DUNE θ_{13} Feldman-Cousins: a story probably about local minima

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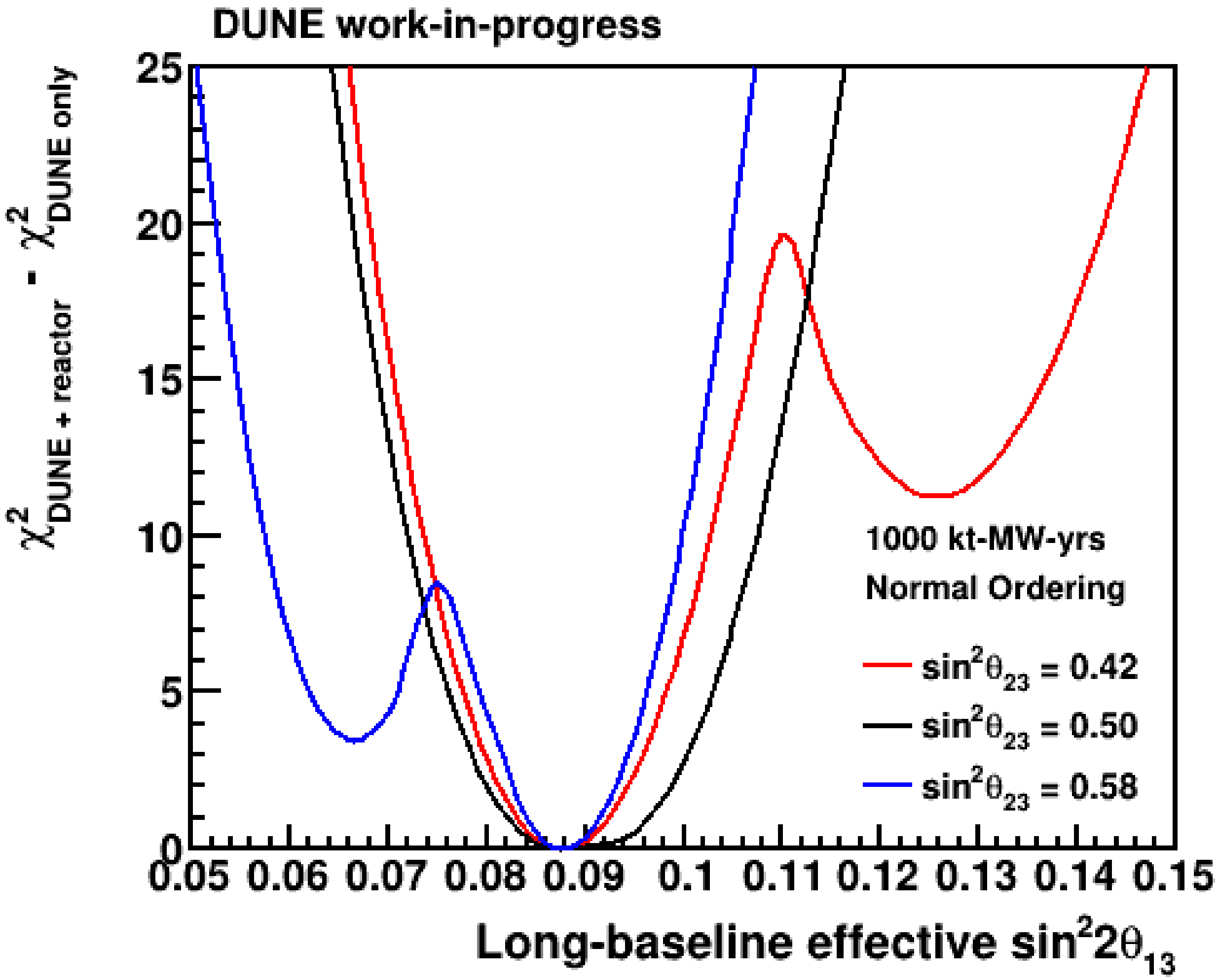
Main θ_{13} result: $\Delta\chi^2$



- Compares a fit with the reactor constraint to a fit without
- When the "true" LBL θ_{13} value is far from the reactor value, the constrained fit introduces an additional penalty, so there should be a higher χ^2
- We don't really know how to interpret this $\Delta\chi^2$, it's not even a "normal" resolution where you fix the parameter
- We want to do a Feldman-Cousins to see what the critical values look like

