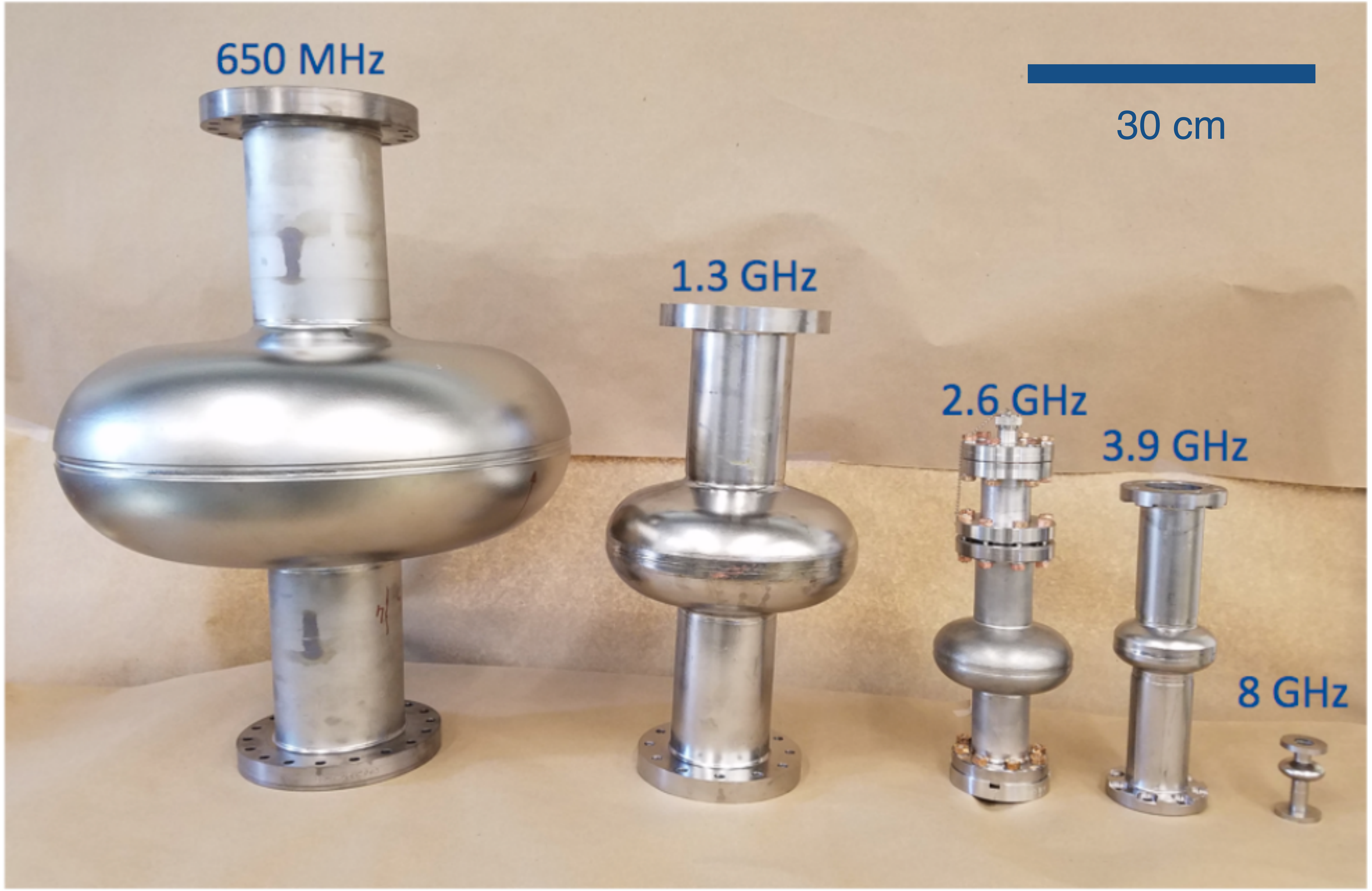
Introduction to Superconductivity

USQIS Summer School
DOE National Quantum Information Science Research Centers

Jim Sauls

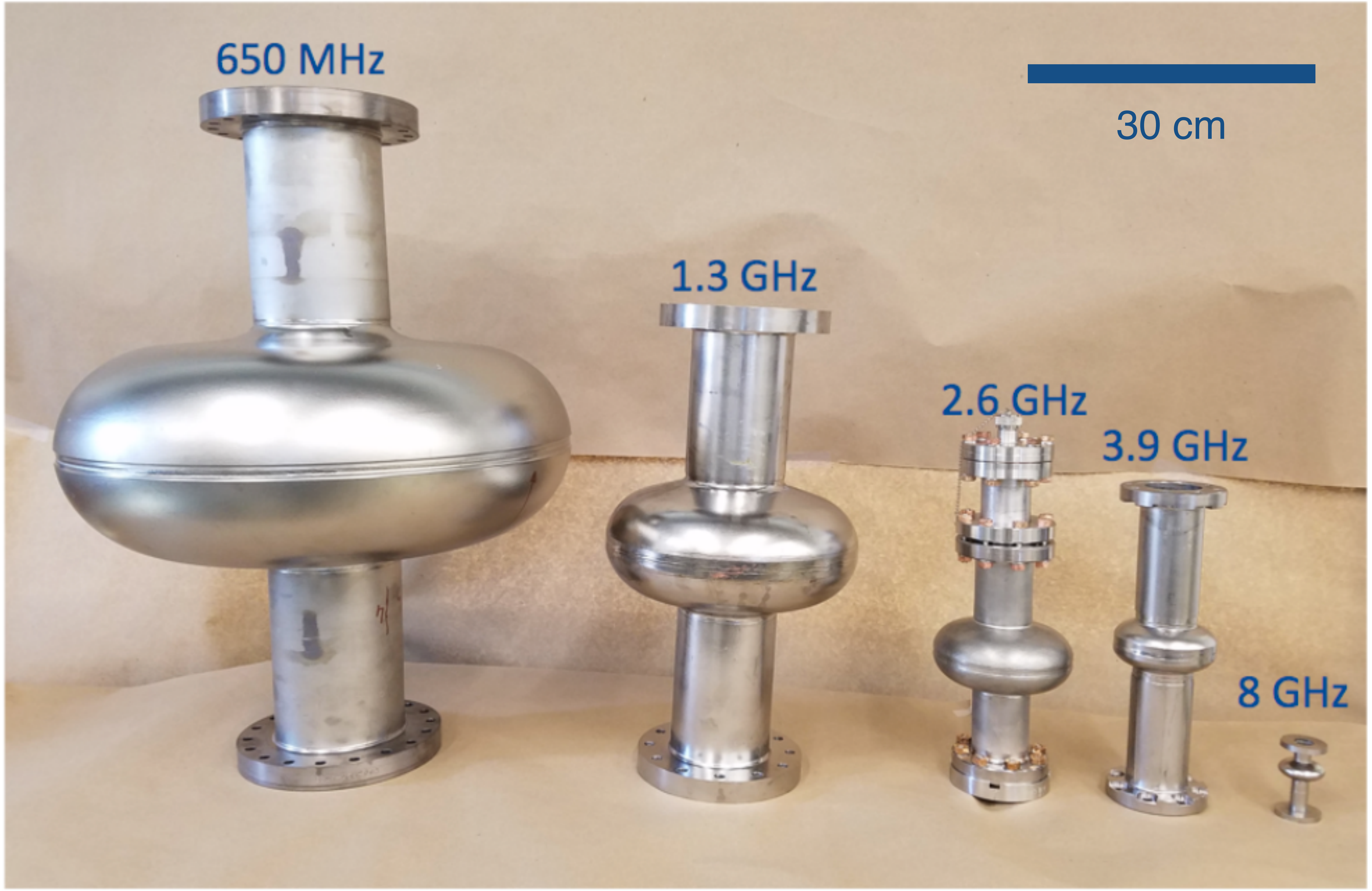
Hearne Institute of Theoretical Physics
Department of Physics and Astronomy
Louisiana State University, Baton Rouge LA

Niobium Superconducting RF (SRF) cavities



Niobium Superconducting RF (SRF) cavities

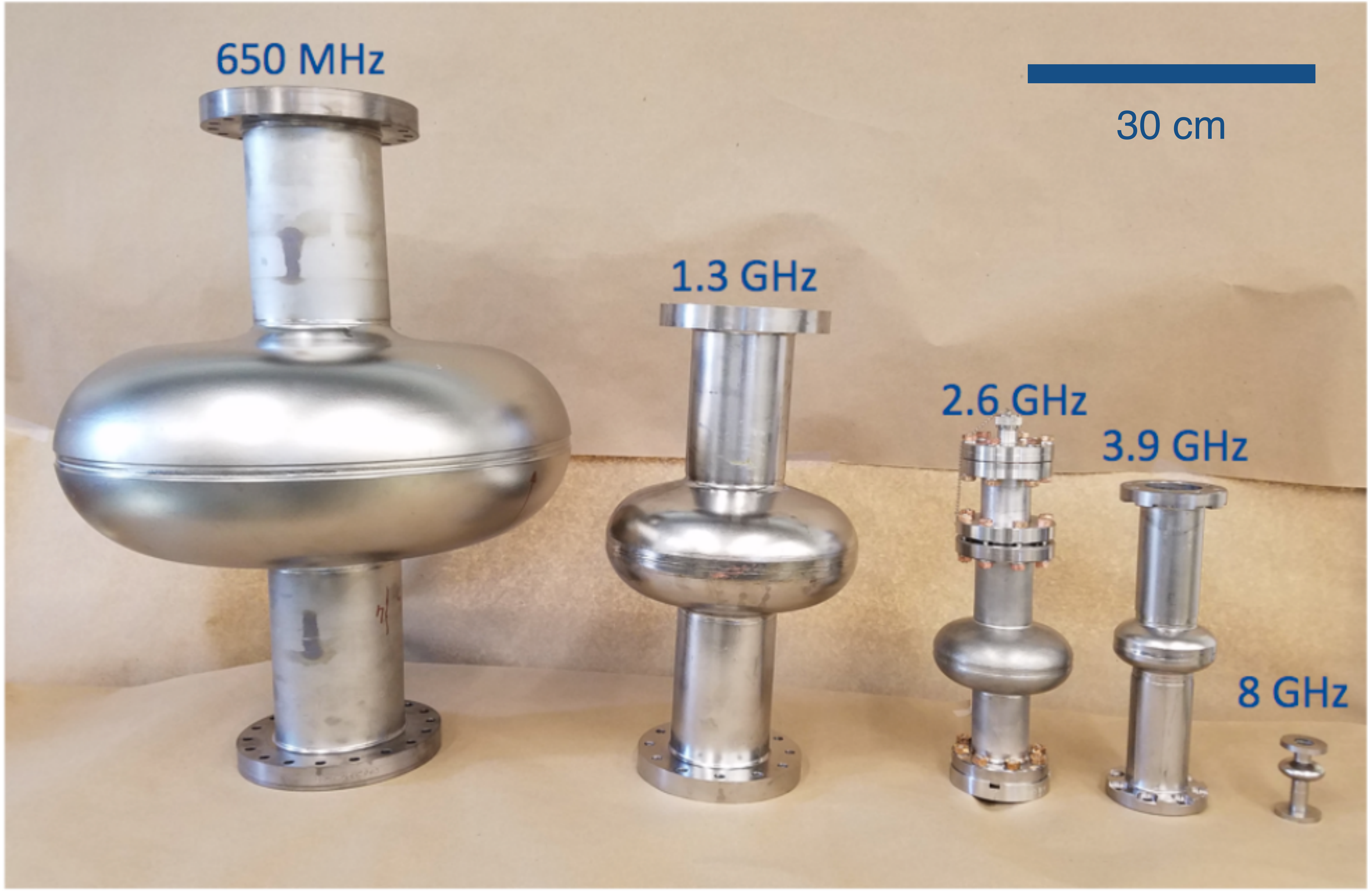
When cooled to temperatures well below the onset of superconductivity …

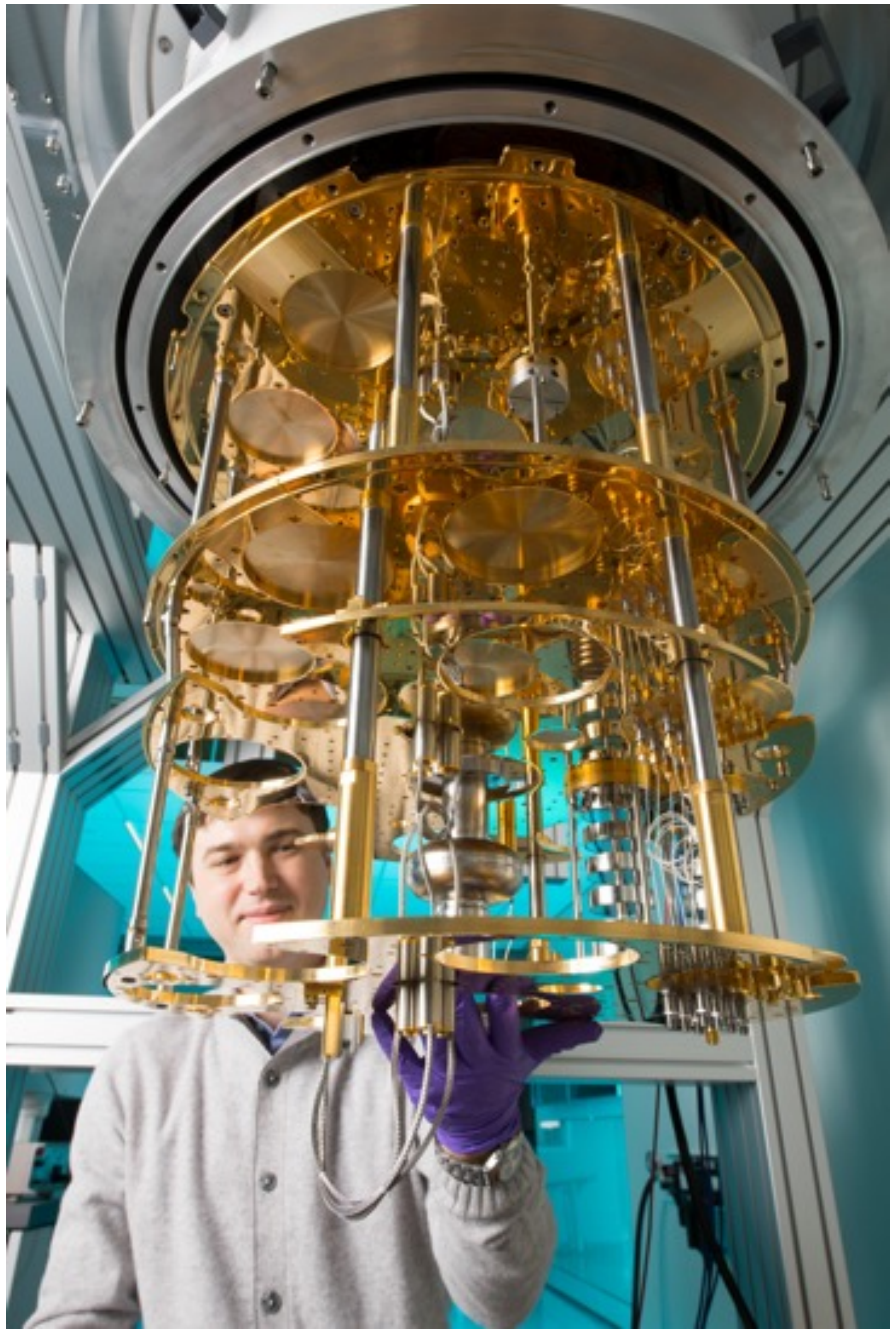


Niobium SRF cavities are the most efficient
Electromagnetic resonators that have been engineered

Niobium Superconducting RF (SRF) cavities

When cooled to temperatures well below the onset of superconductivity …

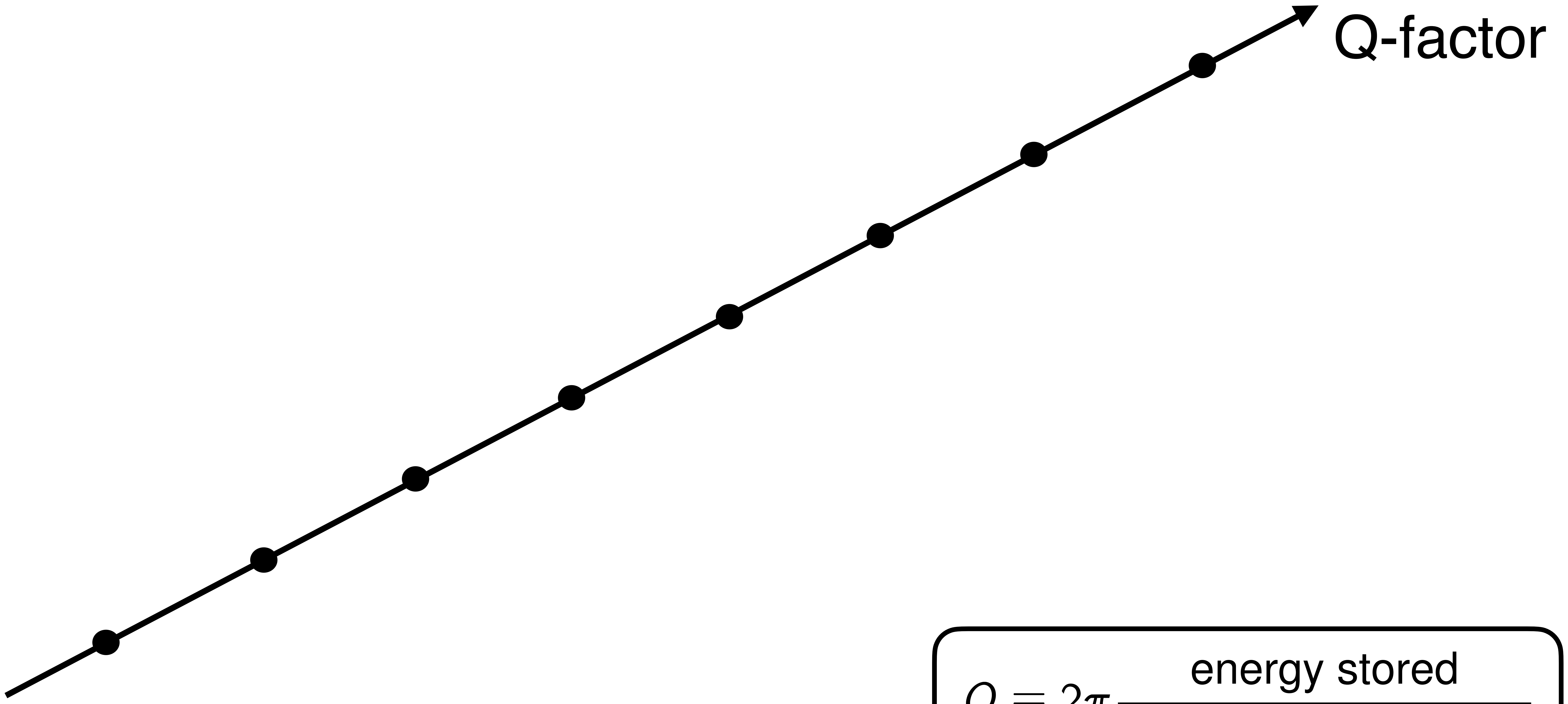




Niobium SRF cavities are the most efficient
Electromagnetic resonators that have been engineered

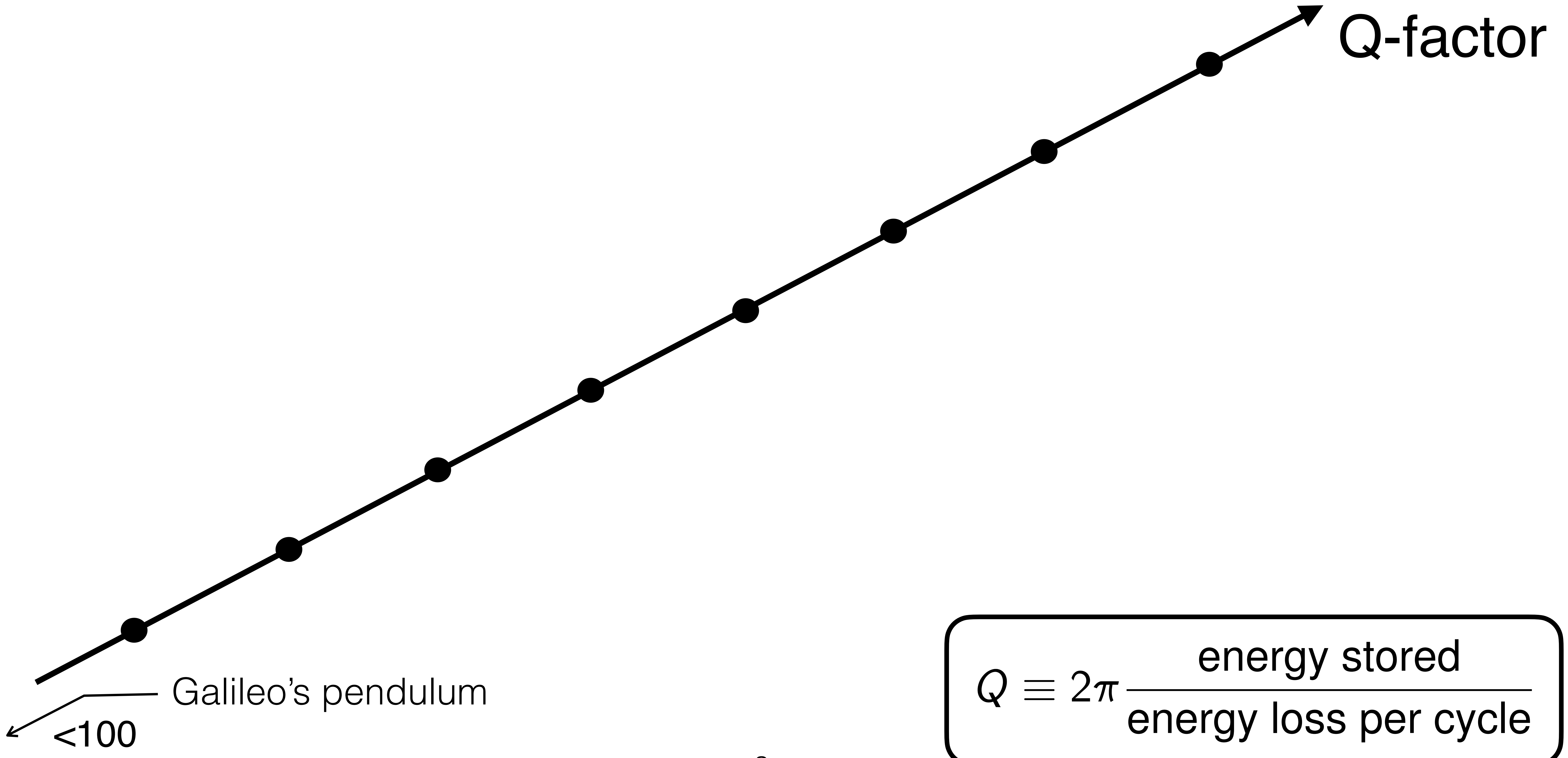
For quantum applications superconducting devices are cooled in Helium dilution refrigerators down to ~10 milli-Kelvin above absolute zero

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



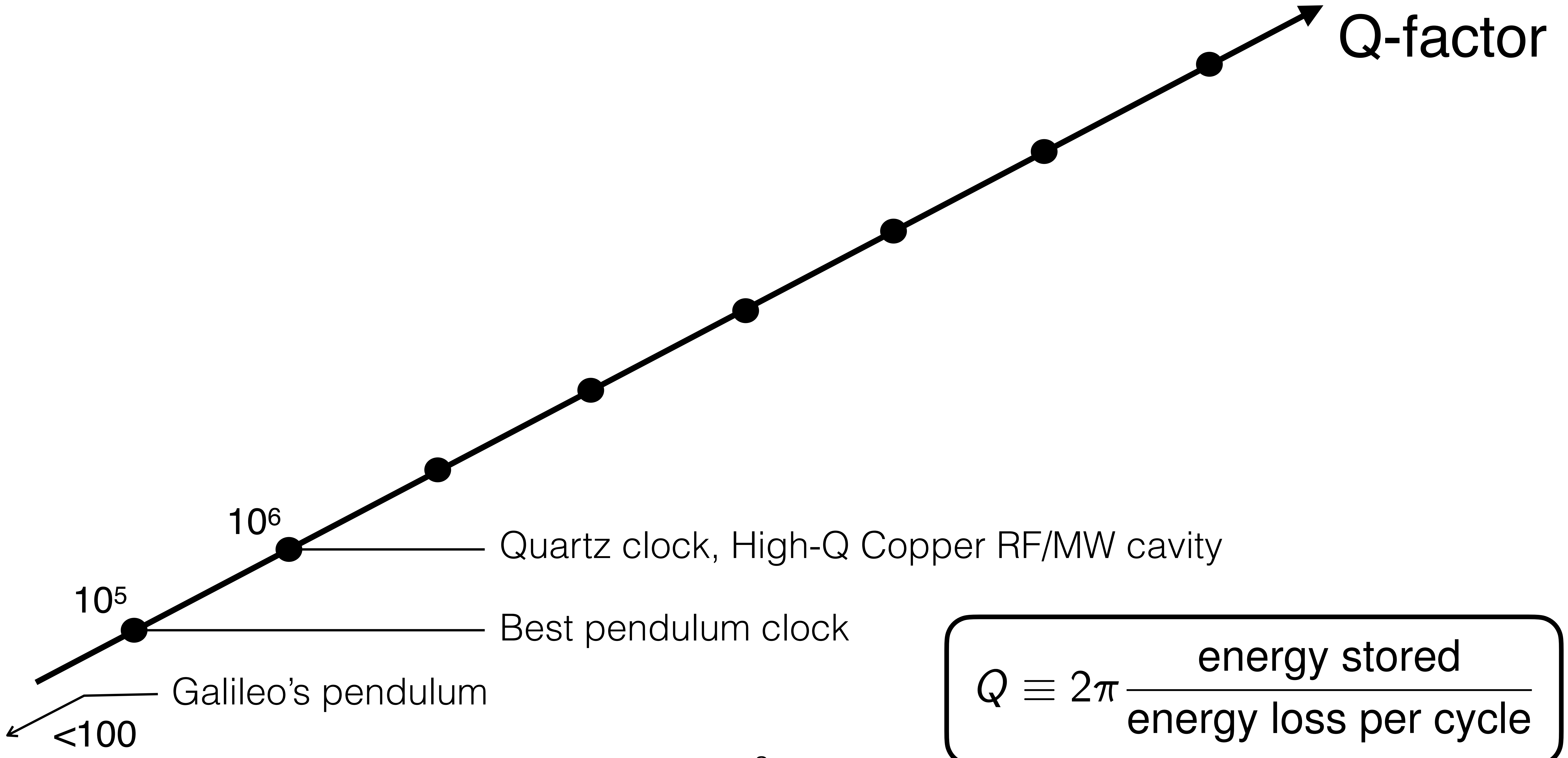
$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



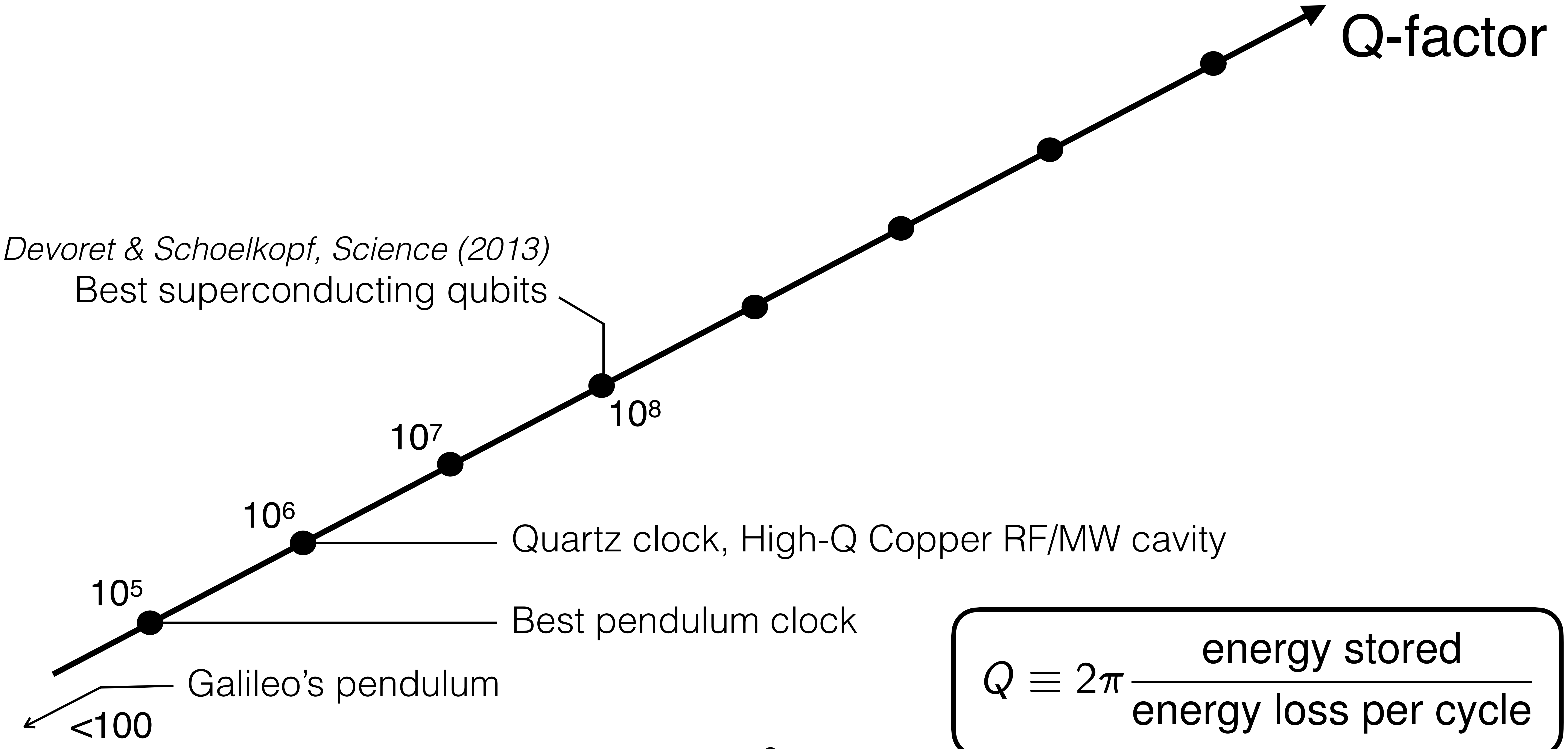
$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



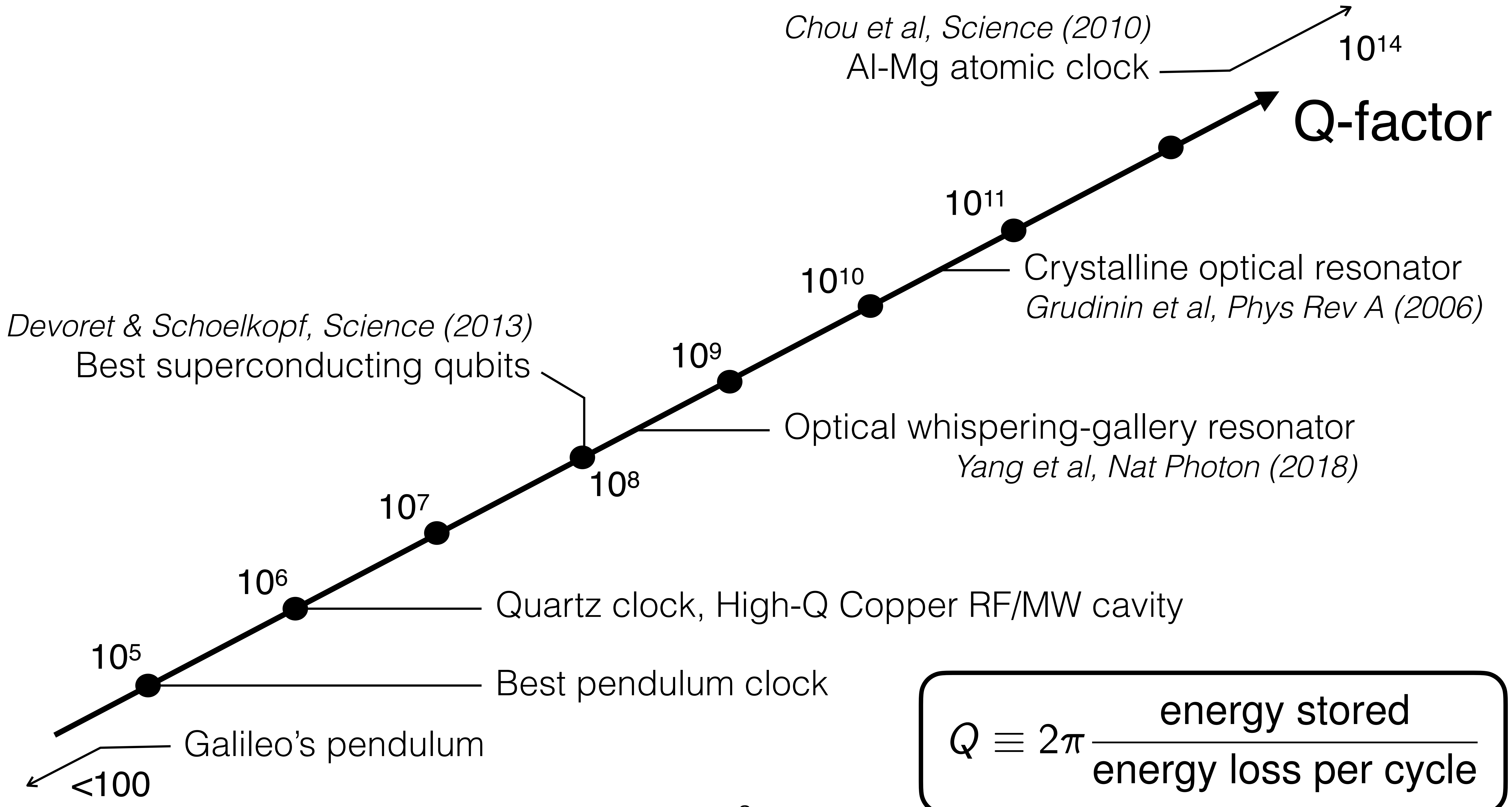
$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



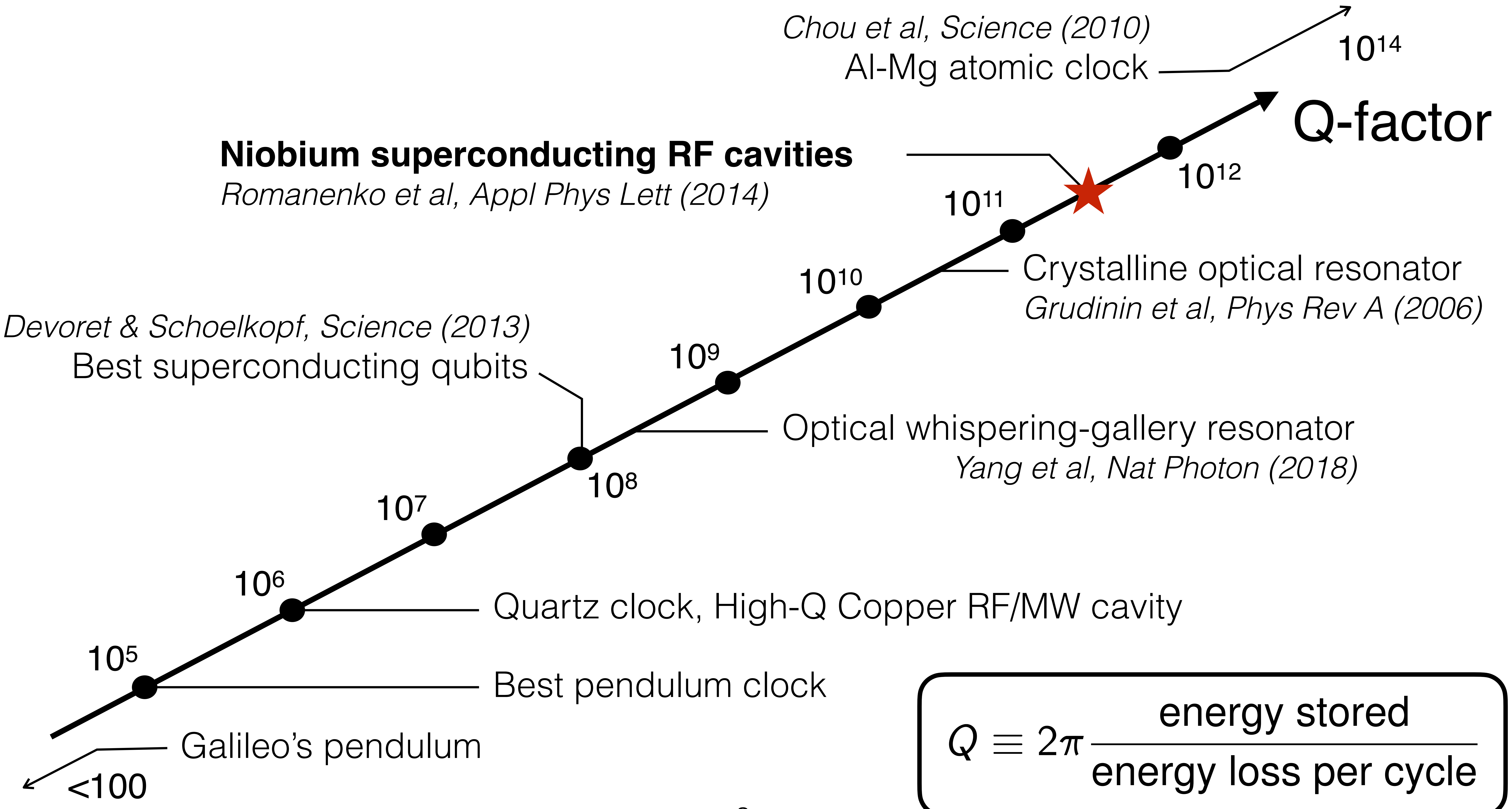
$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

Superconducting RF (SRF) cavities is the most efficient engineered oscillator



$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

100,000 times more efficient than Cu-based RF technology

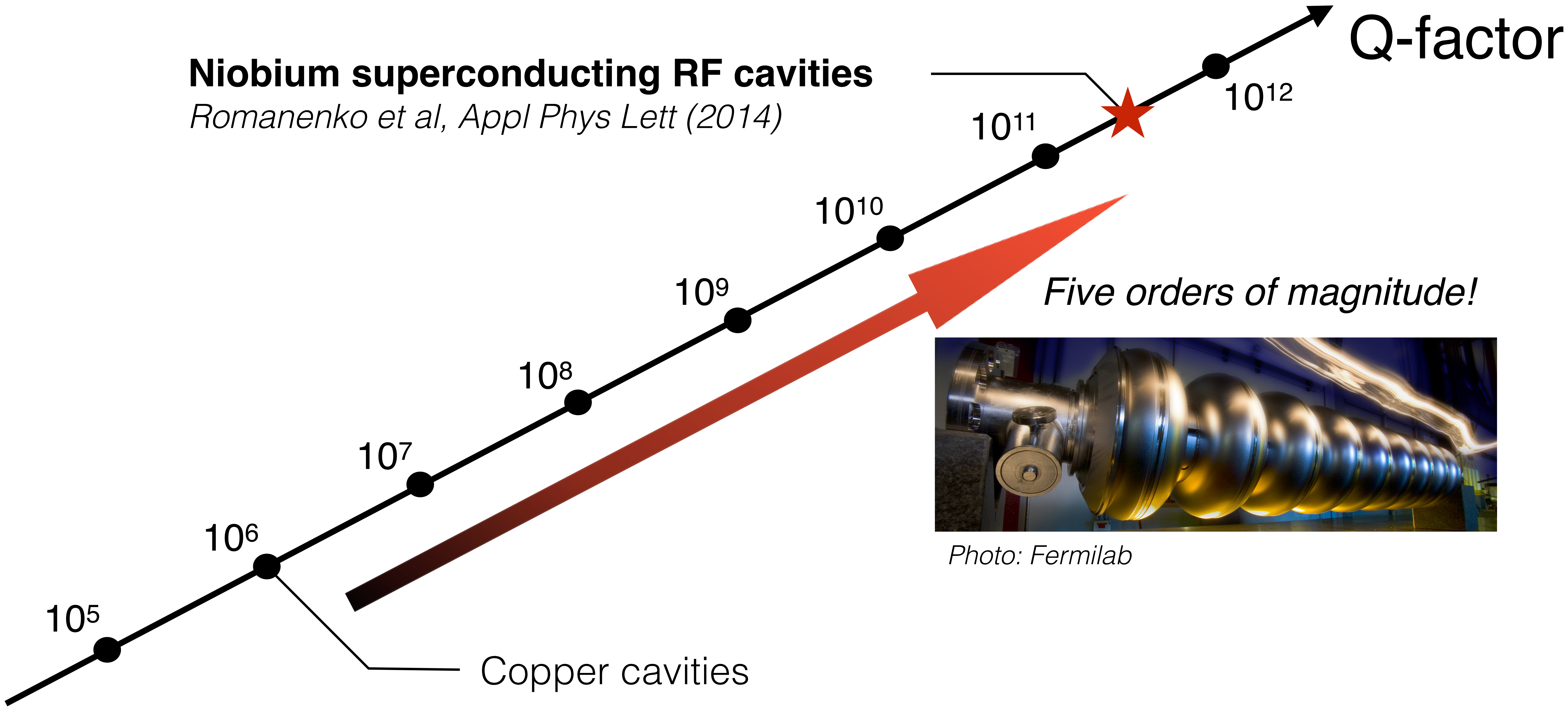


Photo: Fermilab

Credits: wikipedia & Wave Ngampruetikorn

How efficient is $Q = 2 \times 10^{11}$?

Credits: wikipedia & Wave Ngampruetikorn

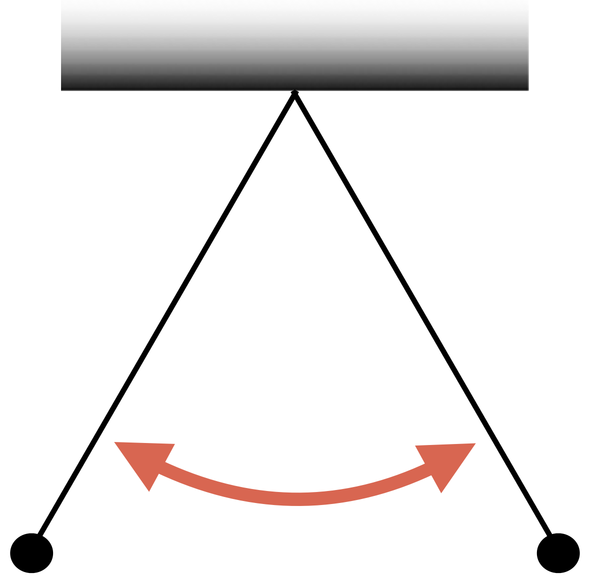
How efficient is $Q = 2 \times 10^{11}$?

