Spectral function approach

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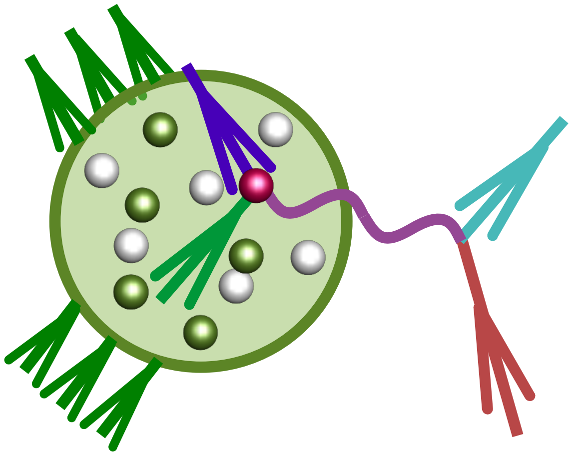
Impulse approximation

$$\frac{d\sigma^{\rm IA}_{\ell A}}{d\omega d\Omega} = \sum_N \int d^3p\, dE\, P^N_{\rm hole}(\mathbf{p}, E) \frac{M}{E_{\mathbf{p}}} \frac{d\sigma^{\rm elem}_{\ell N}}{d\omega d\Omega} P^N_{\rm part}(\mathbf{p}', \mathcal{T}')$$

average over the initial nucleon state

nucleon cross section

final-state interactions



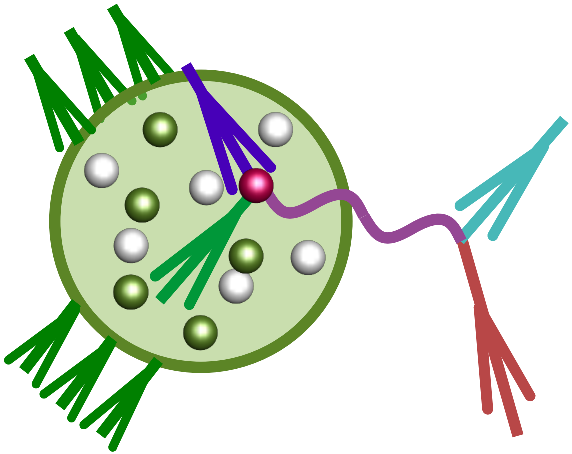
Spectral function approach

$$\frac{d\sigma^{\mathrm{IA}}_{\ell A}}{d\omega\, d\Omega} = \sum_N \int d^3p\, dE\, P^N_{\mathrm{hole}}(\mathbf{p}, E)\, \frac{M}{E_{\mathbf{p}}}\, \frac{d\sigma^{\mathrm{elem}}_{\ell N}}{d\omega\, d\Omega}\, P^N_{\mathrm{part}}(\mathbf{p}', \mathcal{T}')$$

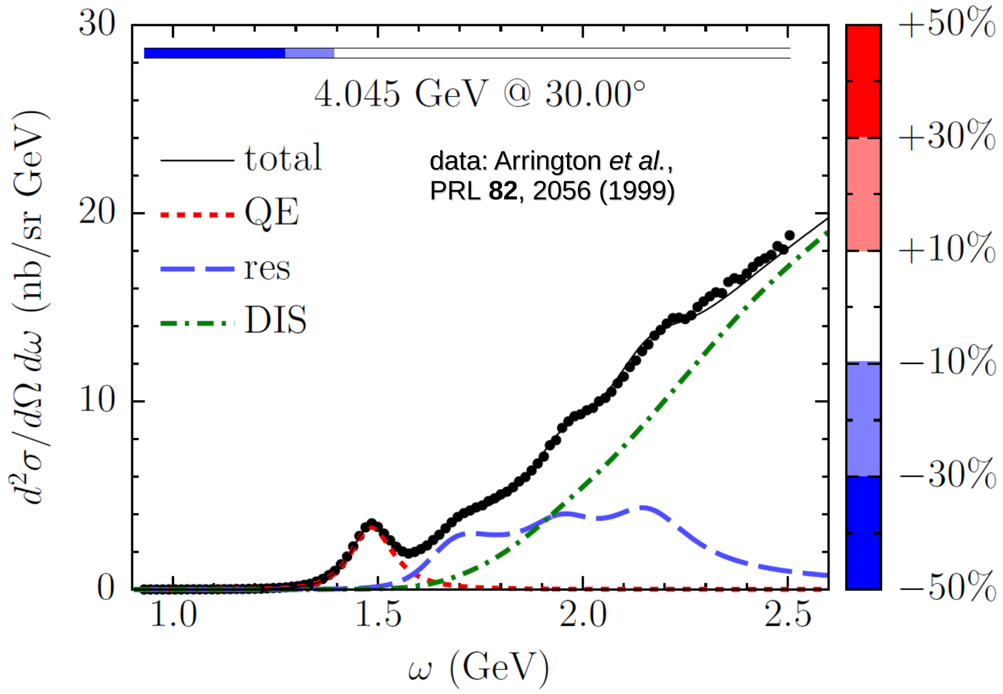
hole spectral function

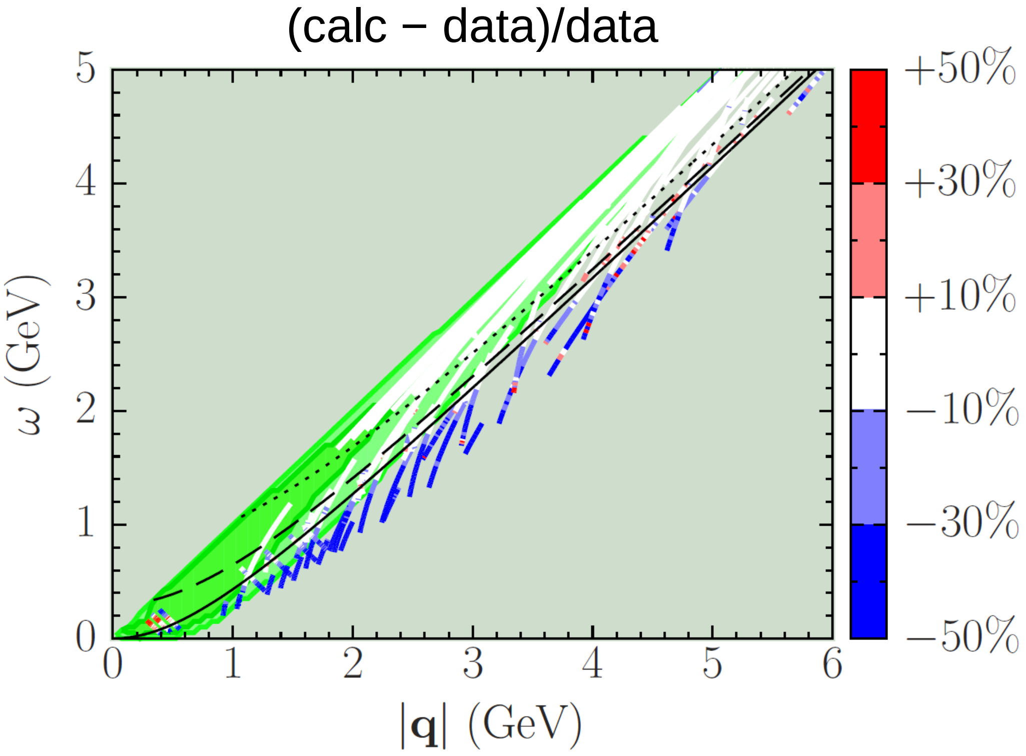
nucleon cross section (relativistic)

particle spectral function (relativistic)



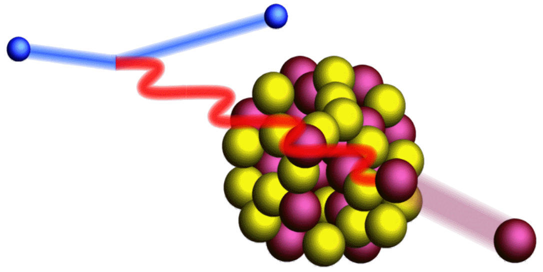
Realistic description of D(*e*,*e'*)



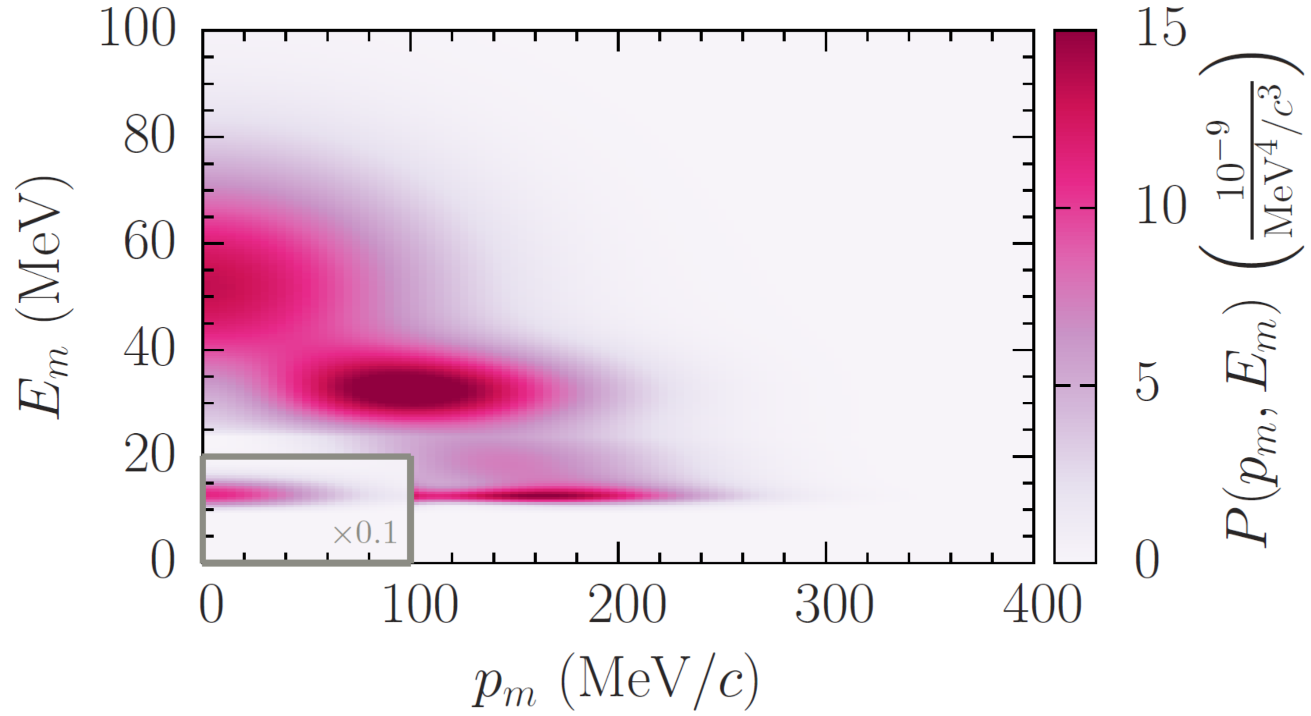


What is the spectral function?

The proton spectral function $P(p_m, E_m)$ describes the probability distribution of removing a proton of momentum p_m from the target nucleus, leaving the residual system with excitation energy $E_m - E_{\mathrm{thr}}$, with E_{thr} being the proton emission threshold.



What is the spectral function?



Universal property of the nucleus, independent of the interaction.

Spectral functions for complex nuclei

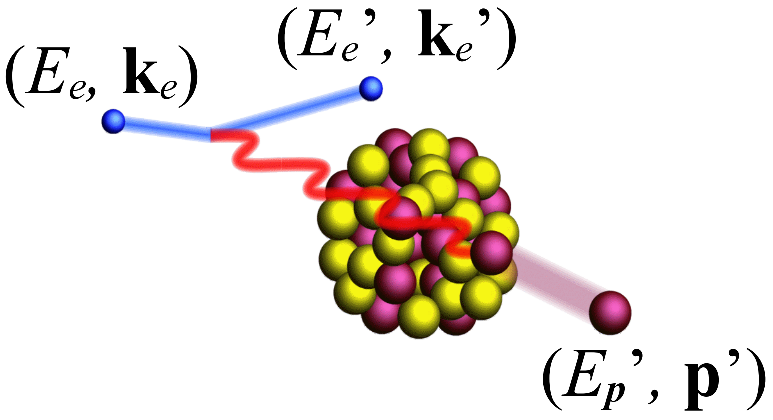
Mean-field part

- describes the shell structure
- can be determined from experimental data
- 70–80% of nucleons

Correlated part

- describes correlated nucleons
- easier to determine from theoretical estimates

Missing energy E_m and missing momentum $\mathbf{p}_m$



$$E_e + M_A = E_e' + E_p' + \underline{E^*_{A-1}}$$

known — determined

$$\mathbf{k}_e + 0 = \mathbf{k}_e' + \mathbf{p}' + \underline{\mathbf{p}_{A-1}}$$

In general,

$$E^*_{A-1} = \sqrt{(M_A - M + E_m)^2 + \boldsymbol{p}^2_{A-1}}$$

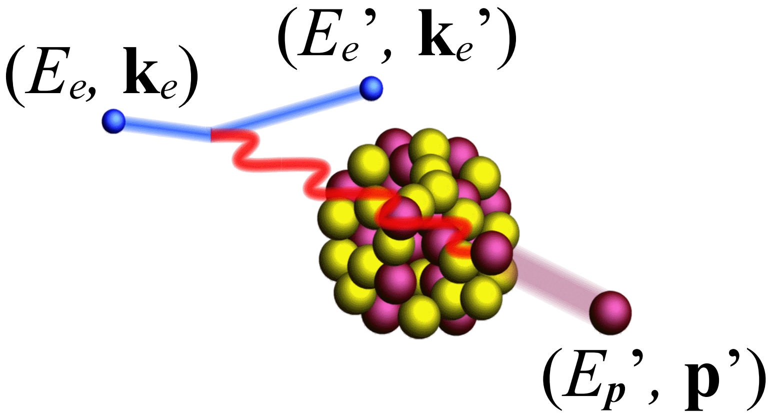
$E_m - E_{\mathrm{thr}}$ is the excitation energy of $^{39}\mathrm{Cl}$

Without final state interactions

$$-\mathbf{p}_{A-1} = \mathbf{p}_m$$

is the initial proton momentum

Missing energy E_m and missing momentum $\mathbf{p}_m$



$$E_e + M - E_m = E_{e'} + E_p$$

known

missing

$$\mathbf{k}_e + \mathbf{p}_m = \mathbf{k}_{e'} + \mathbf{p}'$$

For negligible recoil energy,

$$E^*_{A-1} = M_A - M + E_m$$

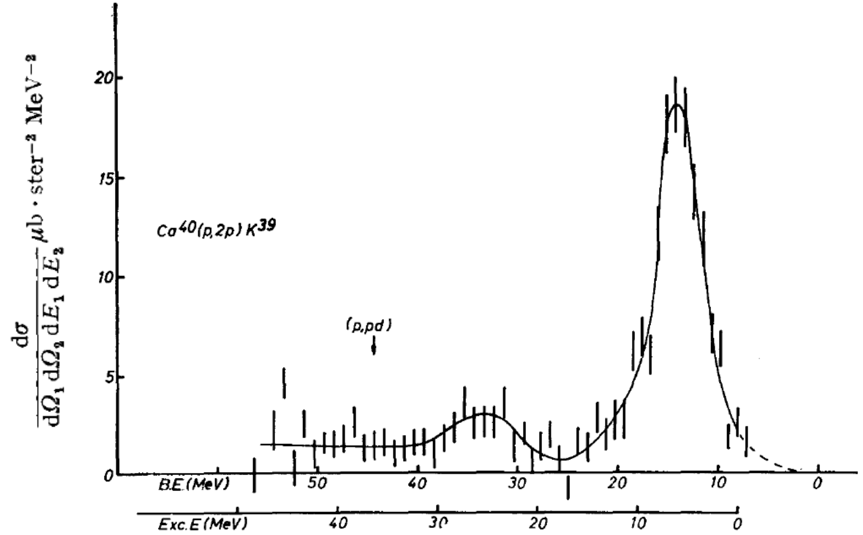
$E_m - E_{\text{thr}}$ is the excitation energy of ^{39}Cl

Without final state interactions

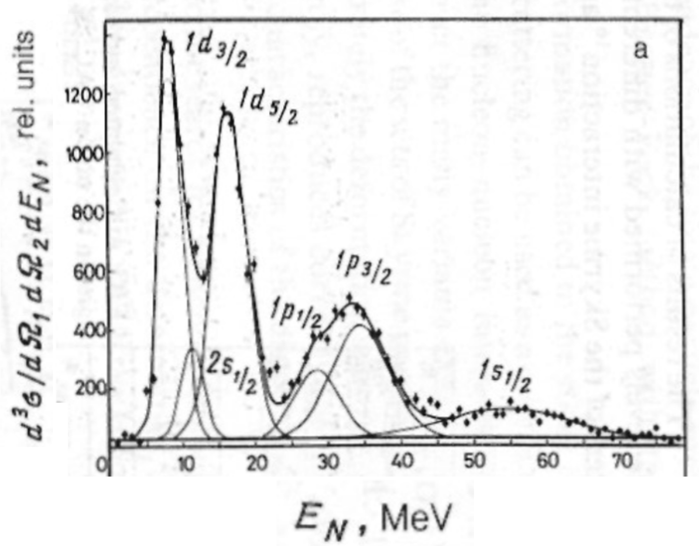
$$-\mathbf{p}_{A-1} = \mathbf{p}_m$$

is the initial proton momentum

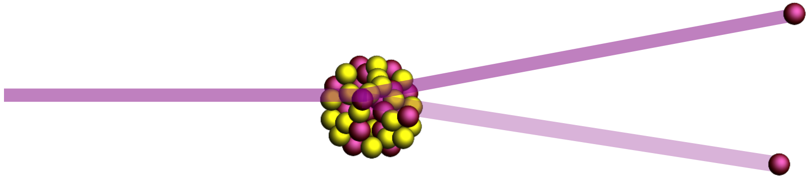
Proton coincidence scattering



Tyren *et al.*, NP 7, 10 (1958)



Volkov *et al.*, SJNP 52, 848 (1990)



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Nuclear Physics **31** (1962) 139—151; © *North-Holland Publishing Co., Amsterdam*

QUASI-FREE ELECTRON-PROTON SCATTERING (I)

GERHARD JACOB † and TH. A. J. MARIS ††

Instituto de Física and Faculdade de Filosofia, Universidade do Rio Grande do Sul, Pôrto Alegre, Brasil

Received 6 July 1961

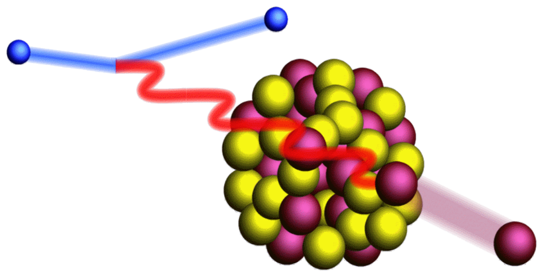
"… quasi-free **(e,e′p) scattering should offer a clear advantage over the (p,2p) processes**. … In a quasi-free (e,e′p) scattering event only the outgoing proton has an appreciable chance of being absorbed in the nucleus. Therefore surface interactions are much less accentuated than in the (p,2p) scattering and **the contributions of the inner shells relatively to those of the upper shell will be much larger**, especially for medium or heavy nuclei."

"The electron-proton angular correlation distributions would, for light and medium nuclei, nearly directly give **the momentum distributions of the separate shells**."