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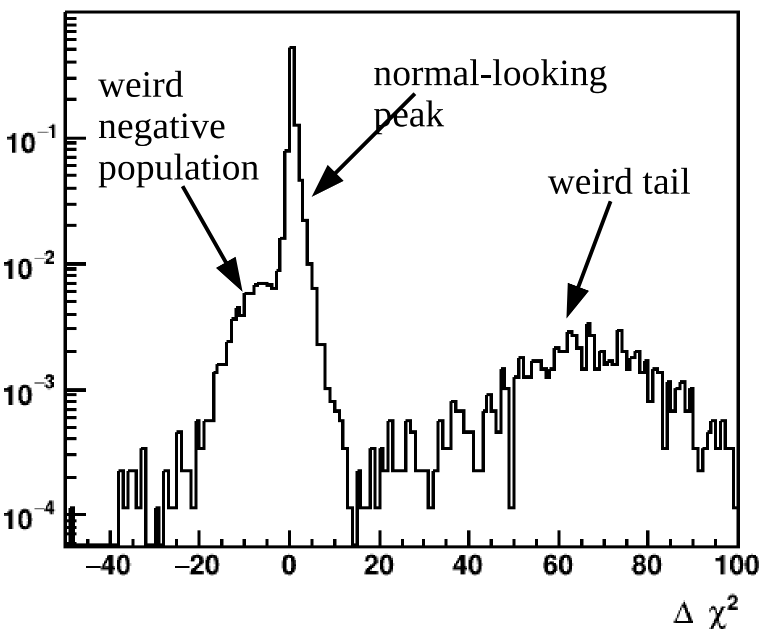
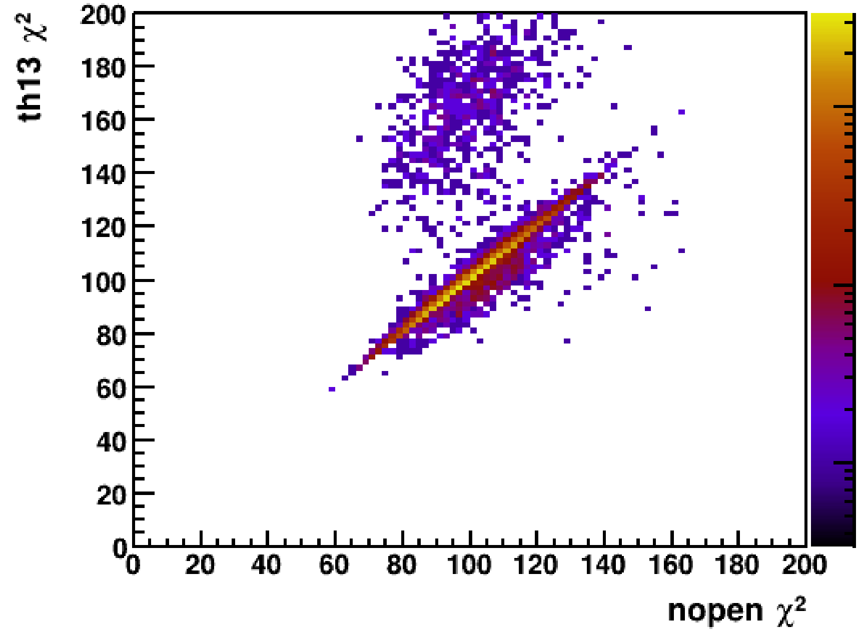
What the F-C does

- At each value of θ_{23}, and setting θ_{13} to the reactor best-fit value, perform thousands of pseudoexperiments, in which we randomly vary:
  - Systematic shifts of every nuisance parameter
  - Some oscillation parameters: solar params, ρ, Δm^2_{32}, δ_{CP}
  - Statistical Poisson throw in every bin
- In each pseudoexperiment, perform two fits:
  - θ_{13} unconstrained ("nopen" = no penalty)
  - θ_{13} constrained by reactor data ("th13" = penalty on θ_{13})
- $\Delta\chi^2 = \chi^2_{constrained} - \chi^2_{unconstrained}$
- Result **should** always be positive → for a given throw, the χ^2 at the same point will always be lower *without* the additional penalty term

What the F-C?