Galileo's pendulum would still be swinging if it were as efficient as today's best oscillators

1600AD

Scientific work on pendulum started

Galileo



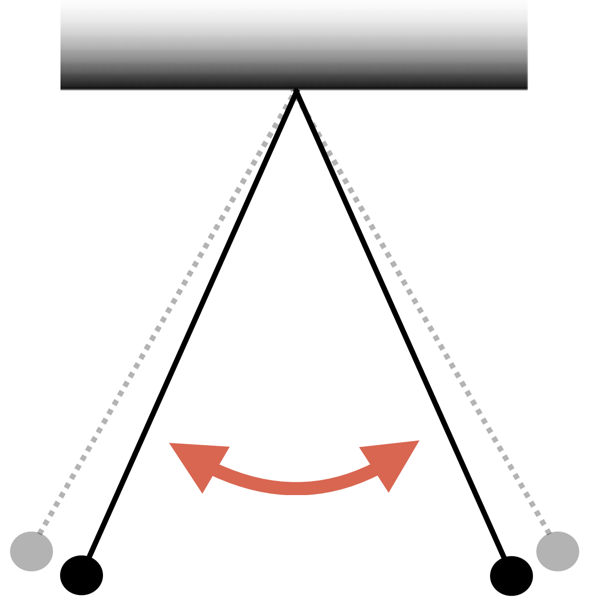
Pendulum

400 years

Today

Galileo

Lost ~20% of its amplitude



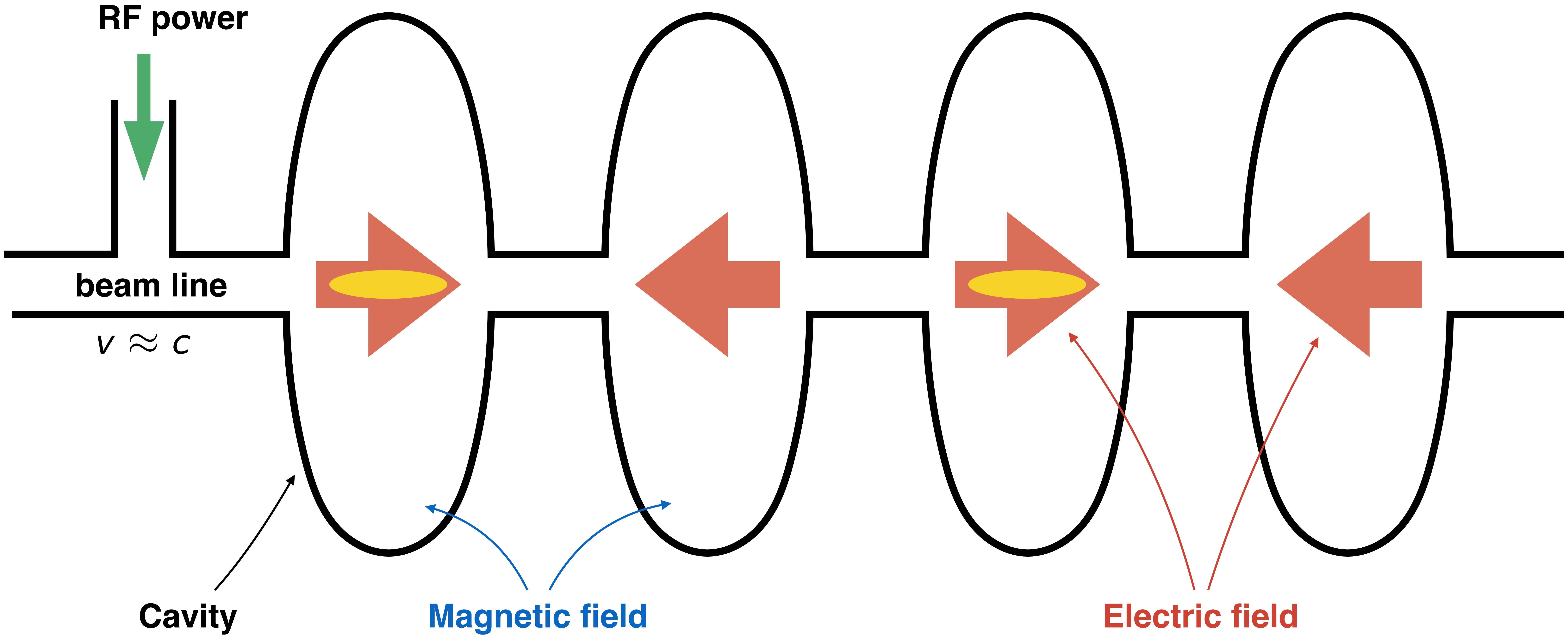
Pendulum

Credits: wikipedia & Wave Ngampruetikorn

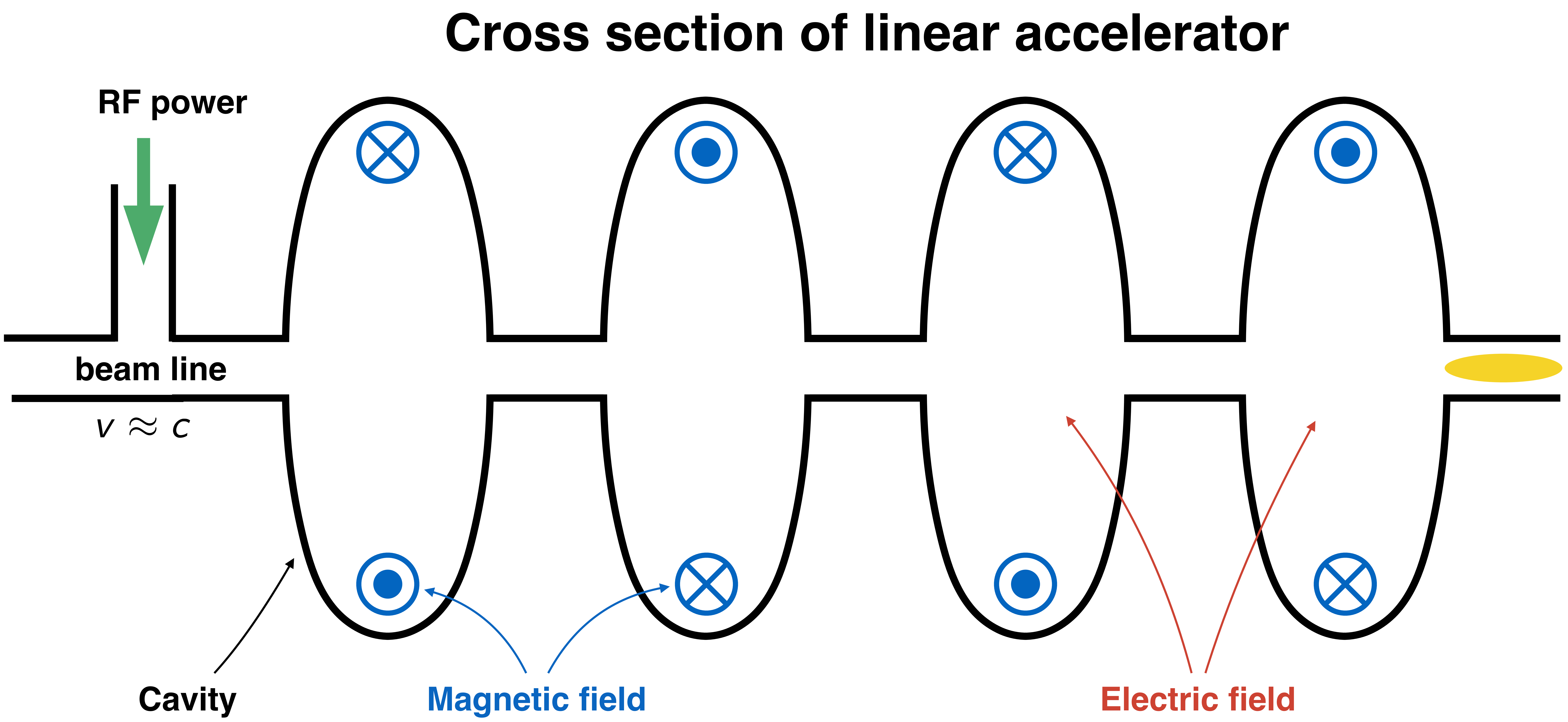
$f = 1\,\mathrm{Hz}, Q = 2 \times 10^{11}$

SRF cavities provide an effective method to accelerate charged particles to high energies

Cross section of linear accelerator



SRF cavities provide an effective method to accelerate charged particles to high energies



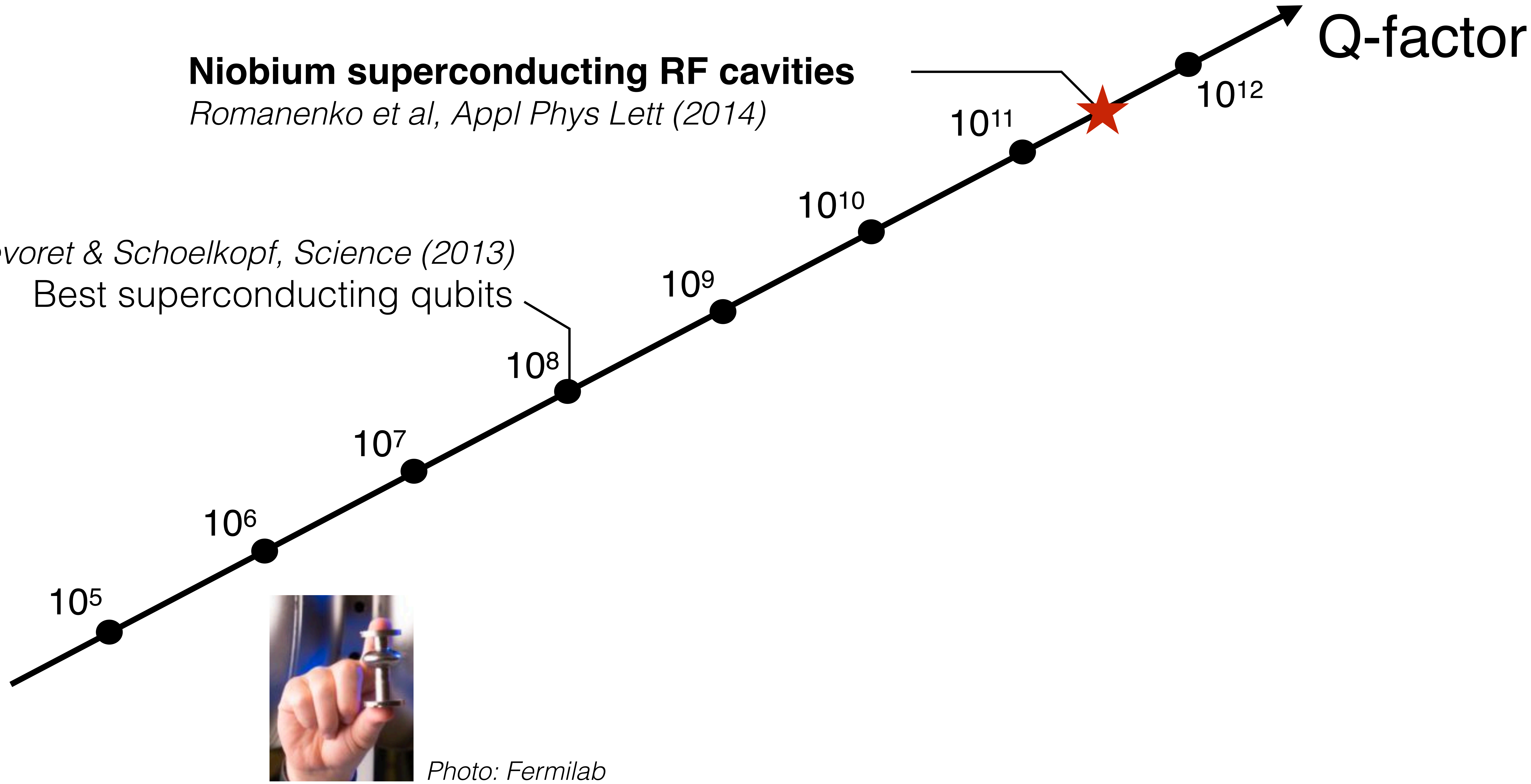
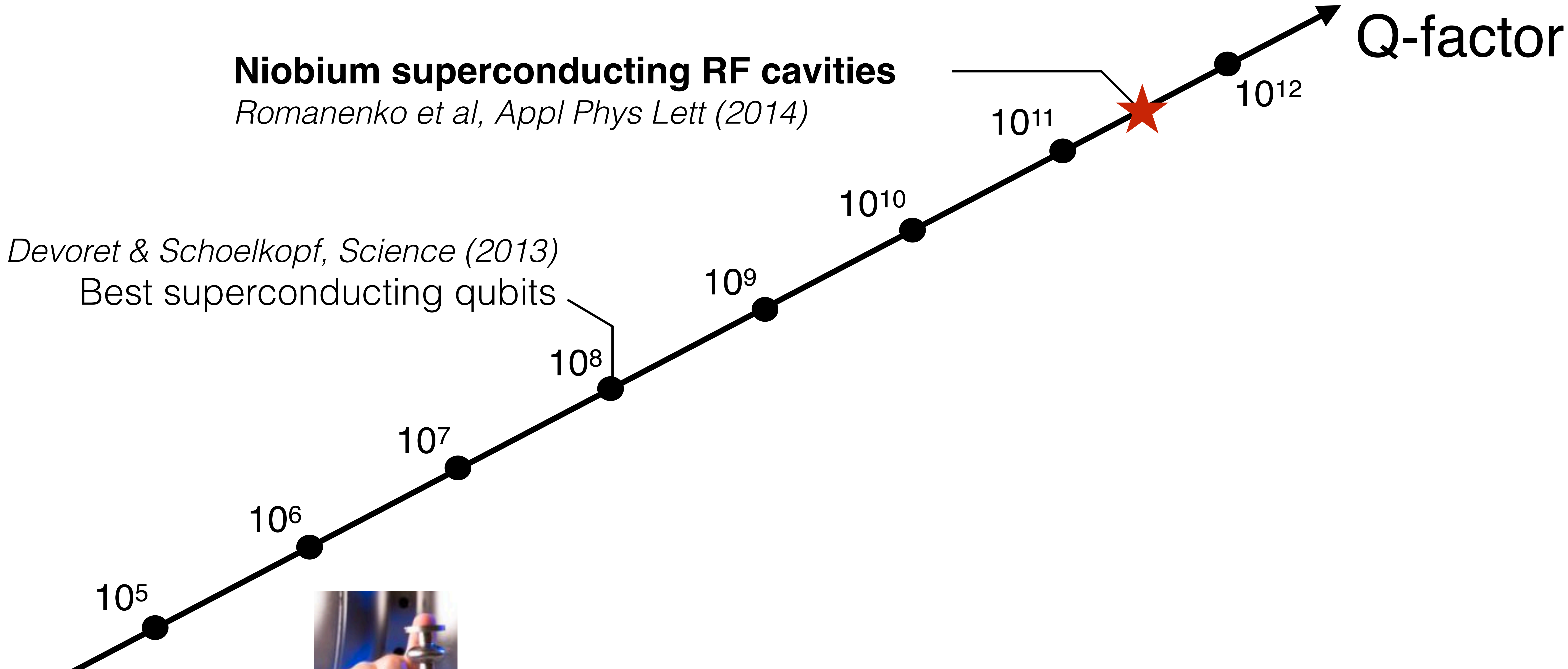


Photo: Fermilab

SRF Cavities in the Quantum Regime



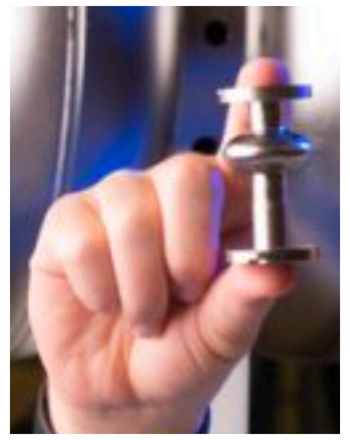
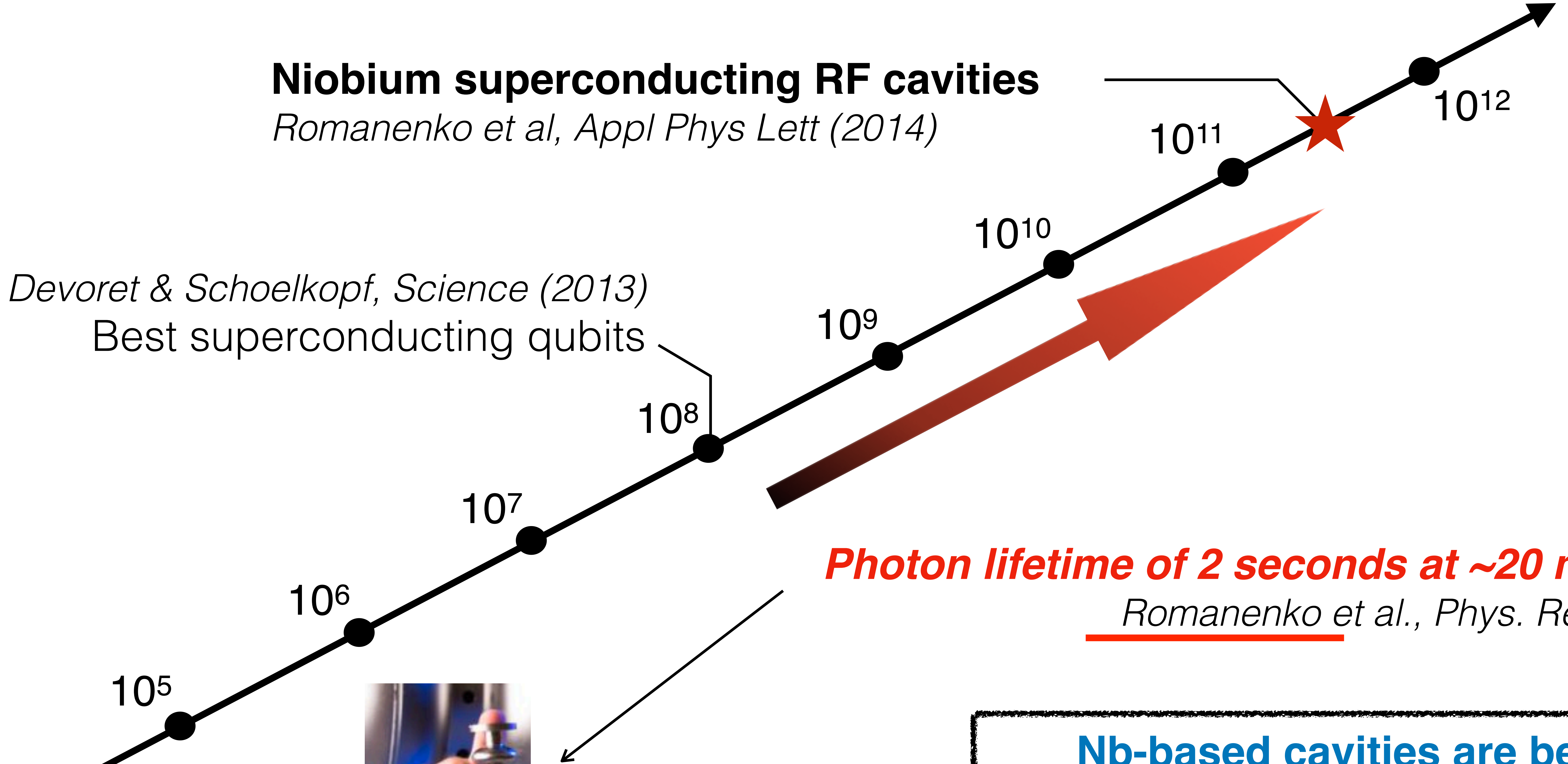


Photo: Fermilab

SRF Cavities in the Quantum Regime

Nb SRF improved *photon* lifetimes by *a factor of 1,000!*



Photon lifetime of 2 seconds at ~20 mK *— cf. 10ms in Al cavity*

Romanenko et al., Phys. Rev. Applied 13, 034032 (2020)

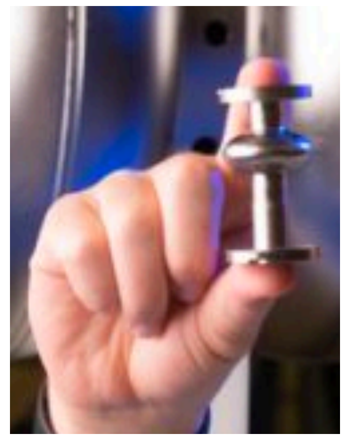


Photo: Fermilab

Nb-based cavities are being developed for Quantum processors!

What else can we do with high Q Superconducting Resonators?

Look for rare events using microwave photons

- Dark Matter Searches
- Tests of Low-Energy QED

SQMS

- What are the key properties of superconductors?

- What are the key properties of superconductors?

- How does superconductivity lead to high Q microwave resonators?

- What are the key properties of superconductors?

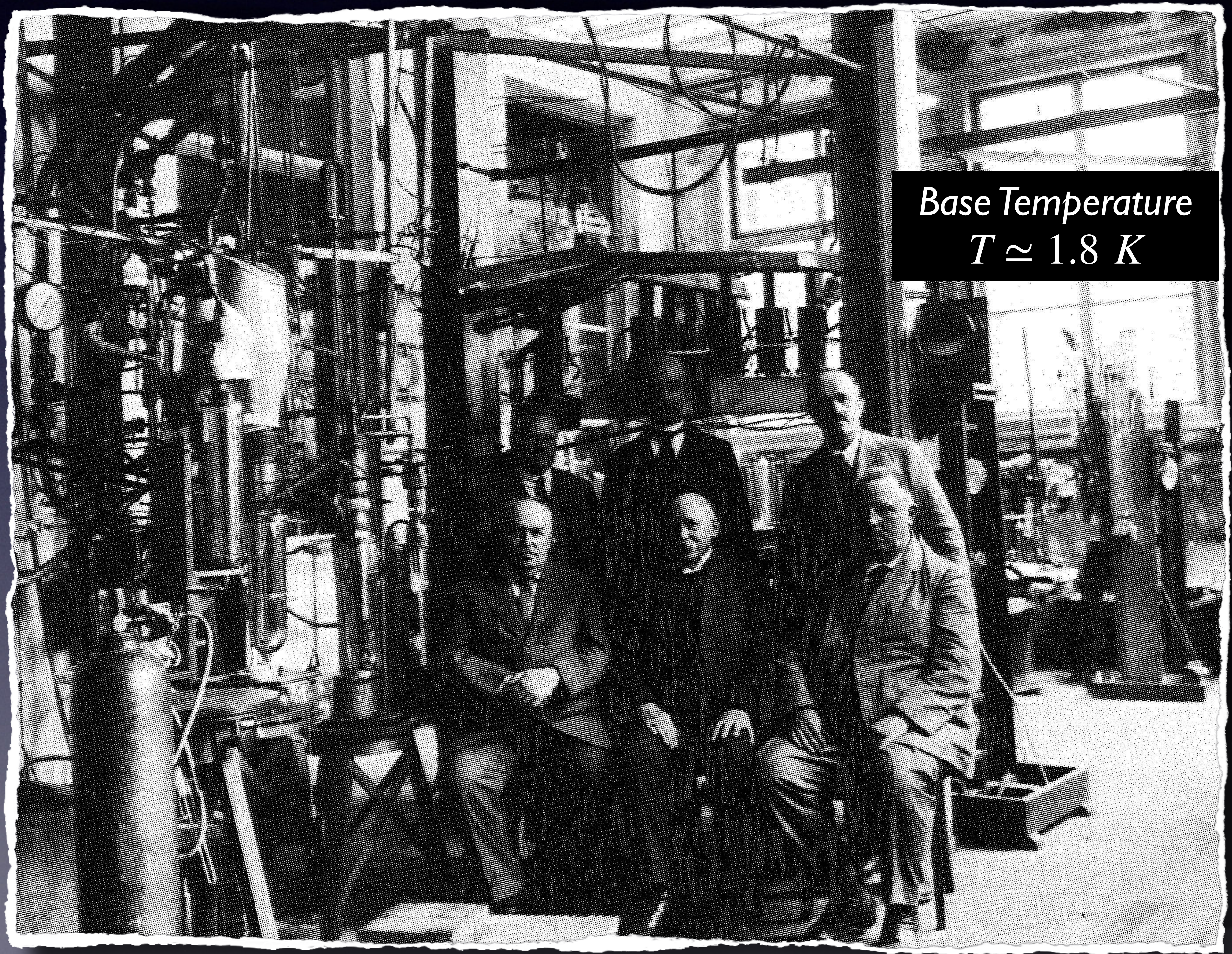
- How does superconductivity lead to high Q microwave resonators?

- What are some of the challenges to improving superconducting quantum devices for processors and sensors?

H. Kamerlingh Onnes' Laboratory

Leiden Institute of Physics

✓1908 - Liquified Helium - New Era in Low Temperature Physics

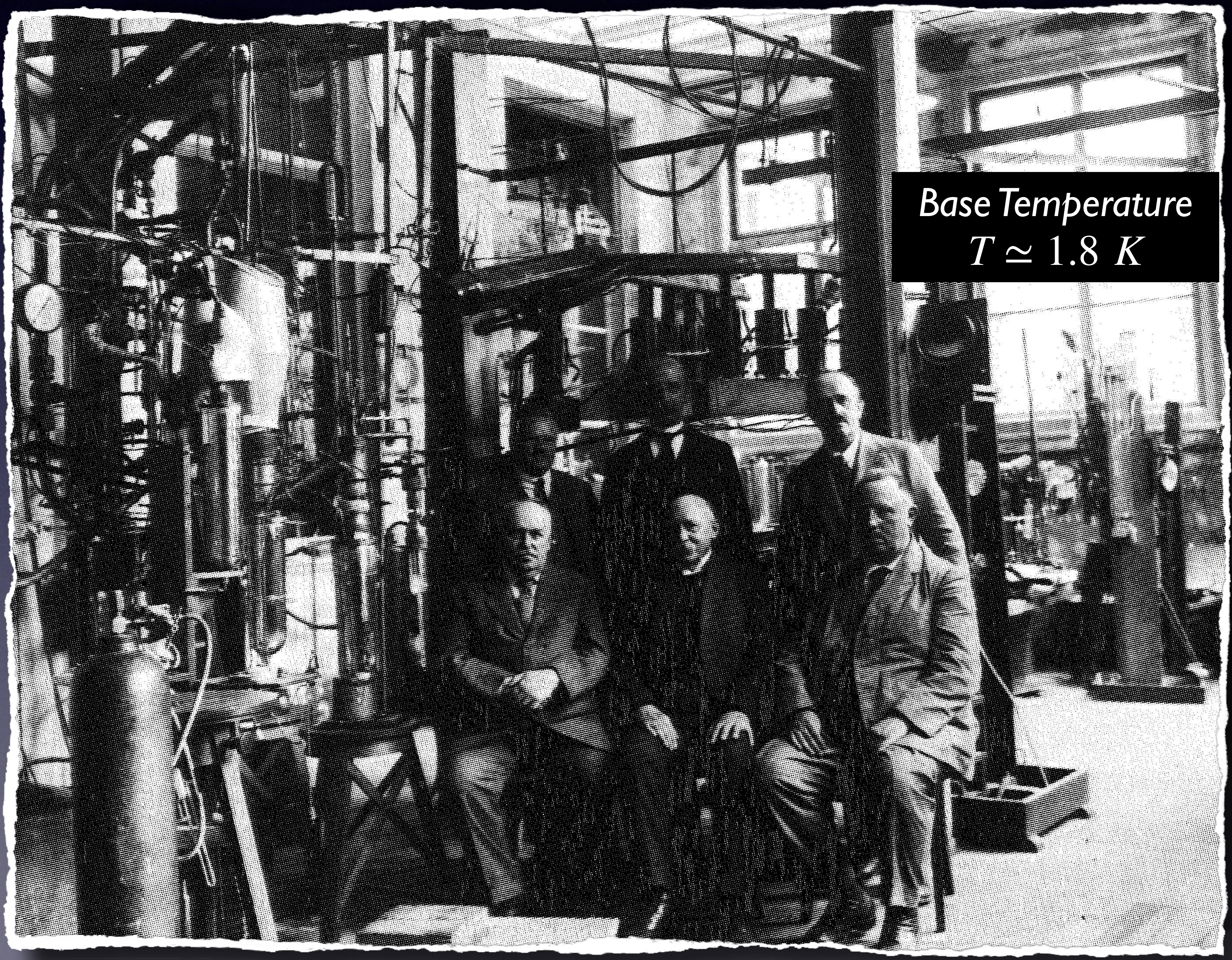


Base Temperature
$T \simeq 1.8\ K$

H. Kamerlingh Onnes' Laboratory

Leiden Institute of Physics

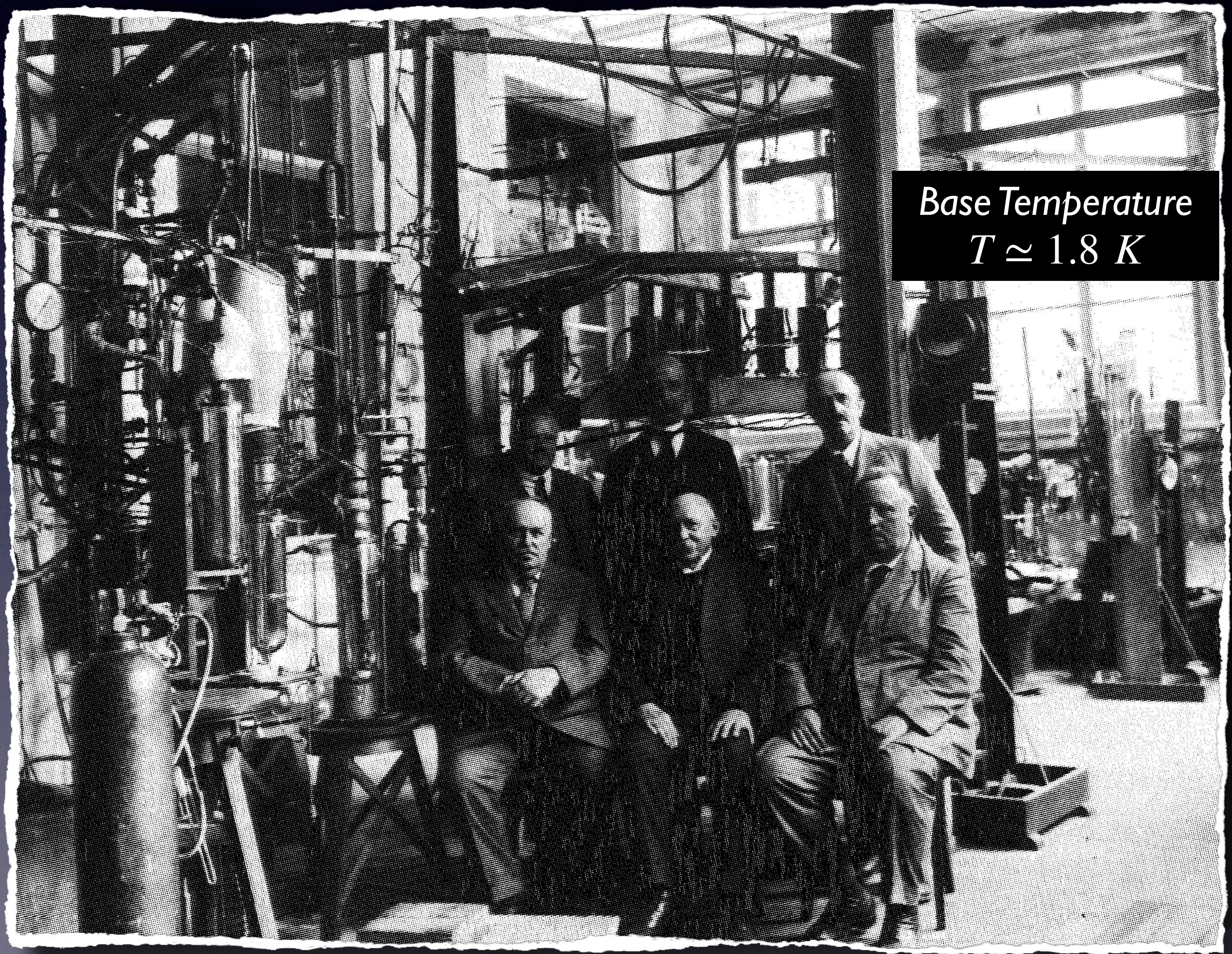
- ✓ 1908 - Liquified Helium - New Era in Low Temperature Physics
- ✓ 1910 - Low Temperature Resistance of Metals



H. Kamerlingh Onnes' Laboratory

Leiden Institute of Physics

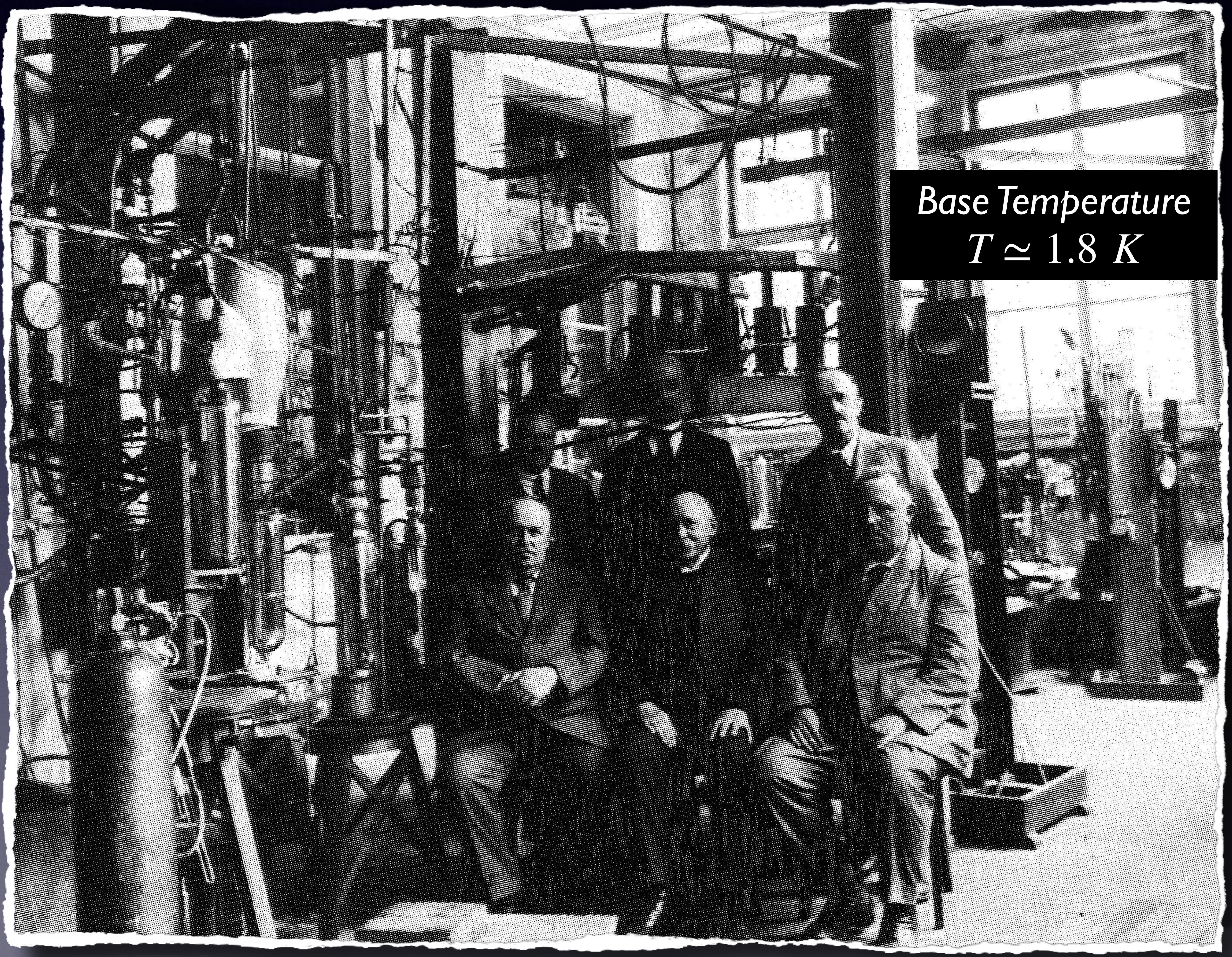
- ✓ 1908 - Liquified Helium - New Era in Low Temperature Physics
- ✓ 1910 - Low Temperature Resistance of Metals
- ✓ 1911 - Discovered Superconductivity in Hg, Pb, Sn



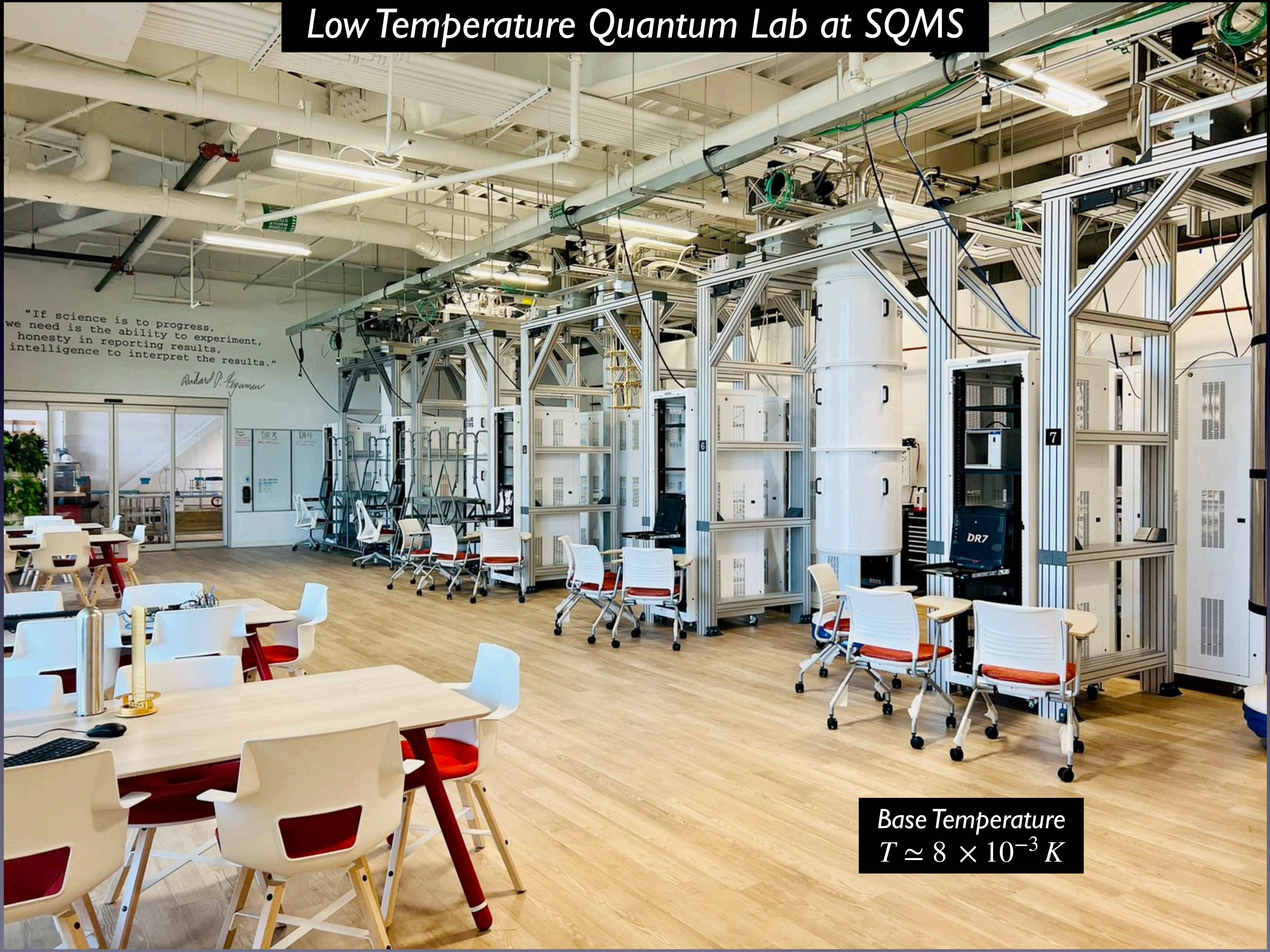
H. Kamerlingh Onnes' Laboratory

Leiden Institute of Physics

- ✓ 1908 - Liquified Helium - New Era in Low Temperature Physics
- ✓ 1910 - Low Temperature Resistance of Metals
- ✓ 1911 - Discovered Superconductivity in Hg, Pb, Sn
- ✓ 1914 - Demonstrated Persistent Currents in a Pb ring.



Low Temperature Quantum Lab at SQMS



Base Temperature
$T \simeq 8 \times 10^{-3}\,K$

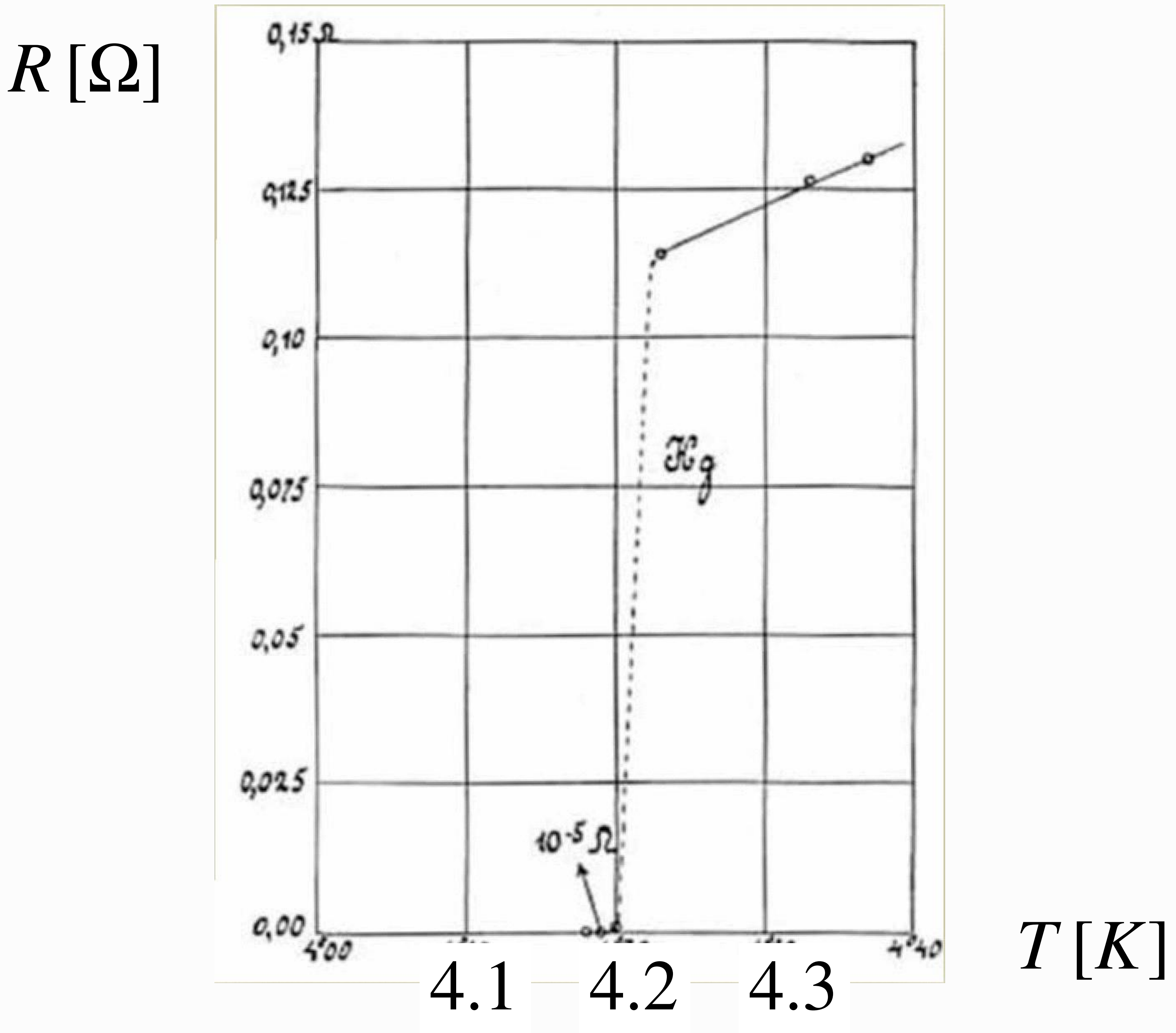
The Discovery of Superconductivity

Kammerlingh Onnes Laboratory, Leiden, April 8, 1911

Physics Today, September 2010, by Dirk van Delft and Peter Kes

- The experiment was started at 7am. Kamerlingh Onnes arrived when helium circulation began at 11:20am.
- The resistance of the mercury fell with the falling temperature. Soon after noon the gas thermometer denoted 5Kelvin.
- Then the team started to reduce the vapor pressure of the helium, and it began to evaporate rapidly. They measured its specific heat and stopped at a vapor pressure of 197 mmHg (0.26 atmospheres), corresponding to about 3K.
- Exactly at 4pm, the resistances of the gold and mercury were determined again. The latter was, in the historic entry, Mercury practically zero.

Transition in Hg



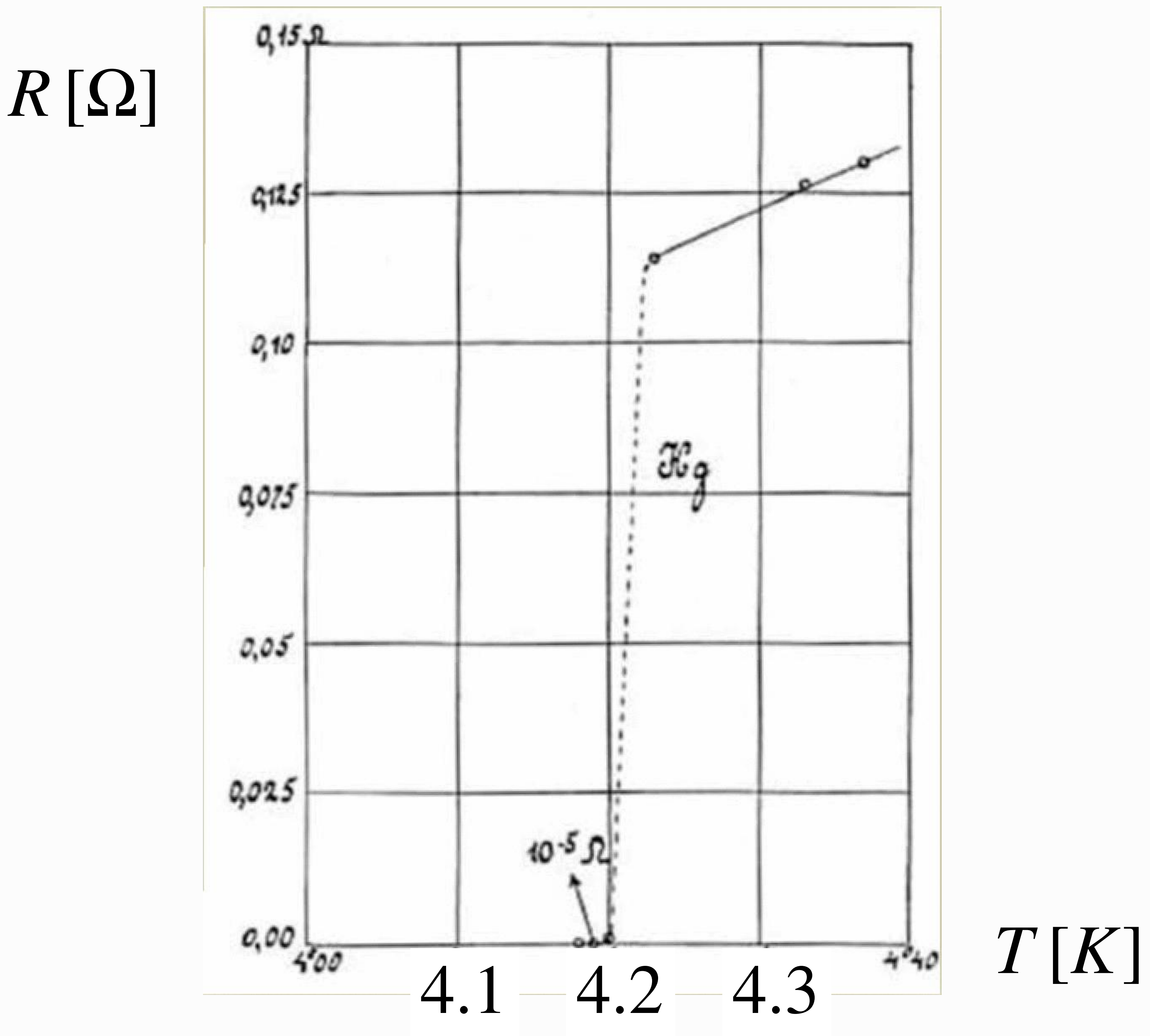
The Discovery of Superconductivity

Kammerlingh Onnes Laboratory, Leiden, April 8, 1911

Physics Today, September 2010, by Dirk van Delft and Peter Kes

- ▶ The experiment was started at 7am. Kamerlingh Onnes arrived when helium circulation began at 11:20am.
- ▶ The resistance of the mercury fell with the falling temperature. Soon after noon the gas thermometer denoted 5Kelvin.
- ▶ Then the team started to reduce the vapor pressure of the helium, and it began to evaporate rapidly. They measured its specific heat and stopped at a vapor pressure of 197 mmHg (0.26 atmospheres), corresponding to about 3K.
- ▶ Exactly at 4pm, the resistances of the gold and mercury were determined again. The latter was, in the historic entry, Mercury practically zero.