Argon spectral functions

E12-14-012 in JLab: (*e*,*e’*) and (*e*,*e’p*) on Ar and Ti

Aim: Obtaining the experimental input indispensable to construct the argon spectral function, thus paving the way for a reliable estimate of the neutrino cross sections in DUNE. In addition, stimulating a number of theoretical developments, such as the description of final-state interactions. [Benhar *et al.*, arXiv:1406.4080]

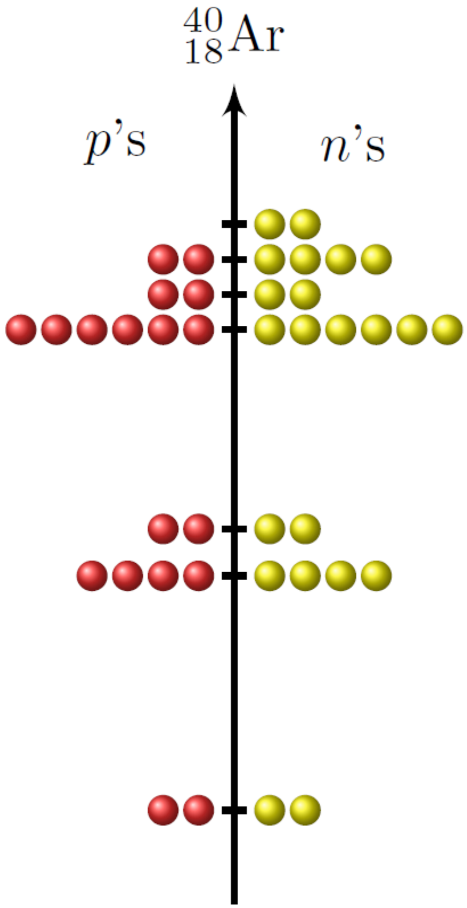


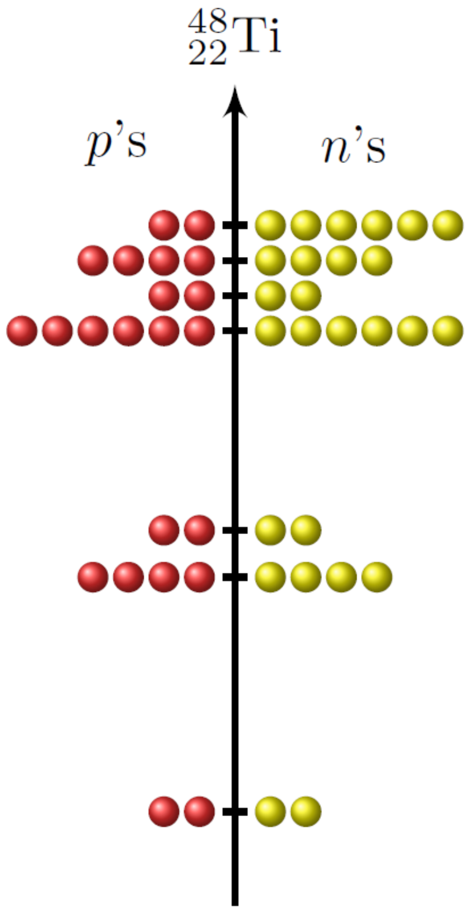
$E_e = 2.222$ GeV

	E'_e (GeV)	θ_e (deg)	$\lvert\mathbf{p}'\rvert$ (MeV)	$\theta_{p'}$ (deg)	$\lvert\mathbf{q}\rvert$ (MeV)	p_m (MeV)	E_m (MeV)
kin1	1.777	21.5	915	−50.0	865	50	73
kin2	1.716	20.0	1030	−44.0	846	184	50
kin3	1.799	17.5	915	−47.0	741	174	50
kin4	1.799	15.5	915	−44.5	685	230	50
kin5	1.716	15.5	1030	−39.0	730	300	50

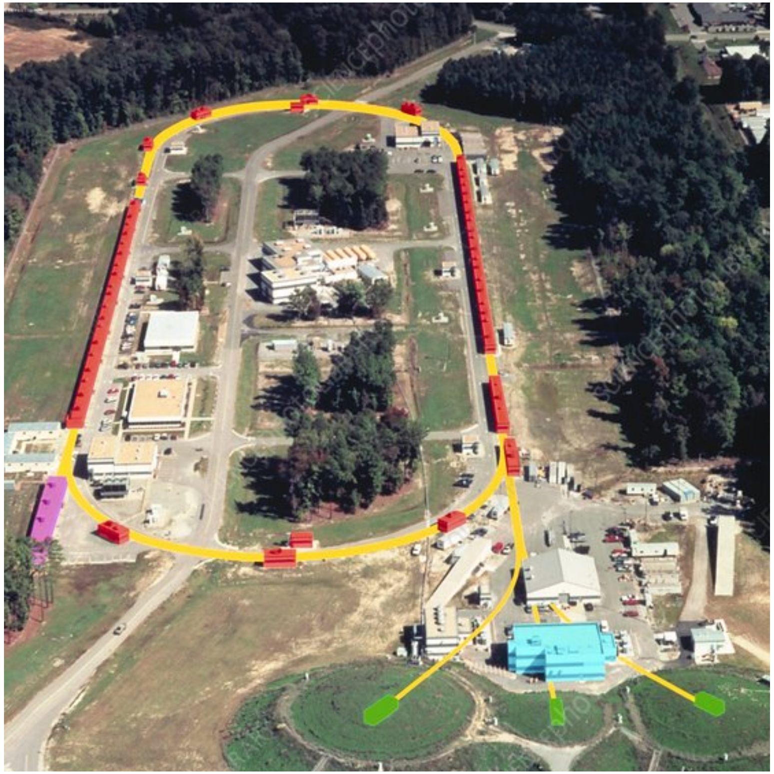
Jiang *et al.*, PRD 105, 112002 (2022); PRD 107, 012005 (2023)

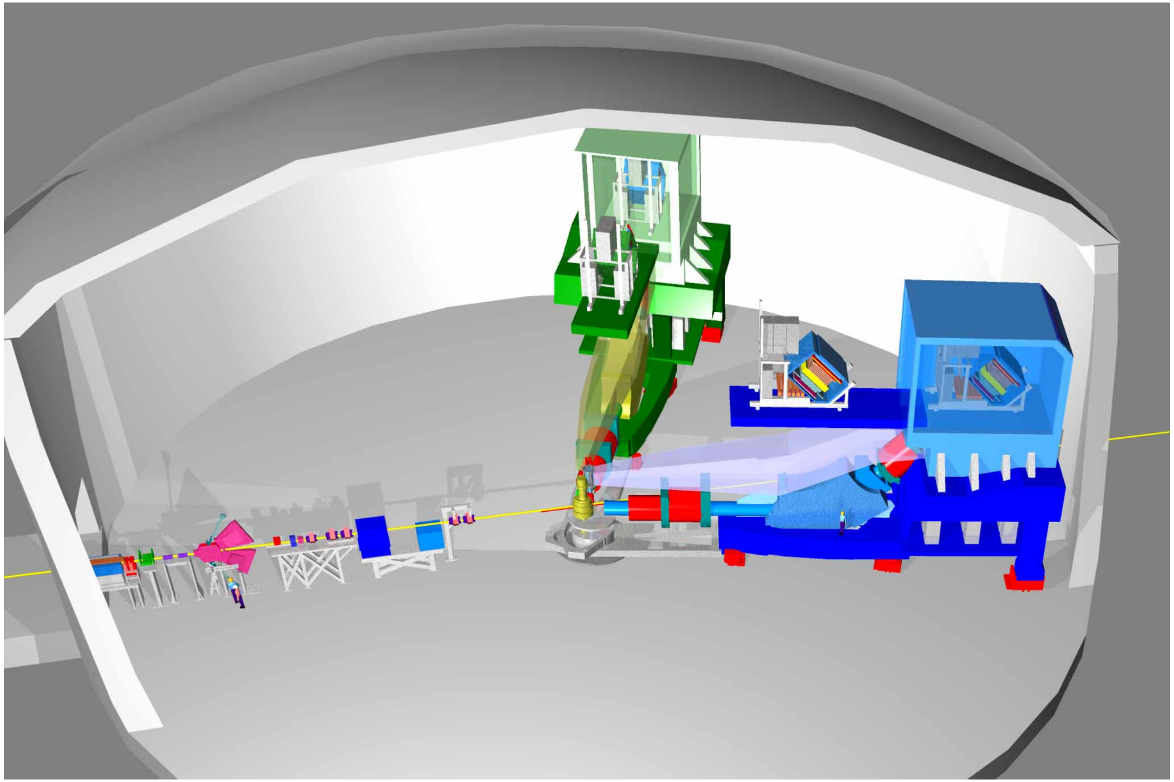
Why titanium?



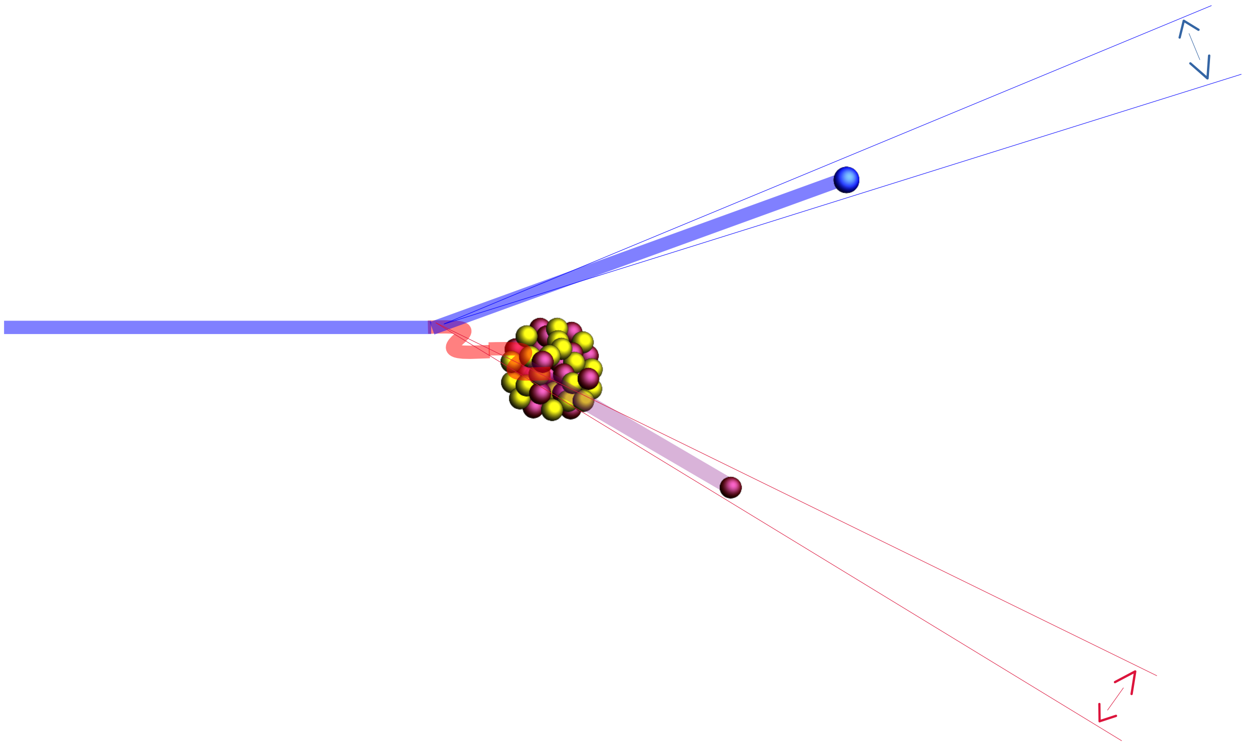


Jefferson Laboratory Hall A





Coincidence scattering



Tracks required to be
±3 mrad (±0.17°) in-plane
±6 mrad (±0.34°) out of plane
…
to reduce the contribution of FSI

(*e*,*e'p*) cross section

elementary cross section

nuclear transparency

$$\frac{d^4\sigma_{\rm IA}}{d\Omega_{k'}dE_{k'}d\Omega_{p'}dE_{p'}} \propto \sigma_{ep}\, S(\mathbf{p}, E)\, T_A(E_{p'})$$

spectral function

T. de Forest Jr., NPA 392, 232 (1983)

Mean-field part of the spectral function

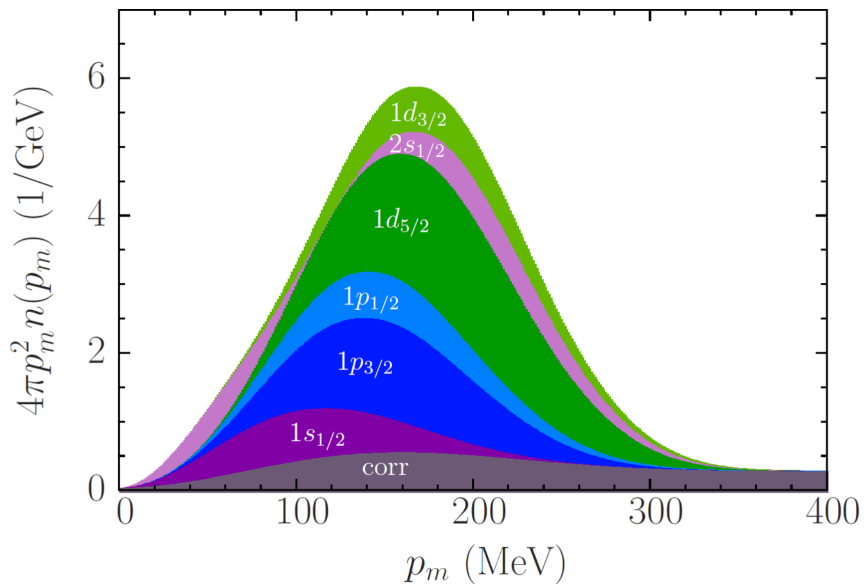
spectroscopic factor

energy distribution

$$P_{\rm MF}(p_m, E_m) = \sum_\alpha S_\alpha |\phi_\alpha(p_m)|^2 f_\alpha(E_m)$$

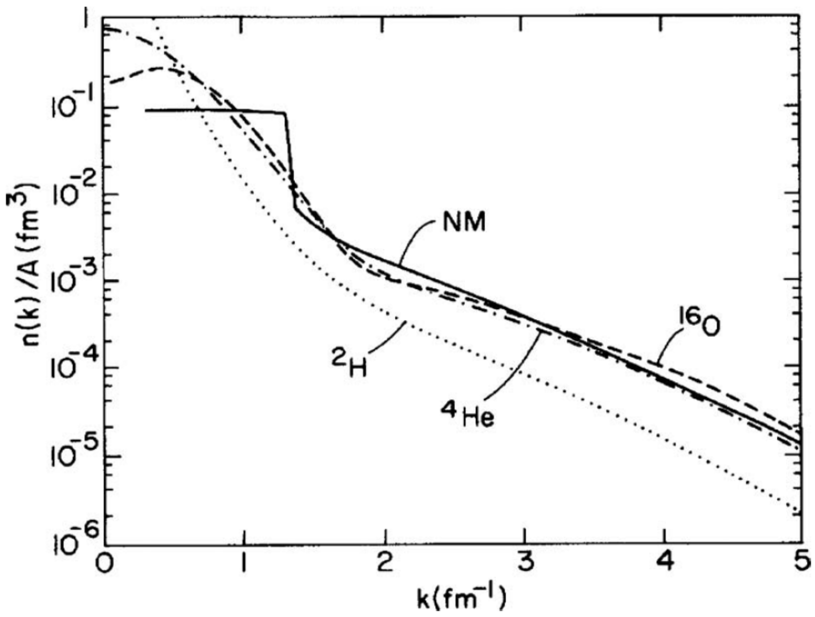
wave function in momentum space

Relativistic MF calculations by C. Giusti



Correlated part of the spectral function

Benhar *et al.*, RMP **80**, 189 (2008)



Ciofi degli Atti and Simula, PRC **53**, 1689 (1996)

- Correlated nucleons form quasi-deuteron pairs, with the relative momentum distributed as in deuteron.
- *NN* pairs undergo CM motion (Gaussian distrib.)
- Excitation energy of the (A − 1)-nucleons is their kinetic energy plus the *pn* knockout threshold

Analysis procedure

1) Extract of the (*e,e'p*) cross section

2) Using σ_{cc1} of de Forest and nuclear transparency, obtain the reduced cross sections as a function of (a) p_m and (b) E_m.

3) Find the parameters of the p_m distribution (*i.e.*, spectroscopic factors) from the fits to the reduced cross sections as a function of p_m (for the E_m ranges of 0–30, 30–54, 54–90 MeV).

4) Using the priors from Step 3), find the parameters of the E_m distribution (*i.e.*, spectroscopic factors, peak positions, distribution widths) from the fits to the reduced cross sections as a function of E_m.

