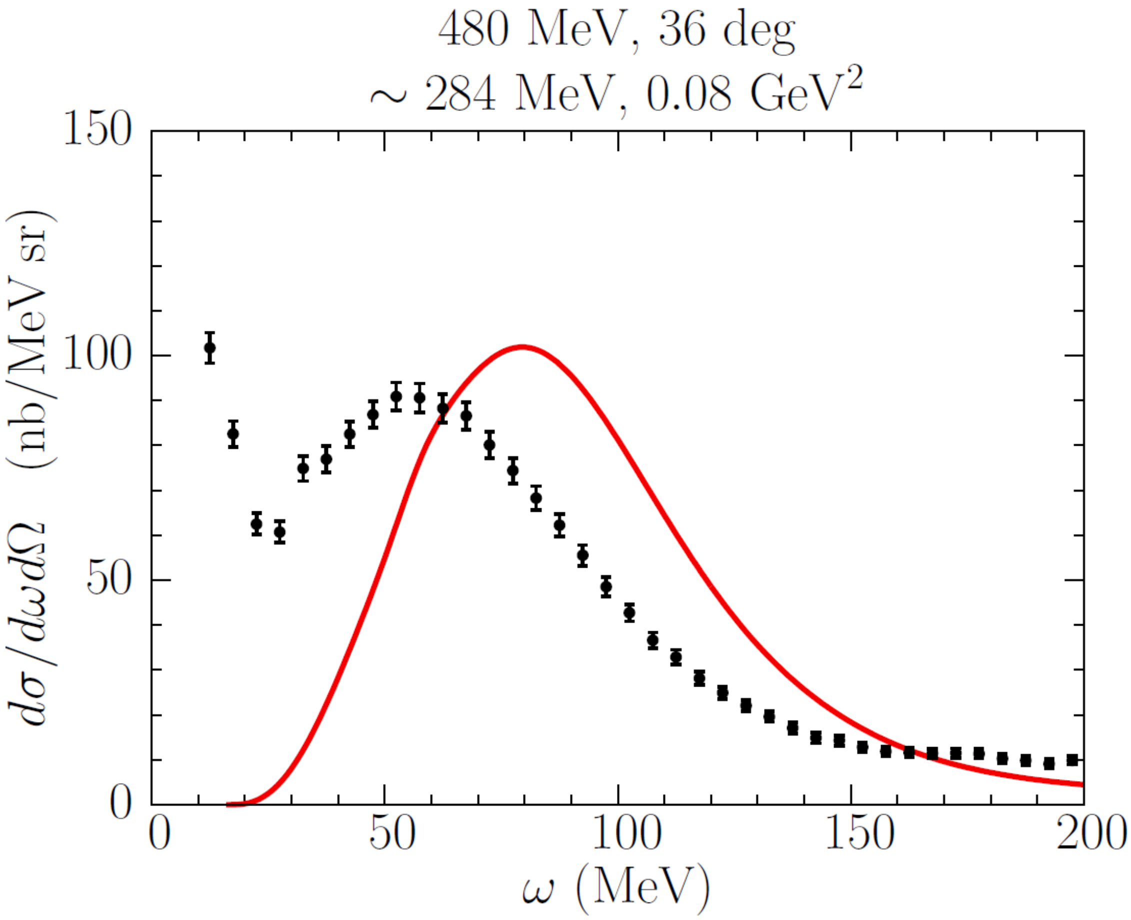
O. Benhar @ NuFact11, PRL 105, 132301 (2010)

Accurate energy reconstruction

- Accurate double differential cross sections for neutrinos are the basic requirement to reconstruct neutrino energy
- They depend on the ground state properties (momentum and energy distributions), elementary cross sections (quasielastic, resonance excitation, deep-inelastic scattering), contributions of scattering on nucleon pairs, and final-state interactions
- The best way to test our cross section models is to confront them with electron scattering data

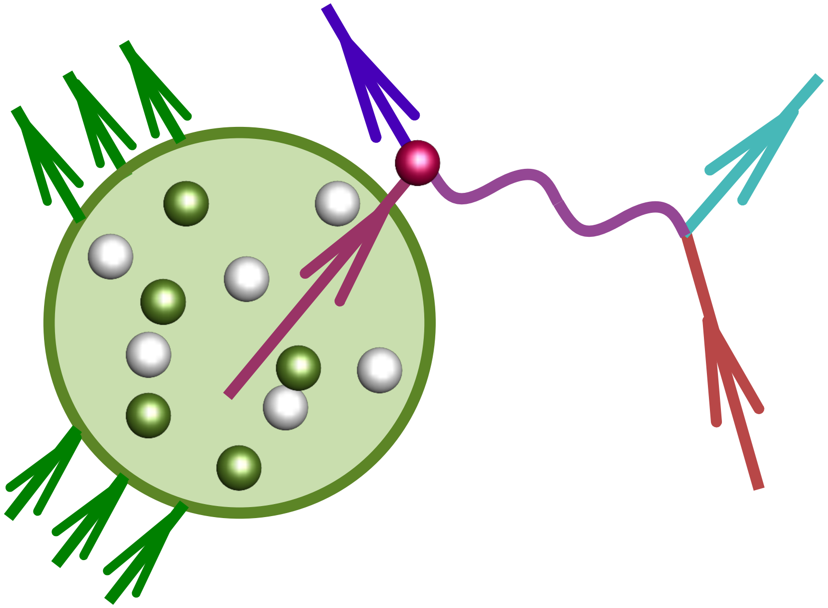
Final state interactions

What is missing?



Energy conservation

$$E_\nu + M_A = E_\mu + E_{A-1} + E_{p'}$$



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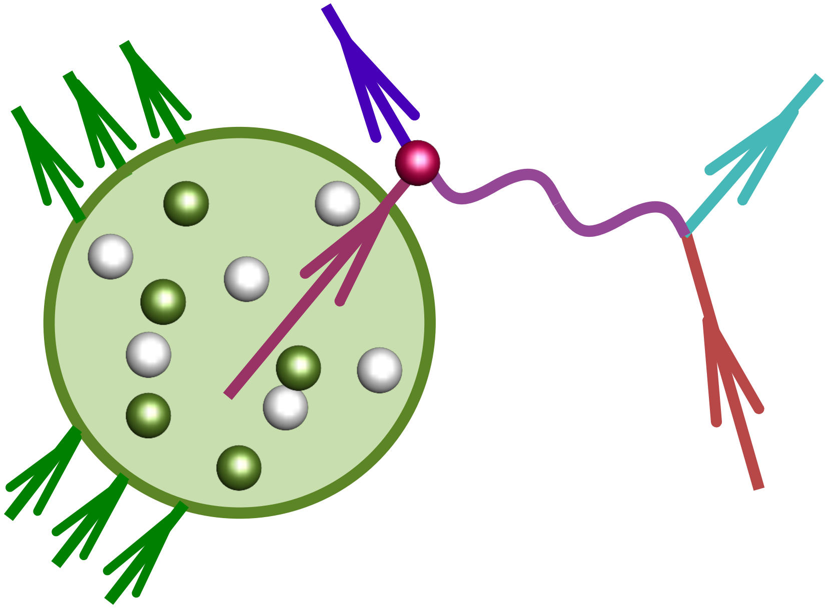
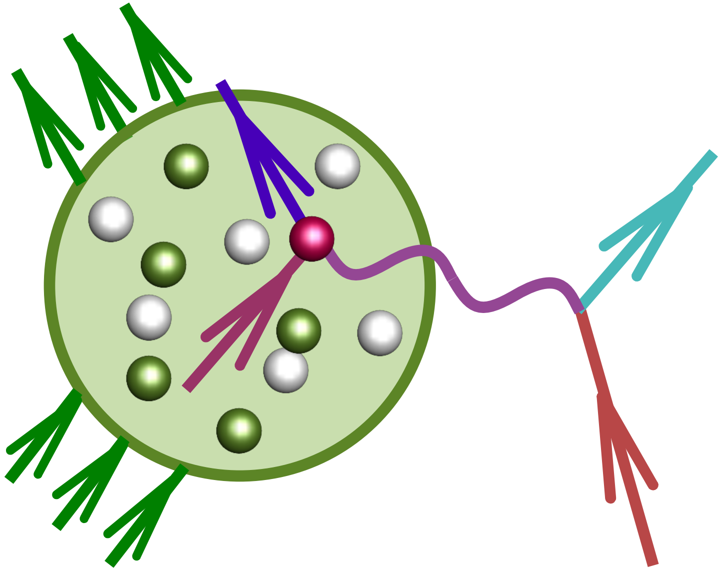


