Lecture 2:

Introduction to Superposition In Quantum Mechanics

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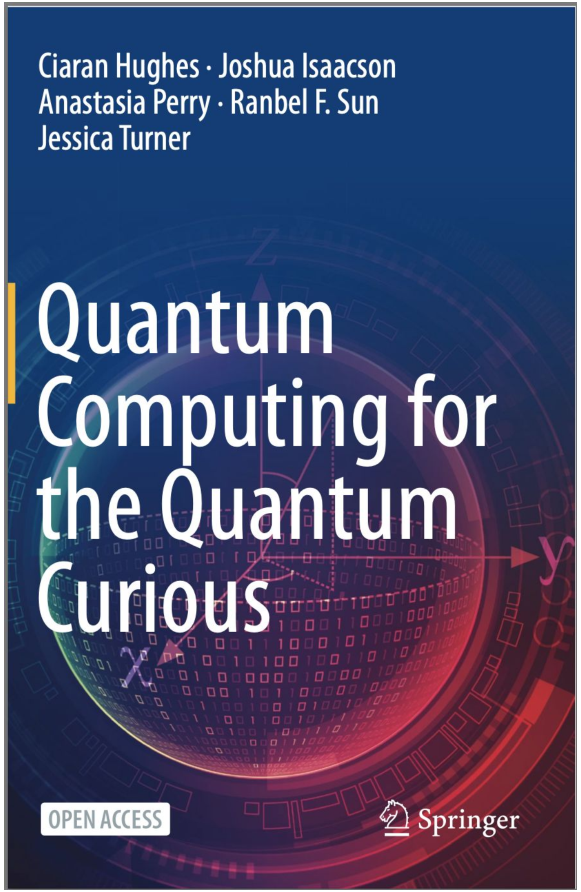
Fermilab

U.S. DEPARTMENT OF ENERGY | Office of Science

0. Today (Hopefully…)



Chapter 1 of *Quantum Computing for the Quantum Curious*

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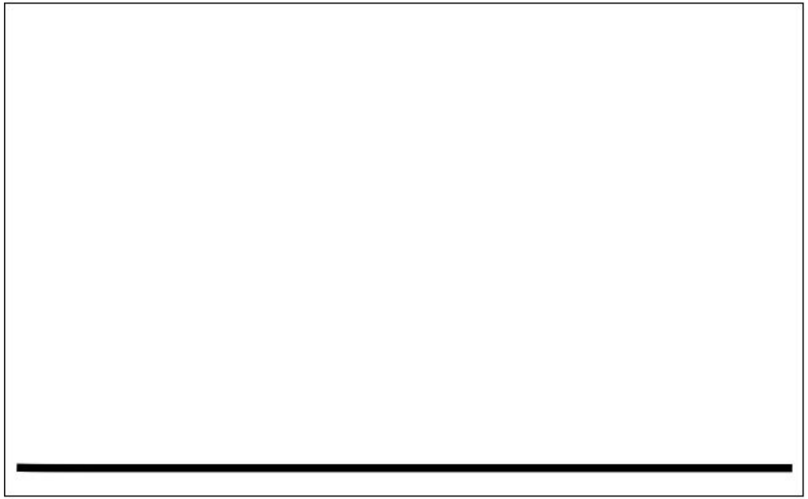
Relevant math we missed yesterday,

e.g. Bra-Ket notation, complex numbers, probability amplitudes, etc.