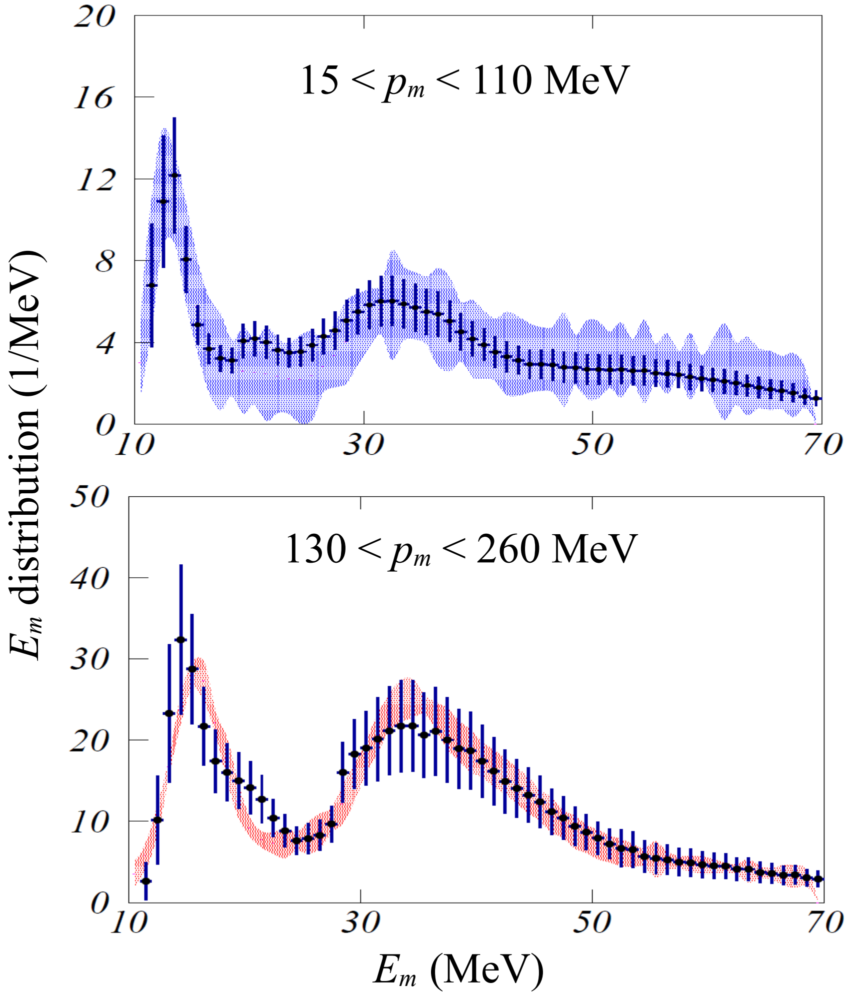
Missing energy distributions for Ar and Ti

α	E_α (MeV) w/ priors	E_α (MeV) w/o priors	σ_α (MeV) w/ priors	σ_α (MeV) w/o priors
$1d_{3/2}$	12.53 ± 0.02	10.90 ± 0.12	1.9 ± 0.4	1.6 ± 0.4
$2s_{1/2}$	12.92 ± 0.02	12.57 ± 0.38	3.8 ± 0.8	3.0 ± 1.8
$1d_{5/2}$	18.23 ± 0.02	17.77 ± 0.80	9.2 ± 0.9	9.6 ± 1.3
$1p_{1/2}$	28.8 ± 0.7	28.7 ± 0.7	12.1 ± 1.0	12.0 ± 3.6
$1p_{3/2}$	33.0 ± 0.3	33.0 ± 0.3	9.3 ± 0.5	9.3 ± 0.5
$1s_{1/2}$	53.4 ± 1.1	53.4 ± 1.0	28.3 ± 2.2	28.1 ± 2.3
corr.	24.1 ± 2.7	24.1 ± 1.7	—	—

Jiang *et al.*, PRD 105, 112002 (2022)

α	E_α (MeV) w/ priors	E_α (MeV) w/o priors	σ_α (MeV) w/ priors	σ_α (MeV) w/o priors
$1f_{7/2}$	11.32 ± 0.10	11.31 ± 0.10	8.00 ± 5.57	8.00 ± 6.50
$1d_{3/2}$	12.30 ± 0.24	12.33 ± 0.24	7.00 ± 0.61	7.00 ± 3.84
$2s_{1/2}$	12.77 ± 0.25	12.76 ± 0.25	7.00 ± 3.76	7.00 ± 3.84
$1d_{5/2}$	15.86 ± 0.20	15.91 ± 0.22	2.17 ± 0.27	2.23 ± 0.29
$1p_{1/2}$	33.33 ± 0.60	33.15 ± 0.65	3.17 ± 0.45	3.03 ± 0.48
$1p_{3/2}$	39.69 ± 0.62	39.43 ± 0.68	5.52 ± 0.70	5.59 ± 0.70
$1s_{1/2}$	53.84 ± 1.86	52.00 ± 3.13	11.63 ± 1.90	13.63 ± 2.59
corr.	25.20 ± 0.02	25.00 ± 0.29	—	—

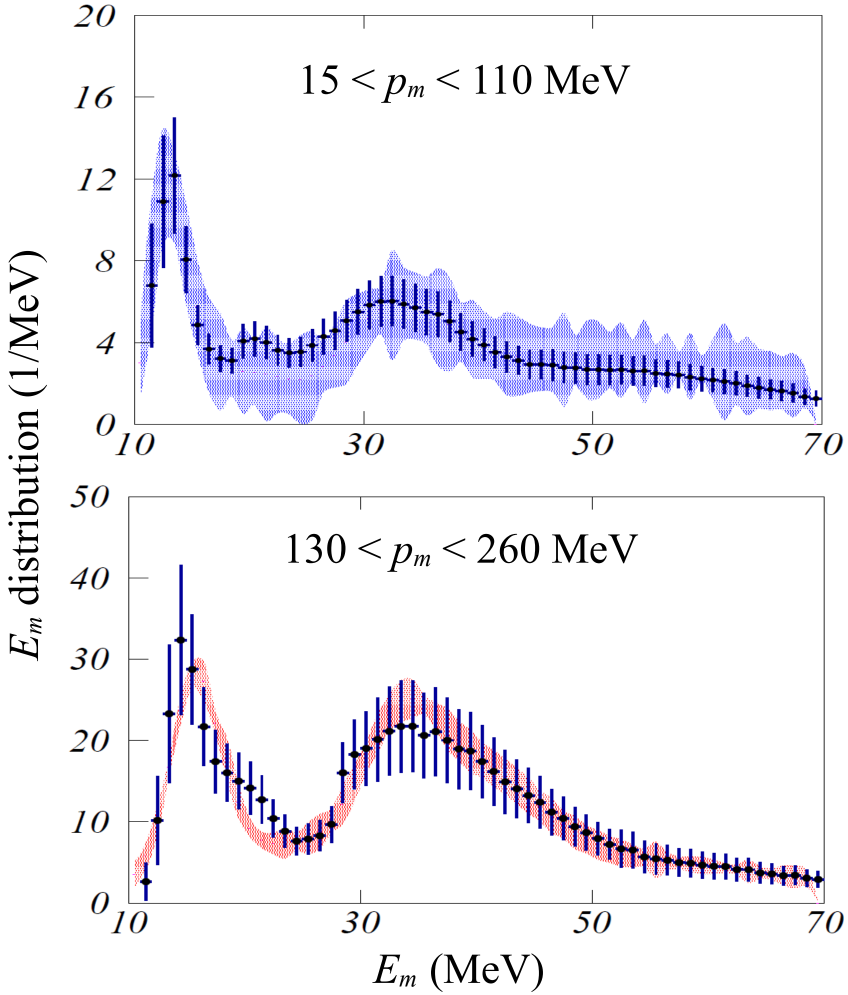
Jiang *et al.*, PRD 107, 012005 (2023)



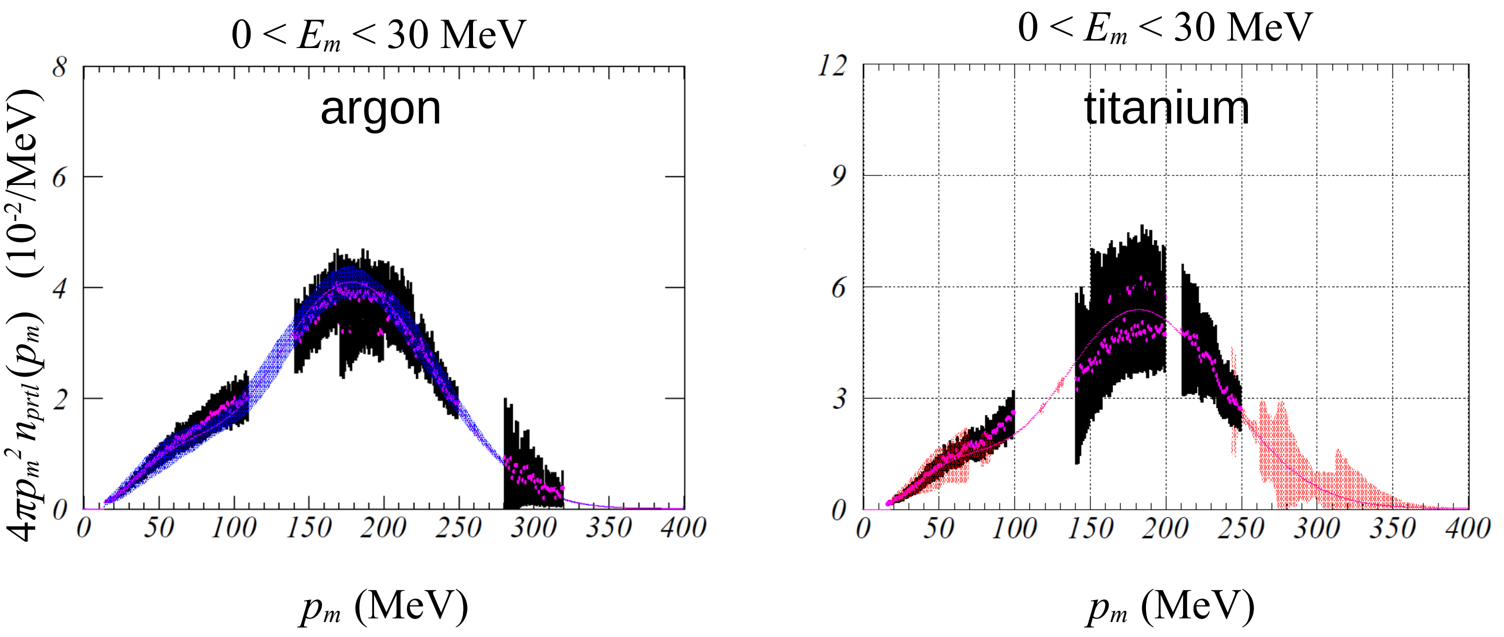
Spectroscopic factors for Ar and Ti

α	N_α	all priors	w/o p_m	w/o corr.
			S_α	
$1d_{3/2}$	2	0.89 ± 0.11	1.42 ± 0.20	0.95 ± 0.11
$2s_{1/2}$	2	1.72 ± 0.15	1.22 ± 0.12	1.80 ± 0.16
$1d_{5/2}$	6	3.52 ± 0.26	3.83 ± 0.30	3.89 ± 0.30
$1p_{1/2}$	2	1.53 ± 0.21	2.01 ± 0.22	1.83 ± 0.21
$1p_{3/2}$	4	3.07 ± 0.05	2.23 ± 0.12	3.12 ± 0.05
$1s_{1/2}$	2	2.51 ± 0.05	2.05 ± 0.23	2.52 ± 0.05
corr.	0	3.77 ± 0.28	3.85 ± 0.25	excluded
$\sum_\alpha S_\alpha$		17.02 ± 0.48	16.61 ± 0.57	14.12 ± 0.42
d.o.f		206	231	232
χ^2/d.o.f.		1.9	1.4	2.0

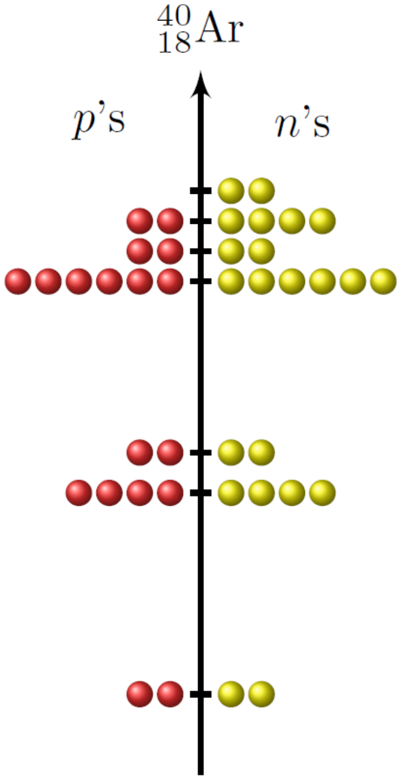
$1f_{7/2}$	2	1.53 ± 0.25	1.55 ± 0.28	1.24 ± 0.22
$1d_{3/2}$	4	2.79 ± 0.37	3.15 ± 0.54	3.21 ± 0.37
$2s_{1/2}$	2	2.00 ± 0.11	1.78 ± 0.46	2.03 ± 0.11
$1d_{5/2}$	6	2.25 ± 0.16	2.34 ± 0.19	3.57 ± 0.29
$1p_{1/2}$	2	2.00 ± 0.20	1.80 ± 0.27	2.09 ± 0.19
$1p_{3/2}$	4	2.90 ± 0.20	2.92 ± 0.20	4.07 ± 0.15
$1s_{1/2}$	2	2.14 ± 0.10	2.56 ± 0.30	2.14 ± 0.11
corr.	0	4.71 ± 0.31	4.21 ± 0.46	excluded
$\sum_\alpha S_\alpha$		20.32 ± 0.65	20.30 ± 1.03	18.33 ± 0.59
d.o.f		121	153	125
χ^2/d.o.f.		0.95	0.71	1.23



Partial momentum distributions

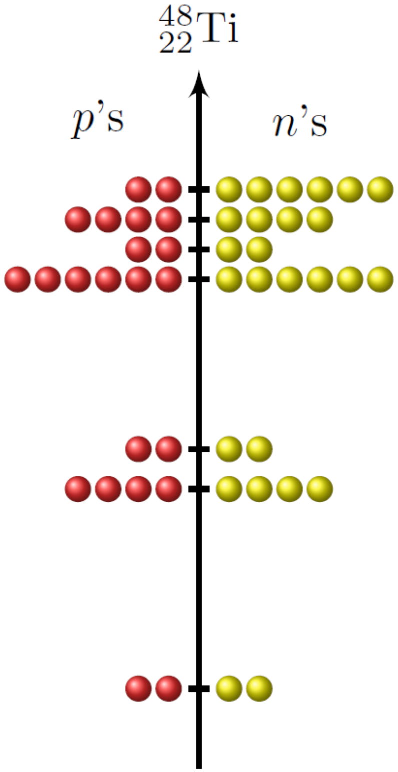


Energy levels



^{40}Ar		^{48}Ti
neutrons		protons
9.87	1f7/2	11.45
11.39	1d3/2	12.21
12.23	2s1/2	12.84
13.23	1d5/2	15.45

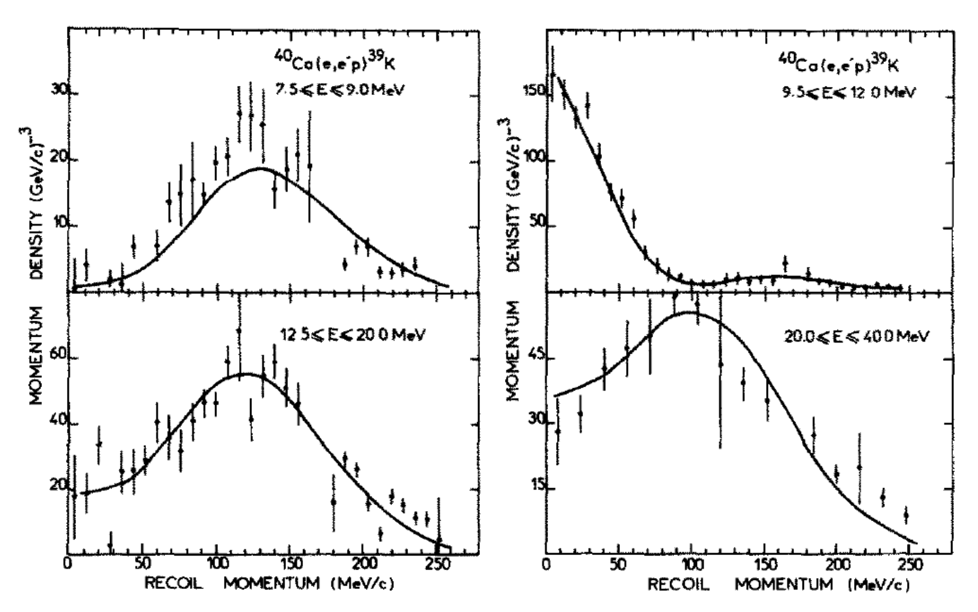
Agreement to 0.6–2.2 MeV

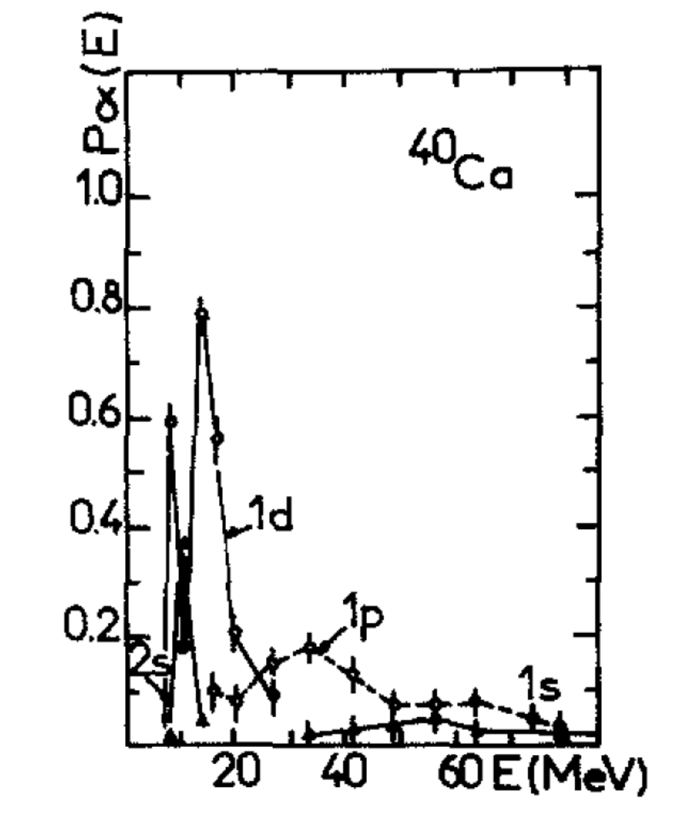


LDA spectral functions

Coincidence electron scattering in Saclay

- Beam energy ~500 MeV
- $0 \leq p_m \leq 250$ MeV, resolution 8 MeV
- $0 \leq E_m \leq 80$ MeV, resolution 1.2 MeV

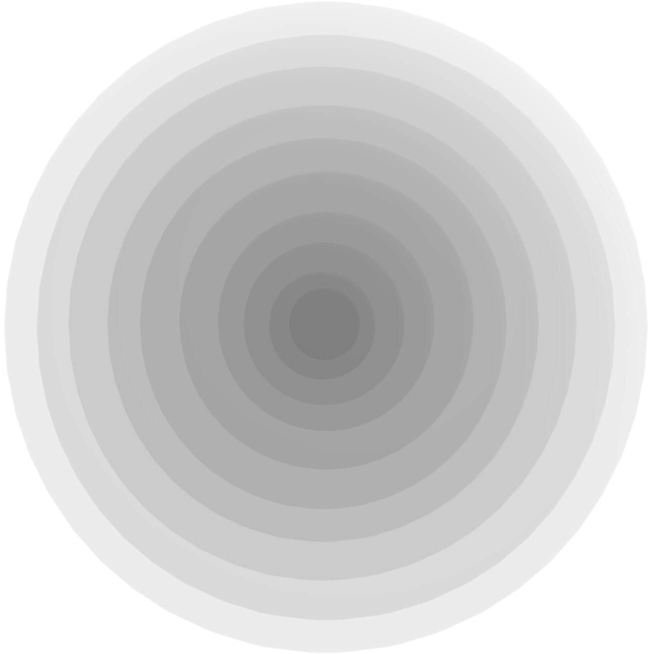




Mougey *et al.*, NPA 262, 461 (1976)

Correlated spectral function

Benhar et al., NPA **579**, 493 (1994)



- Short-range correlation do not depend on shell structure, only on density
- The correlated spectral function for nuclei can be obtained from the results for nuclear matter at different densities

$$P_{\rm corr}(\mathbf{p}, E) = \int d^3r \rho(r) P^{\rm NM}_{\rm corr}(\rho, \mathbf{p}, E)$$

- *NN* interactions: Urbana v_{14}
- *NNN* interactions: Lagaris & Pandharipande
- Shell structure from Saclay: LDA spectral functions for helium, carbon, oxygen, iron

