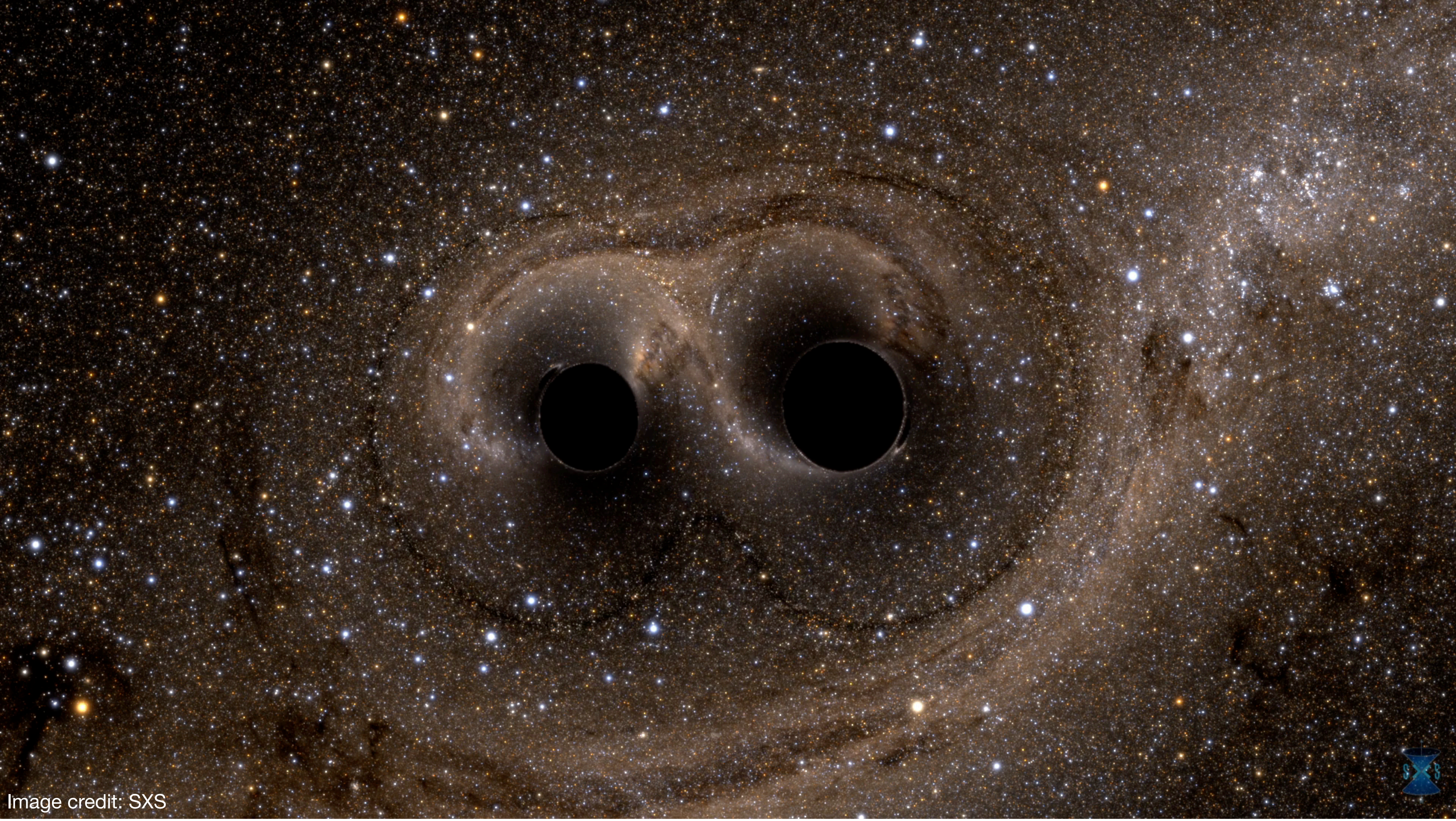


Gravitational Waves Observations by LIGO and Virgo

Vivien Raymond, Cardiff University

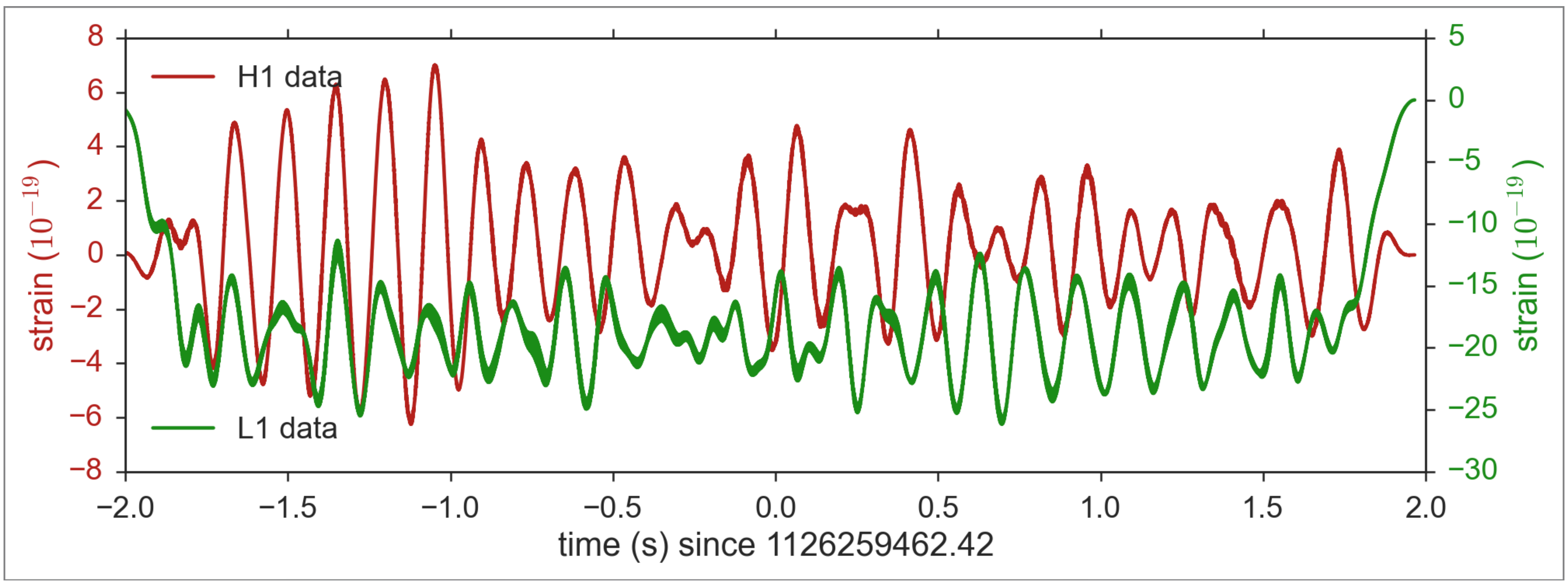
LIGO VIRGO

CARDIFF UNIVERSITY
PRIFYSGOL CAERDYDD

Gravity Exploration Institute
Sefydliad Archwilio Disgyrchiant

Starting from **strain data**

- GW150914: September 14, 2015 at 09:50:45 UTC:

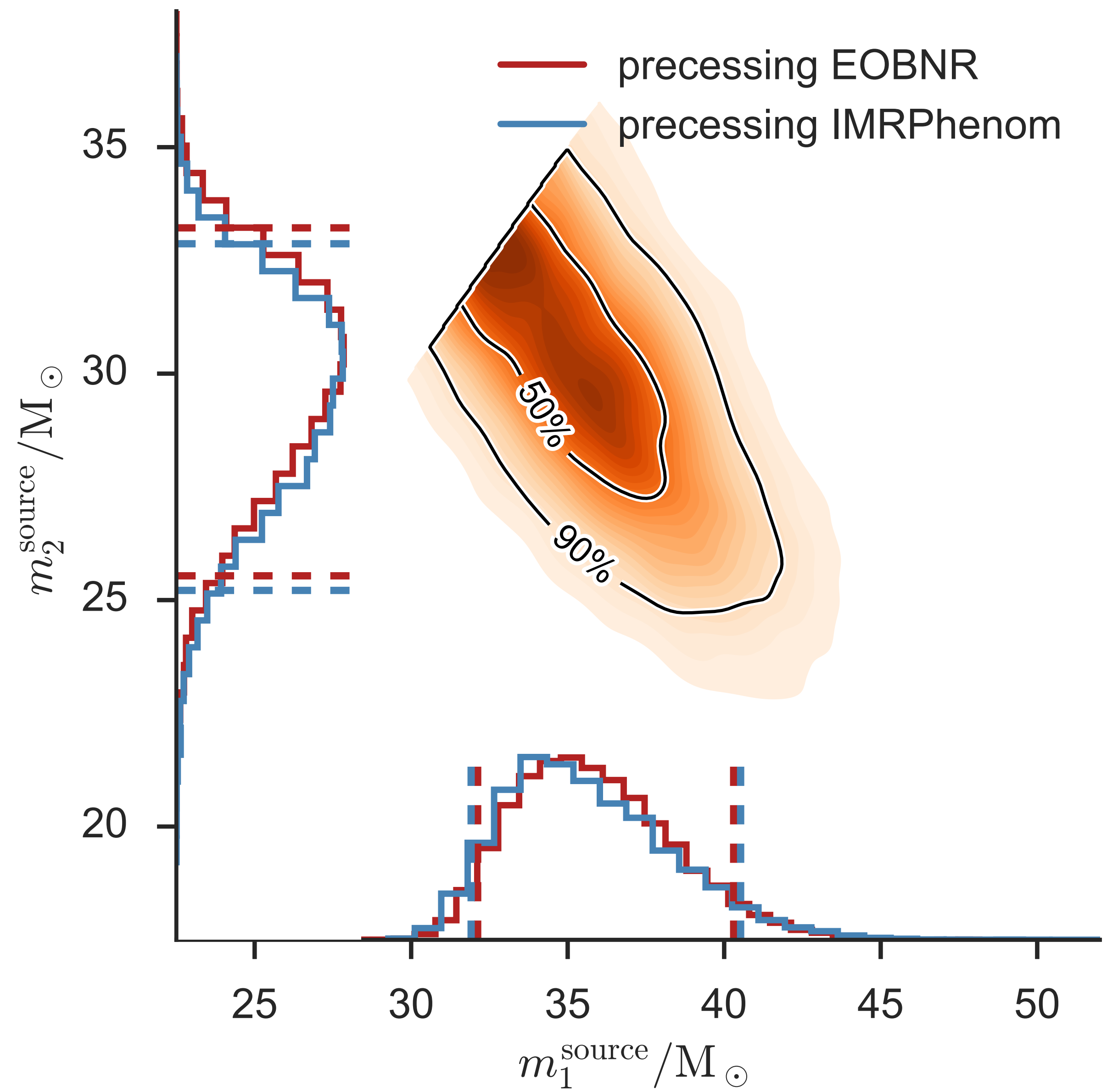


To Probability Density Function

- GW150914 masses estimates:

$$m_1 = 35.4^{+5.0\pm0.1}_{-3.4\pm0.3}\,\mathrm{M}_\odot$$

$$m_2 = 28.9^{+3.3\pm0.3}_{-4.3\pm0.3}\,\mathrm{M}_\odot$$



[LIGO-Virgo Collaboration, 2016]

Parameter Estimation

- We want the **posterior** probability of parameters $\vec{\lambda}$, given the data $\mathbf{d}$. With **Bayes'** theorem:

$$p(\vec{\lambda}\,|\,\mathbf{d}, M) = \frac{p(\vec{\lambda}\,|\,M)\,p(\mathbf{d}\,|\,\vec{\lambda}, M)}{p(\mathbf{d}\,|\,M)}$$

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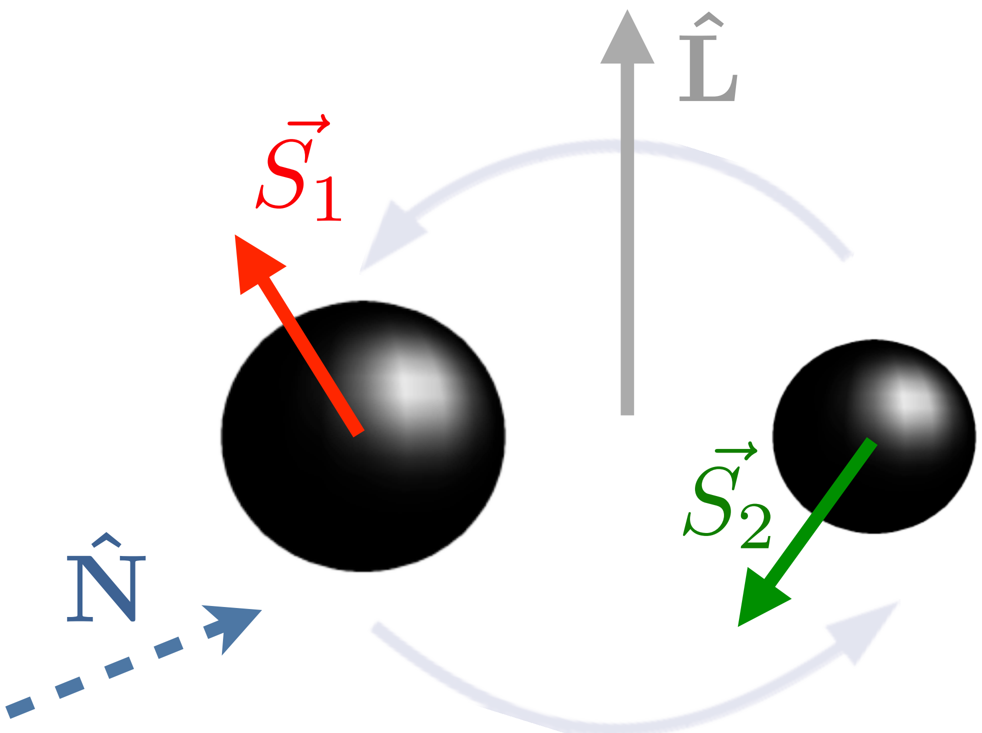
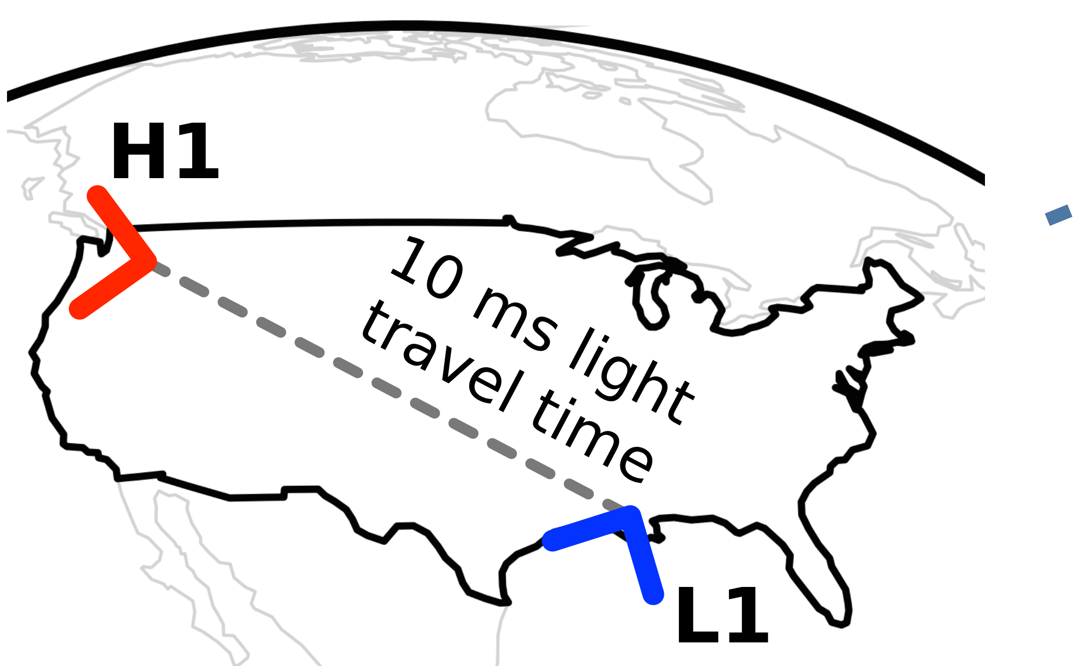
Example: Compact Binary Coalescence