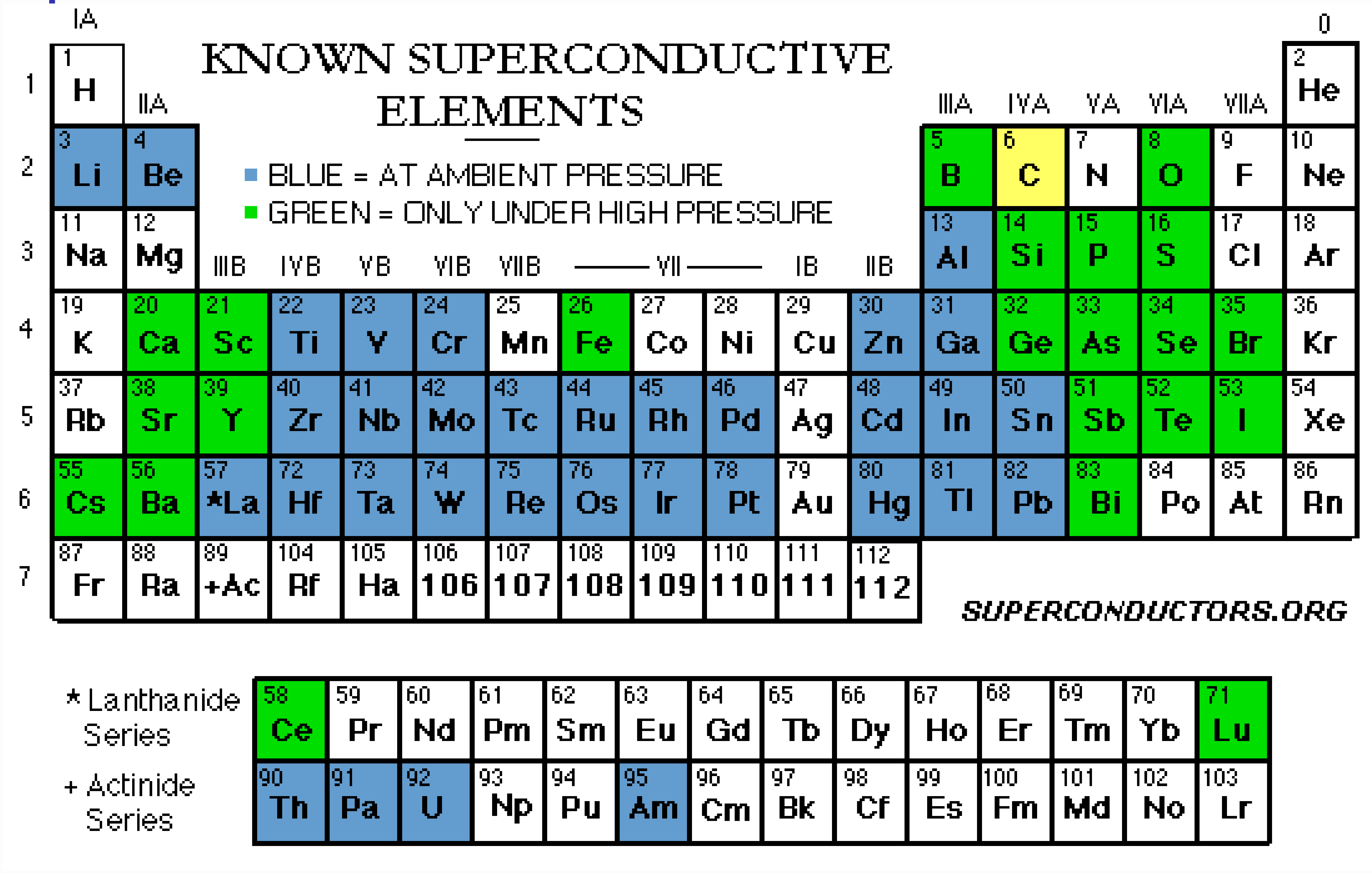
Transition in Hg



▶ ... At the end of the day, Kamerlingh Onnes finished with an intriguing notebook entry: Dorsman [who had controlled and measured the temperatures] really had to hurry to make the observations. The temperature had been surprisingly hard to control. Just before the lowest temperature [about 1.8 K] was reached, the boiling suddenly stopped and was replaced by evaporation in which the liquid visibly shrank. So, a remarkably strong evaporation at the surface.

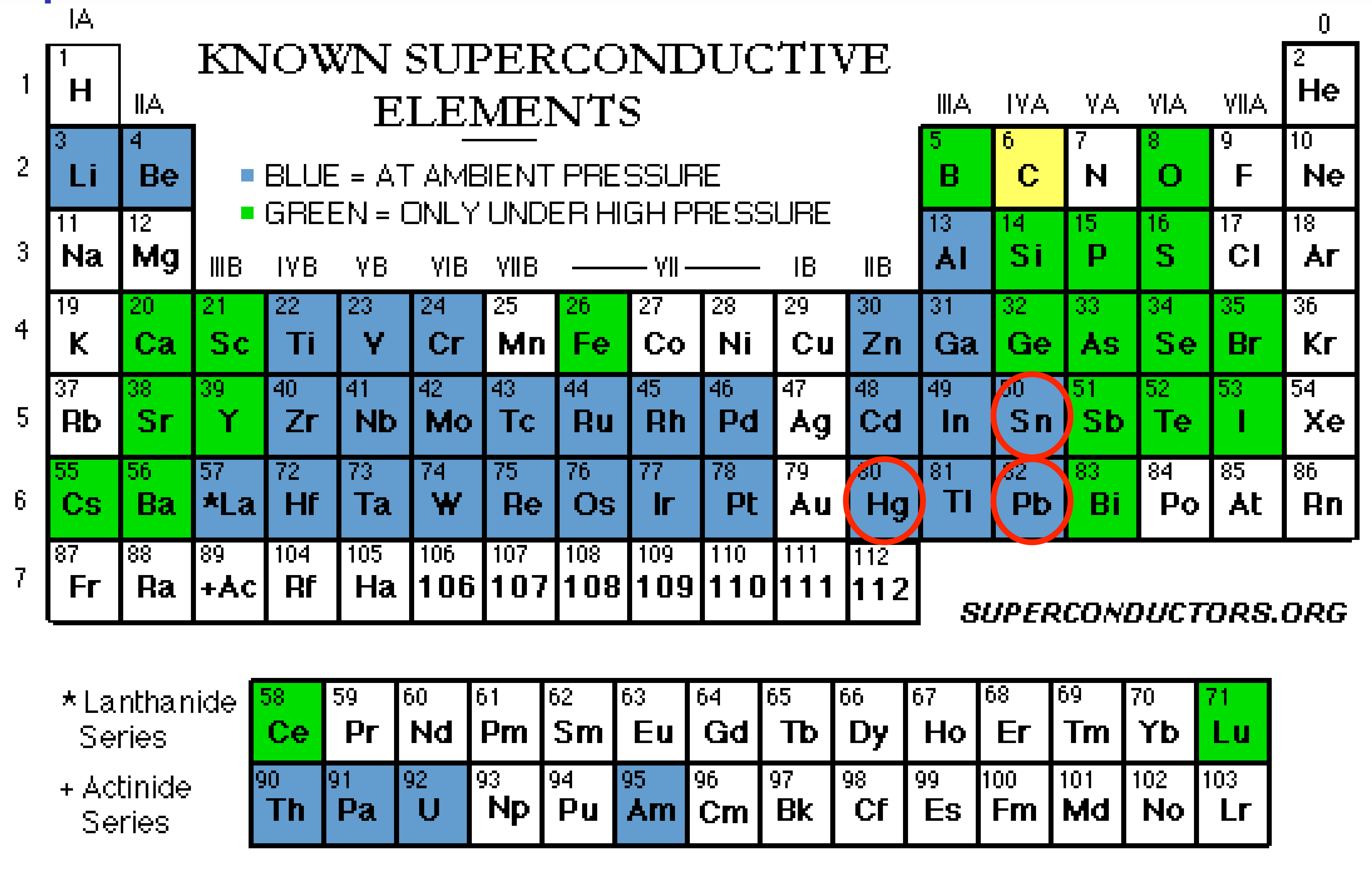
Without realizing the origin, the Leiden team had observed rapid heat transfer in the superfluid phase of liquid helium below 2.2 K discovered by J.F. Allen, A. D. Misener and P. Kapitza in 1937. [Nature, 141, 74 (1938)]

Elemental Superconductors



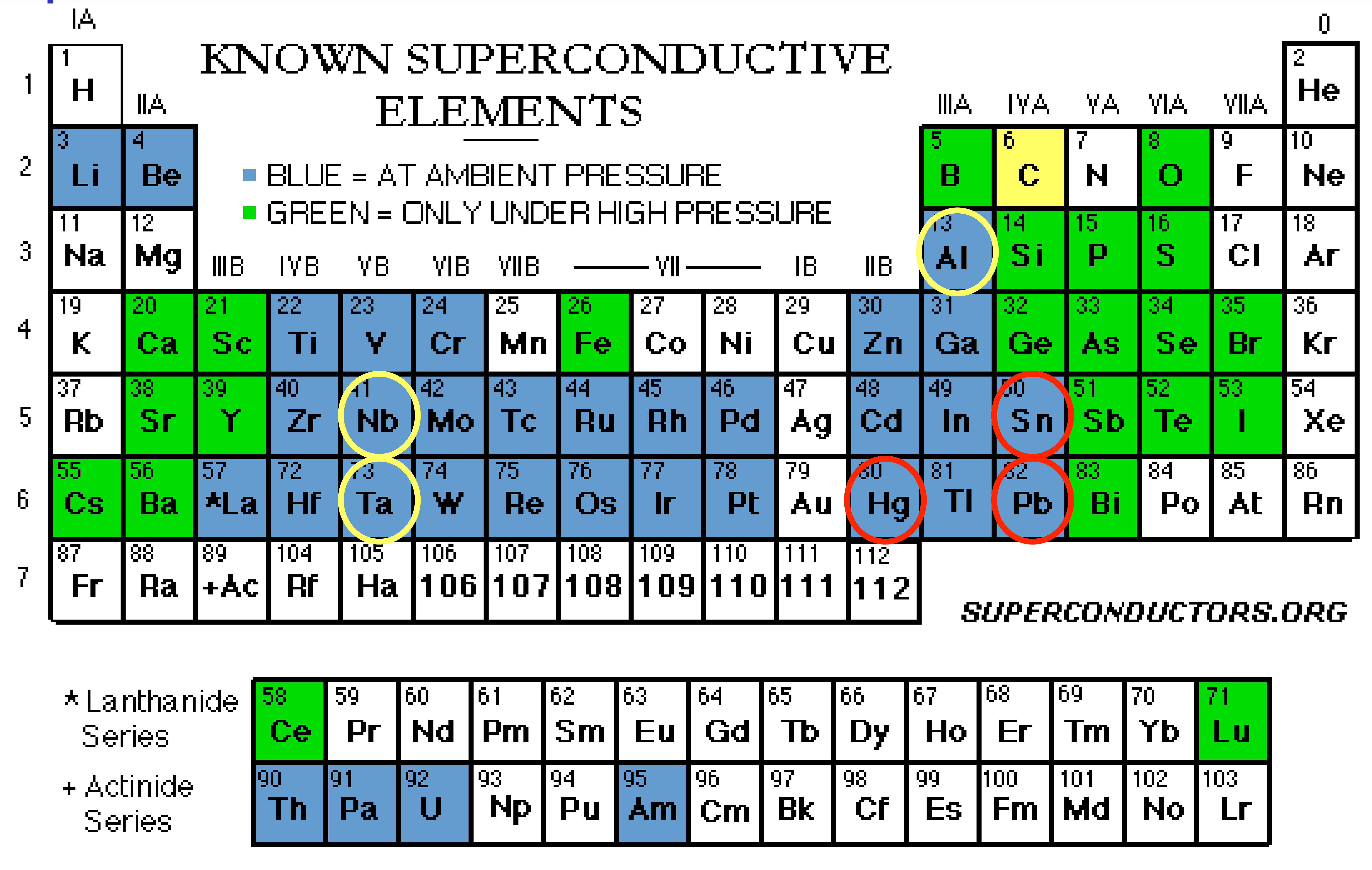
- Lowest T_c Elemental Superconductor: Rh: $T_c = 0.32 \times 10^{-3} K$
- Highest T_c Elemental Superconductor: Nb: $T_c = 9.33\,K$
- Highest T_c Superconducting Compound: $(Hg_{0.8}Tl_{0.2})Ba_2Ca_2Cu_3O_{8.33}$: $T_c = 138\,K$

Elemental Superconductors



- Lowest T_c Elemental Superconductor: Rh: $T_c = 0.32 \times 10^{-3} K$
- Highest T_c Elemental Superconductor: Nb: $T_c = 9.33\,K$
- Highest T_c Superconducting Compound: $(Hg_{0.8}Tl_{0.2})Ba_2Ca_2Cu_3O_{8.33}$: $T_c = 138\,K$

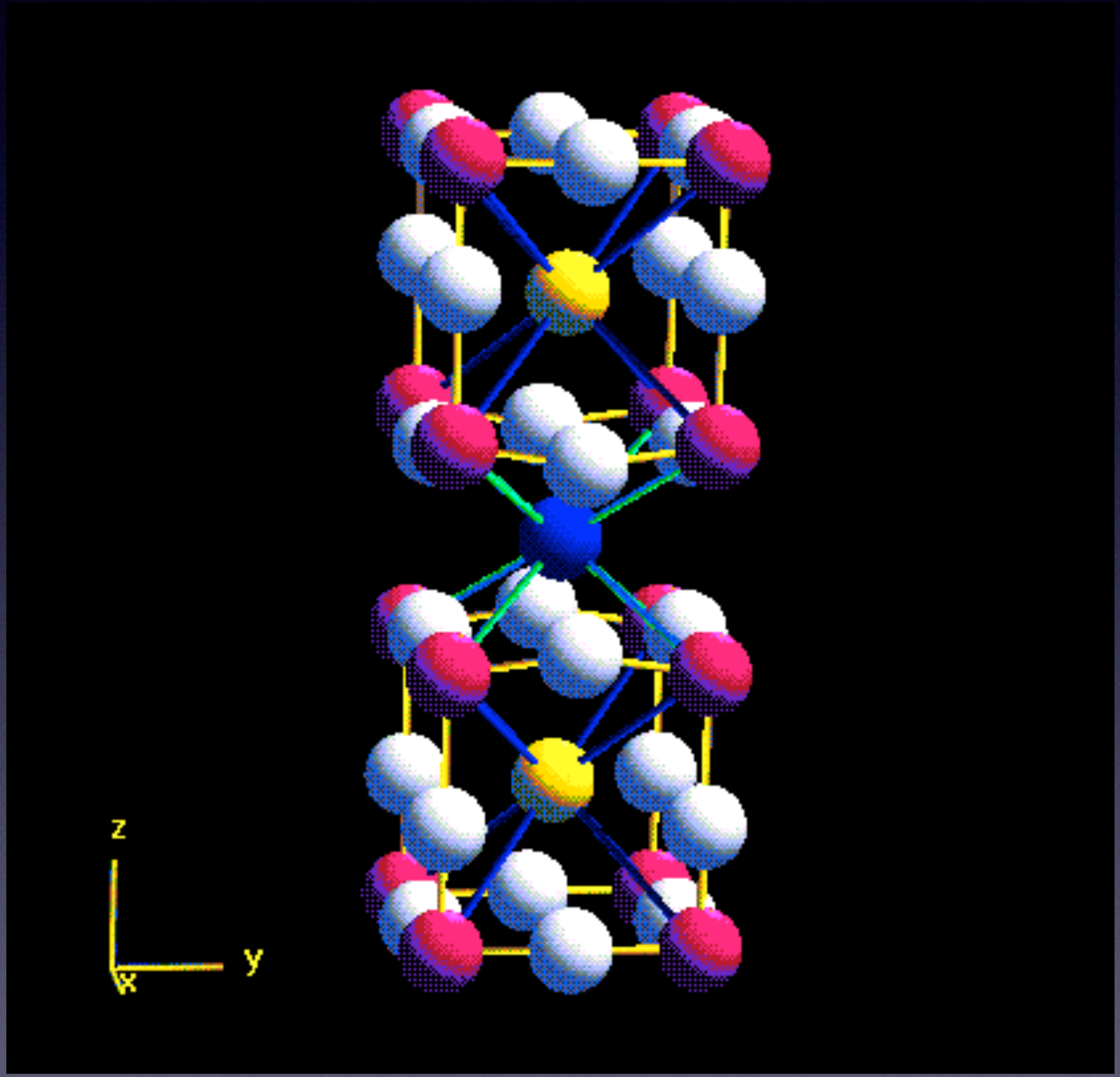
Elemental Superconductors



- Lowest T_c Elemental Superconductor: Rh: $T_c = 0.32 \times 10^{-3} K$
- Highest T_c Elemental Superconductor: Nb: $T_c = 9.33\,K$
- Highest T_c Superconducting Compound: $(Hg_{0.8}Tl_{0.2})Ba_2Ca_2Cu_3O_{8.33}$: $T_c = 138\,K$

High Tc Superconductors

$Y_{0.5}Lu_{0.5})Ba_2Cu_3O_7$	107 K
$(Y_{0.5}Tm_{0.5})Ba_2Cu_3O_7$	105 K
$(Y_{0.5}Gd_{0.5})Ba_2Cu_3O_7$	97 K
$Y_2CaBa_4Cu_7O_{16}$	97 K
$Y_3Ba_4Cu_7O_{16}$	96 K
$NdBa_2Cu_3O_7$	96 K
$Y_2Ba_4Cu_7O_{15}$	95 K
$GdBa_2Cu_3O_7$	94 K
$YBa_2Cu_3O_7$	92 K
$TmBa_2Cu_3O_7$	90 K
$YbBa_2Cu_3O_7$	89 K
$YSr_2Cu_3O_7$	62 K



http://superconductors.org/

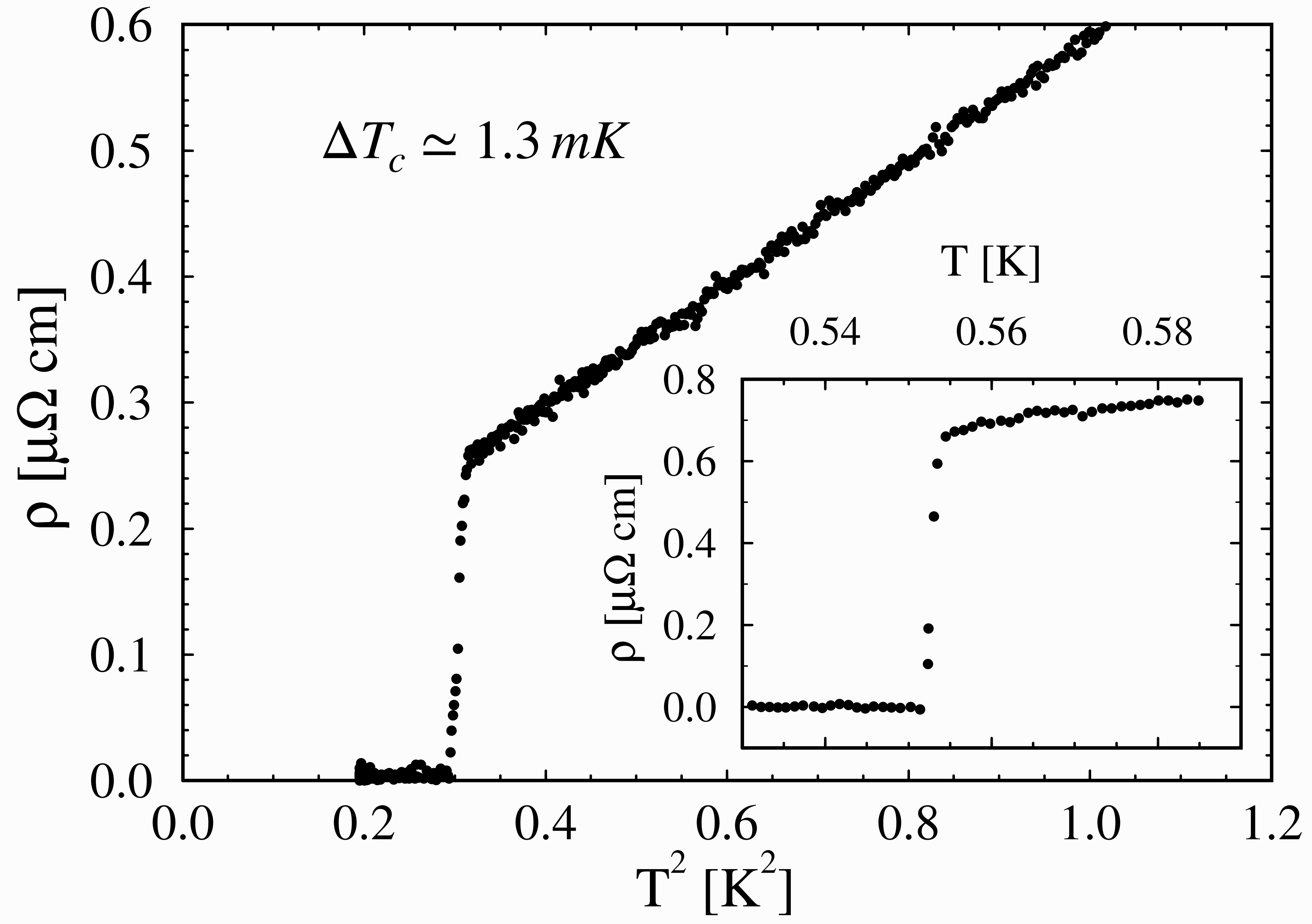
$Bi_{1.6}Pb_{0.6}Sr_2Ca_2Sb_{0.1}Cu_3O_y$	115 K
$Bi_2Sr_2Ca_2Cu_3O_{10}$***	110 K
$Bi_2Sr_2CaCu_2O_9$***	110 K
$Bi_2Sr_2(Ca_{0.8}Y_{0.2})Cu_2O_8$	95-96K
$Bi_2Sr_2CaCu_2O_8$	91-92K

$(Hg_{0.8}Tl_{0.2})Ba_2Ca_2Cu_3O_{8.33}$	138 K*
$HgBa_2Ca_2Cu_3O_8$	133-135 K
$HgBa_2Ca_3Cu_4O_{10+}$	125-126 K
$HgBa_2(Ca_{1-x}Sr_x)Cu_2O_{6+}$	123-125 K
$HgBa_2CuO_4+$	94-98 K

http://phycomp.technion.ac.il/~ira/superconductors.html

Superconductivity in Single Crystals of UPt_3

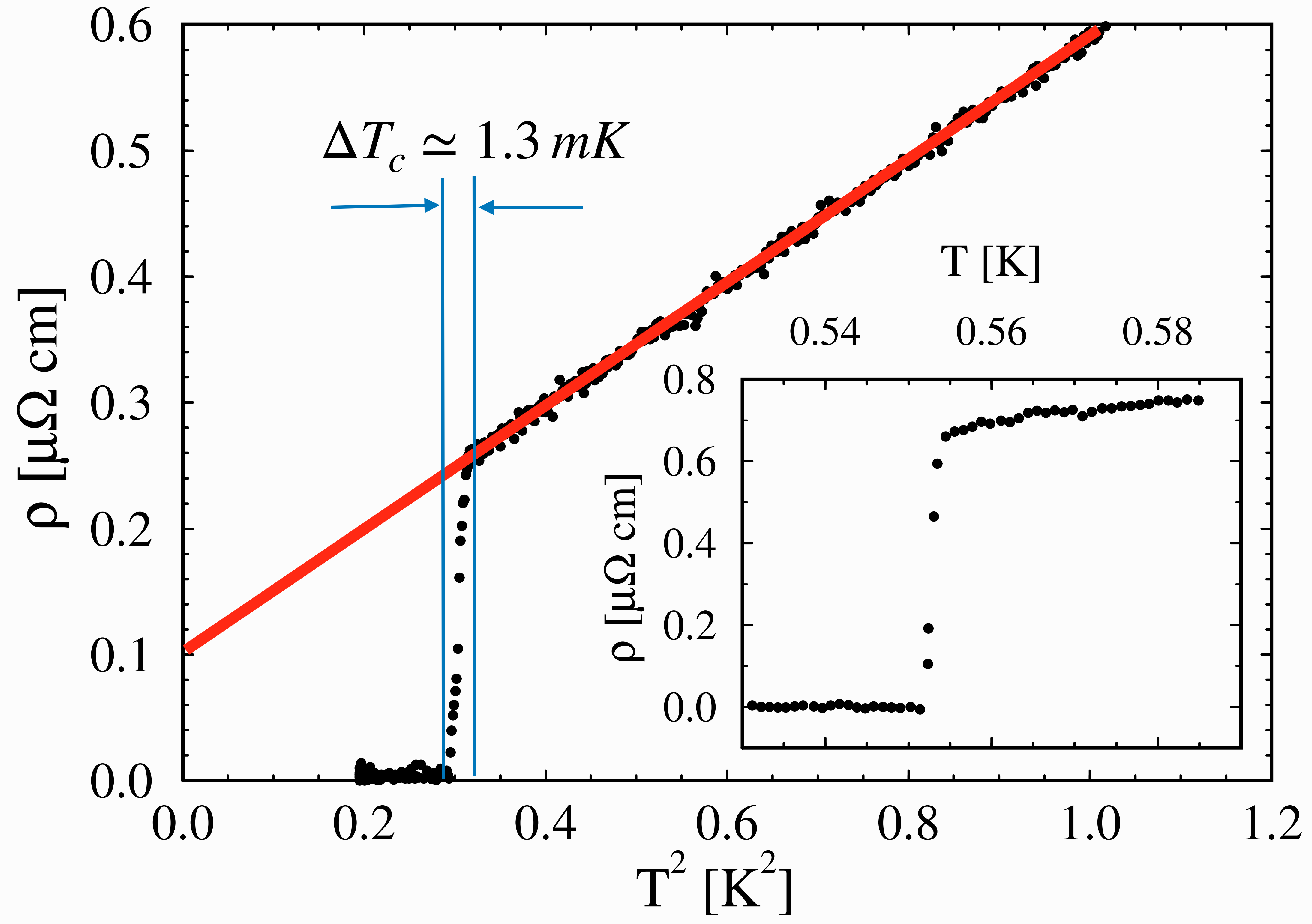
J.B. Kycia et al., Phys. Rev. B 58, R603 (1998).



- Resistivity of UPt_3 annealed at $900^{\rm o}$C. $\rho = \rho_0 + AT^2$
- $T_c = 552\,$mK Inset: Superconducting transition for a sample annealed at $800^{\rm o}$C. The transition *width* is $1.3\,$mK.

Superconductivity in Single Crystals of UPt_3

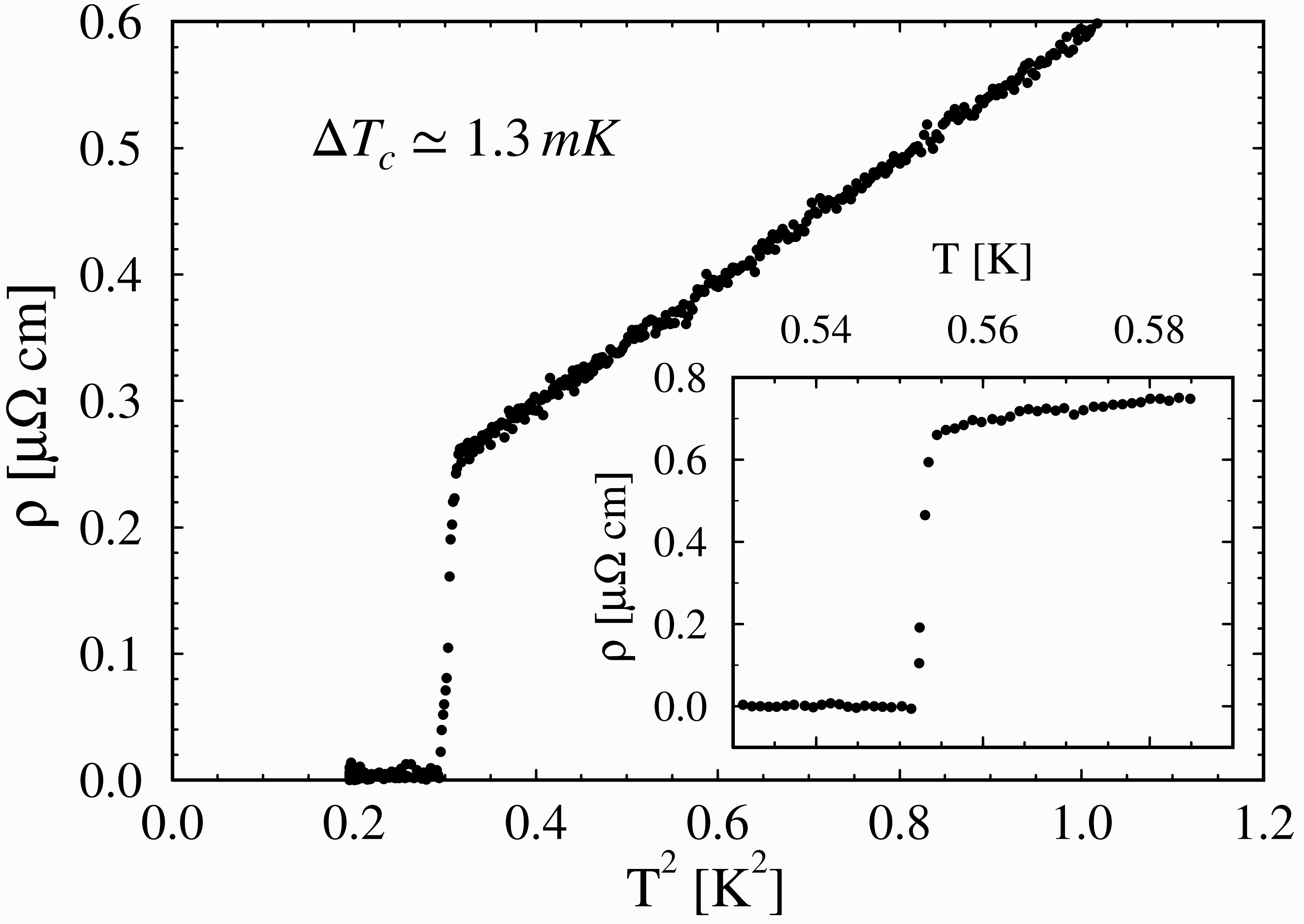
J.B. Kycia et al., Phys. Rev. B 58, R603 (1998).



- Resistivity of UPt_3 annealed at 900°C. $\rho = \rho_0 + AT^2$
- $T_c = 552\,$mK Inset: Superconducting transition for a sample annealed at 800°C. The transition *width* is $1.3\,$mK.

Superconductivity in Single Crystals of UPt_3

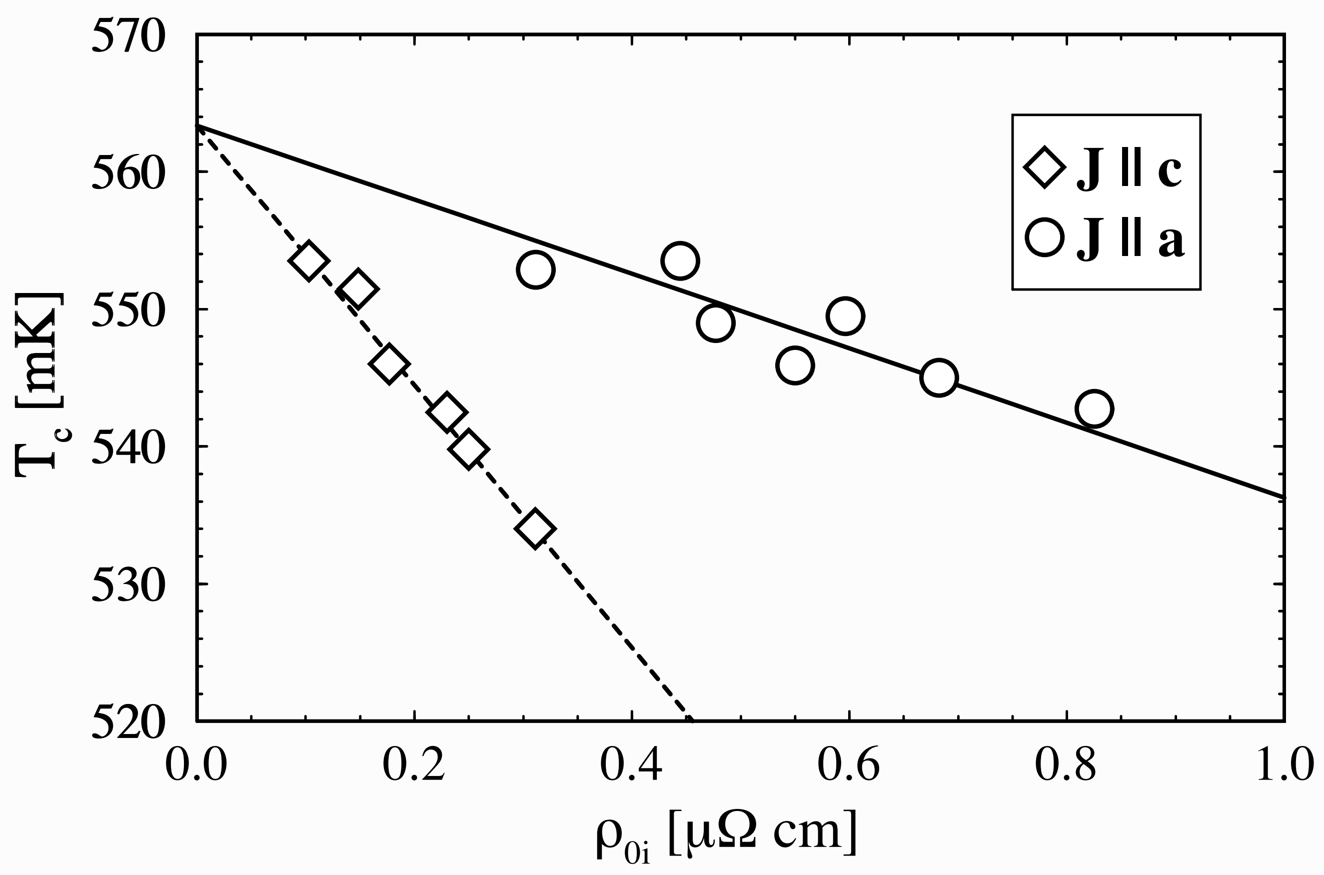
J.B. Kycia et al., Phys. Rev. B 58, R603 (1998).



- Resistivity of UPt_3 annealed at 900^oC. $\rho = \rho_0 + AT^2$
- $T_c = 552\,$mK Inset: Superconducting transition for a sample annealed at 800^oC. The transition *width* is $1.3\,$mK.

- $RRR \equiv \rho(300\,K)/\rho(T \to 0)$
- UPt_3 Crystal ($RRR \approx 1500$)

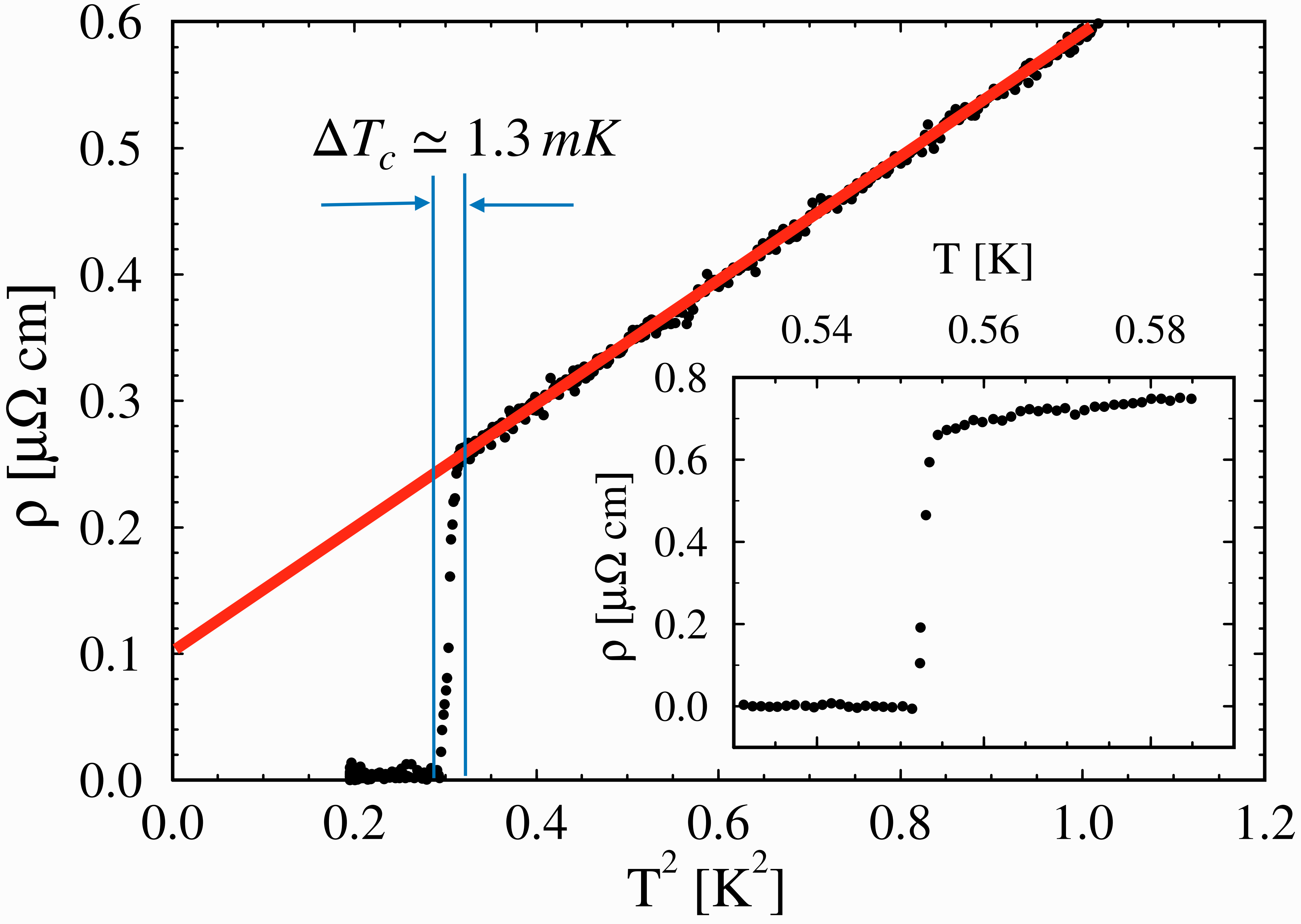
T_c vs. residual resistivity



- Superconductivity in UPt_3 is very sensitive to disorder!

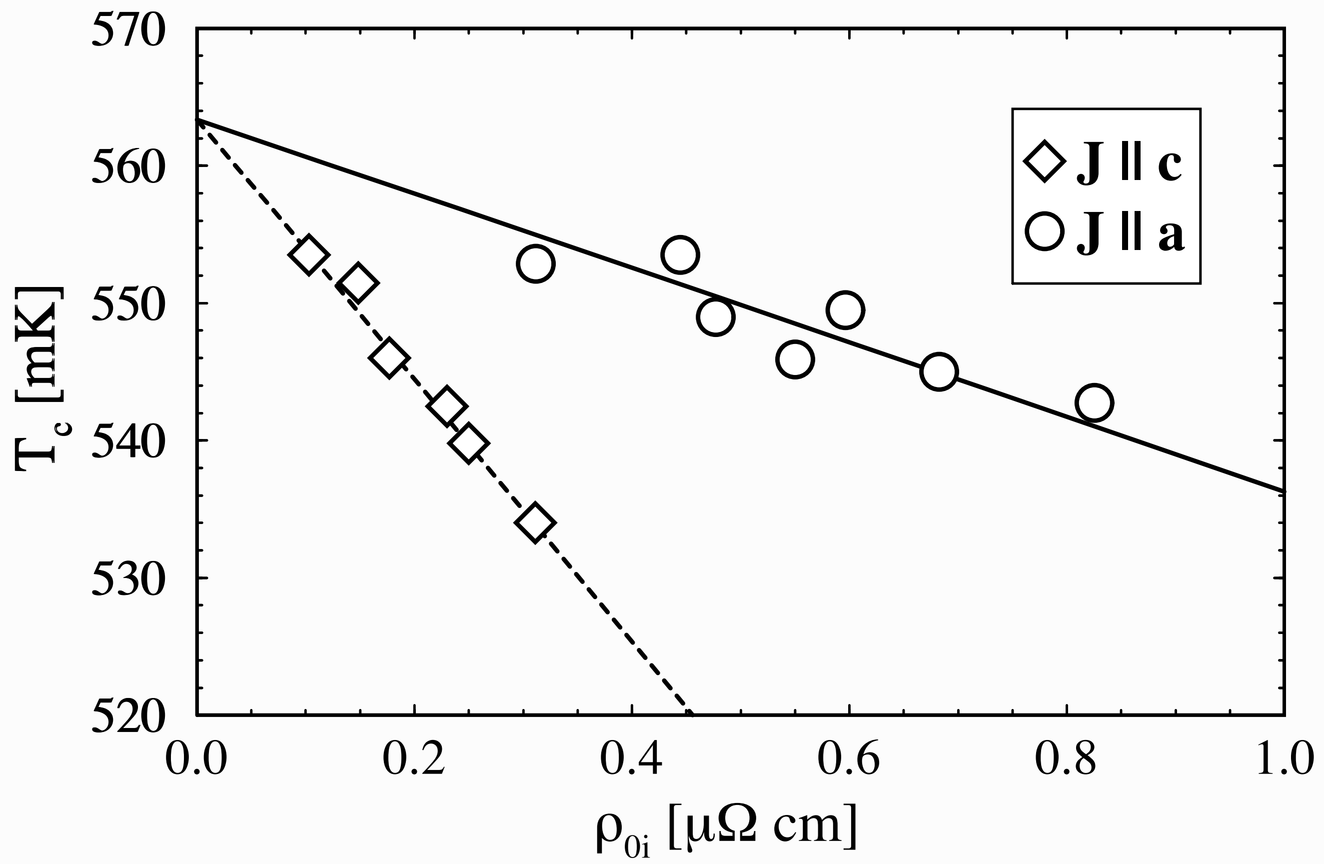
Superconductivity in Single Crystals of UPt_3

J.B. Kycia et al., Phys. Rev. B 58, R603 (1998).



- $RRR \equiv \rho(300\,K)/\rho(T \to 0)$
- UPt_3 Crystal ($RRR \approx 1500$)

T_c vs. residual resistivity



- Superconductivity in UPt_3 is very sensitive to disorder!

- Resistivity of UPt_3 annealed at 900°C. $\rho = \rho_0 + AT^2$
- $T_c = 552\,\mathrm{mK}$ Inset: Superconducting transition for a sample annealed at 800°C. The transition *width* is $1.3\,\mathrm{mK}$.