Application example

Detection of NC events in water

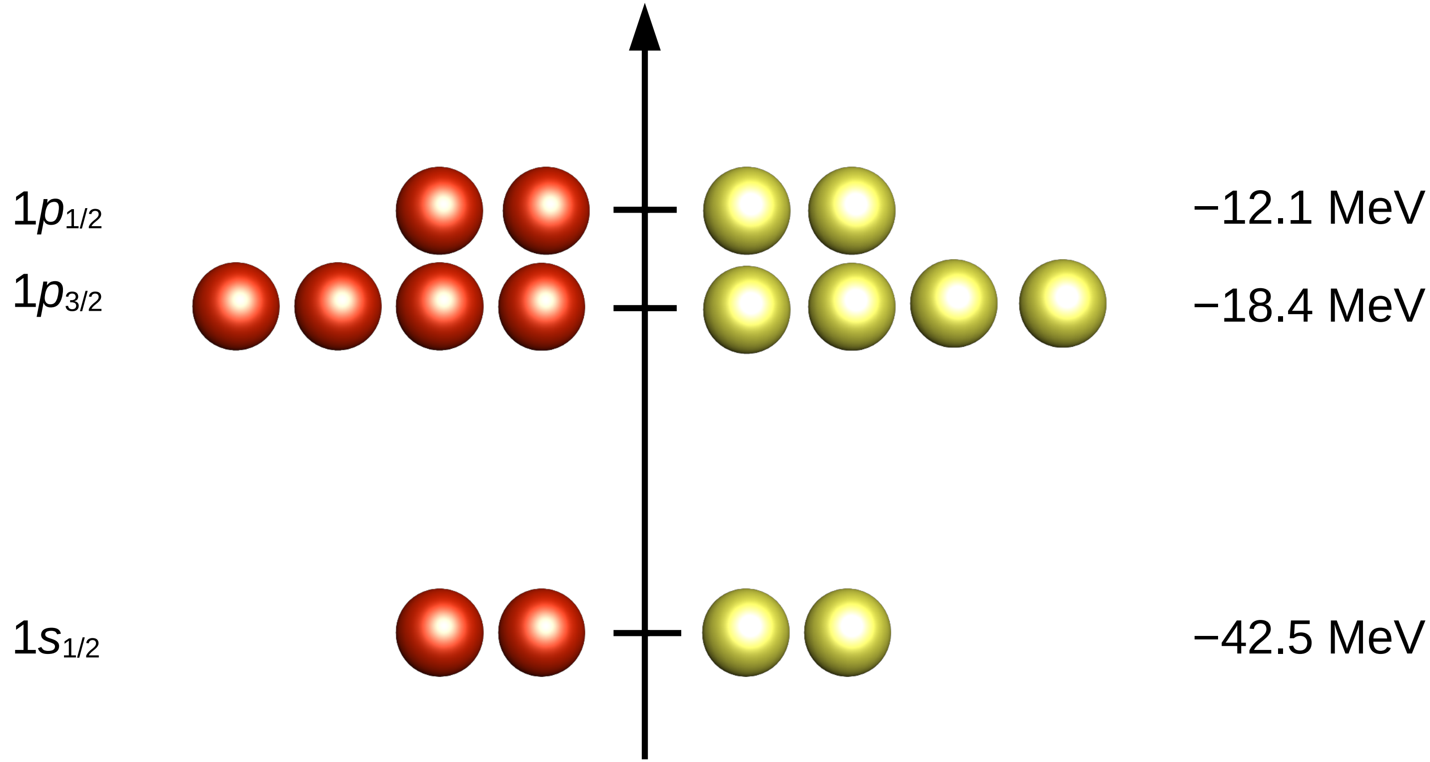
- Relevant for searches for
    - sterile neutrinos
    - diffuse supernova neutrinos
    - light dark matter

- No Cherenkov radiation from neutrons (8/18 ≈ 44% of events)

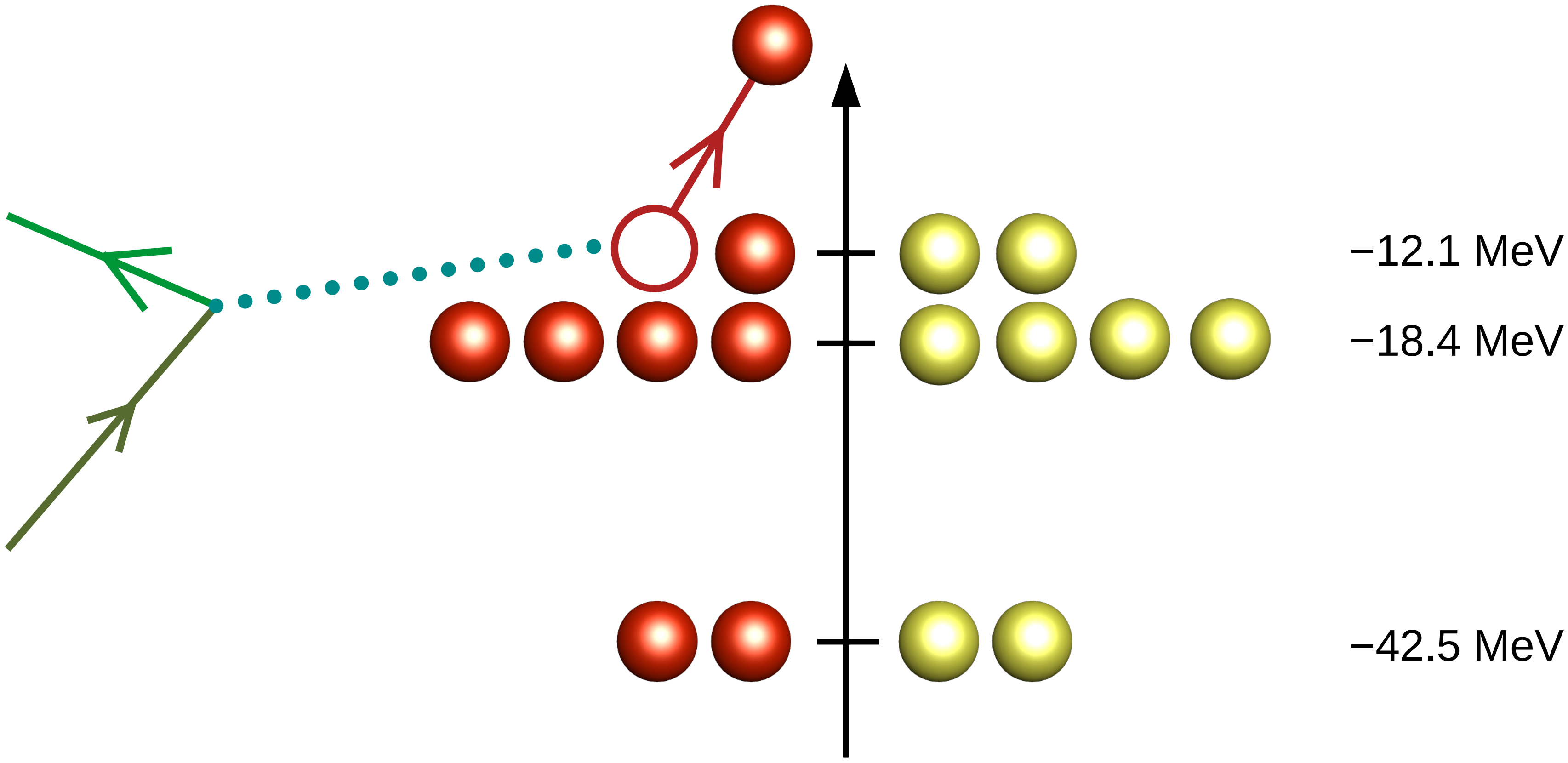
- High threshold for protons (1.07 GeV/*c*)

- Other signature required

Detection of NC events in water

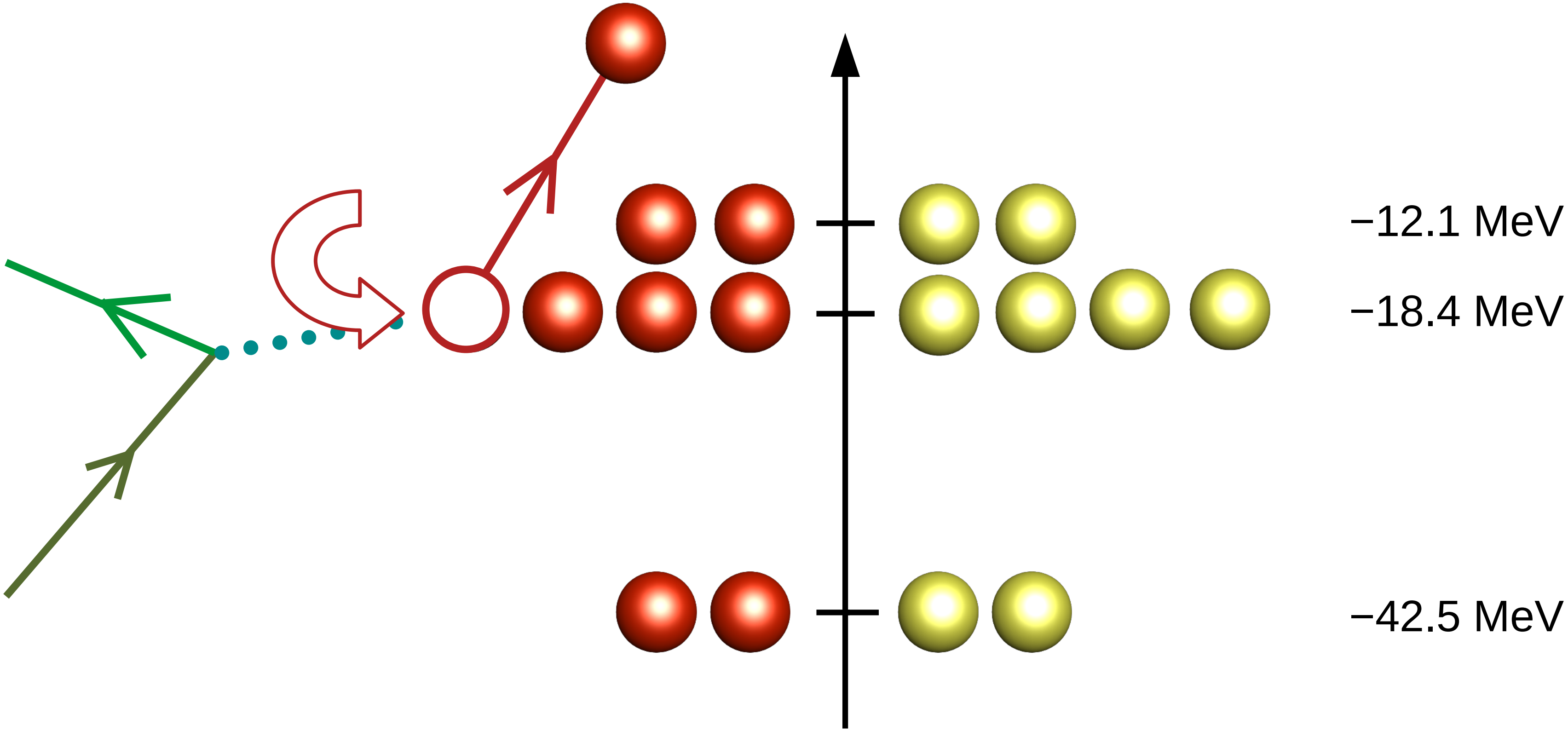


Detection of NC events in water



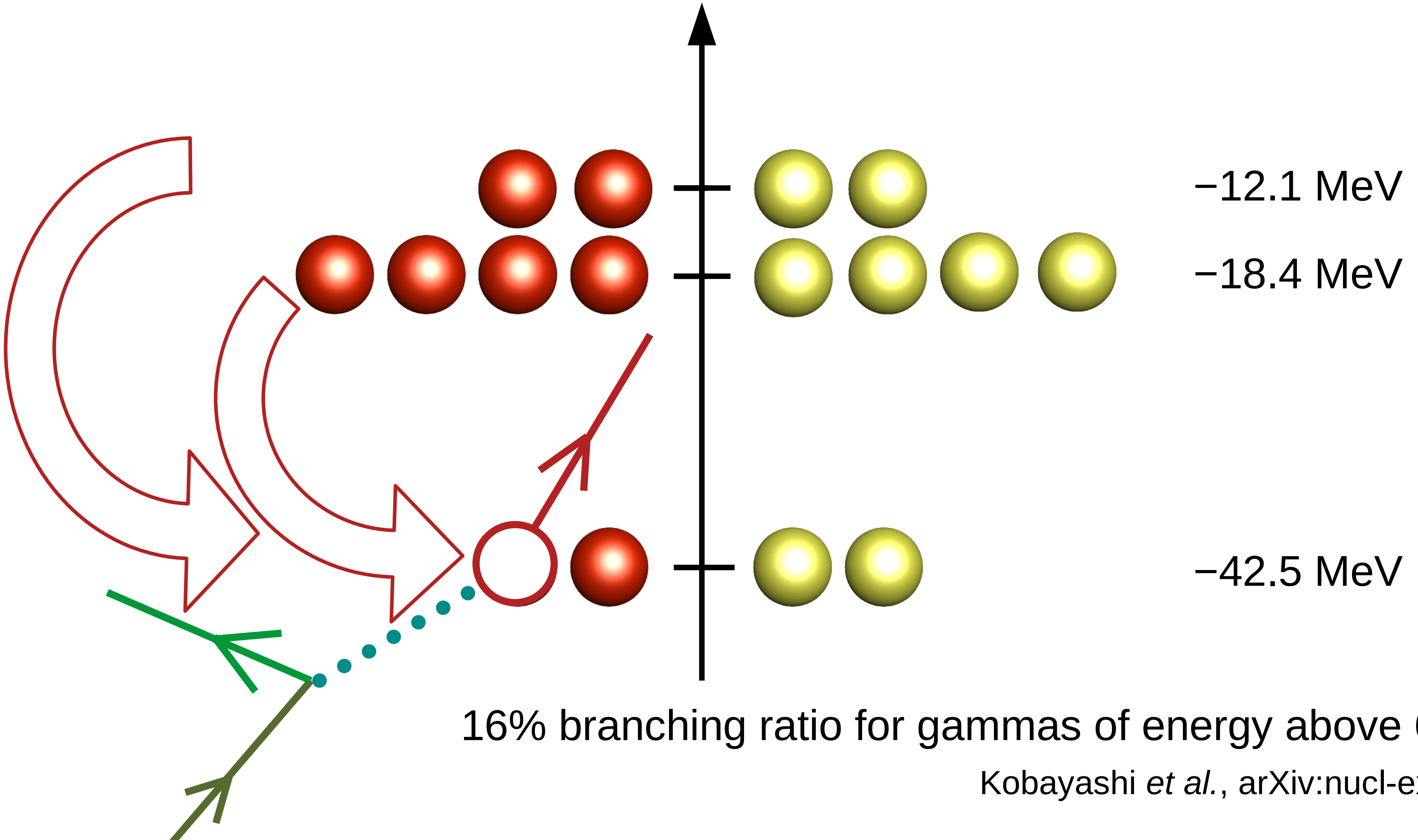
No deexcitation

Detection of NC events in water



6.3-MeV gammas from nuclear deexcitation with 100% branching ratio

Detection of NC events in water

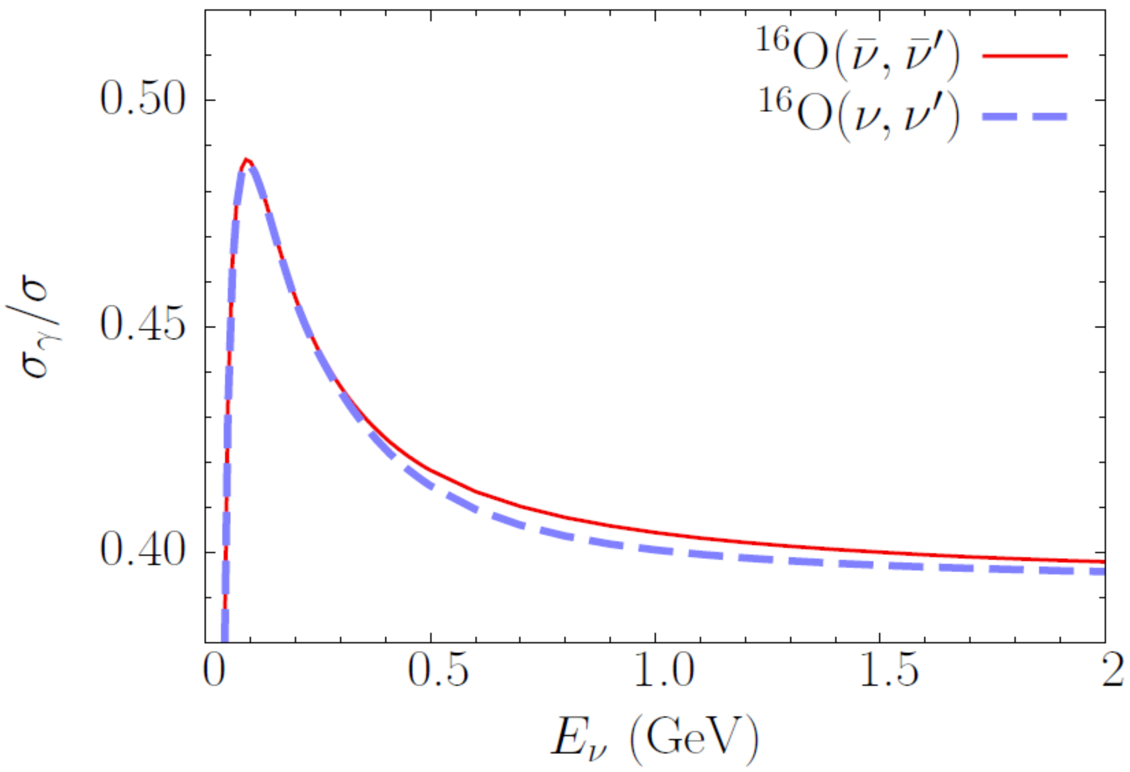


16% branching ratio for gammas of energy above 6.0 MeV

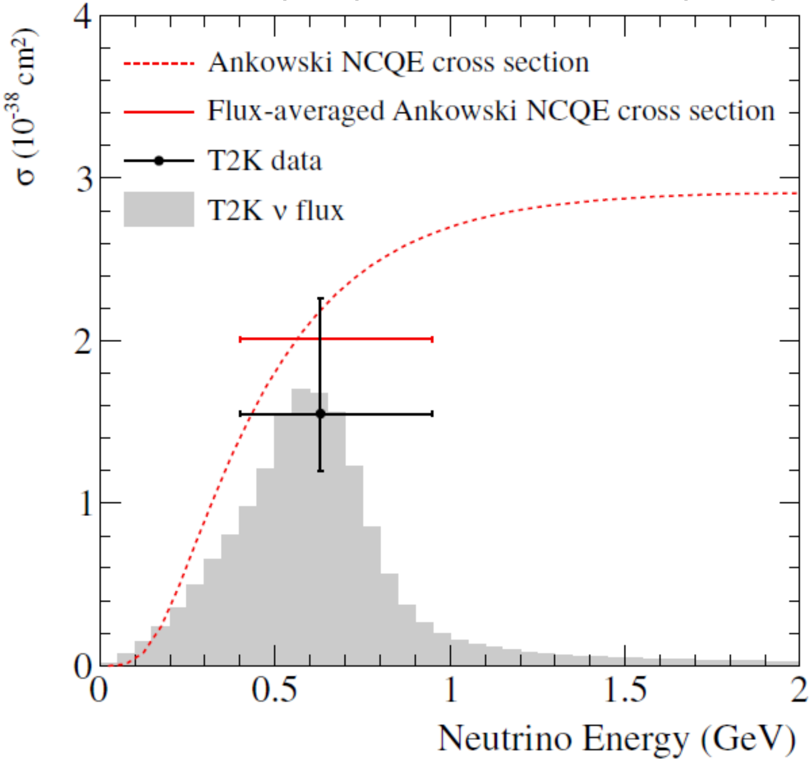
Kobayashi *et al.*, arXiv:nucl-ex/0604006

