Creating Superpositions

Part 1/2: Beam splitters and the Mach-Zehdner interferometer

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Overview

What is quantum superposition **physically**, i.e. Is it observable?

Focus on **binary** cases: quantum particles in a superposition of two states (a qubit)

Simple qubit models (e.g. coin flip) do not capture full picture since amplitudes in superposition not manipulable

We consider two concrete experiments to gain understanding: **beam splitter** and **Mach-Zehdner interferometer**

Brief recap

→ **Def.**: Qubit ⟺ 2-state $|\psi\rangle \in \mathcal{H}$, can be rep.^{d} by vector:

$$|\psi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \equiv \alpha |0\rangle + \beta |1\rangle$$

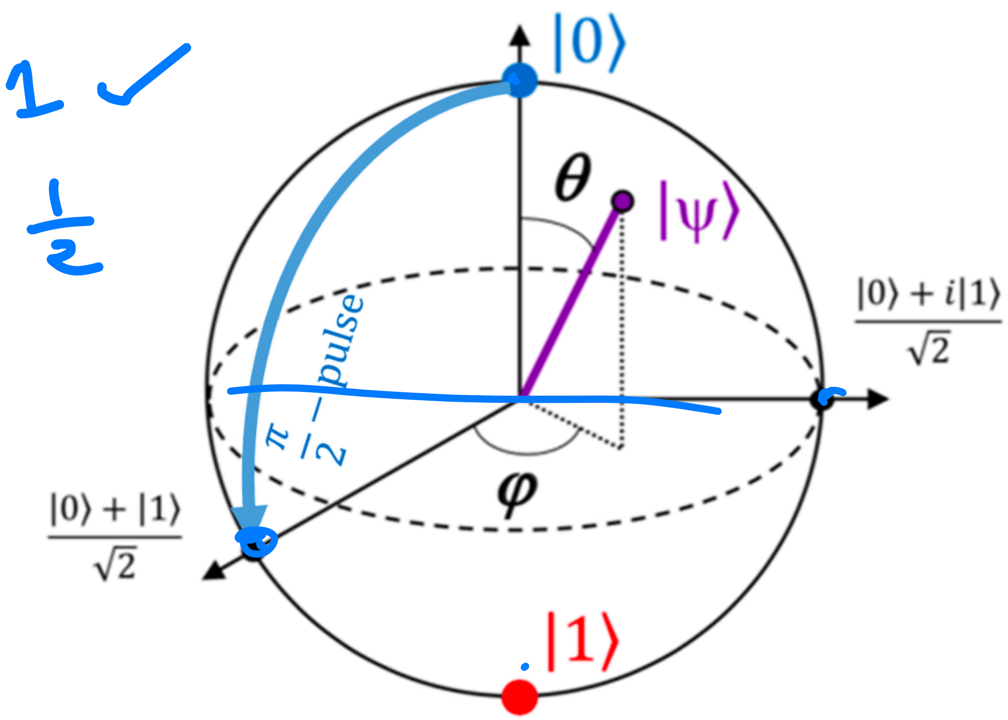
: $|\alpha|^2 + |\beta|^2 \stackrel{!}{=} 1$

$\therefore |\langle\psi|\psi\rangle|^2 \stackrel{!}{=} 1$ (Total prob^y)

→ **E.g.** $|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$

Amps. ⟺ $\begin{cases} |\langle\psi|\psi\rangle|^2 = 1 \checkmark \\ |\langle 0,1|\psi\rangle|^2 = \frac{1}{2} \end{cases}$

→ **Bloch Sphere**:



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\varphi}\sin\frac{\theta}{2}|1\rangle$$

$|\psi\rangle = \cos\theta/2\,|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$

s.t. $\theta \in [0, \pi]$ & $\phi \in [0, 2\pi)$

Proof: $|\psi\rangle = r_1 e^{i\theta_1}|0\rangle + r_2 e^{i\theta_2}|1\rangle$

$= r_1 e^{i(\theta_1+\varphi)}|0\rangle + r_2 e^{i(\theta_2+\varphi)}|1\rangle$

$|\langle\psi|\psi\rangle|^2 \stackrel{!}{=} 1 = r_1^2 + r_2^2 \Rightarrow \begin{cases} r_1 = \cos\theta/2 \\ r_2 = \sin\theta/2 \end{cases}$

