What is a Qubit?

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Classical bits

Bit

- Bit stands for binary digit: 0 or 1.
- It is a a way to represent information on a digitally.

Example

Suppose you have 4 objects to store in a computer. You can use **two** bits.

$$\begin{aligned} \text{First object} &\mapsto 00 \\ \text{Second object} &\mapsto 01 \\ \text{Third object} &\mapsto 10 \\ \text{Fourth object} &\mapsto 11 \end{aligned} \tag{1}$$

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Bra-Ket notation

Notation

- We denote a state Ψ with a ket: $|\Psi\rangle$
  - Kets $|\Psi\rangle$ correspond to the vectors we know and love
- The ‘complex conjugate’ is called bra: $\langle\Psi|$
  - Bras are the ‘dual’ of the kets.
- Bras and Kets belong to different spaces.
- This lecture will focus on Kets.

Quantum bit (Qubit)

Definition

A qubit is a two-level quantum state: $|\Psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

Properties

- α, β are called amplitudes.
- α, β are complex numbers in general.
- A valid qubit state must be **Normalized**: $|\alpha|^2 + |\beta|^2 = 1$

Interpretation

- A qubit state is a superposition of two different states:
  - $|0\rangle$ with amplitude α
  - $|1\rangle$ with amplitudes β

Measurement

'Simple' Measurement: Possible results $|0\rangle$ or $|1\rangle$

Qubit: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

- After measurement, the **superposition** collapses to either $|0\rangle$ or $|1\rangle$
- We obtain $|0\rangle$ with Probability: $P(|0\rangle) = |\alpha|^2$
- We obtain $|1\rangle$ with Probability: $P(|1\rangle) = |\beta|^2$

Surprise :)

- Do you see why a qubit must be normalized $|\alpha|^2 + |\beta|^2 = 1$?

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Vector Representation

From Bra-Ket to Vector

Given a qubit: $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$$\begin{aligned} |0\rangle &\mapsto \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ |1\rangle &\mapsto \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{aligned} \tag{2}$$

A qubit state can be treated as a vector (with everything nice we know about vectors)

$$|\Psi\rangle = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \tag{3}$$

Two qubits in vector form

Vector Form

Can you represent the qubit state

$$|\Psi\rangle_1 \otimes |\Psi\rangle_2 = \alpha_1\alpha_2 |00\rangle + \alpha_1\beta_2 |01\rangle + \beta_1\alpha_2 |10\rangle + \beta_1\beta_2 |11\rangle \tag{4}$$

in vector form?

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What can we do on a qubit? Unitary evolution

We can change the state of a qubit by multiplying with a Unitary matrix: $|\Psi'\rangle = U\,|\Psi\rangle$

Unitary matrix (U)

- A matrix U is unitary if (and only if): $U\,U^\dagger = \mathbb{1}$
- In other words $U^\dagger = U^{-1}$

Example

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad X^\dagger = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad X\,X^\dagger = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

What is the simplest example of a unitary matrix you can think of?

If we don't find it now, I will ask again after the next slide

Unitary evolution – Continued

Properties

- The determinant of a unitary matrix is a complex number with magnitude 1: $|\mathrm{Det}\,(U)| = 1$
- Unitary matrices conserve the normalization of a vector.

Insight :)

Do you see why we will mostly use unitary matrices to change the state of a qubit?

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Yes/No/Maybe

Description	Yes/No/Maybe
$\|\Psi\rangle = \frac{1}{\sqrt{2}}(\|\text{head}\rangle + \|\text{tail}\rangle)$ is a qubit valid state	...
$H^2 = \mathbb{1}$ and $H^\dagger = H$. Is H unitary?	...
$\|\Psi\rangle = \frac{e^{i\phi}}{\sqrt{2}}(\|0\rangle + \|1\rangle)$ is a qubit valid state	...
Every unitary matrix has an inverse	...
A matrix M has 0 determinant. M can be unitary	...
A unitary matrix must be a square matrix	...

Last one: Yes/No/Maybe

You measure a qubit you find $|0\rangle$. If you measure again, you may get $|1\rangle$.
Answer:

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What makes a good qubit?

- Qubits must be easy to control (manipulate)
- Qubits must be isolated from its surrounding

Decoherence time

How long does the qubit take to lose its quantum state.

- The longer the better.
- IBM machines have tens or hundreds of microseconds decoherence time.

How are qubits made?

Superconducting qubits

- Clockwise vs counter-clockwise current are states $|0\rangle$ and $|1\rangle$
- Most popular, pursued by companies such as Google and IBM.
- IBM Osprey has 433 superconducting qubits.

Photonic qubits

- Use light light polarization
- Pursued by companies such as Xanadau
- May resist decoherence but are difficult to scale.

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Conjugate Transpose (Hermitian Conjugate) of a matrix: $M^\dagger$

Definition

The conjugate transpose (Hermitian conjugate) of a matrix M is

$$M^\dagger = (M^*)^T = (M^T)^*$$

Example

$$M = \begin{pmatrix} 1+\mathrm{i} & -\mathrm{i} \\ 1 & 2\mathrm{i} \end{pmatrix} \tag{6}$$

$$\tag{7}$$

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Example

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$$M^* = \begin{pmatrix} 1-\mathrm{i} & \mathrm{i} \\ 1 & -2\mathrm{i} \end{pmatrix} \tag{9}$$

$$M^\dagger = (M^T)^* = \begin{pmatrix} 1-\mathrm{i} & 1 \\ \mathrm{i} & -2\mathrm{i} \end{pmatrix} \tag{10}$$