CP Violation: Invariant and Opportunistic

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@Fermilab, 9/5/23

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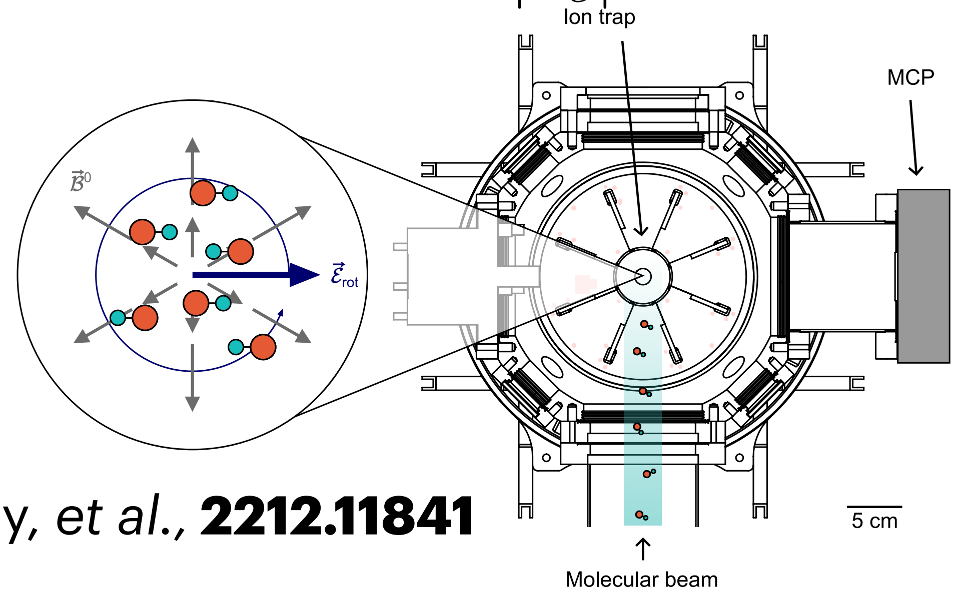
Christophe Grojean, JTR, **2112.03889**, **2302.07288**

Experiment versus CP

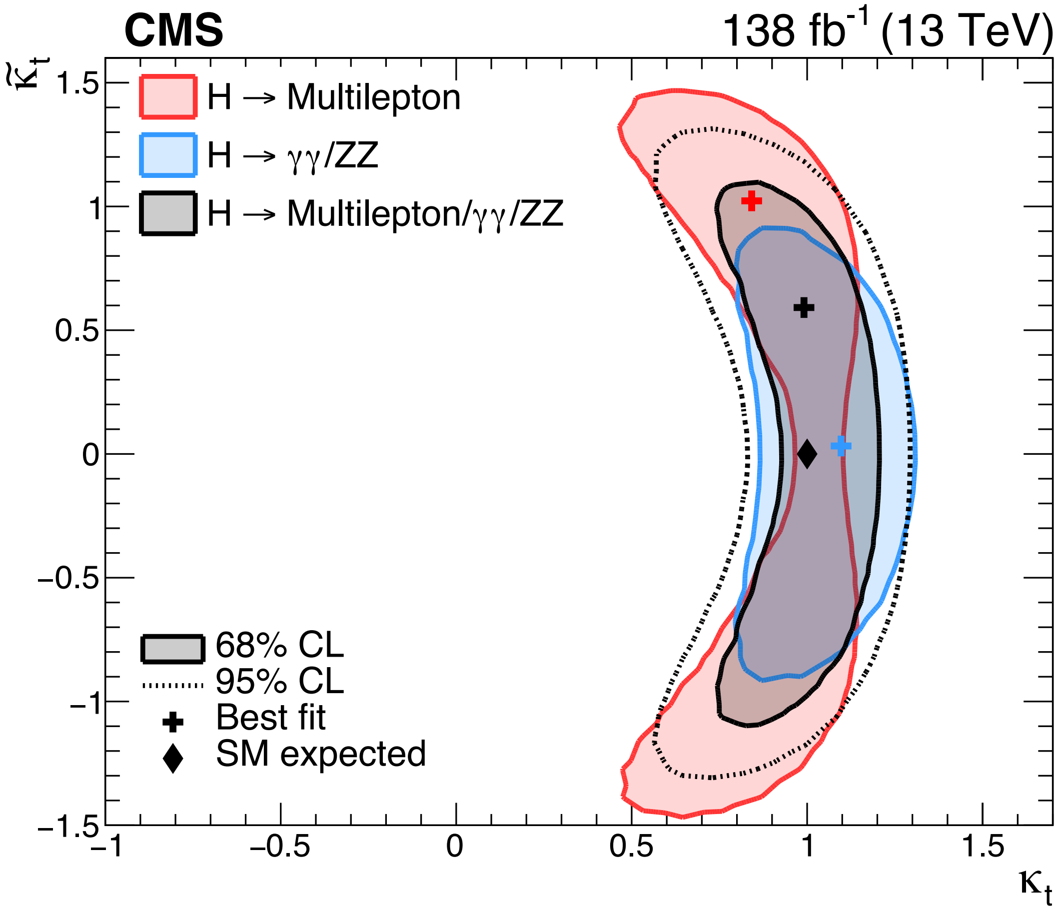
JILA molecular ions: $|d_e| < 4.1 \times 10^{-30}\ e\,\mathrm{cm}$



Roussy, *et al.*, **2212.11841**

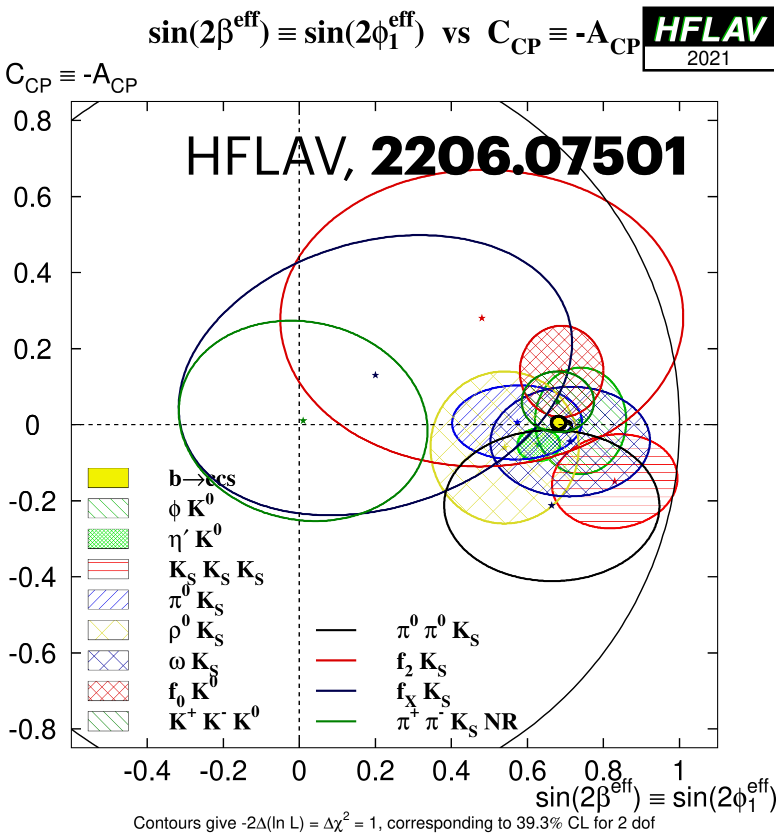
LHC modified top Yukawa:

$$\mathcal{L}_{\mathrm{t\bar{t}H}} = -\frac{m_{\mathrm{t}}}{v}\bar{\psi}_{\mathrm{t}}(\kappa_{\mathrm{t}} + i\gamma_5\tilde{\kappa}_{\mathrm{t}})\psi_{\mathrm{t}}\mathrm{H}$$