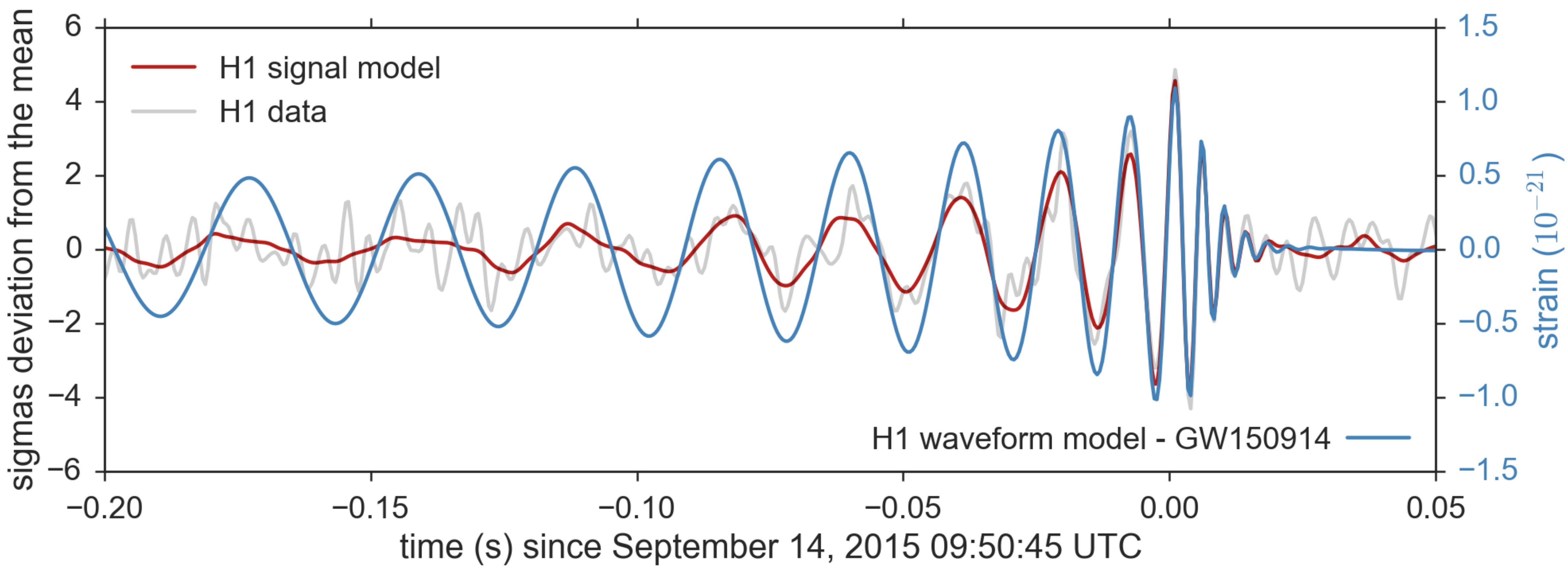
- **Intrinsic** parameters: primary and secondary **masses** and **spins** (and eccentricity)





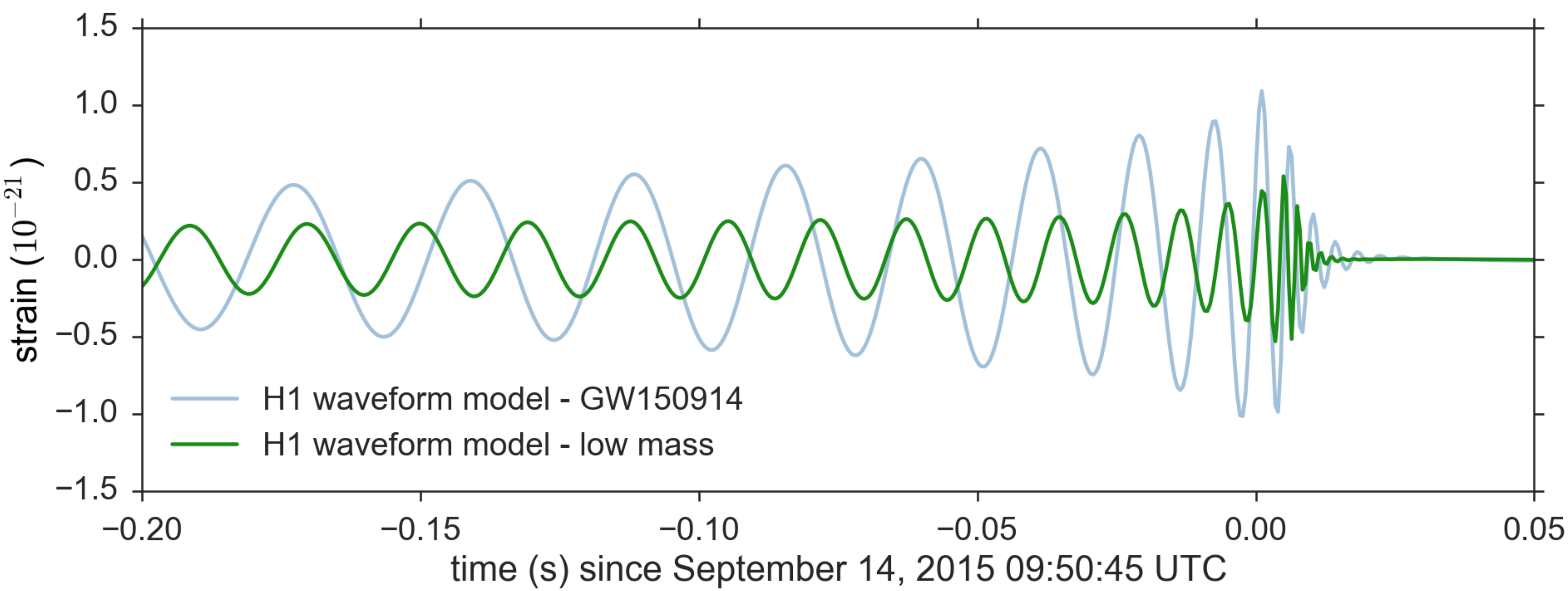
- **Extrinsic**: time, **sky-position**, distance, **orientation**, reference phase

Gravitational waveform models



Parametrisation example: masses

- Chirp mass: $\mathcal{M} = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \simeq \frac{c^3}{G}\left[\frac{5}{96}\pi^{-8/3} f^{-11/3} \dot{f}\right]^{3/5}$
- Mass ratio: $q = \frac{m_1}{m_2}$