$\sin^2\theta_{23} = 0.42$

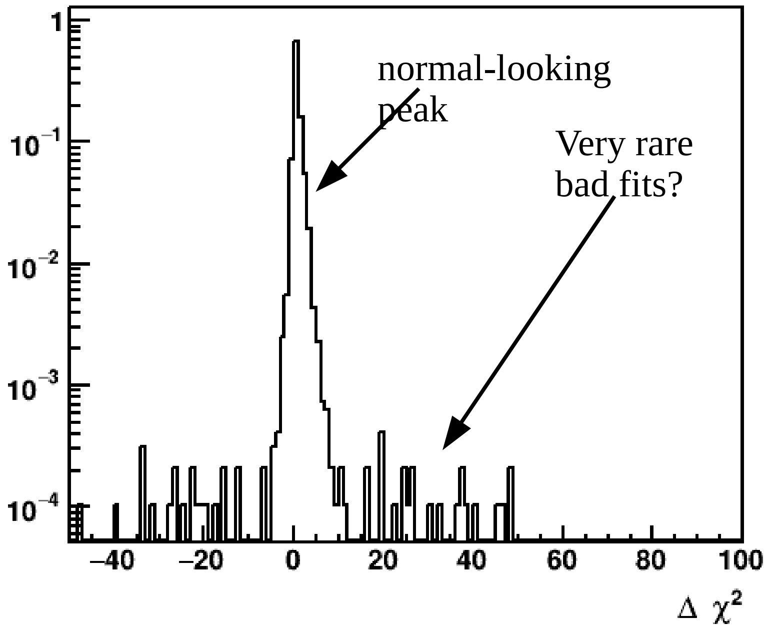


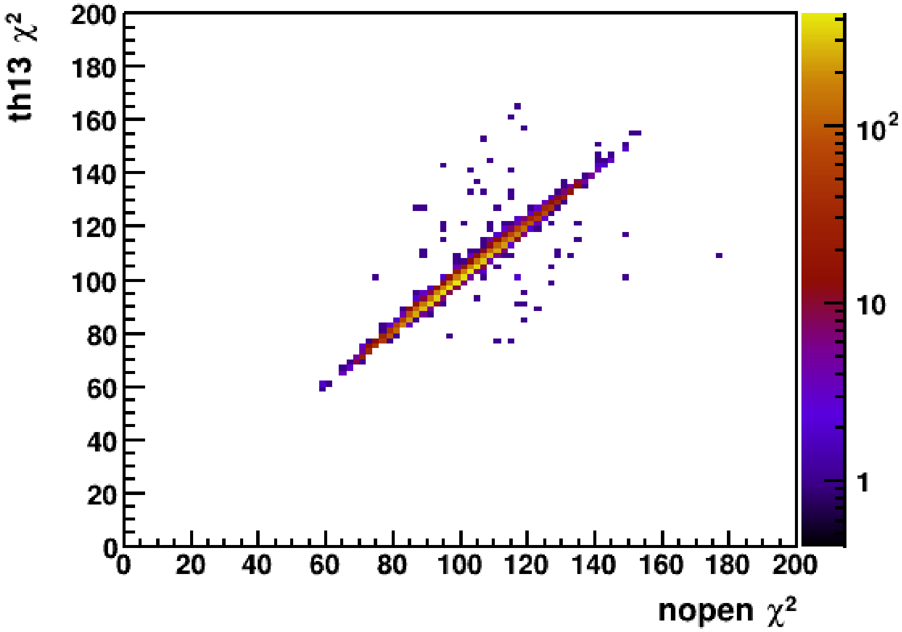


- This looks perfectly normal for 85% of the throws
- But there is a strange tail at very high $\Delta\chi^2$, and a strange population of negative $\Delta\chi^2$s

Things look normal-ish for maximal mixing

$\sin^2\theta_{23} = 0.50$





- For maximal θ_{23}, the two odd populations go away
- Very occasional throws are either negative or in a tail, possibly due to bad fits
- First hint that the oddity is due to the octant

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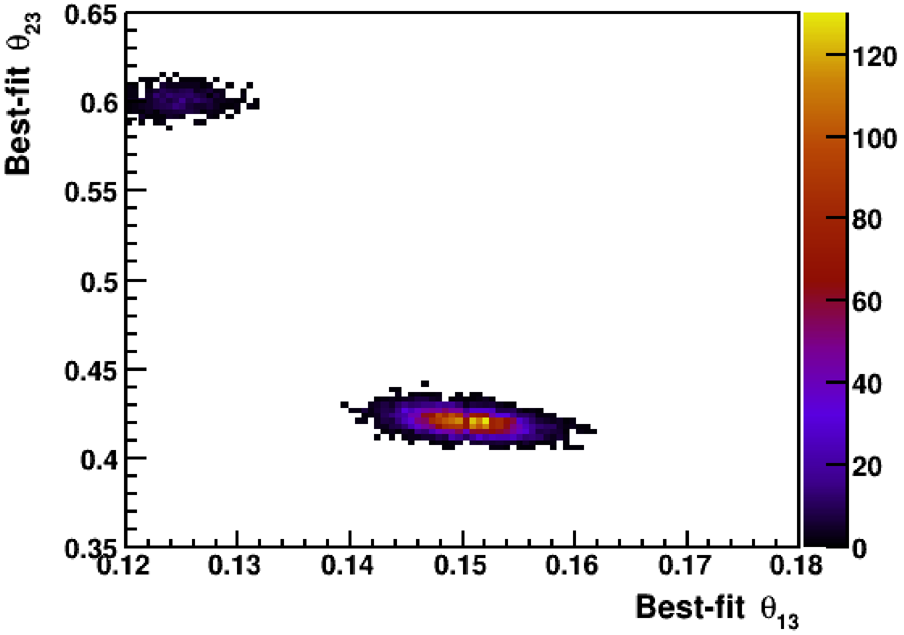
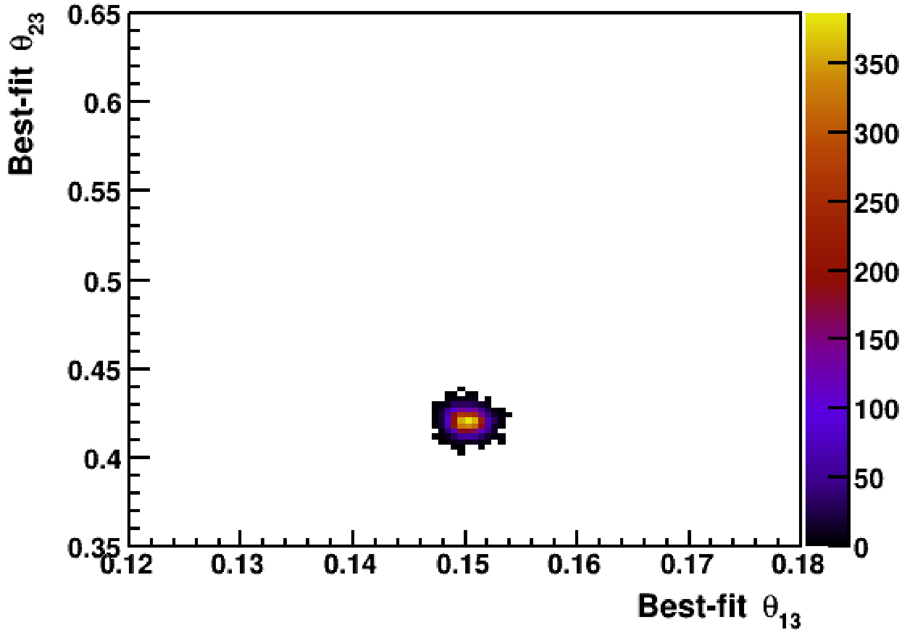
Best fit θ_{13}-θ_{23}: $0 < \Delta\chi^2 < 10$ "normal"