$\varphi = -\theta_1$

$\Rightarrow |\psi\rangle = \cos\theta/2\,|0\rangle + \sin\theta/2\, e^{i\overbrace{(\theta_2-\theta_1)}^{\varphi}}|1\rangle$ ▨

→ Performing actions on a state ⟺ unitary operator → rep.^{d} by matrix

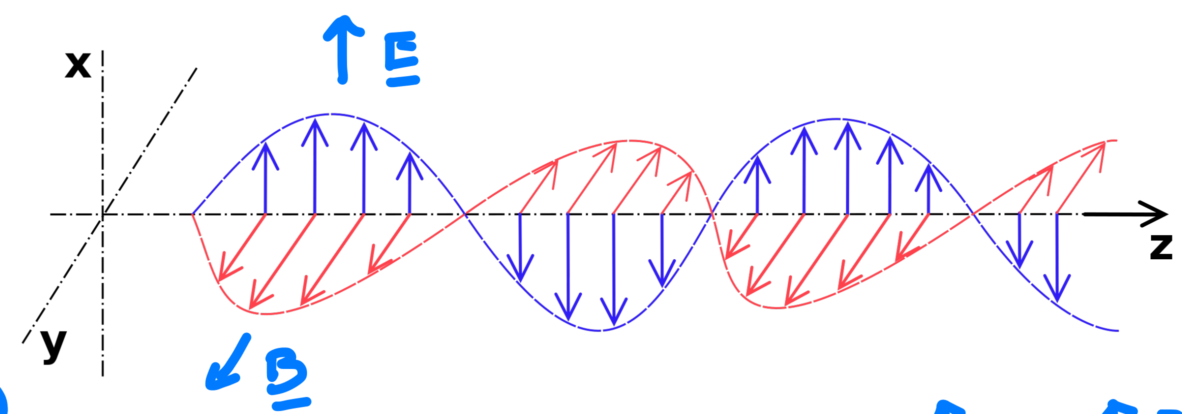
$U|\psi\rangle$ s.t. $UU^\dagger = U^\dagger U = \mathbb{1}$

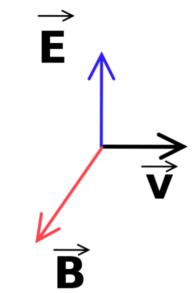
$\Rightarrow U^\dagger = U^{-1}$ (Def. of unitary)

What is light?

→ **Classical**: A propagating EM wave (Maxwell).

→ **Quantum**: A stream of discrete energy packets 'photons' - solution to 'UV catastrophe' (Planck, 1900)





Polarisⁿ: ↑ lin, uni., circ.

→ **Photo-electric effect**:

- EM description: Power $\propto \dfrac{N^{e}_{emitted}}{A_{surf}\,\Delta t}$
- Quantum description (Einstein, 1905):