CMS, **2208.02686**

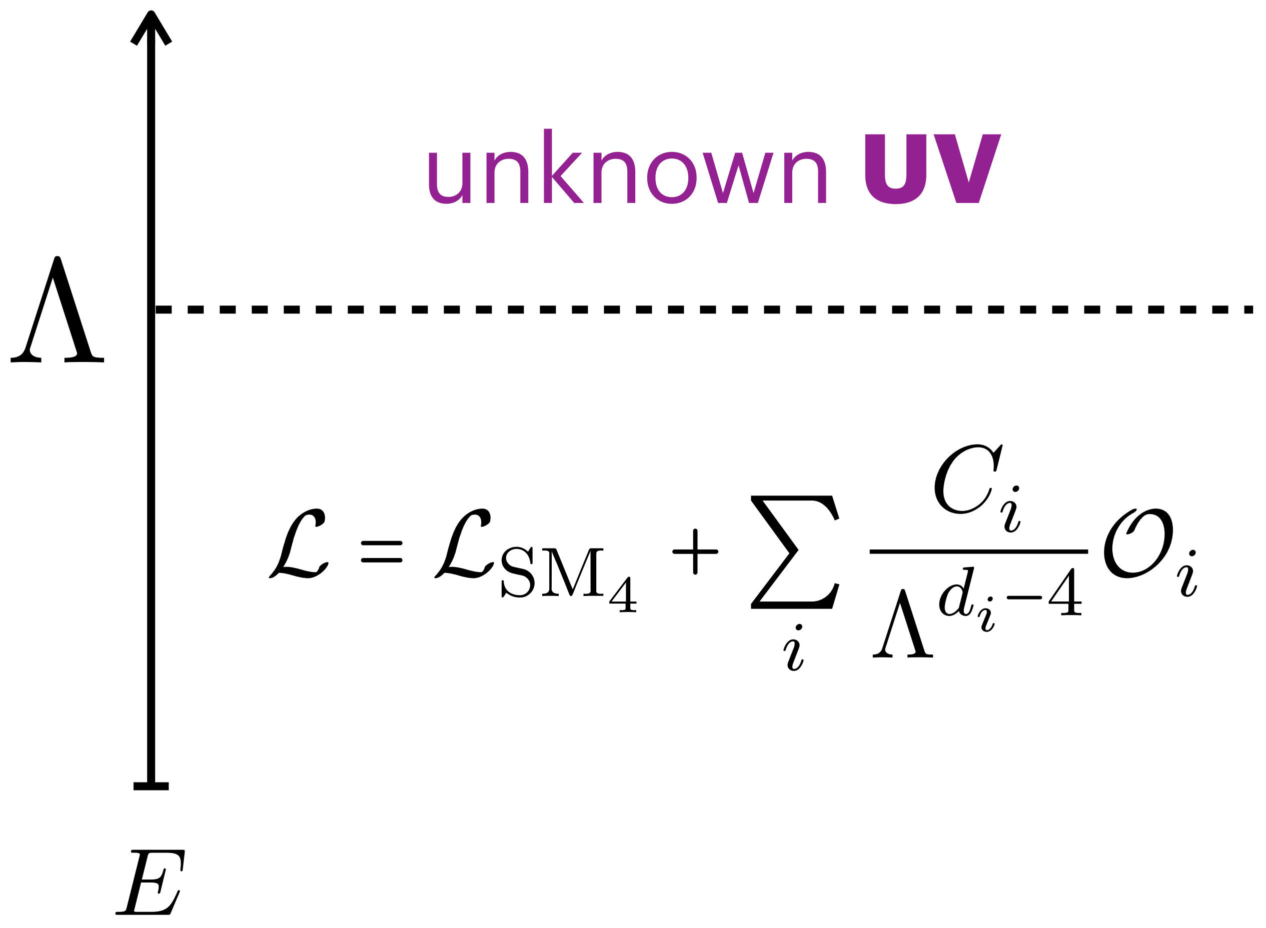
B decays:



HFLAV, **2206.07501**

The Standard Model

(also called “SMEFT”)



$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}_4} + \sum_i \frac{C_i}{\Lambda^{d_i-4}} \mathcal{O}_i$$

- many papers state/assume:

$\mathrm{Re}(C_i)$ = CP conserving

$\mathrm{Im}(C_i)$ = CP violating

- but physics is invariant under rephasing: $\psi \to e^{i\theta}\psi$

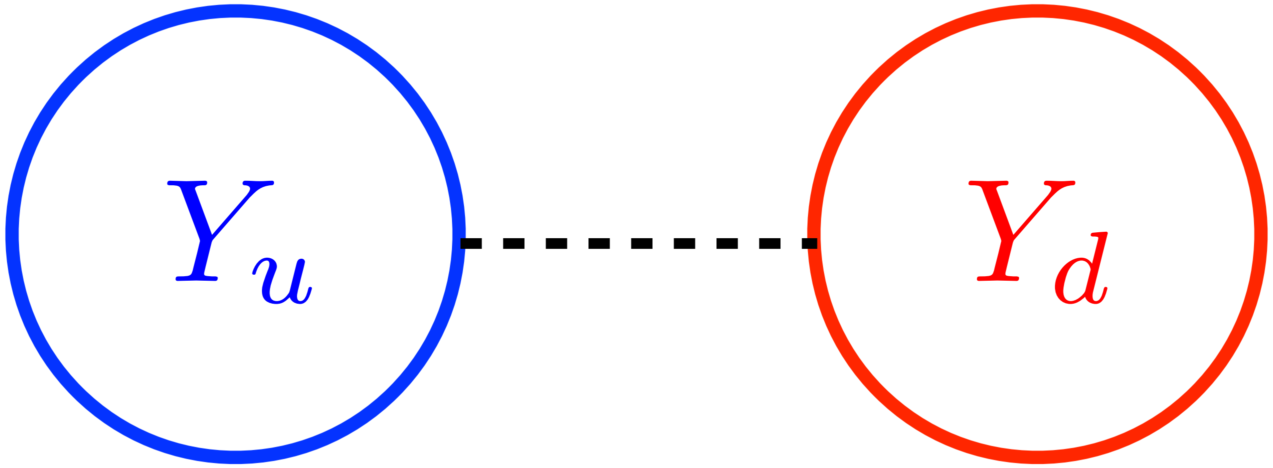
Plan

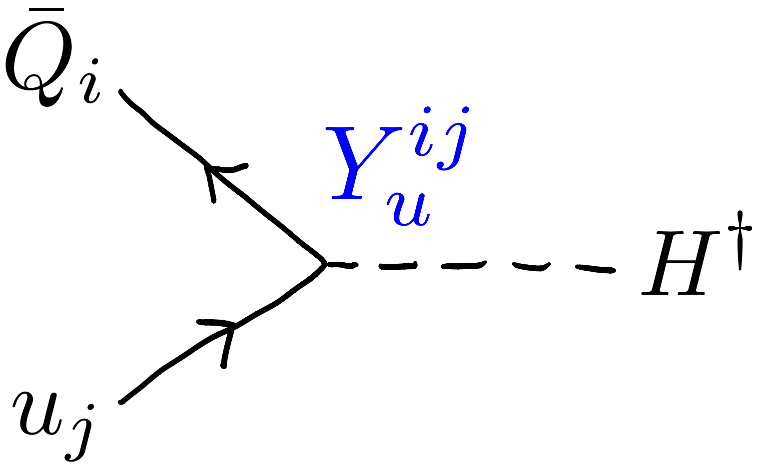
I. CPV is Collective

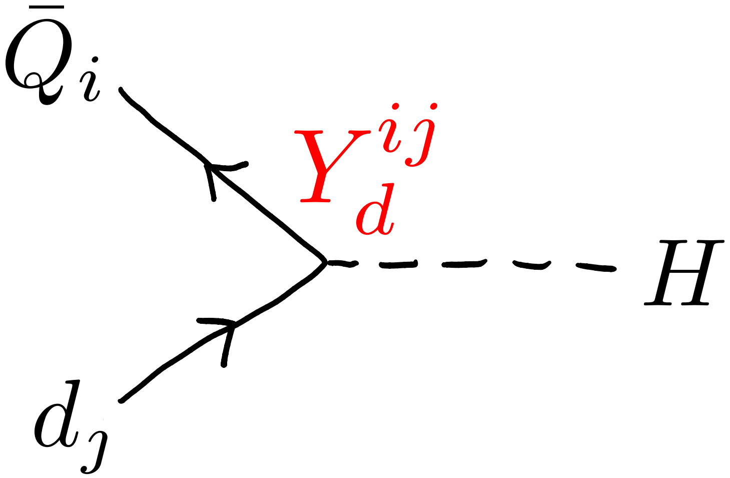
II. Invariant Measures of CPV at dimension 6