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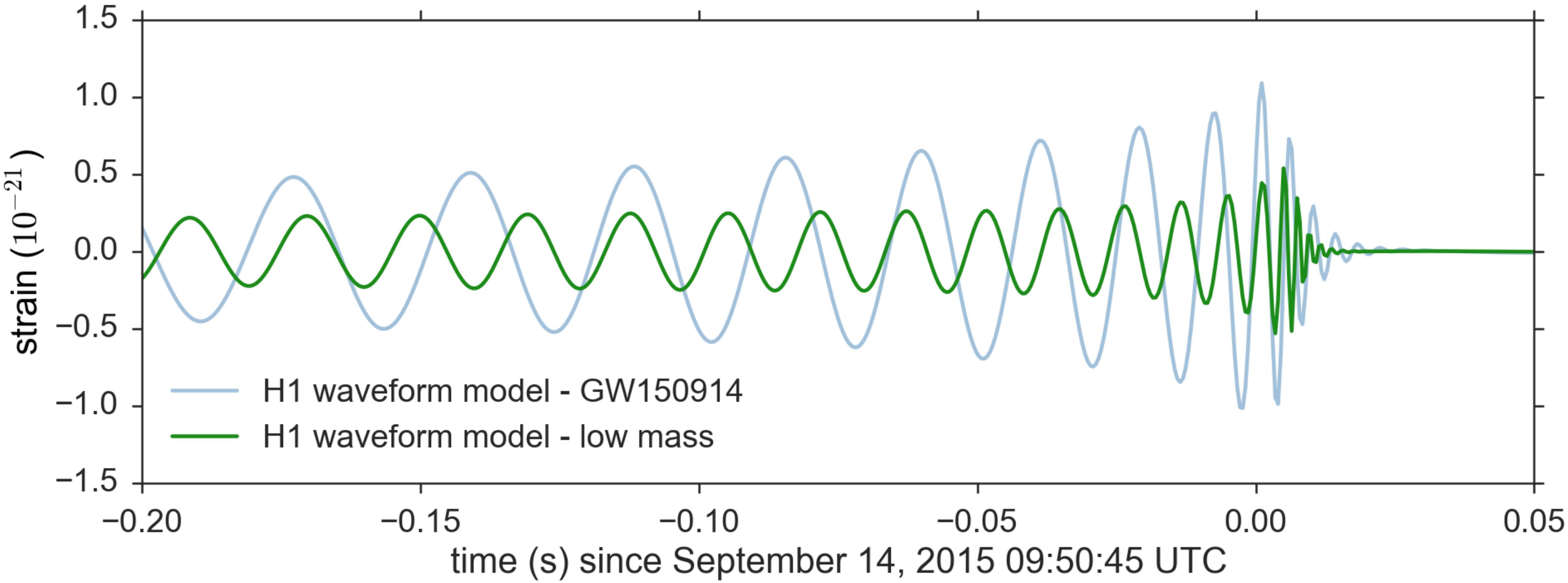
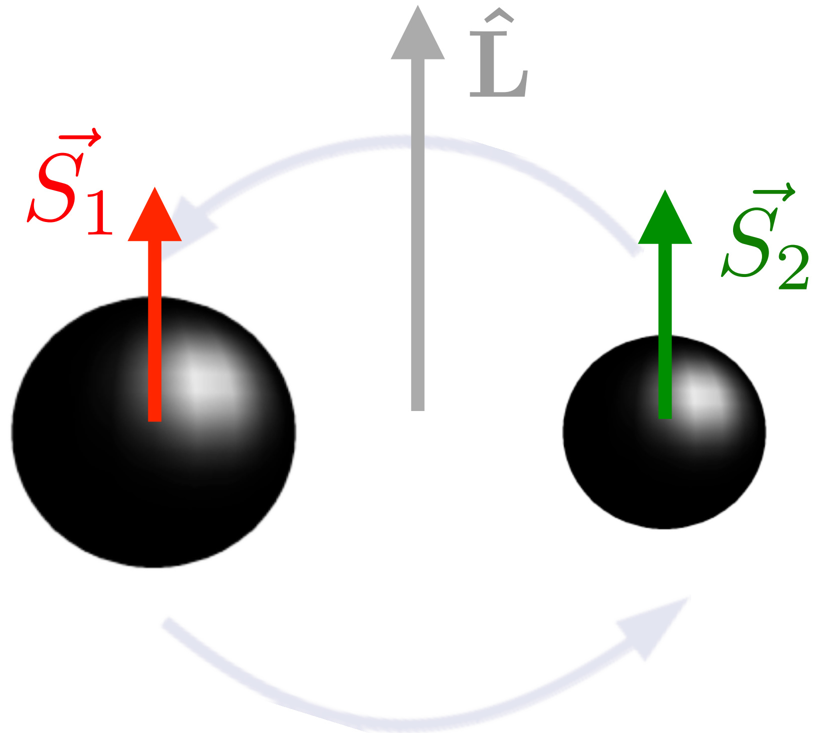
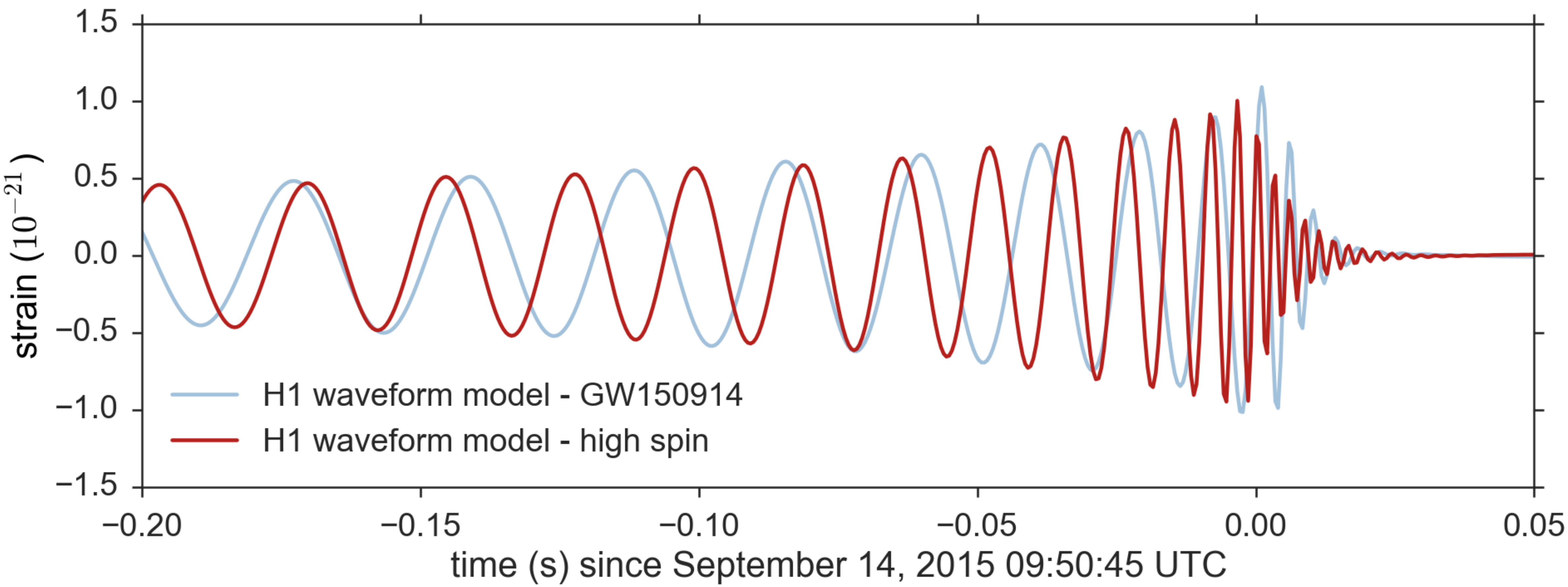
- Mass ratio: $q = \frac{m_1}{m_2}$
- Total mass: $M_{tot} = m_1 + m_2$
- Symmetric mass ratio, chirp times, … And **prior** !



Effects of spins

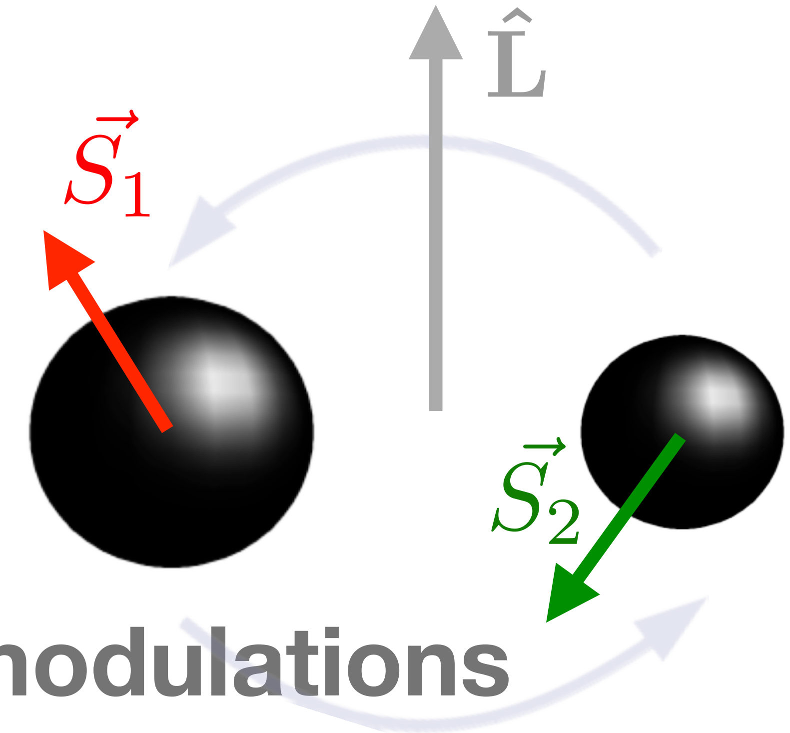
- 2 spin vectors
  - **Magnitude: orbital hang-up**