Light ↔ stream of energy packets w/

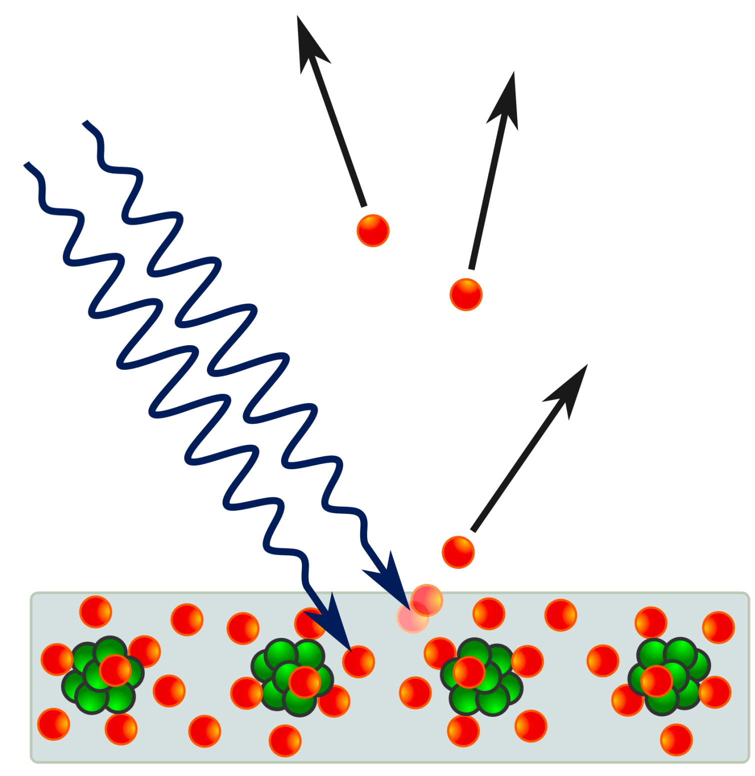
$$\left[E_\gamma = h\nu\right]$$

(h: Planck const., ν: freq)

w/ max KE of ejected e^-:

$$K_{max} = h\nu - W \quad \begin{cases} >0, & \text{emission} \\ <0, & \text{no emission!} \end{cases}$$

W: min. E to remove e^- from surface



What is light?

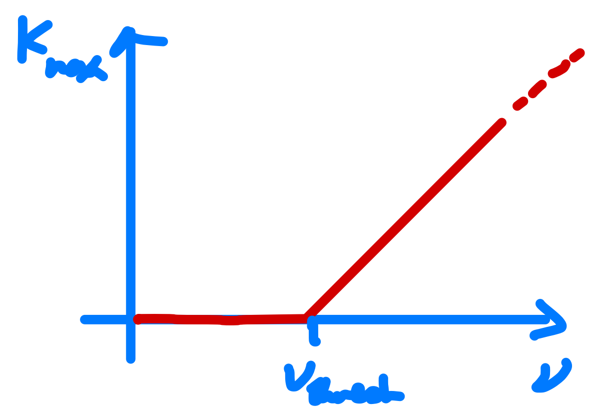
→ Competing descriptions => require experiment !

→ Experiment : Very low intensity laser or monochromatic beam of light (1st exp. used sodium lamp).

↑ single ν

Lower intensity until one is emitting single photons at a time.

Result :