1. Classical Superposition

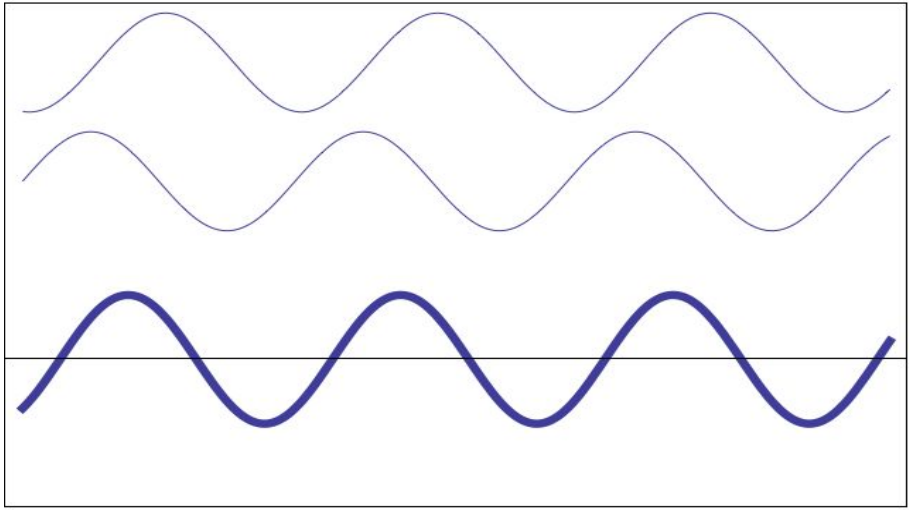
One of the most fundamental pillars of quantum mechanics is the principle of superposition

CLASSICAL SUPERPOSITION



1. Classical Superposition

Superposition in classical physics is the stuff we're used to e.g. water waves, waves on a string

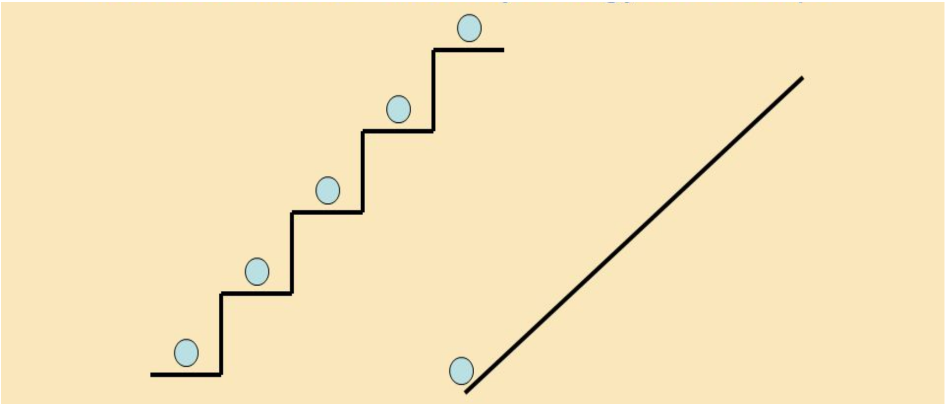


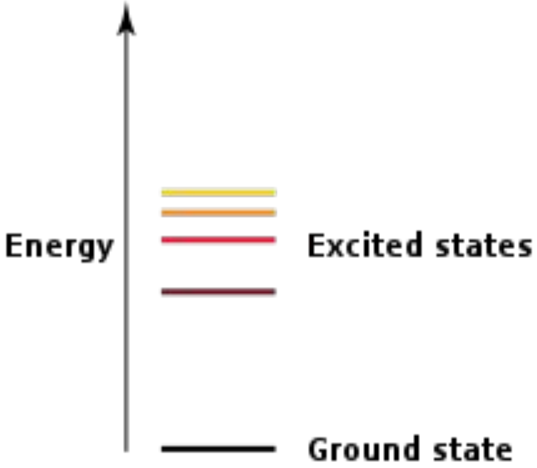
When the waves are exactly out of phase, they cancel!

1. Quantum Systems

How is a quantum system different to a classical system?

