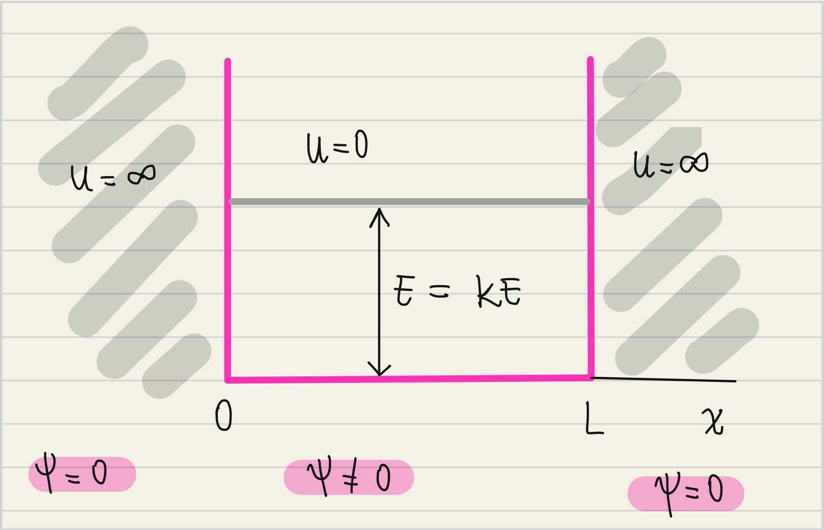
Bound States:

Def. A bound state is one in which a particle's motion is restricted by an external force to a finite region of space

$$V(x) \begin{cases} =0 & 0 \leq x \leq L \\ \infty & \text{otherwise} \end{cases}$$

Infinite Well:



* idea

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\psi(x)$$

→ we choose U to be = 0 between the walls, there we find the time indep. Schrödinger eq

we can rewrite it as

↓

$$\frac{\partial^2}{\partial x^2}\psi(x) = -k^2\psi(x)$$

DO NOT ERASE ME

where we defined:

$$k^2 = \frac{2mE}{\hbar^2}$$

Let's guess the answer:

POLL:

the solution is a $f(x)$ t.c. $\frac{\partial^2 f(x)}{\partial x^2} = - K^2 * f(x)$

A) e^{-kx}

B) $\sin(kx)$

(C) $\cos kx$

(A) $-k e^{-kx} = +k^2 e^{-kx}$ NO

(B) $k\cos(kx) \xrightarrow{\partial/\partial x} -k^2 \cdot \sin(kx)$

we need to impose boundary conditions: $\psi(x=0) = \psi(x=L) = 0$

- for $x=0$ $\psi(x) = 0$

$x = L$ $\psi(x=L) = \sin KL \rightarrow K = \frac{n\pi}{L} = \sqrt{\frac{2mE}{\hbar^2}}$

this eq. allows us to find the eigenstates:

$\psi_n(x) = A \cdot \sin kx$ in q.m. bound states are standing waves

$E = \frac{n^2\pi^2\hbar^2}{2m^2L^2}$ and eigenvalues $n = 1, 2, \ldots$

* $n=0$ is not a sol: $\psi = 0$ & $E=0$ violates uncert. principle

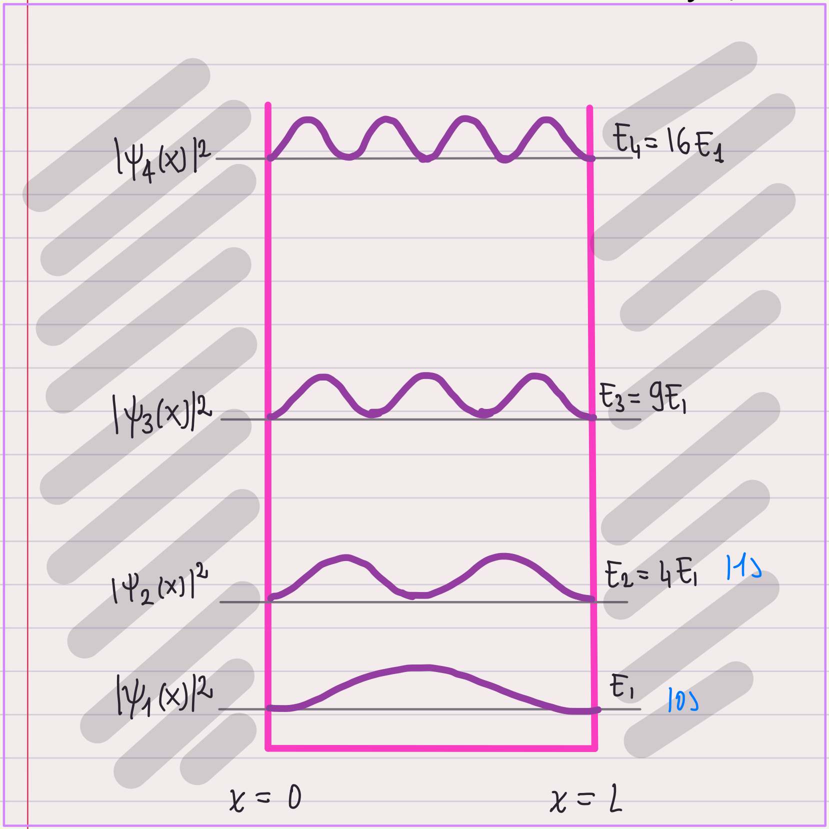
- analogy w classic h.o.

- additional comments: but in this case, a quantum particle CAN'T have ANY value of the energy only special ALLOWED values

A can be det. by normalizing:

$$\int_0^L |A|^2 \cdot \sin^2(kx) = |A|^2 \cdot \frac{L}{2} = 1 \rightarrow \psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} \cdot x\right)$$

This plot is for the probability density of finding the particle in the well.



Solutions:

EIGENSTATES

$$\psi_n(x) = \begin{cases} \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} & 0 < x < L \\ 0 & x < 0, x > L \end{cases}$$

EIGENVALUES

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

↓

Energies are discrete

? NODES: points in which $|\psi(x)|^2 = 0$

How can the particle get from one place in the well to another if there is no possibility of it being found at a point in between?

SOLUTION: has to do with duality particle / wave nature see pg. 155