$$E_\nu + M_A = E_\mu + E_{A-1} + E_{p'} + U_V(p')$$



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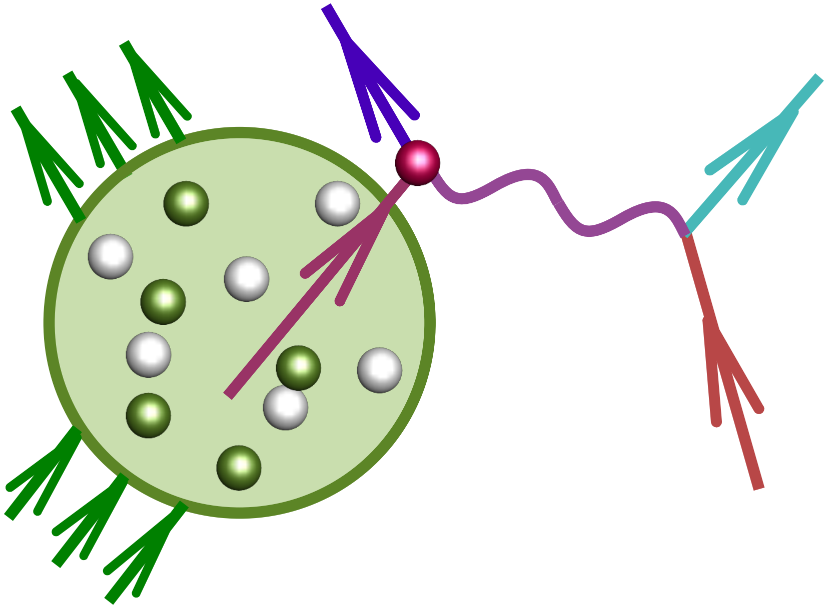
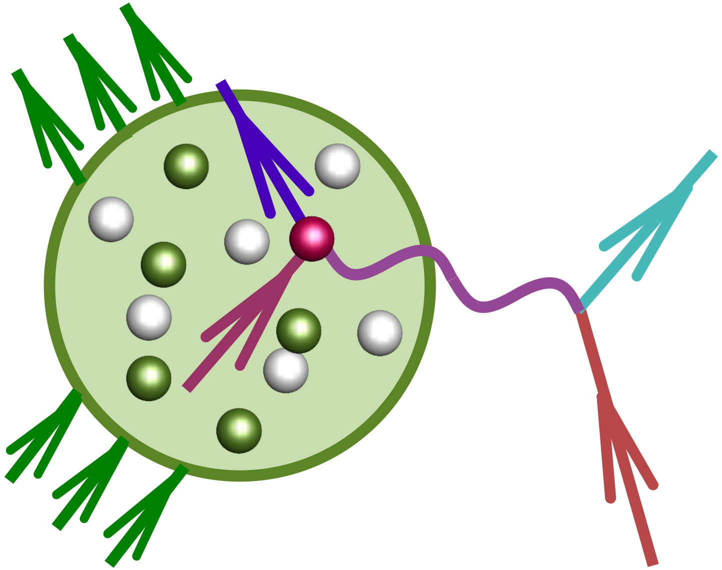


$$E_\nu + M_A \sim E_\mu + E_{A-1} + E_{p'} + U_V(p')$$



Final-state interactions

Soft interactions with the spectator system change the energy spectrum of the struck nucleon. When collisions happen, additional nucleons appear in the final states.

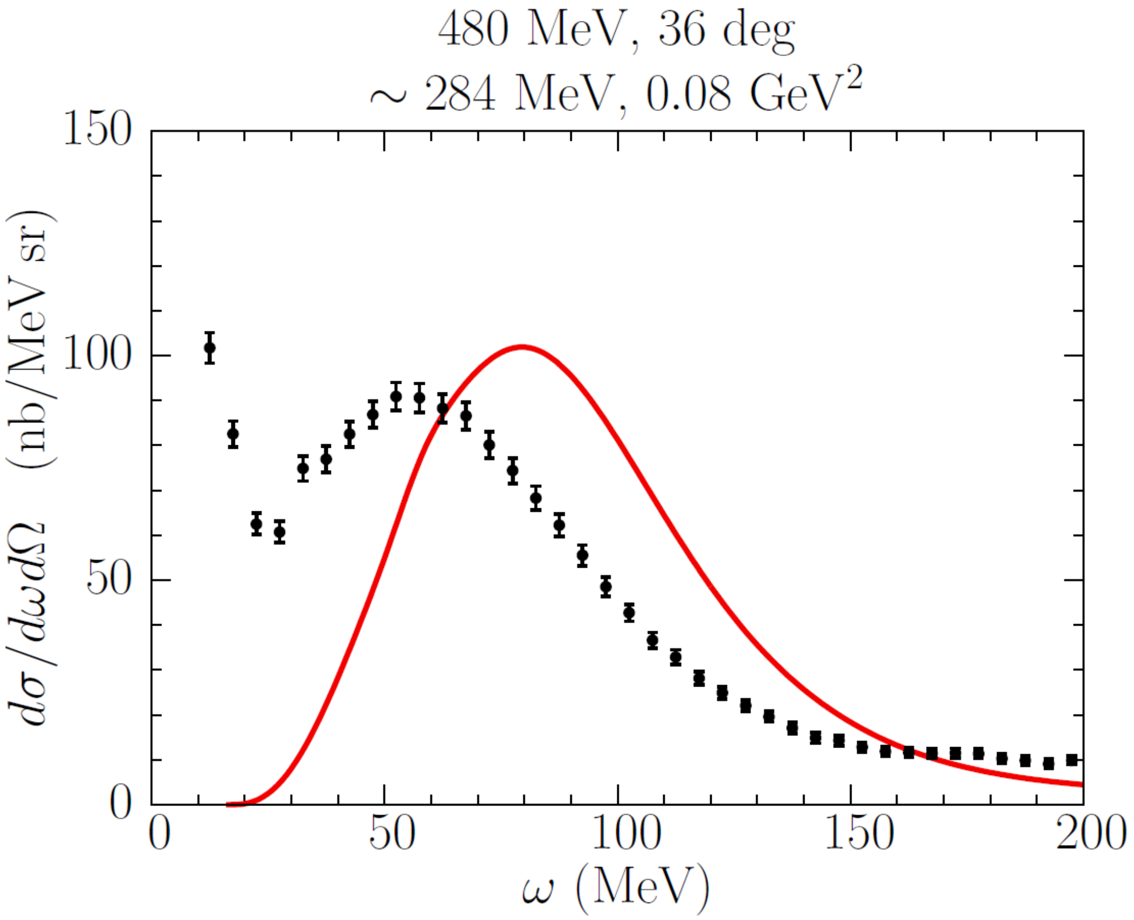
Those effects can be described using an optical potential

