(Relativistic) Mean Field models (and beyond)

Alexis Nikolakopoulos

NuINT school, Sao Paolo, Brazil

11 April 2024

(Relativistic) Mean field models

- Non relativistic mean-field : Hartree-Fock

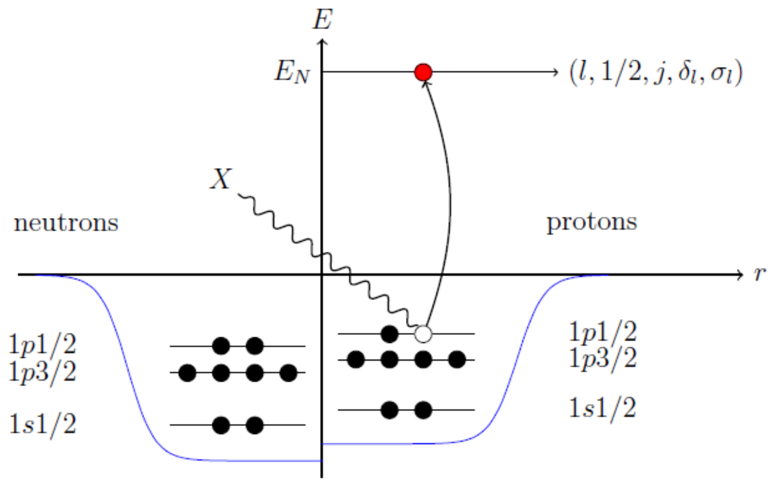
- The mean field idea : energy-density functional

→ beyond the mean-field : HF-CRPA

- Scattering in a relativistic mean field & approximations

Fermilab

Self-consistent mean-field



Mean field nucleus

- Mean field potential
- Single-particle wavefunctions
- Binding energies
- Orthogonal states (→ Pauli-blocking)

RMF

- Non-linear extended sigma-omega model

Extension of the original σ-ω Walecka model (Ann. Phys.83,491 (1974)).

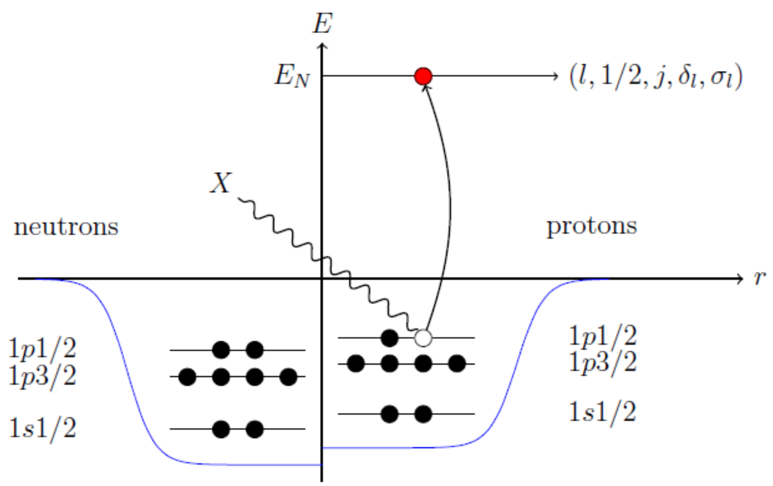
HF-SkE2

- Hartree-Fock with extended Skyrme force

M. Waroquier et al. / Effective Skyrme-type interaction (I)
Nuclear Physics **A404** (1983) 269–297

Fermilab

Self-consistent mean-field



Mean field nucleus

- Mean field potential
- Single-particle wavefunctions
- Binding energies
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<u>Effective interactions</u> constrained by properties of nuclei and nuclear matter

	E/A	r_p	r_n	r_c
		^{16}O		
SkE2	−7.92	2.63	2.60	2.68
SkE4	−7.96	2.65	2.62	2.70
SkIII	−8.03	2.64	2.61	2.70
exp	−7.98			2.71 [a])
		^{90}Zr		
SkE2	−8.67	4.17	4.24	4.21
SkE4	−8.71	4.22	4.29	4.26
SkIII	−8.69	4.26	4.31	4.30
exp	−8.71			4.27 [c])

	t_4 (MeV · fm^8)	K (MeV)	$(E/A)_{\text{n.m.}}$ (MeV)	k_F (fm^{-1})	m^*/m	a_τ (MeV)
SkE2	−15808.79	200	−16.0	1.33	0.72	29.7
SkE4	−12258.97	250	−16.0	1.31	0.75	30.0
SkIII	0.0	356	−15.87	1.29	0.76	28.2

Self-consistent mean-field: Skyrme Hartree-Fock

Particle states solutions of Shrodinger equation with potentials

$$-\nabla\left[\frac{\hbar^2}{2m_q^*(\mathbf{r})}\nabla\phi_{\alpha_q}(\mathbf{r})\right]+[U_q(\mathbf{r})-i\mathbf{W}_q(\mathbf{r})\cdot(\nabla\times\sigma)]\phi_{\alpha_q}(\mathbf{r})=\varepsilon_{\alpha_q}^{\mathrm{HF}}\phi_{\alpha_q}(\mathbf{r}) .$$

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Self-consistent mean-field: Skyrme Hartree-Fock

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The potentials depend on the densities:

$$\begin{aligned}U_q(\mathbf{r})=&\,t_0[(1+\tfrac{1}{2}x_0)\rho_{\mathrm{tot}}-(\tfrac{1}{2}+x_0)\rho_q]+\tfrac{1}{4}(t_1+t_2)\tau_{\mathrm{tot}}+\tfrac{1}{8}(t_2-t_1)\tau_q\\&+\tfrac{1}{8}(t_2-3t_1)\nabla^2\rho_{\mathrm{tot}}+\tfrac{1}{16}(3t_1+t_2)\nabla^2\rho_q+\tfrac{1}{4}t_3(\rho_{\mathrm{tot}}^2-\rho_q^2)\\&-\tfrac{1}{2}W_0'(\nabla\cdot\mathbf{J}_{\mathrm{tot}}+\nabla\cdot\mathbf{J}_q)+\delta_{qp}V^C(\mathbf{r})+\tfrac{1}{24}t_4[2\rho_{\mathrm{tot}}\tau_{\mathrm{tot}}-2\rho_q\tau_q\\&+\tfrac{5}{2}\rho_q\nabla^2\rho_q-\tfrac{5}{2}\rho_{\mathrm{tot}}\nabla^2\rho_{\mathrm{tot}}+\tfrac{5}{4}(\nabla\rho_q)^2-\tfrac{5}{4}(\nabla\rho_{\mathrm{tot}})^2+\tfrac{1}{2}J_{q'}^2]\,,\end{aligned}$$

Which depend on the particle states:

$$\rho_q(\mathbf{r})=\sum_{\alpha_q\gamma_q}\rho_{\alpha\gamma}^{(q)}\phi_{\alpha_q}^*(\mathbf{r})\phi_{\gamma_q}(\mathbf{r})\,,$$

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Self-consistent mean-field: Skyrme Hartree-Fock

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Which depend on the particle states:

$$\rho_q(\mathbf{r})=\sum_{\alpha_q\gamma_q}\rho_{\alpha\gamma}^{(q)}\phi_{\alpha_q}^*(\mathbf{r})\phi_{\gamma_q}(\mathbf{r})\,,$$

→ Iterate until convergence

Self-consistent mean-field: Skyrme Hartree-Fock

$$U_q(\mathbf{r}) = t_0[(1+\tfrac{1}{2}x_0)\rho_{\rm tot} - (\tfrac{1}{2}+x_0)\rho_q] + \tfrac{1}{4}(t_1+t_2)\tau_{\rm tot} + \tfrac{1}{8}(t_2-t_1)\tau_q$$
$$+\tfrac{1}{8}(t_2-3t_1)\nabla^2\rho_{\rm tot} + \tfrac{1}{16}(3t_1+t_2)\nabla^2\rho_q + \tfrac{1}{4}t_3(\rho_{\rm tot}^2 - \rho_q^2)$$
$$-\tfrac{1}{2}W_0'(\boldsymbol{\nabla}\cdot\boldsymbol{J}_{\rm tot} + \boldsymbol{\nabla}\cdot\boldsymbol{J}_q) + \delta_{qp}V^C(\mathbf{r}) + \tfrac{1}{24}t_4[2\rho_{\rm tot}\tau_{\rm tot} - 2\rho_q\tau_q$$
$$+\tfrac{5}{2}\rho_q\nabla^2\rho_q - \tfrac{5}{2}\rho_{\rm tot}\nabla^2\rho_{\rm tot} + \tfrac{5}{4}(\boldsymbol{\nabla}\rho_q)^2 - \tfrac{5}{4}(\boldsymbol{\nabla}\rho_{\rm tot})^2 + \tfrac{1}{2}J_{q'}^2],$$

Remember!

Parameters are fit to reproduce
(static) nuclear properties
Over large mass range

Scattering in non-relativistic mean-field

The relativistic formulation in terms of Dirac spinors

$$\Gamma_V^\mu = F_1(Q^2)\gamma^\mu - \frac{F_2(Q^2)}{2m_N}(\gamma^\mu \not{Q} - \not{Q}\gamma^\mu)$$

Needs to undergoe non-relativistic reduction to act on non-rel single-particle states
In coordinate space

For example:

$$\overline{u} F_A \gamma^\mu \gamma^5 u \quad \rightarrow \quad J_A^0 = \frac{F_A}{2M}\vec{\sigma}\cdot\left(\vec{\nabla} - \overleftarrow{\nabla}\right), \quad \vec{J}_A = F_A \vec{\sigma}$$

By writing $u(p) = \begin{bmatrix} \chi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{2M_N}\chi_s \end{bmatrix}$ and expanding in p/M_N

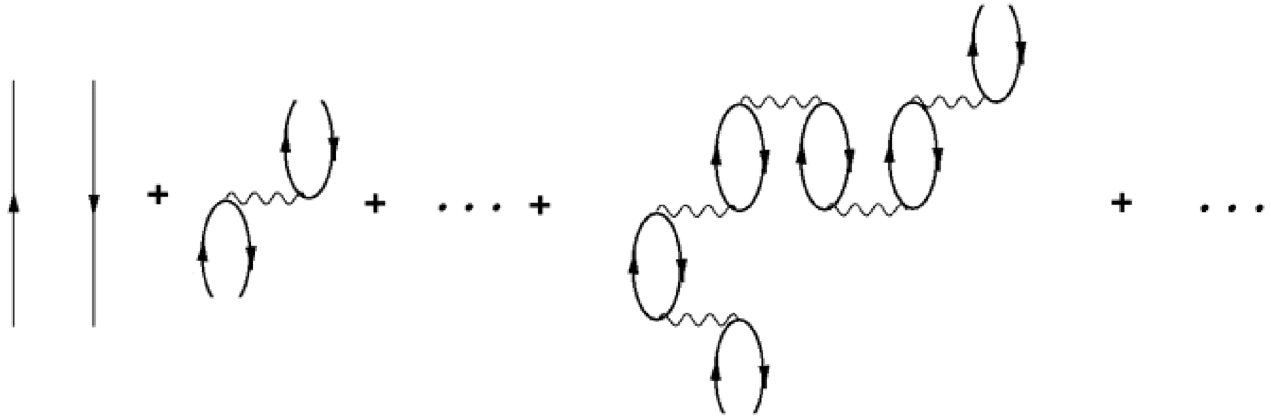
See e.g. [J. D. Walecka, Theoretical Nuclear And Subnuclear Physics]

Fermilab

Beyond the mean field: CRPA calculations in coordinate space

$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar}\int dx \int dx' \, \underline{\Pi^{(0)}(x_1, x; \omega)} \, \widetilde{V}(x, x') \, \Pi^{(RPA)}(x', x_2; \omega)$$

