Nucleon vector form factor implementation in GENIE

Joint Meeting, Fermilab

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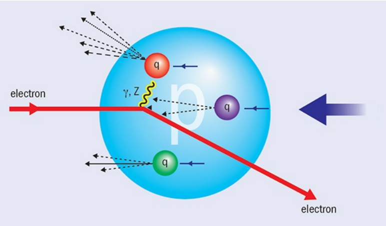
Phys. Rev. D 102, 074012 (2020)

University of Kentucky

June 21, 2023

Outline

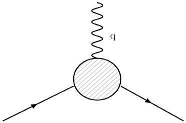
Nucleon structure and its contribution to neutrino and other precision experiments



- Form factor provides the measure of charge and magnetic-moment distributions inside the nucleons.
- Nucleon form factors are important input for neutrino experiments (DUNE and Hyper-K) as well as atomic spectroscopy.
- Discrepancy in the measurement of form factors and charge radii impacts the measurements in neutrino experiments and atomic spectroscopy.
- We present the proton and neutron vector form factors, uncertainties and correlations in a convenient parametric form that is model independent and optimized for $Q^2 \lesssim$ few GeV^2.
- The form factors are determined from a global fit to electron scattering data and precise charge radii measurements.

Nucleon Form Factors in Scattering - I

- The nucleon electromagnetic current is expressed in terms of *Dirac* (F_1) and *Pauli* (F_2) form factors,



$$\Gamma^{\mu}(q^2) = F_1(q^2)\gamma^{\mu} + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2(q^2)$$

- F_1 and F_2 can be written in terms of *Sachs* electric and magnetic form factors G_E and G_M,

$$F_1 = \frac{G_E + \tau G_M}{1+\tau}, \quad F_2 = \frac{G_E - G_M}{1+\tau}, \quad \tau = -\frac{q^2}{4M^2}$$

- The scattering cross-section of a relativistic electron off a recoiling point-like nucleus is given by the *Mott* formula.

- Structure-dependent part is expressed in terms of Sachs electric and magnetic form factors which is given by the *Rosenbluth* formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon}G_M^2\right\}, \qquad \frac{1}{\epsilon} = 1 + 2(2+\tau)\tan^2\frac{\theta}{2}$$

- **NOTE** We can also fit our curve to the isospin rotated basis components isoscalar ($F_{1,2}^S = F_{1,2}^p + F_{1,2}^n$) and isovector ($F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$) form factors. This is important for the application of nucleon form factor in neutrino experiment.

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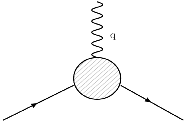
Nucleon Form Factors in Scattering - II

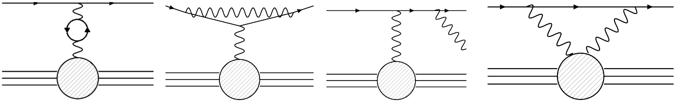
- The form factors are defined from the matrix element of one-photon exchange.



$$\Gamma^{\mu}(q^2) = F_1(q^2)\gamma^{\mu} + \frac{i}{2M}\sigma^{\mu\nu}q_{\nu}F_2(q^2)$$

- To extract them with a percent precision or better, standard QED radiative corrections and modern calculations of structure-dependent two-photon exchange are included.

$$d\sigma_{\rm expt} = d\sigma_{\rm Born}(1+\delta_{\rm RC})$$



Functional Forms for Form Factors

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon}G_M^2\right\}$$

- In the past, a few simple functional forms for G_E and G_M were used by truncating the expansion at some finite k_{max},

$$G_{pol}(q^2) = \sum_{k=0}^{k_{\max}} a_k (q^2)^k, \quad \text{polynomials [Simonetal.(1980), Rosenfelder(2000)]}$$

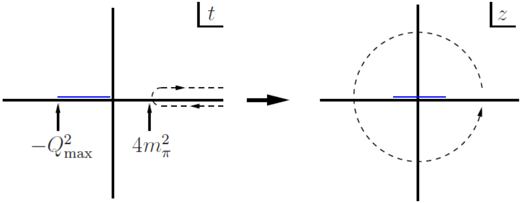
$$G_{invpol}(q^2) = \frac{1}{\sum_{k=0}^{k_{\max}} a_k (q^2)^k}, \quad \text{inverse polynomials [Arrington(2003)]}$$

$$G_{cf}(q^2) = \frac{1}{a_0 + a_1 \frac{q^2}{1+a_2\frac{q^2}{1+\cdots}}}, \quad \text{continued fractions [Sick(2003)]}$$