All subsequent plots: $\sin^2\theta_{23} = 0.42$

unconstrained



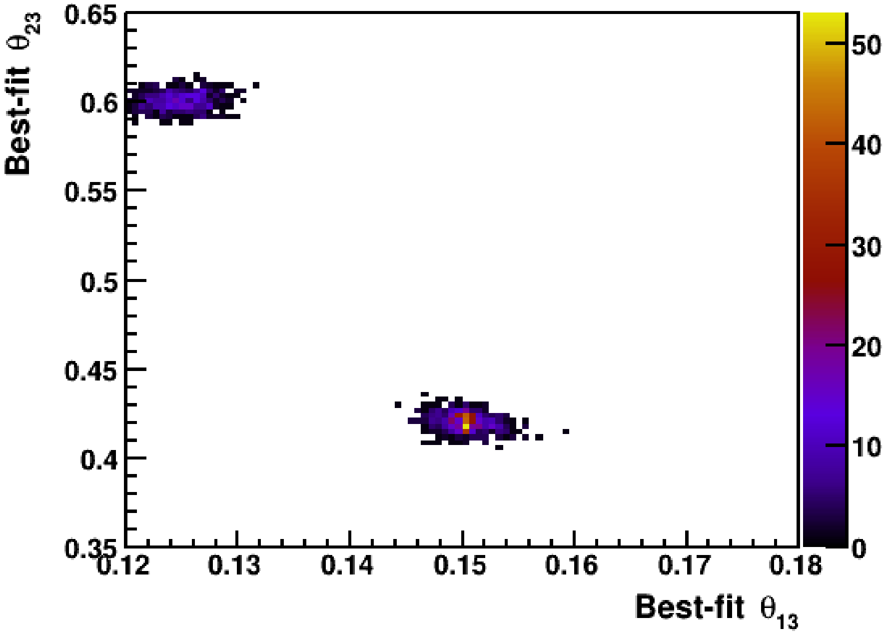
reactor penalty term



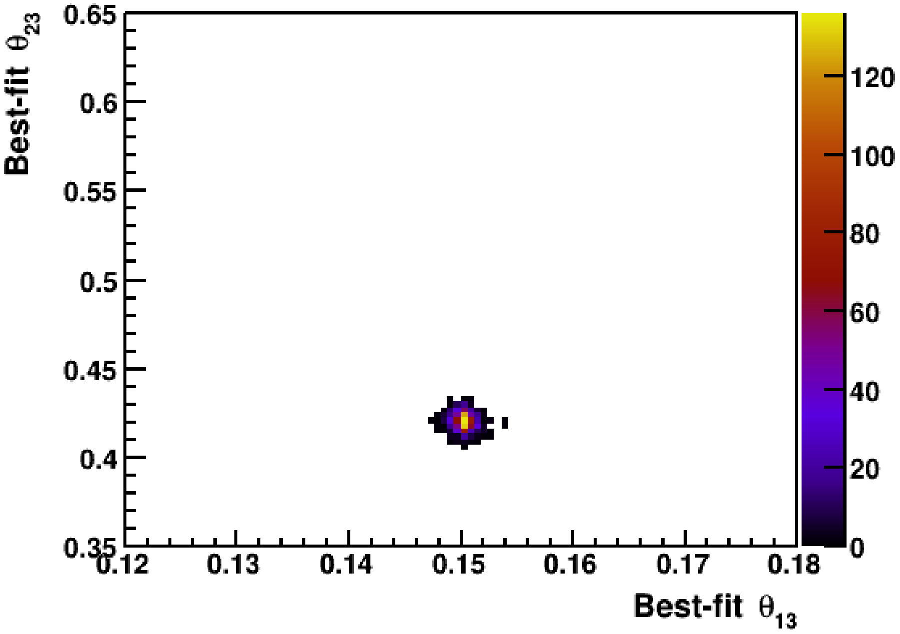
- Recall that without the reactor constraint, there is a second solution in the wrong octant and a different value of θ_{13} – this is normal, and we expect to sometimes prefer it due to other oscillation parameters, statistical bad luck, etc.
- Including the reactor constraint penalizes this other solution very strongly, so it should never be preferred

Best fit θ_{13}-θ_{23}: -10 $\Delta\chi^2 < 0$

unconstrained



reactor penalty term



- Slight negative means slightly lower χ^2 with the penalty
- A larger fraction of these throws are ending up in the wrong octant without the penalty