Detection of NC events in water

A. M. A. *et al.*, PRL 108, 052505 (2012)



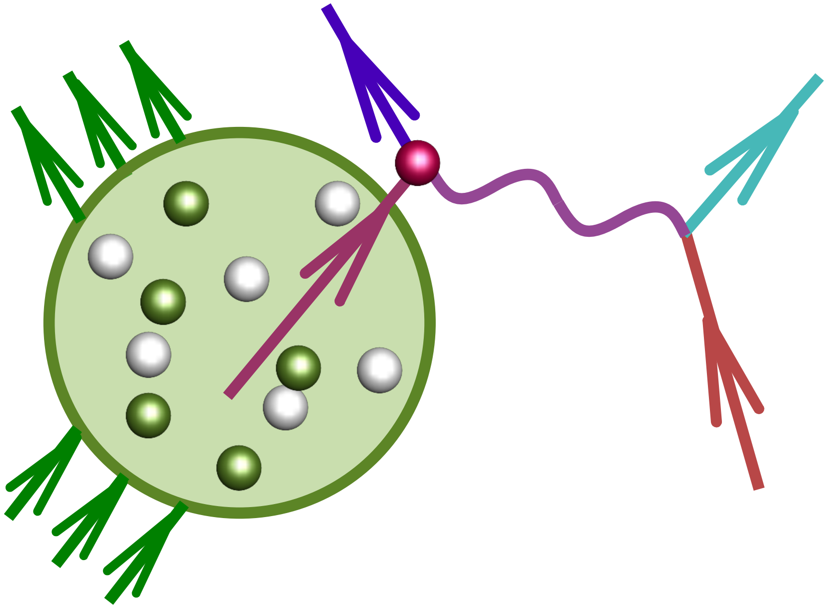
Abe *et al.* (T2K), PRD 90, 072012 (2014)



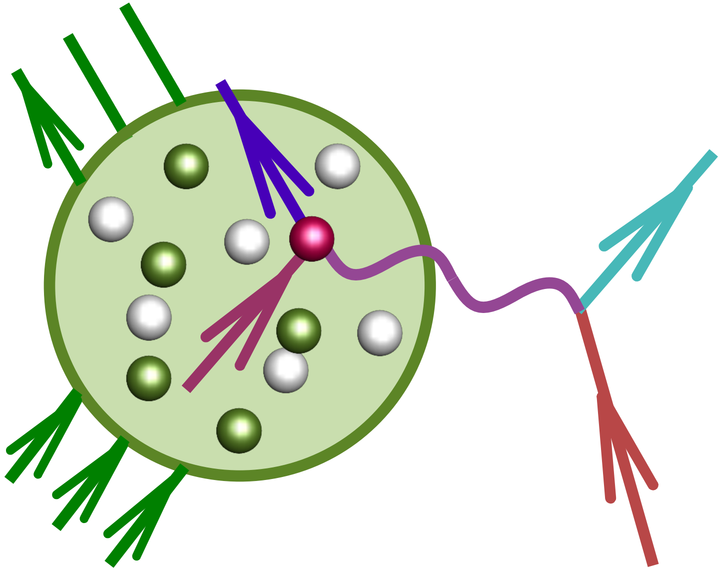
Final-state interactions

Final-state interactions

$$E_\nu + M_A = E_\mu + E_{A-1} + E_{p'}$$



$$E_\nu + M_A \sim E_\mu + E_{A-1} + E_{p'} + U_V(p')$$



Final-state interactions

Soft interactions with the spectator system change the energy spectrum of the struck nucleon. When collisions happen, additional nucleons appear in the final states.

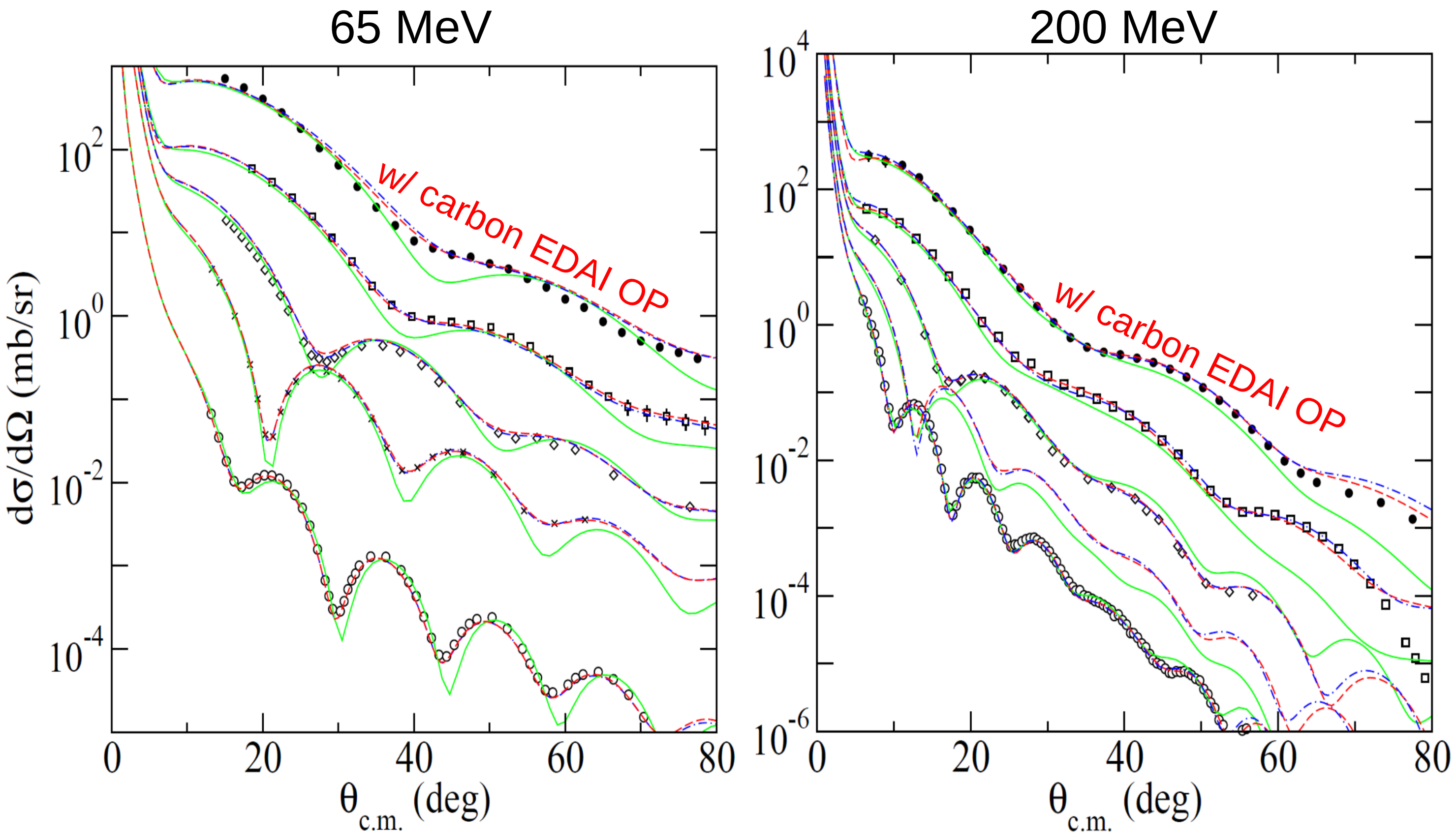
Those effects can be described using an optical potential

- the real part modifies the struck nucleon's energy spectrum, it is $\neq \sqrt{M^2 + p'^2}$
- the imaginary part introduces absorption of the struck nucleons and produces multiple hadrons in the final state (intranuclear cascade)

$$e^{iE_{p'}t} \rightarrow e^{i(E_{p'}+U)t} = e^{i(E_{p'}+U_V+iU_W)t} = e^{i(E_{p'}+U_V)t}\, e^{-U_W t}$$

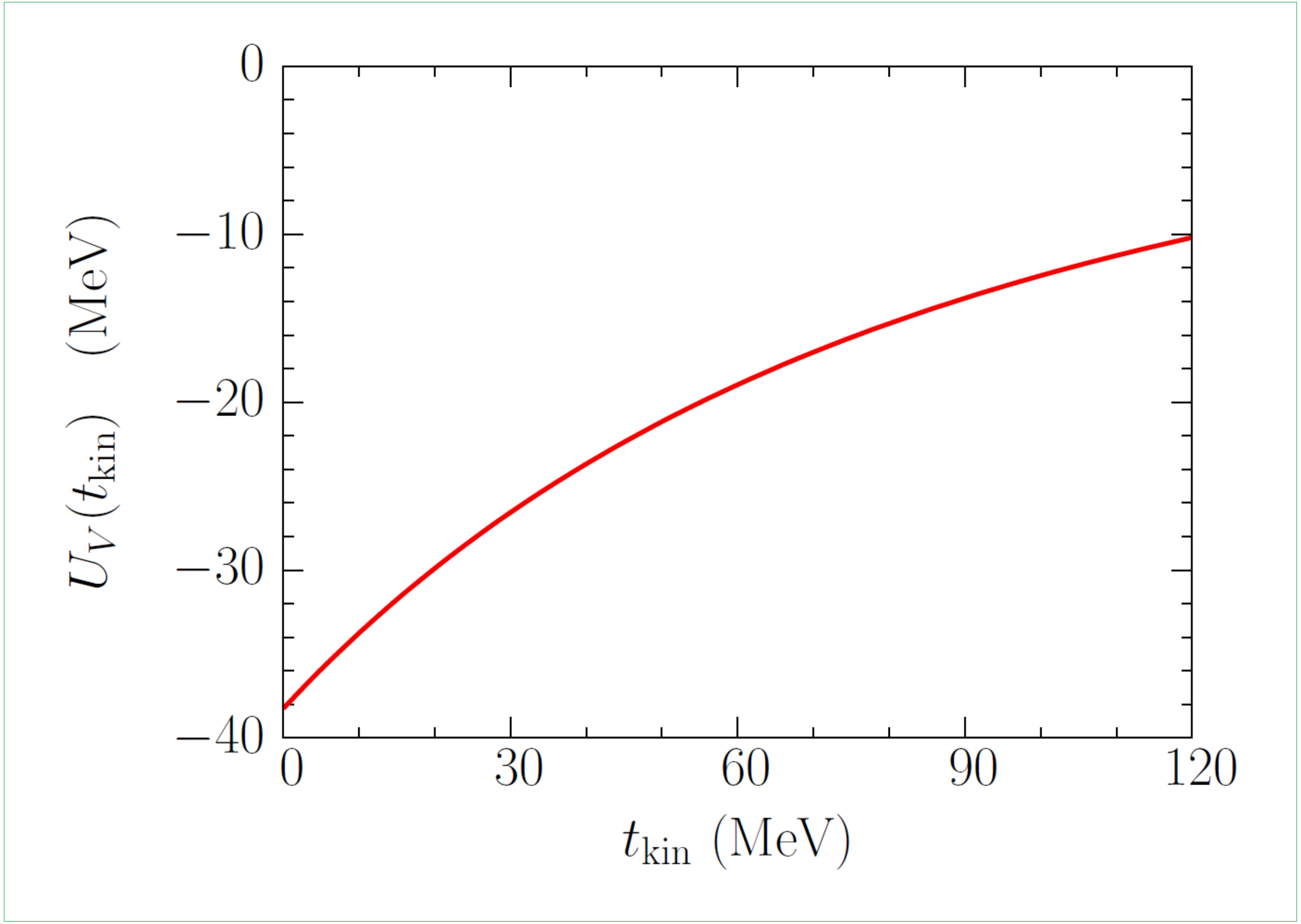
Horikawa *et al.*, PRC 22, 1680 (1980)

Elastic proton scattering on nuclei



Deb *et al.*, PRC 72, 014608 (2005)
EDAI OP by Cooper *et al.*, PRC 47, 297 (1993)

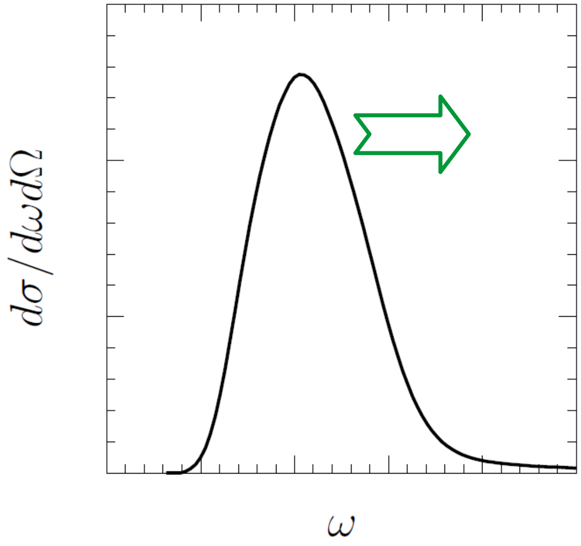
Real part of the optical potential



derived from the optical potential by Cooper *et al.*, PRC 47, 297 (1993)

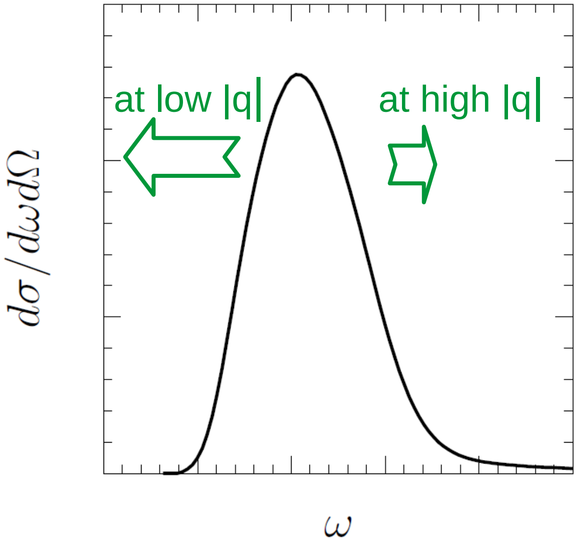
Separation energy in the RFG

- acts in the initial state
- Positive for stable nuclei
- shifts the cross section to high energy transfers ω



Real part of the optical potential

- acts in the final state
- negative at low momentum transfers $|\mathbf{q}|$, positive at high $|\mathbf{q}|$
- shifts the cross section to low ω (high ω) at low |q| (high |q|)



Final-state interactions

The convolution approach,

$$\frac{d\sigma^{\rm FSI}}{d\omega d\Omega} = \int d\omega' f_{\mathbf{q}}(\omega - \omega' - U_V)\frac{d\sigma^{\rm IA}}{d\omega d\Omega}$$

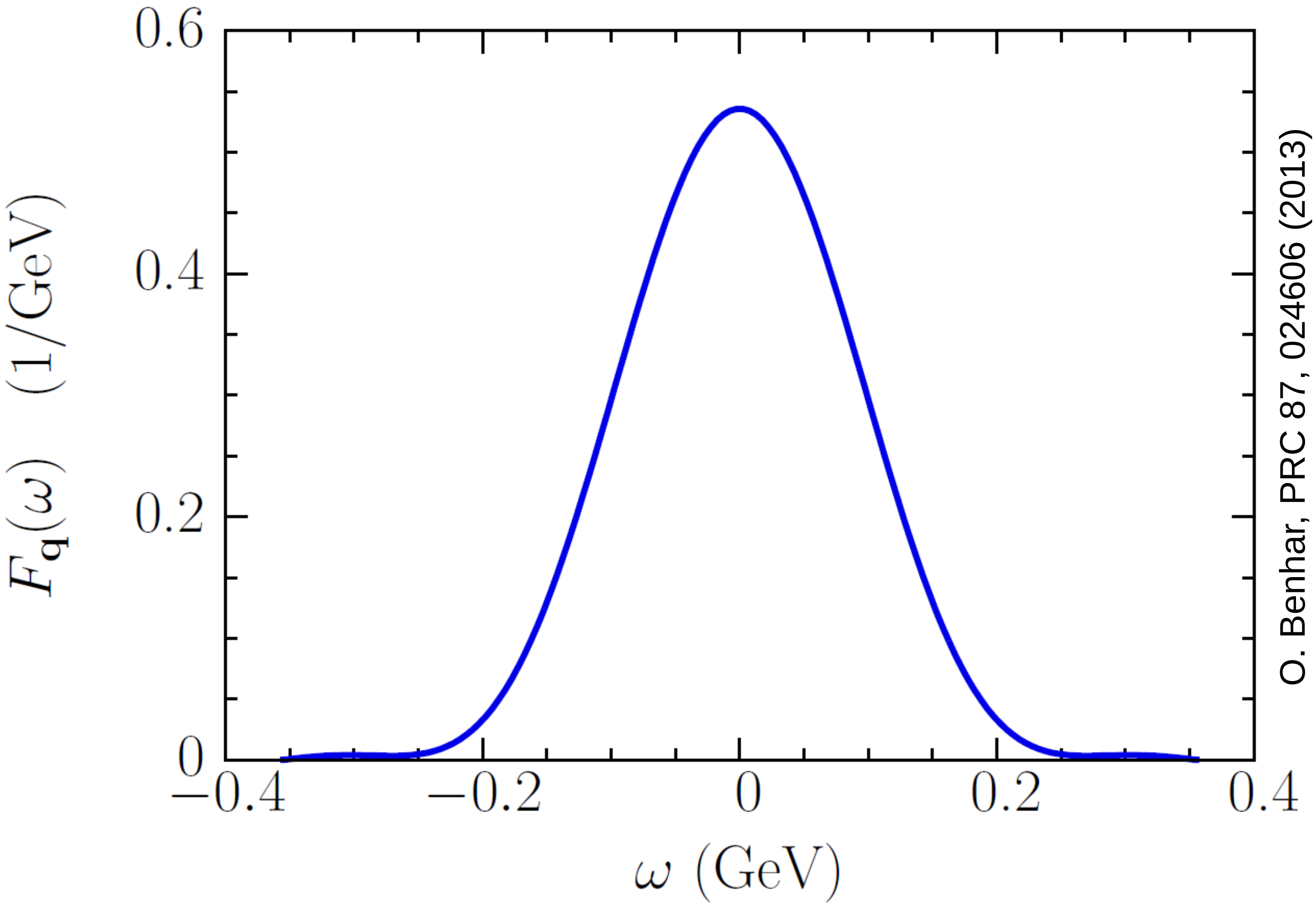
with the folding function

$$f_{\mathbf{q}}(\omega) = \delta(\omega)\sqrt{T_A} + (1 - \sqrt{T_A})F_{\mathbf{q}}(\omega)$$

and nuclear transparency T_A.