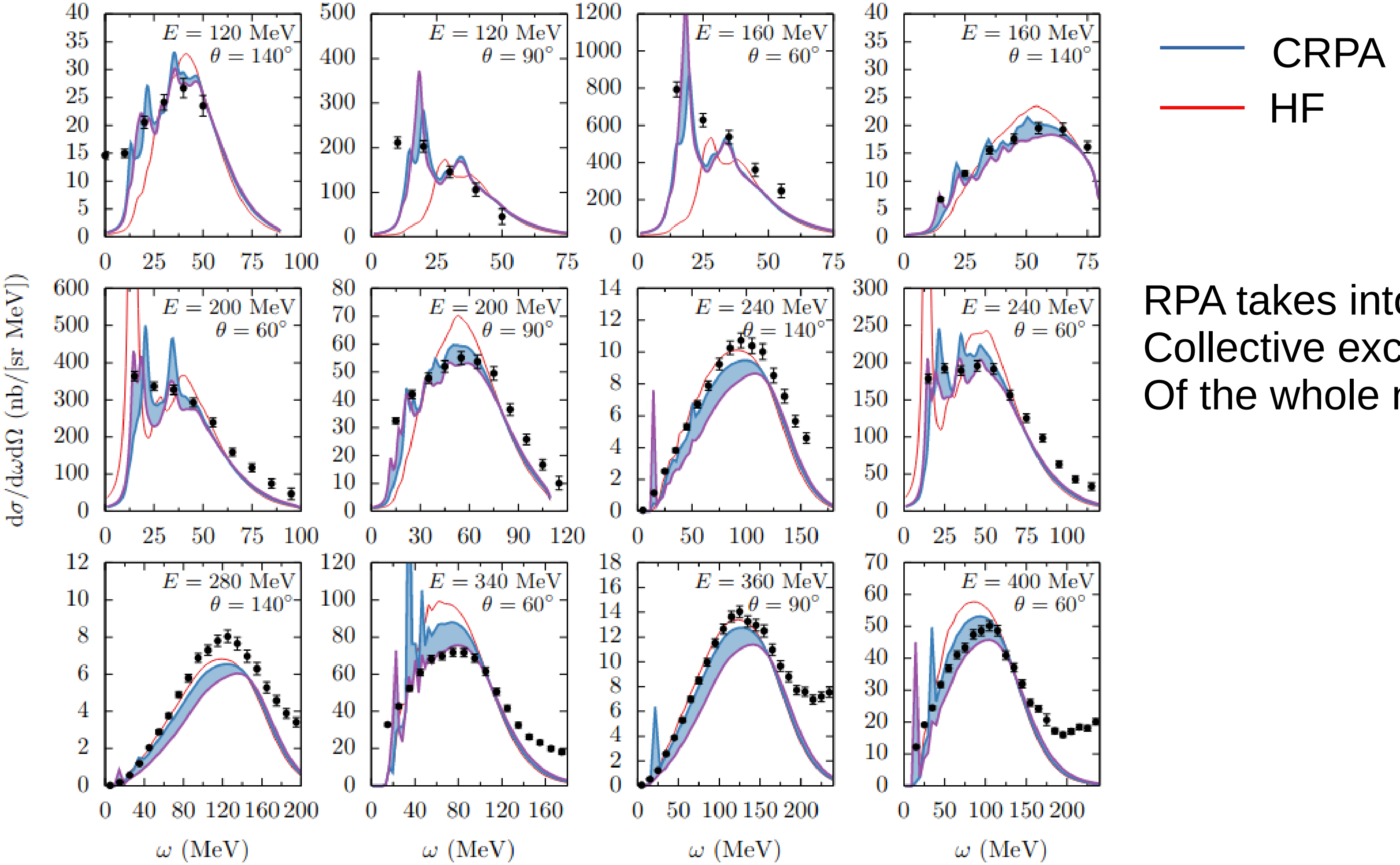
Mean field interaction



For self-consistent calculation: use the same interaction as used for the mean-field

 For all the details: [J. Ryckebusch et al. Nuc. Phys. A, 476]

Beyond the mean field: example for electron scattering of ^{56}Fe



RPA takes into account
Collective excitations
Of the whole nucleus

[Nikolakopoulos et al. Phys. Rev. C 103, 064603 (2021)]

Beyond the mean field: CRPA calculations in coordinate space

$$\Pi^{(RPA)}(x_1, x_2; \omega) = \Pi^{(0)}(x_1, x_2; \omega) + \frac{1}{\hbar} \int dx \int dx' \; \Pi^{(0)}(x_1, x; \omega) \, \widetilde{V}(x, x') \, \Pi^{(RPA)}(x', x_2; \omega)$$

Applications to neutrino-nucleus interactions by the Ghent group:

- νCC on ^{16}O and ^{12}C at low energy N. Jachowicz et al. [Phys.Rev.C 65 (2002) 025501]
- T2K and MiniBooNE V. Pandey et al. [Phys. Rev. C 94, 054609 (2016)]
- Low-E neutrino ^{40}Ar scattering N. Van Dessel et al. [Phys. Rev. C 100, 055503]
- Electron and muon neutrino interactions A.N. et al. [Phys. Rev. Lett. 123 (2019) 5, 052501]
- Many more ...
- → Implementation in GENIE S. Dolan et al. [Phys.Rev.D 106 (2022) 7, 073001]

Relativistic Distorted Wave Impulse Approximation

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

- The wave functions are static solutions of the Dirac equation with potentials:

$$\left[\hat{\alpha}\cdot\hat{\mathbf{p}} + \beta\left(m_N + S\left(r\right)\right) - \left(E - V\left(r\right)\right)\right]\psi = 0,$$

- We can use Γ^{μ} without making use of non-relativistic reduction

Relativistic Distorted Wave Impulse Approximation

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- We can use Γ^{μ} without making use of non-relativistic reduction
- Choices of potential for the wavefunctions:

  - RMF approach : initial and final-state in same potential
  - Optical potential for final state

Relativistic Distorted Wave Impulse Approximation

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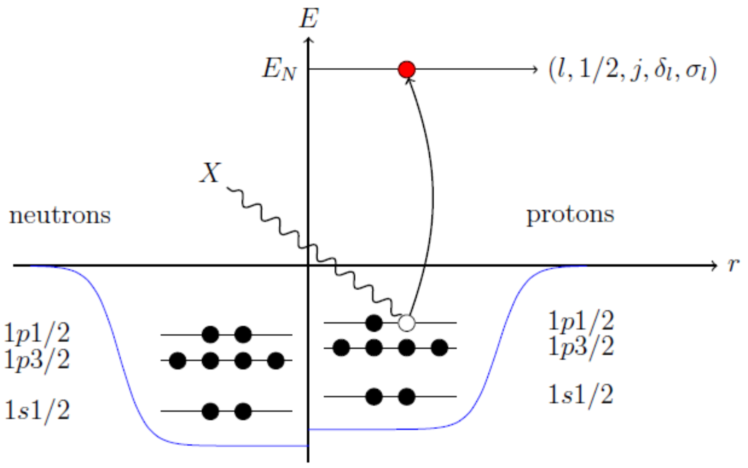

The good quantum numbers are:
- Angular momentum
- Energy

→ **Not momentum**

Relativistic Distorted Wave Impulse Approximation

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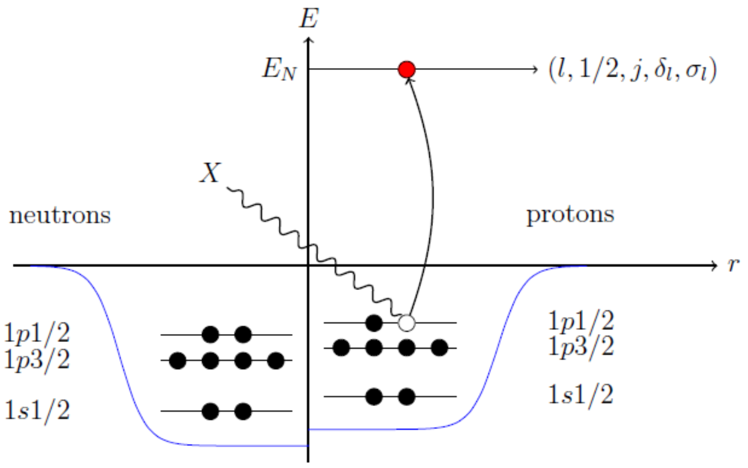

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At large **r** :

$$\psi(\mathbf{r}, E, \hat{\mathbf{p}}) \to u(p) e^{iEt - i\mathbf{p}\cdot\mathbf{r}}, \quad \mathbf{p}^2 = E^2 - M_N^2$$

Relativistic Distorted Wave Impulse Approximation

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{p} \overline{\psi}(\mathbf{p}+\mathbf{q}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{p})$$

Get a 'smearing' in momentum-space

The good quantum numbers:
- Angular momentum
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Relativistic Distorted Wave Impulse Approximation

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

$$\downarrow$$

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{p} \overline{\psi}(\mathbf{p}+\mathbf{q}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{p})$$

The good quantum numbers:
- Angular momentum
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→ **Not momentum**

Get a ‘smearing’ in momentum-space

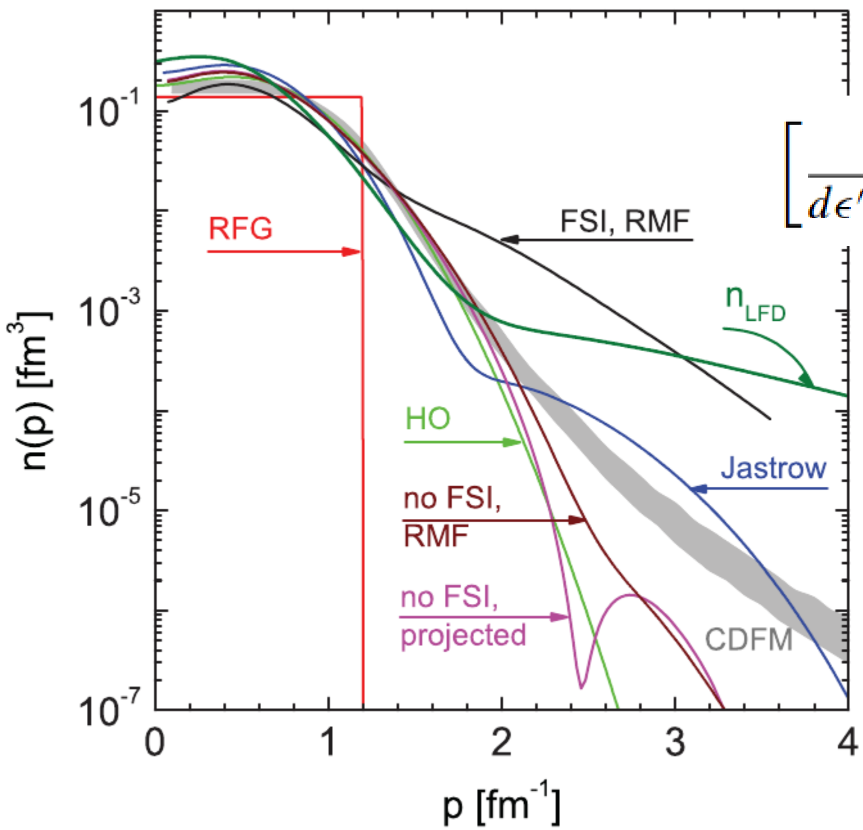
Compare to **RPWIA** expression :

$$\mathcal{J}_{RPWIA} = (2\pi)^{3/2} \overline{u}(p) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{p}-\mathbf{q})$$