- the real part modifies the struck nucleon's energy spectrum, it is $\neq \sqrt{M^2 + p'^2}$
- the imaginary part introduces absorption of the struck nucleons and produces multiple hadrons in the final state (intranuclear cascade)

$$e^{iE_{p'}t} \rightarrow e^{i(E_{p'}+U)t} = e^{i(E_{p'}+U_V+iU_W)t} = e^{i(E_{p'}+U_V)t}\, e^{-U_W t}$$

Horikawa *et al.*, PRC 22, 1680 (1980)

What is missing?



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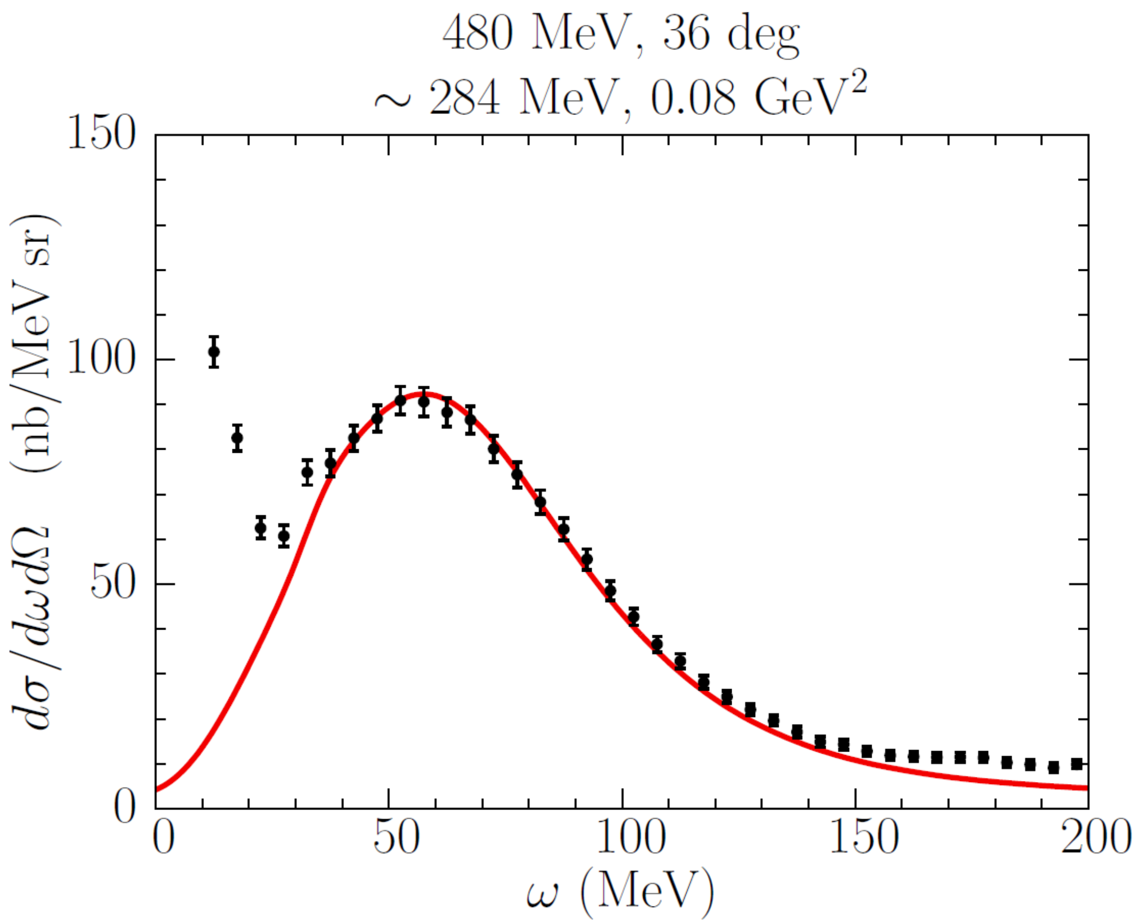


Two-body currents

Multinucleon final states

Two (or more) nucleons in the final state may come from

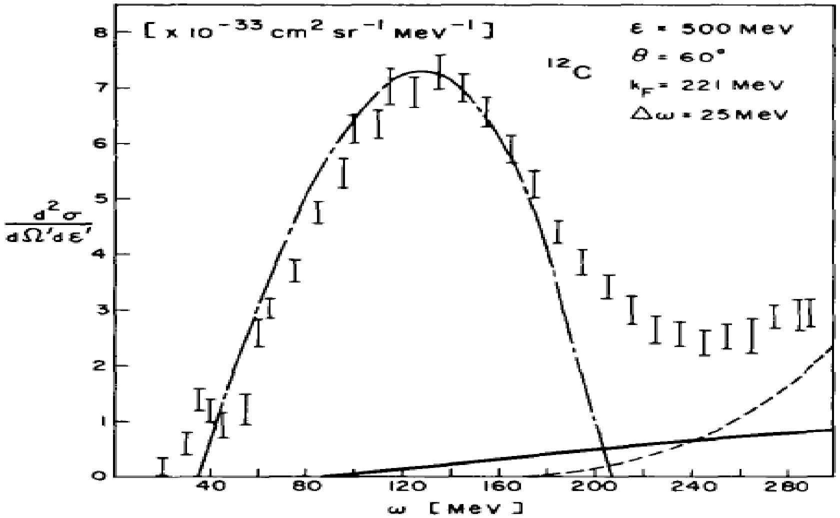
- initial-state correlations
- two-body reaction mechanisms, such as meson exchange currents (MEC)
- final-state interactions (intranuclear cascade)

We cannot distinguish these processes, so they would need to be added at the level of amplitudes, and they interfere.