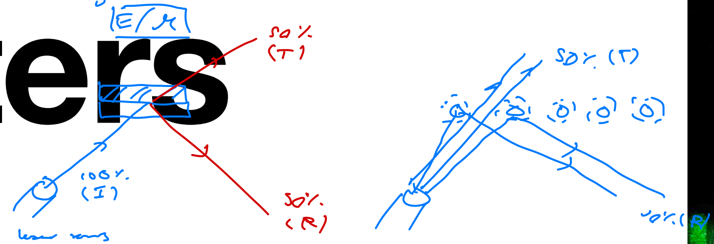


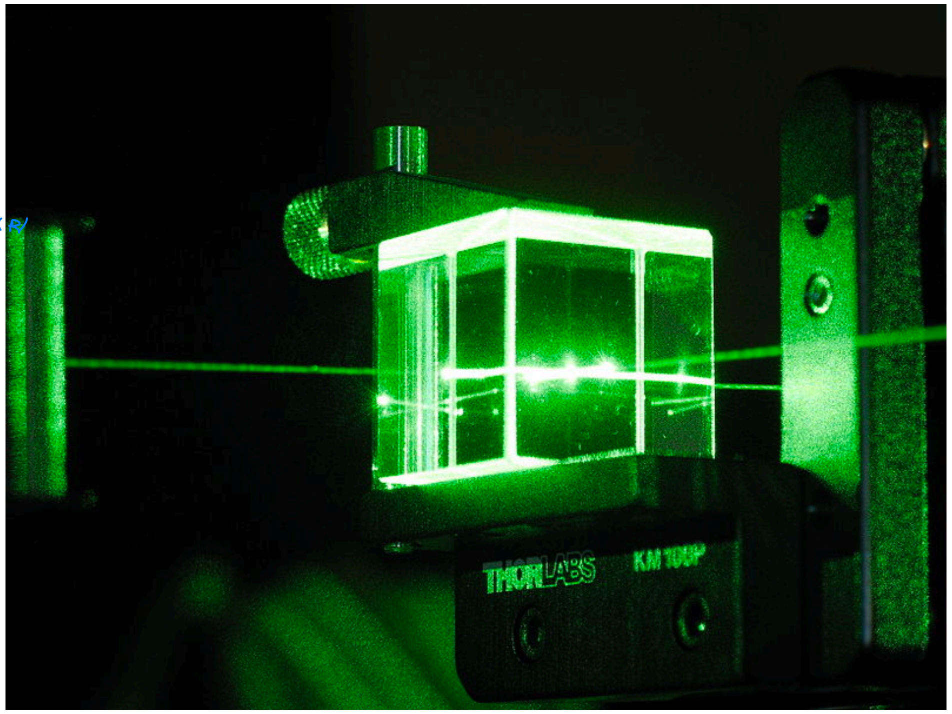
=> Quantum wins !

=> Nobel prizes : Planck, Einstein (1918, 1921)

Beam splitters

Partially ref. mirror - splits light.
→ Made glass w/ Al coating,
e.g. 50/50 B-S :

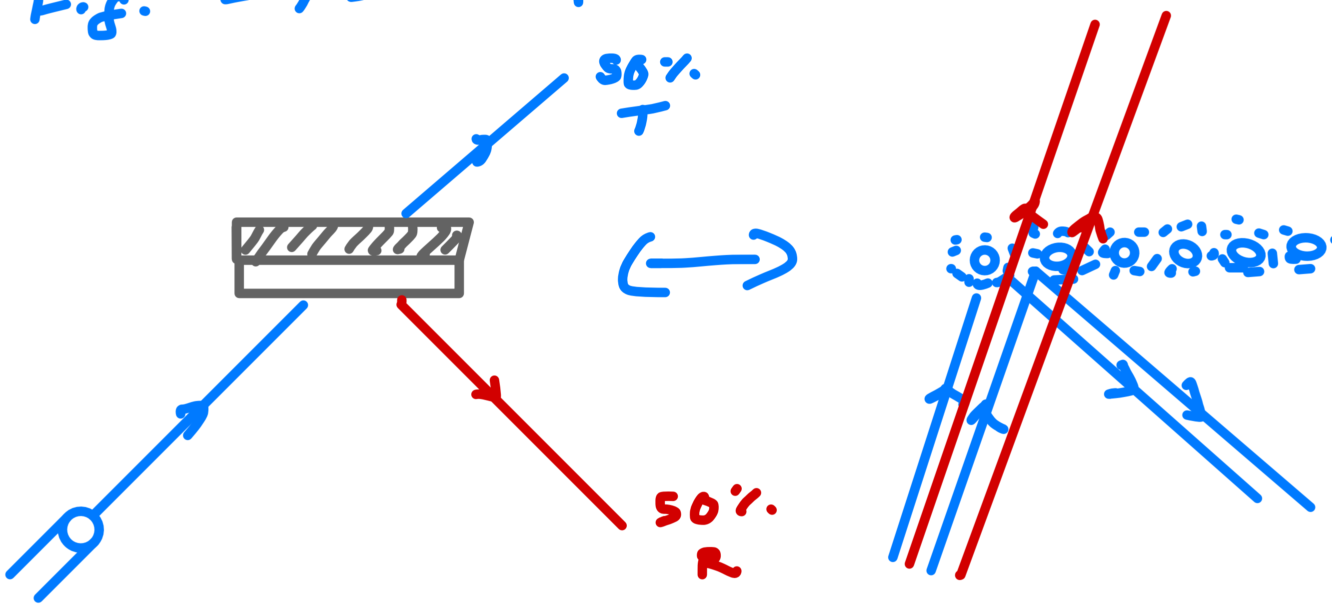




→ Partially reflective mirror that splits light.