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

Supercurrents can persist in *metastable* states

Kamerlingh Onnes

- ✓ 1911 - Superconductivity of Hg T < 4.2K
- ✓ 1913 - Persistent Currents in a Pb ring

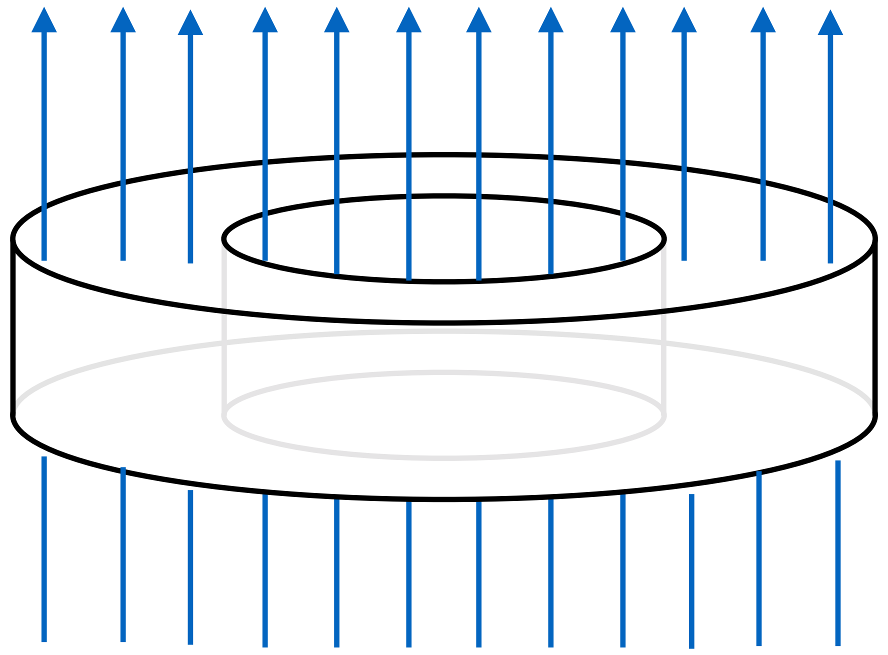
Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

✓ 1913 - Persistent Currents in a Pb ring

applied B-field



$T > T_c$

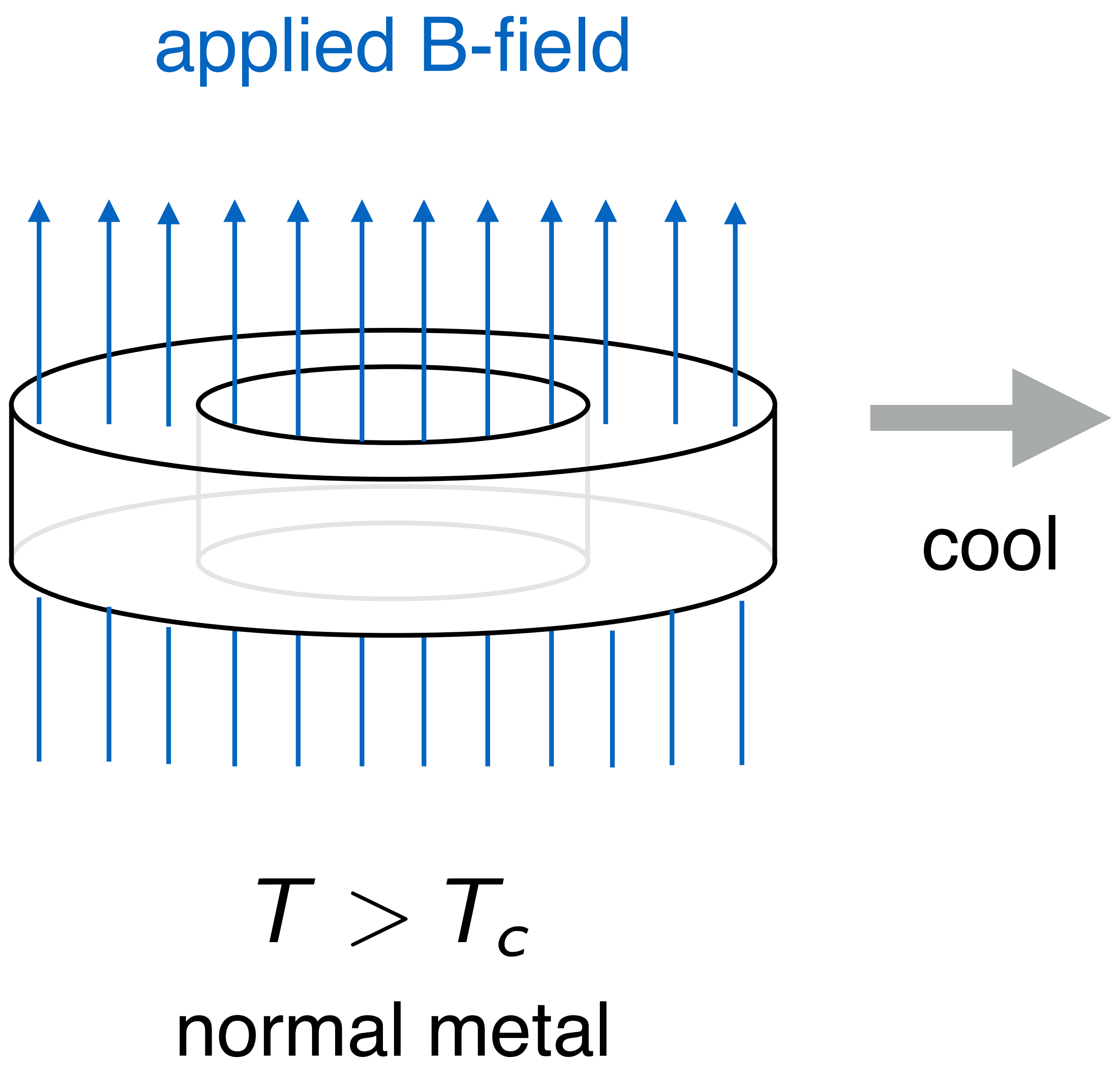
normal metal

Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

✓ 1913 - Persistent Currents in a Pb ring

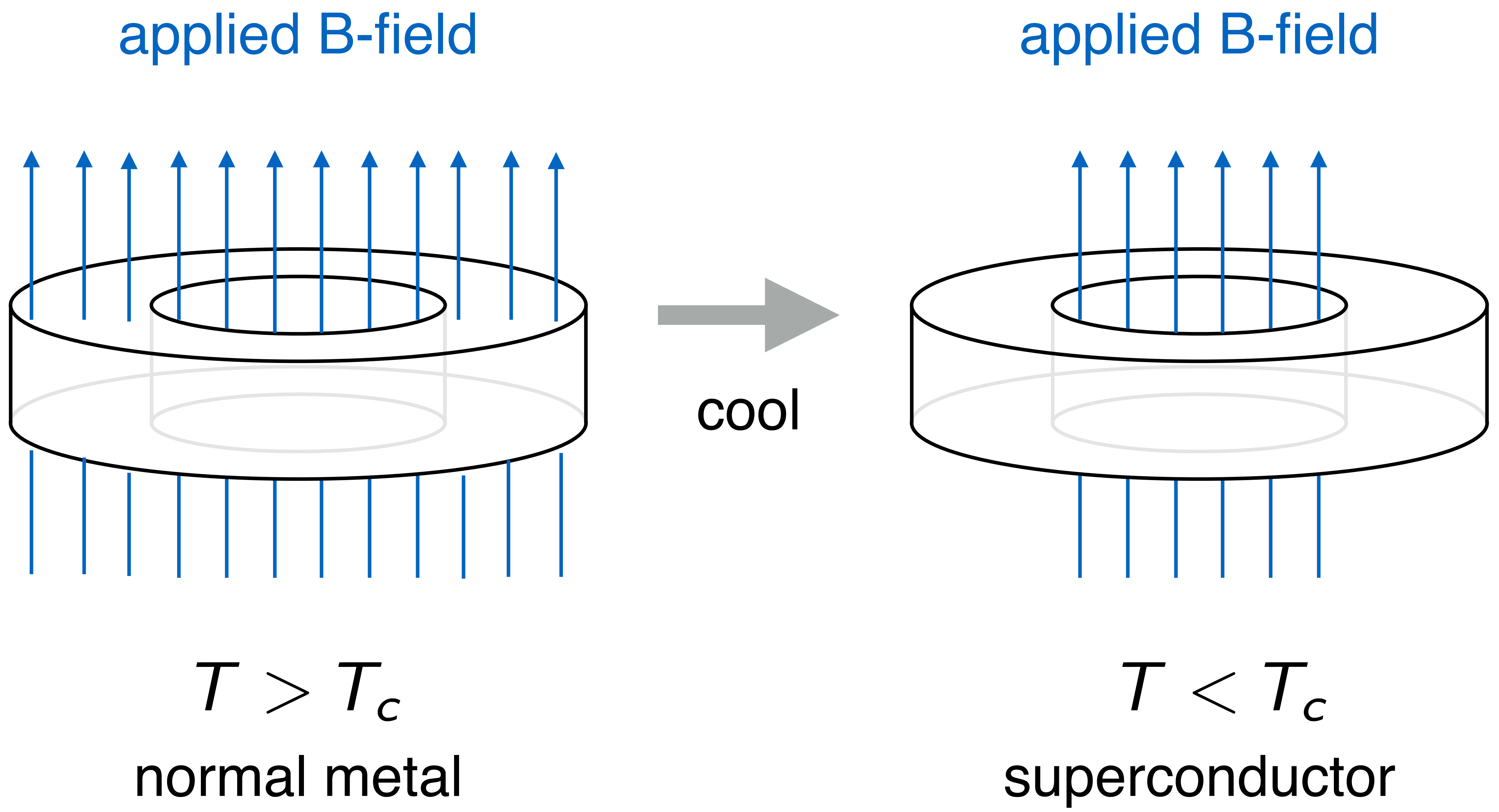


Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

✓ 1913 - Persistent Currents in a Pb ring

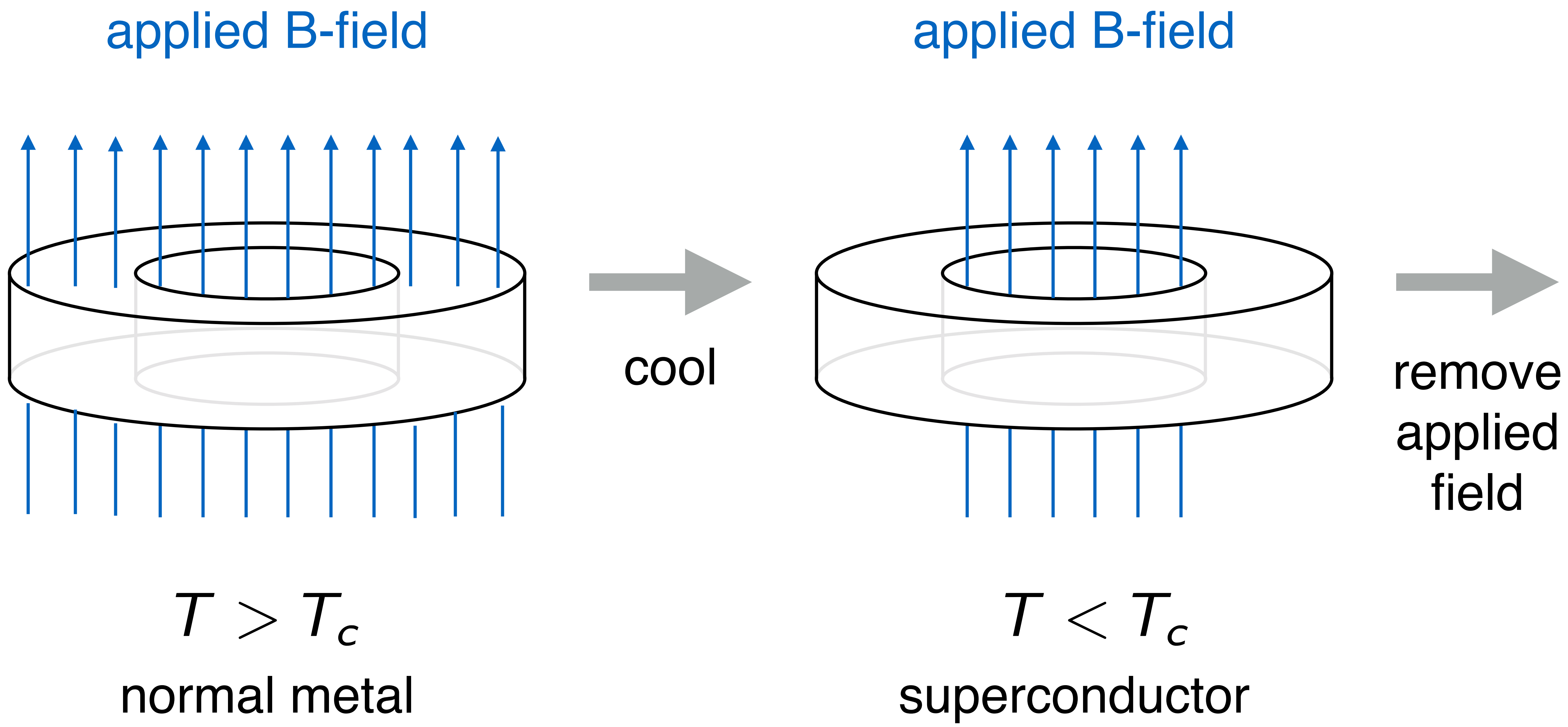


Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

✓ 1913 - Persistent Currents in a Pb ring

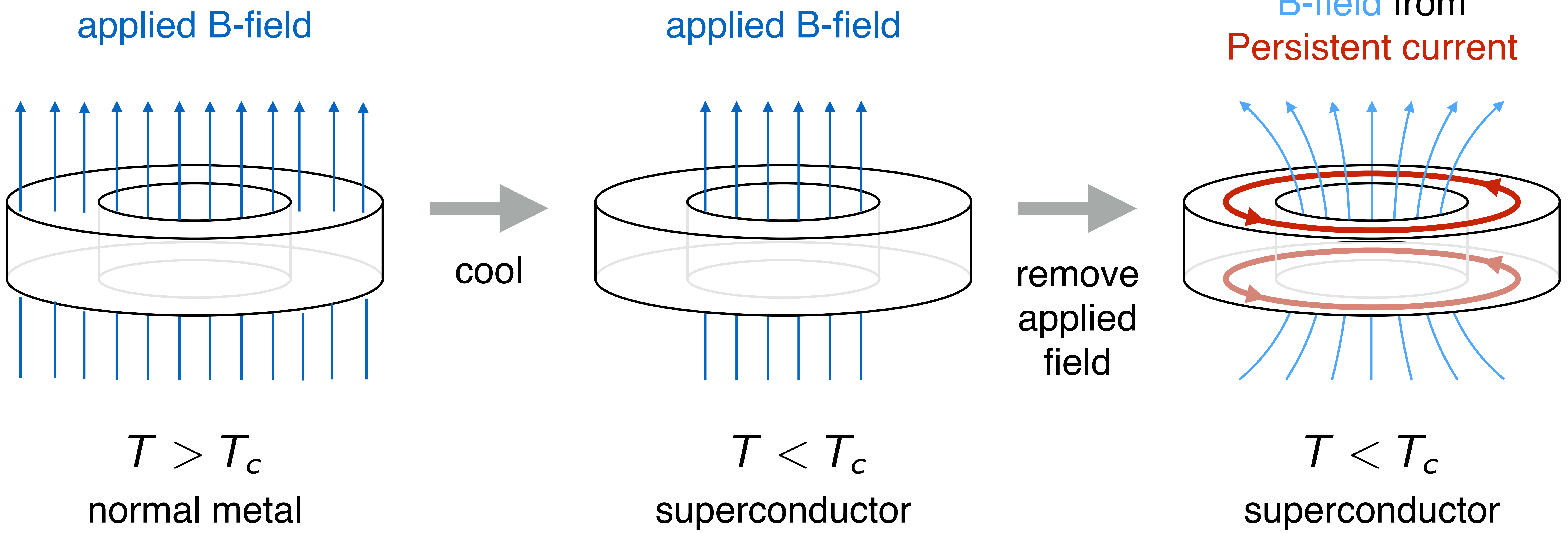


Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

✓ 1913 - Persistent Currents in a Pb ring

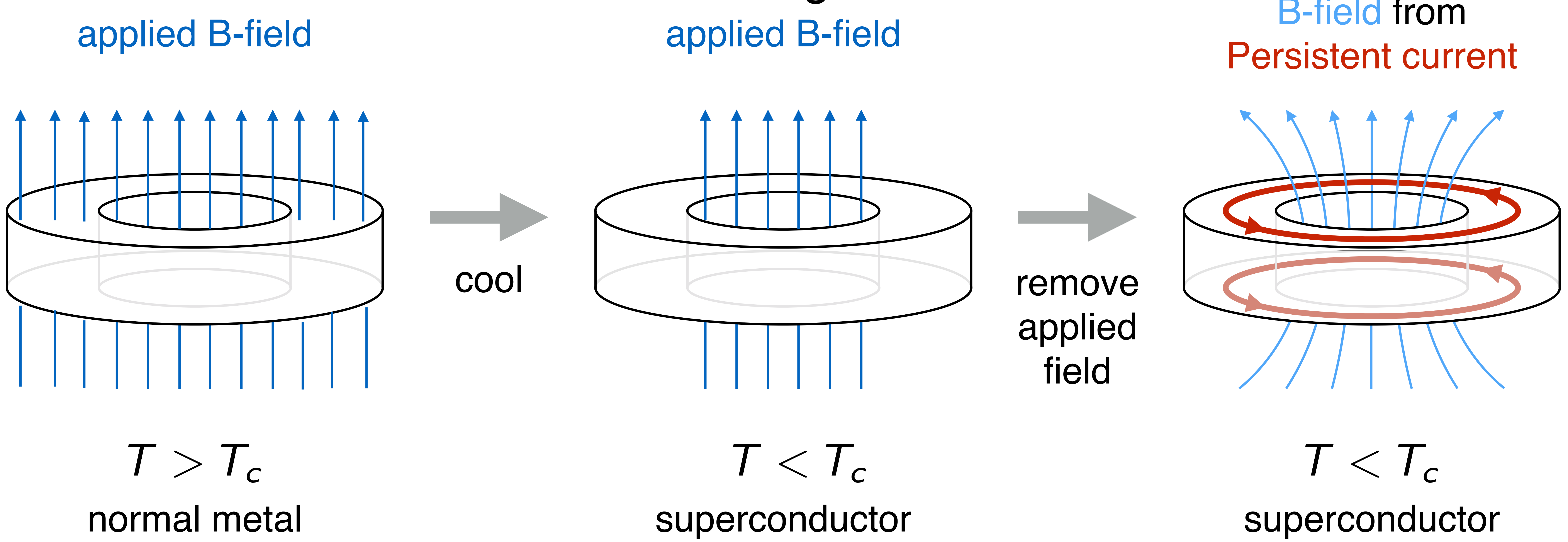


Supercurrents can persist in *metastable* states

Kamerlingh Onnes

✓ 1911 - Superconductivity of Hg T < 4.2K

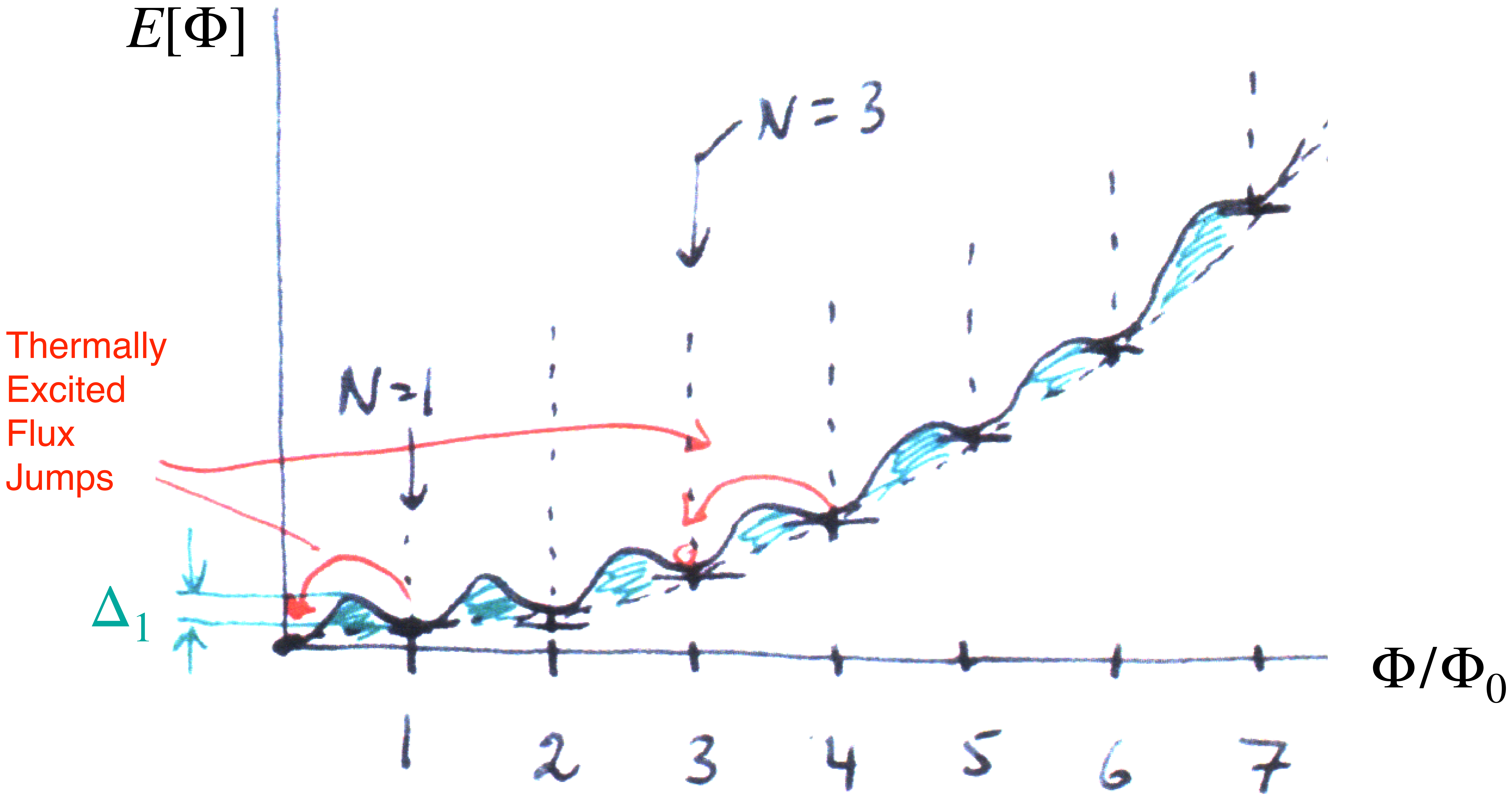
✓ 1913 - Persistent Currents in a Pb ring



❖ *File & Mills, Phys Rev Lett 10, 93 (1963)* - $\tau > 10^5$ *yr*

❖ *Quinn & Ittner, J. Appl. Phys. 33, 748 (2004)* - $\rho(T < T_c) < 3.6 \times 10^{-23} \Omega - cm$

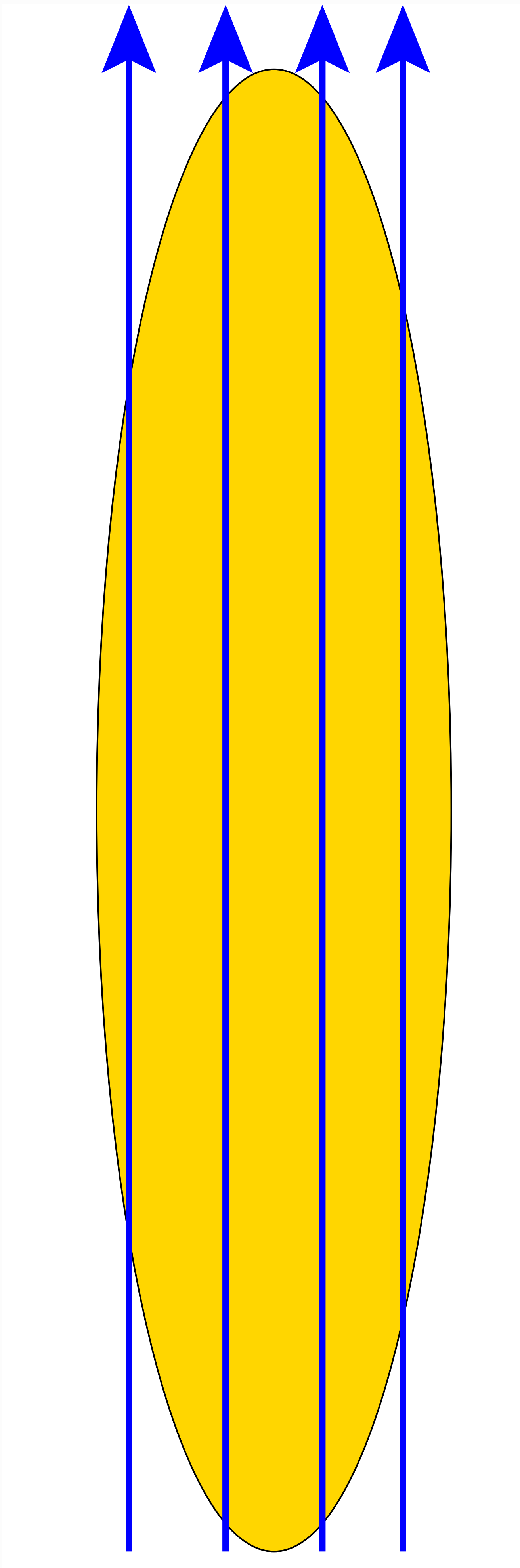
Energy Landscape of Current Carrying States of a Toroidal Ring



❖Metastable Persistent Current States protected by a large activation barrier

Perfect Diamagnetism

1933 - Walther Meissner & Robert Ochsenfeld, T.U. Munich

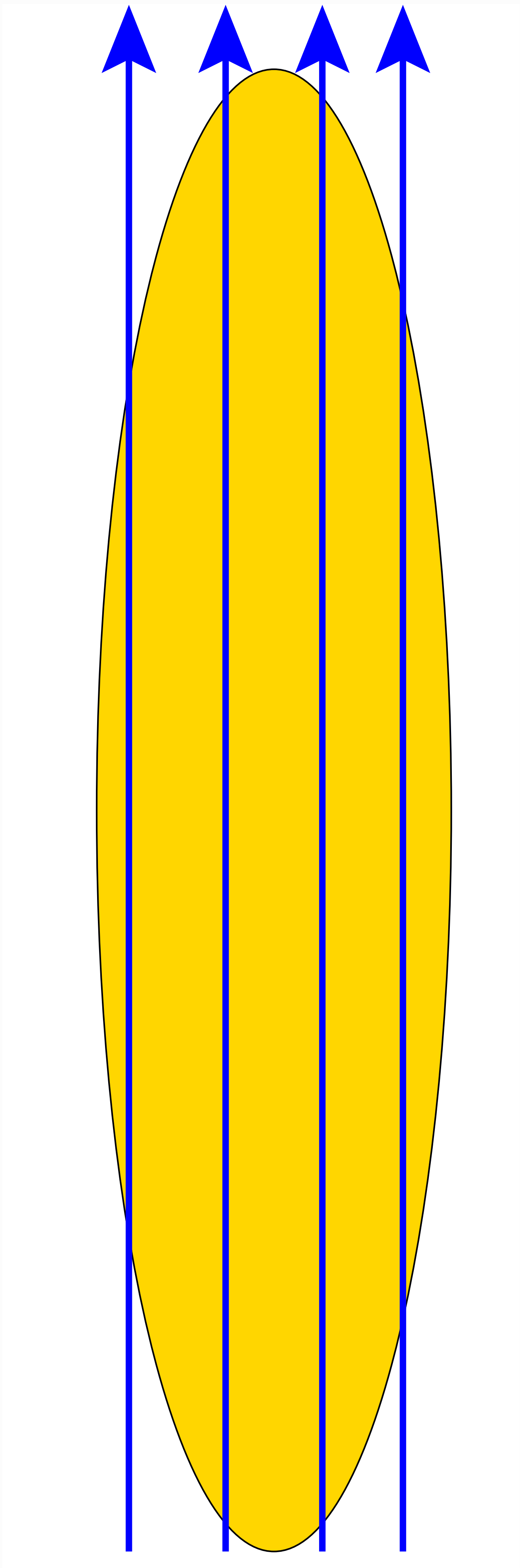


$T > T_c$

$B = H$

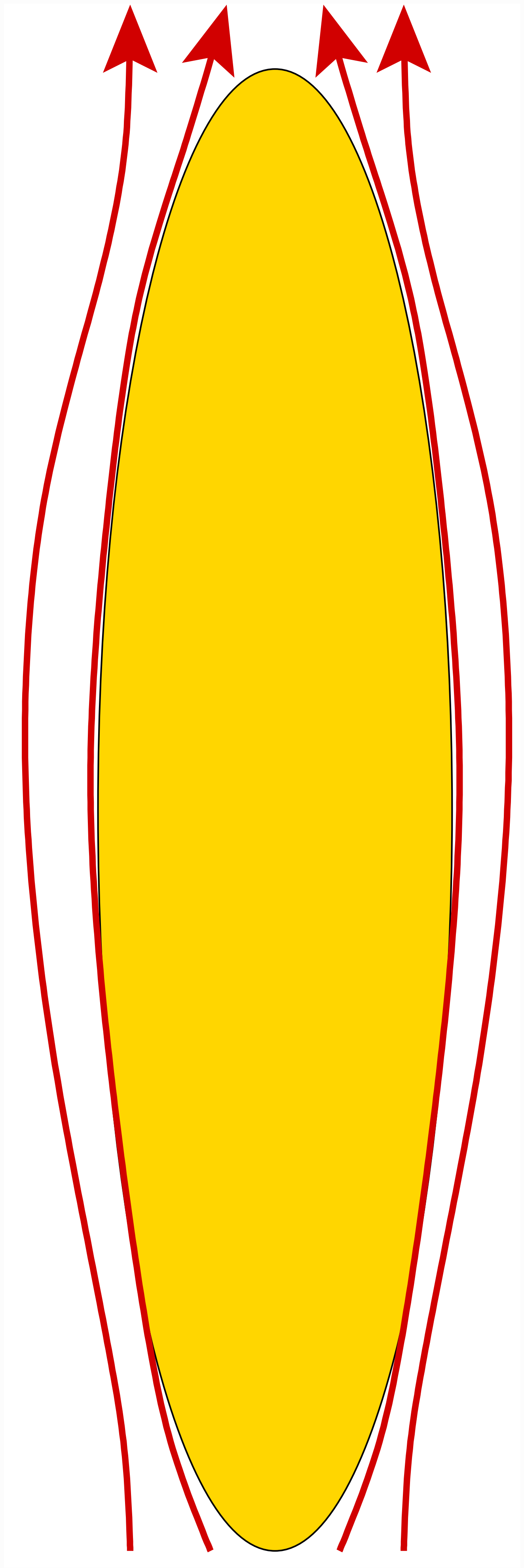
Perfect Diamagnetism

1933 - Walther Meissner & Robert Ochsenfeld, T.U. Munich



$T > T_c$

$B = H$

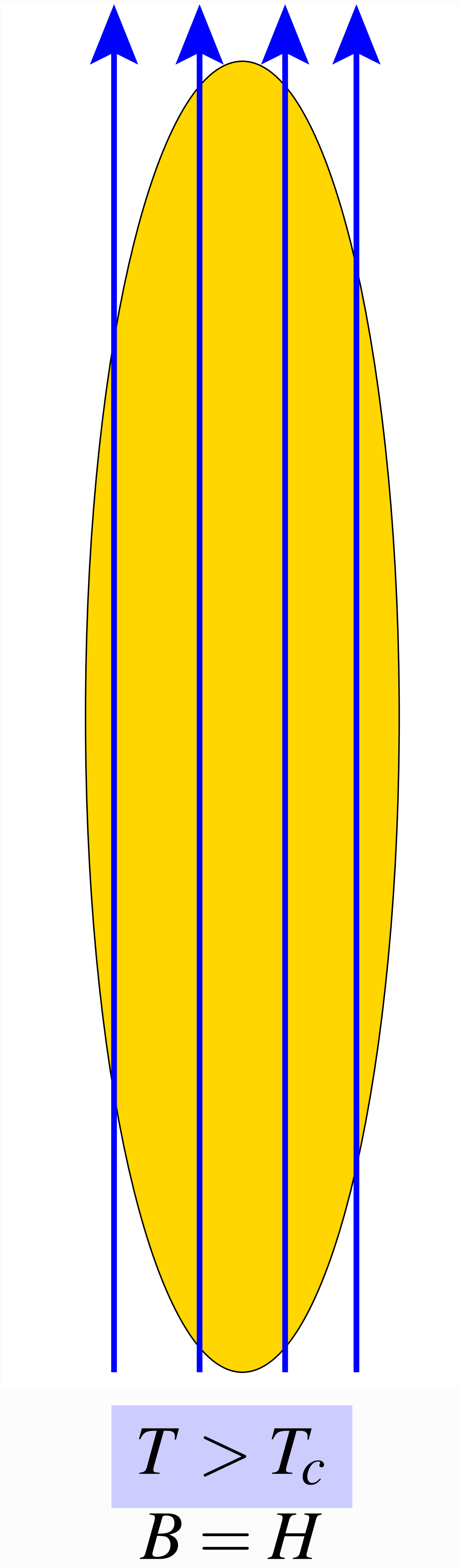


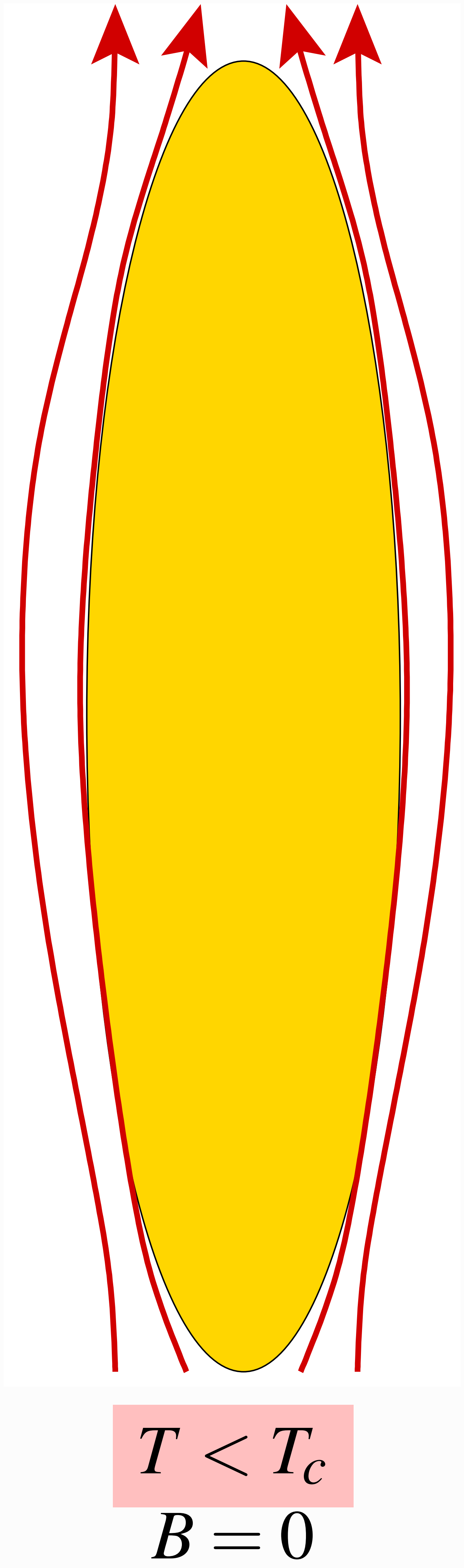
$T < T_c$

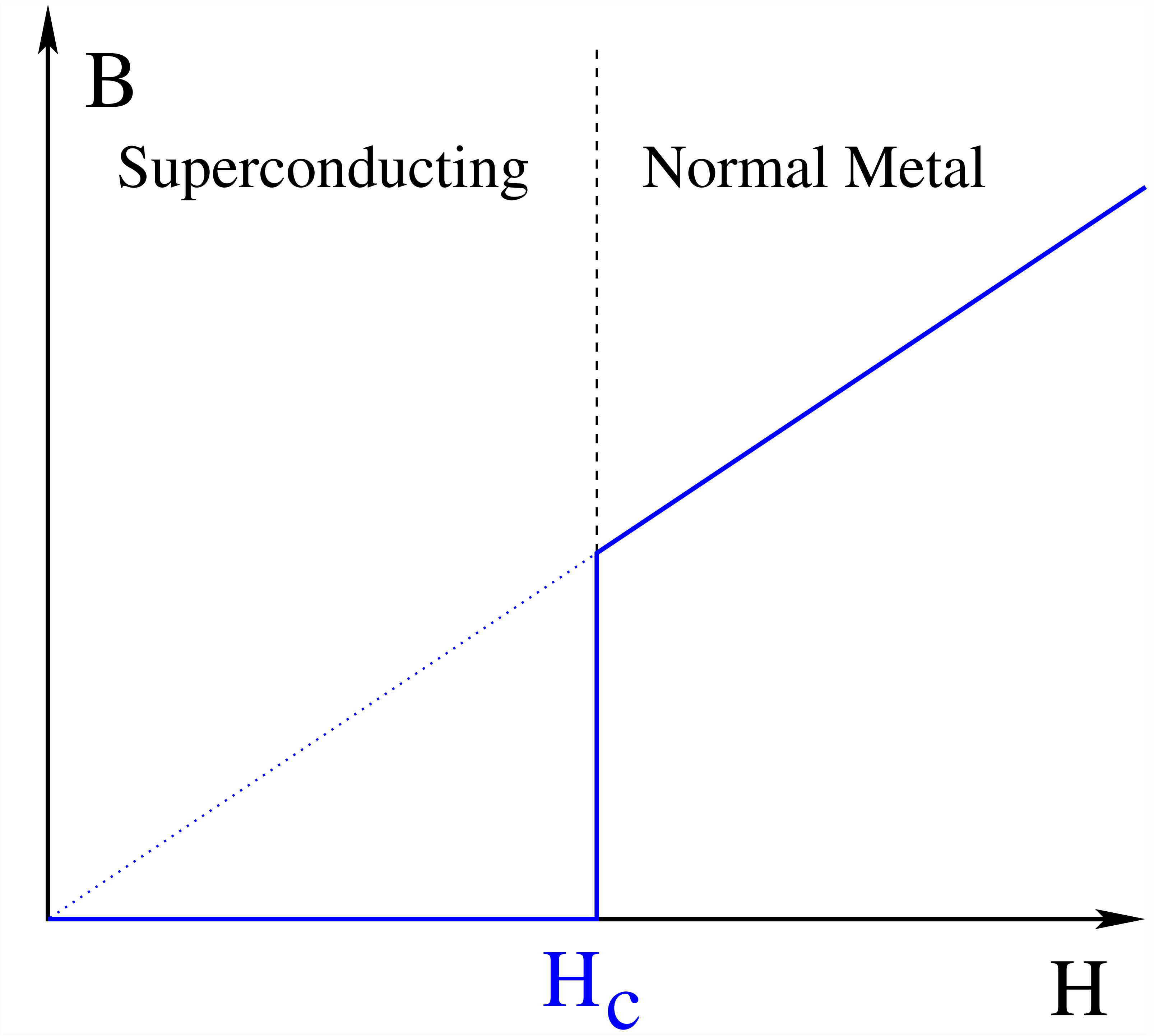
$B = 0$

Perfect Diamagnetism

1933 - Walther Meissner & Robert Ochsenfeld, T.U. Munich

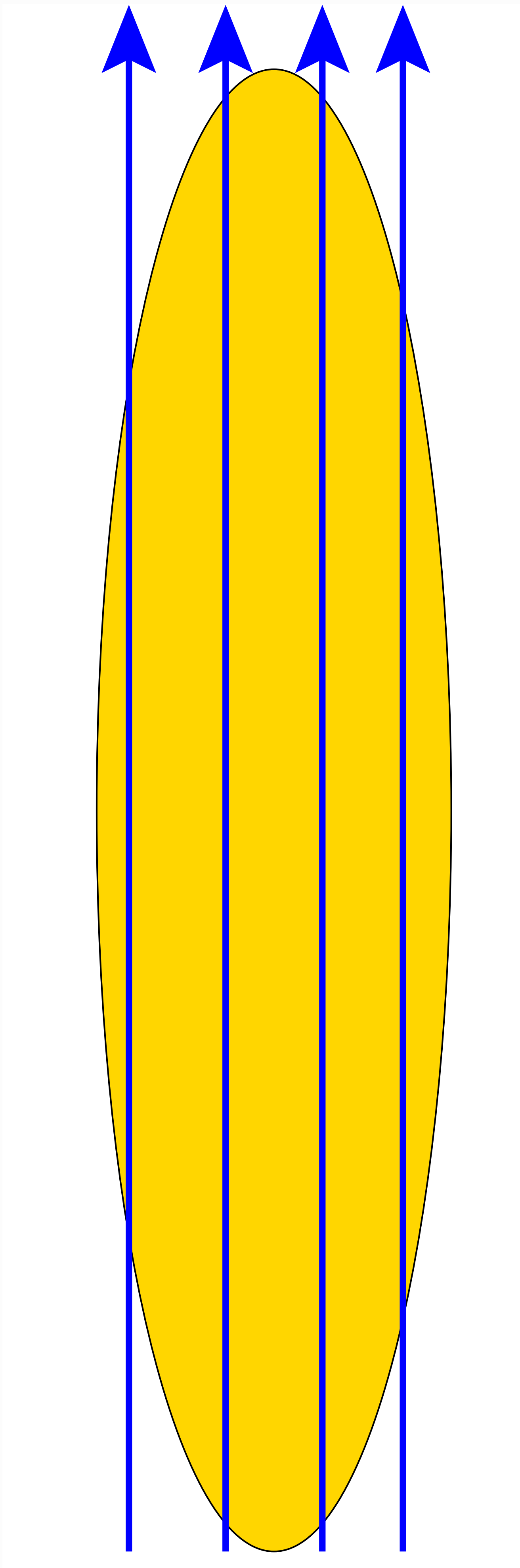




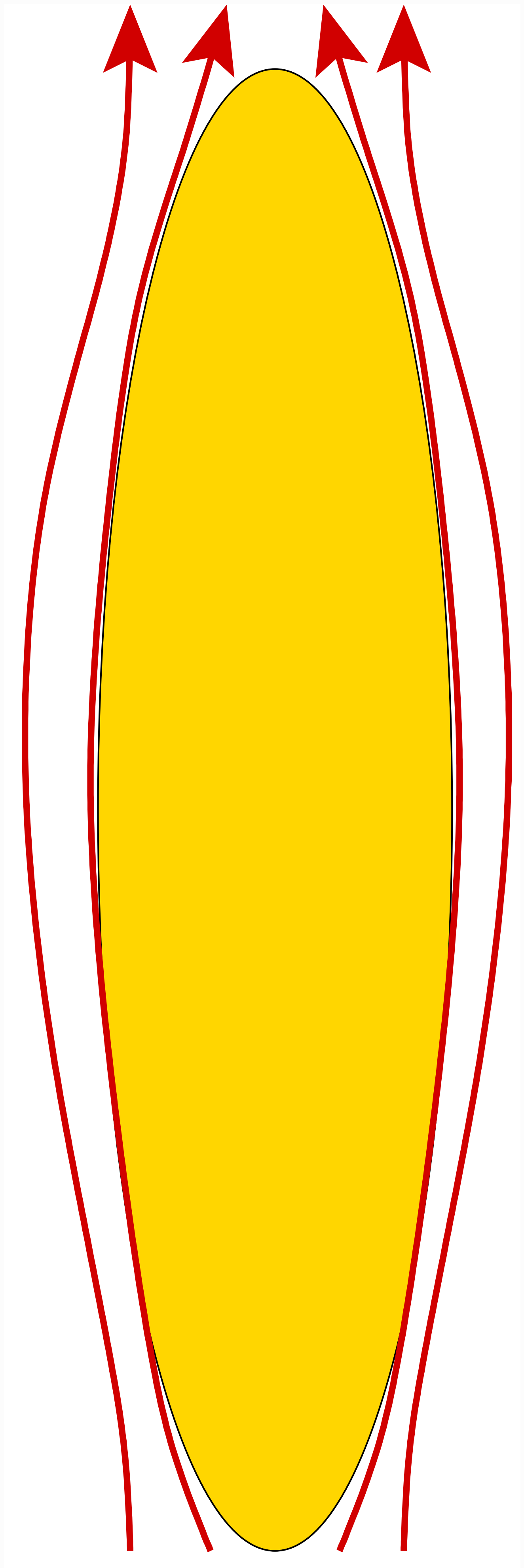


Perfect Diamagnetism

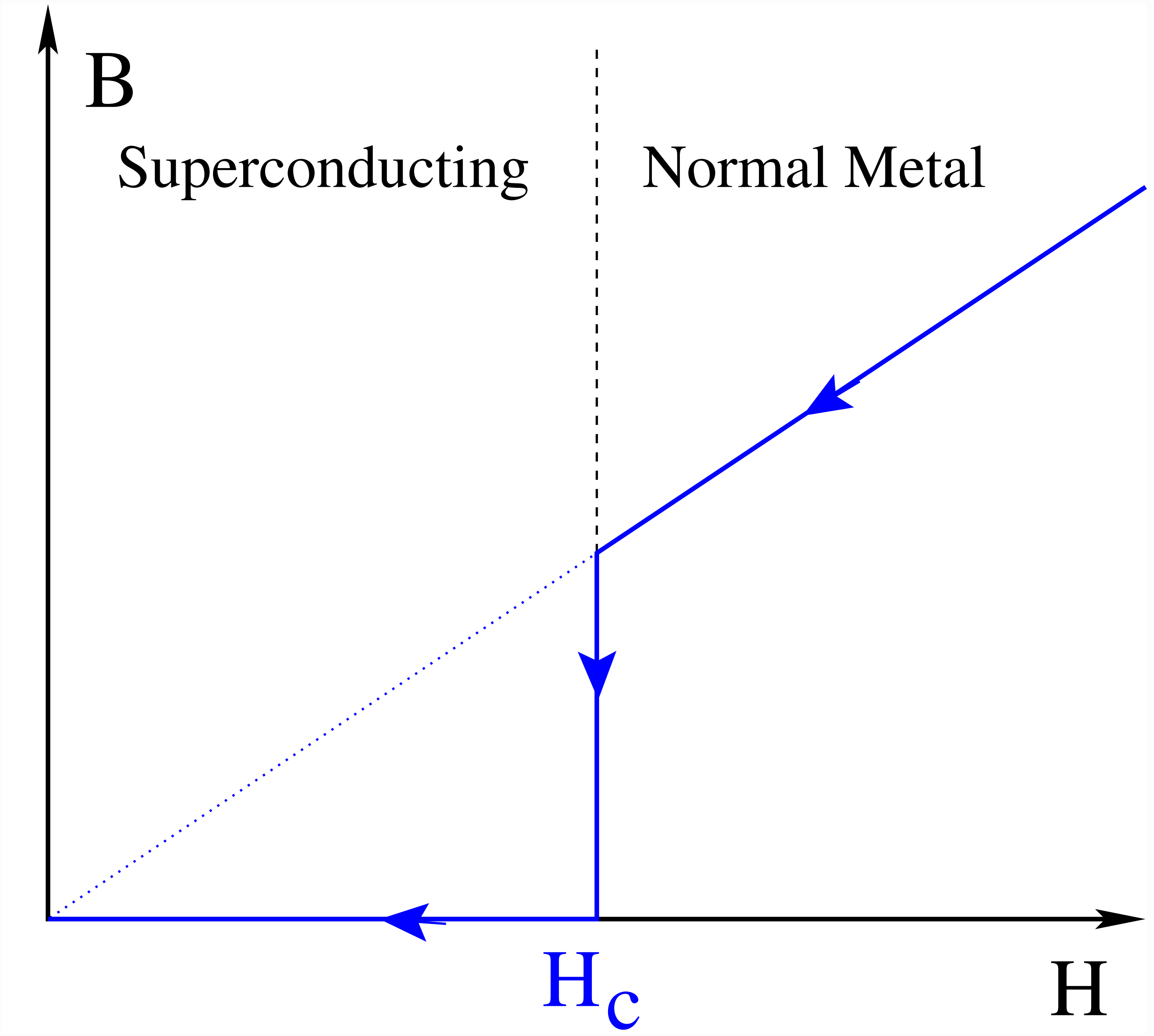
1933 - Walther Meissner & Robert Ochsenfeld, T.U. Munich