Shimizu & Faessler, Nucl. Phys. A 333, 495 (1980)
Alberico *at al.*, Ann. Phys. 154, 356 (1984)

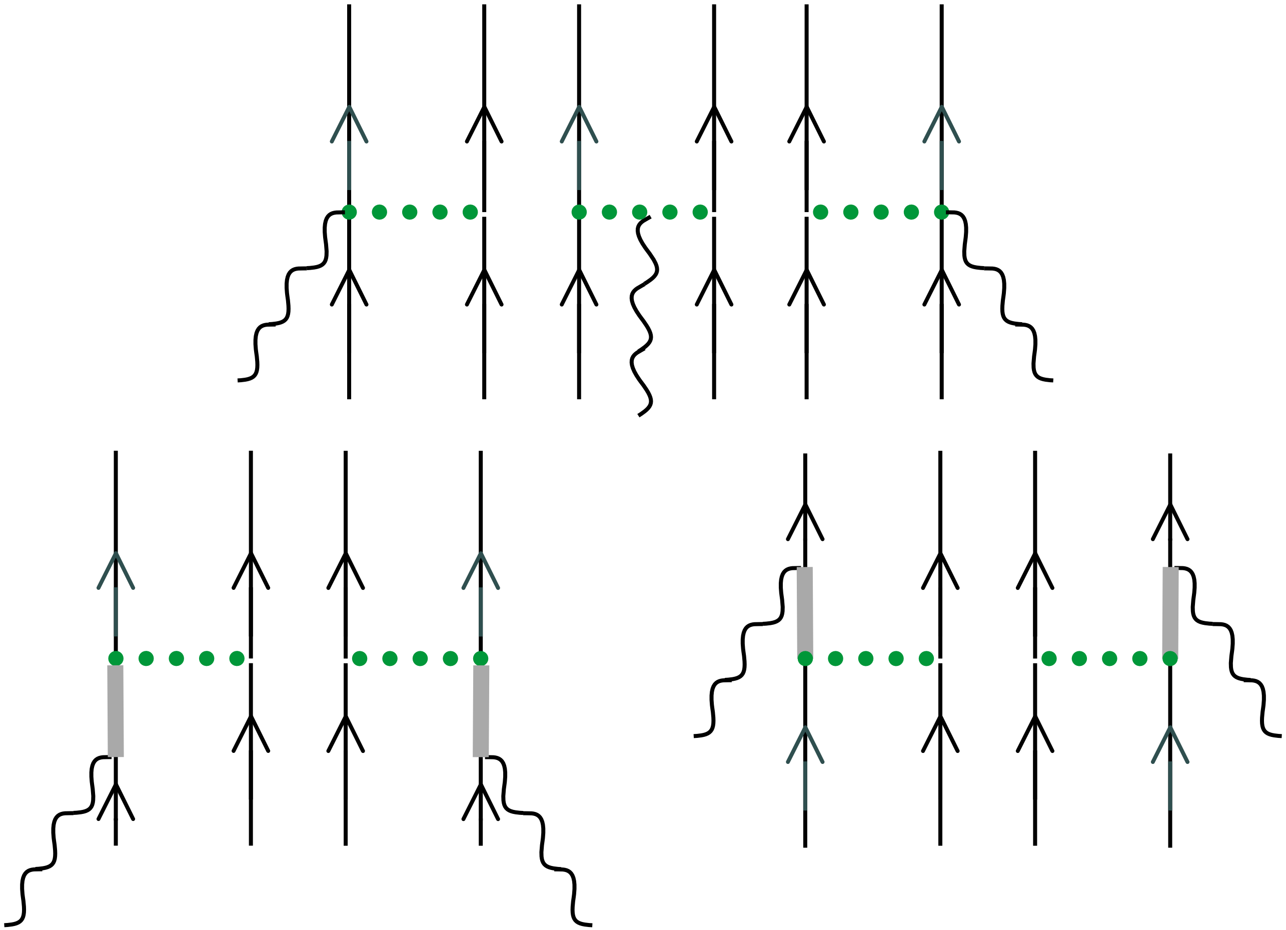
Origins of MEC

“Calculations of [QE + Δ] with a simple uncorrelated Fermi gas model of the nucleus have provided a surprisingly good fit to experiment [Moniz *et al.*(‘71)] … in the region between the peaks, however, has consistently underestimated the experimental values. … we investigate **whether MEC contributions can fill in this `dip’ region**.”



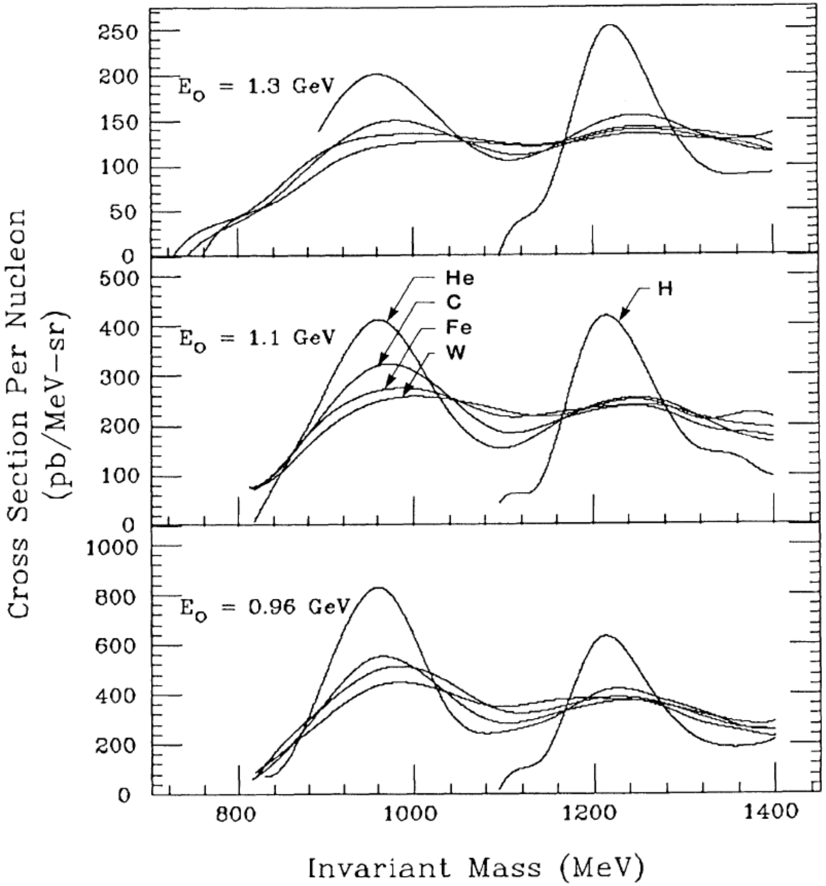
Donnelly, van Orden, de Forest & Hermans, PLB 76, 393 (1978)