→ Bound-wavefunction evaluated at fixed missing momentum
→ Allows for ‘factorized’ expression with momentum distribution

Relativistic Distorted Wave Impulse Approximation

PWIA
Allows for factorized expression with momentum distribution
(see also Artur's talk)



[A. N. Antonov et al. PRC83, 045504]

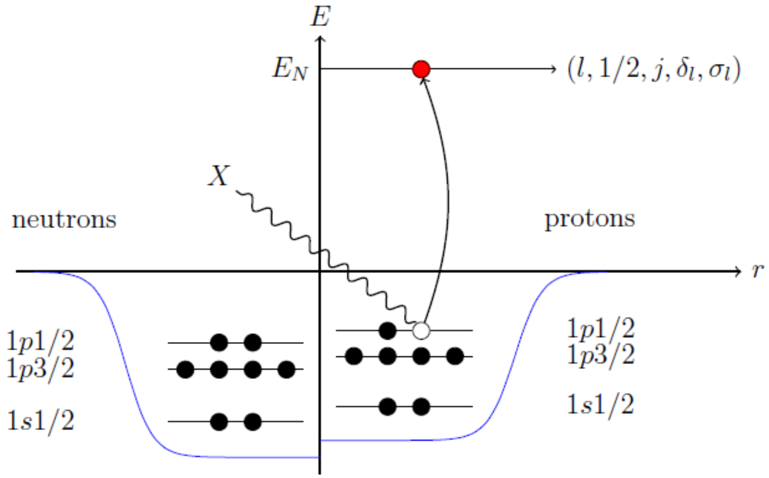
$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N}\right]^{\mathrm{PWIA}}_{(e,e'N)} = K\sigma^{eN}(q,\omega;p,\mathcal{E},\phi_N)S(p,\mathcal{E}),$$

$$\rho(p) = n^{\mathrm{dist}}(p) = \frac{\left[\frac{d\sigma}{d\Omega' d\epsilon' d\Omega_N}\right]_{\mathrm{FSI}}}{K\sigma^{eN}}.$$

Relativistic Distorted Wave Impulse Approximation: RMF

- The wave functions are static solutions of the Dirac equation with potentials:

$$[\hat{\alpha} \cdot \hat{\mathbf{p}} + \beta (m_N + S(r)) - (E - V(r))] \psi = 0,$$



From **consistent** initial-final states:

- Current conservation
- Pauli-blocking

Dirac current is conserved

For free nucleons, the Dirac current is conserved:

$$Q_\mu \overline{u}(p')\ \gamma^\mu u(p) = \overline{u}(p')\left[\not{p}' - \not{p}\right] u(p) = (M_N - M_N)\overline{u}(p')u(p)$$

For nuclear matrix elements:

$$Q \cdot J = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}(E_f - E_i)\overline{\psi}(\mathbf{r})\gamma^0 \psi_\kappa^{m_j}(\mathbf{r}) - i\nabla_i(\overline{\psi}(\mathbf{r})\gamma^i \psi_\kappa^{m_j}(\mathbf{r}))$$

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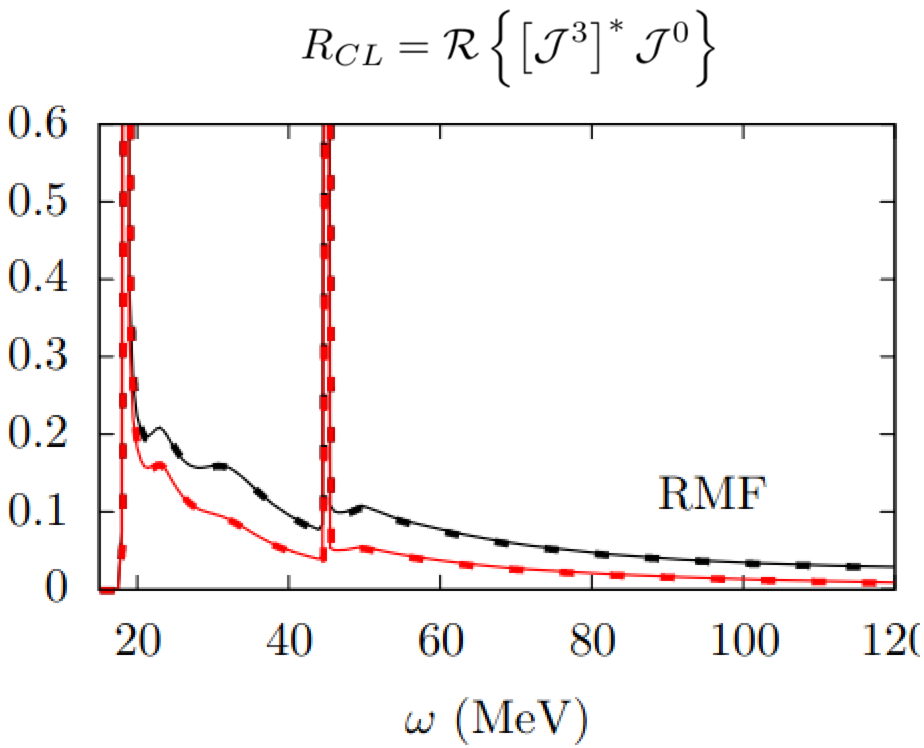
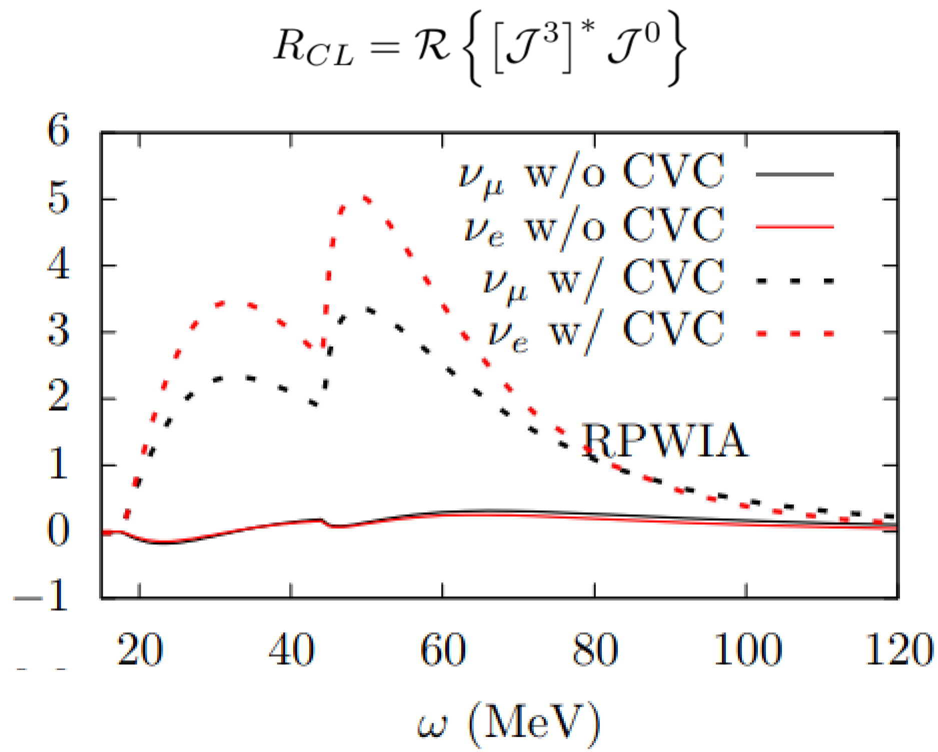
If both states are solutions with energy E_i, E_f of **the same** Dirac equation:

$$\left[i\nabla_i\gamma^i + M_N + S(r) + \gamma^0(V(r) - E)\right]\psi(r) = 0$$

Dirac current is conserved: low energy scattering with vector current

$$Q \cdot \mathcal{J} = \omega \mathcal{J}^0 - |\vec{q}|\mathcal{J}^3 = 0$$





$$\mathcal{J}^0 \equiv \frac{\omega}{|\vec{q}|}\mathcal{J}^3$$ → Putting in CVC 'by hand'

Orthogonality of states : Pauli-blocking

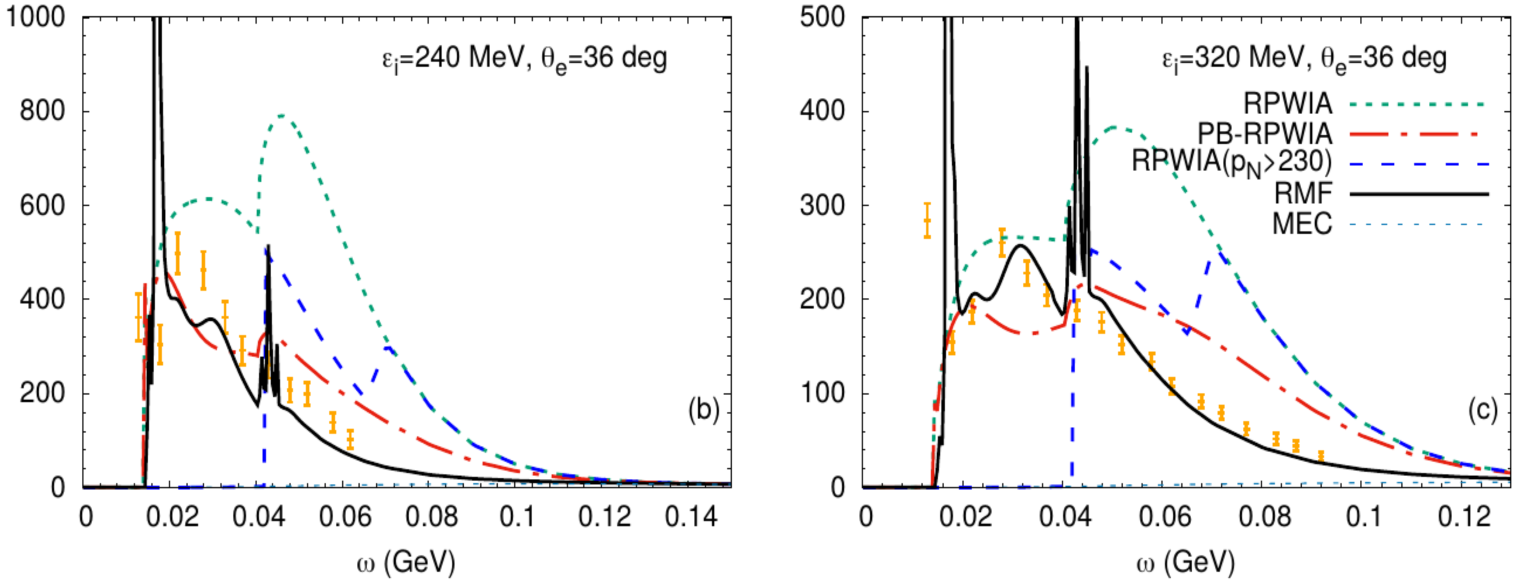
Different energy solution of Dirac equation are orthogonal

→ This leads to a proper implementation of Pauli-blocking

Orthogonality of states : Pauli-blocking

R. Gonzalez-Jimenez et al. [Phys. Rev. C 100, 045501 (2019)]

Electron-scattering at low-energies



- RPWIA : final-state are Dirac plane-waves
- RMF : same potential for initial and final states