$T > T_c$
$B = H$



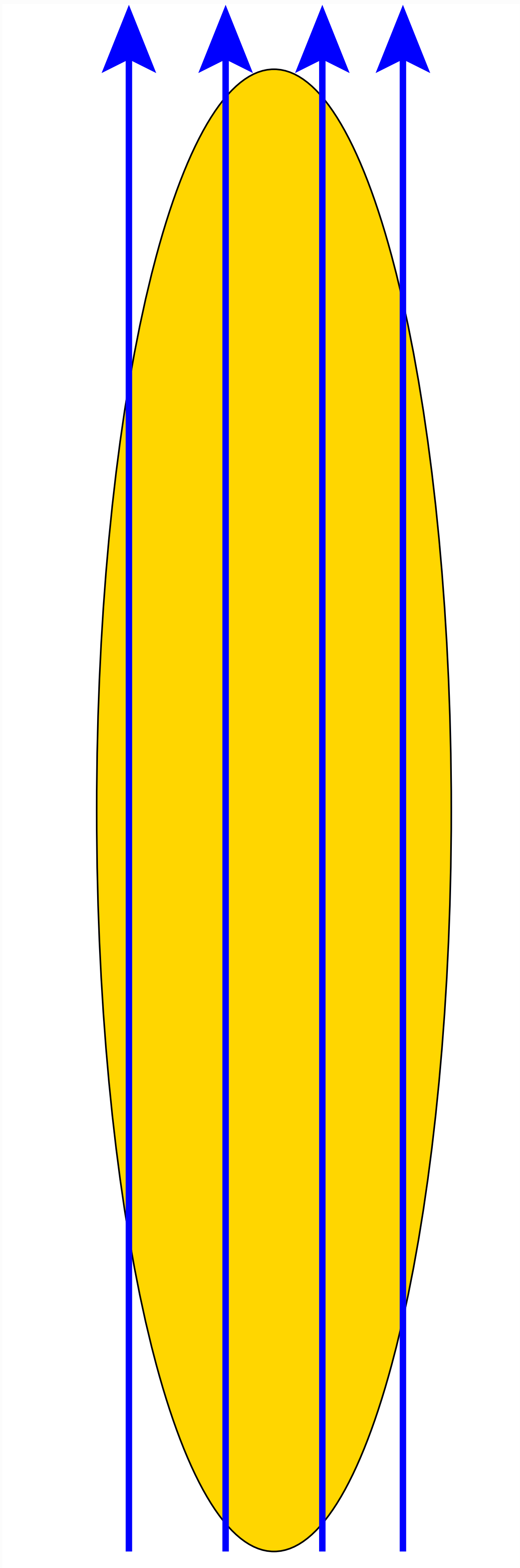
$T < T_c$
$B = 0$



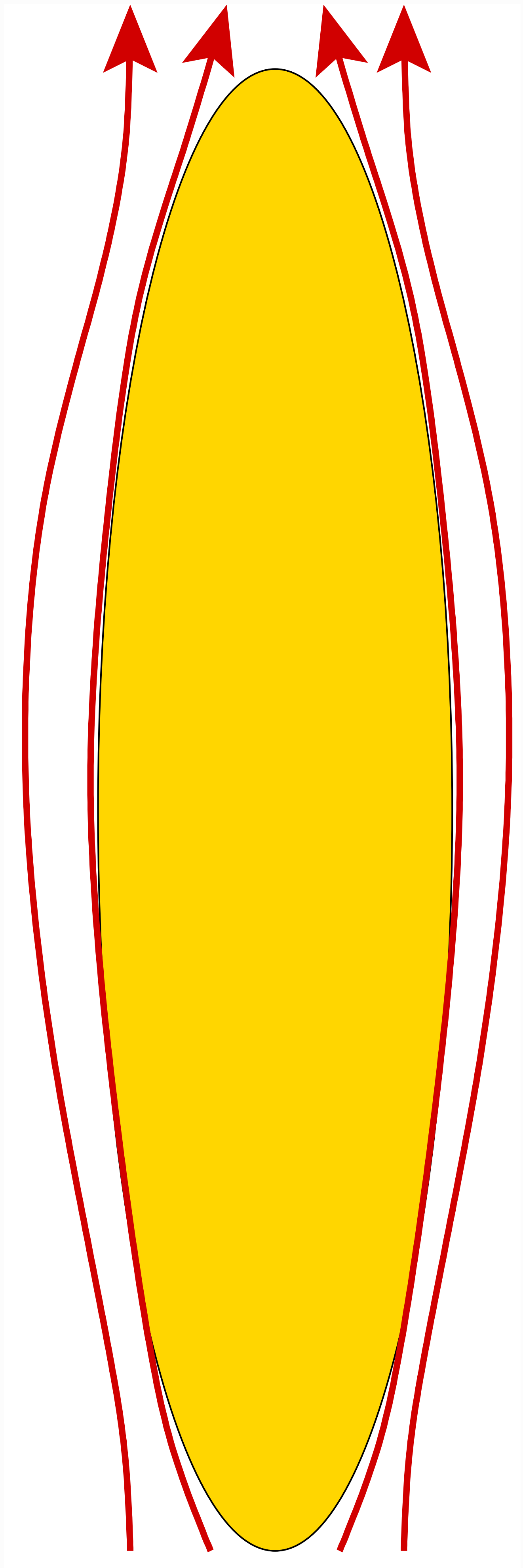
Perfect Diamagnetism

Equilibrium Current Carrying State of Supercurrents

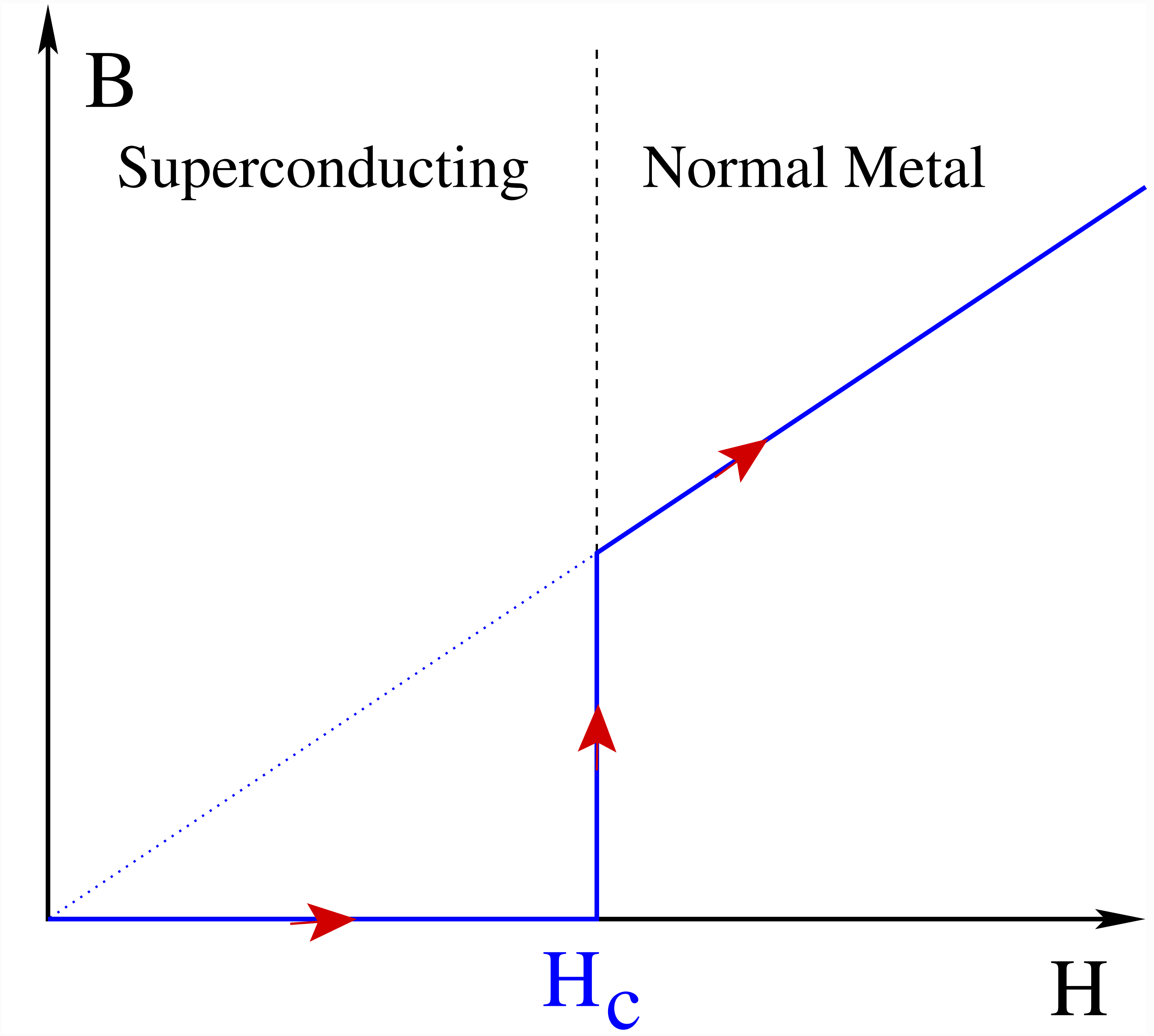
1933 - Walther Meissner & Robert Ochsenfeld, T.U. Munich



$T > T_c$
$B = H$



$T < T_c$
$B = 0$



$$B = H + \boxed{4\pi M} = 0$$

- Screening Currents on the Boundary!

Electrodynamics of Superconductors

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". *Proc. Roy. Soc. A.* **149,** 866 (1935)

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

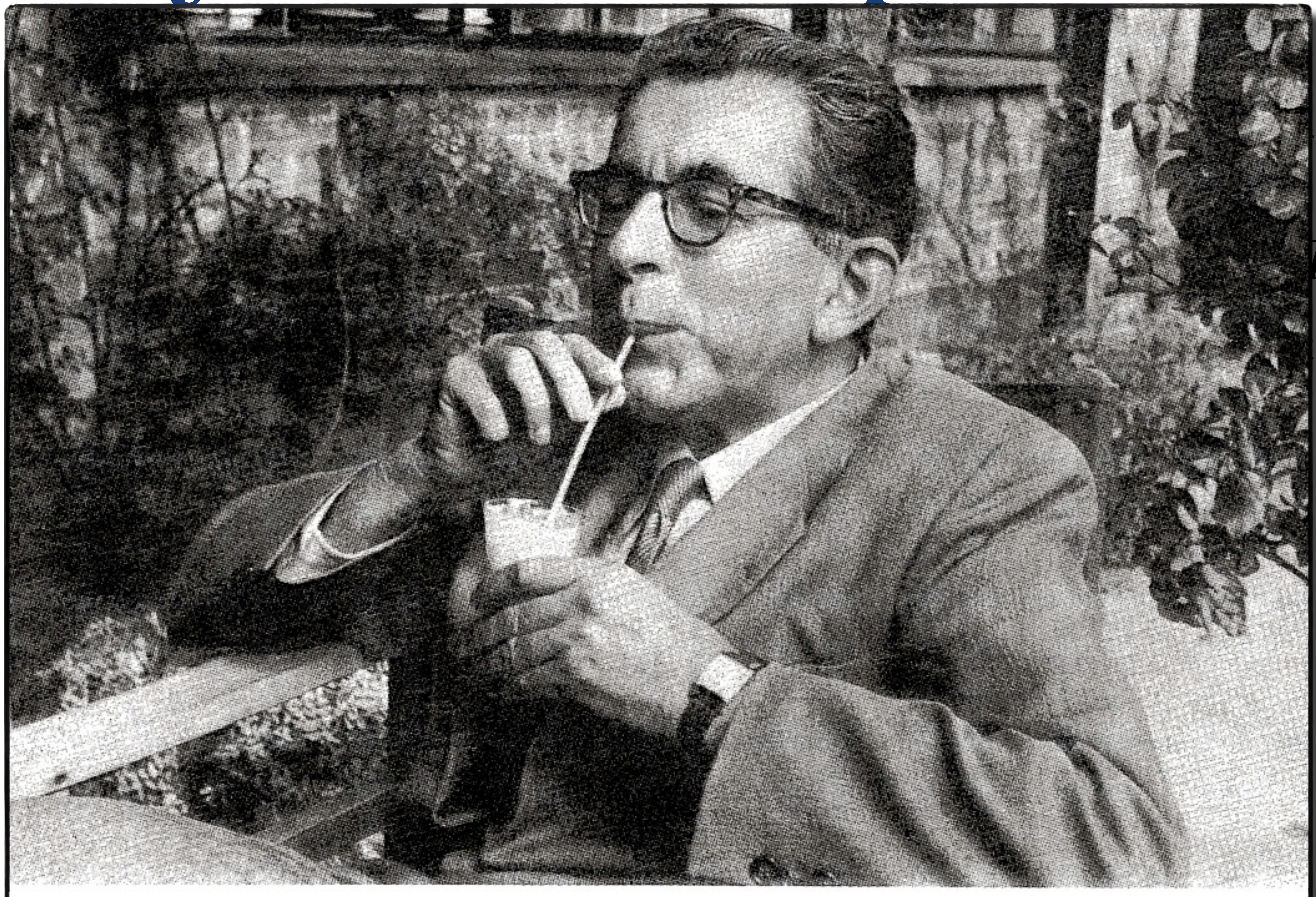
Two Fluid Theory: n_n normal e^- and n_s *super* e^-

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors



Fritz London (1900–1954). (AIP Niels Bohr Library, Francis Simon Collection)

$T \ll T_c$

$-\Delta/T \rightarrow 0$

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B} = \mathbf{J}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". *Proc. .Roy. Soc. A.* ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B} = \sigma_n\,\mathbf{E}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\nabla \times \mathbf{B} = \sigma_n \mathbf{E} \boxed{-n_s\left(\frac{e^2}{c}\right)\mathbf{A}}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

★ Macroscopic Quantum State of $n_s \sim \mathcal{O}(n)$ electrons

London, F., London, H. ["The Electromagnetic Equations of the Supraconductor"]. Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B} = \cancel{\sigma_n \mathbf{E}} \boxed{-n_s\left(\frac{e^2}{c}\right)\mathbf{A}}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

Screening of Magnetic Fields

★ Macroscopic Quantum State of $n_s \sim \mathcal{O}(n)$ electrons

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B} = \cancel{\sigma_n \mathbf{E}} \boxed{-n_s\left(\frac{e^2}{c}\right)\mathbf{A}}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

Screening of Magnetic Fields

$$\left(-\nabla^2\mathbf{B} + \frac{1}{\lambda_L^2}\right)\mathbf{B} = 0$$

★ Macroscopic Quantum State of $n_s \sim \mathcal{O}(n)$ electrons

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. **149,** 866 (1935)

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\nabla\times\mathbf{B} = \cancel{\sigma_n\mathbf{E}} \boxed{-n_s\left(\frac{e^2}{c}\right)\mathbf{A}}$$

Low Temperatures: $T \ll T_c$

$$n_s \to n \quad n_n \sim e^{-\Delta/T} \to 0$$

Screening of Magnetic Fields

$$\left(-\nabla^2\mathbf{B} + \frac{1}{\lambda_{\mathrm{L}}^2}\right)\mathbf{B} = 0$$

$$\lambda_{\mathrm{L}} = \sqrt{\frac{c^2}{4\pi n_s e^2}} \approx 50 - 100\,\mathrm{nm}$$

★ Macroscopic Quantum State of $n_s \sim \mathcal{O}(n)$ electrons

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

Electrodynamics of Superconductors

Fritz London's Theory c.a. 1935

Two Fluid Theory: n_n normal e^- and n_s *super* e^-

$$\frac{c}{4\pi}\boldsymbol{\nabla}\times\mathbf{B} = \cancel{\sigma_n \mathbf{E}}\ \boxed{-n_s\left(\frac{e^2}{c}\right)\mathbf{A}}$$

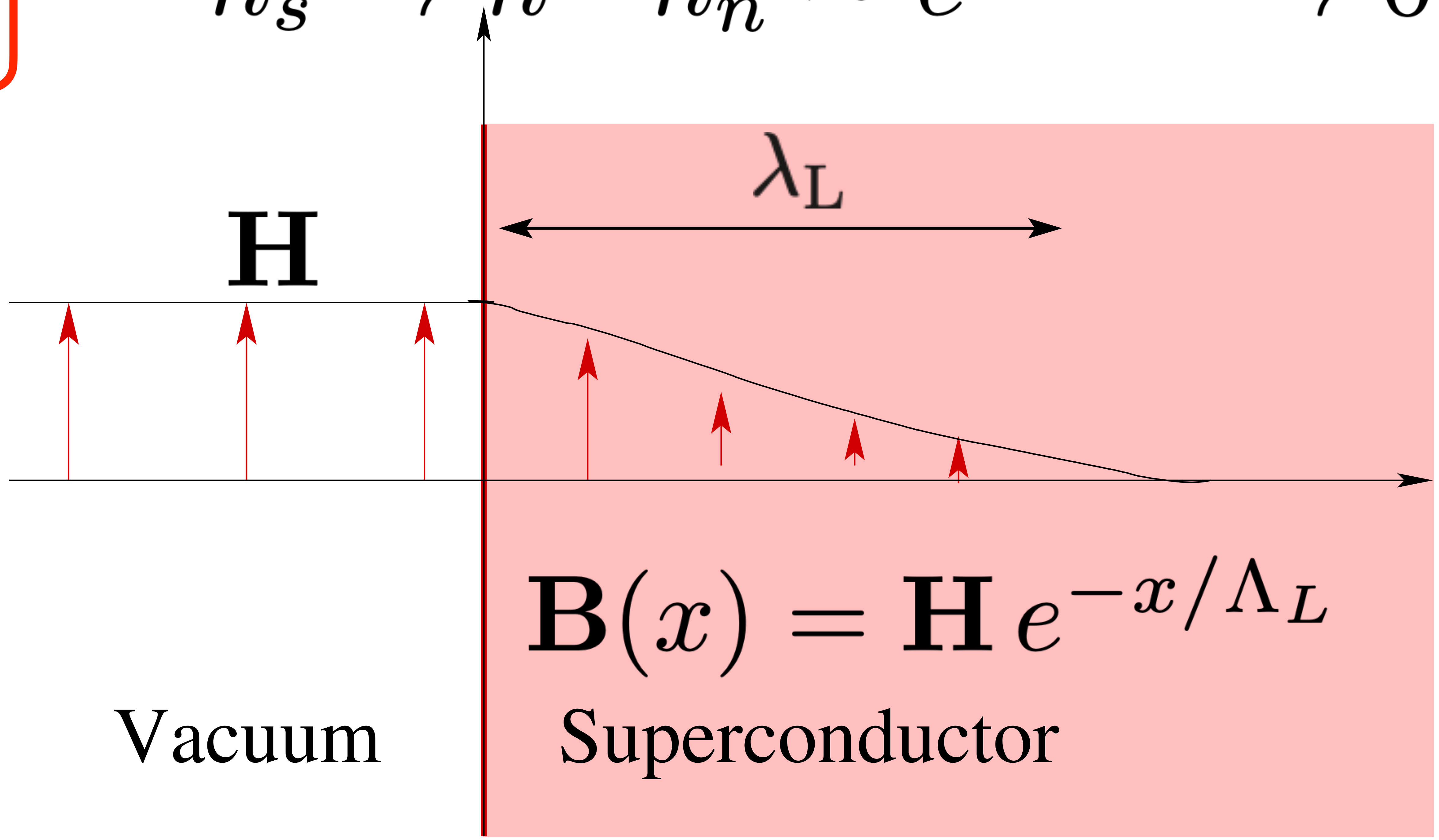
Low Temperatures: $T \ll T_c$

$$n_s \to n \qquad n_n \sim e^{-\Delta/T} \to 0$$

Screening of Magnetic Fields

$$\left(-\nabla^2\mathbf{B} + \frac{1}{\lambda_\mathrm{L}^2}\right)\mathbf{B} = 0$$

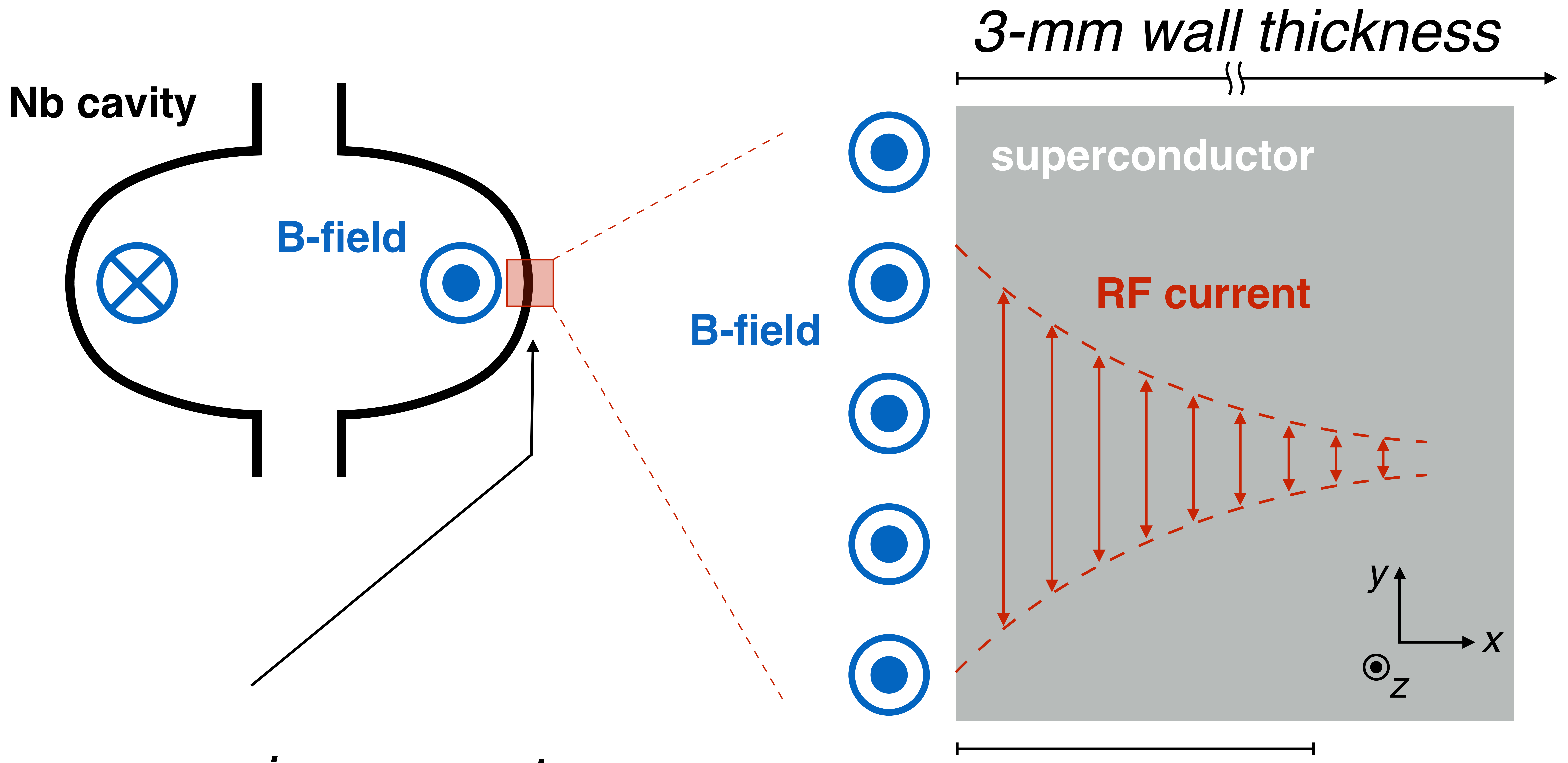
$$\lambda_\mathrm{L} = \sqrt{\frac{c^2}{4\pi n_s e^2}} \approx 50 - 100\,\mathrm{nm}$$



★ Macroscopic Quantum State of $n_s \sim \mathcal{O}(n)$ electrons

London, F., London, H. "The Electromagnetic Equations of the Supraconductor". Proc. .Roy. Soc. A. ***149,*** *866 (1935)*

SRF Cavities with High Q work because of a 100-nm layer of superconductor!!



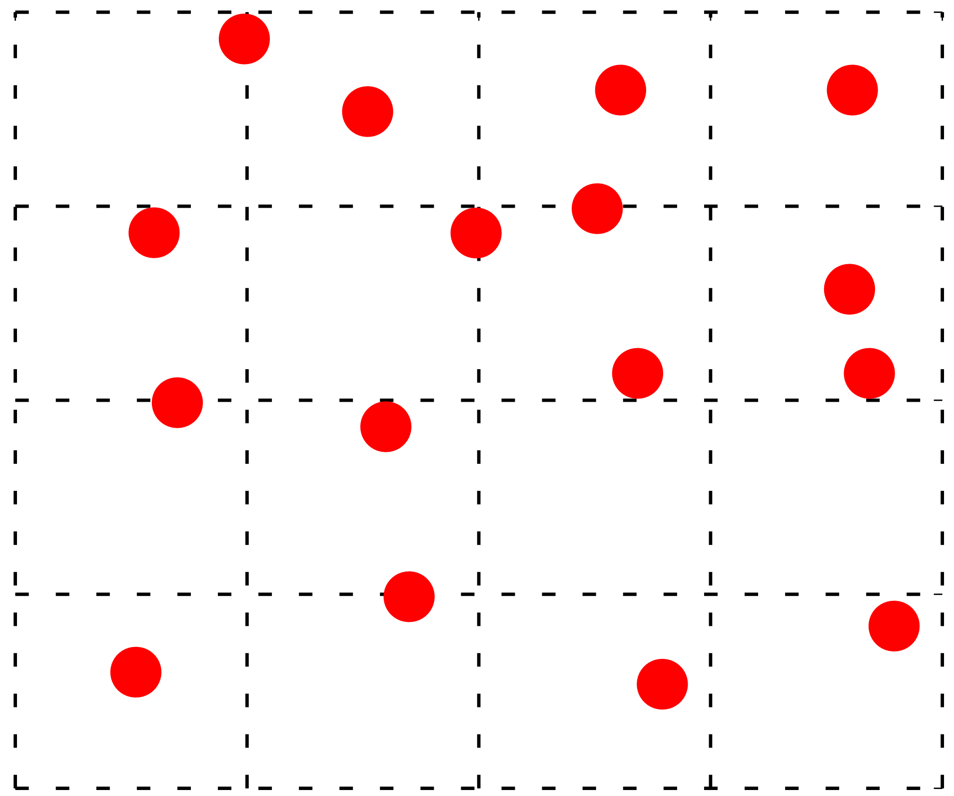
This region on screening currents confines the EM field to the cavity and a small layer of superconductor

RF field penetrates only ~100nm (London penetration depth of Nb)

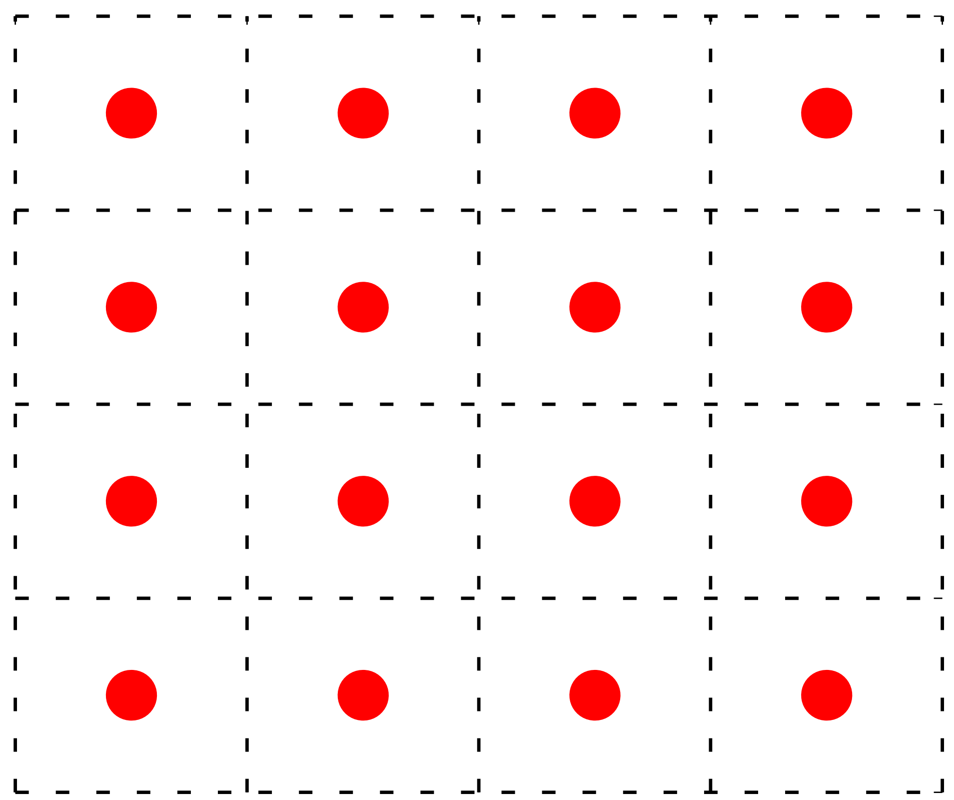
Broken Symmetry & Ordered Phases

Broken Symmetry & Ordered Phases

Liquid

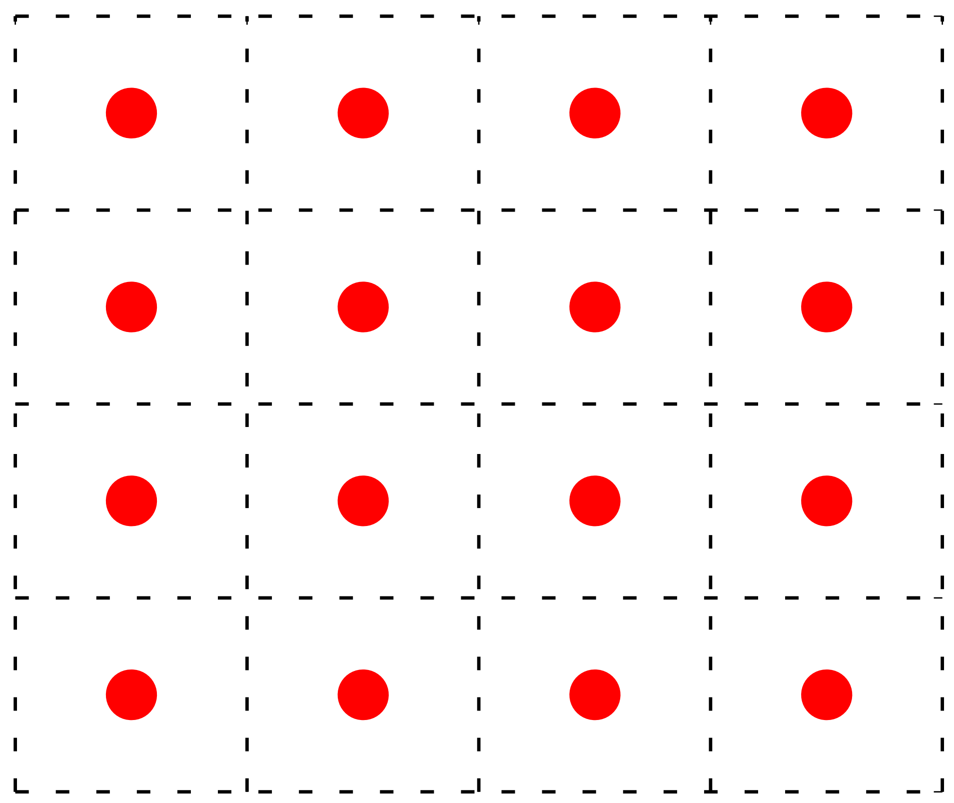


Broken Symmetry & Ordered Phases



Solid

Broken Symmetry & Ordered Phases



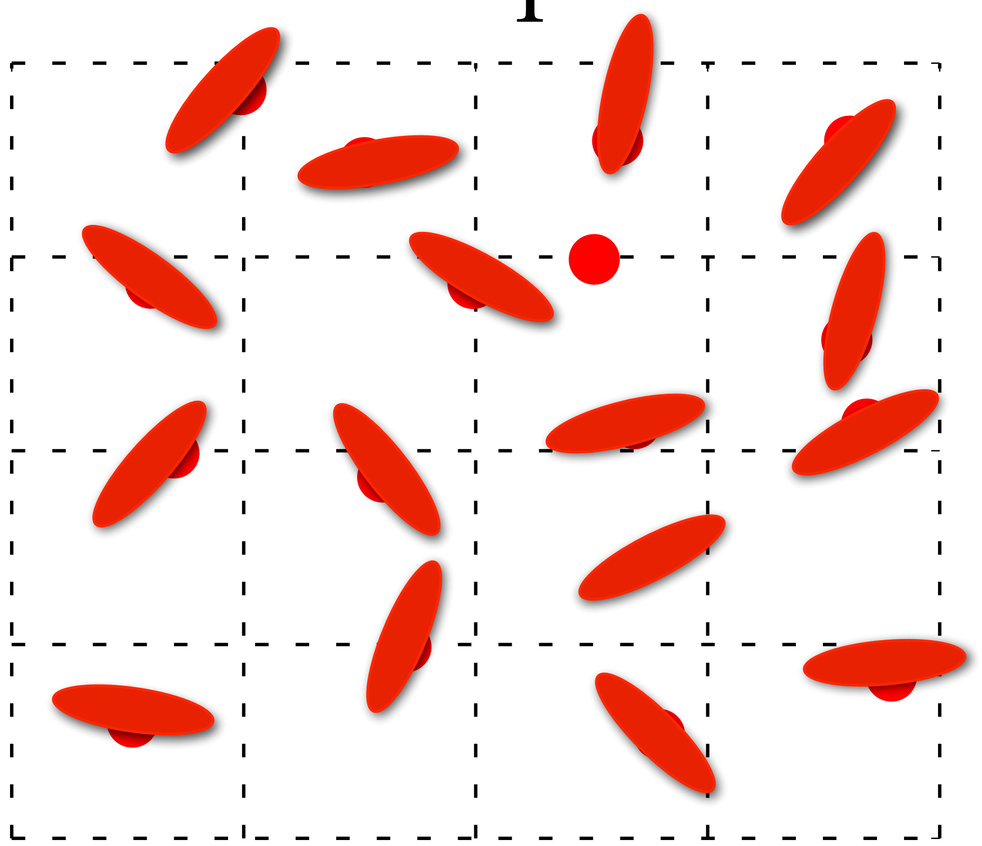
Solid

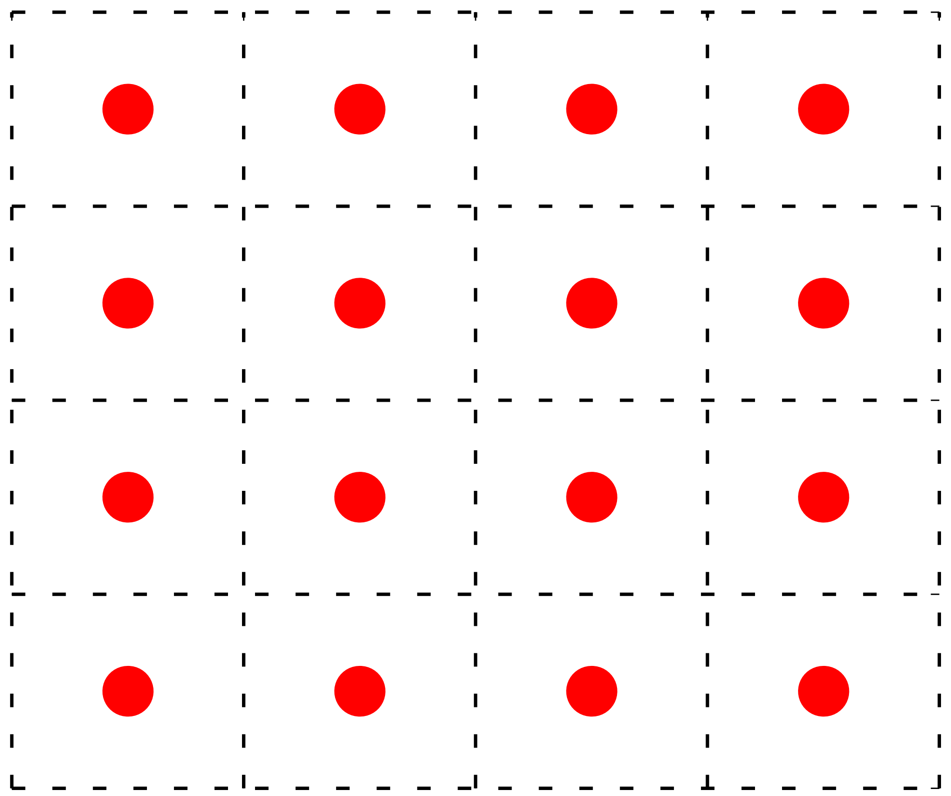
Translations

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

Broken Symmetry & Ordered Phases



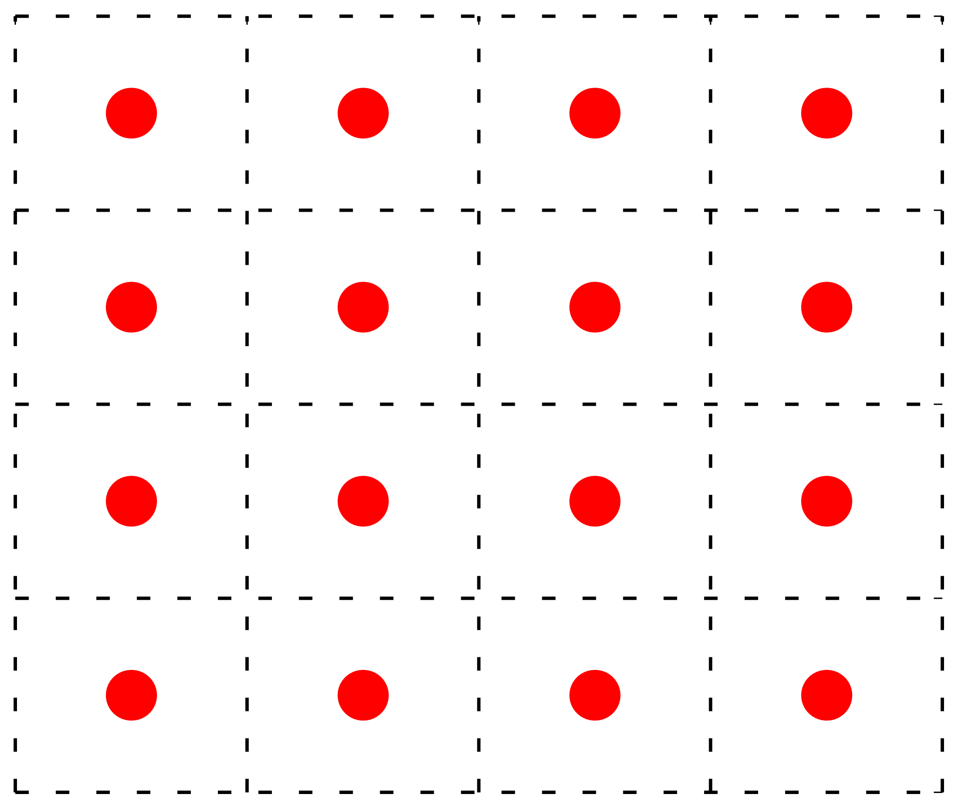


Translations

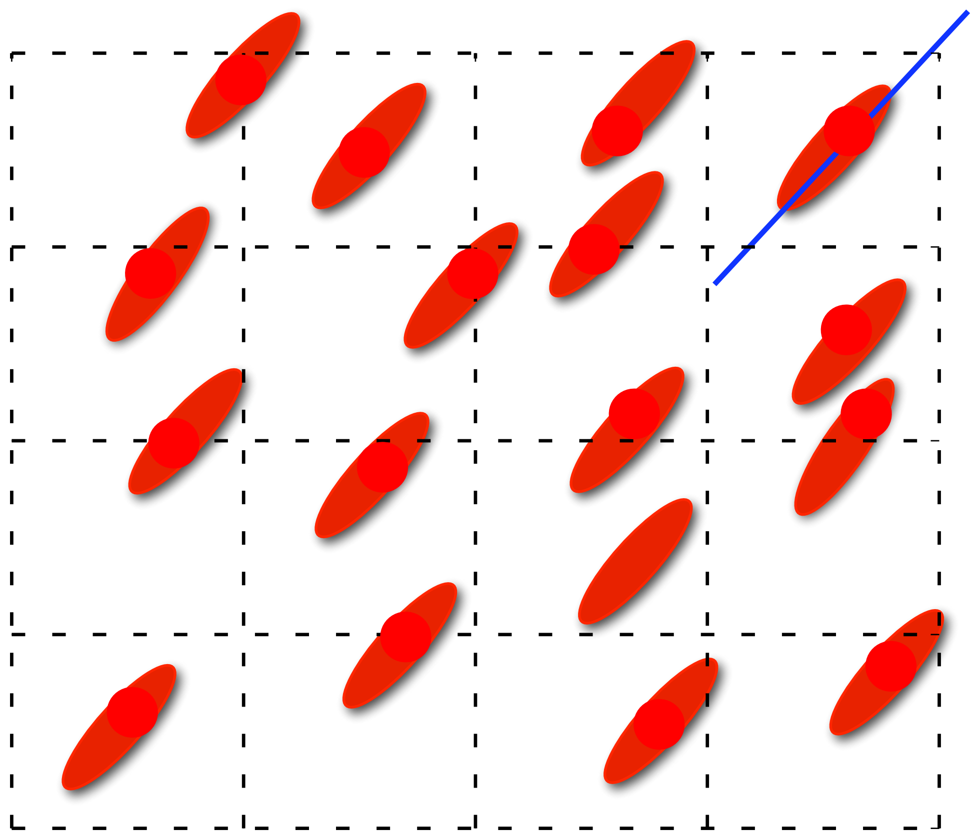
$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction
peaks