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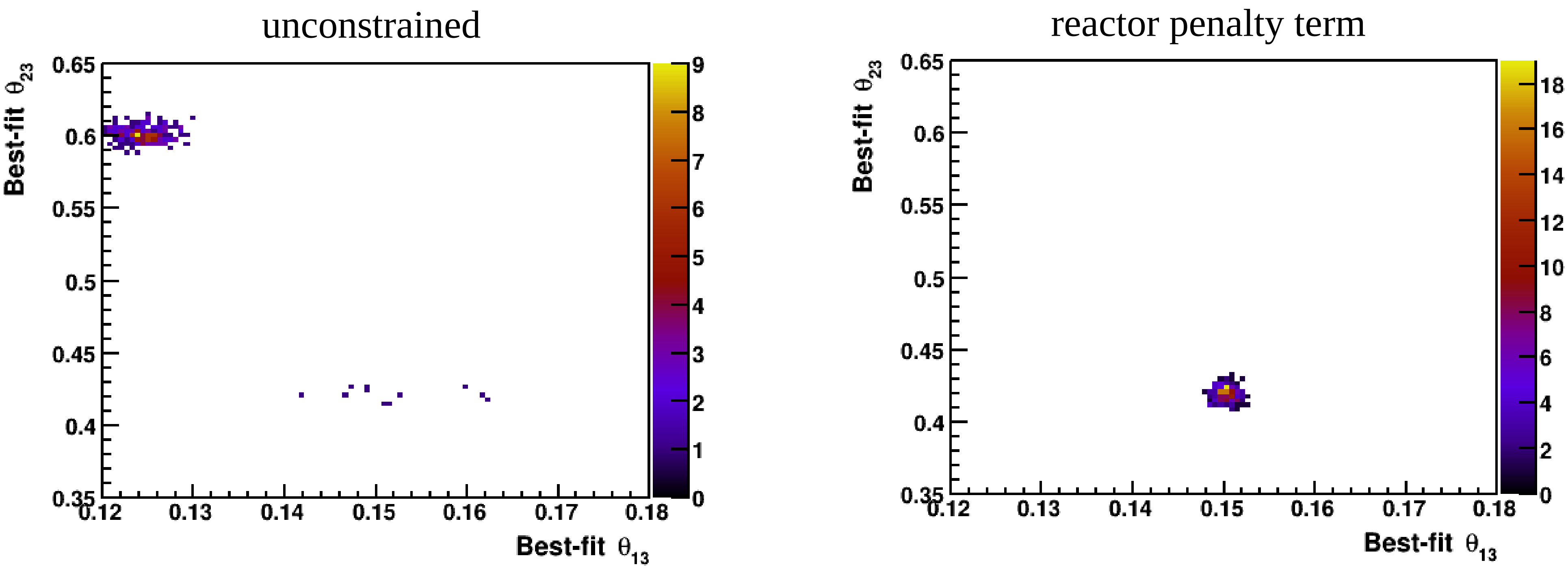
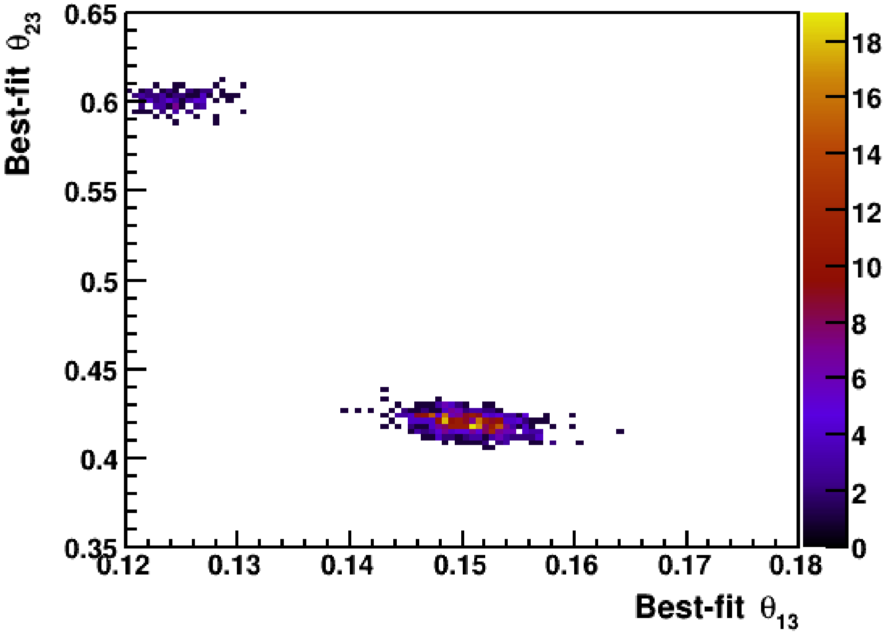
Best fit θ_{13}-θ_{23}: $\Delta\chi^2 < -10$