Orthogonality of states : Pauli-blocking

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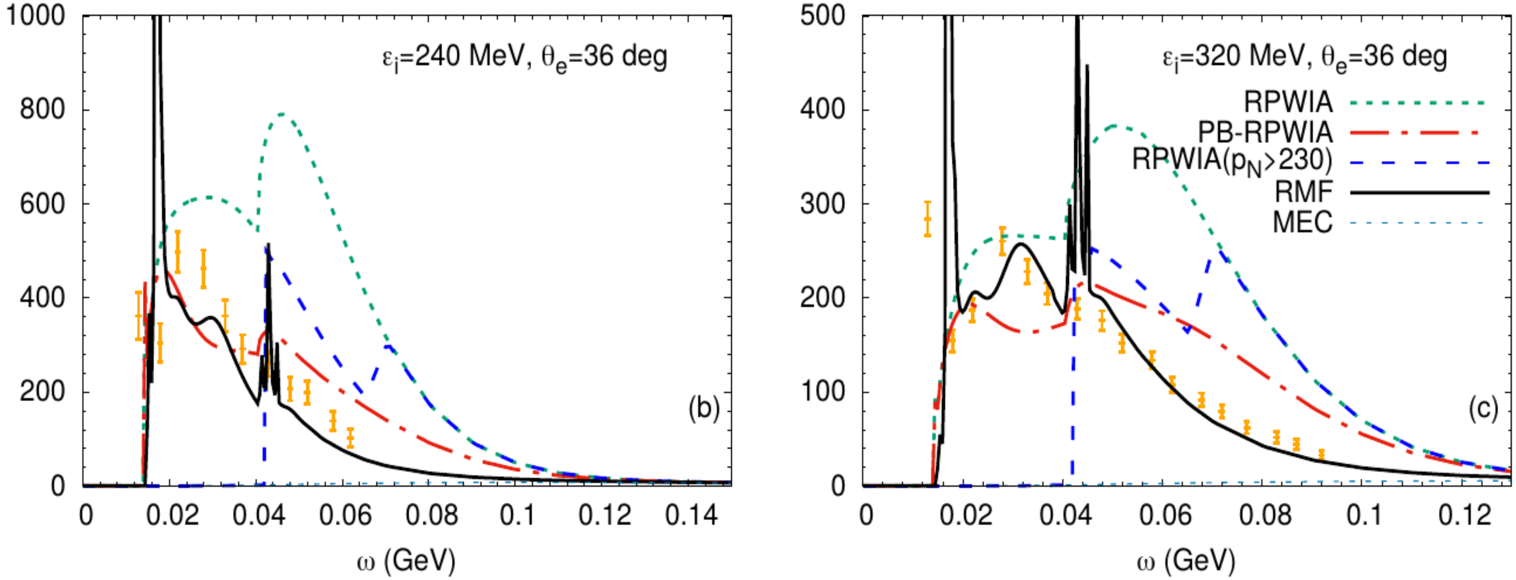
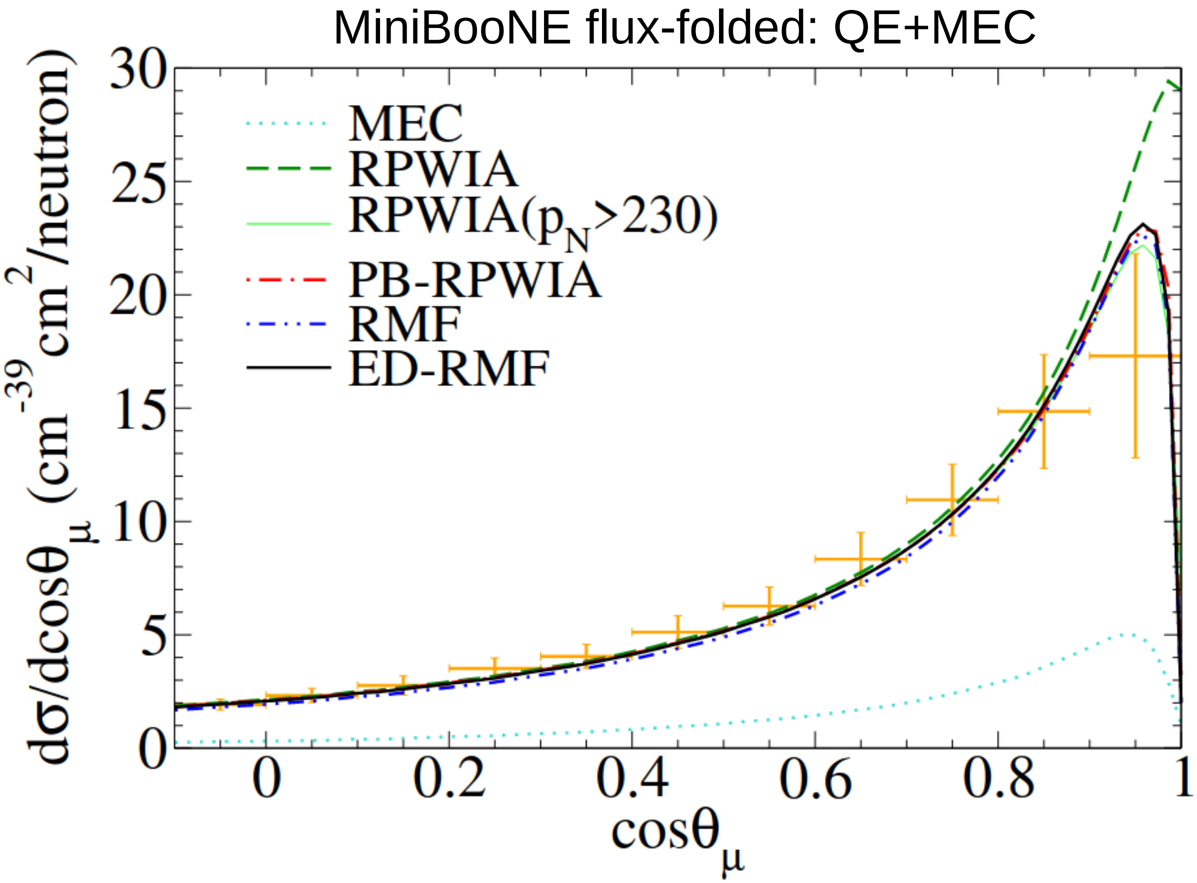


- RPWIA : final-state are Dirac plane-waves
- RMF : same potential for initial and final states
- RPWIA(p_N > 230): ‘Fermi gas-type’ Pauli-blocking
- PB-RPWIA : $|\Psi^{s_N}(\mathbf{p}_N)\rangle = |\psi^{s_N}_{pw}(\mathbf{p}_N)\rangle - \sum_{\kappa,m_j} [C^{m_j,s_N}_{\kappa}(\mathbf{p}_N)]^\dagger \, |\psi^{m_j}_{\kappa}\rangle$

Orthogonality of states : Pauli-blocking

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

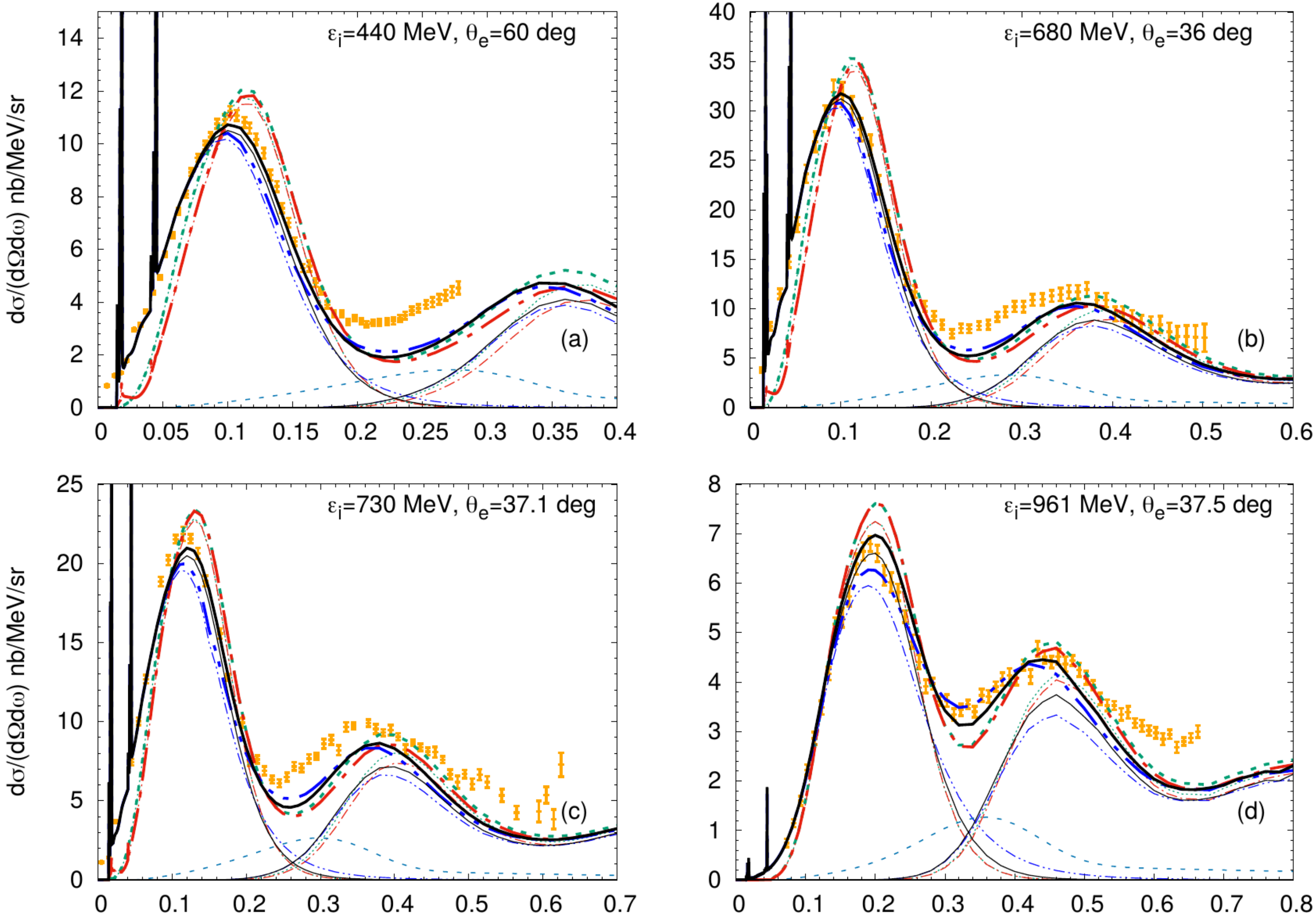


Pauli-blocking leads to a suppression at small angles

→ **For neutrino-data all approaches give similar results**

RDWIA for electron scattering at intermediate energies

[R. Gonzalez-Jimenez, A. Nikolakopoulos, N. Jachowicz, J.M. Udias PRC 100, 045501 (2019)]