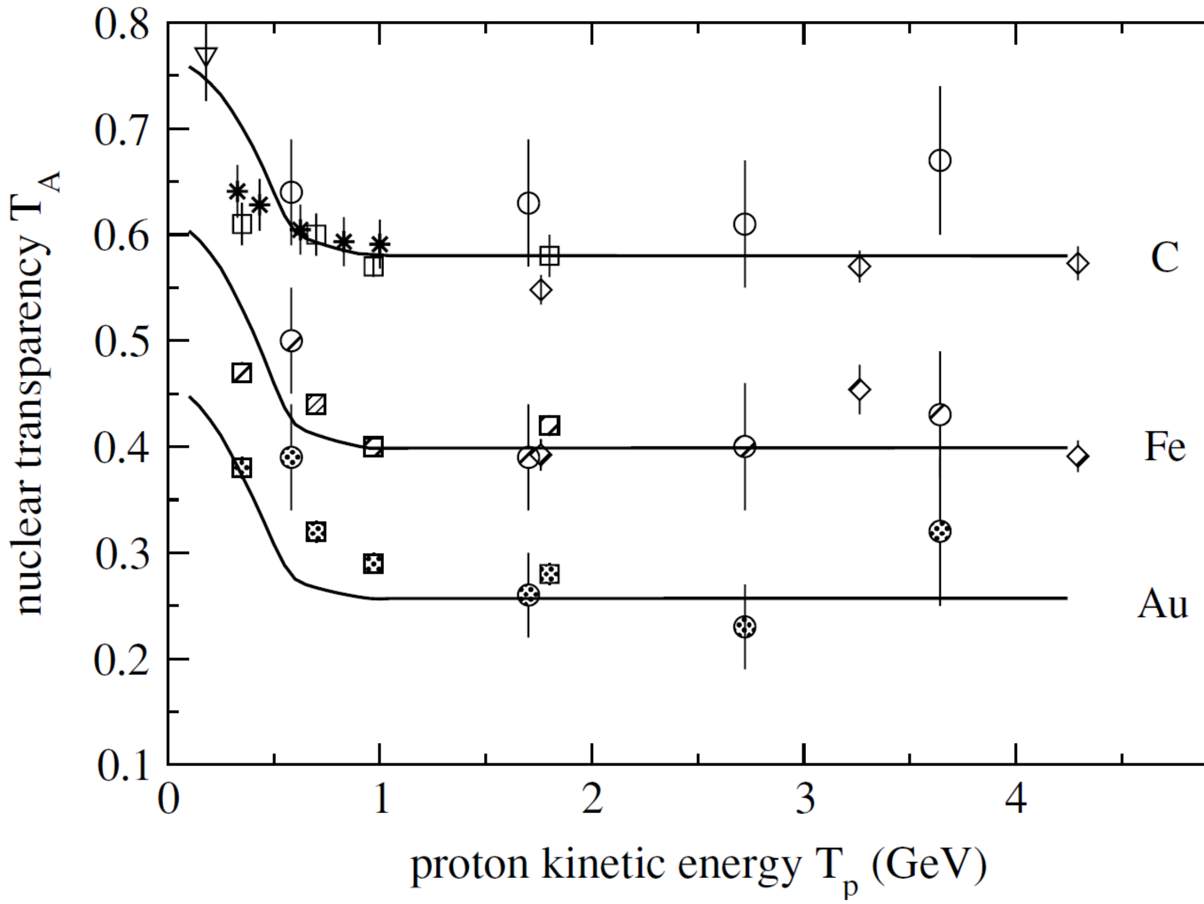
O. Benhar, PRC 87, 024606 (2013)

$F_{\mathbf{q}}(\omega)$



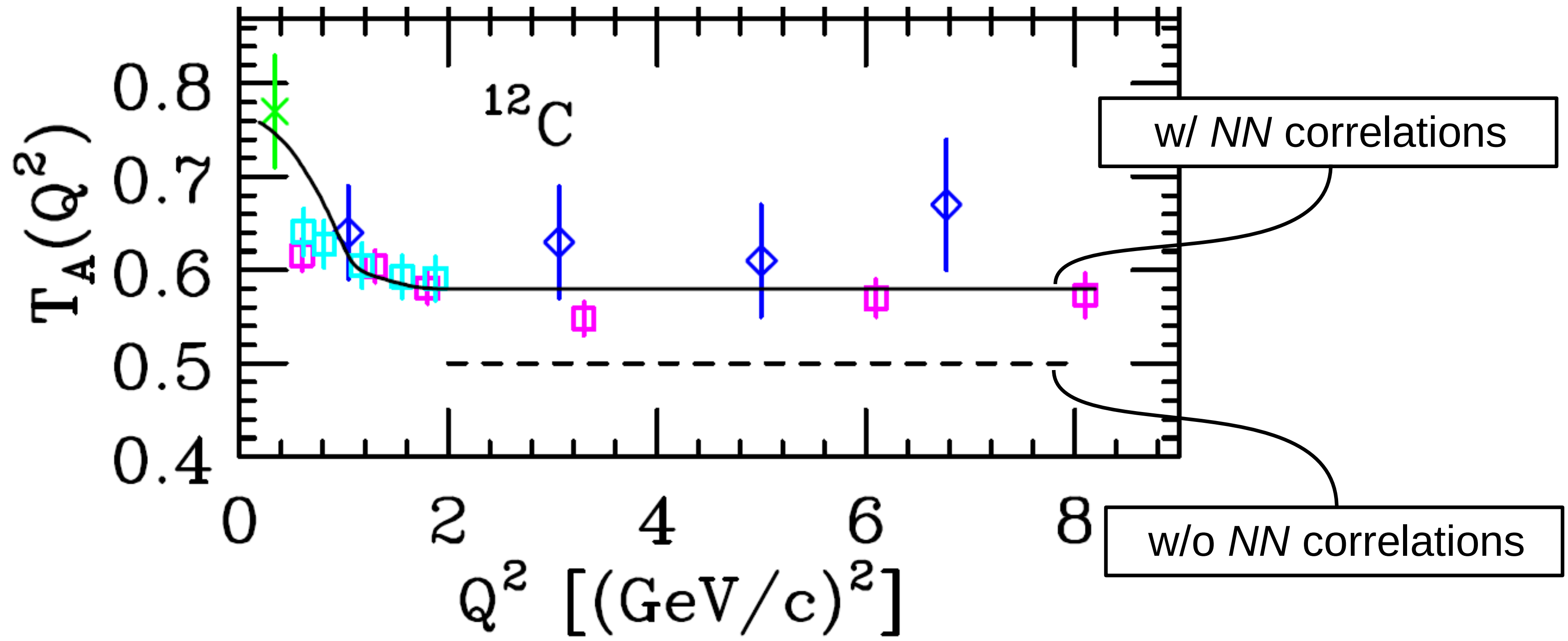
O. Benhar, PRC 87, 024606 (2013)

Nuclear transparency



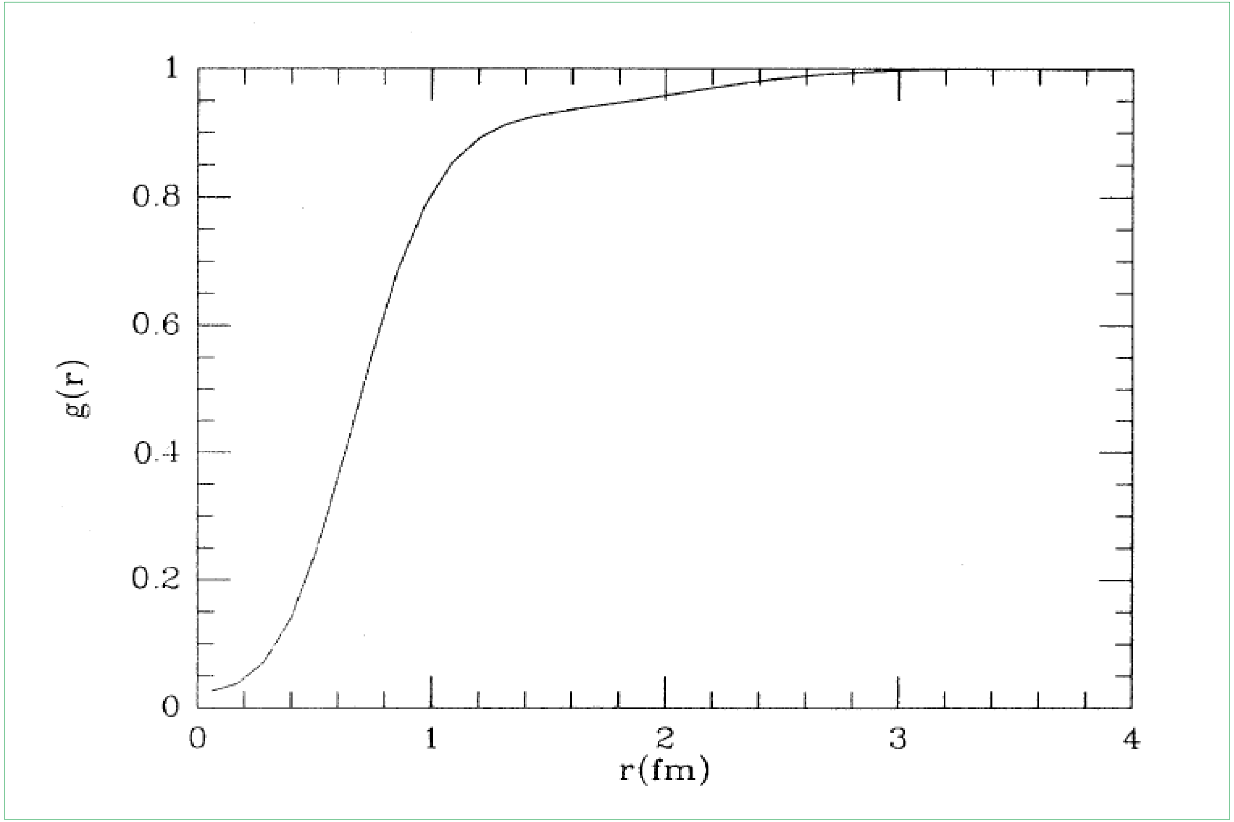
Rohe *et al.*, PRC 72, 054602 (2005)

Nuclear transparency



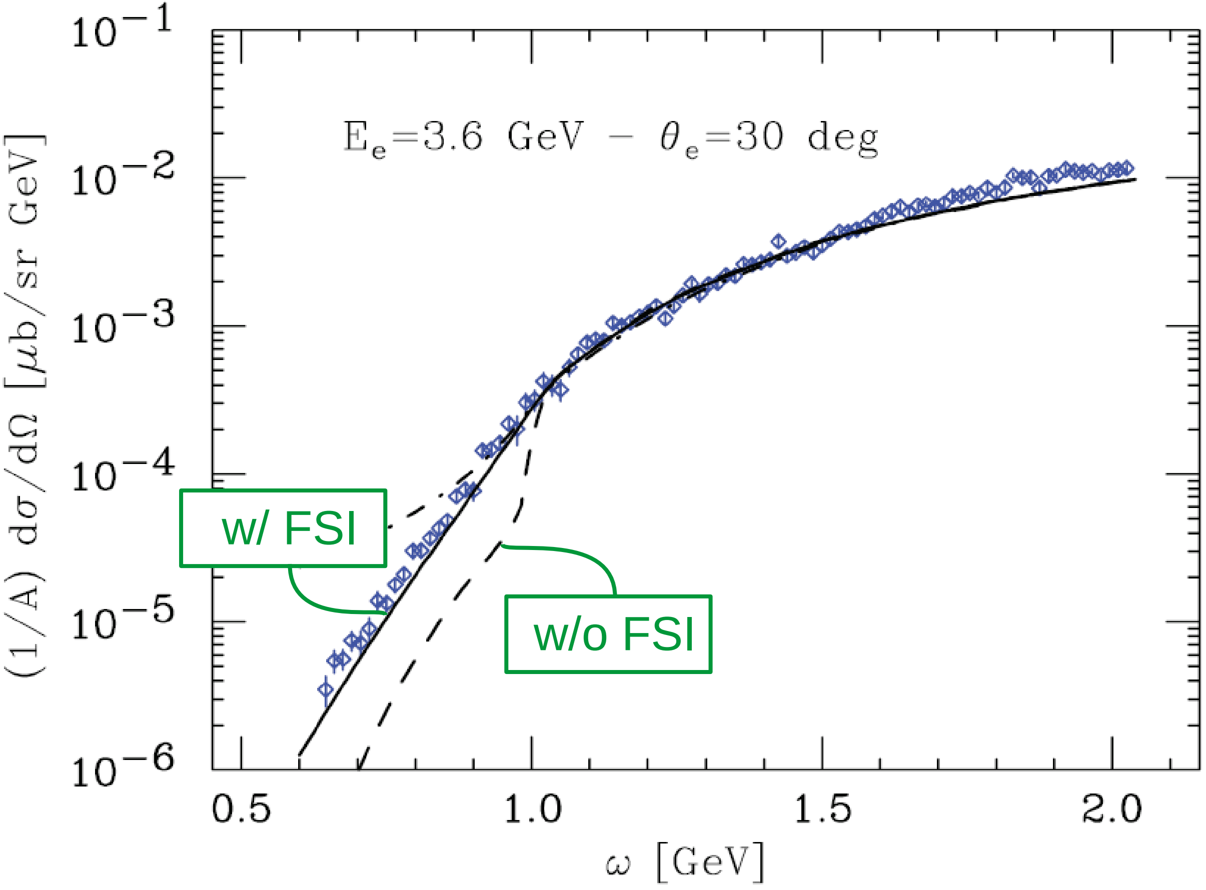
Omar Benhar at NuInt'05

NN pair distribution in nuclear matter



O. Benhar *et al.*, PRC 44, 2328 (1991)

Electron scattering on nuclear matter

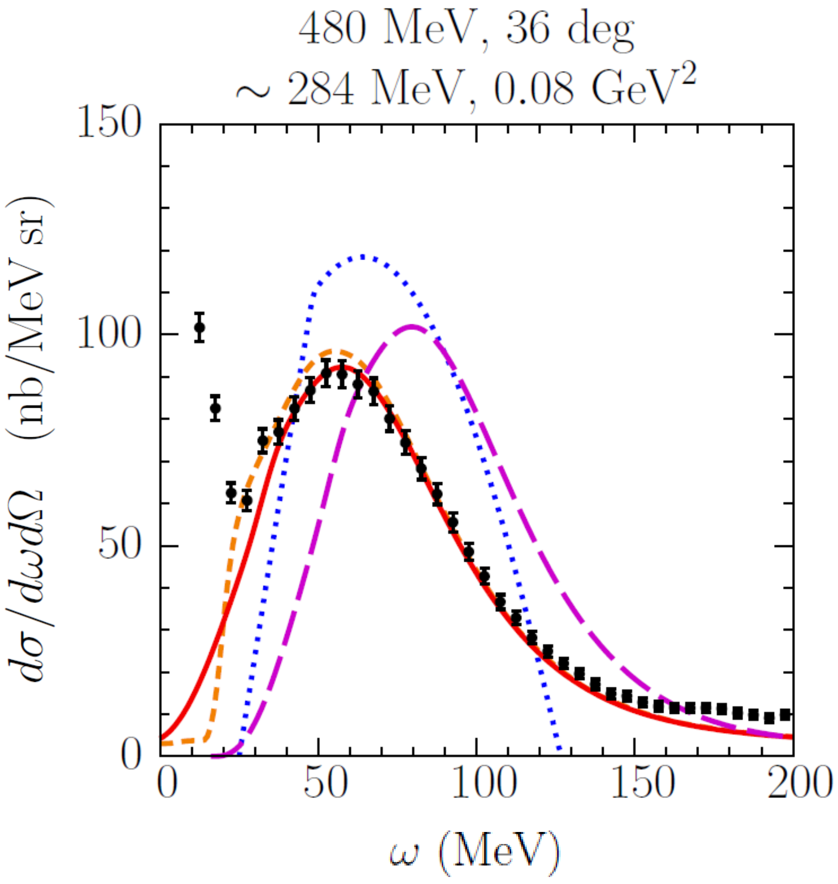


O. Benhar, PRC 87, 024606 (2013)

Realistic description of the nucleus: C(*e*,*e*')

SF w/ FSI,
LDA treatment
of Pauli blocking

SF w/ FSI,
step function

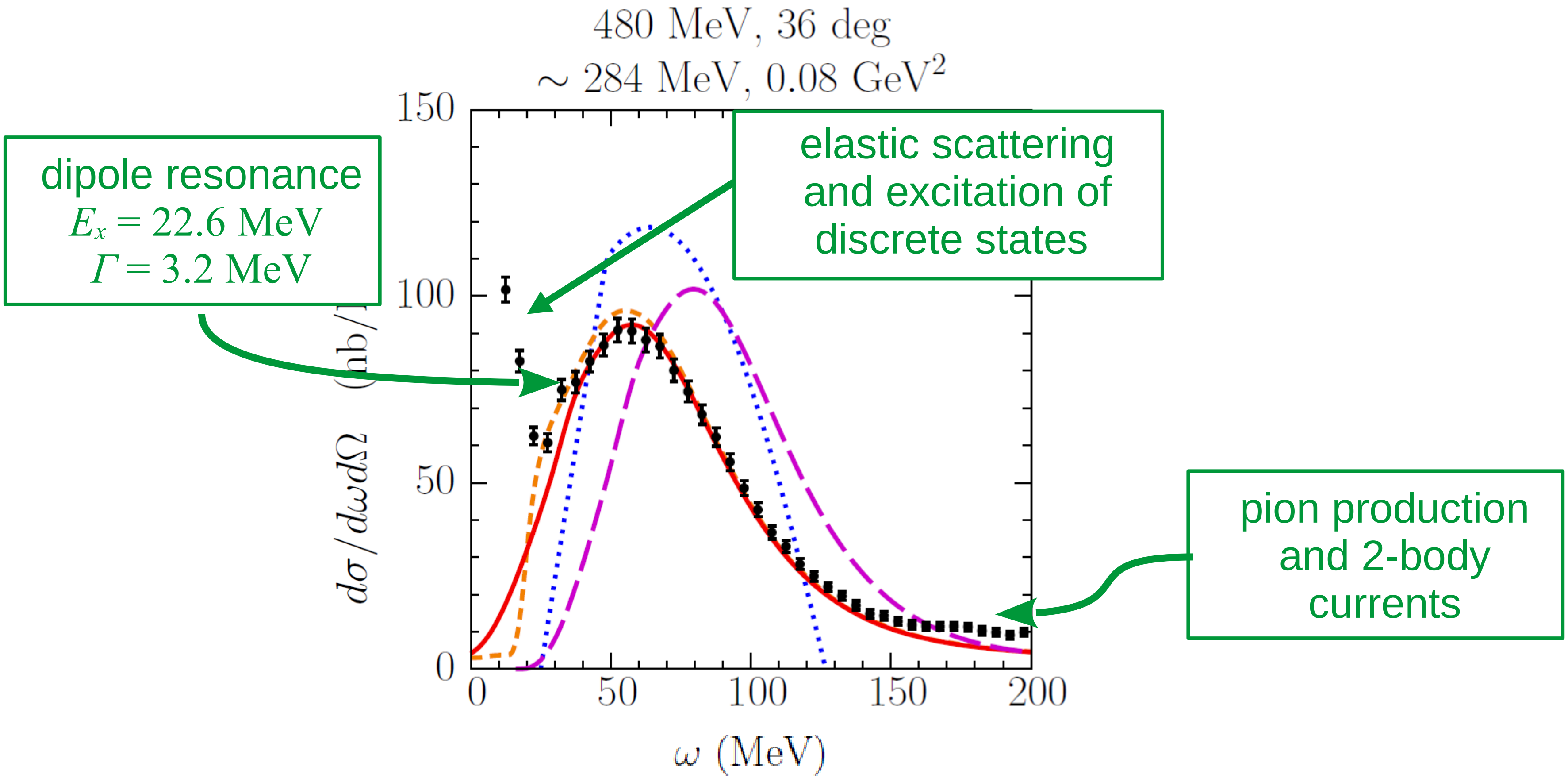


RFG

SF w/o FSI

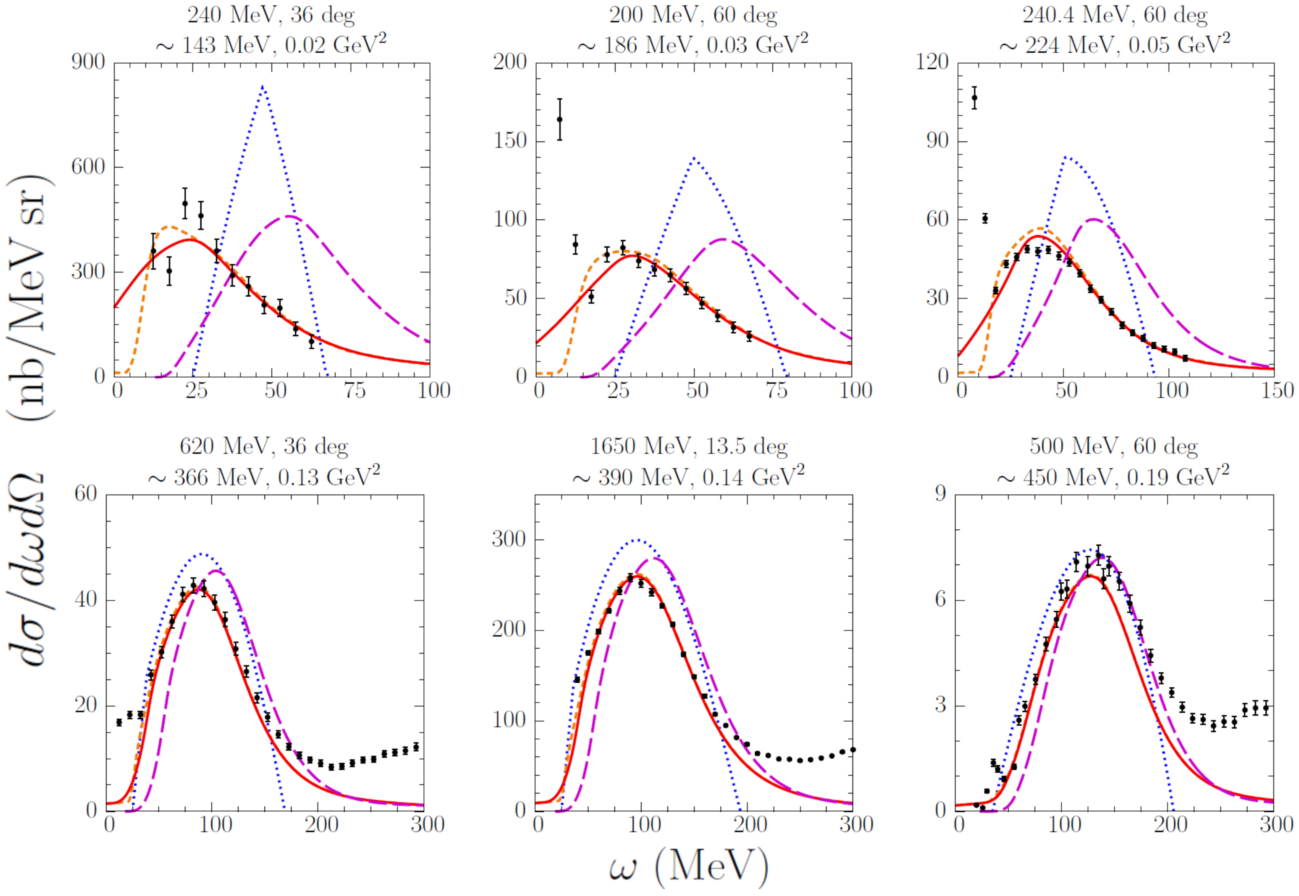
A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

What is not included?