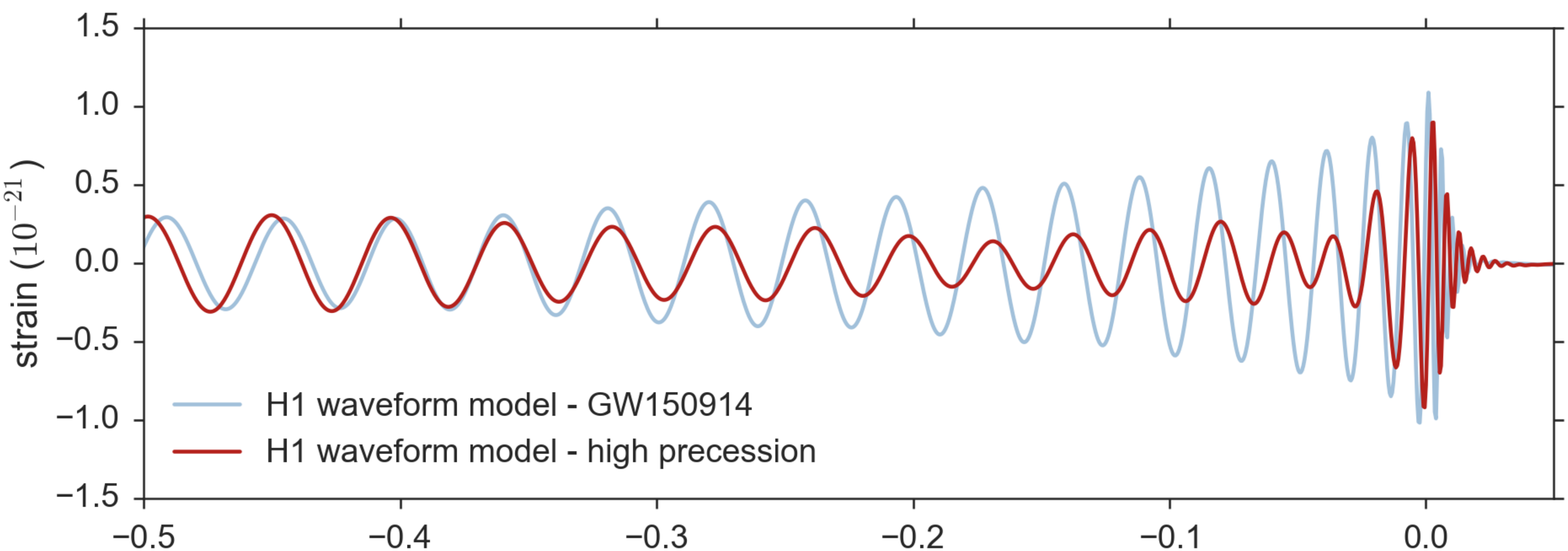




Effects of spins

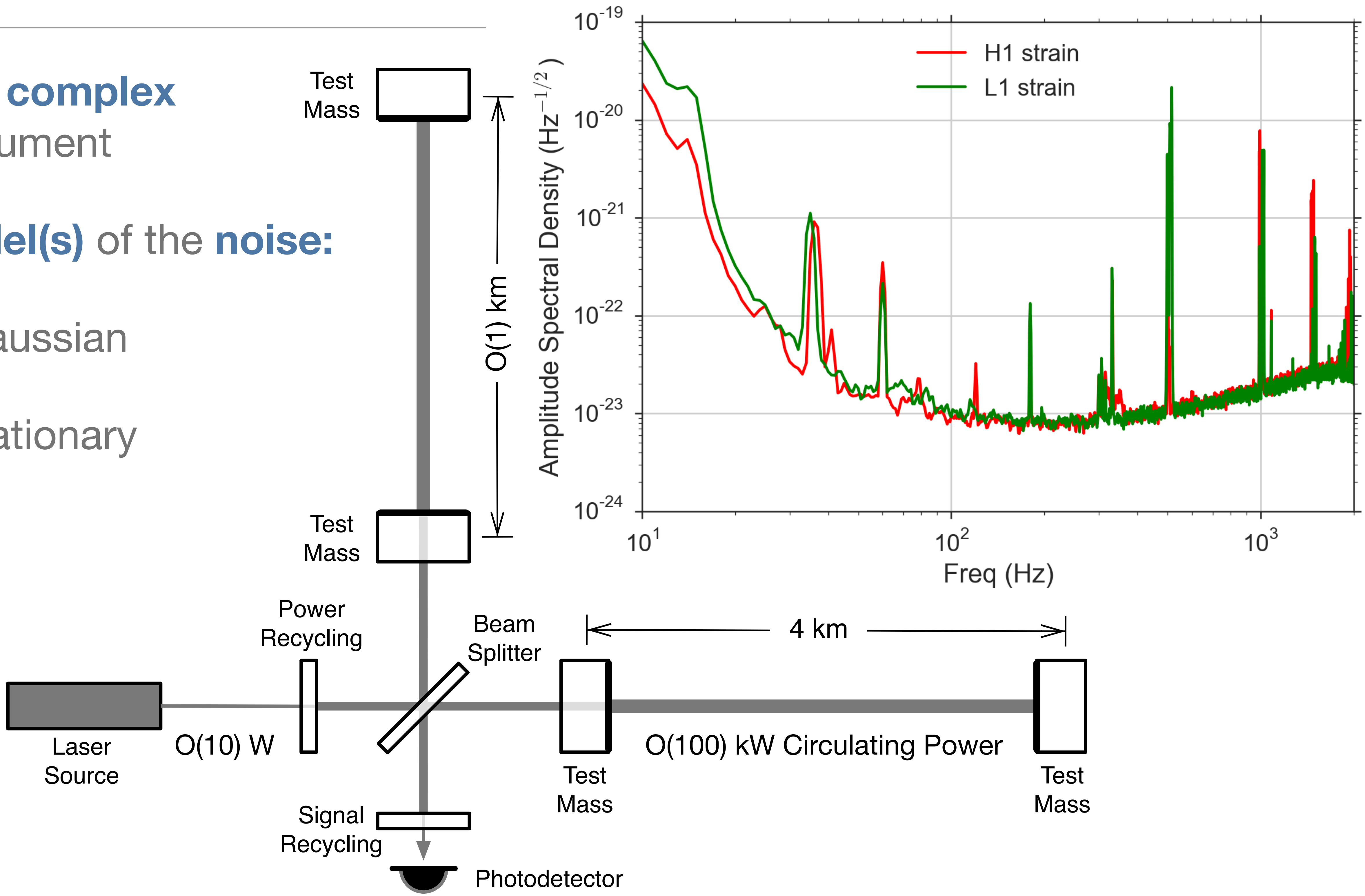
- 2 spin vectors
  - **Mis-alignment: precession and modulations**
  - Harmonic decomposition





Observatories

- Very **complex** instrument
- **Model(s)** of the **noise:**
    - Gaussian
    - stationary



[LIGO-Virgo Collaboration, 2016]

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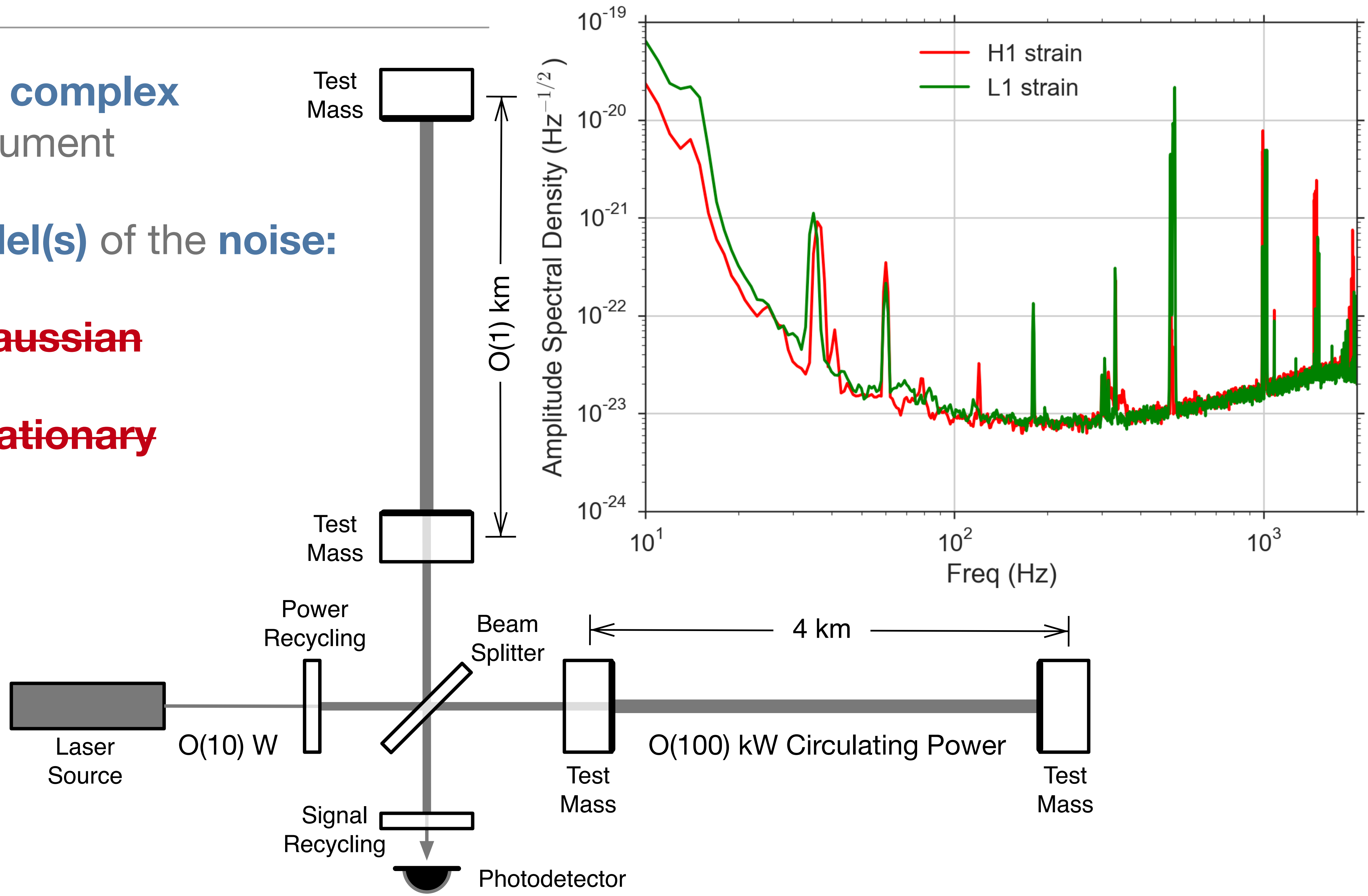
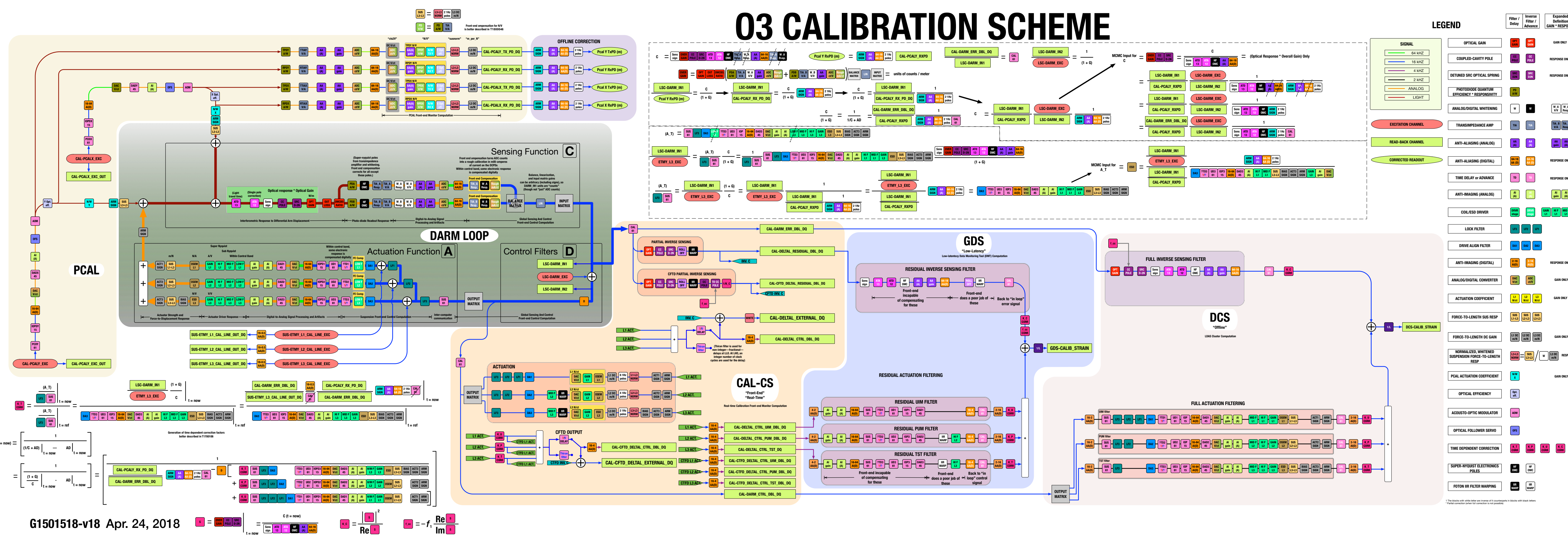