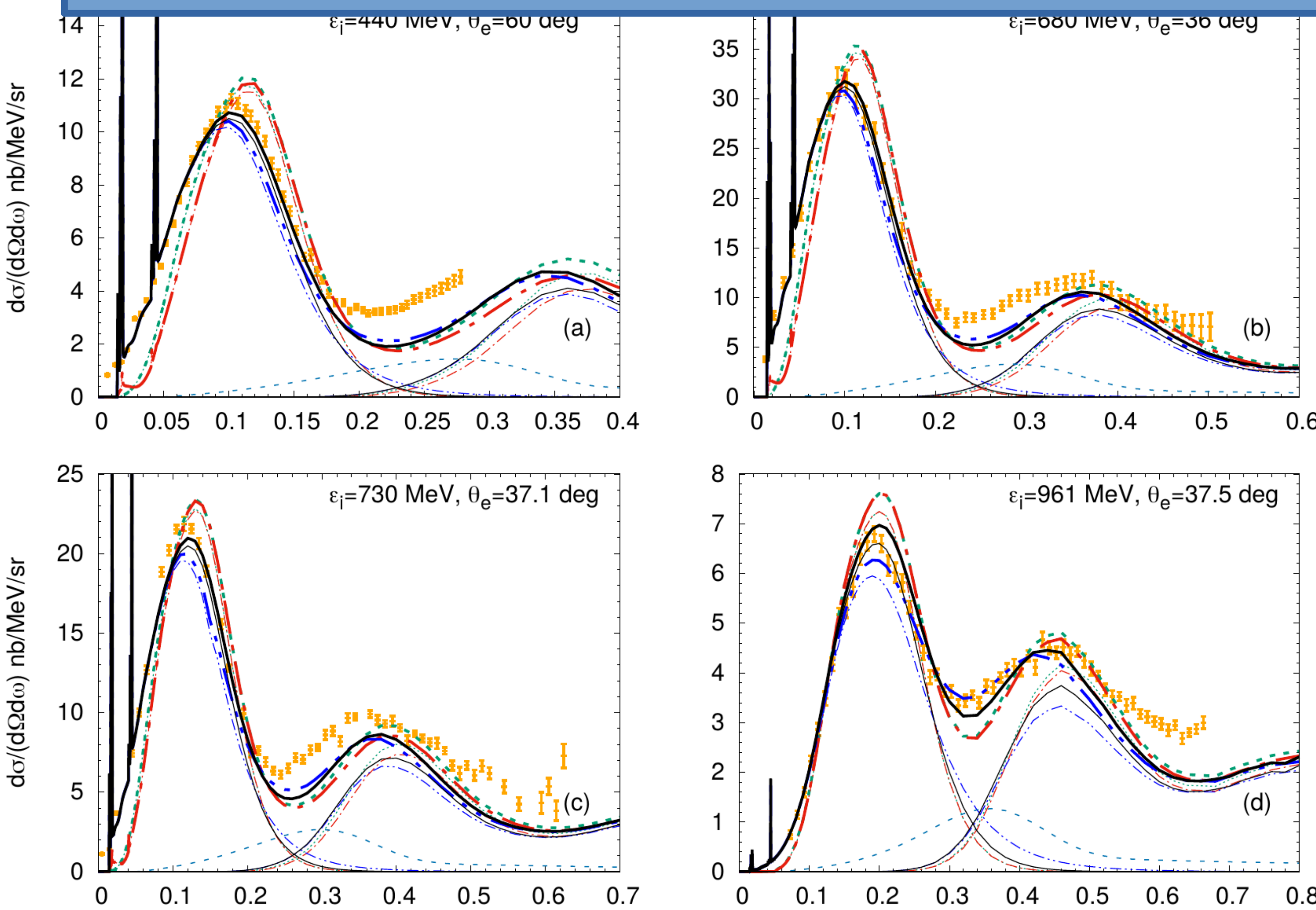


RPWIA

RDWIA

RDWIA for electron scattering at intermediate energies

The effect of final-state interactions is also important for inclusive CS

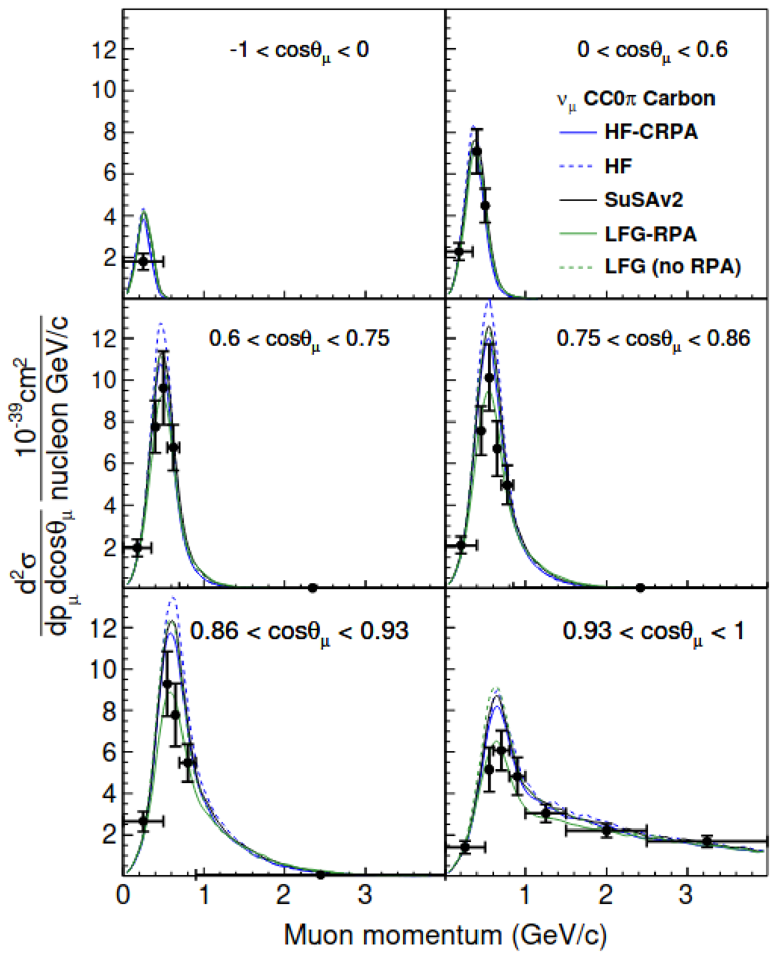


RPWIA

RDWIA

Mean field implementations in event generators

The effect of final-state interactions is also important for inclusive CS



Implementation of the CRPA model in the GENIE event generator and analysis of nuclear effects in low-energy transfer neutrino-nucleus interactions

S. Dolan,[1] A. Nikolakopoulos,[2] O. Page,[3] S. Gardiner,[4] N. Jachowicz,[2] and V. Pandey[5]

[1] *CERN, European Organization for Nuclear Research, Geneva, Switzerland**
[2] *Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium*†
[3] *School of Physics, University of Bristol, Bristol BS8 1TL, United Kingdom*
[4] *Fermi National Accelerator Laboratory, Batavia, IL 60502, USA*
[5] *Department of Physics, University of Florida, Gainesville, FL 32611, USA*‡

(Dated: November 2, 2021)

Implementation of the SuSAv2-MEC 1p1h and 2p2h models in GENIE and analysis of nuclear effects in T2K measurements

S. Dolan,[1,2,3] G.D. Megias,[1,2,4] and S. Bolognesi[2]

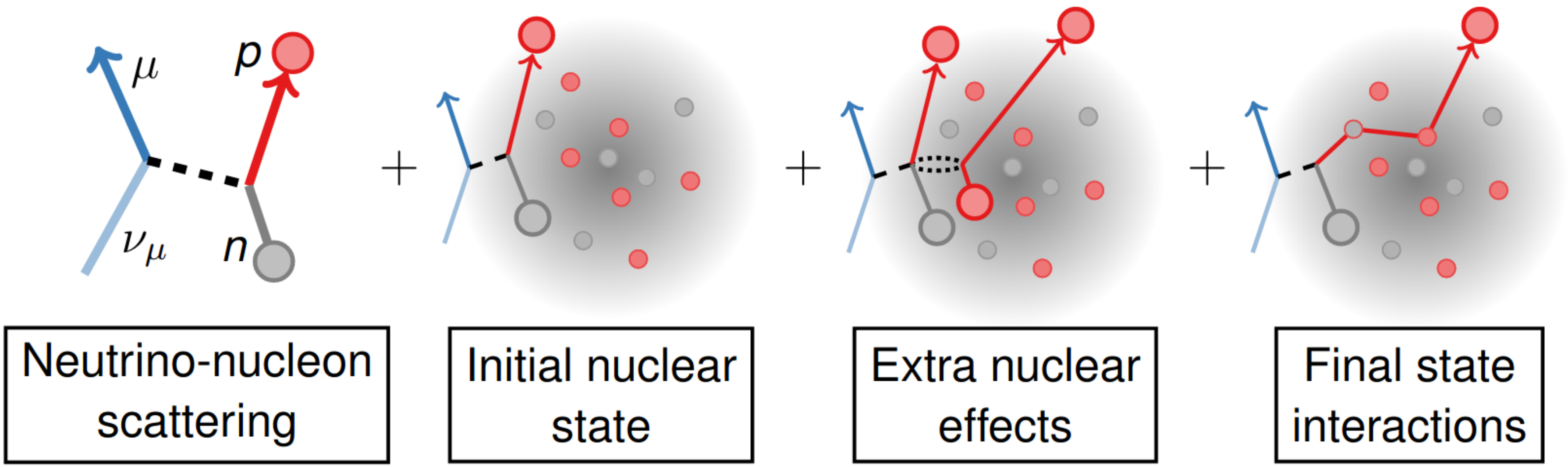
[1] *IN2P3-CNRS, Laboratoire Leprince-Ringuet, Palaiseau 91120, France*
[2] *DPhP, IRFU, CEA Saclay, 91191 Gif-sur-Yvette, France*
[3] *CERN, European Organization for Nuclear Research, Geneva, Switzerland*
[4] *University of Tokyo, Institute for Cosmic Ray Research, Research Center for Cosmic Neutrinos, Kashiwa, Japan*

(Dated: February 21, 2020)

The implementation in generators is often **Only** the inclusive cross section

Neutrino interactions in event generators

The idealized version of the event generator:



[Fig. From K. Niewczas]

This idea is based off 'factorized' approach

In reality:
The inclusive cross section already includes nuclear effects
And final-state interactions

Neutrino interactions based on inclusive CS in GENIE

Input to the generator is inclusive cross section:

$$\frac{d\sigma(E_\nu)}{dE_l \, d\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H^{\mu\nu}_{n,\kappa}(\omega, q, \Omega_N, E_{n,\kappa})$$

Lost nucleon information → Need to generate it in GENIE

1. Draw initial nucleon $\mathbf{p}_m$ from $p^2 n(p)$ (e.g. LFG)

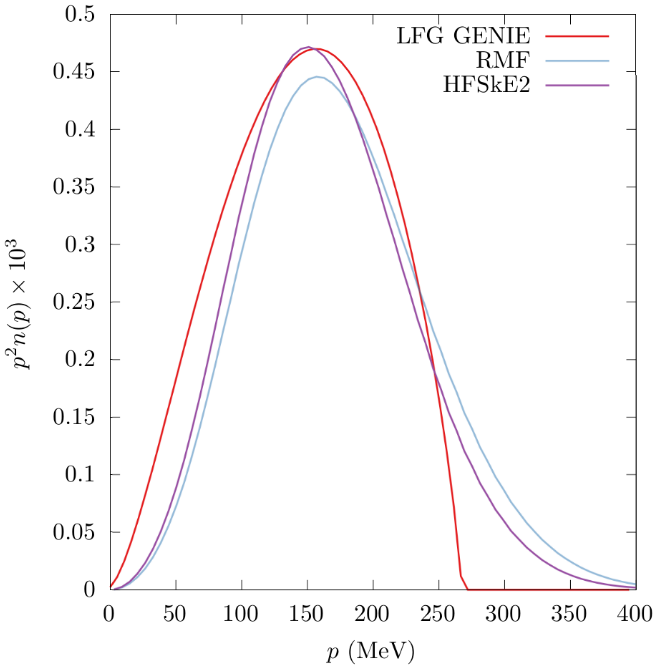
!! 2. Compute $E_m^2 = p_m^2 + M_N^2$

3. $E_N = E_m + \omega - E_b(q)$

4. $k_N^2 = E_N^2 - M_N^2$

!! $|\mathbf{p}_m + \mathbf{q}| \neq k_N = \sqrt{E_N^2 - M_N^2}$

→ $$\mathbf{k}_N = \frac{k_N}{|\mathbf{p}_m + \mathbf{q}|} (\mathbf{p}_m + \mathbf{q})$$