III. Opportunistic CPV

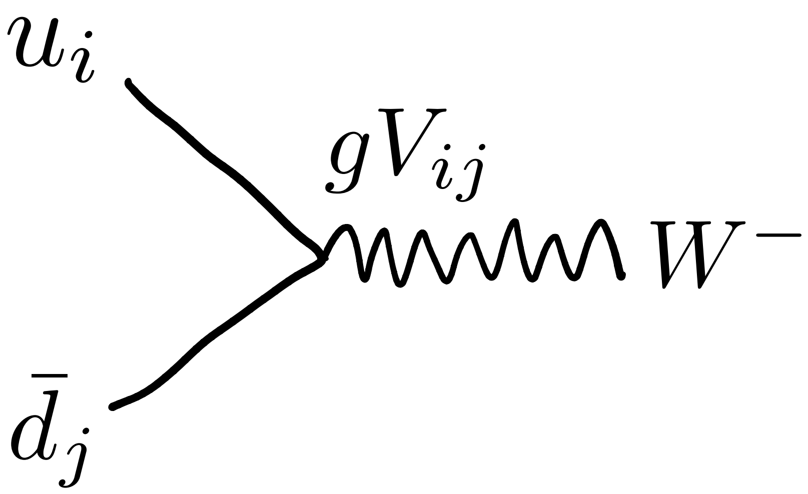
I. CPV is Collective







CP Violation in SM$_4$



$$\delta \approx 1.1$$

CKM matrix: $$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Wolfenstein parameterization:

$$V_{\rm CKM} = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\lambda \approx 0.23 \qquad \begin{aligned} \eta &\approx 0.36 \\ \rho &\approx 0.16 \\ A &\approx 0.82 \end{aligned}$$

can assign Wolfenstein-scaling to Yukawas:

$$y_{t,c,u} \sim \lambda^{0,4,8} \qquad y_{b,s,d} \sim \lambda^{3,5,7} \qquad y_{\tau,\mu,e} \sim \lambda^{3,5,9}$$

Do Complex Parameters Imply CP Violation?

Does the following CKM matrix imply CP violation?

$$V_{\text{CKM}} = \begin{pmatrix} \frac{72-21i}{325} & \frac{4}{13} & -\frac{12i}{13} \\ -\frac{12}{13} & \frac{576+168i}{1625} & \frac{49-168i}{1625} \\ -\frac{96-28i}{325} & -\frac{57}{65} & -\frac{24i}{65} \end{pmatrix}$$

All phases can be removed by field redefinitions:

$$\begin{pmatrix} \frac{72-21i}{325} & \frac{4}{13} & -\frac{12i}{13} \\ -\frac{12}{13} & \frac{576+168i}{1625} & \frac{49-168i}{1625} \\ -\frac{96-28i}{325} & -\frac{57}{65} & -\frac{24i}{65} \end{pmatrix} = \begin{pmatrix} \frac{3-4i}{5} & 0 & 0 \\ 0 & \frac{4-3i}{5} & 0 \\ 0 & 0 & \frac{3-4i}{5} \end{pmatrix} \begin{pmatrix} \frac{3}{13} & \frac{4}{13} & \frac{12}{13} \\ -\frac{12}{13} & \frac{24}{65} & \frac{7}{65} \\ -\frac{4}{13} & -\frac{57}{65} & \frac{24}{65} \end{pmatrix} \begin{pmatrix} \frac{4+3i}{5} & 0 & 0 \\ 0 & \frac{3+4i}{5} & 0 \\ 0 & 0 & \frac{4-3i}{5} \end{pmatrix}$$

ex) suppose in “up basis”:

$$Y_u = Y_u^{\text{diag}}$$

$$Y_d = V_{\text{CKM}} Y_d^{\text{diag}}$$

remove by redefining:

$$Q_i \to e^{i\delta_i} Q_i$$

remove by redefining:

$$d_j \to e^{-i\delta_j} d_j$$

complex parameters do not always imply CP violation!

Why do we need CP Violating Invariants?

- physical quantities are invariant under rotating fermions by phases:

$$\psi \to e^{i\theta}\psi$$

- more generally, physical quantities are invariant under unitary redefinitions of fermions in flavor space:

ex) $$Q_i \to U_Q^{ij}\, Q_j$$

U_Q is any 3x3 unitary matrix

- it should be possible to write all CP violating observables in terms of objects that are invariant under unitary flavor transformations

Jarlskog Invariant

invariant measure of CP violation:

$$J_4 = 3\,\mathrm{Im}\,\mathrm{Det}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right] = \mathrm{Im}\,\mathrm{Tr}\left[Y_u Y_u^\dagger, Y_d Y_d^\dagger\right]^3$$

evaluating using typical CKM parameterization:

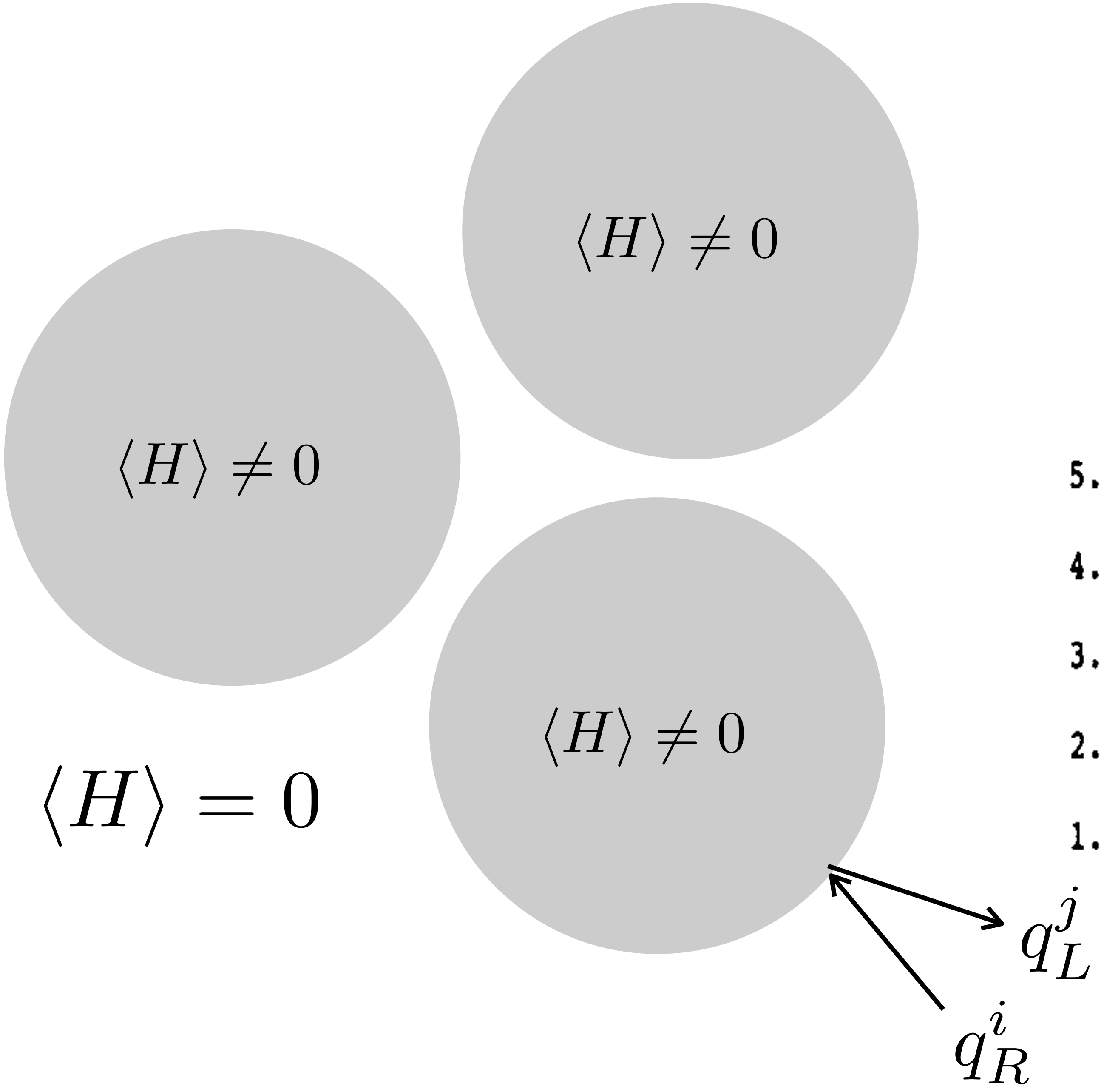
$$J_4 = 6(y_t^2 - y_c^2)(y_t^2 - y_u^2)(y_c^2 - y_u^2)(y_b^2 - y_s^2)(y_b^2 - y_d^2))(y_s^2 - y_d^2) \\ \times \cos\theta_{12}\cos\theta_{23}\cos^2\theta_{13}\sin\theta_{12}\sin\theta_{23}\sin\theta_{13}\sin\delta$$

expanding using Wolfenstein: $J_4 \approx 6.4\eta\,\lambda^{36} \approx 8\times 10^{-24}$