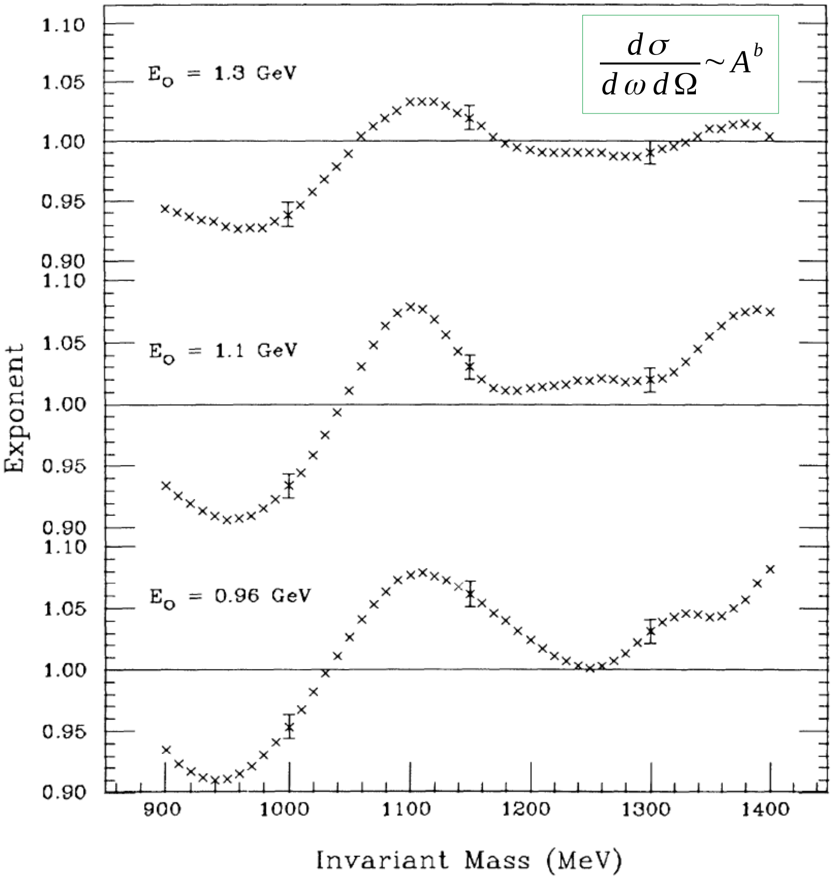
Contributing processes



see e.g. Dekker, Brussaard & Tijon, PLB 266, 249 (1991)

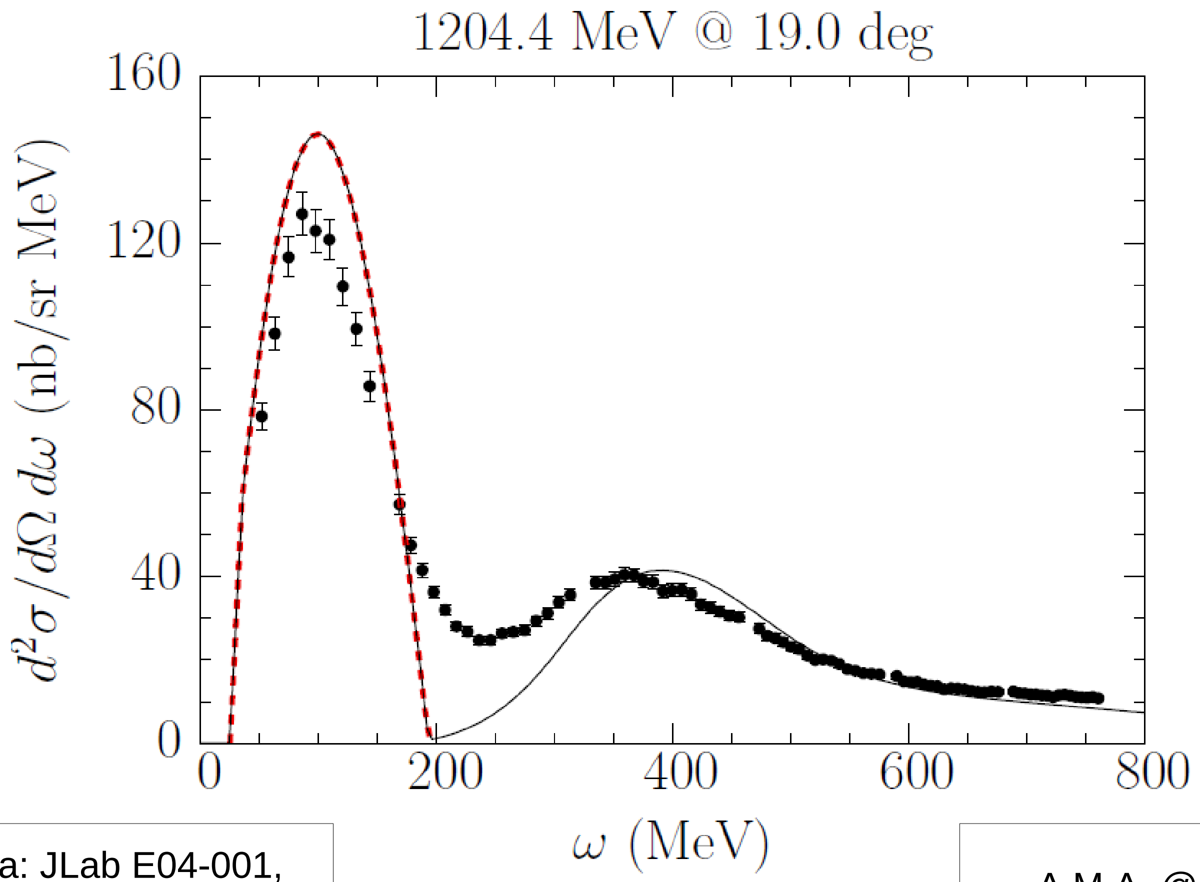
Cross section scaling





Sealock *et al.*, PRL 62, 1350 (1989)