[LIGO-Virgo Collaboration, 2016]

Calibration

- From **voltage** to **photon count** to **displacement** to **strain**



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- ~~Build~~ **Learn** a **likelihood** function
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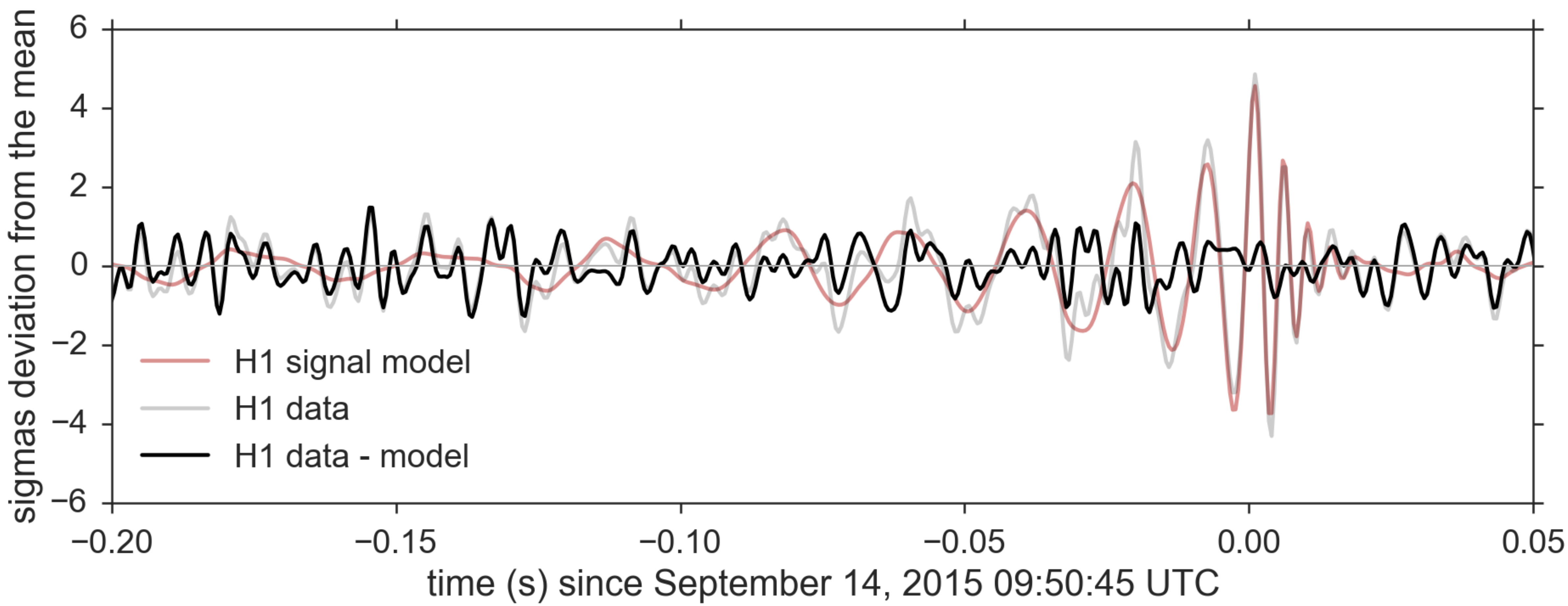
The Likelihood



- How close is the **remainder** to the **mean**?
  - Assumptions: **gaussianity** and **stationarity**

Likelihood: noise model

Noise correlation matrix

- Gaussian noise $$p(n_1, n_2, \ldots, n_{N_f}) \approx e^{-\frac{1}{2} n_i C^{-1}_{ij} n_j}$$
- Stationary noise $$C_{ij} = \frac{1}{2} S_n(f_i)\, \delta_{ij}$$

n^{th} detector PSD

- **Note**: many things do break those assumptions, e.g. finite-length segments, windowing…

Likelihood

- Probability of obtaining data $\mathbf{d}$ assuming signal $h(\vec{\lambda})$ and that the noise is **stationary** and **gaussian**:

$$p(\mathbf{d}\,|\,\vec{\lambda}, M) \approx \exp\left(-\frac{1}{2}(\mathbf{d} - R[h(\vec{\lambda})]\,|\,\mathbf{d} - R[h(\vec{\lambda})])\right)$$

- With the discrete bins now continuous:

$$(a\,|\,b) = 2\int_0^{+\infty} df\, \frac{a^*(f)\,b(f) + a(f)\,b^*(f)}{S_n(f)}$$

Likelihood

- In practice, we use:

$$p(\mathbf{d}\,|\,\vec{\lambda}, M) \approx \exp\left(-2\sum_{n=1}^{N_{det}}\int_{f_{low}}^{f_{high}} df\,\frac{\left|\mathbf{d}_n(f) - R_n\left[h(f;\vec{\lambda}), f;\vec{\lambda}\right] - g_n(f;\vec{\lambda})\right|^2}{S_n(f;\vec{\lambda})}\right)$$