CP is conserved (at dimension 4) if and only if: $J_4 = 0$

Jarlskog, Phys. Rev. Lett. **55** (1985) 1039
Bernabeu, Branco, Gronau, Phys. Lett. B **169** (1986) 243

Application: Electroweak Baryogenesis



$\langle H \rangle \neq 0$

$\langle H \rangle \neq 0$

$\langle H \rangle \neq 0$

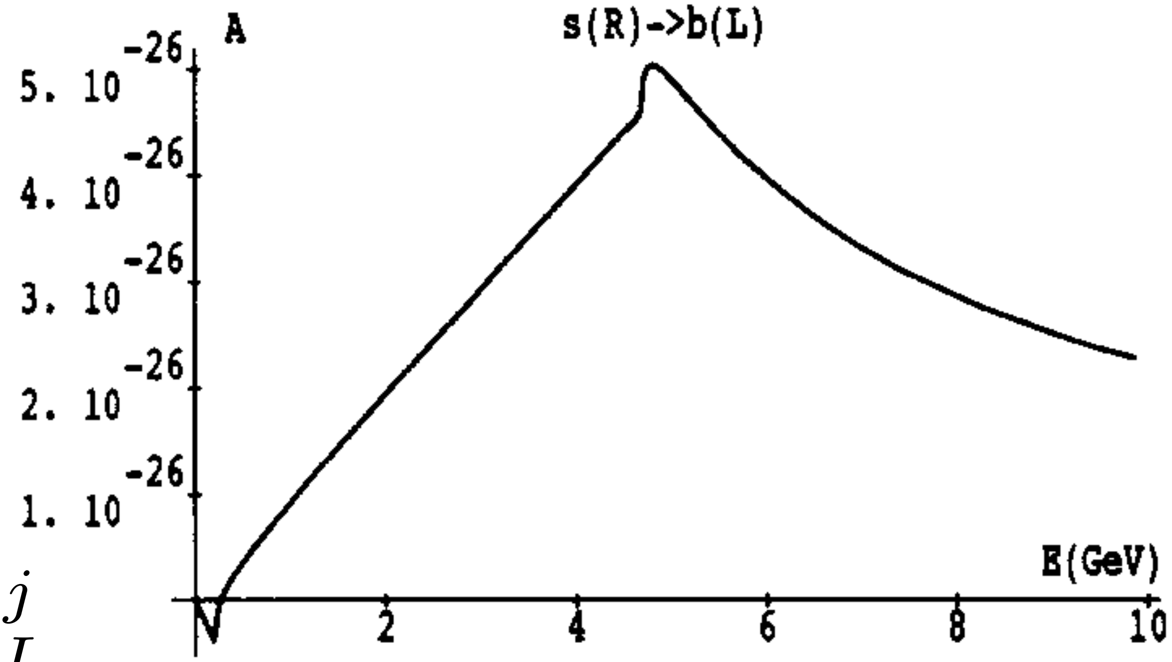
$\langle H \rangle = 0$

q_L^j

q_R^i

in SM$_4$, asymmetry suppressed by:

$J_4 \approx 8 \times 10^{-24}$

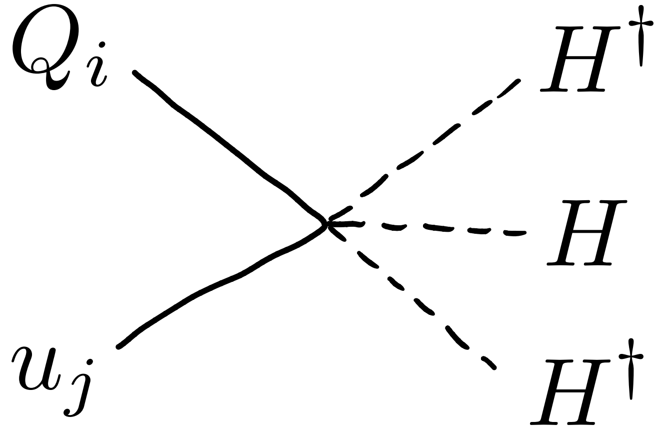


Gavela, Lozano, Orloff, Pene, **hep-ph/9406288**

An Example Dimension 6 Operator

complex Yukawas can be generated by:

$$\mathcal{L} \supset \frac{C_{ij}}{\Lambda^2} |H|^2 \bar{Q}_i H^\dagger u_j$$



C has, *a priori*, 18 real parameters, 9 are phases:

$$C = \begin{pmatrix} |c_{11}|e^{i\delta_{11}} & |c_{12}|e^{i\delta_{12}} & |c_{13}|e^{i\delta_{13}} \\ |c_{21}|e^{i\delta_{21}} & |c_{22}|e^{i\delta_{22}} & |c_{23}|e^{i\delta_{23}} \\ |c_{31}|e^{i\delta_{31}} & |c_{32}|e^{i\delta_{32}} & |c_{33}|e^{i\delta_{33}} \end{pmatrix}$$

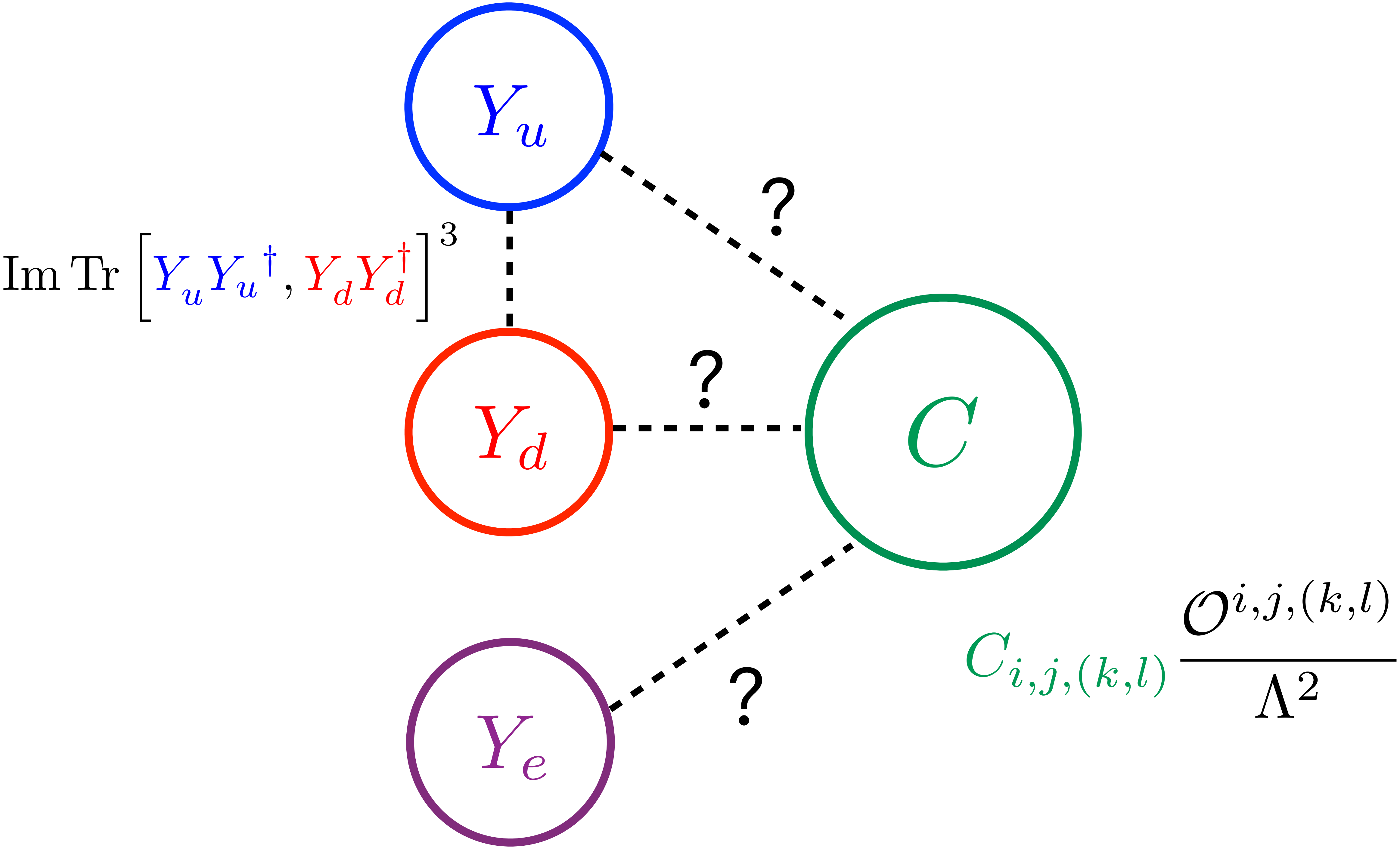
but can set C real and diagonal using unitary transformations:

$$Q_i \to U_Q^{ij}\, Q_j$$

$$u_i \to U_u^{ij}\, u_j$$

does this operator violate CP?

II. Invariant Measures of CPV at dimension 6



$$\mathrm{Im}\,\mathrm{Tr}\left[Y_u Y_u{}^\dagger, Y_d Y_d^\dagger\right]^3$$

$$C_{i,j,(k,l)} \frac{\mathcal{O}^{i,j,(k,l)}}{\Lambda^2}$$

Quentin Bonnefoy, Emmanuel Gendy, Christophe Grojean, JTR, **2112.03889**

CP Violation at Dimension 6

consider an amplitude: $$\mathcal{A} = \underbrace{\mathcal{A}^{(4)}}_{\frac{1}{\Lambda^0}} + \underbrace{\mathcal{A}^{(6)}}_{\frac{1}{\Lambda^2}} + \ldots$$

observable: $$|\mathcal{A}|^2 = \underbrace{|\mathcal{A}^{(4)}|^2}_{\frac{1}{\Lambda^0}} + \underbrace{2\,\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)\,*}\right)}_{\frac{1}{\Lambda^2}} + \ldots$$

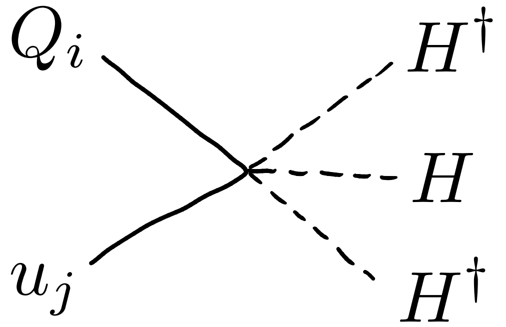
CP conserved at order $\frac{1}{\Lambda^0}$ if and only if: $J_4 = 0$

invariant measures of CP violation at order $\frac{1}{\Lambda^2}$?

Sample Operator: Bilinear

- example operator: $\mathcal{L} \supset \frac{C_{ij}}{\Lambda^2} |H|^2 \bar{Q}_i H^\dagger u_j$

(9 phases)



- invariant measures of CP violation:

$$\mathrm{Im}\,\mathrm{Tr}\left(Y_u^\dagger C\right) \qquad \mathrm{Im}\,\mathrm{Tr}\left(Y_u^\dagger Y_u Y_u^\dagger C\right) \qquad \mathrm{Im}\,\mathrm{Tr}\left(Y_u^\dagger (Y_u Y_u^\dagger)^2 C\right)$$

(invariants with higher powers of Y_u are redundant)

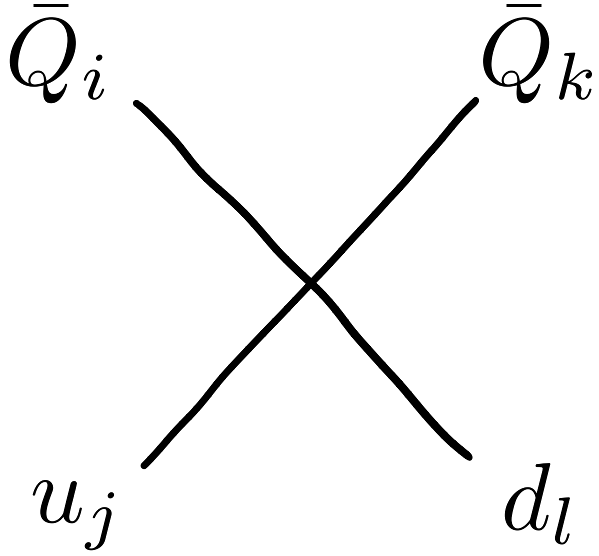
- additional invariants by including both Yukawas :

ex)
$$\mathrm{Im}\,\mathrm{Tr}\left(Y_u^\dagger Y_d Y_d^\dagger C\right)$$

Sample Operator: 4-Fermi

$$\mathcal{L} \supset \frac{C_{ijkl}}{\Lambda^2} \bar{Q}_i u_j \bar{Q}_k d_l$$

(81 phases)



example invariants:

$$\mathrm{Im}\, Y^\dagger_{u,ij} Y^\dagger_{d,ik} C_{ljlk}$$

$$\mathrm{Im}\, Y^\dagger_{u,ij} Y^\dagger_{d,kl} C_{kjil}$$

$$\mathrm{Im}\, \left(Y_u Y_u^\dagger\right)_{ij} Y^\dagger_{u,kl} Y^\dagger_{d,km} C_{jlim}$$

•••

How Many Phases at order $1/\Lambda^2$?

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\,\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)^*}\right) + \ldots$$

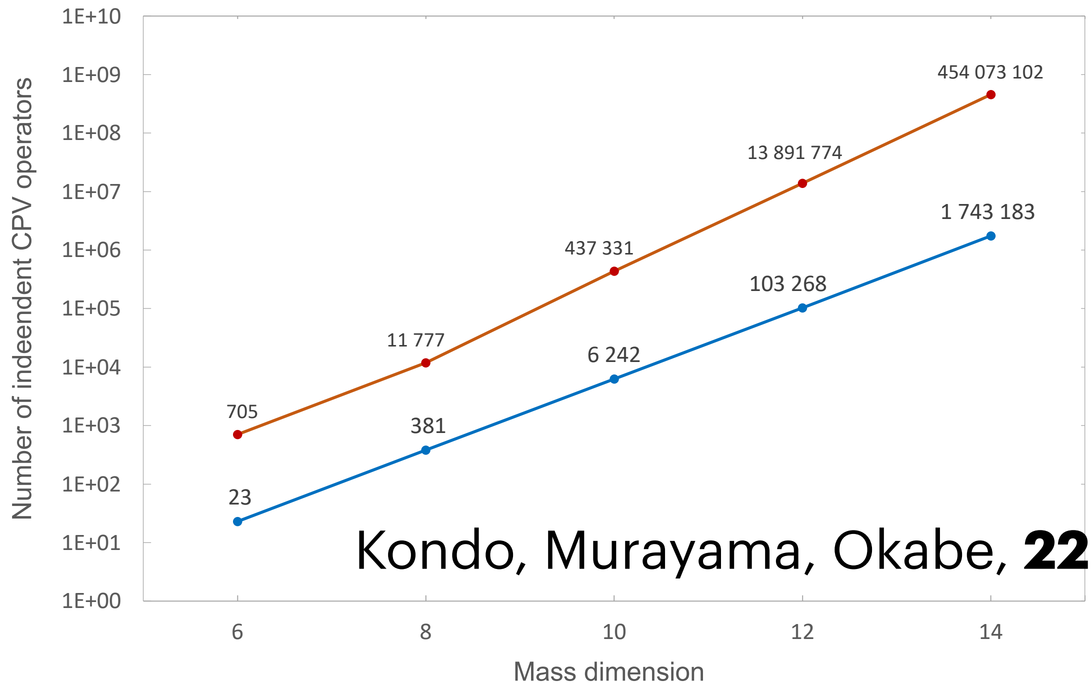
phases transforming under $U(1)_{L_i - L_j}$ do not contribute in limit: $m_\nu \to 0$