Broken Symmetry & Ordered Phases



Solid



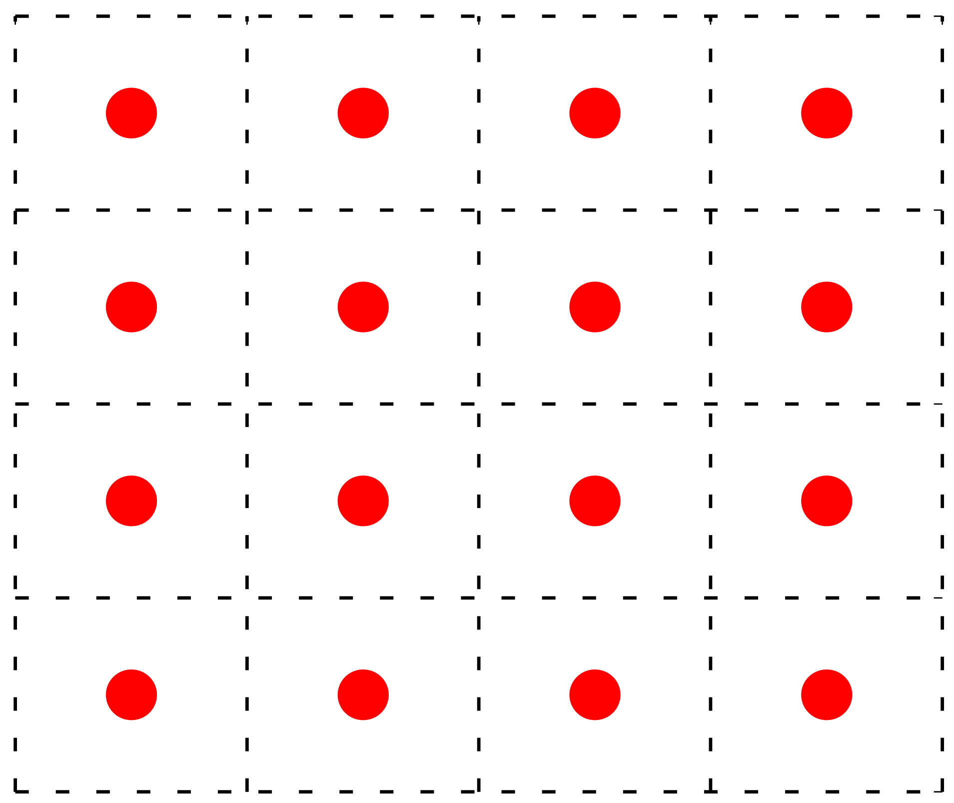
Nematic

Translations

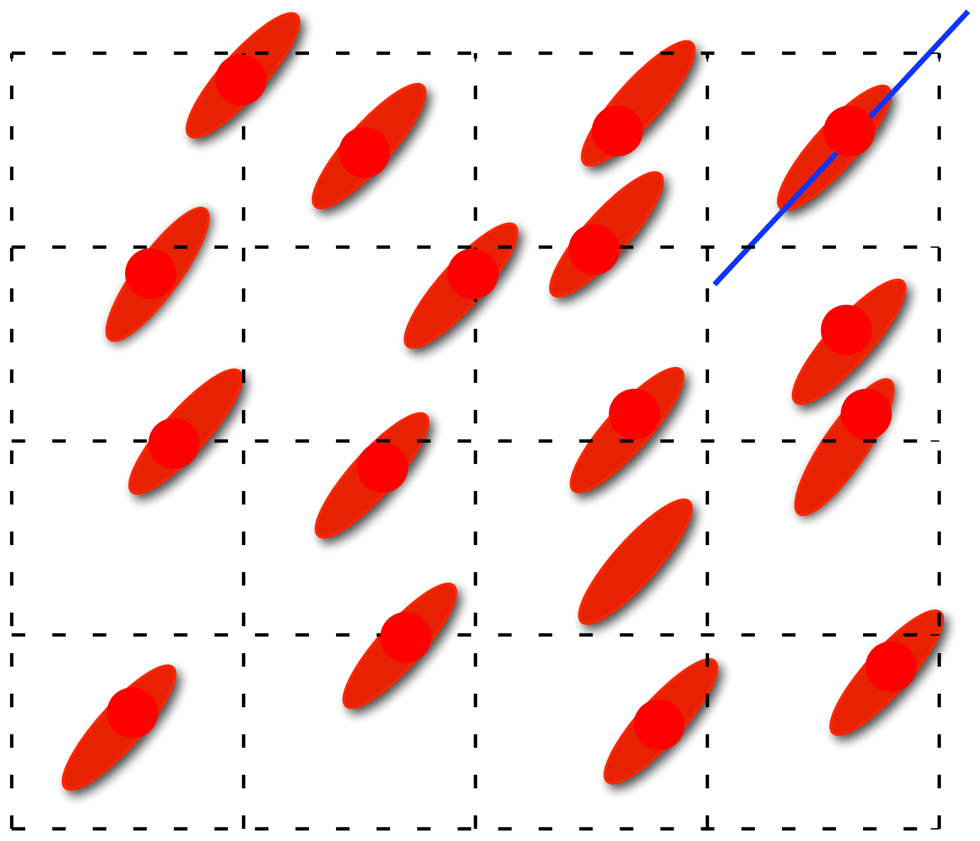
$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

Broken Symmetry & Ordered Phases



Solid



Nematic

Translations

Space Rotations

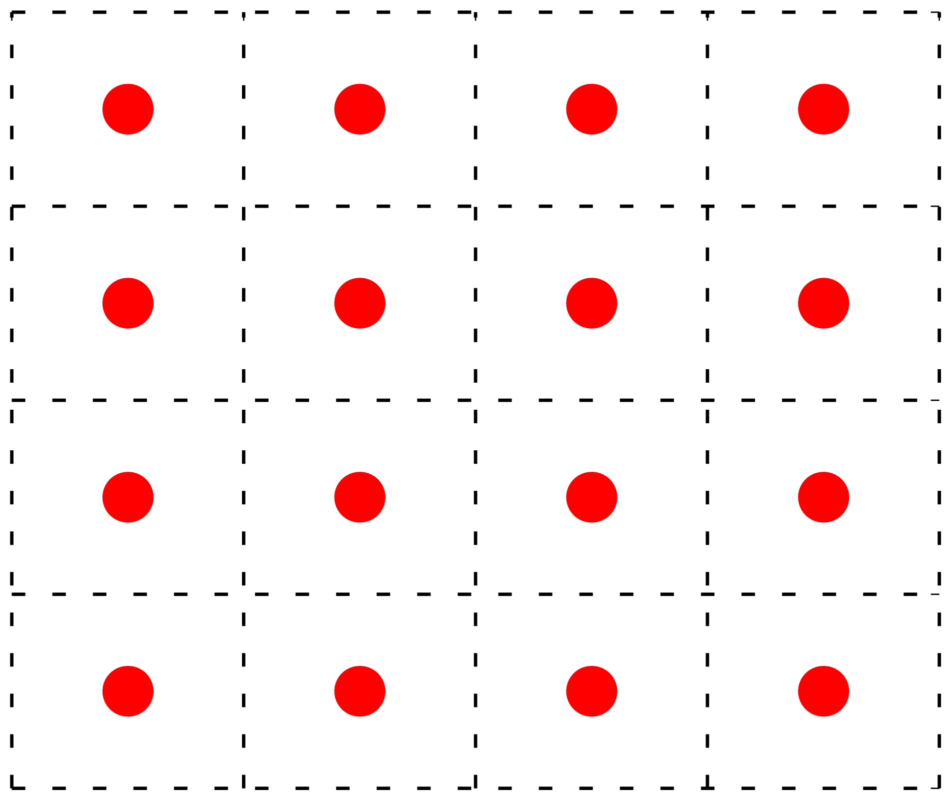
$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction
peaks

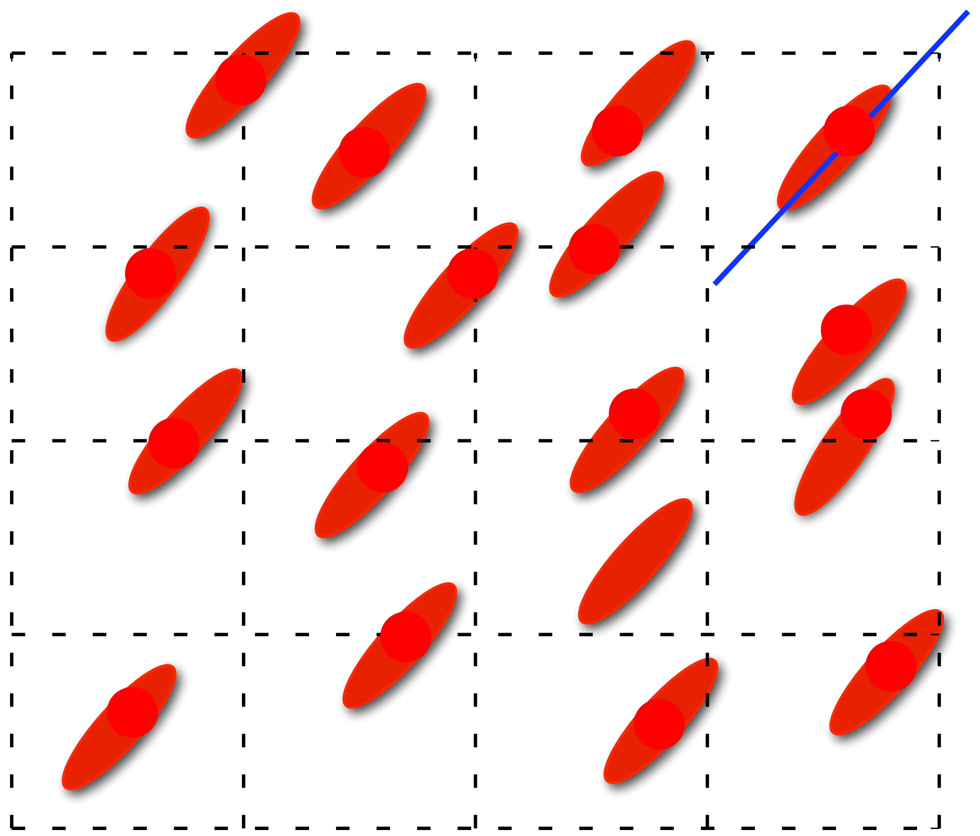
$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

Anisotropic
dielectric function

Broken Symmetry & Ordered Phases

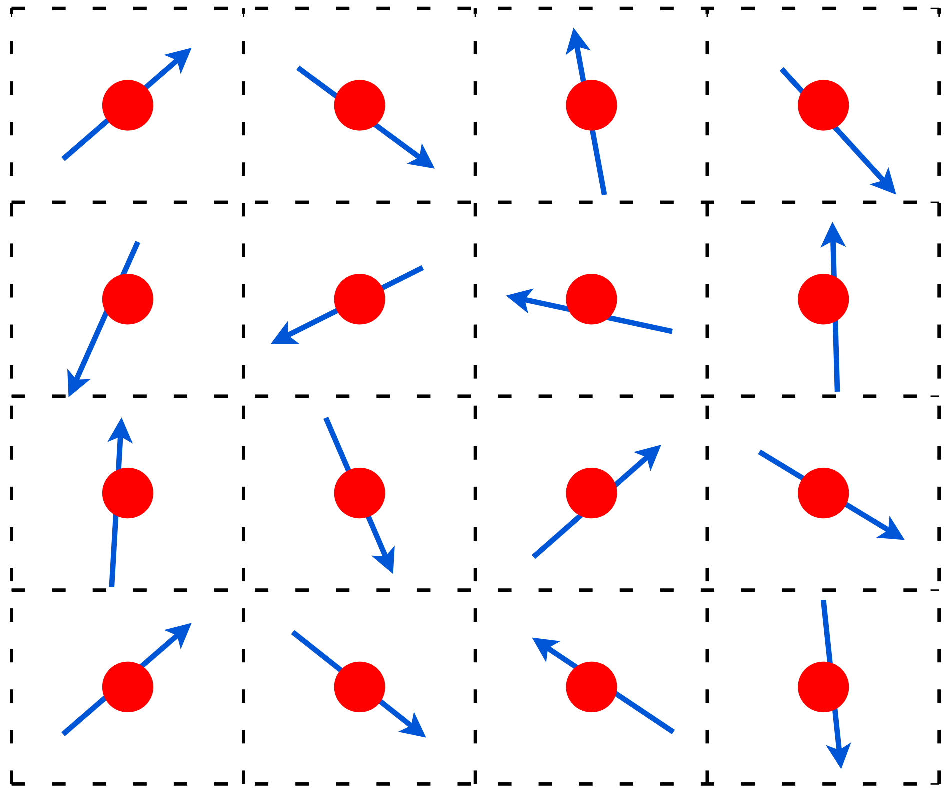


Solid



Nematic

Paramagnet



Translations

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

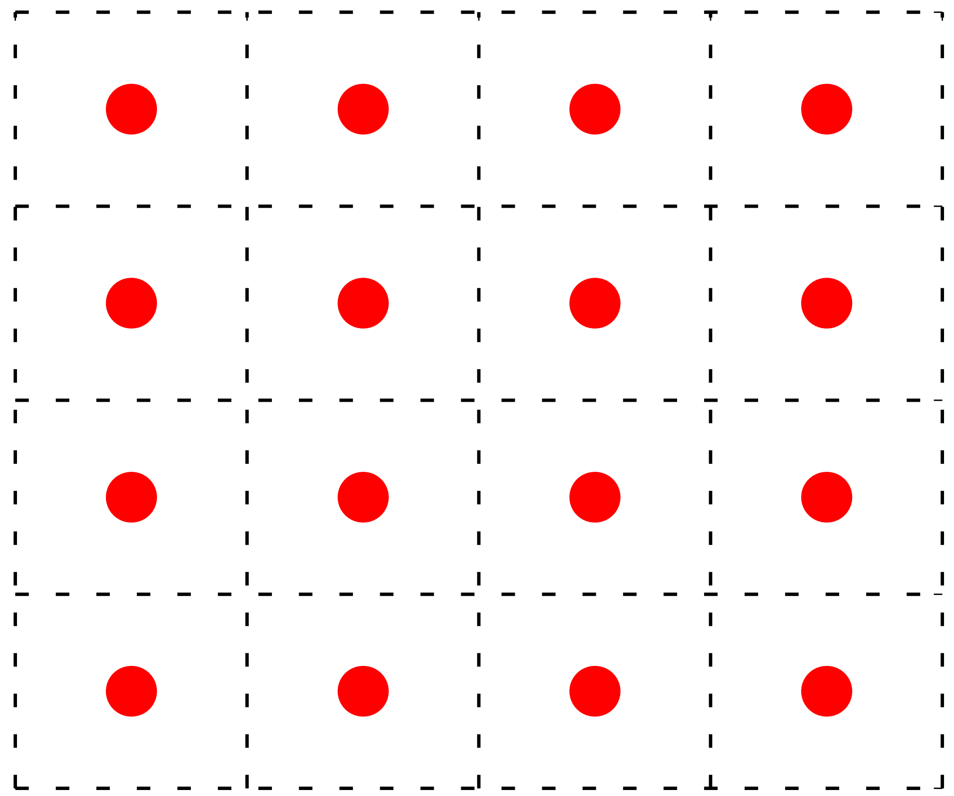
X-ray diffraction peaks

Space Rotations

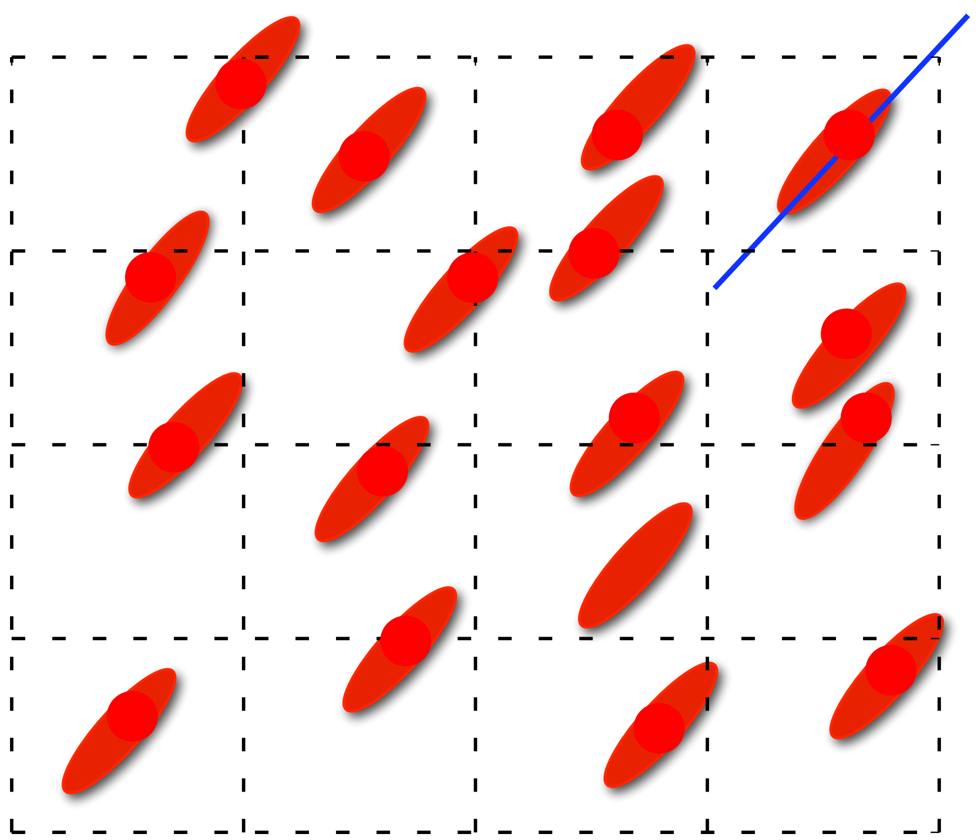
$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

Anisotropic dielectric function

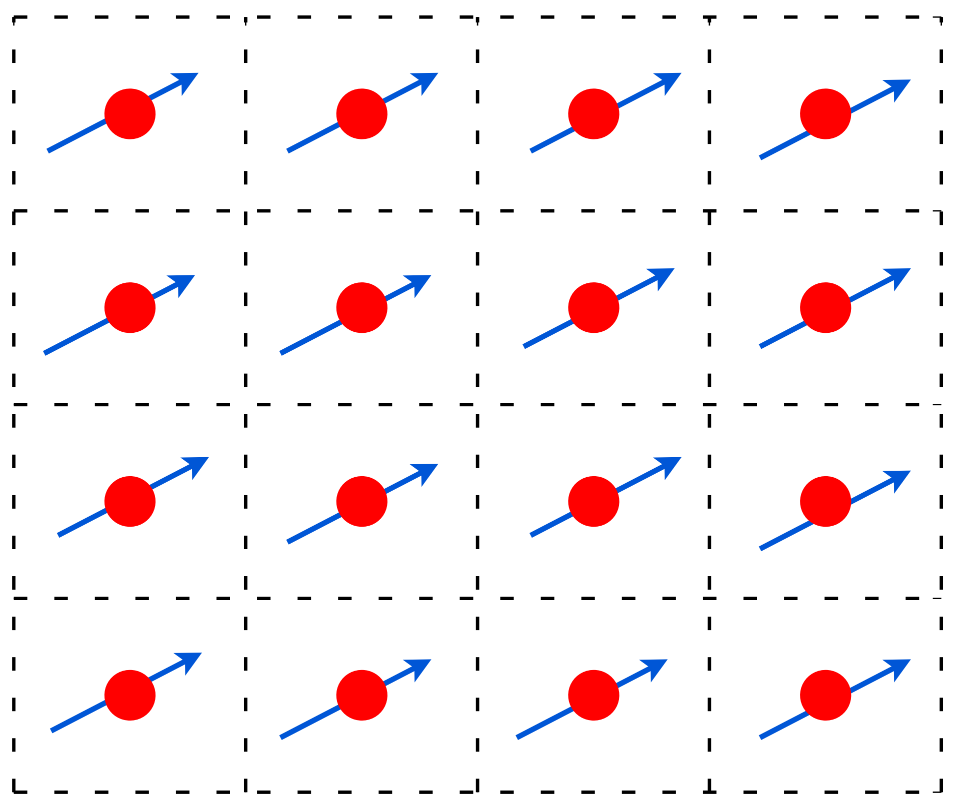
Broken Symmetry & Ordered Phases



Solid



Nematic



Ferromagnet

Translations

Space Rotations

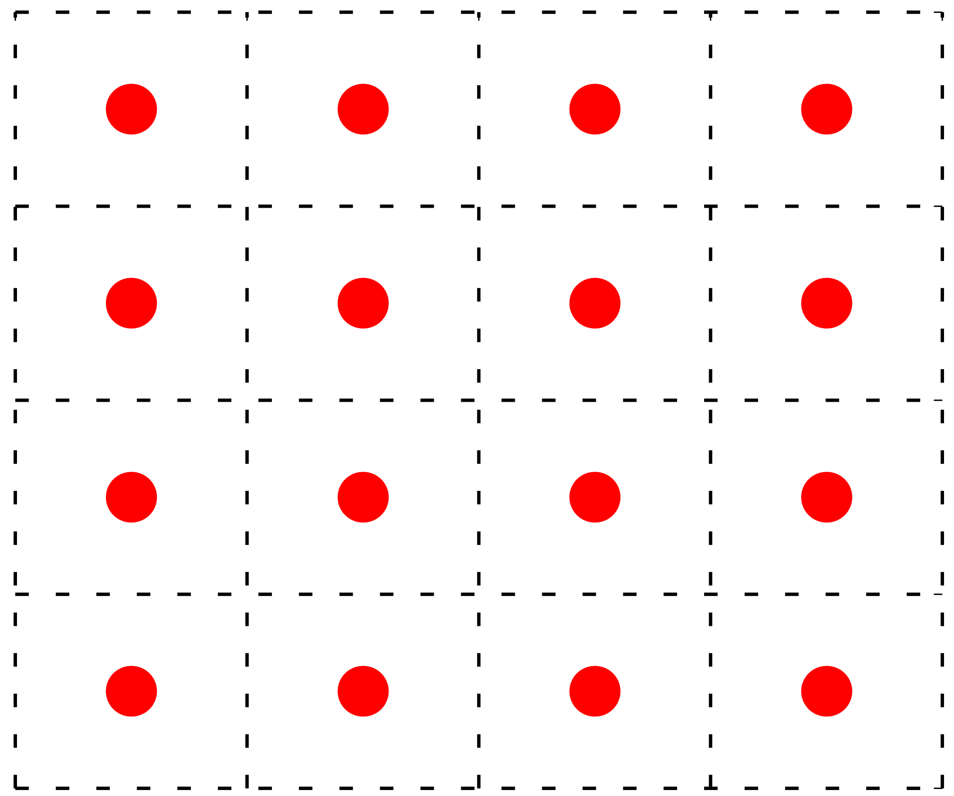
$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

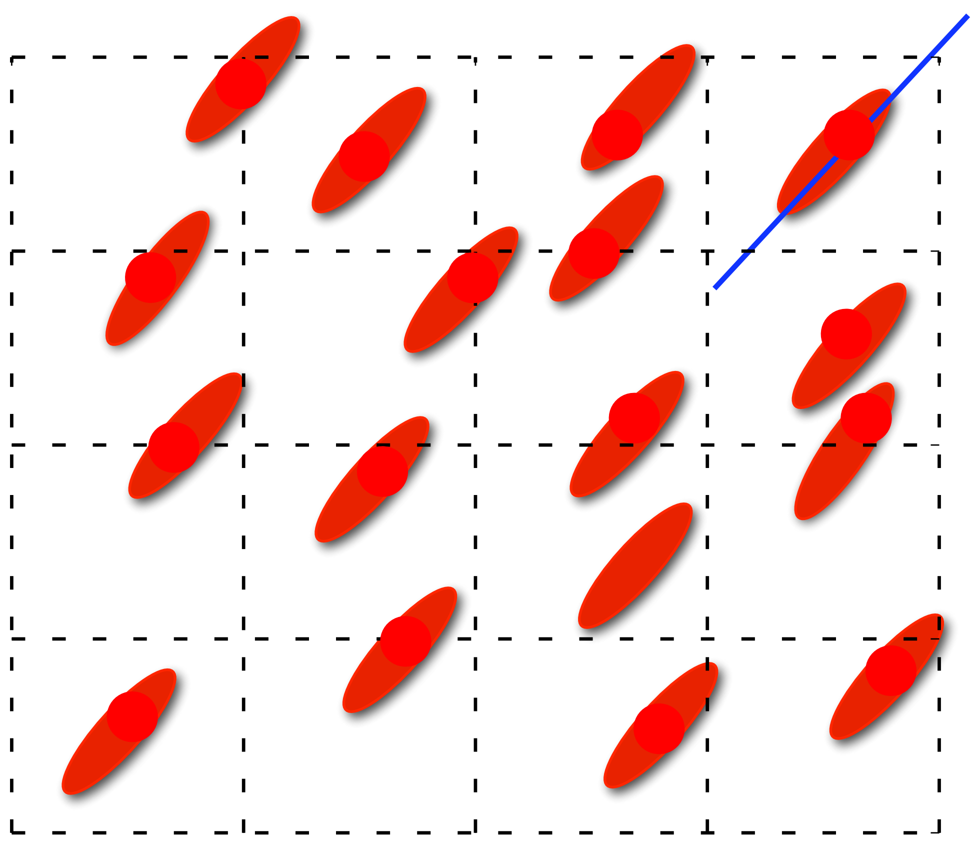
$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

Anisotropic dielectric function

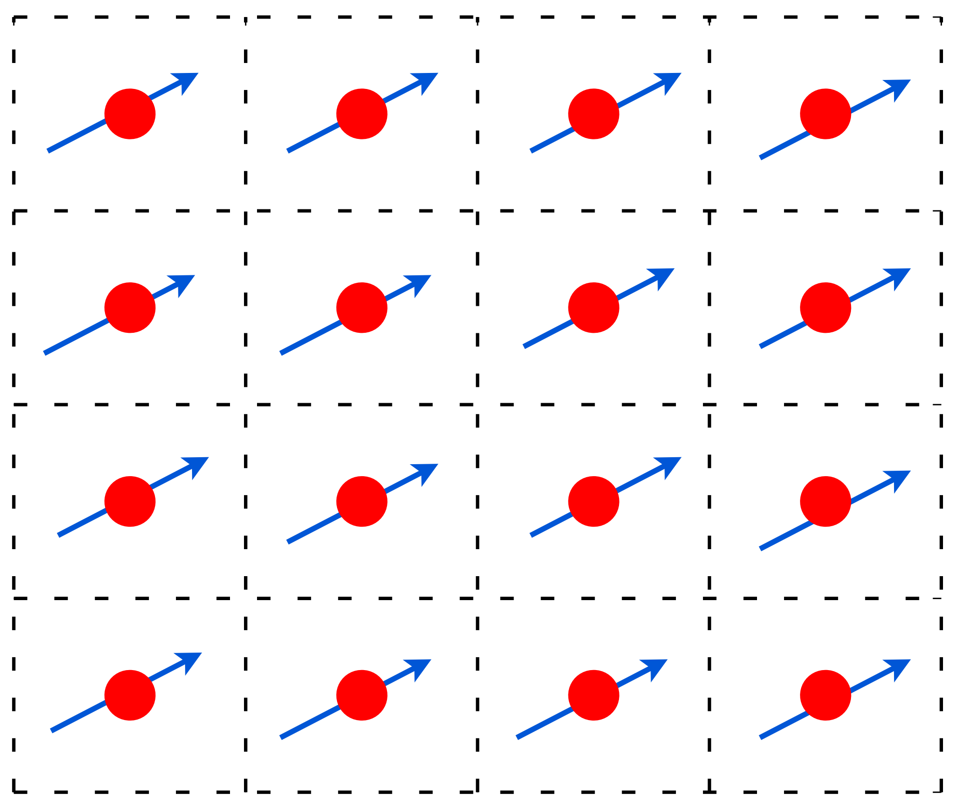
Broken Symmetry & Ordered Phases



Solid



Nematic



Ferromagnet

Translations

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

Space Rotations

$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

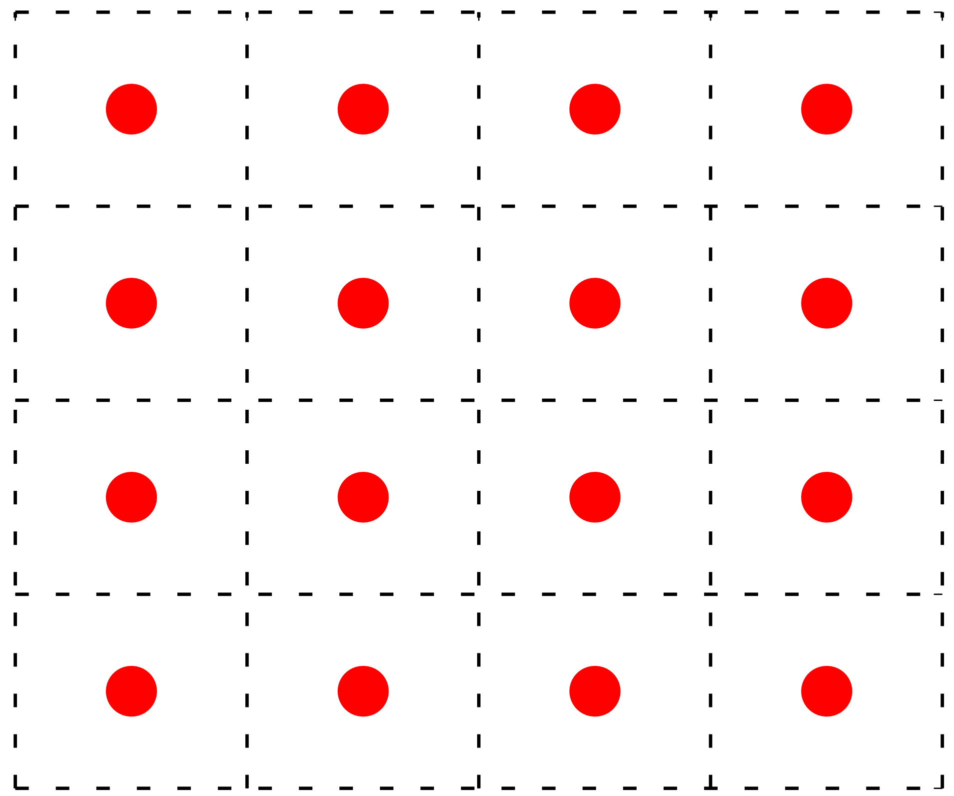
Anisotropic dielectric function

Spin Rotation

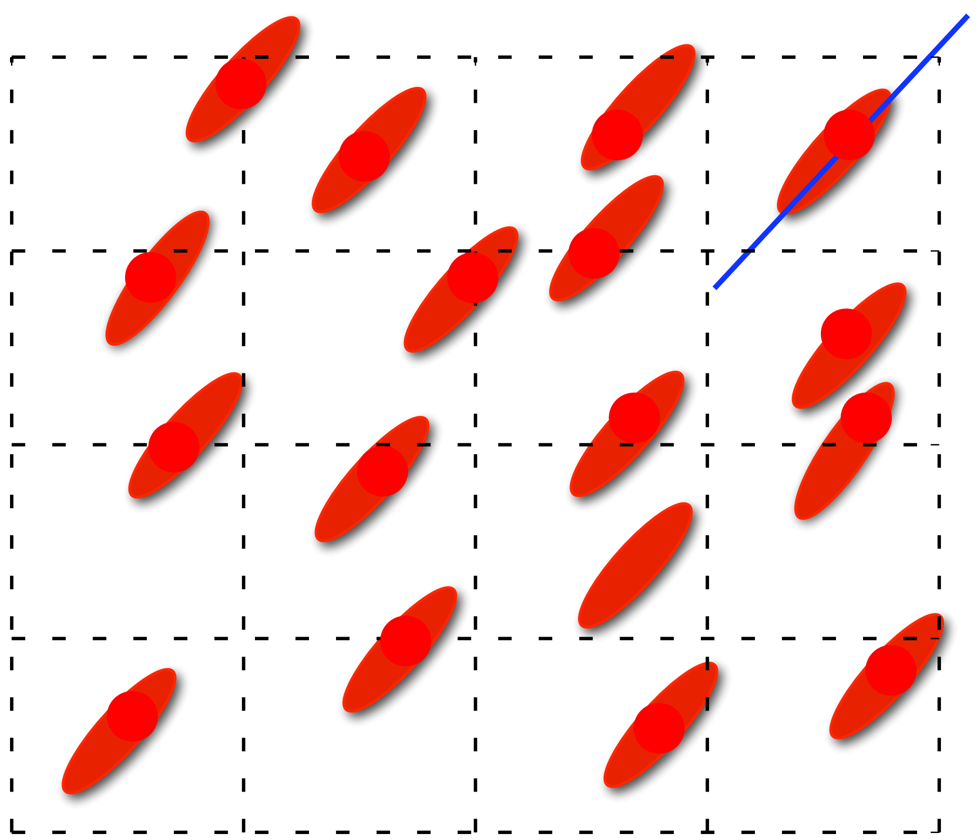
$$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

Spontaneous Magnetization

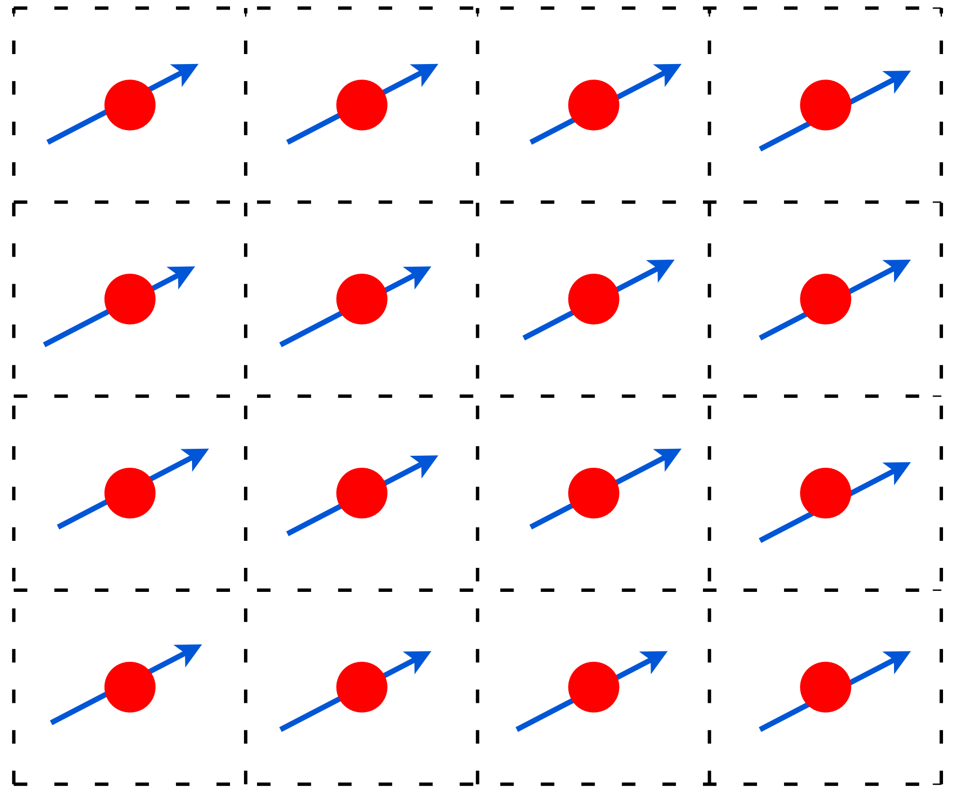
Broken Symmetry & Ordered Phases



Solid

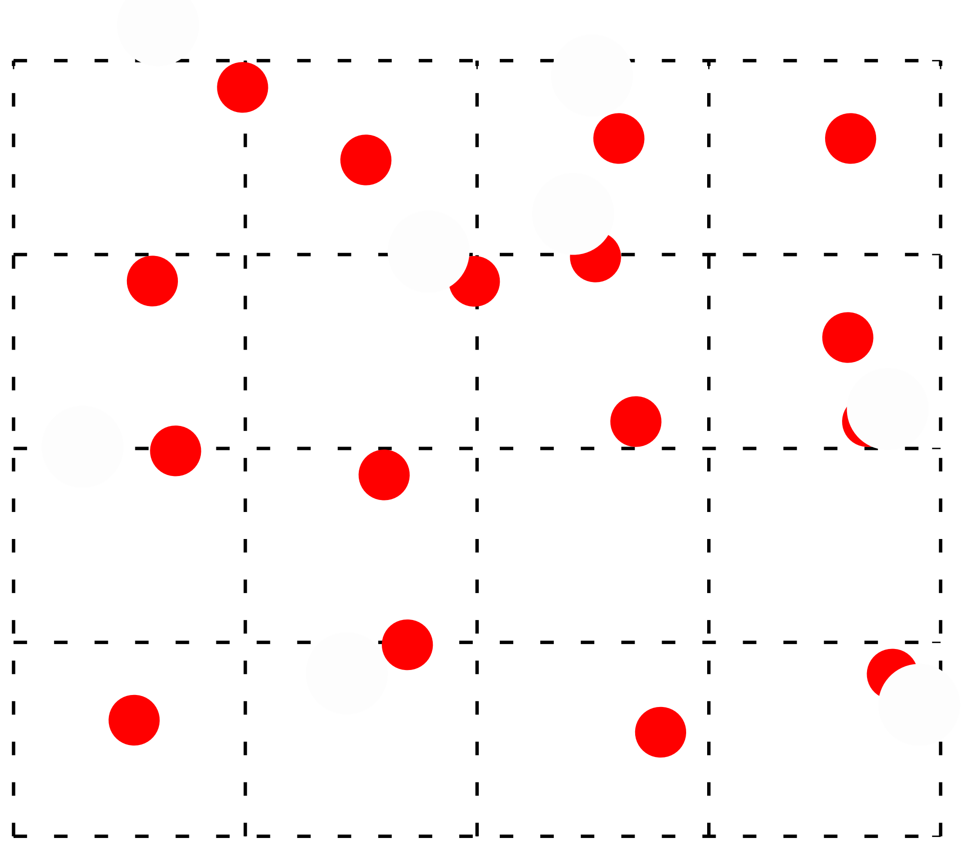


Nematic



Ferromagnet

Helium & Metals



Translations

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

Space Rotations

$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

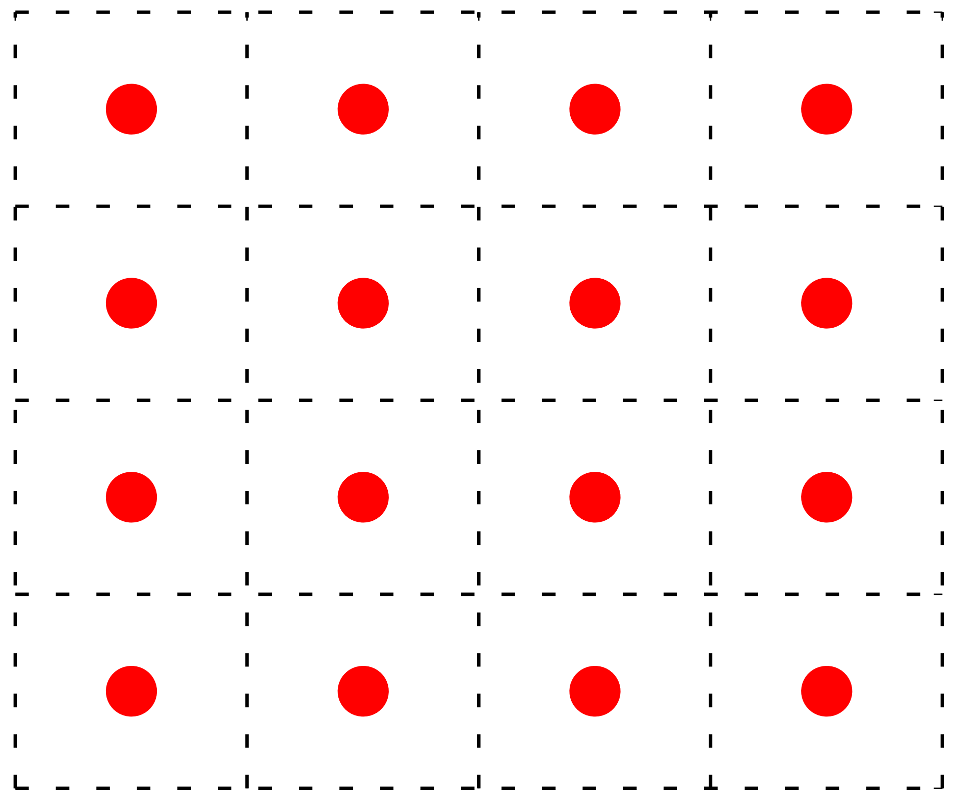
Anisotropic dielectric function

Spin Rotation

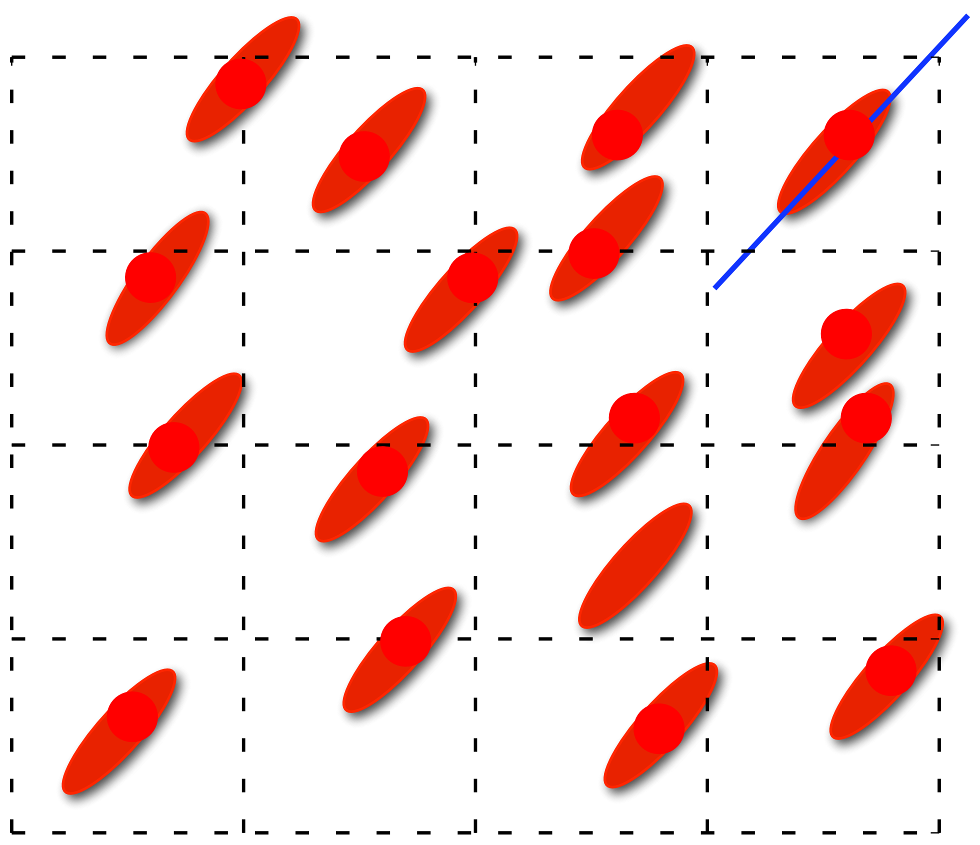
$$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