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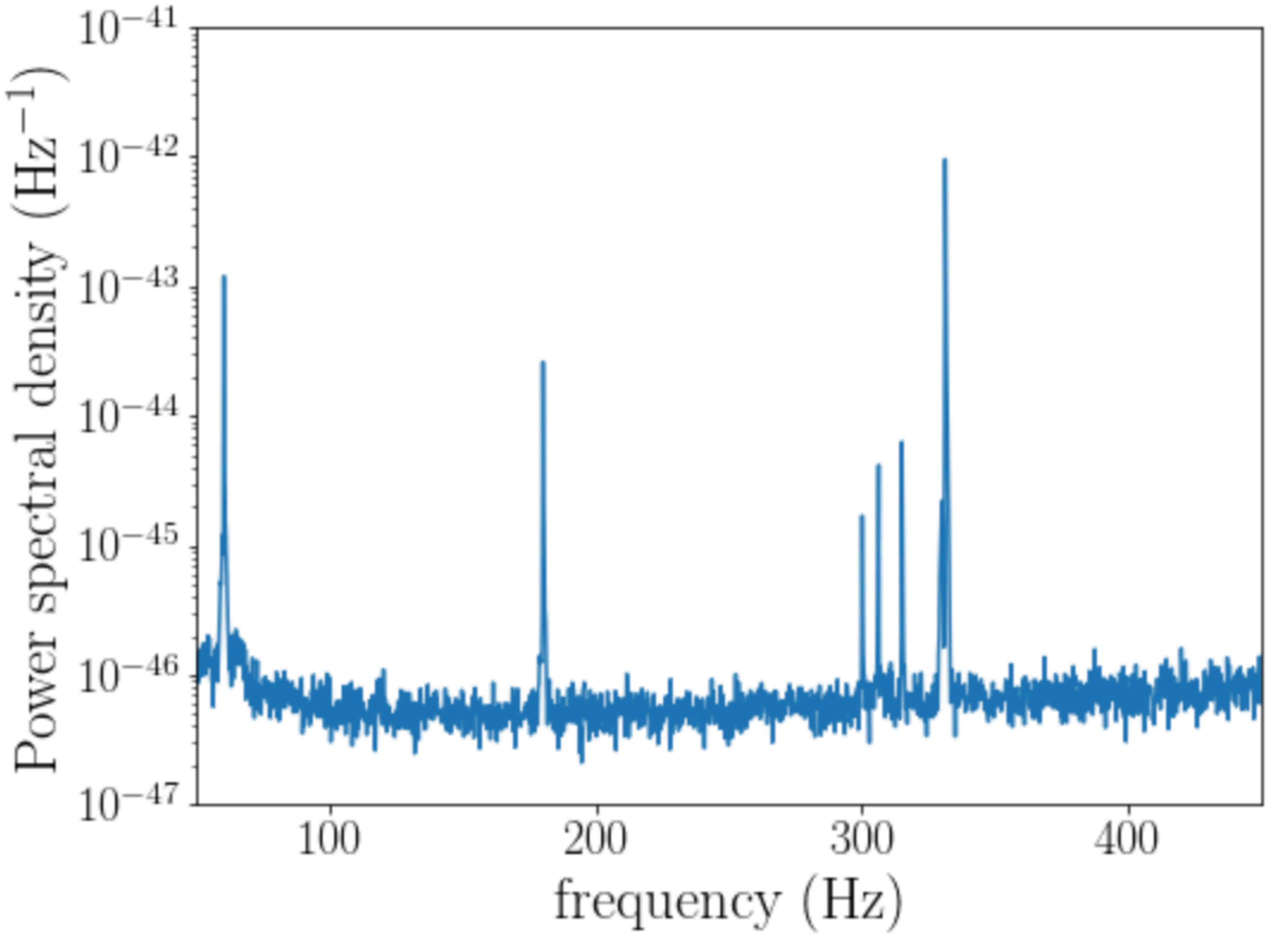
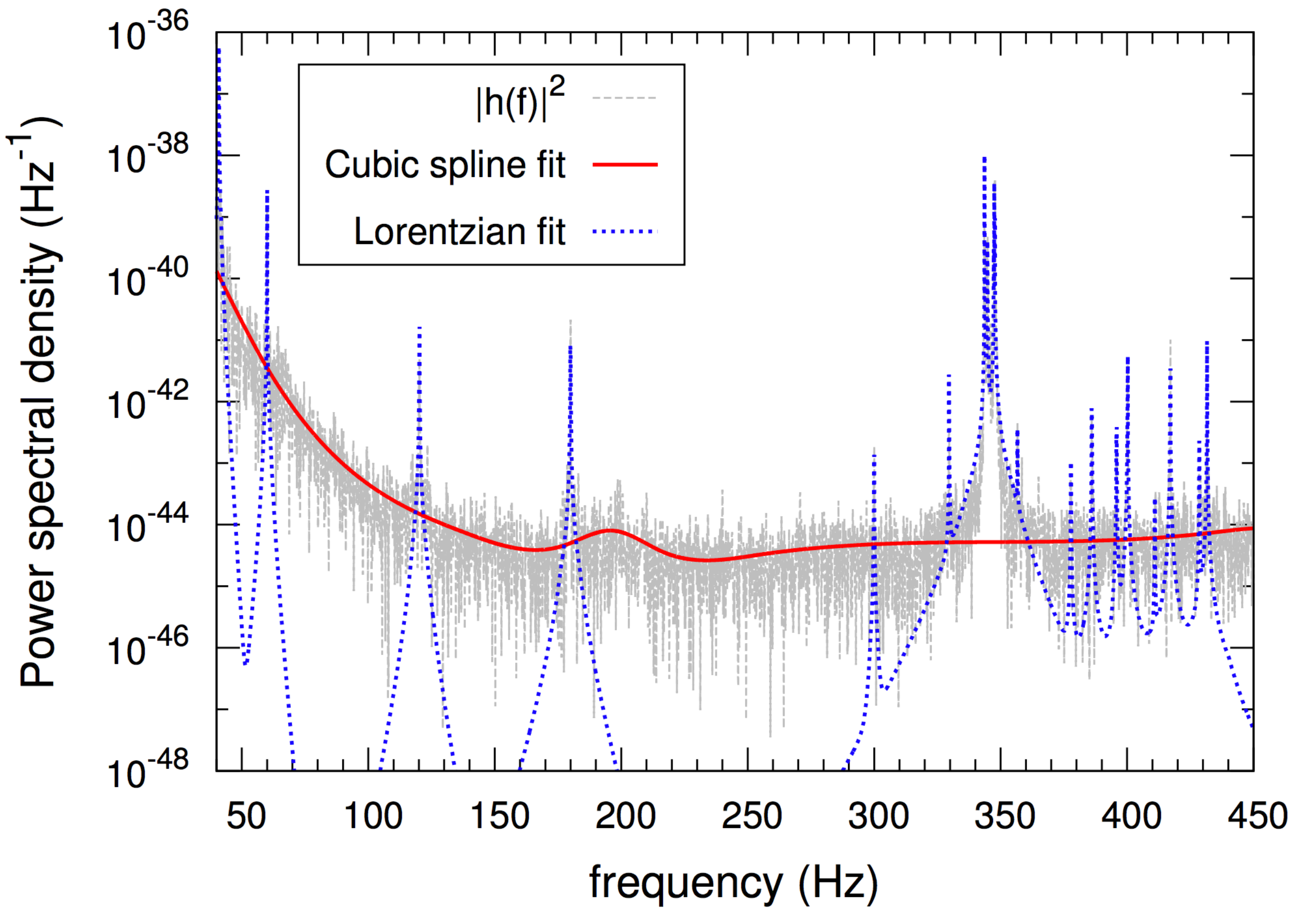
Noise model: PSD

Off-source (average)



On-source



[Littenberg and Cornish, 2014]

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Noise model: Gaussianity

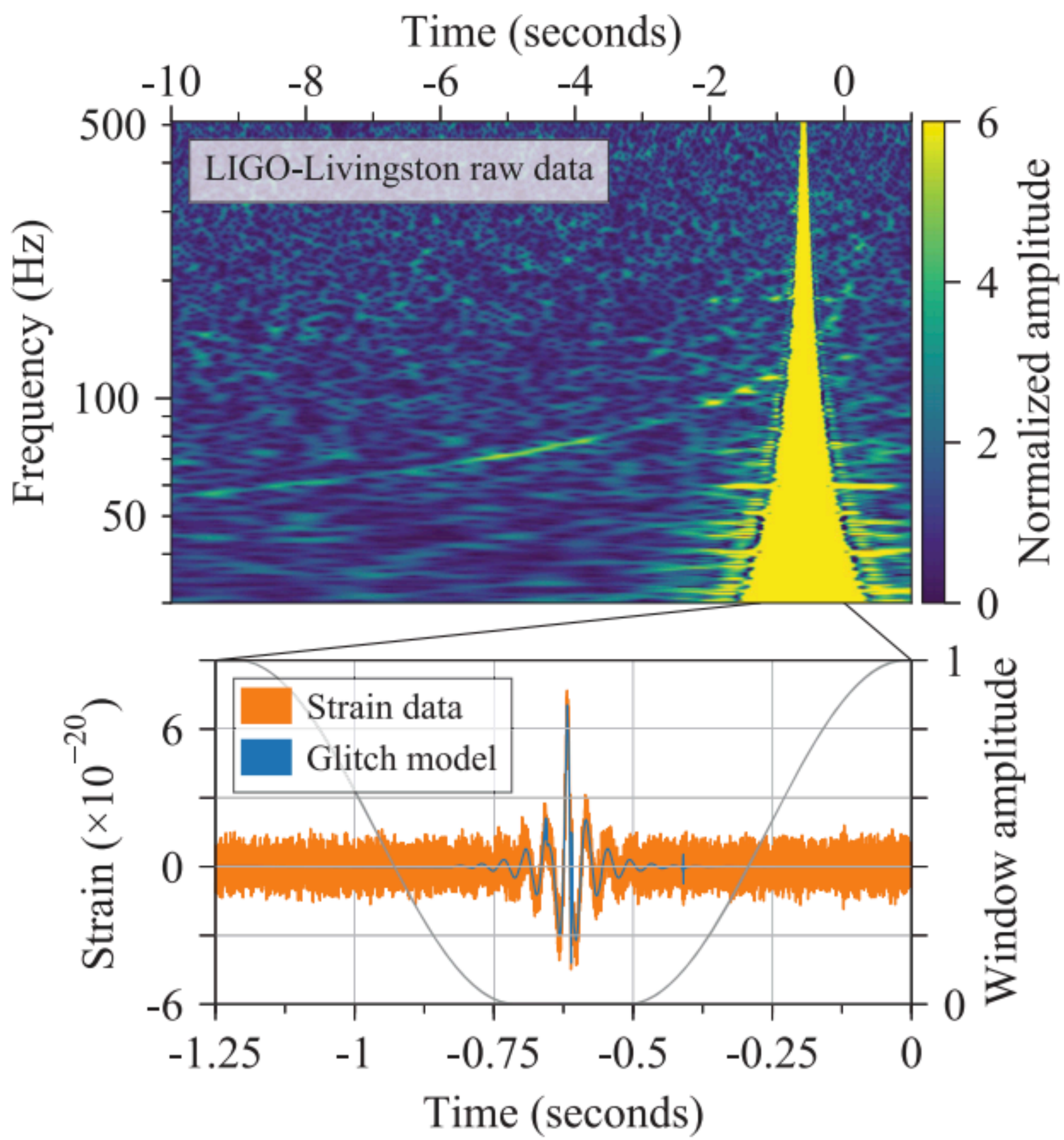
- Include glitch in the likelihood

or

- remove it from the data

or

- learn a glitch model…



[LIGO-Virgo Collaboration, 2017]