Relativistic Fermi gas

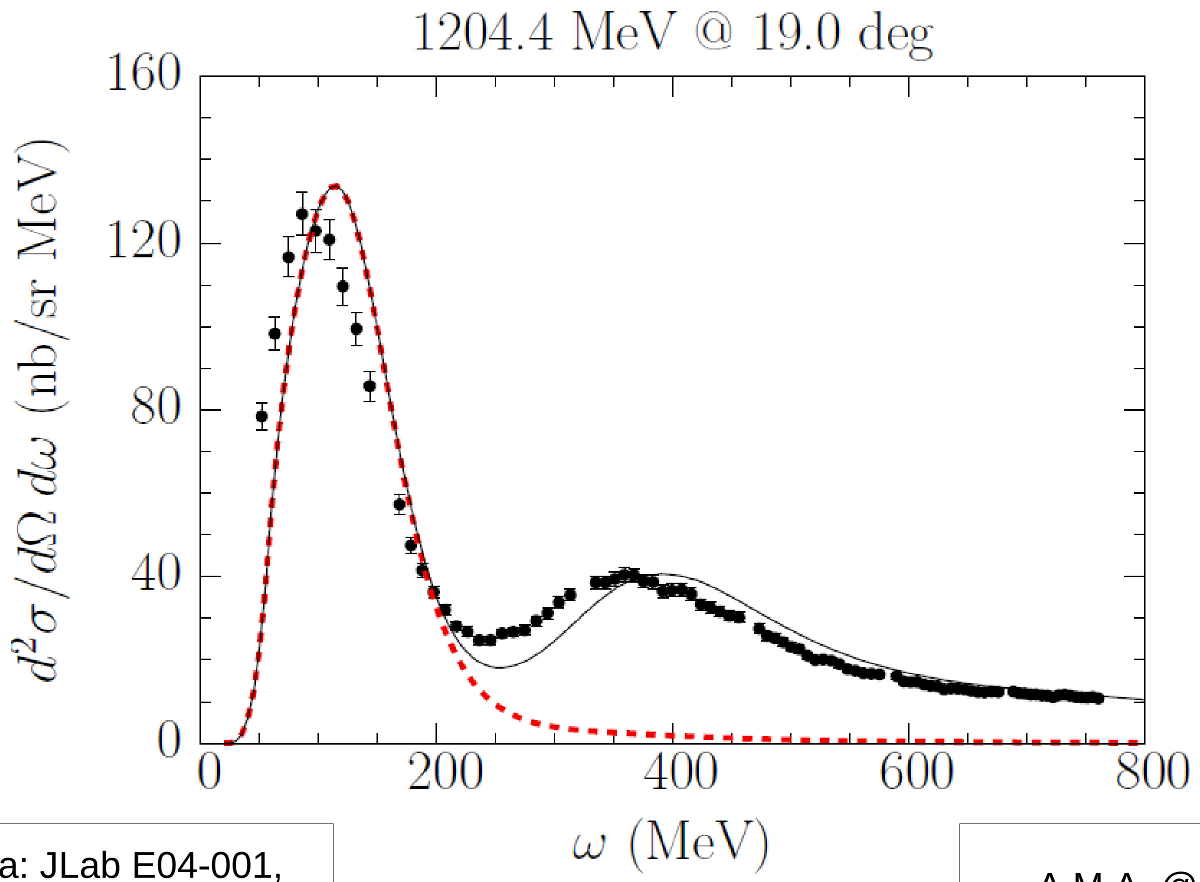


data: JLab E04-001, preliminary

A.M.A. @ NuInt18

Spectral function, no FSI

QE treated as in
A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

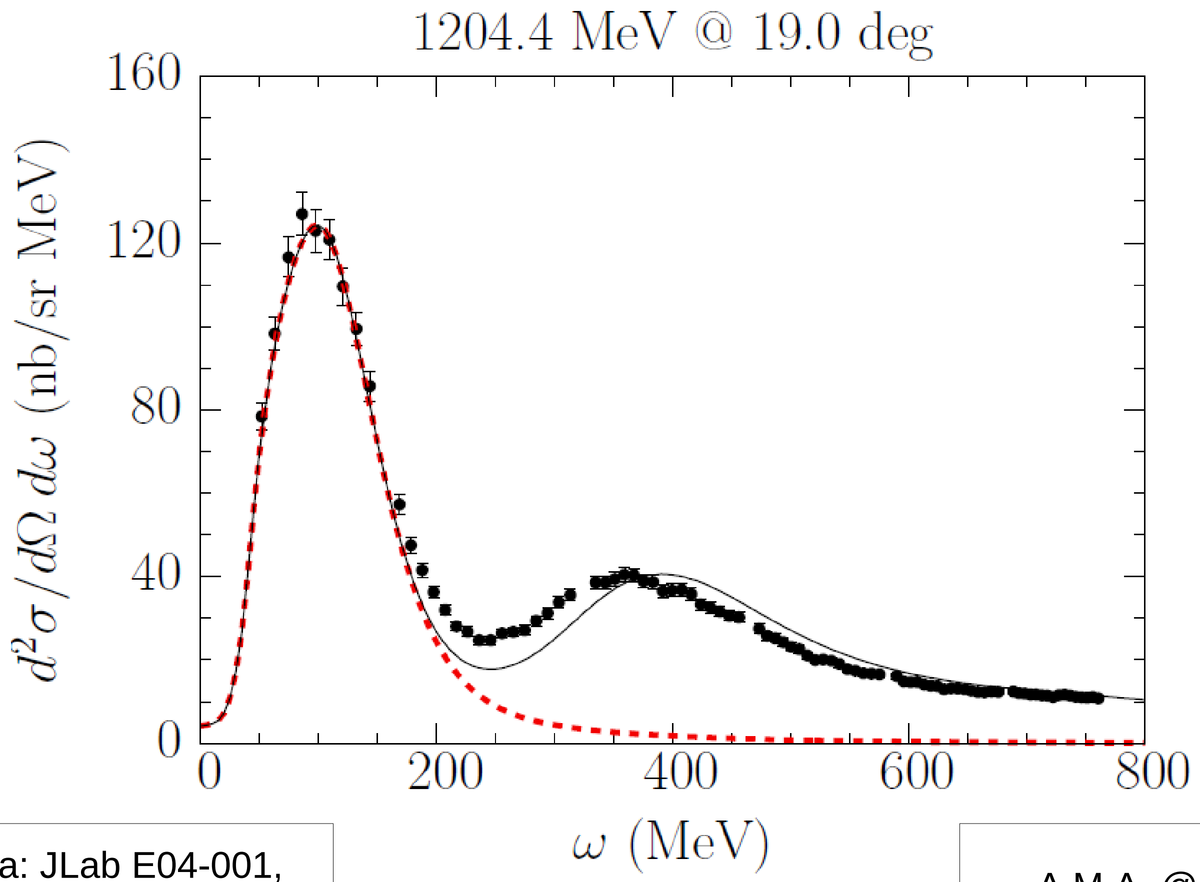


data: JLab E04-001,
preliminary

A.M.A. @ NuInt18

Spectral function, FSI for QE

QE treated as in
A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

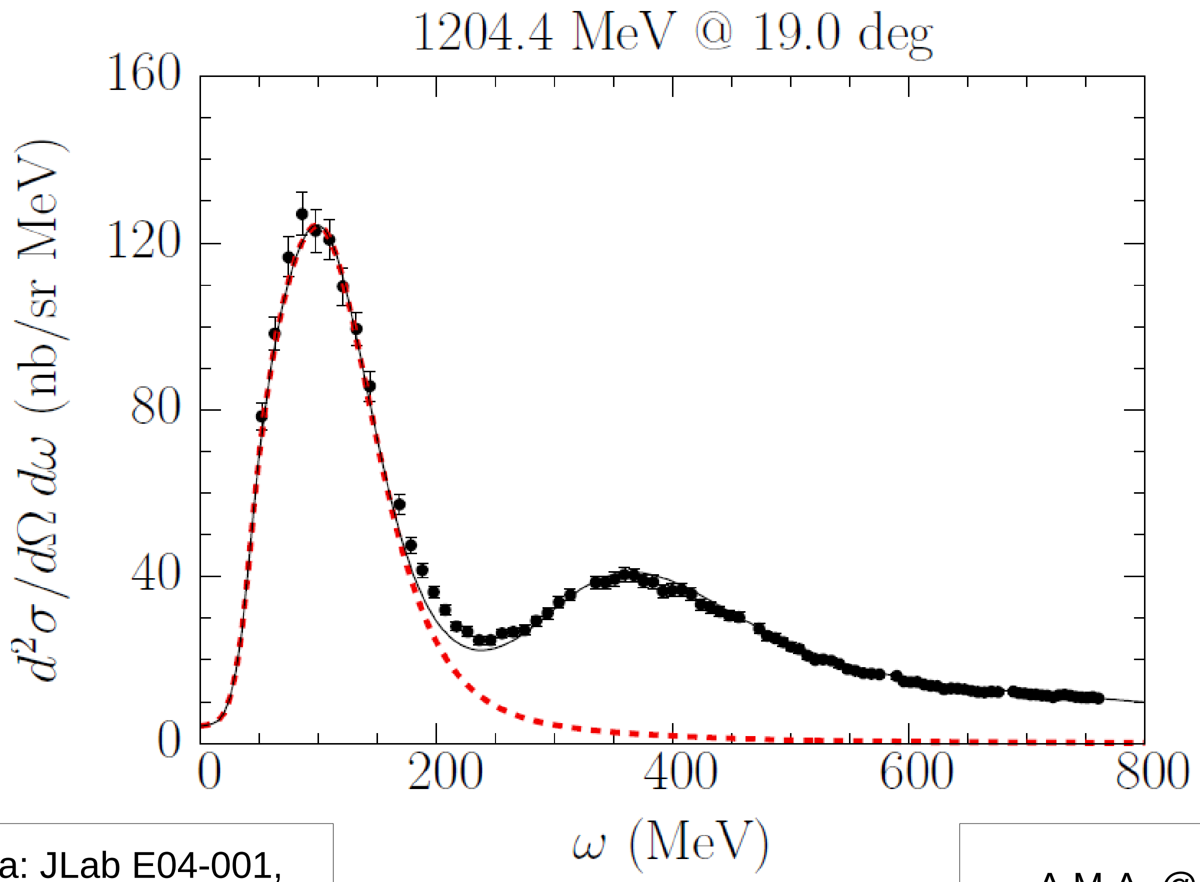


data: JLab E04-001, preliminary

A.M.A. @ NuInt18

Spectral function, FSI for QE, shifted Δ

QE treated as in
A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

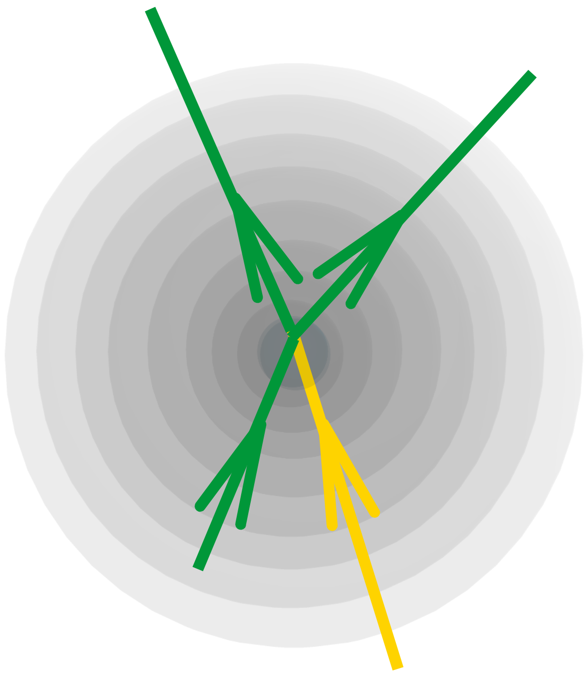


data: JLab E04-001, preliminary

A.M.A. @ NuInt18

Miscellanea

Coulomb effects



deeper binding

deceleration

$$\nu_\ell + n \to \ell^- + p,$$
$$\bar{\nu}_\ell + p \to \ell^+ + n,$$

acceleration

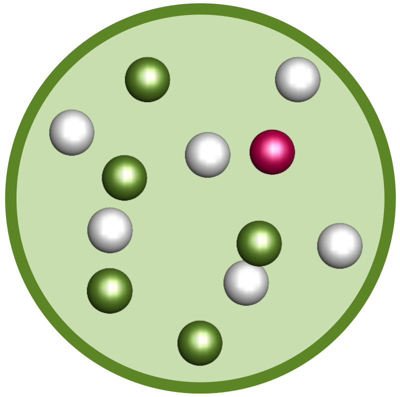
deeper potential

ν-$\bar{\nu}$ difference relevant for CP violation measurements:
O(10 MeV) for carbon, *O*(20 MeV) for argon

Off-shell effects

Our targets (carbon, oxygen, argon, iron) are stable against emission of nucleons. These nuclei cannot lower their energy by emission of a nucleon:

$$M_A = E_{A-1} + E_p < E_{A-1} + M$$



This means that every nucleon’s energy must be lower than its mass.