5. Give residual momentum to remnant

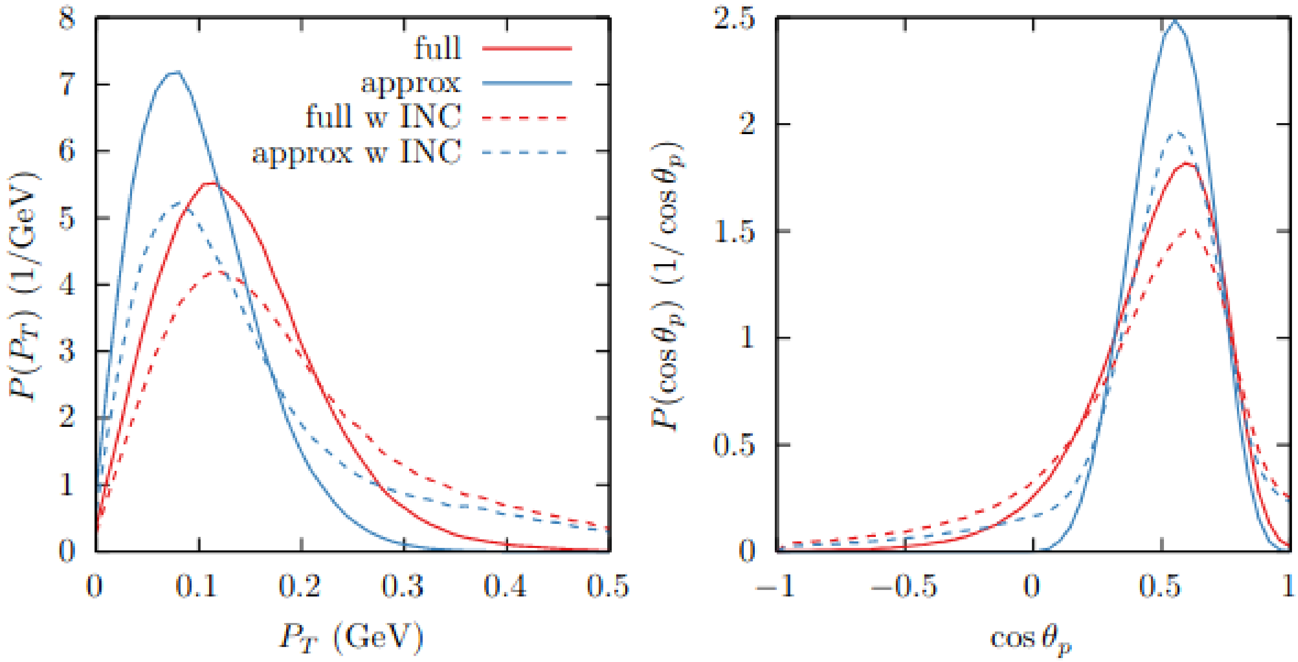
Fermilab

Neutrino interactions based on inclusive CS in GENIE

Input to the generator is inclusive cross section:

$$\frac{\mathrm{d}\sigma(E_\nu)}{\mathrm{d}E_l \mathrm{d}\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int \mathrm{d}\Omega_N \sum_{n,\kappa} H^{\mu\nu}_{n,\kappa}(\omega, q, \Omega_N, E_{n,\kappa})$$

Lost nucleon information → Need to generate it in GENIE



Compare full result with GENIE approximation → same inclusive CS!

Relativistic Distorted Wave Impulse Approximation

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

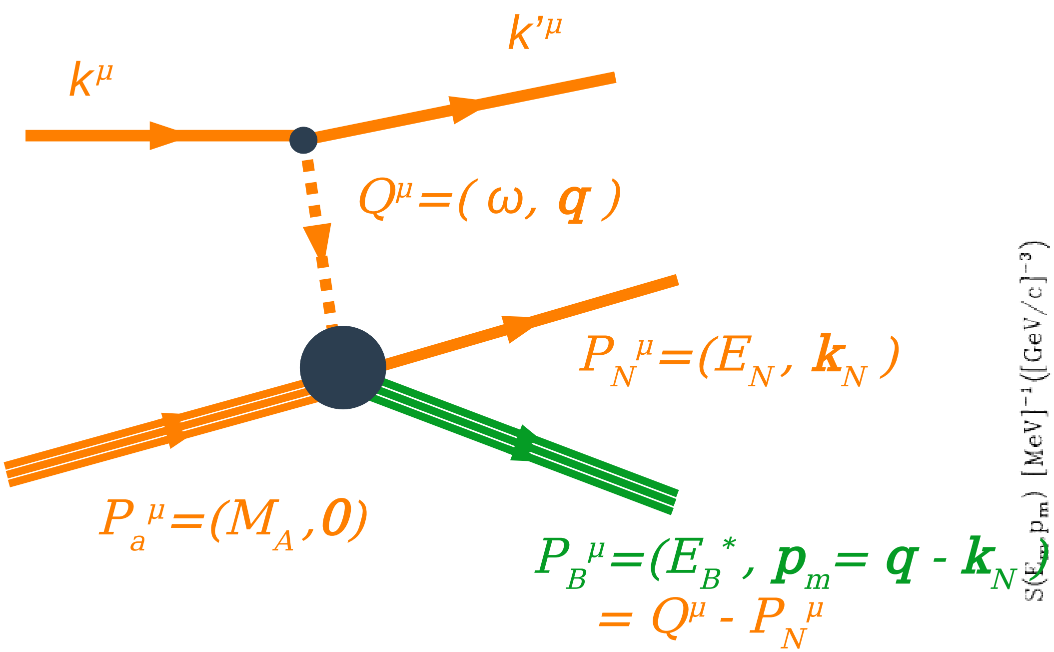
- The wave functions are static solutions of the Dirac equation with potentials:

$$\left[\hat{\alpha}\cdot\hat{\mathbf{p}} + \beta\left(m_N + S\left(r\right)\right) - \left(E - V\left(r\right)\right)\right]\psi = 0,$$

- We can use Γ^{μ} without making use of non-relativistic reduction
- RMF approach → initial and final-state in same (real) potential
- **We can use different description for final-state wavefunction → For exclusive interactions : Optical potential approach**

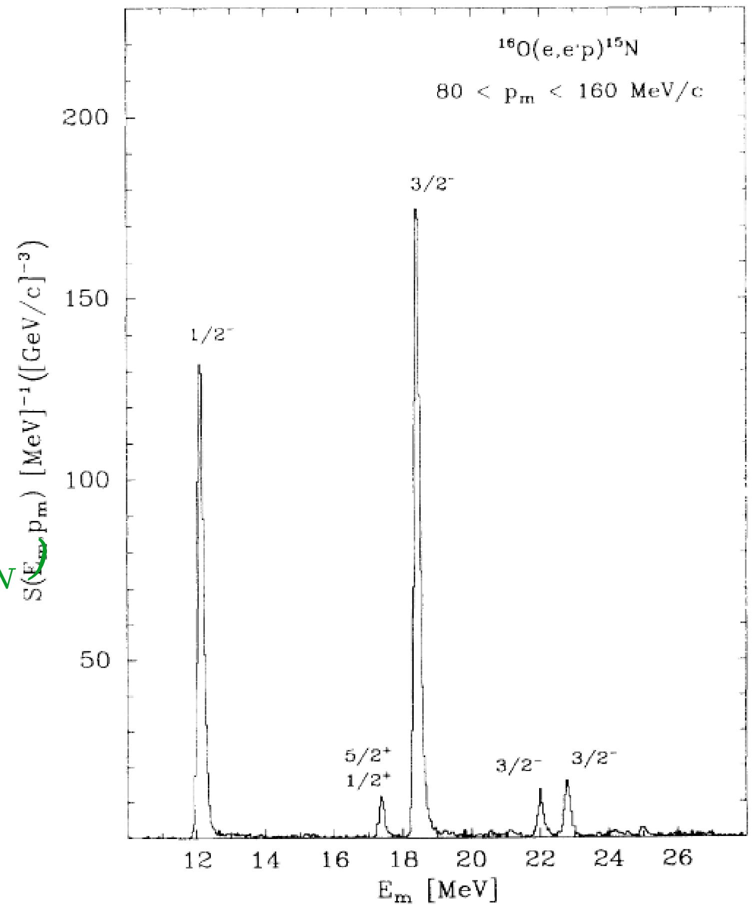
Exclusive cross sections in the RDWIA

Exclusive scattering:



know/measured

Kinematics of residual system known

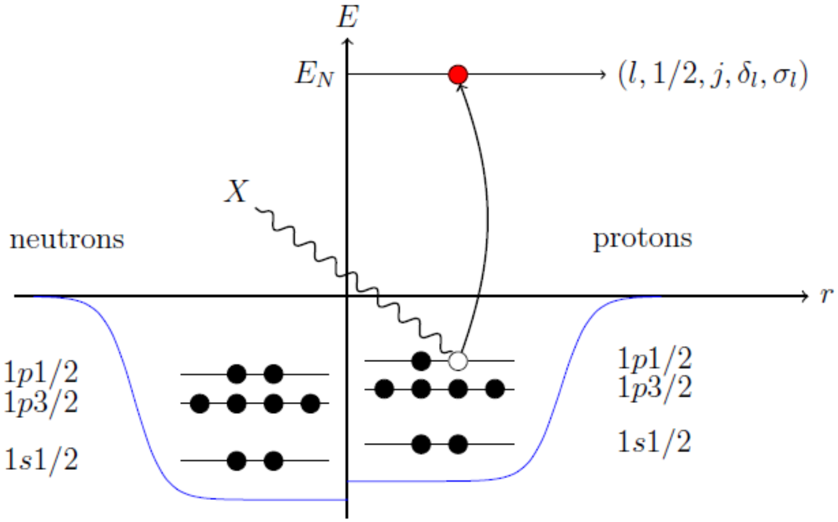


[M. Leuschner et al. PRC49, 955 (1994)]

Exclusive cross sections in the RDWIA

Exclusive electron scattering:

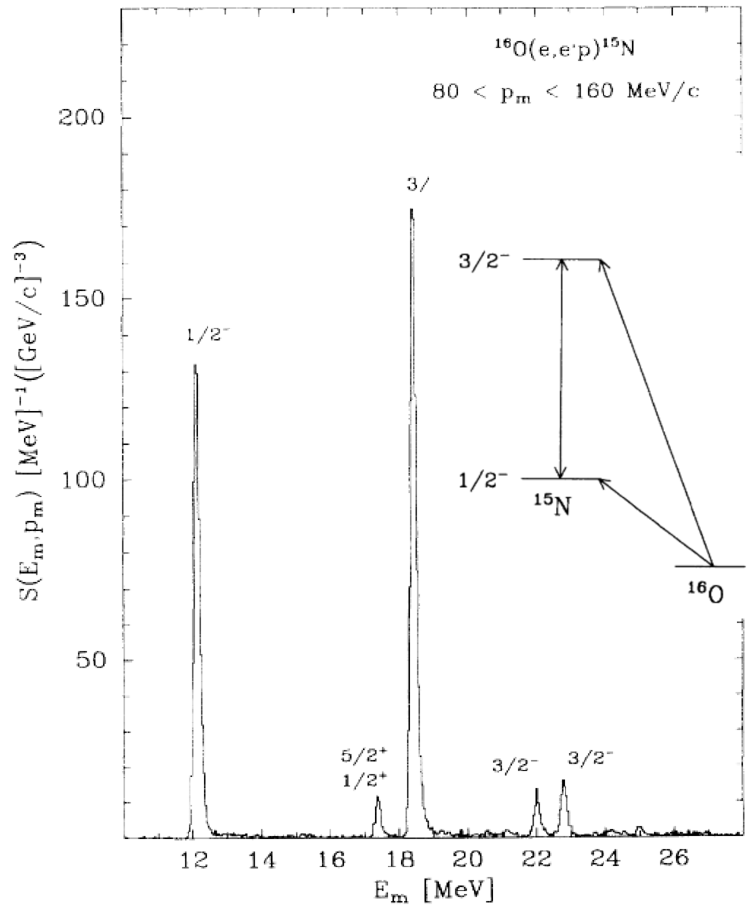
In the 'mean-field' region



$$\mathcal{M} = j_{lep,\mu} \langle \Psi_f | \mathcal{O}^{\mu} | \Psi_i \rangle$$