- there are 1143 phases appearing in fermionic dimension 6 operators

Alonso, Jenkins, Manohar, Trott, **1312.2014**

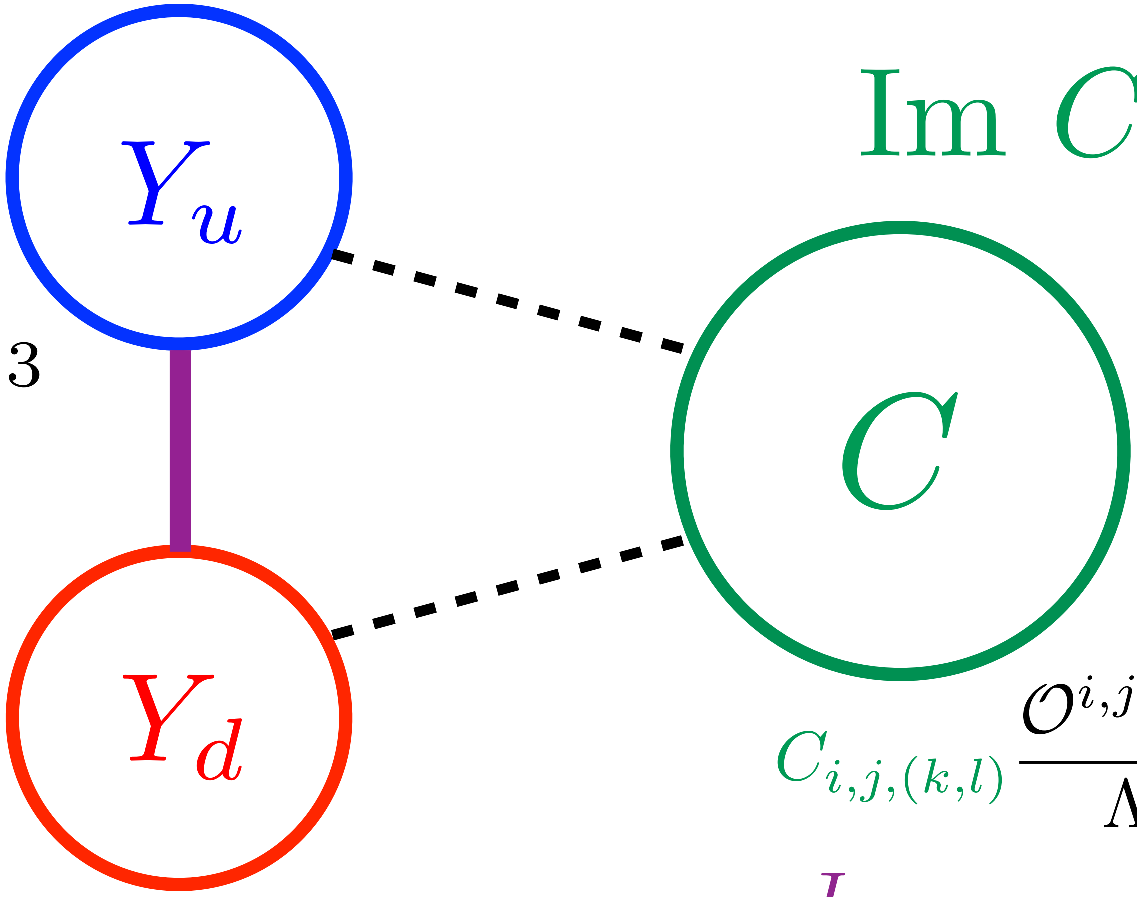
- 699 phases can appear in observables at order $1/\Lambda^2$ (444 phases do not!)

Quentin Bonnefoy, Emmanuel Gendy,
Christophe Grojean, JTR, **2112.03889**,



Kondo, Murayama, Okabe, **2212.02413**

III. Opportunistic CPV



$$\mathrm{Im}\, C = 0$$

$$J_4 = \mathrm{Im}\,\mathrm{Tr}\left[Y_u {Y_u}^\dagger, Y_d Y_d^\dagger\right]^3$$

$$C_{i,j,(k,l)} \frac{\mathcal{O}^{i,j,(k,l)}}{\Lambda^2}$$

$$\mathrm{CPV} \propto \frac{J_4}{\Lambda^2}$$

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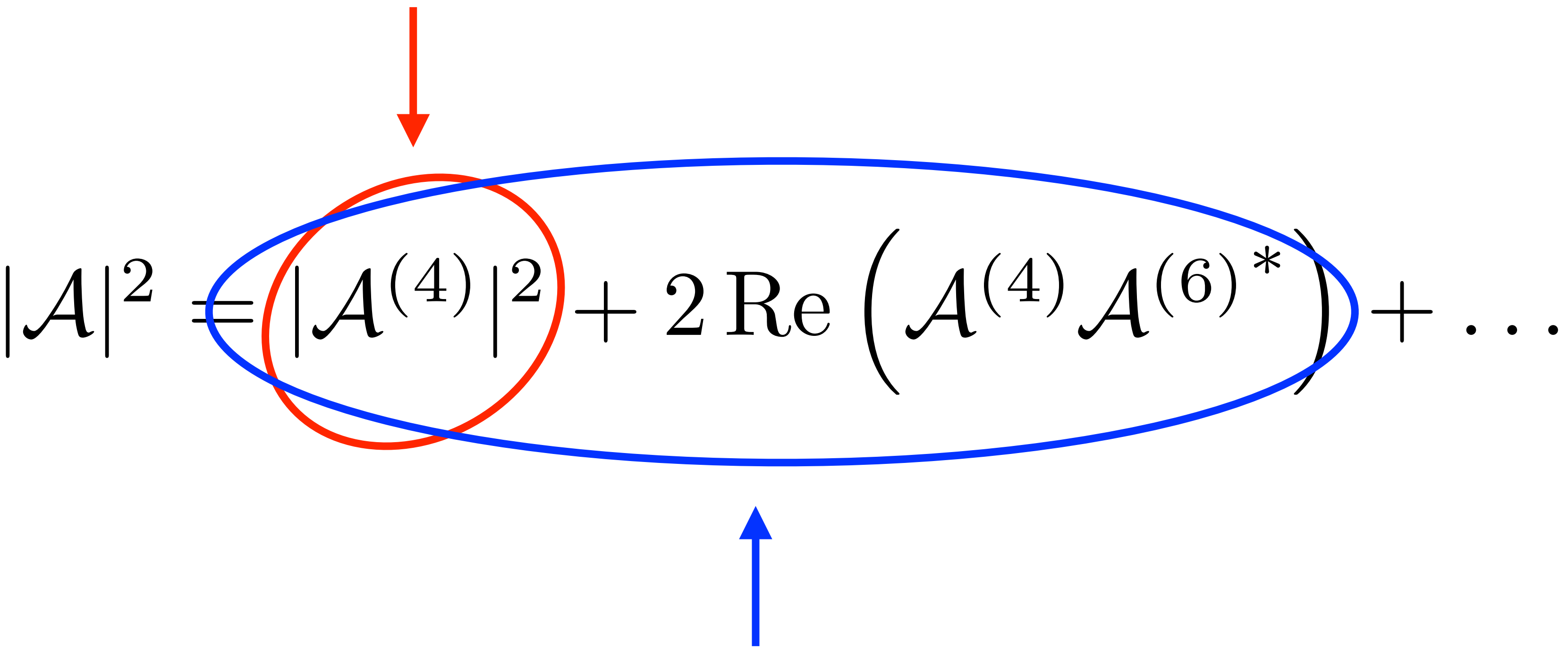
Preserving CP at Dimension 6



preserves CP if and only if: $J_4 = 0$

$$|\mathcal{A}|^2 = |\mathcal{A}^{(4)}|^2 + 2\,\mathrm{Re}\left(\mathcal{A}^{(4)}\mathcal{A}^{(6)\,*}\right) + \ldots$$

preserves CP if and only if:

$$J_4 = 0$$

$$L_i = \mathrm{Im}\,\mathrm{Tr}\,f(Y_u, Y_d, Y_e)\,C$$

where $i = 1, 2, \ldots, 699$

When do dimension 6 operators violate CP?