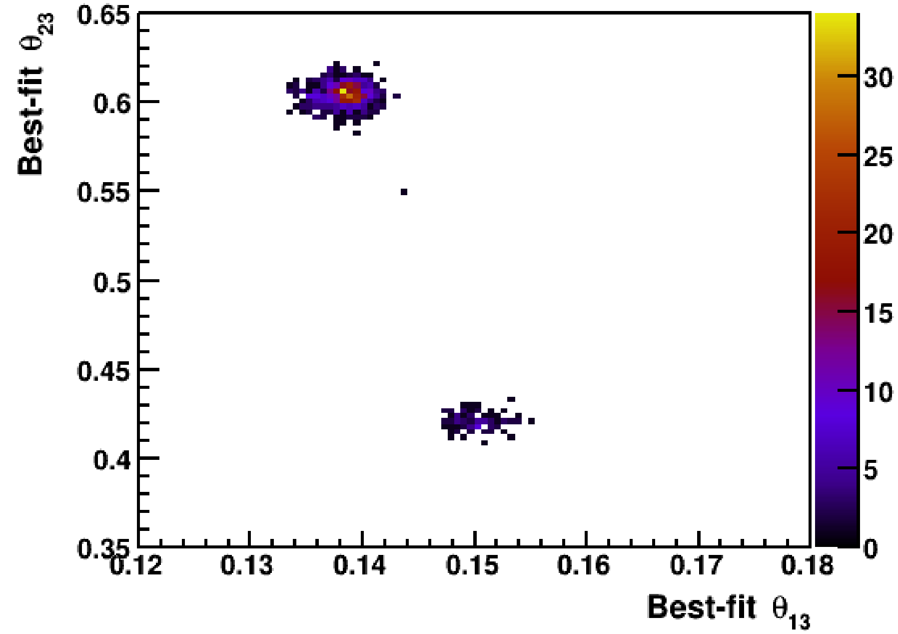
- Nearly all of these prefer the wrong octant with no penalty
- My theory: Most of these would prefer the correct

Best fit θ_{13}-θ_{23}: $\Delta\chi^2 > 10$ "tail"

unconstrained



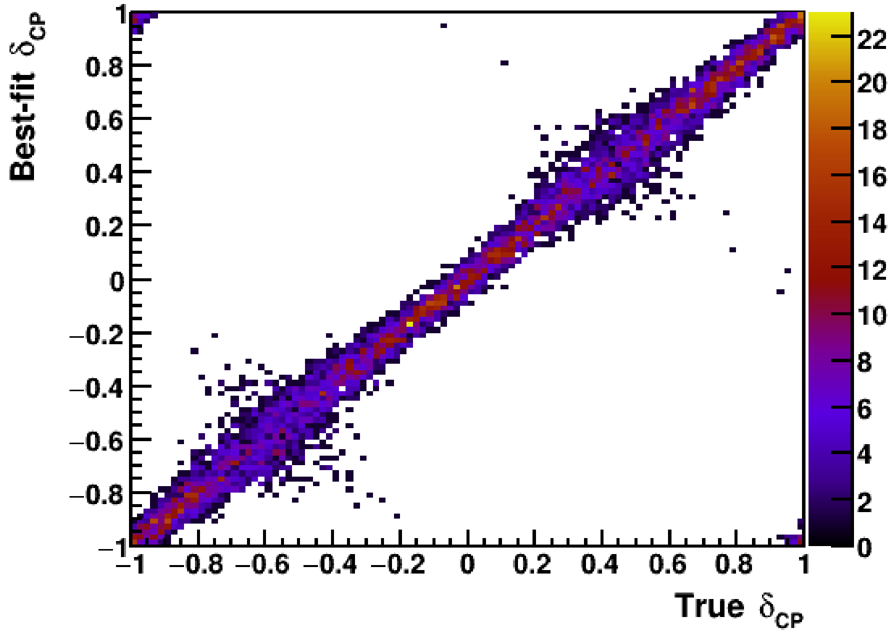
reactor penalty term



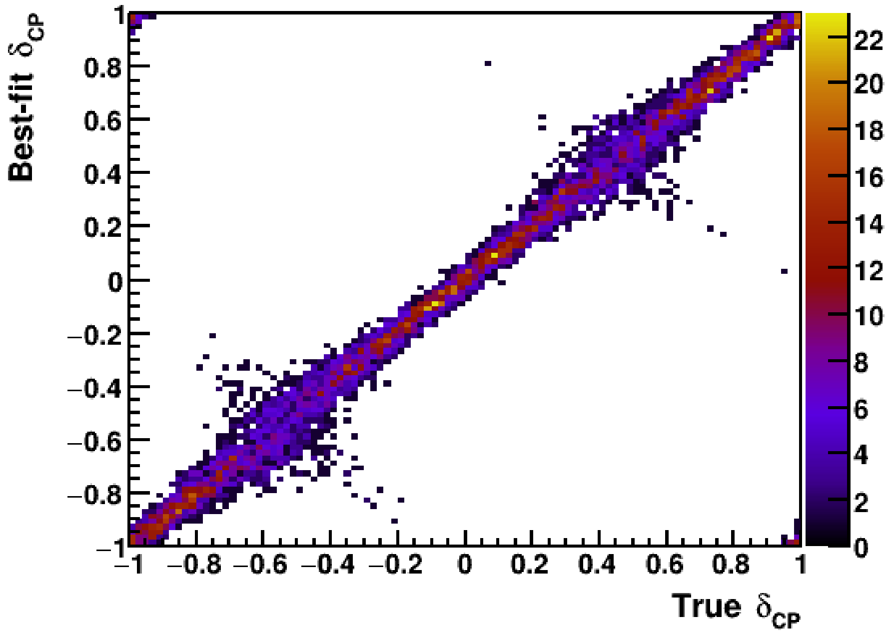
- These are in the wrong octant *despite the penalty*
- It is almost certainly not real → there is a better fit in the correct octant, but the fitter is getting stuck in a local minimum in the wrong octant