1. **Objects at the very small “particulate” scales exhibit peculiar properties, e.g. their measurable quantities (energy, momenta, …) are discretely quantised**

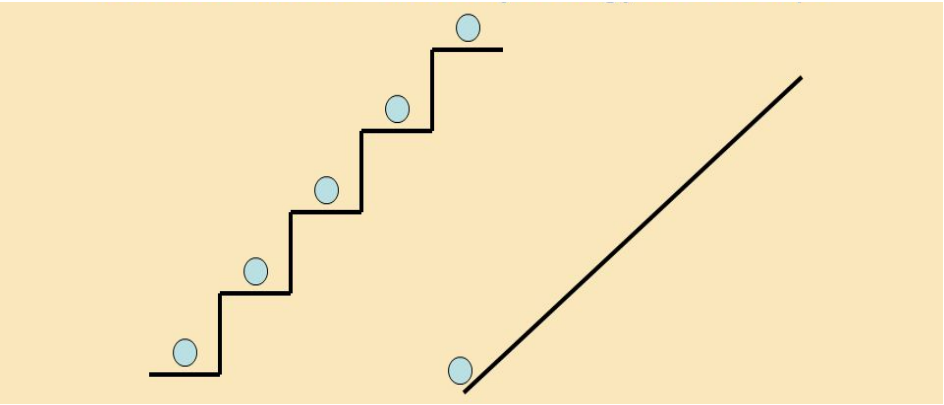


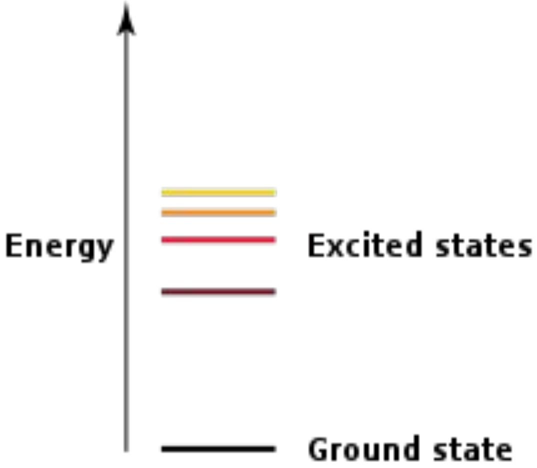


1. Quantum Systems

How is a quantum system different to a classical system?

2. A quantum system can be characterised by any of its measurable quantities.





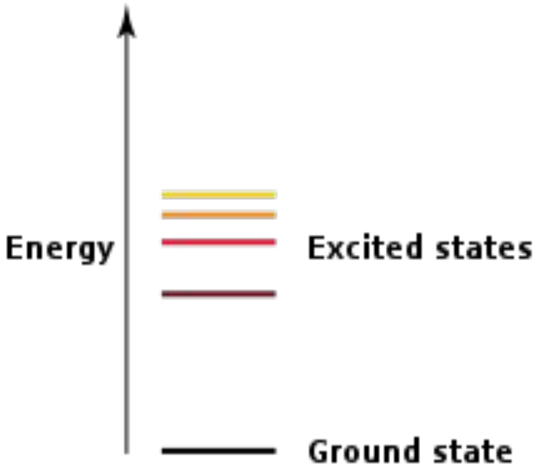
1. Quantum Systems

How is a quantum system different to a classical system?

i.e. $\mathbf{E = 0, 1, 2, 3, \ldots hf} \rightarrow$ each energy taken by the particle

would be described as a **definite state**

The particle may take any of its possible states with some probability!

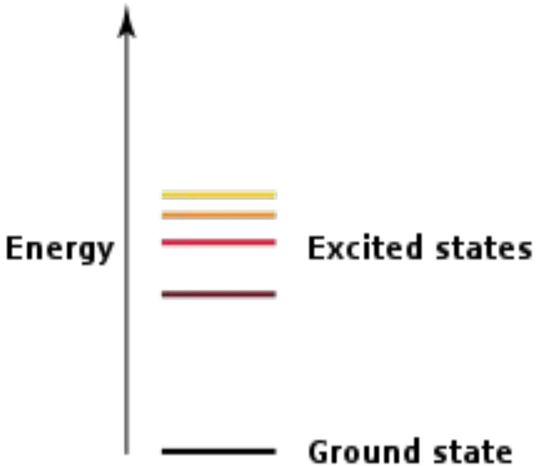


How is a quantum system different to a classical system?