Spontaneous Magnetization

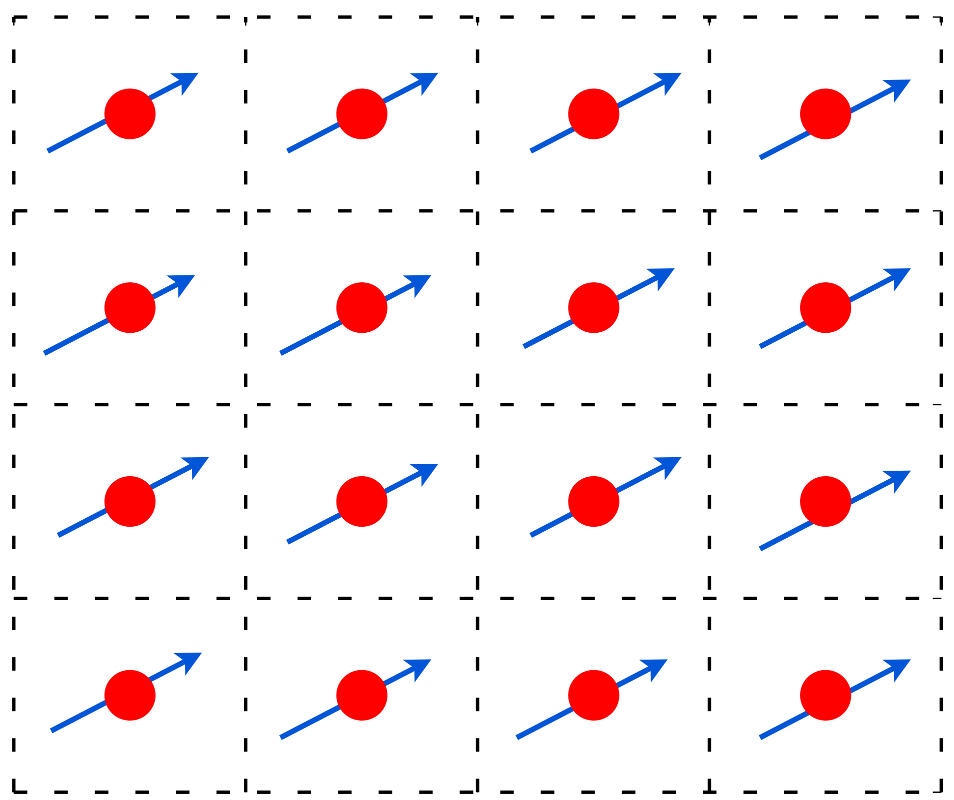
Broken Symmetry & Ordered Phases



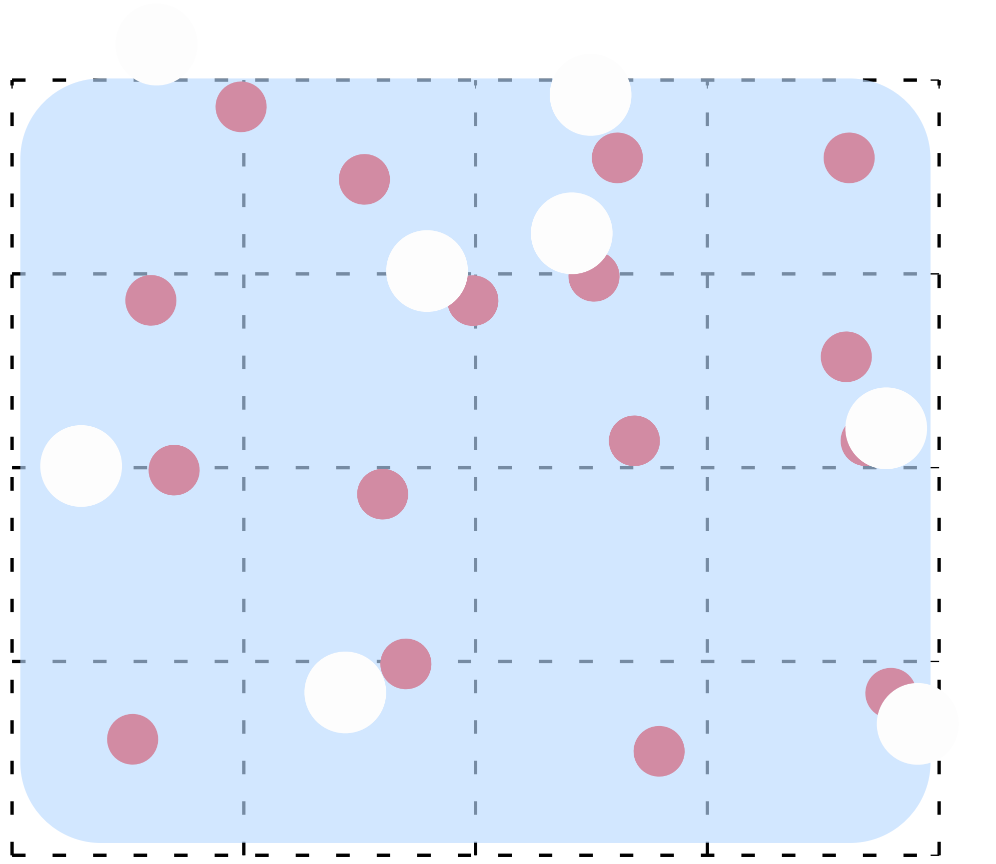
Solid



Nematic



Ferromagnet



Superfluids
Superconductors

Translations

Space Rotations

Spin Rotation

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

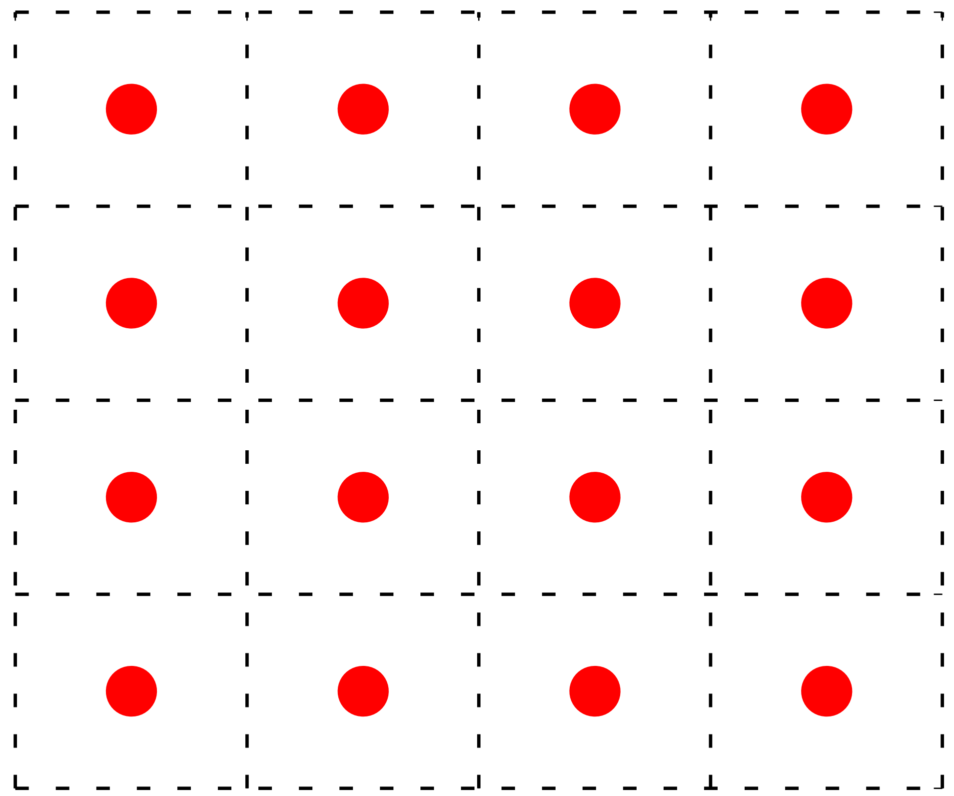
$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

Anisotropic dielectric function

$$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

Spontaneous Magnetization

Broken Symmetry & Ordered Phases

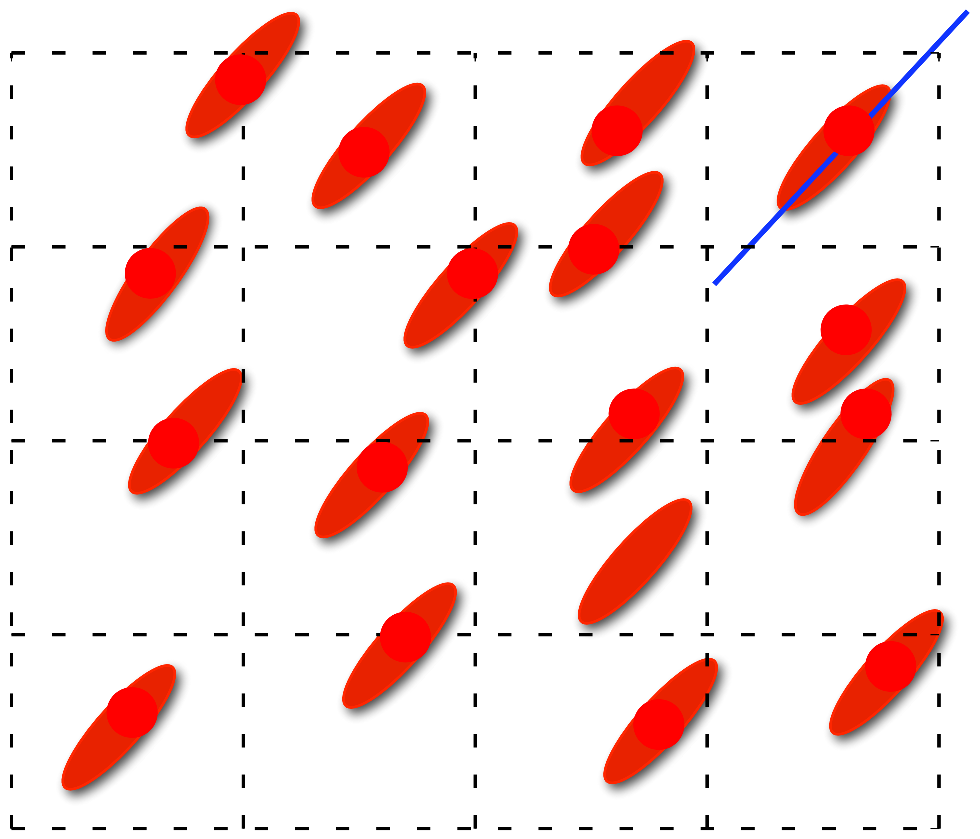


Solid

Translations

$$\rho(\mathbf{q}) = \int d\mathbf{r} e^{-i\mathbf{q}\cdot\mathbf{r}}\, \delta n(\mathbf{r})$$

X-ray diffraction peaks

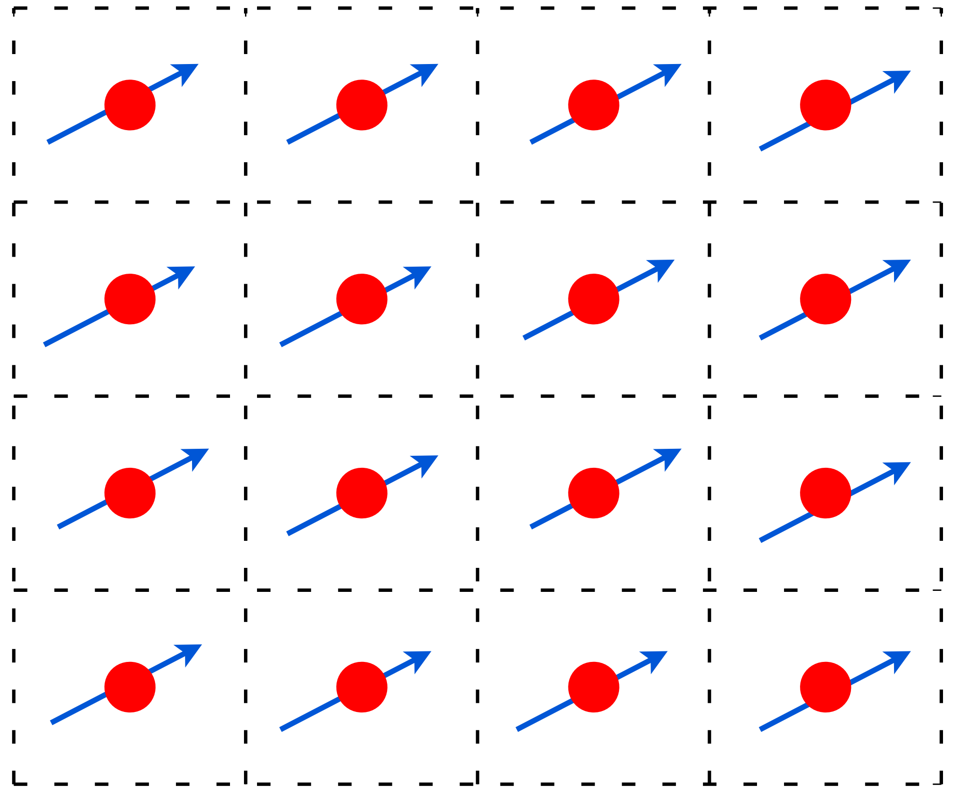


Nematic

Space Rotations

$$\Delta\varepsilon_{ij} = \varepsilon(T)\, \hat{\mathbf{n}}_i \hat{\mathbf{n}}_j$$

Anisotropic dielectric function

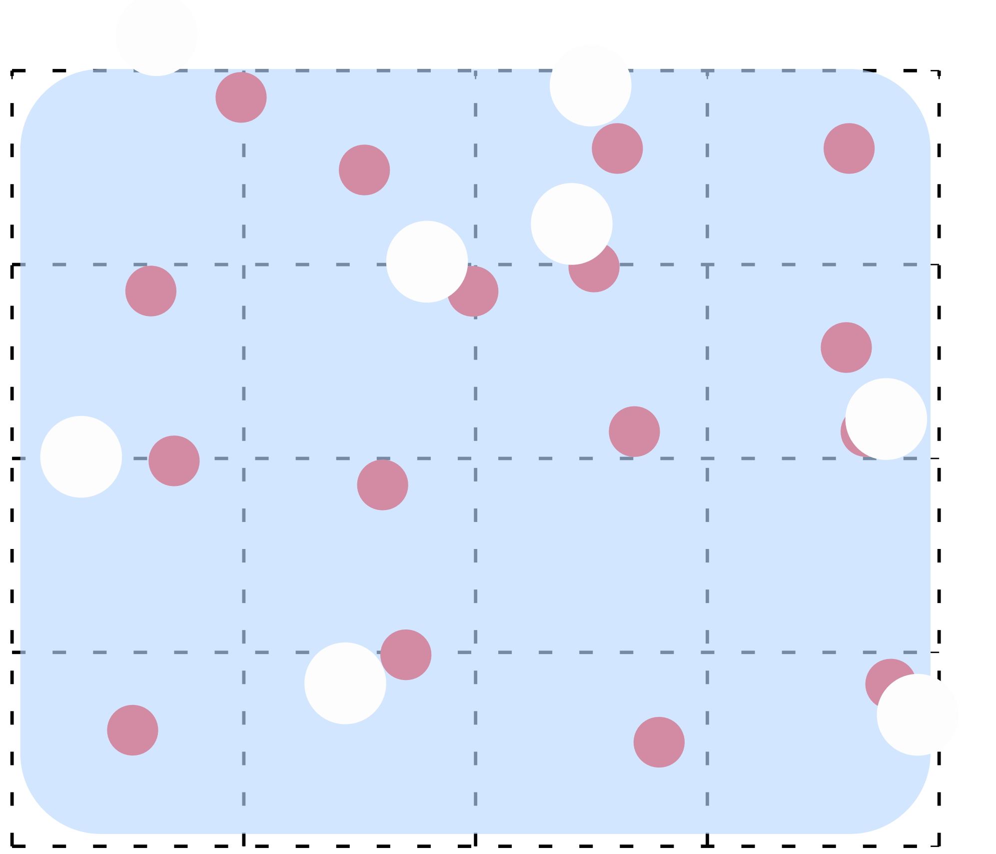


Ferromagnet

Spin Rotation

$$\mathbf{M} = \gamma \langle \mathbf{S} \rangle$$

Spontaneous Magnetization



Superfluids
Superconductors

Gauge

$$\Psi = \langle \psi(\mathbf{r}) \rangle \simeq \sqrt{N/V}\ \boxed{e^{i\vartheta}}$$

Condensate Wave function

V. L. Ginzburg and L. D. Landau, JETP ... (1950).

- Superconductivity originates from a macroscopic quantum state, $\Psi(\mathbf{r})$ with,

$$n_{\mathrm{S}} \equiv |\Psi(\mathbf{r})|^2 = \text{density of “super” electrons}$$

- $\Psi(\mathbf{r}) = |\Psi(\mathbf{r})| \, e^{i\vartheta(\mathbf{r})} \leadsto$ $\vec{J}(\mathbf{r}) \propto |\Psi(\mathbf{r})|^2 \nabla\vartheta(\mathbf{r}) = \text{“supercurrents”}$
- $\Psi(\mathbf{r})$ is a thermodynamic variable of state at finite T and p

V. L. Ginzburg and L. D. Landau, JETP ... (1950).

- Superconductivity originates from a macroscopic quantum state, $\Psi(\mathbf{r})$ with,

$$n_S \equiv |\Psi(\mathbf{r})|^2 = \text{density of "super" electrons}$$

- $\Psi(\mathbf{r}) = |\Psi(\mathbf{r})|\, e^{i\vartheta(\mathbf{r})} \rightsquigarrow$ $\vec{J}(\mathbf{r}) \propto |\Psi(\mathbf{r})|^2\, \nabla\vartheta(\mathbf{r}) = \text{"supercurrents"}$
- $\Psi(\mathbf{r})$ is a thermodynamic variable of state at finite T and p

- Equilibrium: Landau's theory of symmetry breaking phase transitions
  - Continuous Transition: $\Psi \to 0$ for $T \to T_c - 0^+$
  - Discontinuous Change in Symmetry: $G' \subset G$

V. L. Ginzburg and L. D. Landau, JETP ... (1950).

- Superconductivity originates from a macroscopic quantum state, $\Psi(\mathbf{r})$ with,

$$n_S \equiv |\Psi(\mathbf{r})|^2 = \text{density of “super” electrons}$$

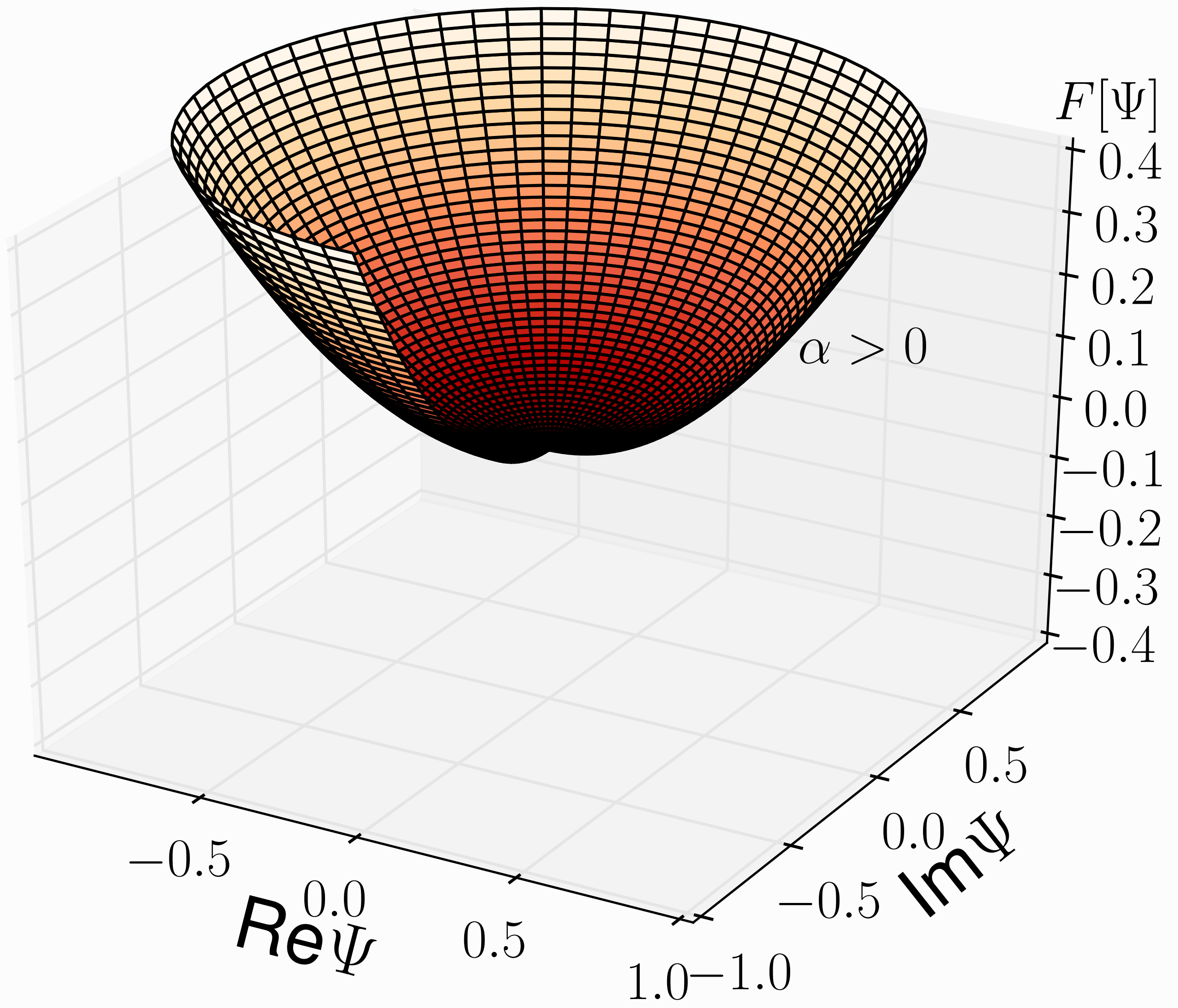
- $\Psi(\mathbf{r}) = |\Psi(\mathbf{r})|\, e^{i\vartheta(\mathbf{r})} \rightsquigarrow$ $\vec{J}(\mathbf{r}) \propto |\Psi(\mathbf{r})|^2\, \nabla\vartheta(\mathbf{r}) = \text{“supercurrents”}$
- $\Psi(\mathbf{r})$ is a thermodynamic variable of state at finite T and p

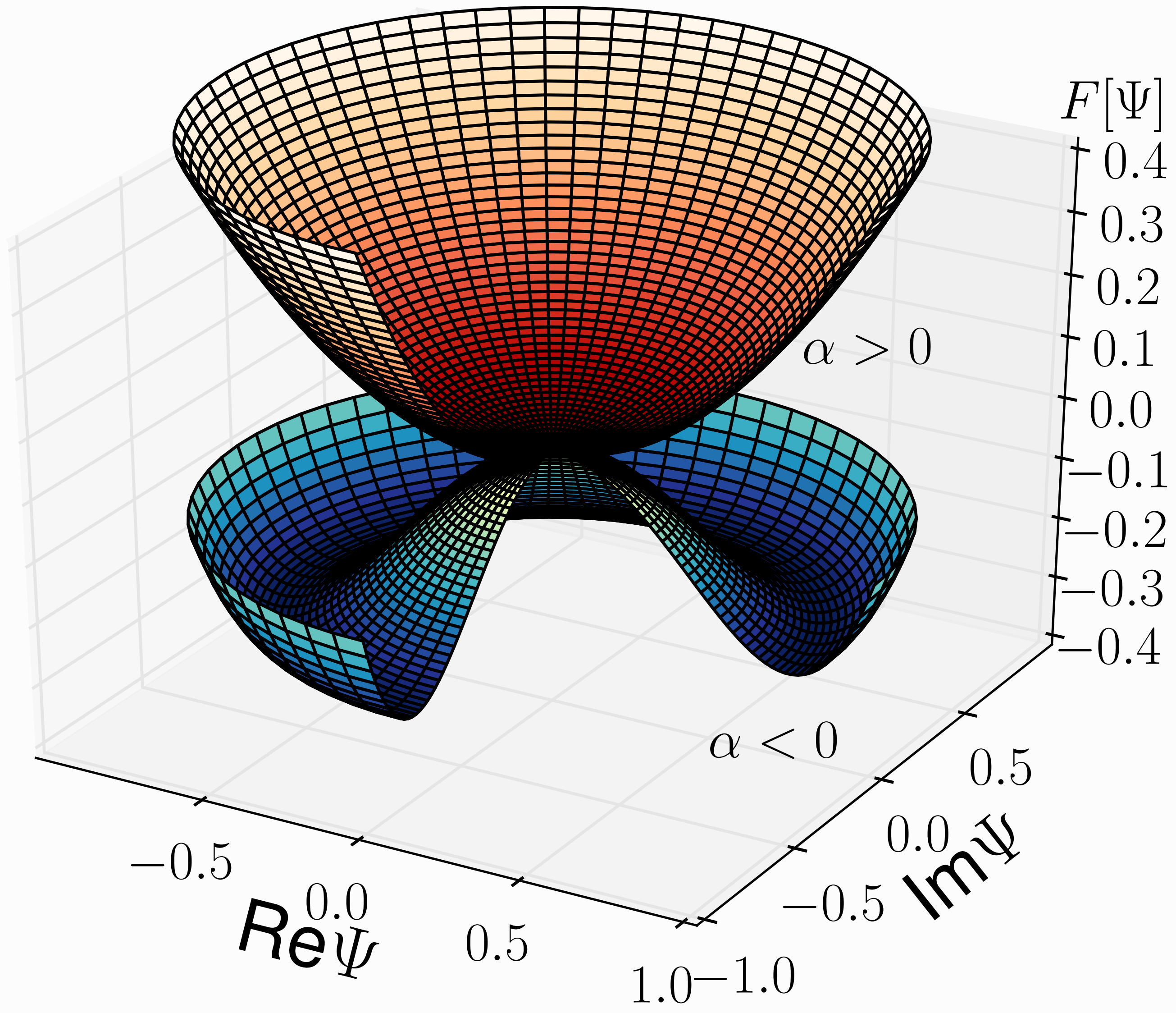
- Equilibrium: Landau's theory of symmetry breaking phase transitions
  - Continuous Transition: $\Psi \to 0$ for $T \to T_c - 0^+$
  - Discontinuous Change in Symmetry: $G' \subset G$
- Ginzburg-Landau: $\mathtt{U(1)}_\mathsf{N}$ gauge symmetry is broken: $\Psi \xrightarrow{\alpha\in\mathtt{U(1)}_\mathsf{N}} \Psi\, e^{i\alpha} \neq \Psi$
- Taylor Expansion of the Free Energy in Maximal Symmetry Invariants: $\Psi\Psi^* = |\Psi|^2$ $\Psi\Psi\Psi^*\Psi^* = |\Psi|^4$ $\nabla_i\Psi\nabla_i\Psi^* = |\nabla\Psi|^2$

- $F = E - TS$ for a homogeneous Superconductor ($\vec{B} = 0$):

$$F[\Psi; T, p] = \overbrace{F_N(p,T)}^{\text{Normal State}} + \int dV \left\{ \alpha(p,T)\, |\Psi|^2 + \beta(p,T)\, |\Psi|^4 \right\} \quad (1)$$

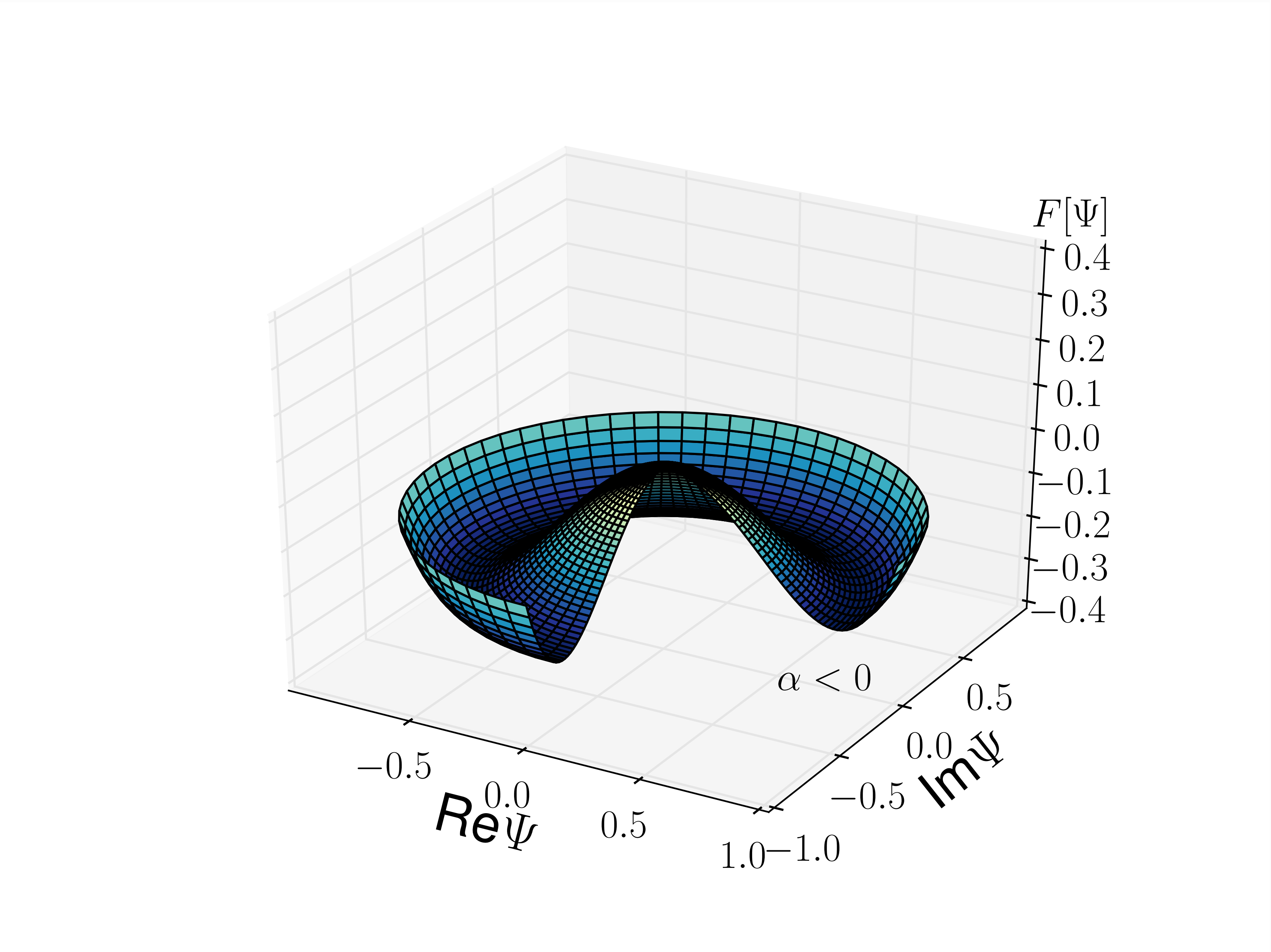
- Equlibrium: Minimum of F with respect to Ψ
- Global Stability: $\beta > 0$ ▶ Transition: $\alpha(T_c, p) \equiv 0 \rightsquigarrow \alpha(T,p) \approx \alpha'(T - T_c)$





- $T > T_c$: $\Psi = 0$ (normal state)
- $T < T_c$: $|\Psi| = \sqrt{\dfrac{|\alpha(T,p)|}{2\beta}} \sim (1 - T/T_c)^{\frac{1}{2}}$

- Condensate Amplitude for $T < T_c$: $\Psi_{\text{eq}} = \sqrt{\frac{\alpha' T_c}{2\beta}}(1 - T/T_c)^{\frac{1}{2}}$



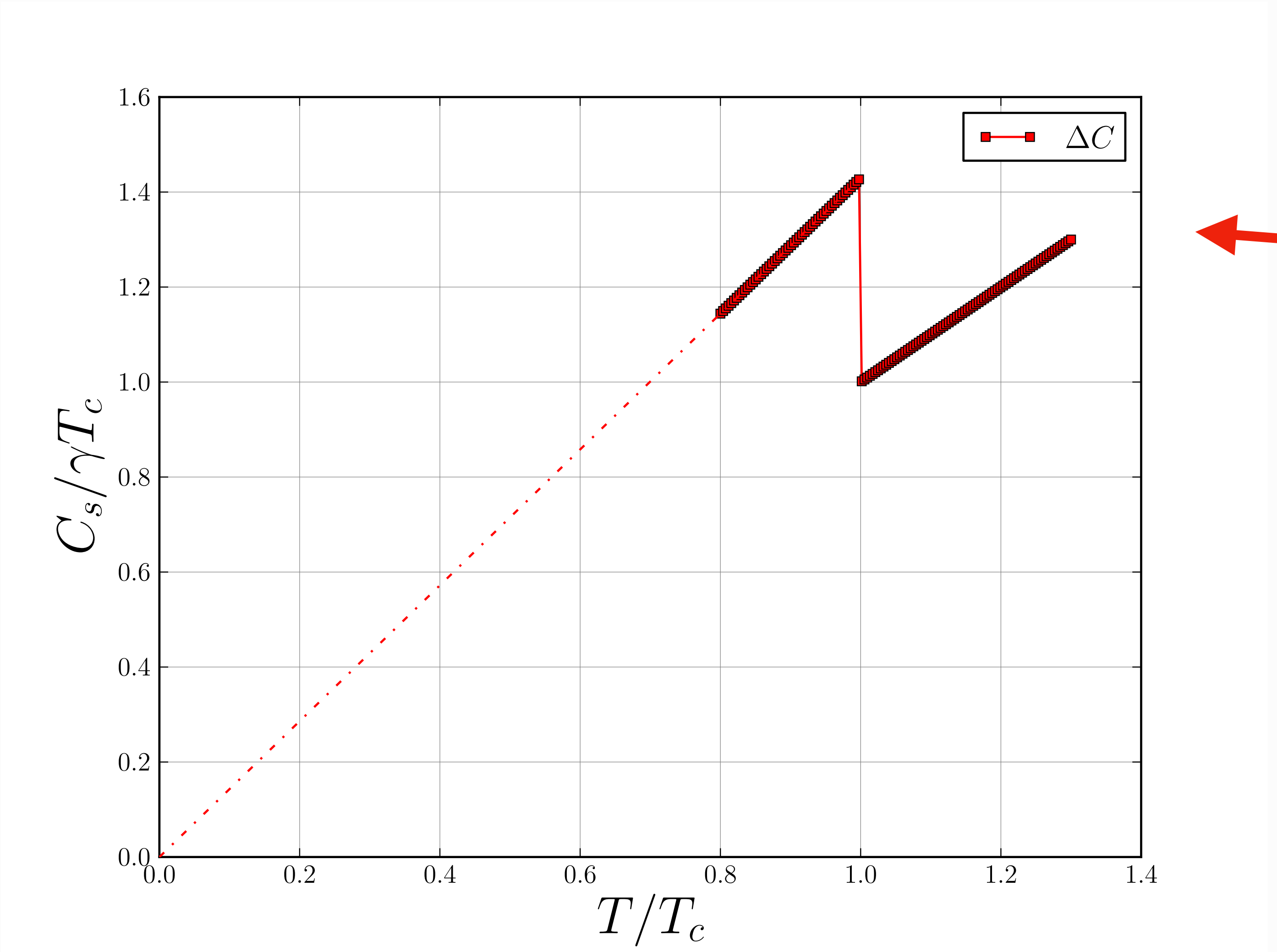
- Condensation Energy Density:

$$\Delta F_{\text{eq}} = \frac{1}{2}\alpha(T,p)\Psi_{\text{eq}}^2 \tag{2}$$

$$= -\frac{(\alpha')^2}{4\beta}(T - T_c)^2 \tag{3}$$

- Entropy Reduction:

$$\Delta S = -\frac{\partial \Delta F}{\partial T} = \frac{(\alpha')^2}{2\beta}(T - T_c) \tag{4}$$



- Heat Capacity Jump:

$$\Delta C = T\frac{\partial \Delta S}{\partial T} = T\frac{(\alpha')^2}{2\beta} \tag{5}$$

- External Field - $\vec{H}$, Magnetization $\vec{M}$ and Total Field $\vec{B}$ $= \nabla \times \vec{A}$ (Vector Potential)
- $\Psi(\mathbf{r})$ is the wave function for "super" electrons of charge e^*
- BCS theory: $e^* = 2e$ (Cooper Pairs of Electrons)
- Local Gauge Invariant Coupling: $\frac{\hbar}{i}\nabla\Psi \rightsquigarrow \left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)\Psi$

GL Functional with Coupling to $\vec{A}$ plus Field Energy

$$\boldsymbol{F}[\Psi,\vec{A}] = \int dV \left\{ \alpha(T)|\Psi|^2 + \tfrac{1}{2}\beta|\Psi|^4 + \frac{1}{2M^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)\Psi\right|^2 + \frac{1}{8\pi}\left(\nabla\times\vec{A}-\vec{H}\right)^2 \right\}$$

- External Field - $\vec{H}$, Magnetization $\vec{M}$ and Total Field $\vec{B}$ $= \nabla \times \vec{A}$ (Vector Potential)
- $\Psi(\mathbf{r})$ is the wave function for "super" electrons of charge e^*
- BCS theory: $e^* = 2e$ (Cooper Pairs of Electrons)

Local
- Gauge Invariant Coupling: $\frac{\hbar}{i}\nabla\Psi \rightsquigarrow \left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)\Psi$

GL Functional with Coupling to $\vec{A}$ plus Field Energy

$$\boldsymbol{F}[\Psi,\vec{A}] = \int dV \left\{ \alpha(T)|\Psi|^2 + \tfrac{1}{2}\beta|\Psi|^4 + \frac{1}{2M^*}\left|\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)\Psi\right|^2 + \frac{1}{8\pi}\left(\nabla\times\vec{A} - \vec{H}\right)^2 \right\}$$

Ginzburg-Landau Field Equations

Euler-Lagrange
Equations of GL Theory

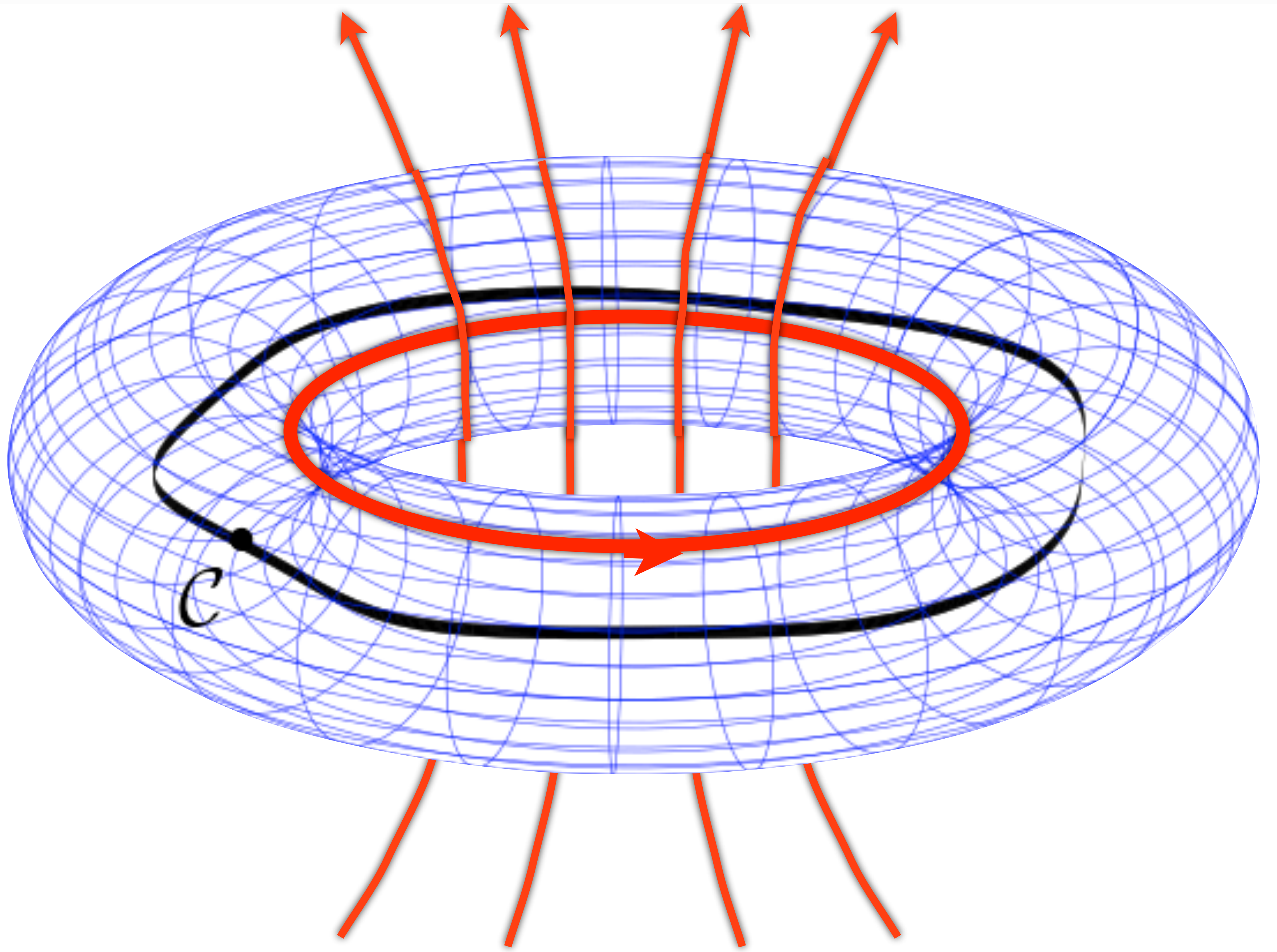
$$\frac{1}{2M^*}\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)^2\Psi + \beta|\Psi|^2\Psi = -\alpha(T)\Psi$$

Local Gauge Invariance

$$\nabla\times\nabla\times\vec{A} = \frac{4\pi}{c}\vec{J} = \frac{4\pi}{c}\frac{e^*}{2M^*}\left[\Psi^*\left(\frac{\hbar}{i}\nabla - \frac{e^*}{c}\vec{A}\right)\Psi + c.c.\right] = \boxed{\frac{4\pi}{c}\frac{e^*}{M^*}|\Psi(\mathbf{r})|^2\left(\hbar\nabla\vartheta - \frac{e^*}{c}\vec{A}\right)}$$

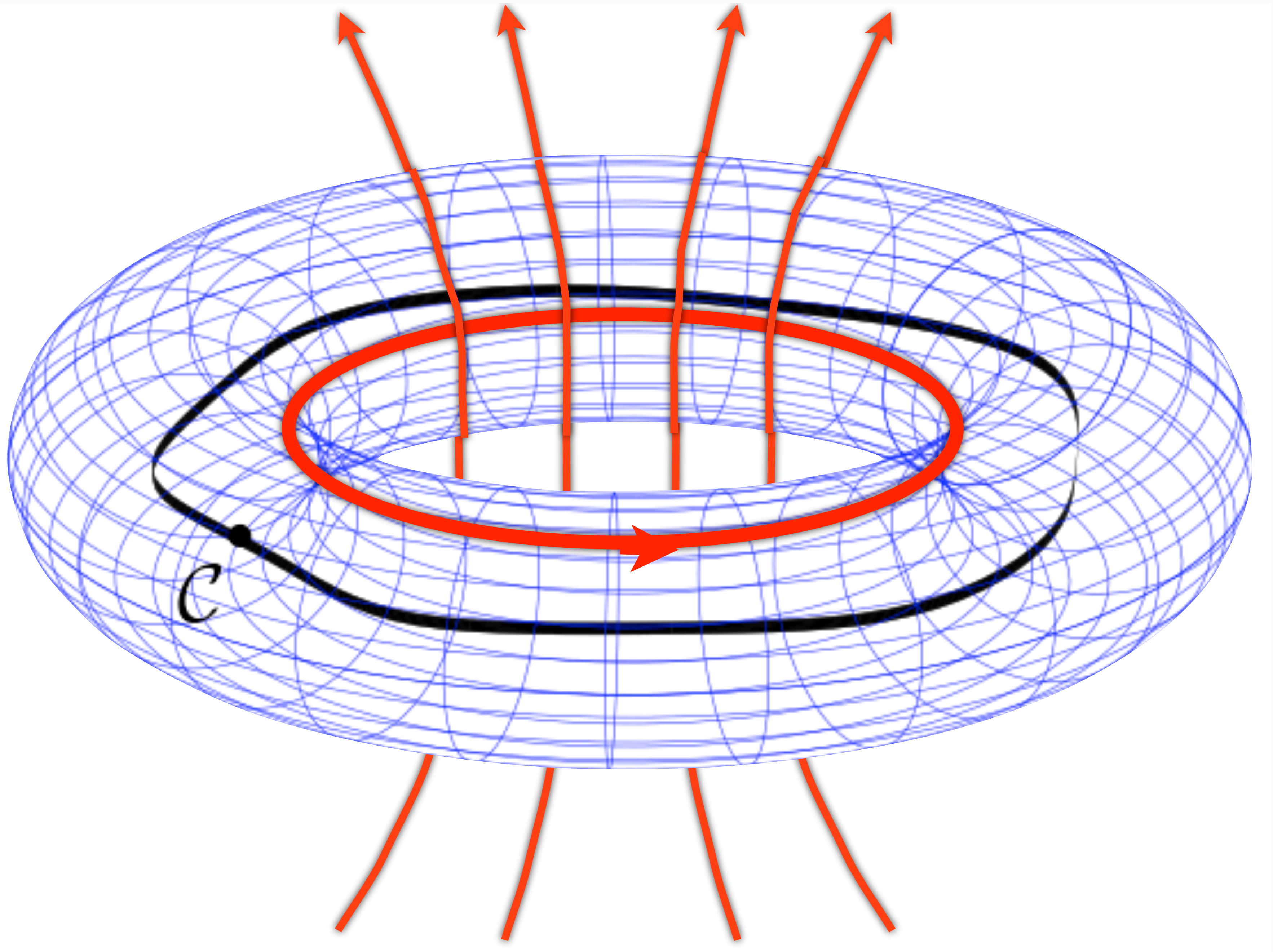
- Gauge change: $\vec{A} \to \vec{A} + \nabla\Lambda \rightsquigarrow \vartheta \to \vartheta + \frac{e^*}{\hbar c}\Lambda$ (broken $\mathtt{U(1)_N}$ symmetry)

- $T < T_c$ in an External Magnetic Field
- $T < T_c$ Turn off External Field
- Trapped $\vec{B}$ threads the hole in the Torus



- $\vec{B} = 0$ inside the Superconductor

- $T < T_c$ in an External Magnetic Field
- $T < T_c$ Turn off External Field
- Trapped $\vec{B}$ threads the hole in the Torus