Best fit θ_{13}-θ_{23}: $0 < \Delta\chi^2 < 10$ "normal"

unconstrained



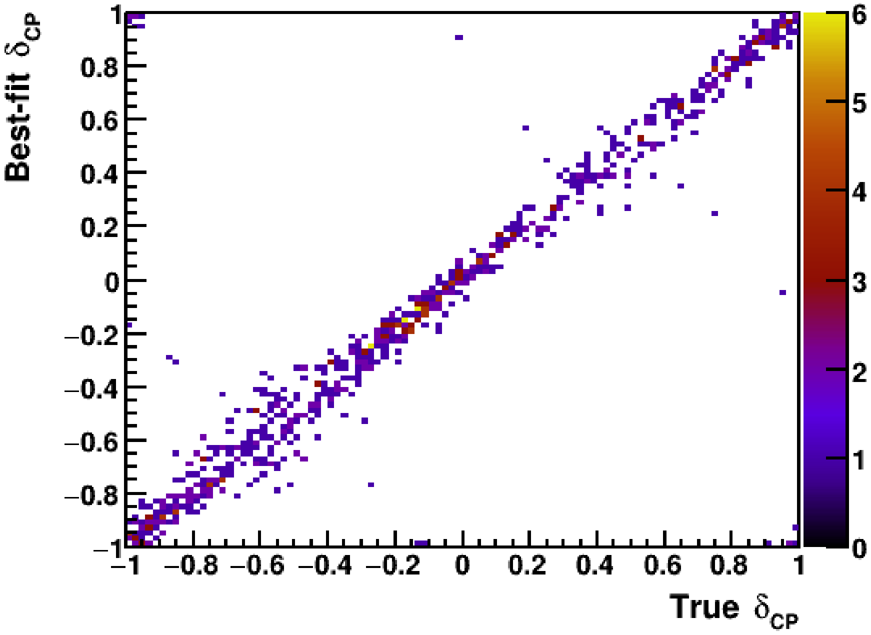
reactor penalty term



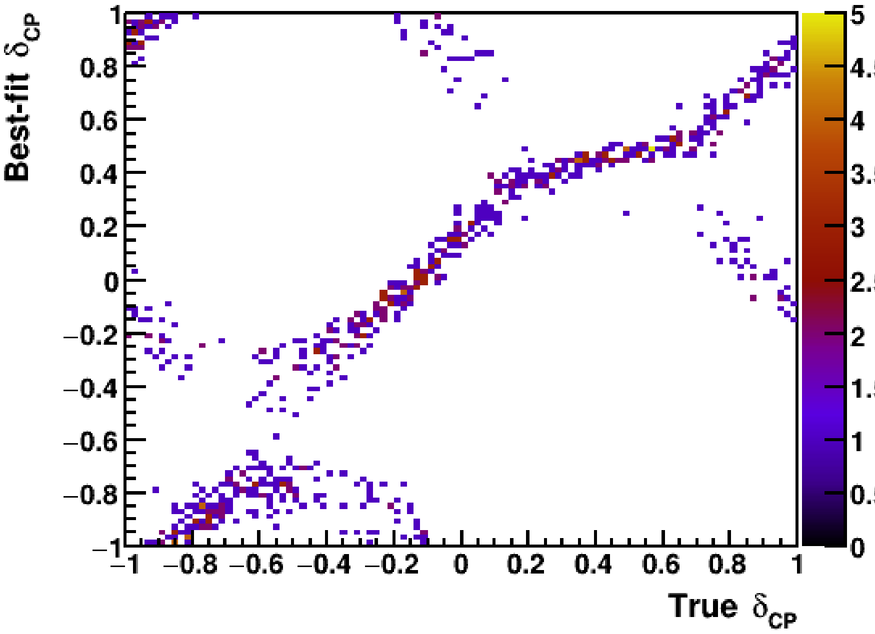
- We throw δ_{CP} flat in these fits, this is showing the true and best-fit values for fits that are normal-seeming