- In 2010, Hill & Paz showed that the above functional forms exhibit pathological behavior with increasing k_{max}

The Bounded z Expansion

- According to QCD constraint, nucleon form factors must be analytic in $t \equiv q^2 \equiv -Q^2$ outside of a time-like cut starting at $t_{\rm cut} = 4m_\pi^2$, the two-pion production threshold ($t_{\rm cut} = 9m_\pi^2$ for isoscalar combinations). [Hill & Paz (2010)]



- A conformal map gives a small expansion variable t_0 in kinematic region of scattering experiments that lies on the negative real axis. It is represented by the blue line for a set of data with maximum momentum transfer $Q^2_{\rm max}$.

$$z(q^2) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$G_E = \sum_{k=0}^{k_{\rm max}} a_k [z(q^2)]^k, \quad G_M = \sum_{k=0}^{k_{\rm max}} b_k [z(q^2)]^k$$

The Sum Rule

- Perturbative QCD requires that the form factors must fall off faster than $1/Q^3$ in the large Q^2 limit

$$Q^n G(-Q^2)\Big|_{Q^2\longrightarrow\infty} \longrightarrow 0$$

Therefore,

$$\frac{d^n G}{dz^n}\Big|_{z\longrightarrow 1} \longrightarrow 0, \quad n = 0, 1, 2, 3.$$

- In order to implement the above constraints on a form factor we can enforce the following four sum rules [Lee, Arrington, Hill (2015)]

$$\sum_{k=n}^{k_{\mathrm{max}}} k(k-1)\cdots(k-n+1)a_k = 0, \quad n = 0, 1, 2, 3.$$

- We choose $k_{\mathrm{max}} = 8$ and estimate fitting uncertainty as a difference to $k_{\mathrm{max}} \rightarrow k_{\mathrm{max}} + 1$. Four parameters (e.g., $a_1,\ a_2,\ a_3,\ a_4$) are determined by fitting to data.

νN CCQE Cross Section

- Neutrino-nucleon charged-current quasielastic cross section is expressed in terms of form factors as [Llewellyn-Smith (1972)]

$$\frac{d\sigma}{dQ^2}(Q^2, E_\nu) = \frac{G_F^2|V_{ud}|^2}{8\pi}\frac{M^2}{E_\nu^2}\left[A(q^2)\frac{m_l^2 - q^2}{M^2} - B(q^2)\frac{s-u}{M^2} + C(q^2)\left(\frac{s-u}{M^2}\right)^2\right]$$

- The functions A, B and C depend on the nucleon isovector form factors $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$, axial form factor F_A and pseudoscalar form factor F_P

$$A(q^2) = 2\tau(F_1^V + F_2^V)^2 - (1+\tau)\left\{(F_1^V)^2 + \tau(F_2^V)^2 - (F_A)^2\right\}$$
$$-r_l^2\left\{(F_1^V + F_2^V)^2 + (F_A + 2F_P)^2 - 4(1+\tau)F_P^2\right\}$$

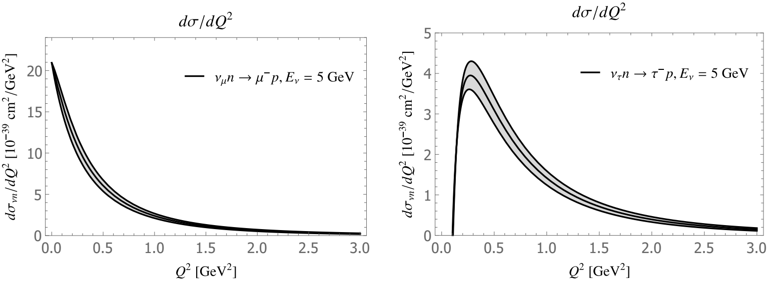
$$B(q^2) = 4\tau F_A(F_1^V + F_2^V)$$

$$C(q^2) = \frac{1}{4}\left\{(F_1^V)^2 + \tau(F_2^V)^2 + (F_A)^2\right\}$$

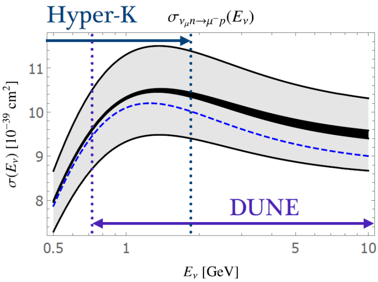
- Hence the nucleon electric and magnetic form factors are important input for the neutrino cross section.