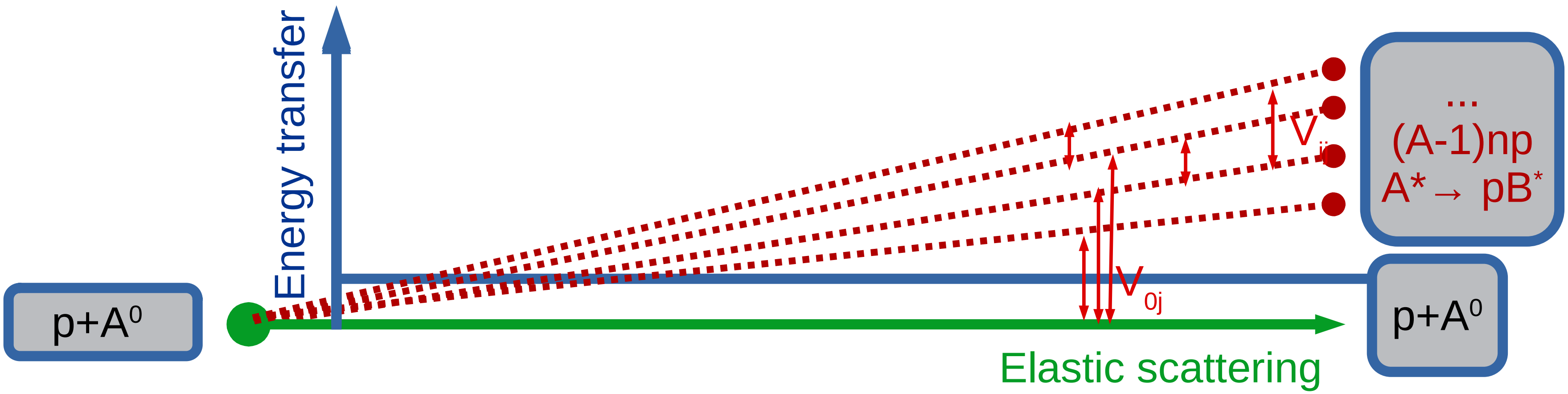
$$\langle \Psi_f | = \langle \phi_p | \langle ^{15}N^* |$$

Direct 1-proton knockout from a nuclear shell



[M. Leuschner et al. PRC49, 955 (1994)]

Exclusive cross sections in the RDWIA: optical potential



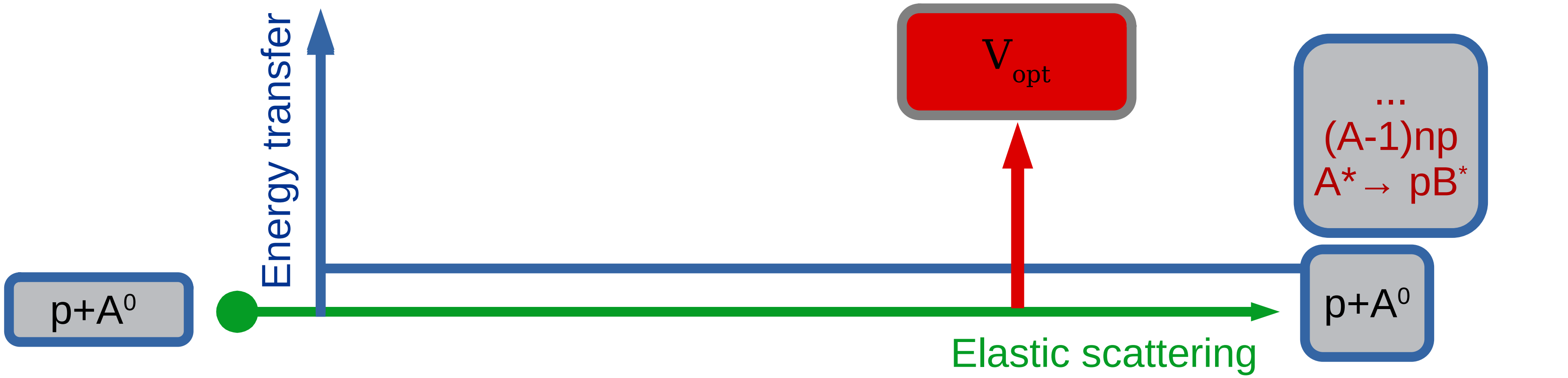
$$(H_{00} - E)|\phi_0\rangle = -V_{0j}|\phi_j\rangle \ (j > 0)$$

$$(H_{ij} - E)|\phi_j\rangle = -V_{j0}|\phi_0\rangle \ (i, j > 0)$$

$$\left[H^{free} + V_{00}^{nA} + V_{0j}\frac{1}{E - H_{ij} + i\eta}V_{j0} - E\right]|\phi_0\rangle$$

Coupled channels problem → Effective one-body problem as a formal solution

Exclusive cross sections in the RDWIA: optical potential



$$\left[H^{free} + V_{00}^{nA} + V_{0j}\frac{1}{E - H_{ij} + i\eta}V_{j0} - E\right]|\phi_0\rangle$$

$$\approx \left[H^{free} + \mathcal{V}^{opt} - E\right]|\phi_0\rangle$$

Coupled channels problem → Effective one-body problem with optical potential

The (empirical) relativistic optical potential

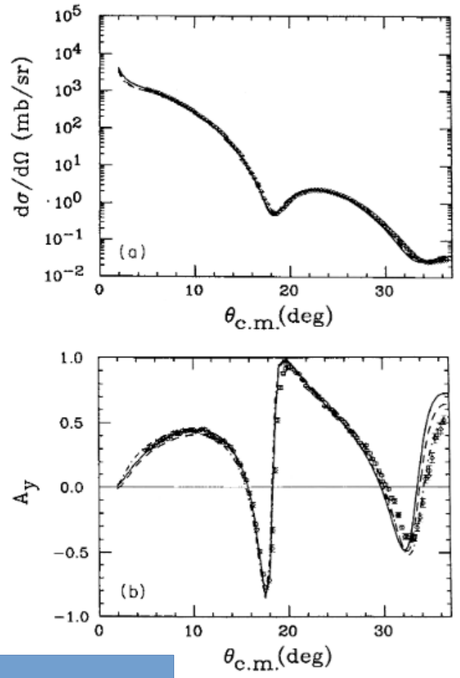
PHYSICAL REVIEW C VOLUME 47, NUMBER 1 JANUARY 1993

Global Dirac phenomenology for proton-nucleus elastic scattering

E. D. Cooper, S. Hama, and B. C. Clark
Department of Physics, The Ohio State University, Columbus, Ohio 43210

R. L. Mercer
IBM Thomas J. Watson Research Center, Yorktown Heights, New York 10598
(Received 31 August 1992)

		σ_R(mb)				
			EDAD-fit			
Target	T_p (MeV)	EDAI-fit	fit 1	fit 2	fit 3	Reference
^{12}C	29.00	420.2	435.5	433.1	422.7	[6]
	30.30	415.9	429.0	425.6	414.2	[7]
	49.00	358.8	363.0	348.4	327.7	[6]
	49.48	357.4	361.8	347.0	326.1	[8]
	61.40	323.3	335.6	317.0	294.8	[9]
	65.00	313.5	329.0	309.7	287.4	[10]
	122.00	202.2	269.0	254.4	230.5	[11]
	160.00	177.8	252.3	246.4	215.2	[11]
	200.00	177.6	243.0	243.9	205.0	[11–13]
	300.00	201.1	233.0	235.4	194.9	[14]
	398.00	215.8	227.4	218.6	199.1	[15]
	494.00	227.2	223.7	203.0	211.6	[16]
	797.50	238.4	235.3	209.9	250.0	[17,18]
	1040.00	198.6	259.4	243.8	232.2	[19,20]



Fit to elastic proton-nucleus scattering data

Get a potential with imaginary part → 'absorption' of nucleons
Actually nucleon inelastic rescattering

Exclusive electron scattering with Optical potential: ‘standard approach’

$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

[Meucci et al. PRC64, 014604]

Solution of Dirac equation with optical potential

The optical potential
Removes all the flux lost in
Inelastic final-state interactions

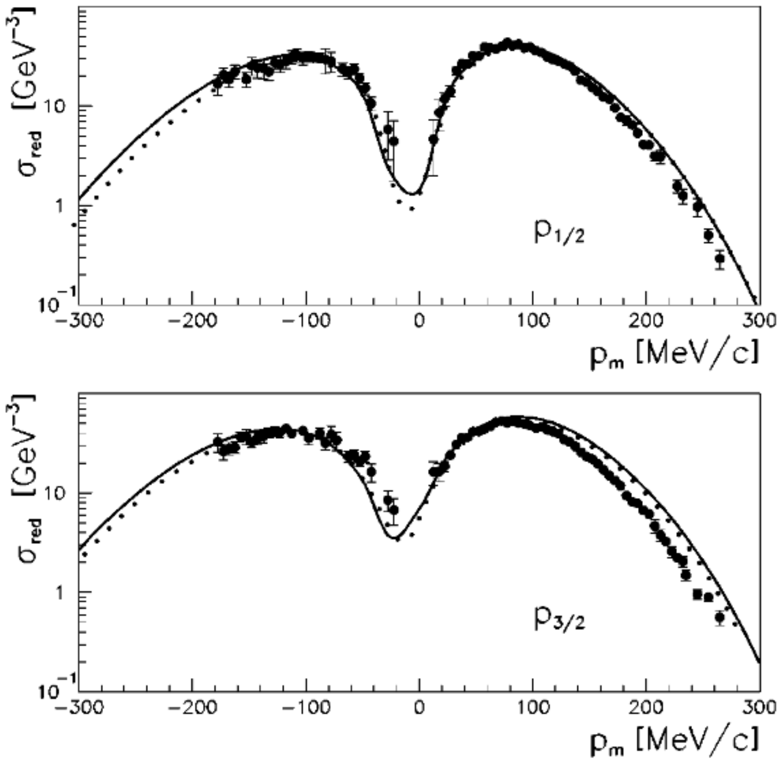


FIG. 11. The reduced cross section (σ_{red}) of the ^{16}O$(e,e'p)$ reaction as a function of the recoil momentum p_m for the transitions to the $1/2^-$ ground state and to the $3/2^-$ excited state of ^{15}N, in

Exclusive electron scattering with Optical potential: 'standard approach'