V.R. Pandharipande, Nucl. Phys. B (Proc. Suppl.) 112, 51 (2002)

Off-shell effects

In a nuclear model, the initial nucleon's energy may

- differ from the on-shell energy by a constant

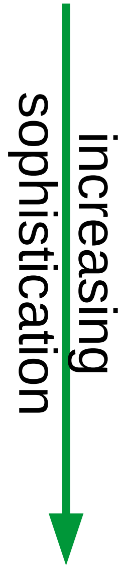
$$E_p = \sqrt{M^2 + \boldsymbol{p}^2} - \epsilon\,,$$

- be a function of the momentum

$$E_p = \sqrt{M^2 + \boldsymbol{p}^2} - \epsilon(|\boldsymbol{p}|)\,,$$

- lack 1:1 correspondence with momentum

$$E_p = M - E\,.$$



Off-shell effects

The elementary cross section

$$\frac{d\sigma_{\ell N}^{\mathrm{elem}}}{dE_{\mathbf{k}'}d\Omega dE_{\mathbf{p}'}d\Omega_{\mathbf{p}'}} \propto L_{\mu\nu}H^{\mu\nu}$$

contains the leptonic and hadronic tensors

$$L_{\mu\nu} \propto j_{\mu}^{\mathrm{lept}} j_{\nu}^{\mathrm{lept}*} \quad \text{and} \quad H^{\mu\nu} \propto j_{\mathrm{hadr}}^{\mu} j_{\mathrm{hadr}}^{\nu *}$$

With only the hadronic one affected by the off-shell effects.

Nucleon current

On the mass shell,

$$j^{\mu}_{\mathrm{hadr}} = \overline{u}(\mathbf{p}', s') \left(\gamma^{\mu} F_1 + i\sigma^{\mu\kappa} \frac{q_{\kappa}}{2M} F_2 + \dots \right) u(\mathbf{p}, s)$$

or equivalently

$$j^{\mu}_{\mathrm{hadr}} = \overline{u}(\mathbf{p}', s') \left(\gamma^{\mu}(F_1 + F_2) - \frac{(p + p')^{\mu}}{2M} F_2 + \dots \right) u(\mathbf{p}, s)$$