i.e. $\mathbf{E = 0, 1, 2, 3, \ldots hf} \rightarrow$ each energy taken by the particle would be described as a **definite state**

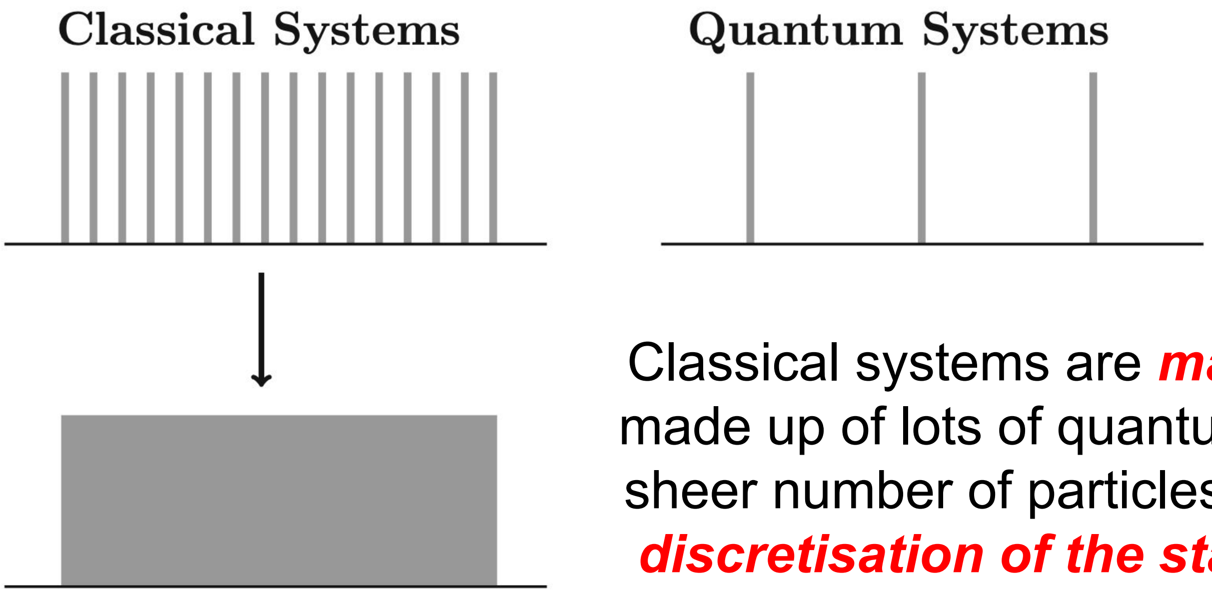
In bra-ket notation, we may write this is as the sum

$$|E\rangle = A_1|E_1\rangle + A_2|E_2\rangle$$



1. Quantum Systems

How is a quantum system different to a classical system?

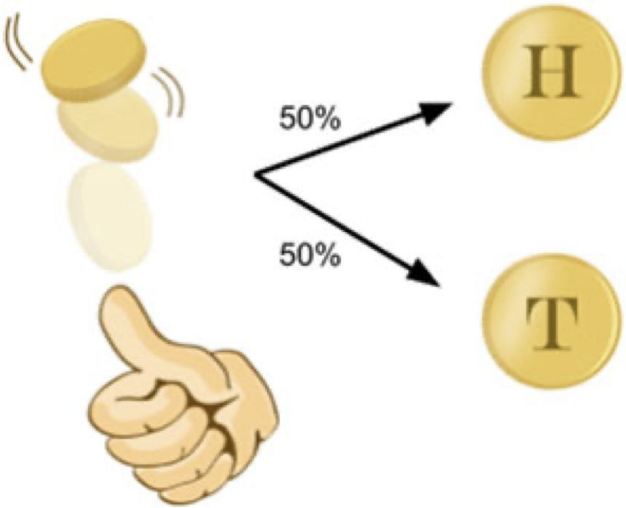


Classical systems are ***macroscopic*** and made up of lots of quantum particles. The sheer number of particles means that the ***discretisation of the states tends to a continuum***