→ Made w/ glass + vaporized Al coating.

→ E.g. 50/50 splitter :



→ Simulation time : Let us fire single photon laser at 50/50 splitter. (pg. 19 QCQC)

Beam splitters

→ once measured SP collapses either T, R respectively.
→ Q: B-S & measurement?

→ What is happening?

(A) Photon split in 1/2
(B) Photon either R or T
(C) Photon both R & T

Answer:

× both detectors would trigger
(B) & (C) indistinguishable by experiment
∵ both descriptions are same ⇒ Need more!

→ What do we mean in (C)?

Let $\{|0\rangle, |1\rangle\} \leftrightarrow \{T, R\}$ ⇒ initially photon in $|0\rangle$ state. Beam-splitter ⇔ unitary operator BS s.t.

$$BS\,|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\Rightarrow BS = \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix}$$

→ Check unitarity: $\frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}-1 & 1\\ 1 & 1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$ ✓

→ Probability of T/R: $|\langle 1|BS|0\rangle|^2 = 1/2$, $|\langle 0|BS|0\rangle|^2 = 1/2$ } as classical particle...

Mach-Zehdner Interferometer

→ B-S experiment not sufficient to dist. (B) from (C)

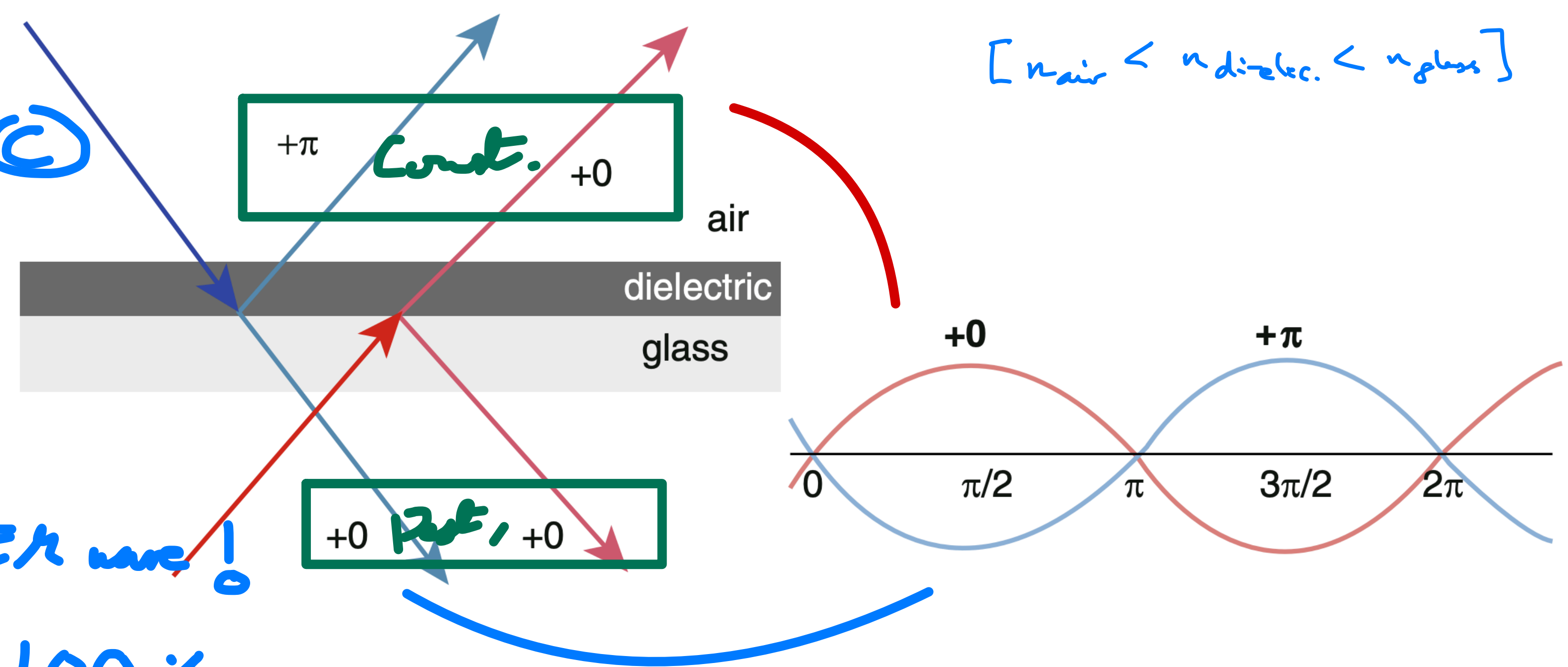
→ Add another B-S !