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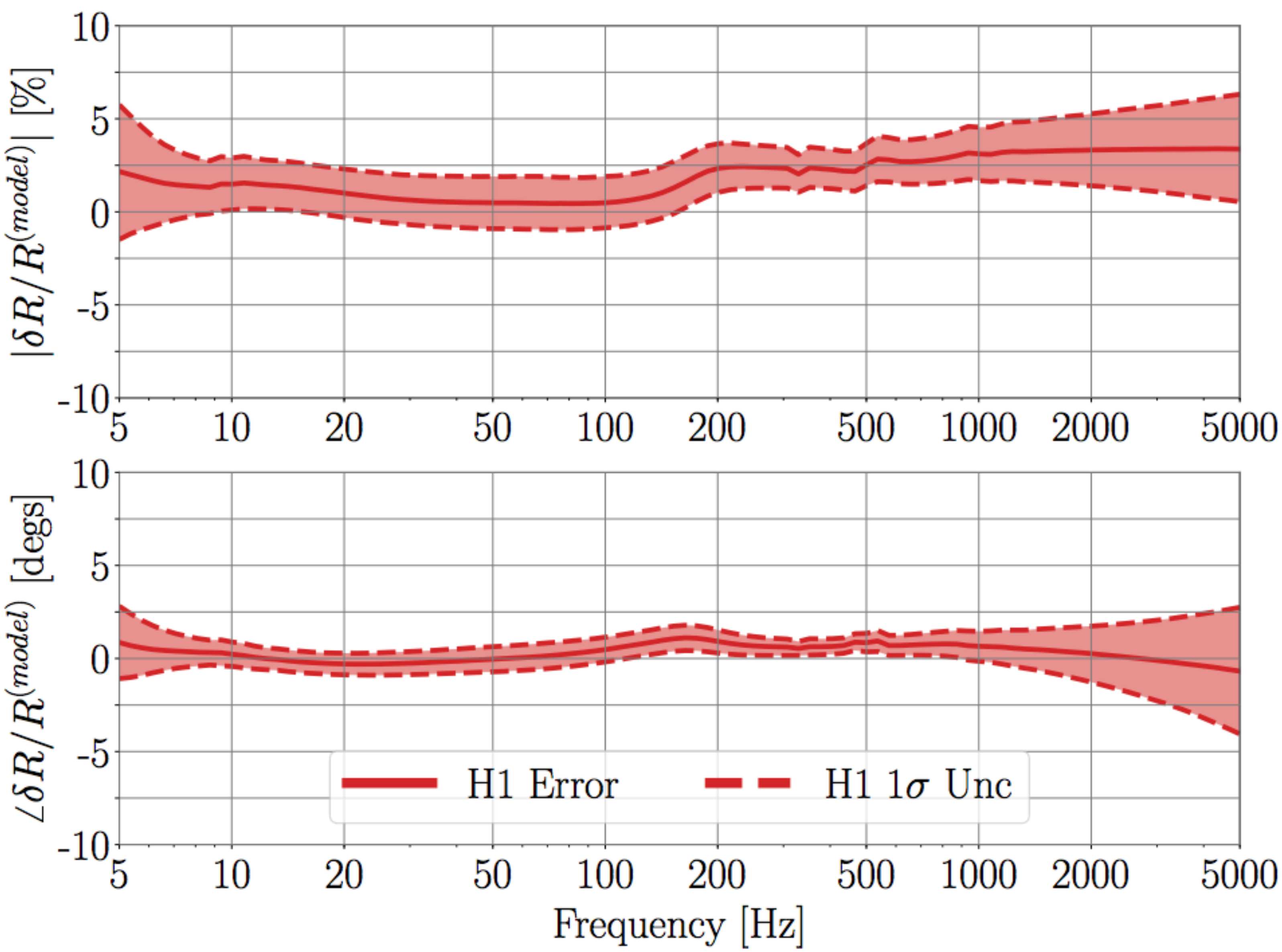
Noise model: Calibration

- Interpolate with cubic splines and marginalise over the calibration error.



$$h' \to h'(1+\delta A)e^{i\delta\phi}$$

$$\delta A(f) = p_s(f; \{f_i, \delta A_i\})$$
$$\delta\phi(f) = p_s(f; \{f_i, \delta\phi_i\})$$

[Cahillane et al., 2017]

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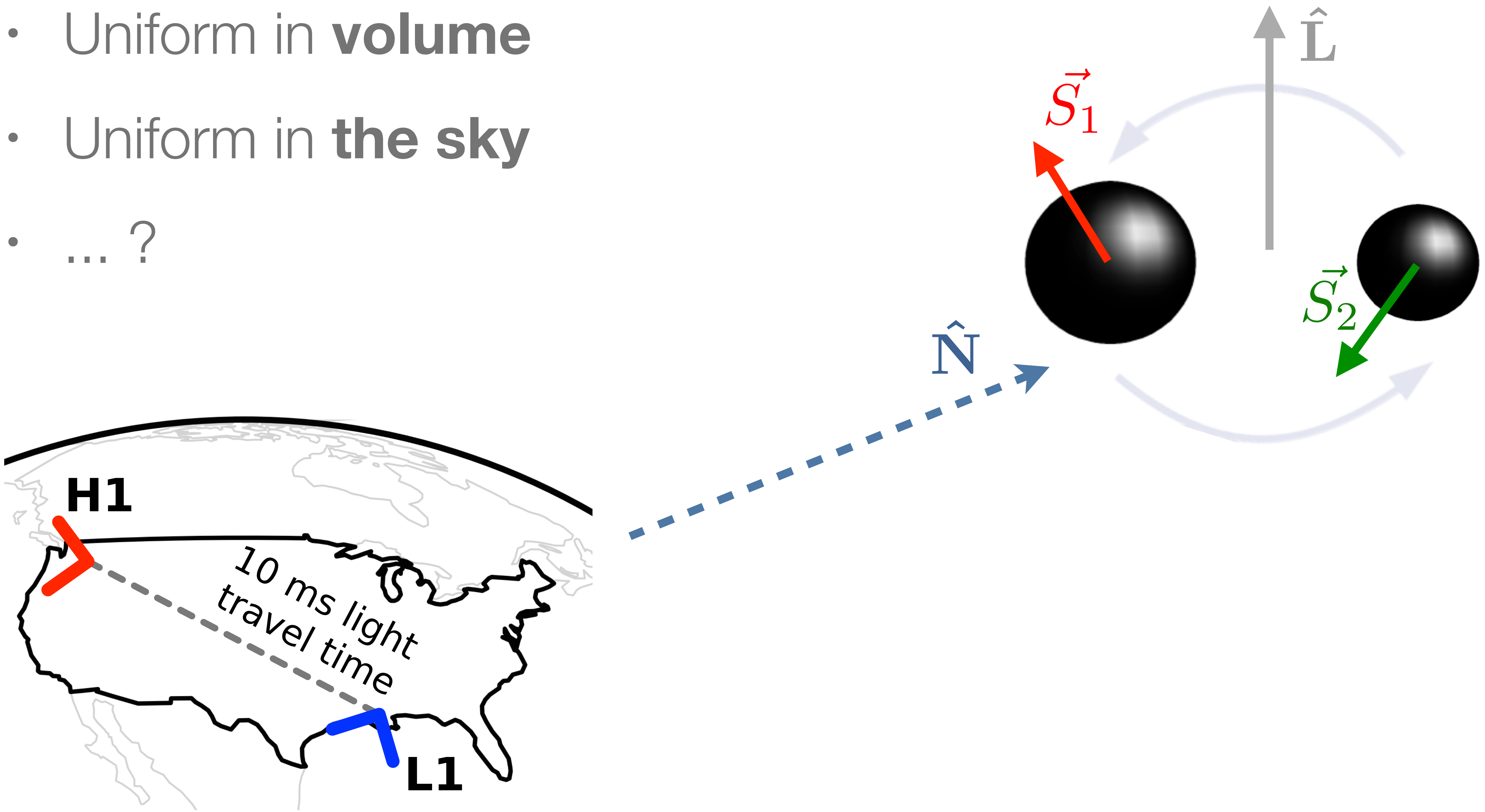
Prior: Compact Binary Coalescence

- Uniform in **volume**
- Uniform in **the sky**
- ... ?



Credit: LIGO/Virgo