1. Example: Coin Toss (non-QM)

How is a quantum system different to a classical system?

$$|\mathrm{coin}\rangle = \frac{1}{\sqrt{2}}|\mathrm{H}\rangle + \frac{1}{\sqrt{2}}|\mathrm{T}\rangle$$



1. Example: Coin Toss (non-QM)

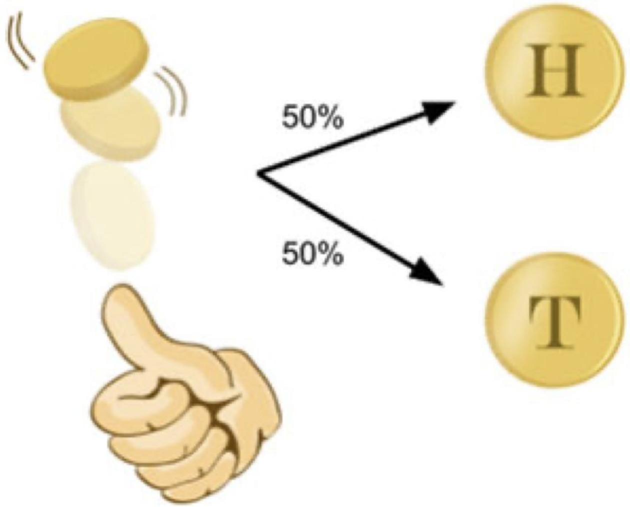
How is a quantum system different to a classical system?

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}|\text{H}\rangle + \frac{1}{\sqrt{2}}|\text{T}\rangle$$

$$|\Psi_f\rangle = |\text{H}\rangle \quad |\Psi_i\rangle = |\text{coin}\rangle$$

$$|\langle\Psi_f|\Psi_i\rangle|^2 = |\langle\text{H}|\text{coin}\rangle|^2$$

$$= \left(\frac{1}{\sqrt{2}}\right)^2$$



Group problems (take home)

1.7 An electron can be in one of two potential wells that are so close that it can "tunnel" from one to the other (see §5.2 for a description of quantum-mechanical tunnelling). Its state vector can be written

$$|\psi\rangle = a|A\rangle + b|B\rangle, \tag{1.45}$$

where $|A\rangle$ is the state of being in the first well and $|B\rangle$ is the state of being in the second well and all kets are correctly normalised. What is the probability of finding the particle in the first well given that: (a) $a = \mathrm{i}/2$; (b) $b = \mathrm{e}^{\mathrm{i}\pi}$; (c) $b = \frac{1}{3} + \mathrm{i}/\sqrt{2}$?

1.8 An electron can "tunnel" between potential wells that form a chain, so its state vector can be written

$$|\psi\rangle = \sum_{-\infty}^{\infty} a_n|n\rangle, \tag{1.46a}$$

where $|n\rangle$ is the state of being in the n^{th} well, where n increases from left to right. Let

$$a_n = \frac{1}{\sqrt{2}}\left(\frac{-\mathrm{i}}{3}\right)^{|n|/2} \mathrm{e}^{\mathrm{i}n\pi}. \tag{1.46b}$$

a. What is the probability of finding the electron in the n^{th} well?
b. What is the probability of finding the electron in well 0 or anywhere to the right of it?

Problems from chapter 1 of

https://www-thphys.physics.ox.ac.uk/people/JamesBinney/qb.pdf

Very good read!