Best fit θ_{13}-θ_{23}: $\Delta\chi^2 > 10$ "tail"

unconstrained



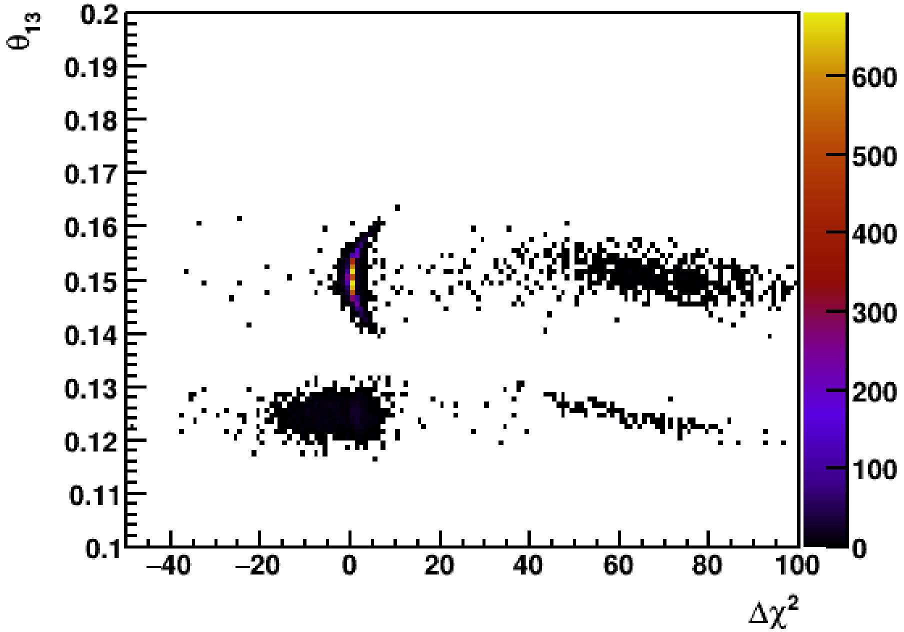
reactor penalty term



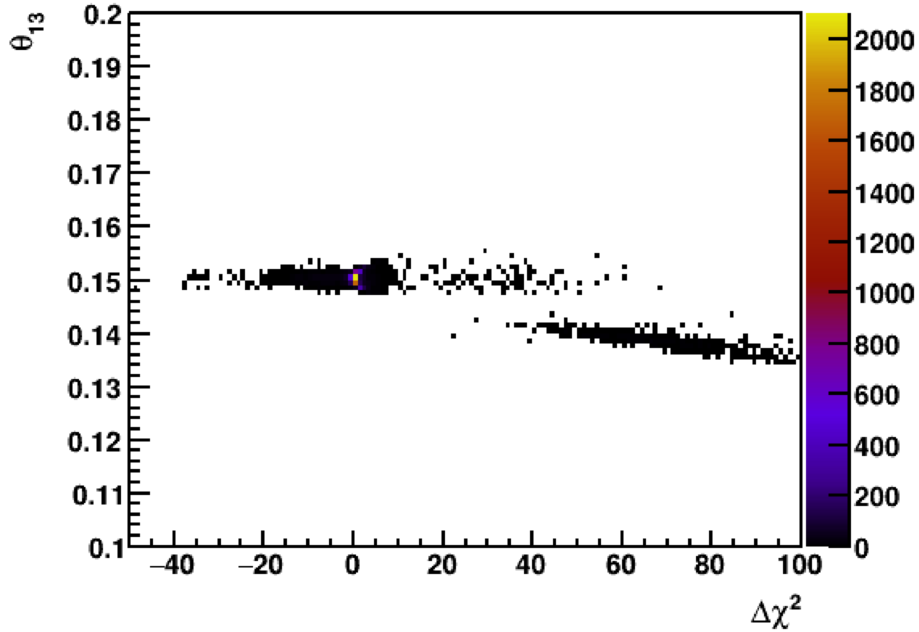
- The wrong-octant flip is also correlated with δ_{CP} being wrong in the fits with the reactor penalty

Best fit θ_{13} vs. $\Delta\chi^2$

unconstrained



reactor penalty term



- Can see that majority of the tail throws are getting wrong-octant despite penalty
- Most negative throws are wrong octant without the penalty and right octant with
- Some wrong-octant throws *have a lower χ^2 for the same throw* in the right octant with an additional penalty term → I can't think of any reason why they would not also have a lower χ^2 in the right octant without that additional term

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What we learned

- This is a hard experiment
- We know there are often local minima in:
  - Upper and Lower mass ordering
  - Upper and lower octant of θ_{23}
  - Upper and lower octant of δ_{CP} (e.g. $-\pi/4$ and $-3\pi/4$)
- One hypothetical solution is to run eight fits where these parameters are constrained to particular combinations of octants, then take the global minimum
- This is not great $\rightarrow$ unlike for Δm^2, where 0 is disfavored at 1000000σ, $\theta_{23} = 0.5$ and $\delta_{CP} = -\pi/2$ are perfectly sensible values, so constraining fits around them is sketchy

Constrained fits attempt

- Ran three fits with constraints on θ_{23}:
  - (0, 0.5)
  - (0.5, 1)
  - (0, 1)
- But very often, the constrained fits create a new local minimum at 0.5 and the best fit ends up there when it is trying to explore the other octant

Something that would work

- Run 100 fits with random seed values, take the minimum
- This would be computationally horrible, but would produce the correct answer without cheating

Proposal for now

- Seed the fits with the true parameter values
- This will give a sensible result (?)
- This will not waste massive amounts of resources
- We understand the problem is with local minima, and that it has solutions, they are just complicated to implement

Smart solution

- A smart algorithm might run one fit, then run a few others with intelligent seed values based on our knowledge of where the large degeneracies in the likelihood surface are
- It could learn from the results whether it has probably found the right answer or needs to keep going
- Somebody smart could work on this and write a nice paper for DUNE