consider a dimension 6 operator: $\frac{C_6}{\Lambda^2}\mathcal{O}_6$

naively this is CP conserving if: $\mathrm{Im}\, C_6 = 0$

(possible if invariants capturing all phases vanish)

but the dimension-4 sector already violates CP: $J_4 \neq 0$

opportunistic CP violation: $\mathcal{O}_4^*\mathcal{O}_6$ can violate CP even when $\mathrm{Im}\, C_6 = 0$

Opportunistic CP Violation

minimal basis: if $J_4 = 0$ there are 699 independent CP violating invariants

$$\mathrm{Im}\,\mathrm{Tr}\, f(Y_u, Y_d, Y_e)\, C$$

maximal basis: if $J_4 \neq 0$ there are 1551 independent invariants

opportunistic CP violation: interference between dim. 4 and 6 operators that relies on the CKM phase

is opportunistic CP violation suppressed by $J_4 \approx 6.4\eta\,\lambda^{36} \approx 8 \times 10^{-24}$?

An Opportunistic Example in K-Kbar

consider the operator: $C^{(1)}_{HQ,ij}(H^\dagger i\overleftrightarrow{D}H)(\bar{Q}_i\gamma^\mu Q_j)$

(3 phases
and 6 real param)

minimal set of invariants:

$$L_1 = \operatorname{Im}\operatorname{Tr}\Big(X_u X_d C^{(1)}_{HQ}\Big),\quad L_5 = \operatorname{Im}\operatorname{Tr}\Big(X_u^2 X_d^2 C^{(1)}_{HQ}\Big),\quad L_7 = \operatorname{Im}\operatorname{Tr}\Big(X_u X_d X_u^2 X_d^2 C^{(1)}_{HQ}\Big)$$

if the minimal set vanishes, then can take real C_{HQ}

$$X_u \equiv Y_u Y_u^\dagger$$
$$X_d \equiv Y_d Y_d^\dagger$$

opportunistic CPV:

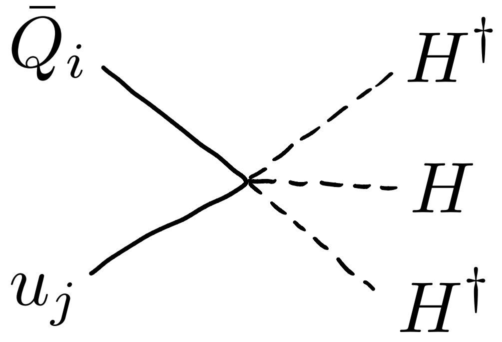
$$\left(\frac{\epsilon'}{\epsilon}\right)_{\mathrm{BSM}} \propto L_5 + \operatorname{Tr}(X_u)\operatorname{Tr}(X_d)L_1 - \operatorname{Tr}(X_d)\underset{\uparrow}{L_2} - \operatorname{Tr}(X_u)\underset{\uparrow}{L_3}$$

opportunistic

$$L_2 = \operatorname{Im}\operatorname{Tr}\Big(X_u^2 X_d C^{(1)}_{HQ}\Big) \qquad L_3 = \operatorname{Im}\operatorname{Tr}\Big(X_u X_d^2 C^{(1)}_{HQ}\Big)$$

Size of Opportunistic CP Violation

consider for example: $\frac{C_{ij}}{\Lambda^2}|H|^2\bar{Q}_i H^\dagger u_j$



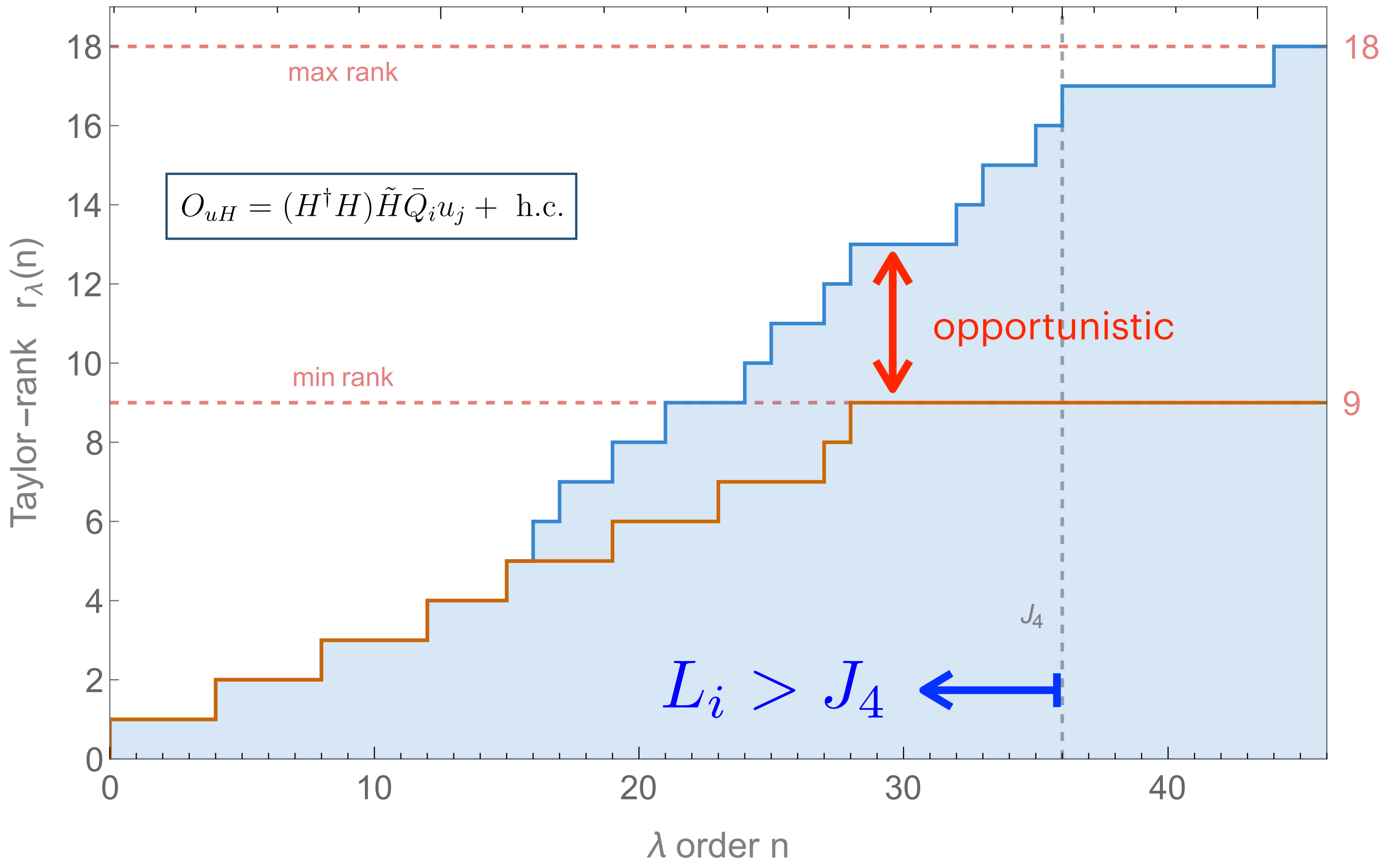
(18 real parameters, 9 phases)

consider the “up basis”:

$$Y_u = Y_u^{\text{diag}}$$

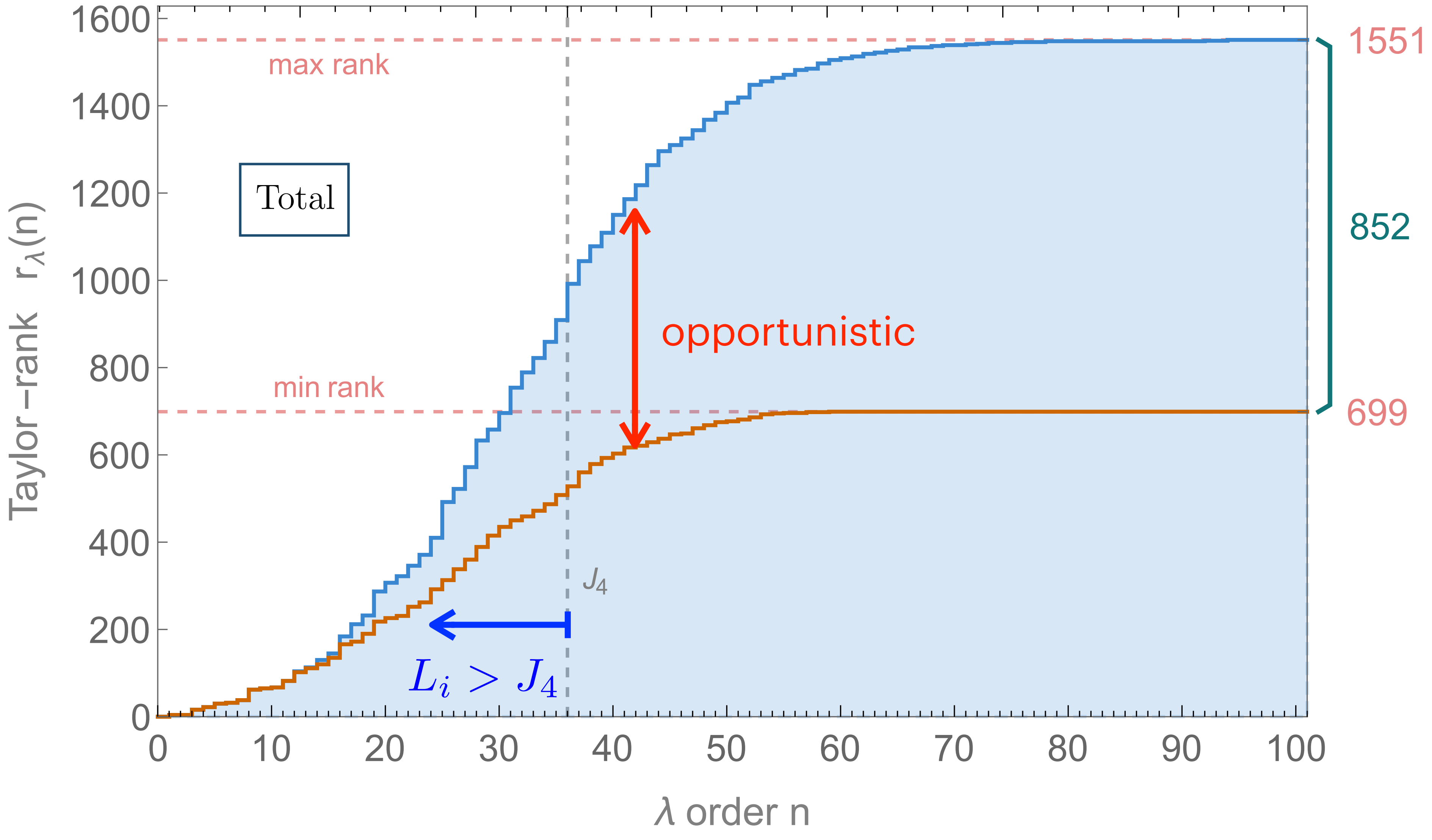
$$Y_d = V_{\text{CKM}} Y_d^{\text{diag}}$$

counting the number of independent invariants at order λ^n



Size of Opportunistic CP Violation

all dimension 6 operators:



opportunistic CP violation can be larger than J_4 → dimension 6 operators can amplify the CKM phase!

Practical Comments (“so *what?*”)

- Flavor invariants allow for precise, linear, parameterizations of flavor violating observables.
- Invariants capture how some phases have suppressed impacts on baryogenesis at high temperatures

Q&A

I’m constraining CP violation in SMEFT (at leading $1/\Lambda^2$ order).
What does my measurement have to do with invariants?

You’re constraining a linear combination of invariants!

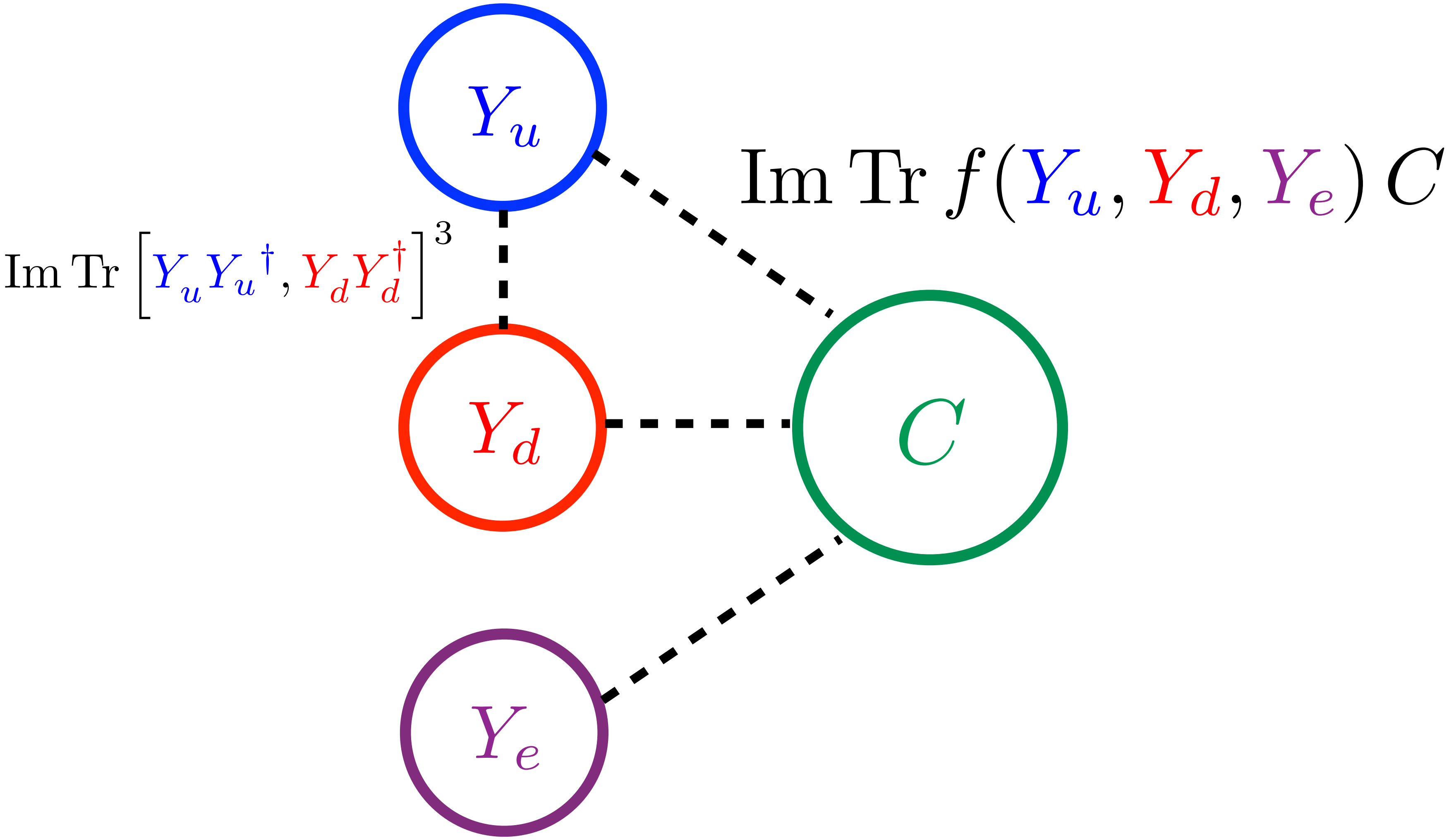
What linear combination of invariants am I constraining ?

Contact us! We might be able to help figure that out.

Is my measurement sensitive to opportunistic CP violation?

Maybe.

Take Away



future directions:

- invariants as parameters in global fits
- invariants in electroweak baryogenesis
- RG mixing of invariants
- invariants at dimension >6 (nonlinear)