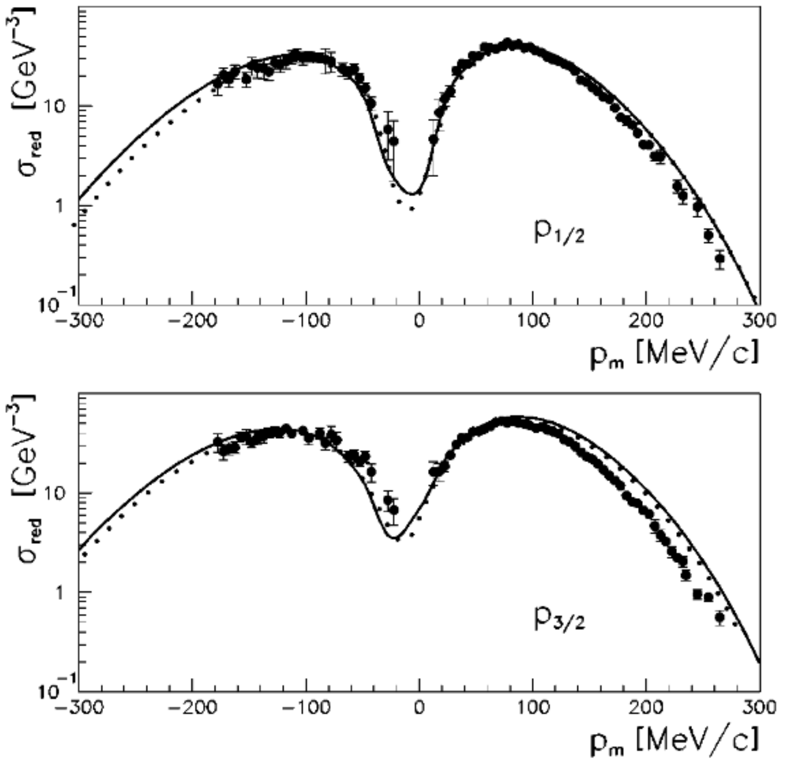
$$\mathcal{J}^{\mu}_{RDWIA} = \int \mathrm{d}\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}} \overline{\psi}(\mathbf{r}, \vec{k}', s_N) \Gamma^{\mu} \psi^{m_j}_{\kappa}(\mathbf{r})$$

[Meucci et al. PRC64, 014604]

Solution of Dirac equation with optical potential

The optical potential
Removes all the flux lost in
Inelastic final-state interactions

In neutrino experiments we want to
Describe **explicitly**
Inelastic final-state interactions



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Where do the protons go ?: explicit modeling of 'rescattering'

Neutrino event generators use intra-nuclear cascade models (INCs)

- Hadrons move in the nucleus on classical trajectories
- Density and in-medium cross section determine mean-free-path
- Interaction produces secondary hadrons that are propagated through
- Stochastically determines final-state content

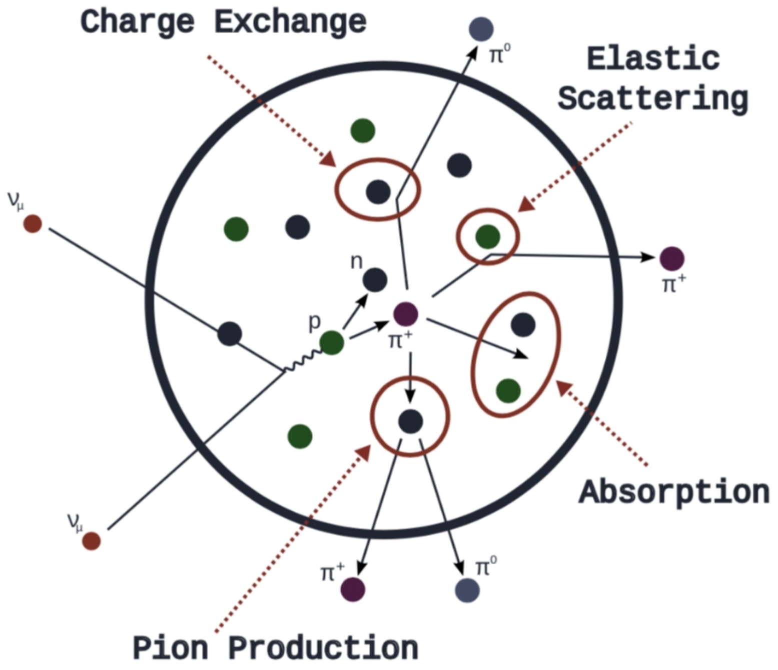

Fig: T. Golan

Where do the protons go ?: explicit modeling of 'rescattering'

Neutrino event generators use intra-nuclear cascade models (INCs)

- Hadrons move in the nucleus on classical trajectories
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Optical potential can not do this!

But can be compared to part of the signal

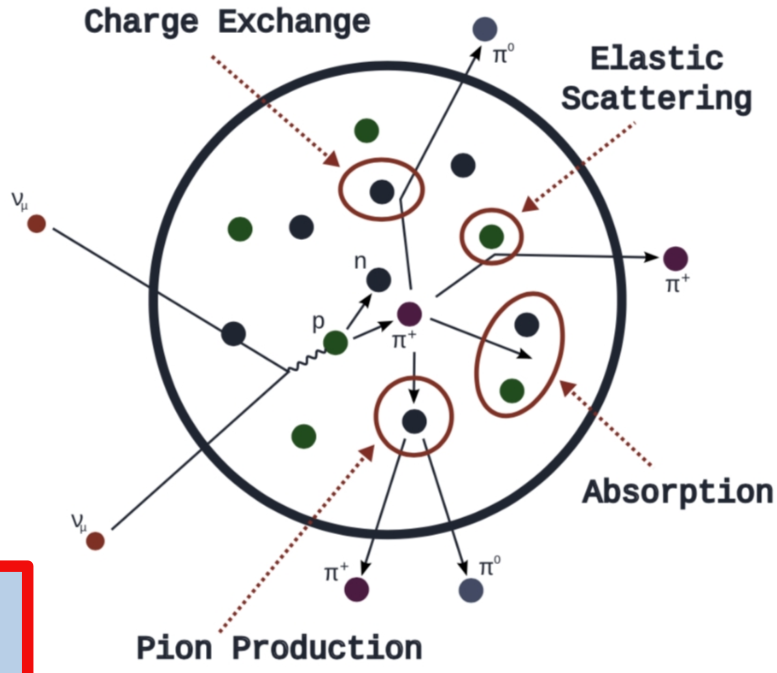


Fig: T. Golan

Use the RDWIA to benchmark the cascade model

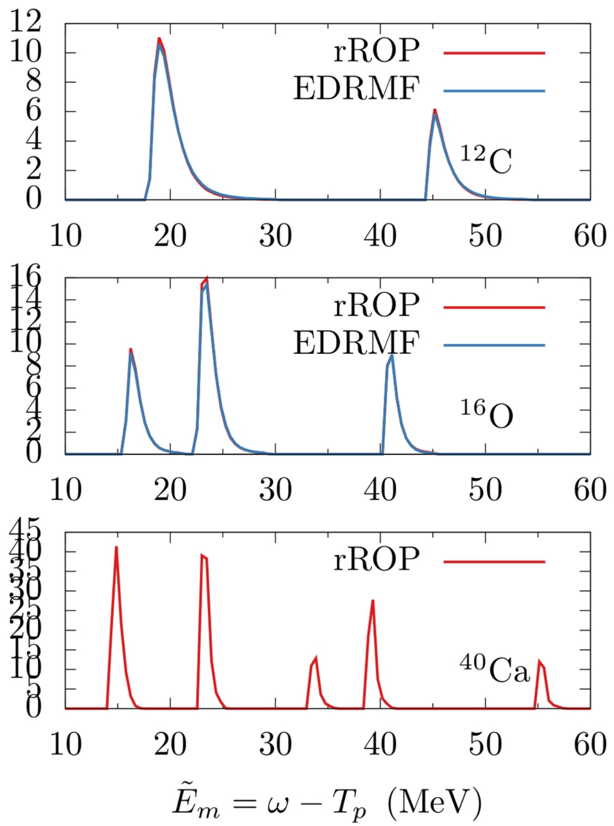
1. Input to the cascade:

Events from **unfactorized**
five-fold differential nucleon knockout
cross section
=RDWIA results in mean-field

2. Kinematic cuts on results

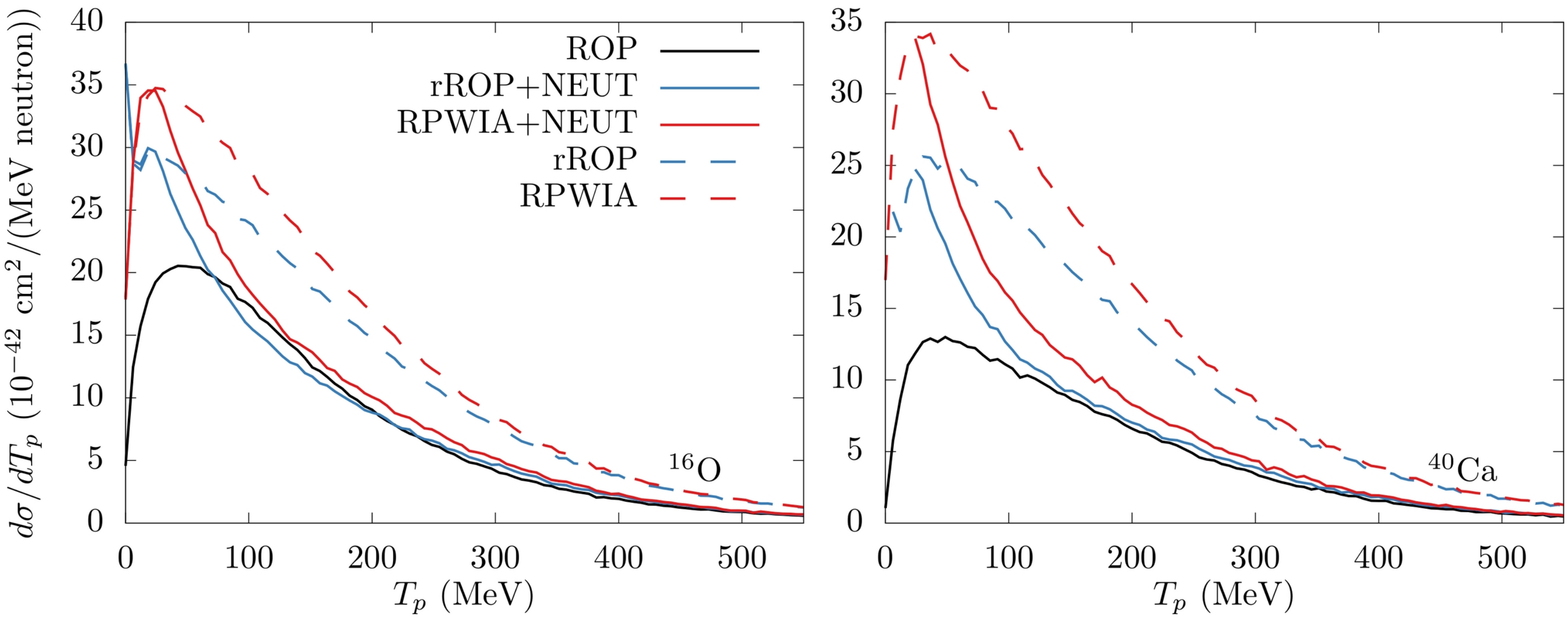
Select events with E_m in shell-model
Is an exclusive cross section

Cascade can be compared
To optical potential calculations



Use the RDWIA to benchmark the cascade model

Direct comparison for T2K flux



Cascade and RDWIA agree at high energies
Disagree strongly at low energies!

Take-aways:

- Make use of **effective interactions** fitted to nuclei at mean-field level

- If we restrict to mean field we get **consistent description**
  - Orthogonality ↔ Pauli-exclusion
  - **Final-state interactions**
  - Current conservation
  - …

- In reality: **Beyond mean-field contributions are important**

- The mean-field can be used as a **basis for these contributions**

  - CRPA results for long-range correlations
  - Short range correlations:
  - **LCA** [V. Cuyck, PhysRevC.94.024611]
  - Effective shells [R. Gonzalez-Jimenez, Phys. Rev. C 105, 025502]

LCA: [J. Ryckebusch et al. Phys. Lett. B, 792, (2019)]

Questions ? Answers ?