A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

Realistic description of the nucleus: C(*e*,*e'*)



A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

Monte Carlo simulations

Coming soon to a generator near you

- ACHILLES

  J. Isaacson, W. I. Jay, A. Lovato, P.A.N. Machado, and N. Rocco, PRD 107, 033007 (2022)

- GENIE

  M. Betancourt, S. Gardiner, N. Rocco, and N. Steinberg, PRD 108, 113009 (2023)

- NEUT
- NuWro:

  A.M.A. and J.T. Sobczyk, PRC 77, 044311 (2008)

  R. Dharmapal Banerjee, A.M. Ankowski, K. M. Graczyk, B.E. Kowal, H. Prasad, J.T. Sobczyk, PRD 109, 073004 (2024)

NuWro

- LDA spectral functions for carbon, oxygen, and iron

  Benhar *et al.*, NPA 579 493, (1994); Benhar *et al.*, PRD 72, 053005 (2005)

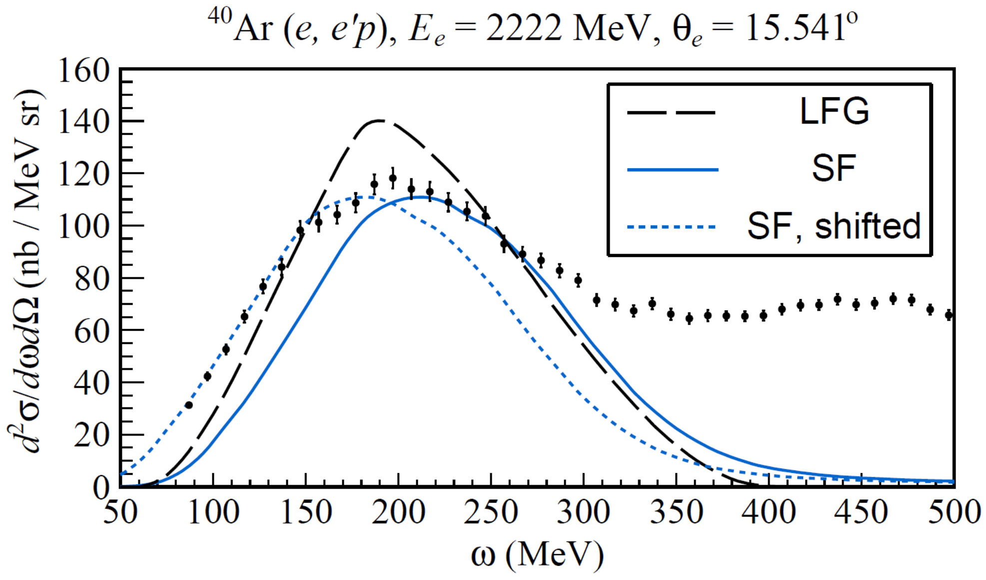
- FSI for carbon

  A.M.A., O. Benhar & M. Sakuda, PRD 91, 033005 (2015)

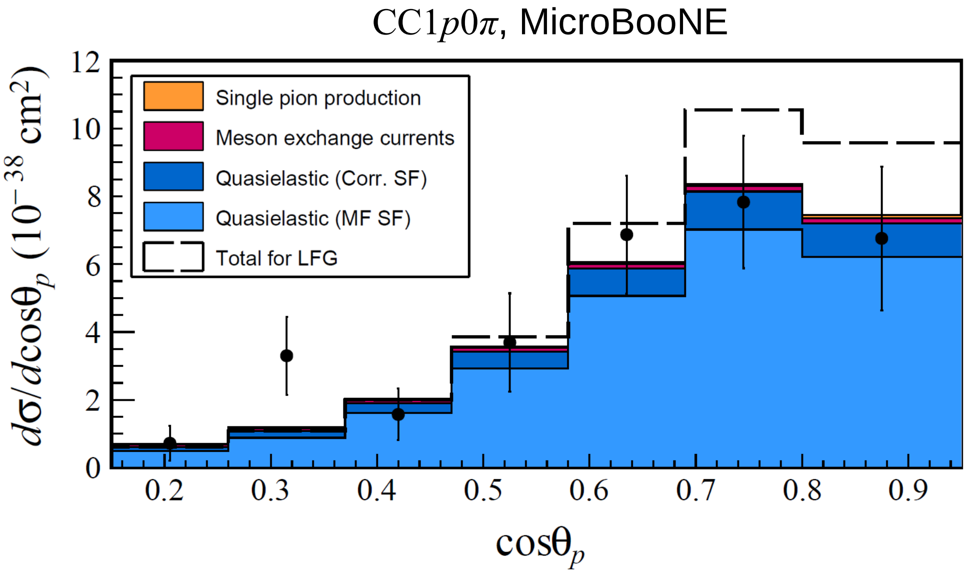
- JLab spectral functions for protons and neutrons in argon
- Low energy developments (Coulomb effects & nuclear recoil)
- New axial form factors (MINERvA, Meyer *et al.*)

  R. Dharmapal Banerjee *et al.*, PRD 109, 073004 (2024)

JLab spectral functions in NuWro



data:Dai *et al.*, PRC 99, 054608 (2019)



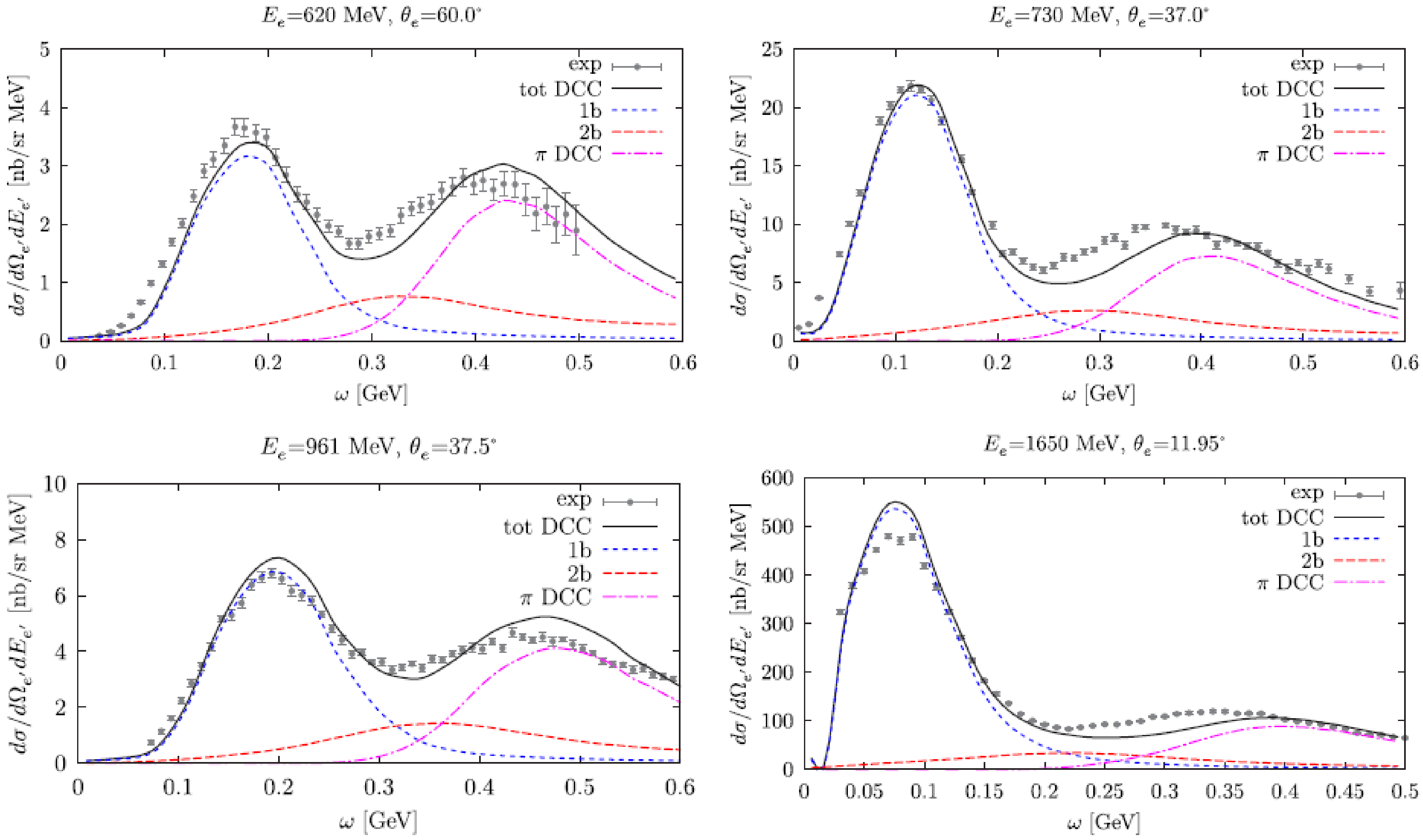
data: Abratenko *et al.*, PRL 125, 201803 (2020)

Combined χ^2/d.o.f. for the MicroBooNE CC$1p0\pi$ data for the restricted phase space ($\cos\theta_\mu < 0.8$) is **1.0 for the local Fermi gas** and **0.7 for the spectral function approach**.

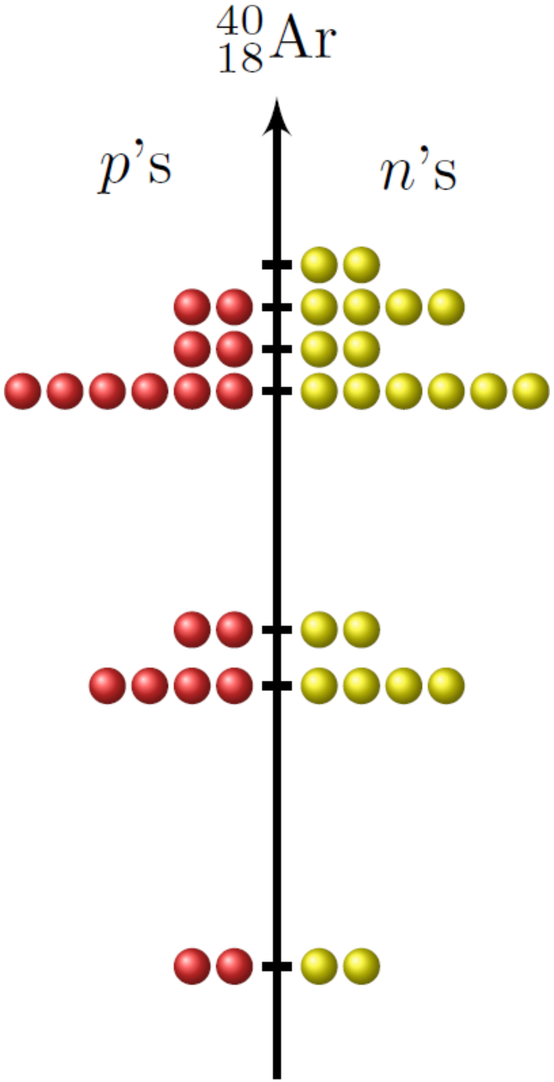
Summary

- The success of the long-baseline neutrino program requires reliable cross sections.
- The spectral function approach is a viable option to account for the shell structure and correlations between nucleons in a realistic manner.
- Inclusive and exclusive cross sections can be calculated for relativistic final states.
- Readily applicable to deuteron, helium, carbon, oxygen, argon, and iron.
- Implemented in ACHILLES, GENIE, NEUT, and NuWro.
- Give it a try!

Thank you!



Rocco *et al.*, PRC 100, 045503 (2019)

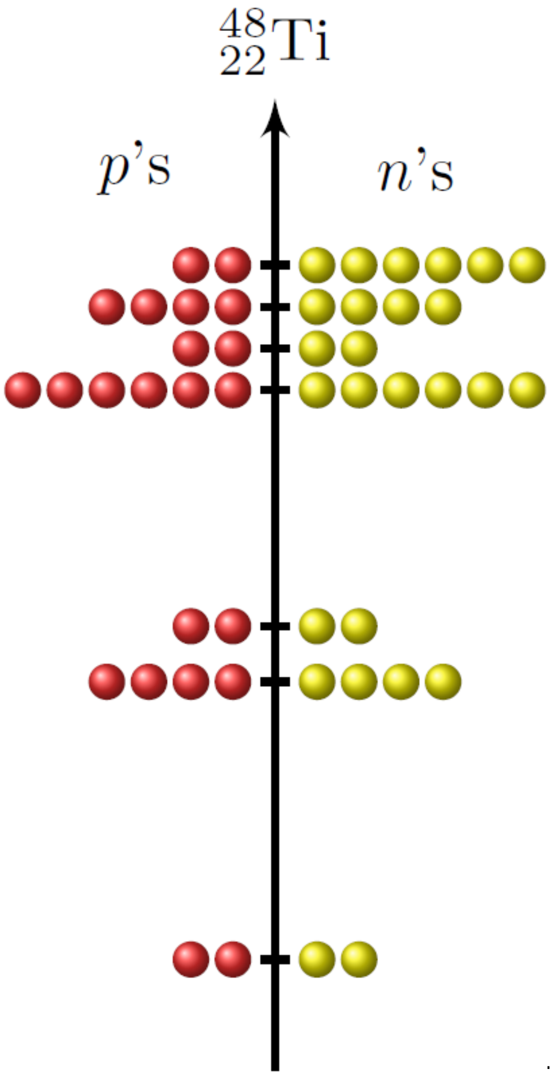


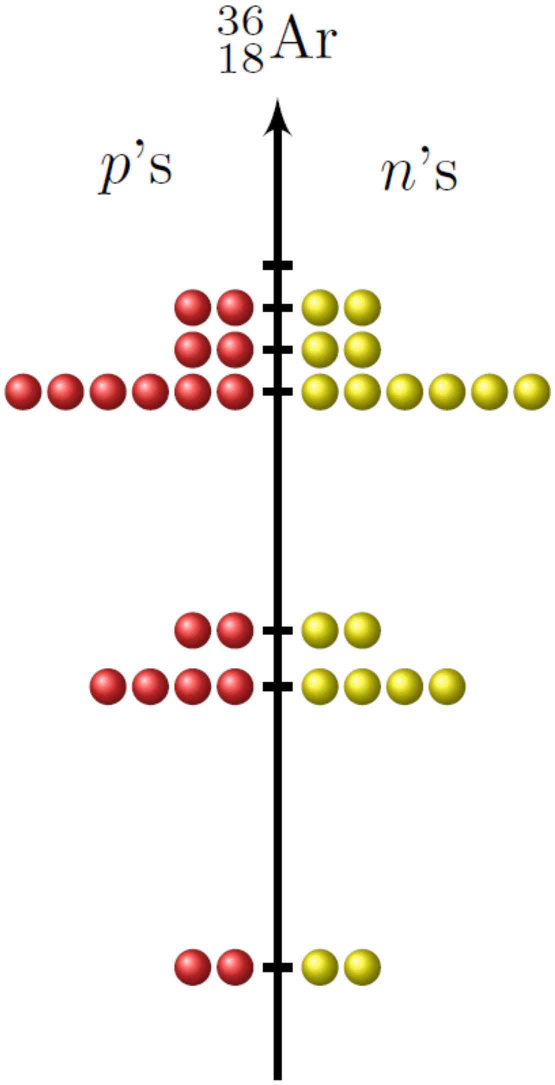
proton energy levels

Ar		Ti
	1f7/2	11.32(10)
12.53(2)	1d3/2	12.30(24)
12.92(2)	2s1/2	12.77(25)
18.23(2)	1d5/2	15.86(20)
28.8(7)	1p1/2	33.3(6)
33.0(3)	1p3/2	39.7(6)
33.0(3)	1p3/2	39.7(6)
53.4(1.1)	1s1/2	53.8(1.9)

Jiang *et al.*,
PRD 105, 112002 (2022)

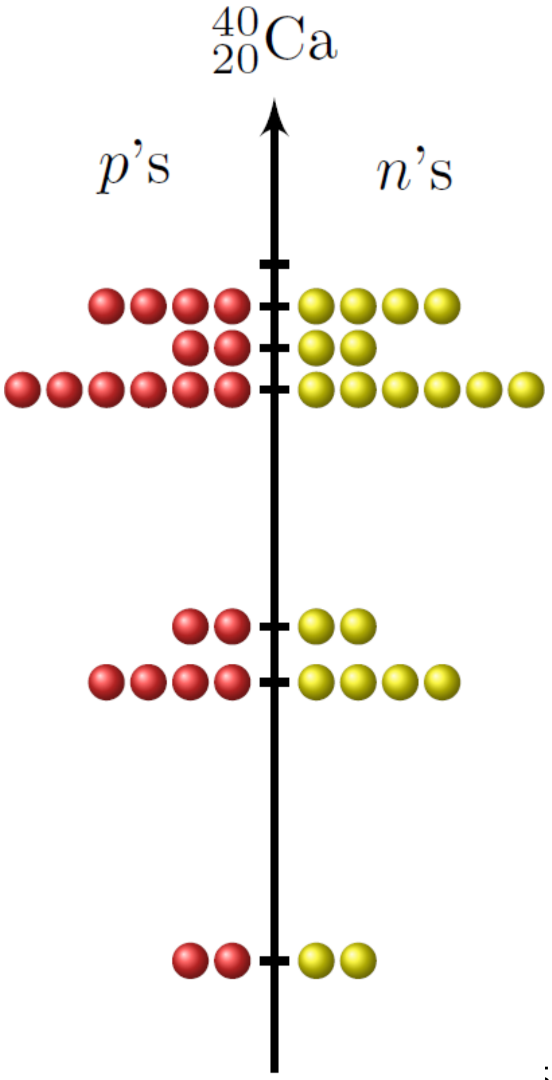
Jiang *et al.*
PRD 107, 012005 (2023)