what happens?

1) Light now behaves like EM wave!

2) Only 1 detector gets 100%

3) Both EM & photon Const./Dest. interfere !



[$n_{air} < n_{dielec.} < n_{glass}$]

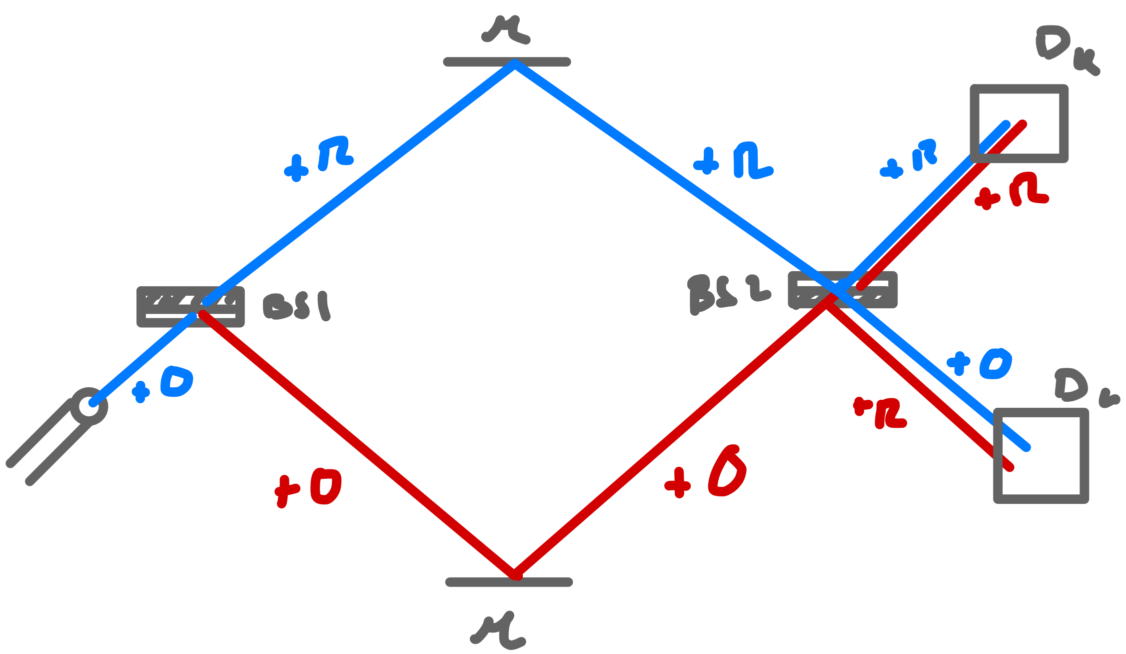
→ Why interference ?