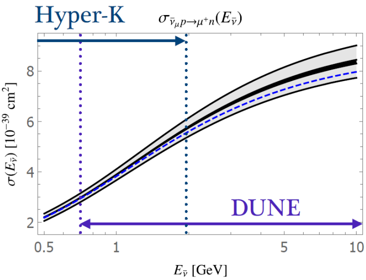
Relevant kinematics for DUNE and HYPER-K

- Electron and muon neutrino cross sections are sensitive to $Q^2 \lesssim 1\ \mathrm{GeV}^2$ while tau neutrino requires larger Q^2.
- Two isospin-decomposed fits with data below $Q^2 < 1\ \mathrm{GeV}^2$ and $Q^2 < 3\ \mathrm{GeV}^2$ are performed.



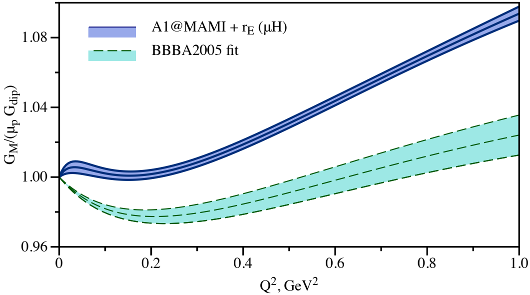
νN CCQE Cross Section Results





- **Dark band** : uncertainty of vector form factor.
- **Light band** : uncertainty of axial form factor.
- **Blue line** : BBBA2005 fit (currently-used fit) of electromagnetic form factors.
- CCQE cross section differs by 3–5% compared to currently-used form factor models (BBBA2005) when the vector form factors are constrained by recent high-statistics electron–proton scattering data from A1@MAMI.

Proton Magnetic Form Factors including A1@MAMI data



- G_M^p from A1@MAMI is significantly different to previous result.

Neutrino Event Generator, GENIE

- Most of the neutrino event generators generate an event in the following two steps -
  - Firstly, a nucleon level neutrino interaction is simulated using various physics models (e.g., Llewellyn Smith calculation is used for quasi-elastic interaction).
  - Secondly, the interactions of the generated particles in the nucleus are simulated.
- The charged-current quasi-elastic scattering off a free nucleon used in the GENIE simulation is described by Llewellyn Smith calculation with nucleon vector and axial form factors.
- Several vector form factor models are available in GENIE (e.g. Dipole, BBBA2005, etc.)
- The z-expansion axial form factor from Phys. Rev. D 93, 113015 (2016) [A. S. Meyer, et al.] has already been implemented in GENIE.

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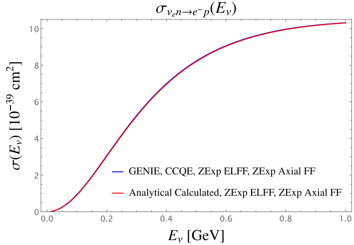
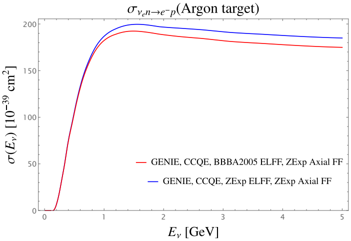
GENIE - New Implementation Status

- Our z-expansion parametrization of nucleon vector form factors has been successfully implemented in GENIE.





- Our implementation has been validated by comparing the cross-section from GENIE vs Analytical Calculation

- A nuclear level comparison has been made by calculating the cross-section for the Argon target.(3–5% discrepancy between BBBA2005 and z-expansion)

Conclusion and Future Steps

- Including data of A1@MAMI Collaboration, CCQE cross sections shift by 3-5 % triggered by proton magnetic form factor.
- The implementation of our new vector form factors in the neutrino event generator, GENIE, has been done successfully.
- A discrepancy between GENIE's representation and the conventional representation of the nucleon pseudo-scalar form factor has been observed. (GENIE uses a different form factor for neutron versus proton initial state, complicating validations).
- This discrepancy should be documented and the GENIE community should be made aware of it.
- If the community agrees, I will implement the standard convention for the pseudo-scalar form factor in GENIE.
- Next, I will guide the validation and incorporation of the new vector form factor into the in-development version, GENIE 3.6.0.

Thank You