De Forest approximation

In an event with the energy transfer ω to a nucleon of energy E_p, described in the impulse approximation

$$\mathbf{q}+\mathbf{p}=\mathbf{p}'$$
$$\omega+E_p=E_{p'}$$

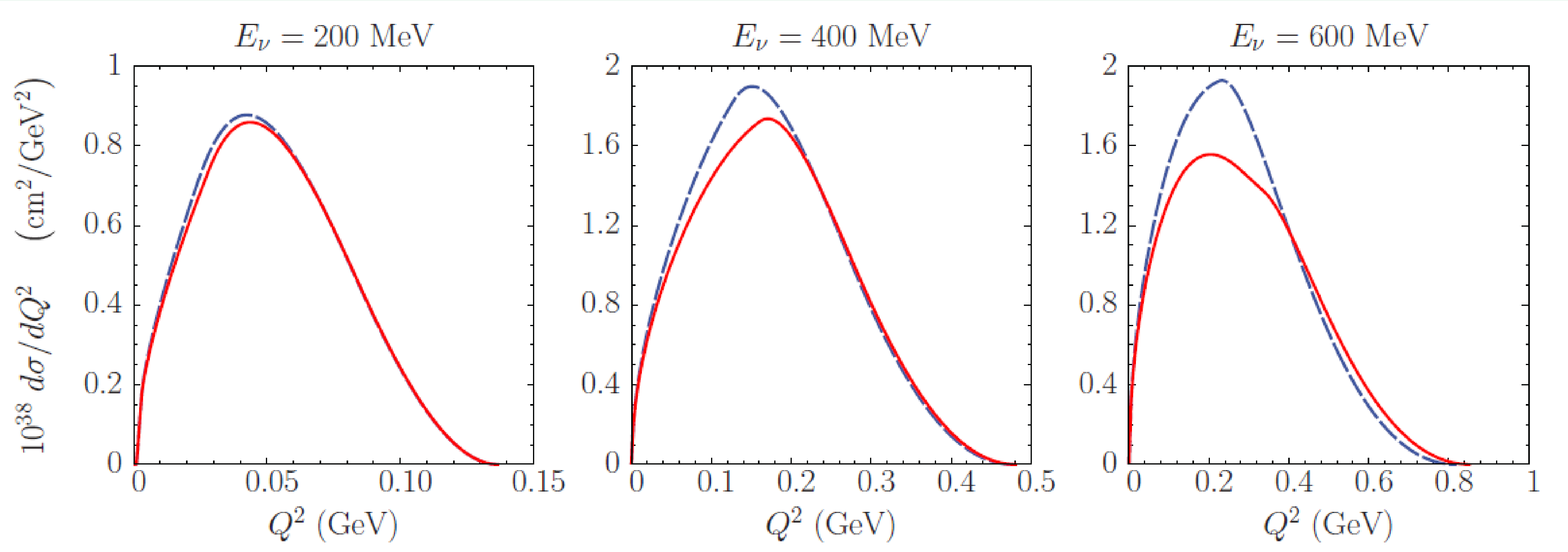
one can approximate the nuclear off-shell current by the on-shell current with a different energy transfer $\widetilde{\omega}$,

$$\mathbf{q}+\mathbf{p}=\mathbf{p}'$$
$$\widetilde{\omega}+\widetilde{E}_p=\widetilde{\omega}+\sqrt{M^2+\mathbf{p}^2}=E_{p'}$$

and $\omega-\widetilde{\omega}$ is assumed to be transferred to the $A-1$ nucleon spectator system.

T. de Forest, Jr, NPA 392, 232 (1983)

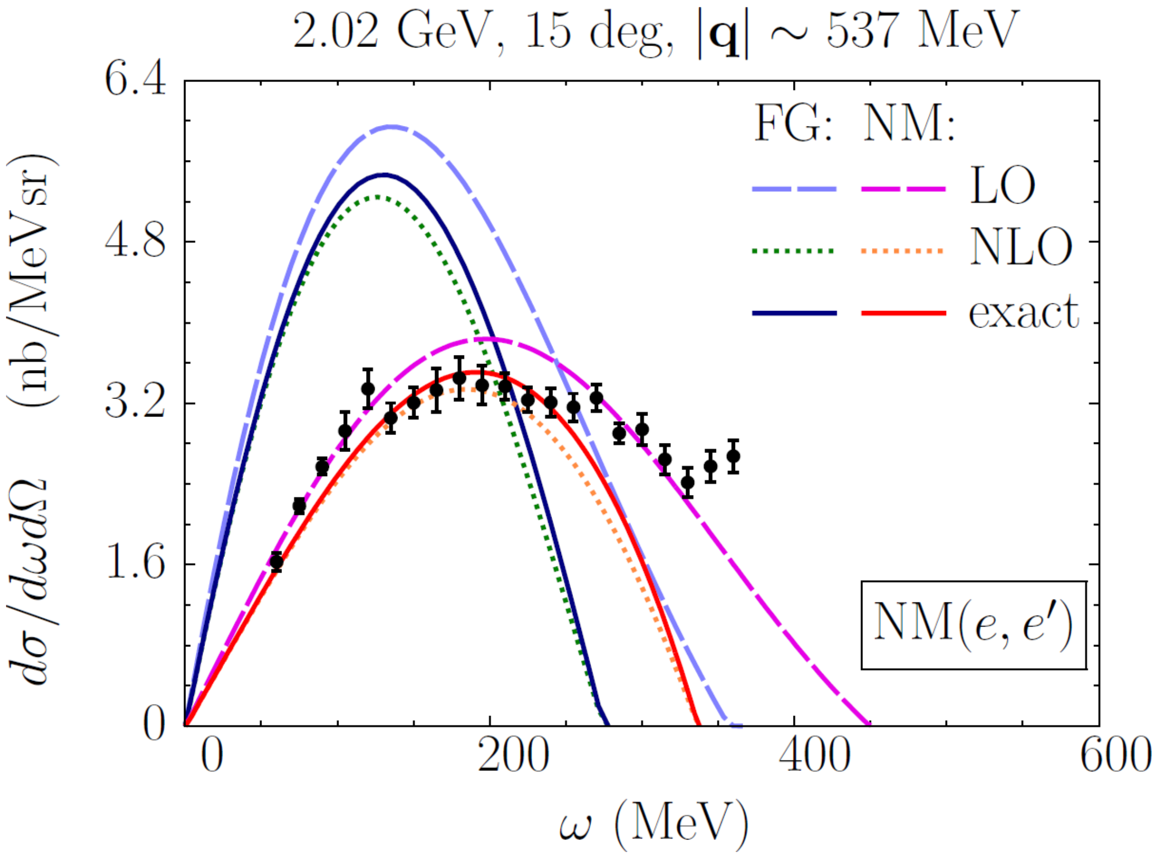
Importance of fully relativistic kinematics



A.M.A. & O. Benhar, PRC 83, 054616 (2011)

Sizable differences between the **relativistic** and **nonrelativistic** cross sections for neutrino energies *O*(500 MeV).

Importance of fully relativistic kinematics



A.M.A. & O. Benhar, PRC 83, 054616 (2011)

At |q| ~ 500 MeV, semi-relativistic result **5% lower** than the relativistic one.

Summary

- The success of the long-baseline neutrino program (DUNE and Hyper-Kamiokande) requires reliable cross sections.
- Theory and generator development needed for many years to come. New experimental analyses to propose and perform.
- Plenty of new experimental data coming soon from the Short-Baseline Neutrino program, T2K, NOvA, and MINERvA.
- It is an excellent time to start your involvement in neutrino physics!

Thank you!

Electrons and neutrinos

For scattering in a given angle and energy, ν's and e's differ almost exclusively due to the elementary cross sections.

Comparing to electron-scattering data we are sensitive to any problems with

- the energy and momentum distributions of the nucleons bound in the target
- final-state interactions
- the vector part of the cross sections, channel by channel

Electron data allow MC validation, reduction of systematic uncertainties, as well as their rigorous determination.

A.M.A., A. Friedland, S. W. Li, O. Moreno, P. Schuster, N. Toro & N. Tran, PRD 101, 053004 (2020)