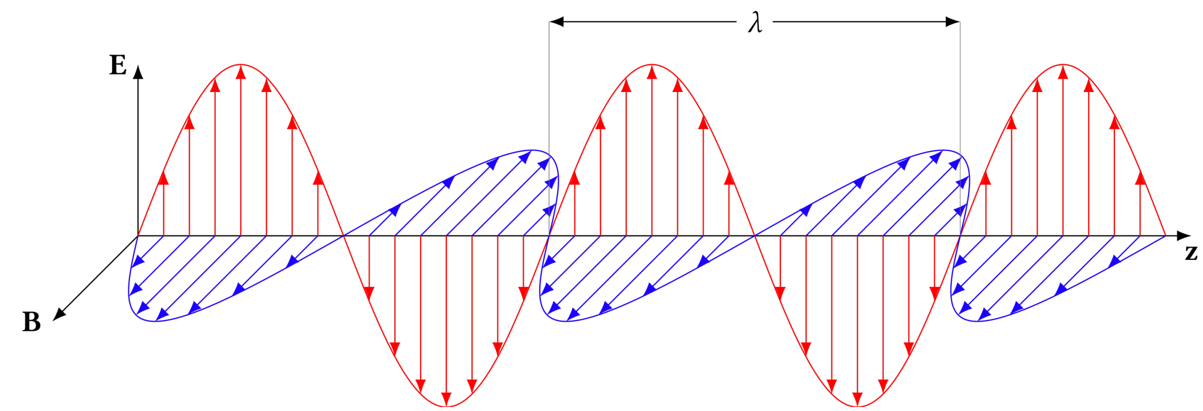
- Metal Oxide coating (Al_2O_3, Ti_2O_3, ...) slows light ⇒ phase shift & in simulation shift = π.

- Leads to const./dest. interference.

Mach-Zehdner Interferometer

→ Classical EM wave:





⇒ 100% D_1 & 0% D_2 ✓

Bonus Q: what happens if you flip BS2?

→ Photon description:

- Define states to be upper or lower ⇒ seen by D_U / D_L: $|U\rangle \equiv |1\rangle$ ⇒ initial source is $|L\rangle = |0\rangle$

  $|L\rangle \equiv |0\rangle$

Mach-Zehdner Interferometer

→ In detail:

1) 1st B-S: Operator $BS_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} e^{i\pi} & 1 \\ 1 & 1 \end{pmatrix}$

acts on $|0\rangle$,

$$|\psi_1\rangle = BS_1|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

2) Mirrors: Operator $M = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ since it crosses the paths & $|\psi_2\rangle = M|\psi_1\rangle = |\psi_1\rangle$ (check!)

3) 2nd B-S: Operator $BS_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & e^{i\pi} \end{pmatrix}$

∵ lower beam gets shifted,

$$|\psi_{fin}\rangle = BS_2 \cdot \cancel{M} \cdot BS_1|0\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$ ✓

$$= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

⇒ 100% in D_u ✓

→ { Quantum desc. works for both expts ✓
Classical works in one way for one & another way for other ✗

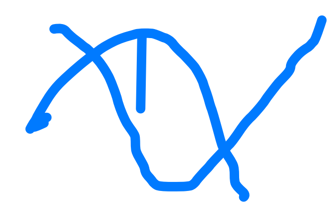
Tuning Superposition

→ **Bonus**: This apparatus has cool property.

→ Add a phase shifter in lower path!

Q: What happens? A: You can tune superpos"

→ How does this work? $PS_L = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$



$$|\psi_{fin}\rangle = BS_2 \cdot M \cdot PS_L \, BS_1 \, |0\rangle$$

$$= \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & e^{i\theta} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \frac{1}{2}(1+e^{i\theta}) \\ 0 & \frac{1}{2}(1-e^{i\theta}) \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1+e^{i\theta} \\ 1-e^{i\theta} \end{pmatrix}$$

$$= \begin{cases} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & \theta = 0 \ ✓ \\ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & \theta = \pi \ ✓ \\ \frac{1}{2}\begin{pmatrix} 1+i \\ 1-i \end{pmatrix}, & \theta = \pi/2 \Rightarrow 50/50 \end{cases}$$

check! ←

A: $\left| \frac{1}{2} \begin{pmatrix} 1+i \\ 1-i \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right|^2 = \frac{1}{4} \cdot 2 = \frac{1}{2}$

$\Rightarrow$ 50/50

Summary

Beam splitters can put photons into **quantum superpositions**

Single beam splitter showed photon indistinguishable from **classical corpuscle**

Double beam splitter showed photon indistinguishable from **classical E&M wave**

Quantum superposition accurately describes both scenarios, therefore quantum/photon description wins!