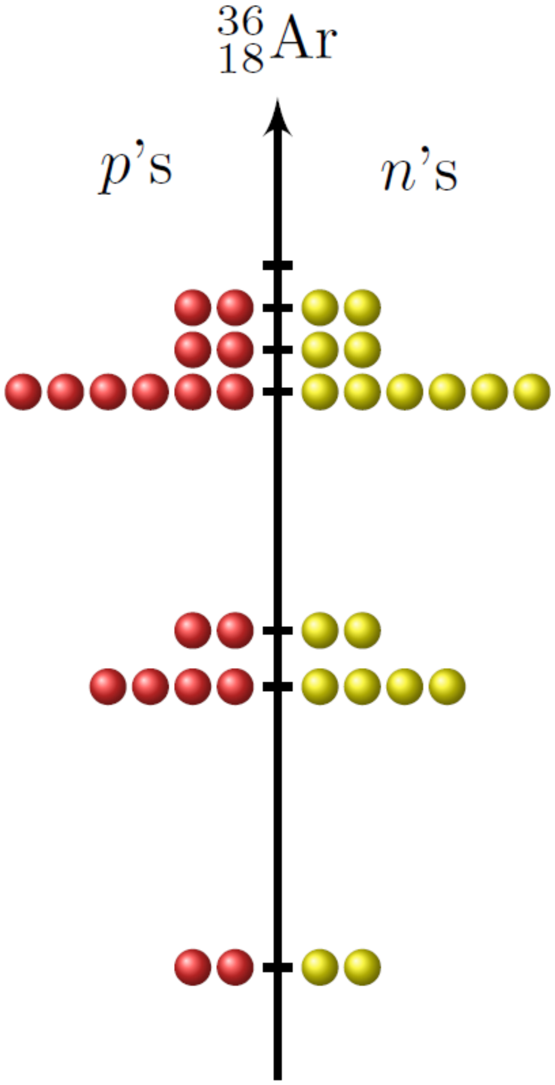




proton energy levels

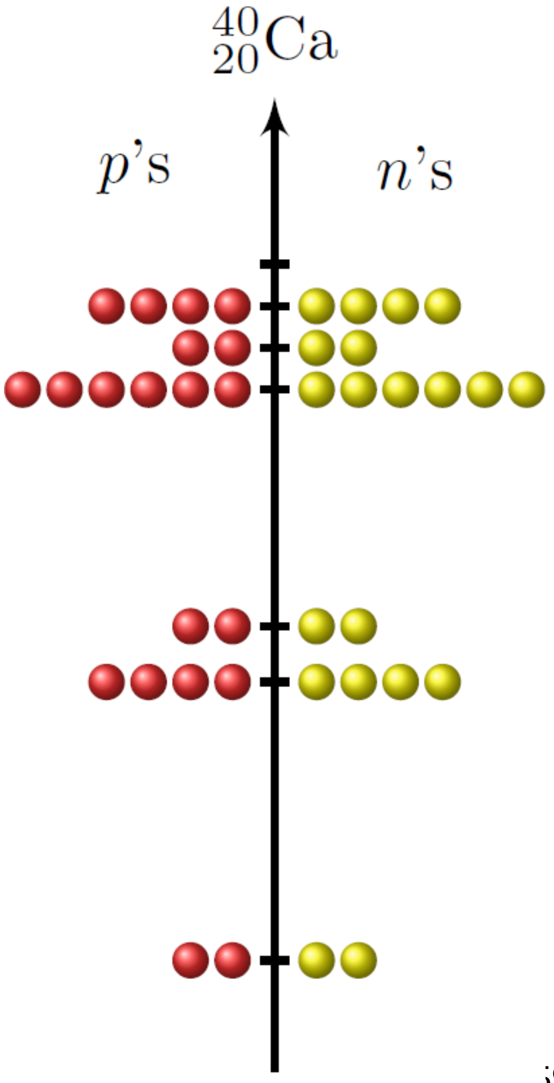
40Ar		40Ca
12.53	1d3/2	8.33
12.92	2s1/2	10.85
18.23	1d5/2	14.66
	1p1/2	
	1p3/2	
	1p3/2	
	1s1/2	



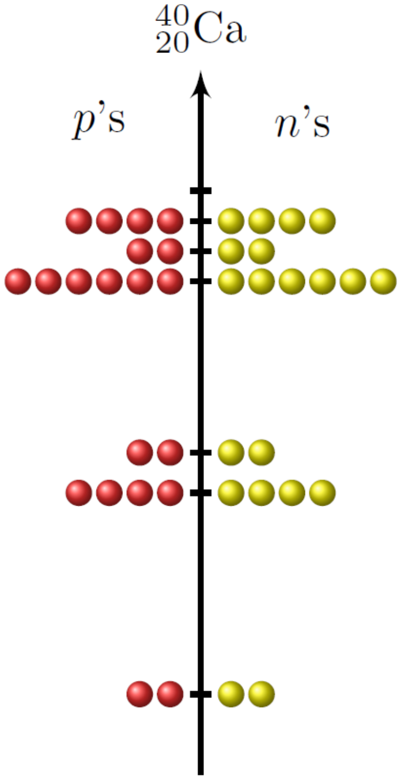


proton energy levels

36Ar		40Ca
8.51	1d3/2	8.33
9.73	2s1/2	10.85
14.23	1d5/2	14.66
	1p1/2	
	1p3/2	
	1p3/2	
	1s1/2	

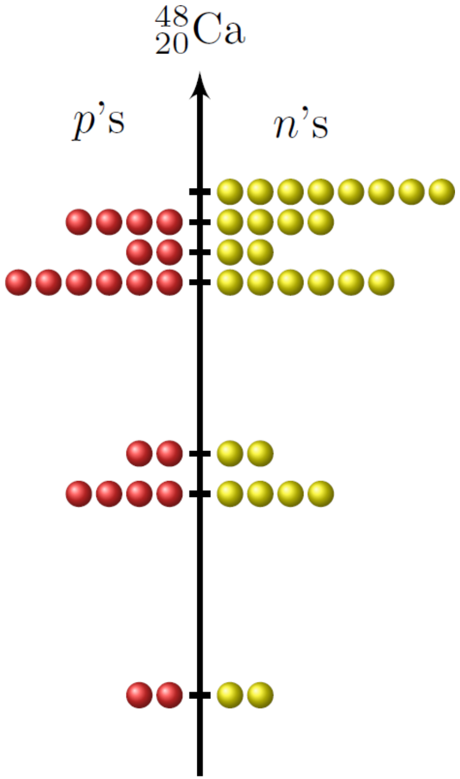


Calcium isotopes



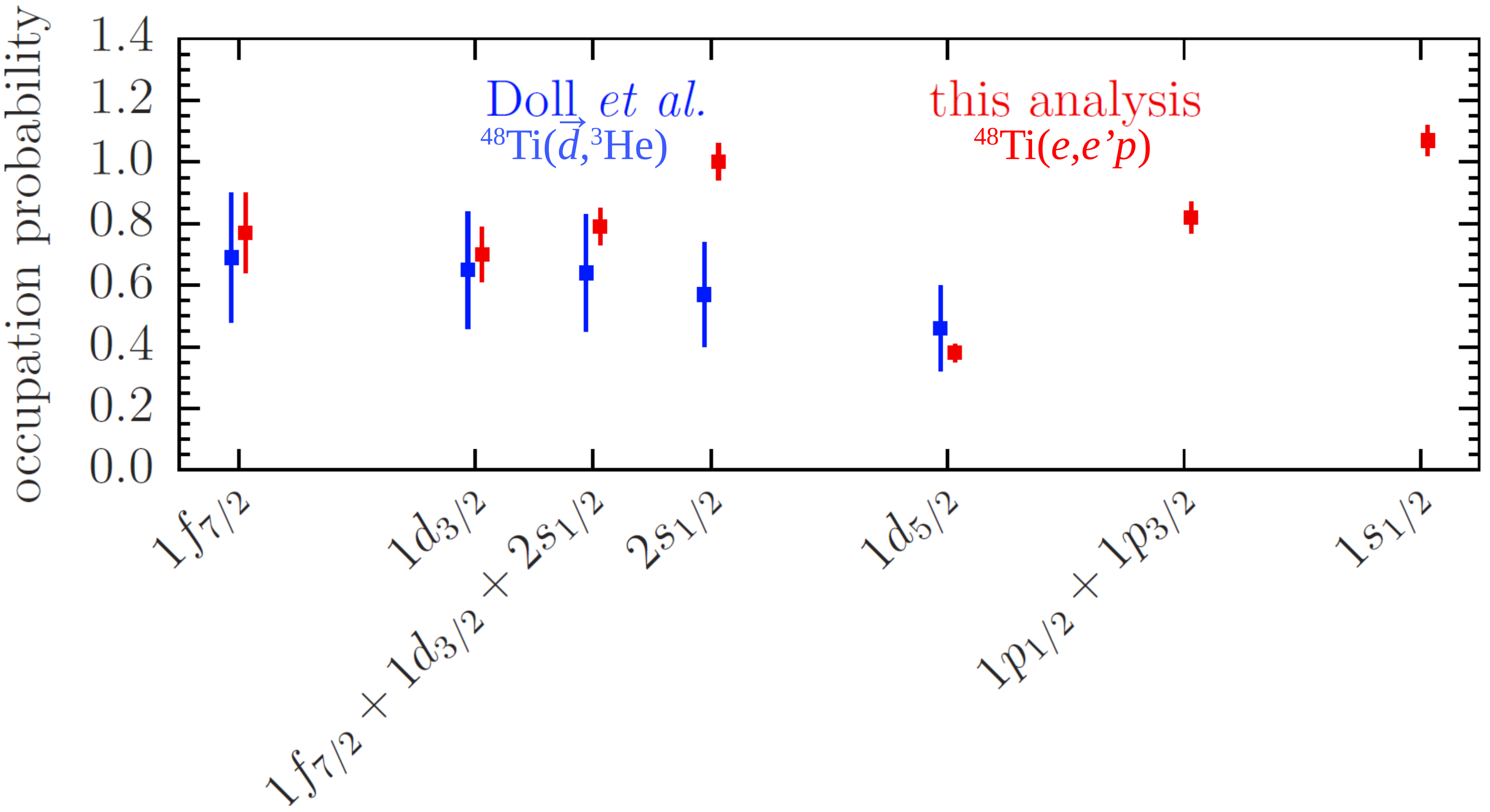
^{40}Ca		^{48}Ca
8.3(3)	1d3/2	16.8(3)
11.1(3)	2s1/2	17.1(3)
16.8(4)	1d5/2	23.9(7)

Kramer, Ph.D. thesis (1990)



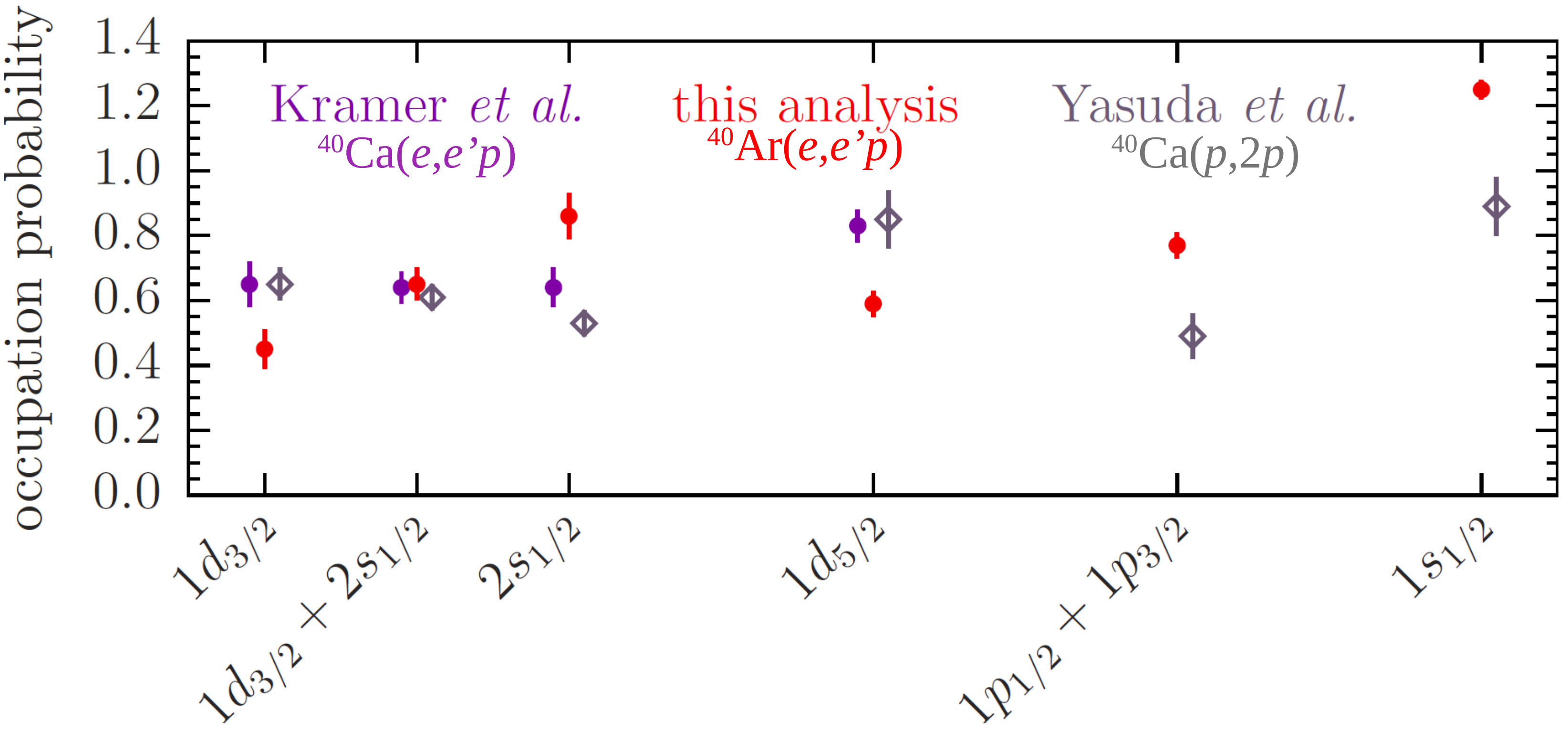
6–8.5 MeV differences

Occupation probability



52-MeV polarized [Doll *et al.*, JPG **5**, 1421 (1979); E_x < 7.54 MeV] deuteron beam at Karlsruhe

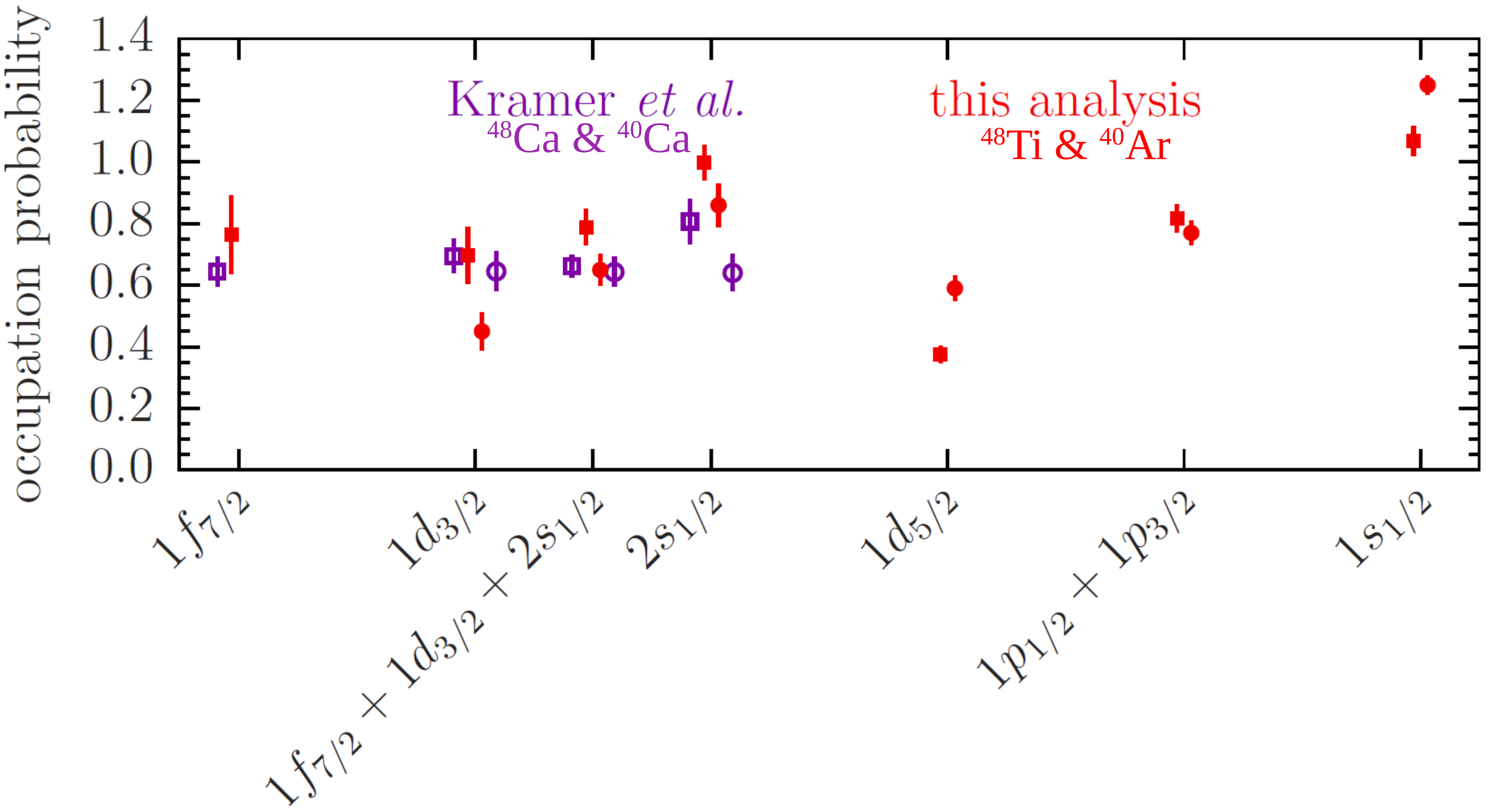
Occupation probability



Kramer *et al.* [Ph.D. thesis (1990)]: ~340–440-MeV electron beam at NIKHEF-K

Yasuda *et al.* [Ph.D. thesis (2012)]: 392-MeV polarized proton beam at RCNP

Occupation probability



Kramer *et al.* [Ph.D. thesis (1990)]: ~340–440-MeV electron beam at NIKHEF-K