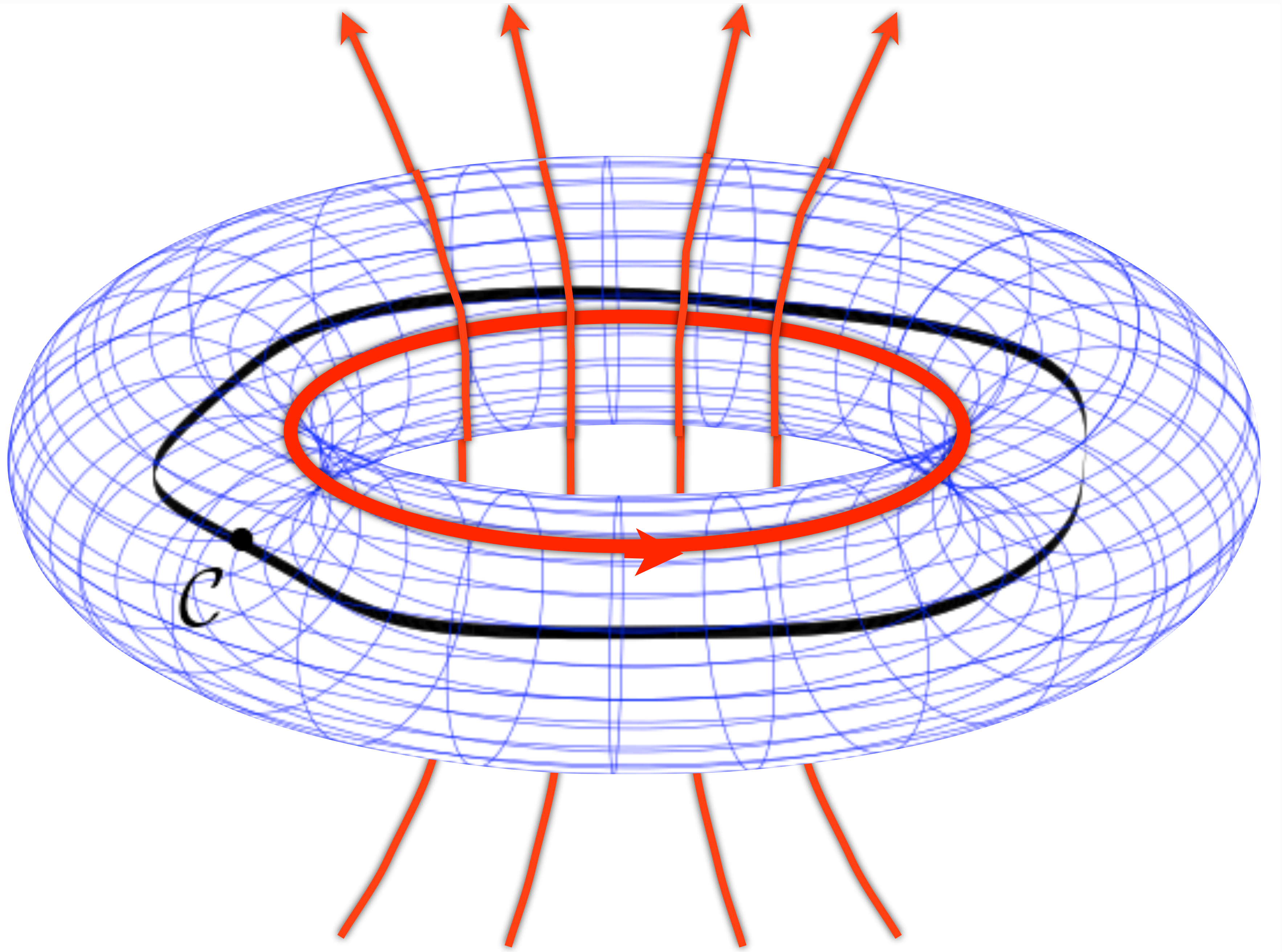


- $\vec{B} = 0$ inside the Superconductor

- $\vec{J} = \frac{e^*}{M^*} |\Psi_0|^2 \left(\hbar \nabla \vartheta - \frac{e^*}{c} \vec{A} \right)$
- $\boxed{\vec{J} \neq 0}$ on the Inner surface
- Meissner Screening inside the SC

$\rightsquigarrow \boxed{\vec{J} = 0} \qquad \therefore \boxed{\oint_{\mathscr{C}} \vec{J} \cdot d\vec{l} \equiv 0}$

- $T < T_c$ in an External Magnetic Field
- $T < T_c$ Turn off External Field
- Trapped $\vec{B}$ threads the hole in the Torus



- $\vec{B} = 0$ inside the Superconductor

- $\vec{J} = \frac{e^*}{M^*}|\Psi_0|^2\left(\hbar\nabla\vartheta - \frac{e^*}{c}\vec{A}\right)$
- $\vec{J} \neq 0$ on the Inner surface
- Meissner Screening inside the SC

  $\rightsquigarrow \vec{J} = 0 \qquad \therefore \oint_{\mathscr{C}} \vec{J}\cdot d\vec{l} \equiv 0$

- $\therefore \oint_{\mathscr{C}} \vec{A}\cdot d\vec{l} = \frac{\hbar c}{e^*}\oint_{\mathscr{C}} \nabla\vartheta\cdot d\vec{l}$
- Phase Quantization: $\oint_{\mathscr{C}} \nabla\vartheta\cdot d\vec{l} = N\,2\pi$

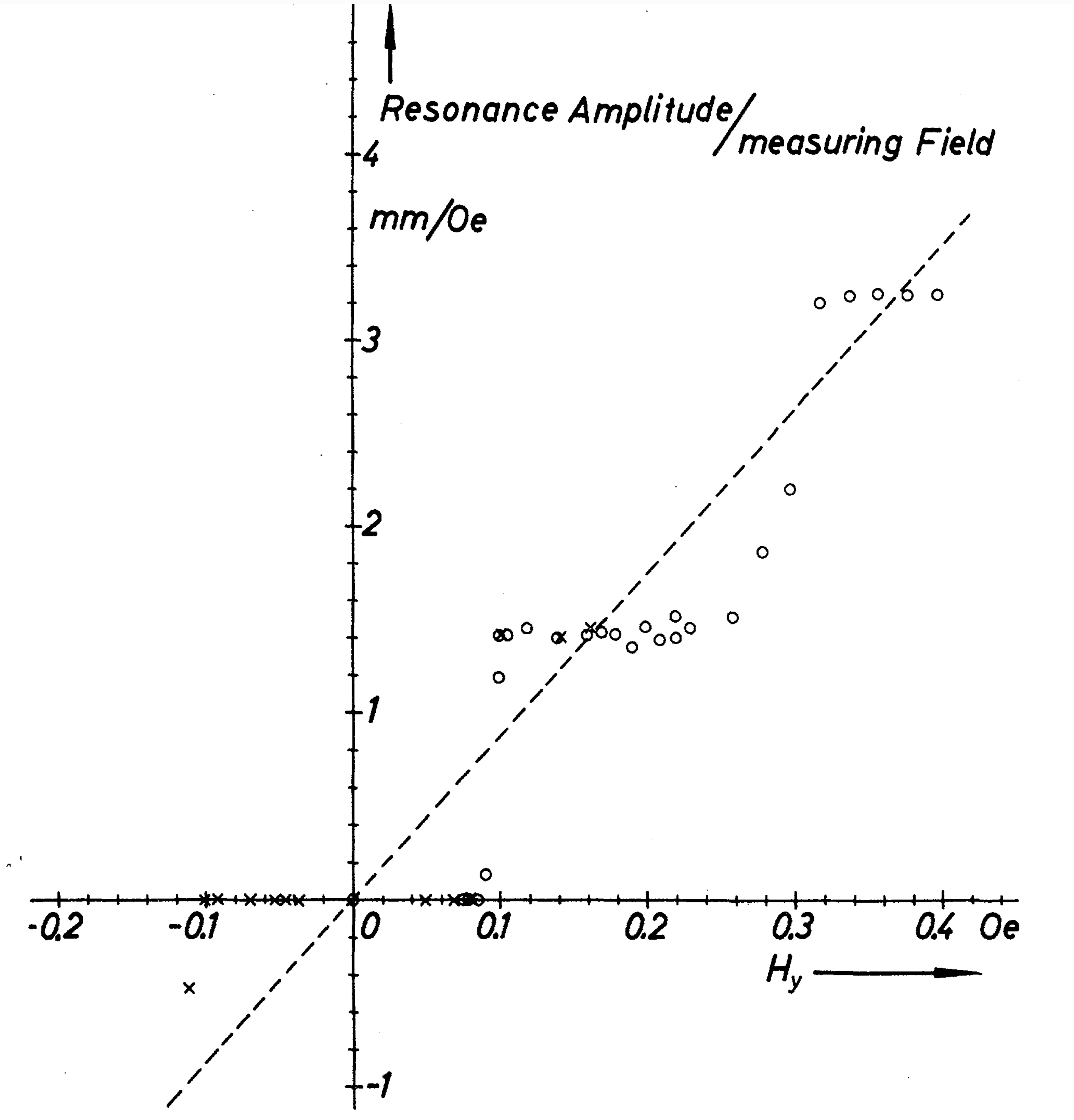
  $N = 0, \pm 1, \pm 2, \ldots$

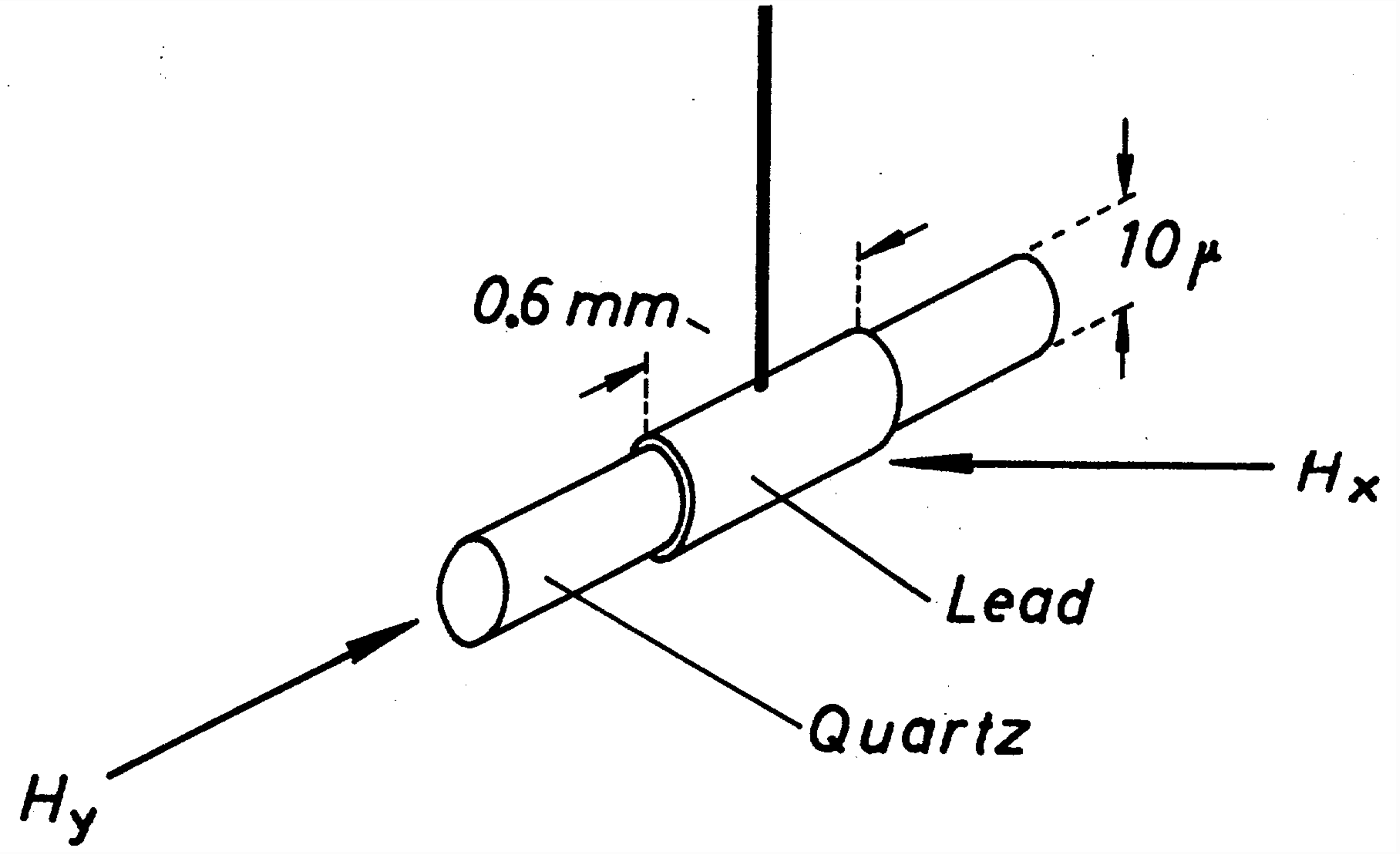
- Flux Quantization: $\Phi \equiv \iint_{S_{\mathscr{C}}} \vec{B}\cdot d\vec{S} = \oint_{\mathscr{C}} \vec{A}\cdot d\vec{l} = N\frac{hc}{e^*}$
- Superconducting electrons are *bound electron pairs*: $\rightsquigarrow e^* = 2e$

  $\therefore \Phi = N\frac{hc}{2e}$ with *Flux Quantum* $\Phi_0 \equiv \frac{hc}{2e} \approx 2\times 10^{-7}$ G-cm^2

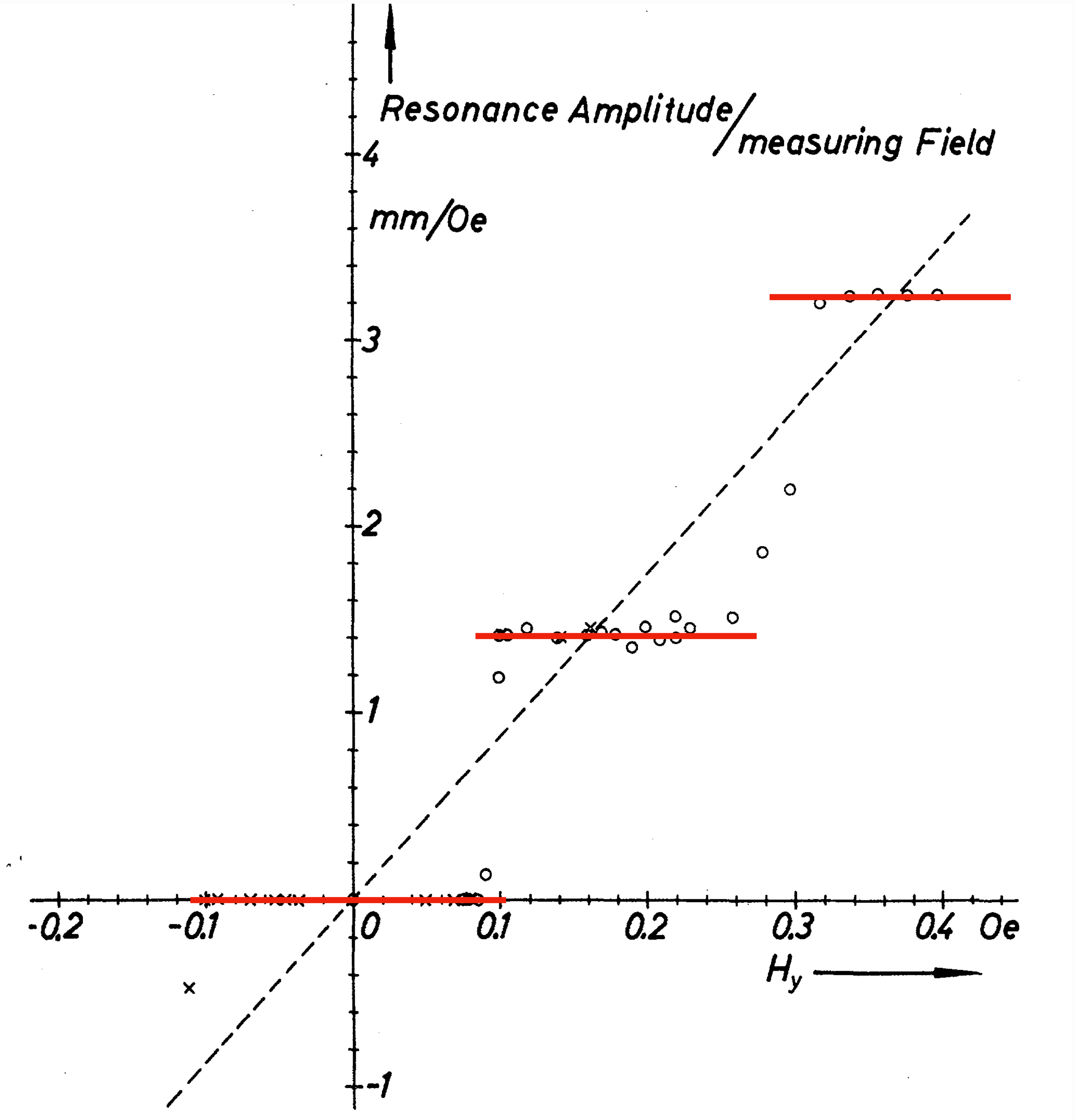
Observation of Magnetic Flux Quantization August 6, 2023

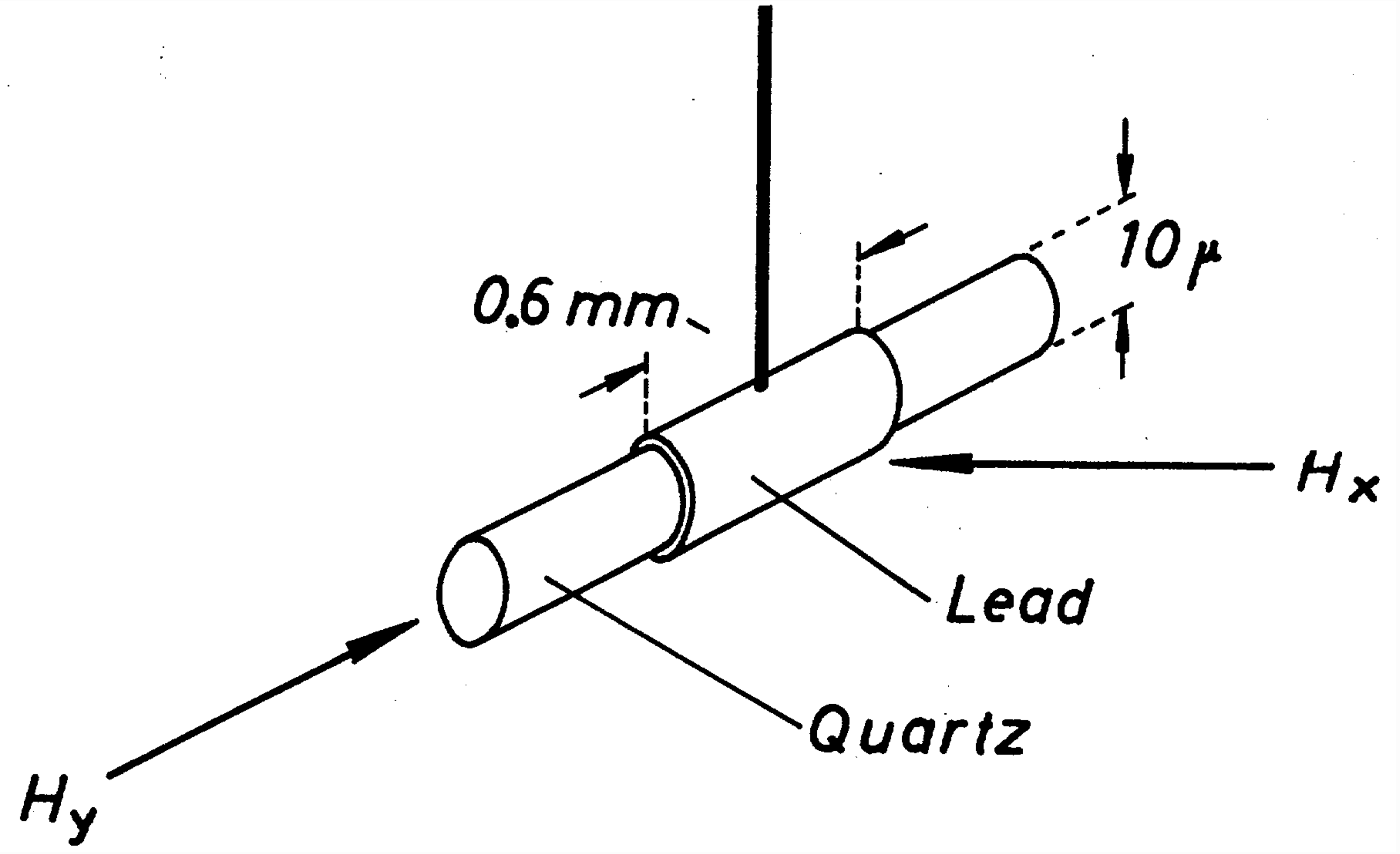
R. Doll and M. Näbauer, Physical Review Letters 7, 51 (1961)





R. Doll and M. Näbauer, Physical Review Letters 7, 51 (1961)





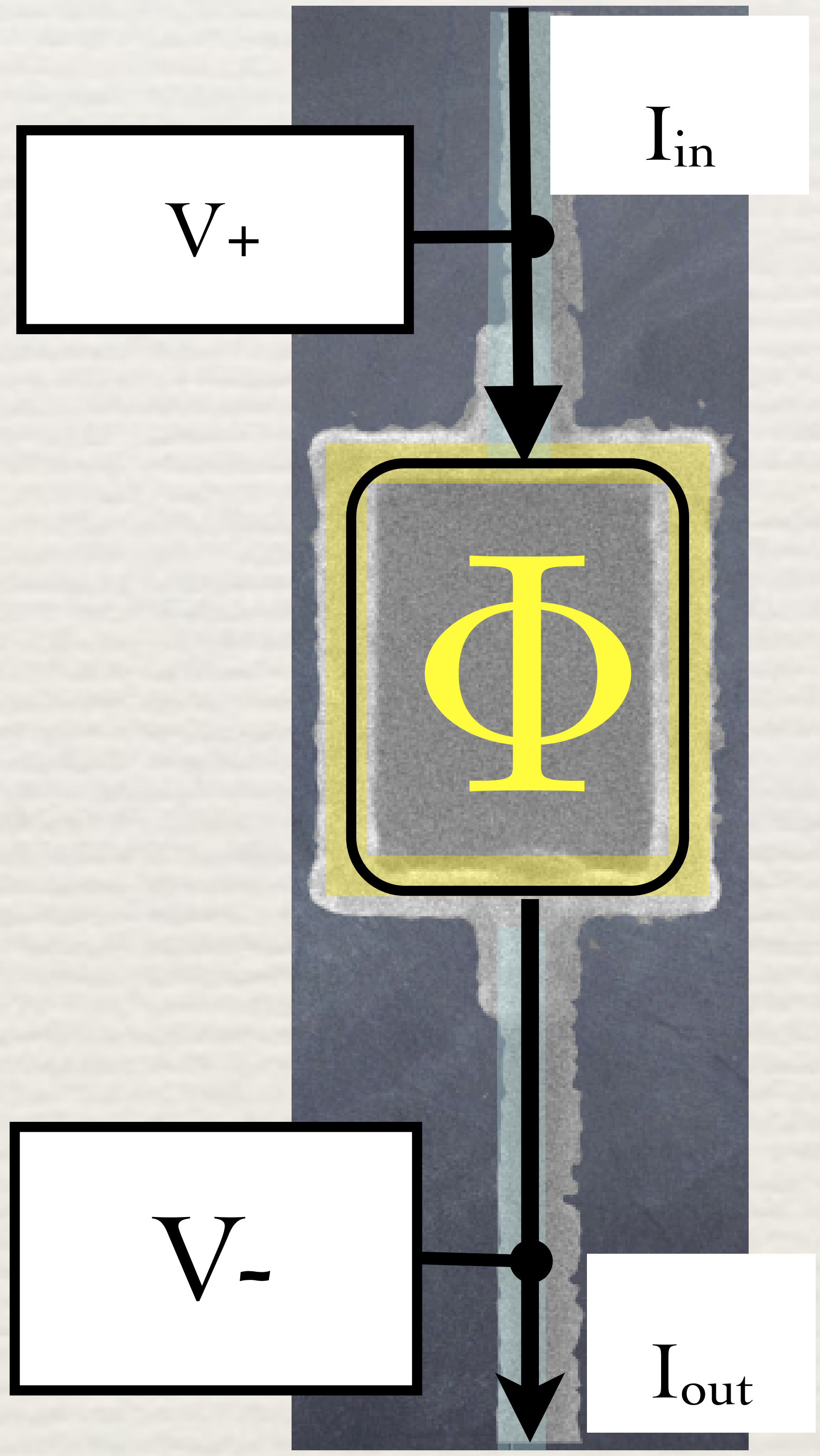
- Doll and Näbauer thought they had a systematic error ("$\approx \times 2$") as they were looking to find steps of size $\frac{hc}{e}$ - London's prediction prior to BCS theory!

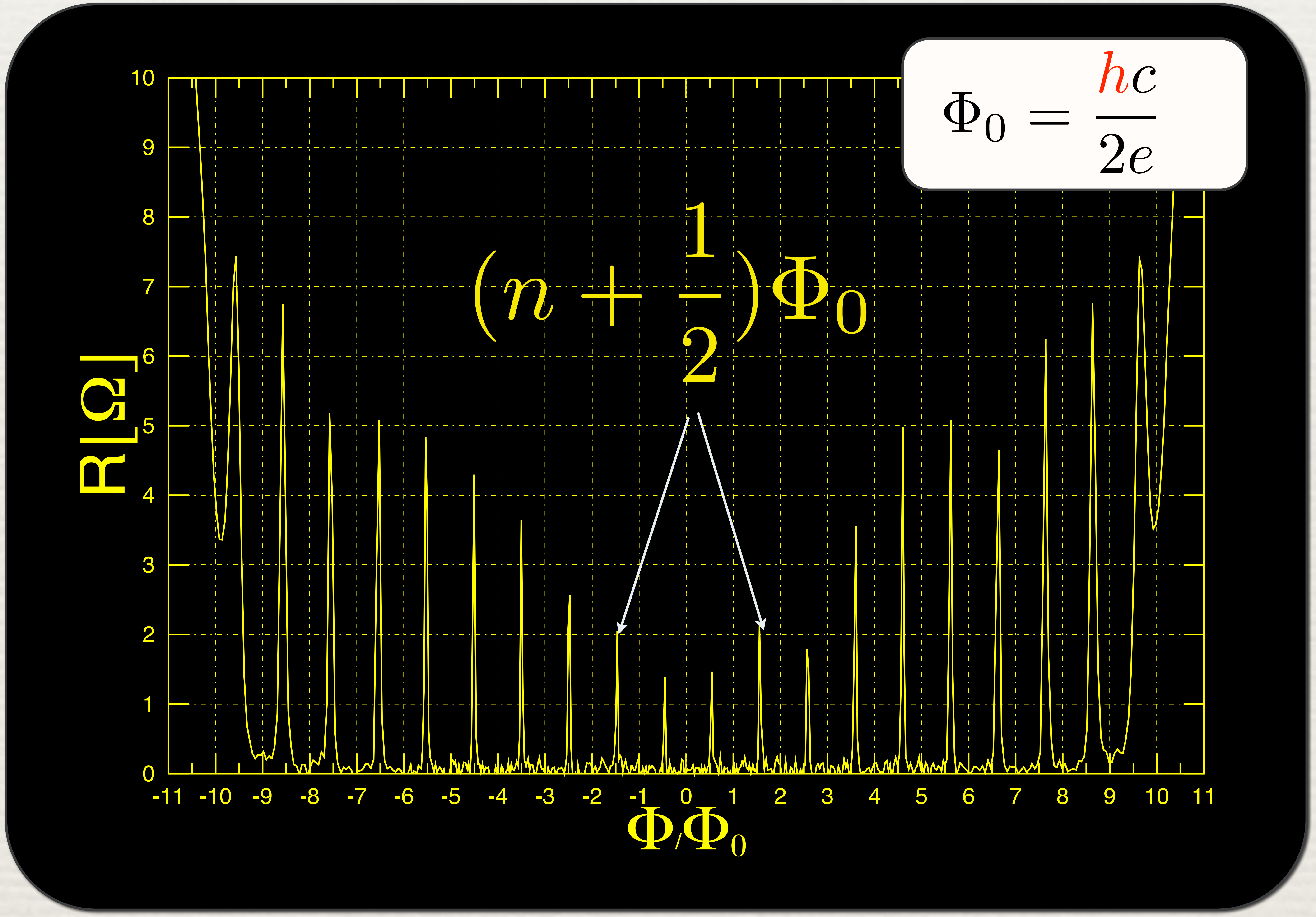
Superconductor-Normal-Superconductor Quantum Interference Device

- Au – normal conductor (in *contact* to superconducting leads)
- Al – superconductor
- Silicon oxide substrate

V. Chandrasekhar's Lab
Northwestern University

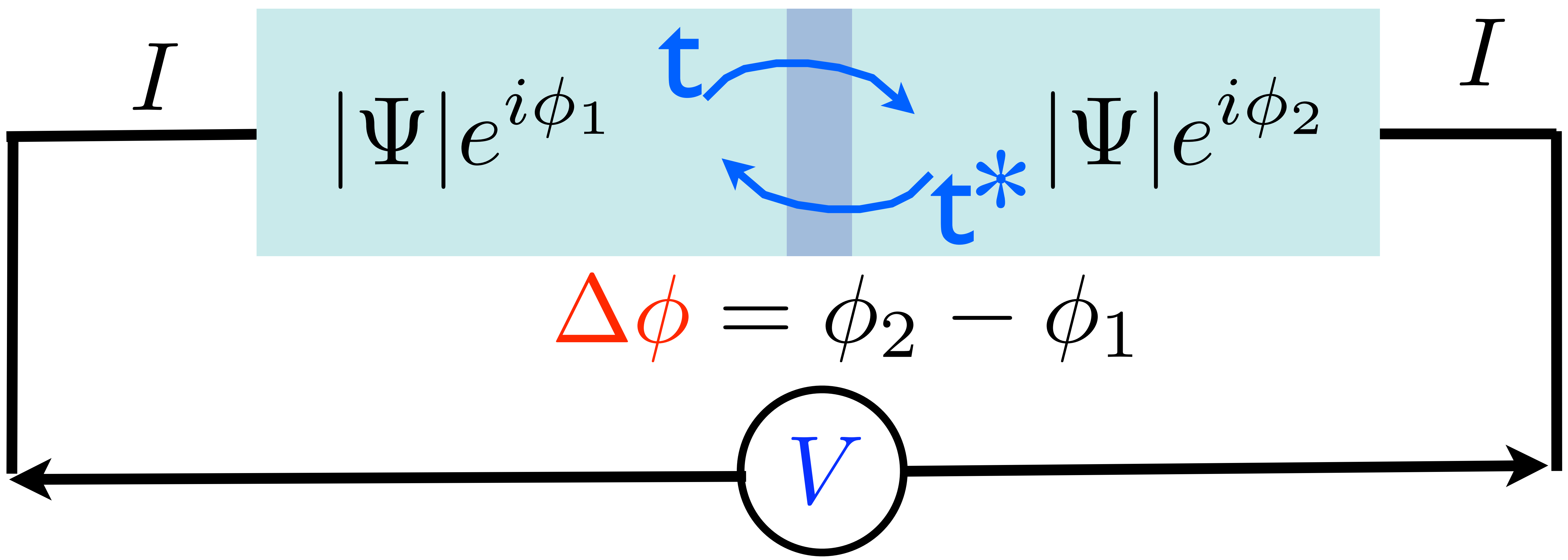
J. Wei, et al. 2008 (J. Appl. Phys.)





$$\Delta\vartheta = 2\pi\Phi/\Phi_0$$

Josephson Effect



d.c. Josephson current

$$I_c \propto |\mathsf{t}|^2$$

$$I = I_c(T)\,\sin(\Delta\phi)$$

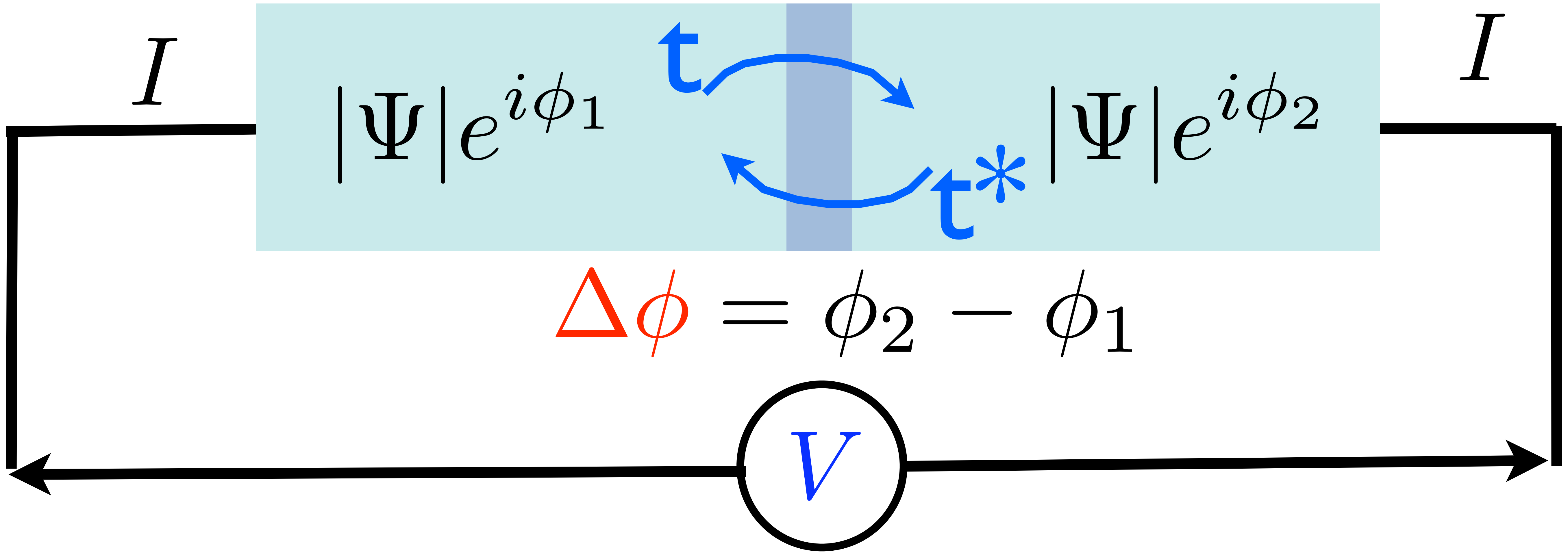
a.c. Josephson effect

$$I > I_c$$

$$\Delta\phi_t = \frac{2e}{\hbar} V\, t$$

B. Josephson, Phys. Lett. 1, 251 (1962);
Adv. Phys. 14, 419 (1965)

Josephson Effect



Key element of quantum circuits for qubits

$\Downarrow$

Next Lecture by Prof. Jens Koch

d.c. Josephson current

$$I = I_c(T)\,\sin(\Delta\phi)$$

$$I_c \propto |\mathrm{t}|^2$$

a.c. Josephson effect

$$I > I_c$$

$$\Delta\phi_t = \frac{2e}{\hbar} V\, t$$

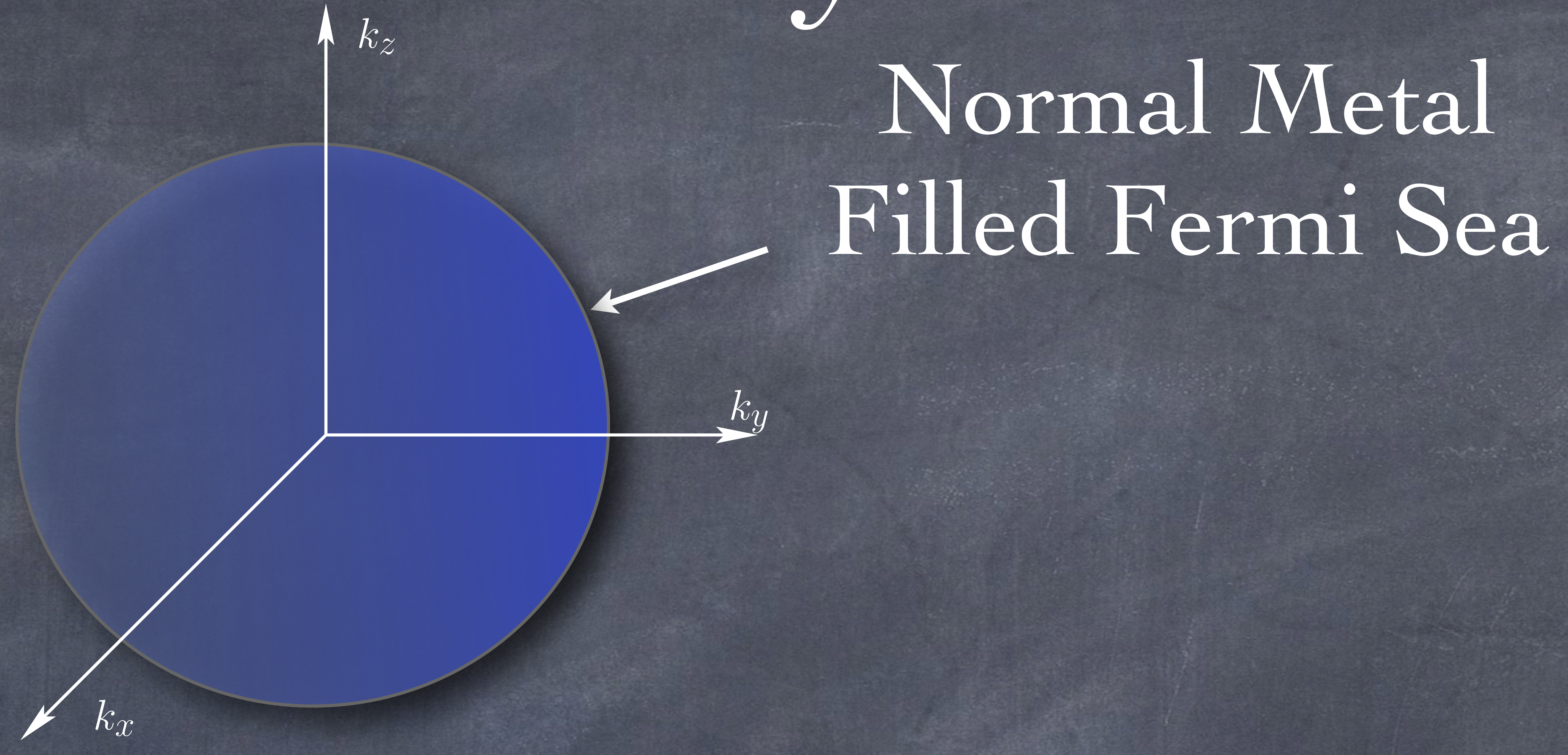
B. Josephson, Phys. Lett. 1, 251 (1962);
Adv. Phys. 14, 419 (1965)

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

Cooper's Instability

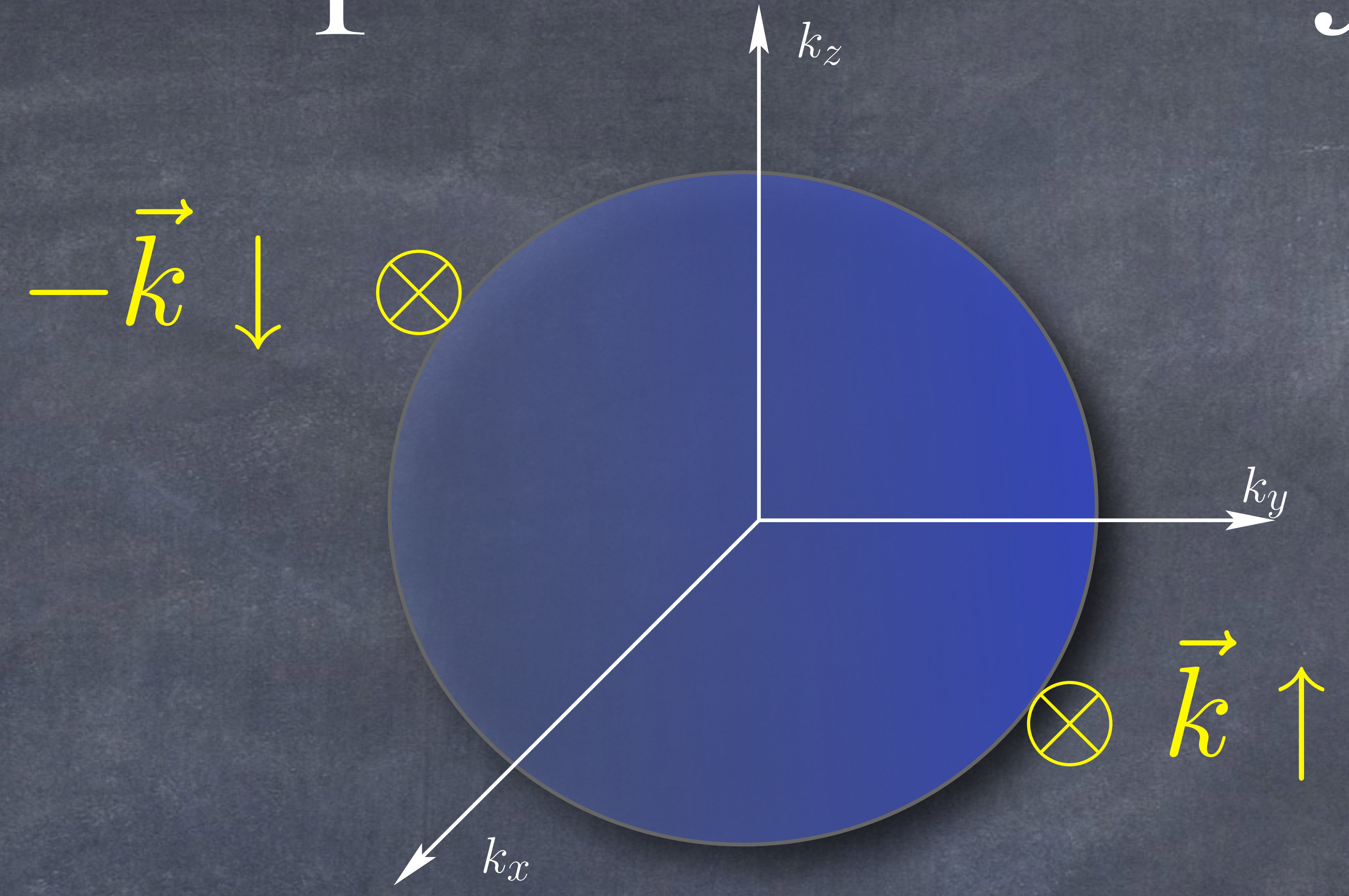
L. N. Cooper, Phys. Rev. 104, 1189 (1956)

Cooper's Instability



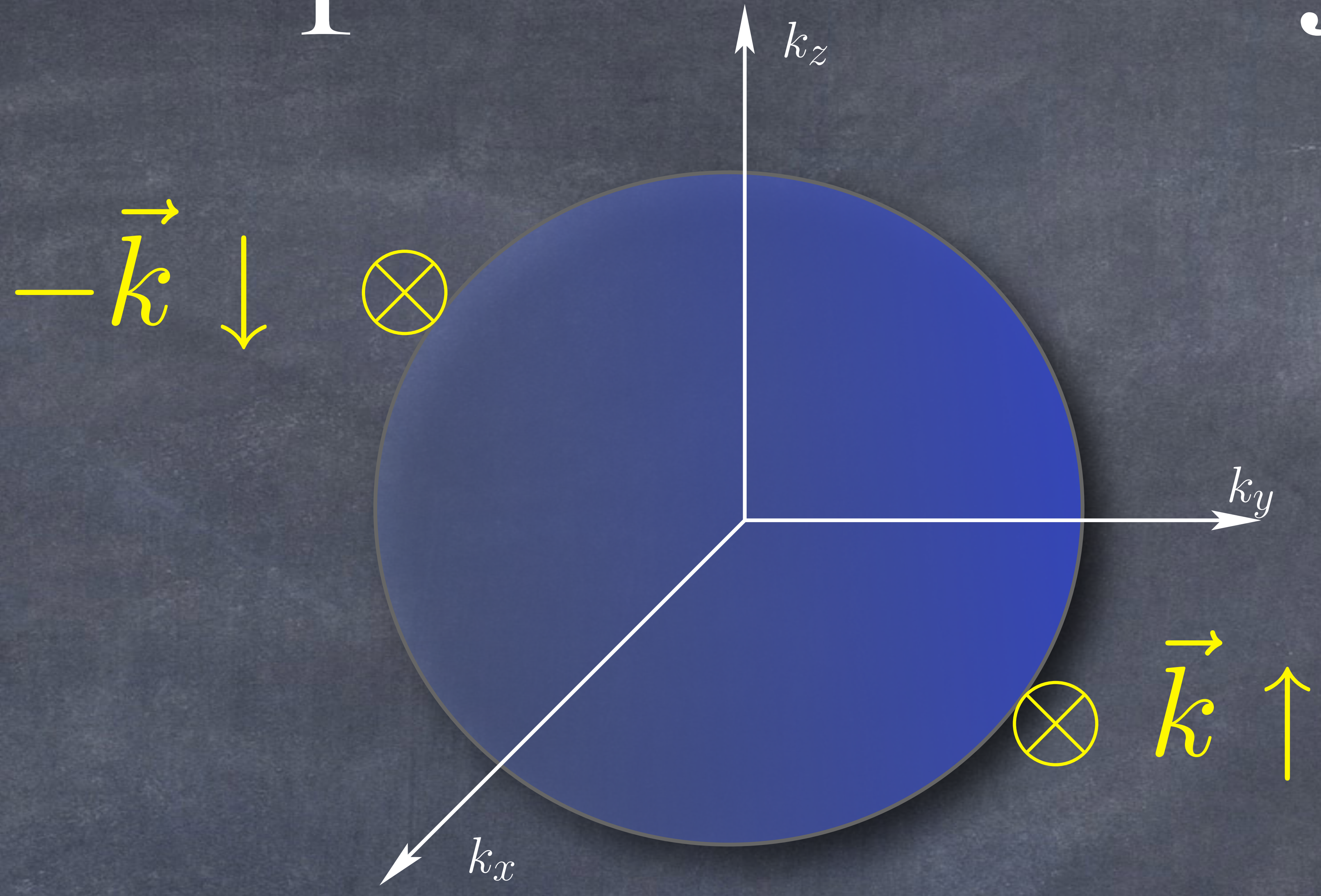
L. N. Cooper, Phys. Rev. 104, 1189 (1956)

Cooper's Instability



L. N. Cooper, Phys. Rev. 104, 1189 (1956)

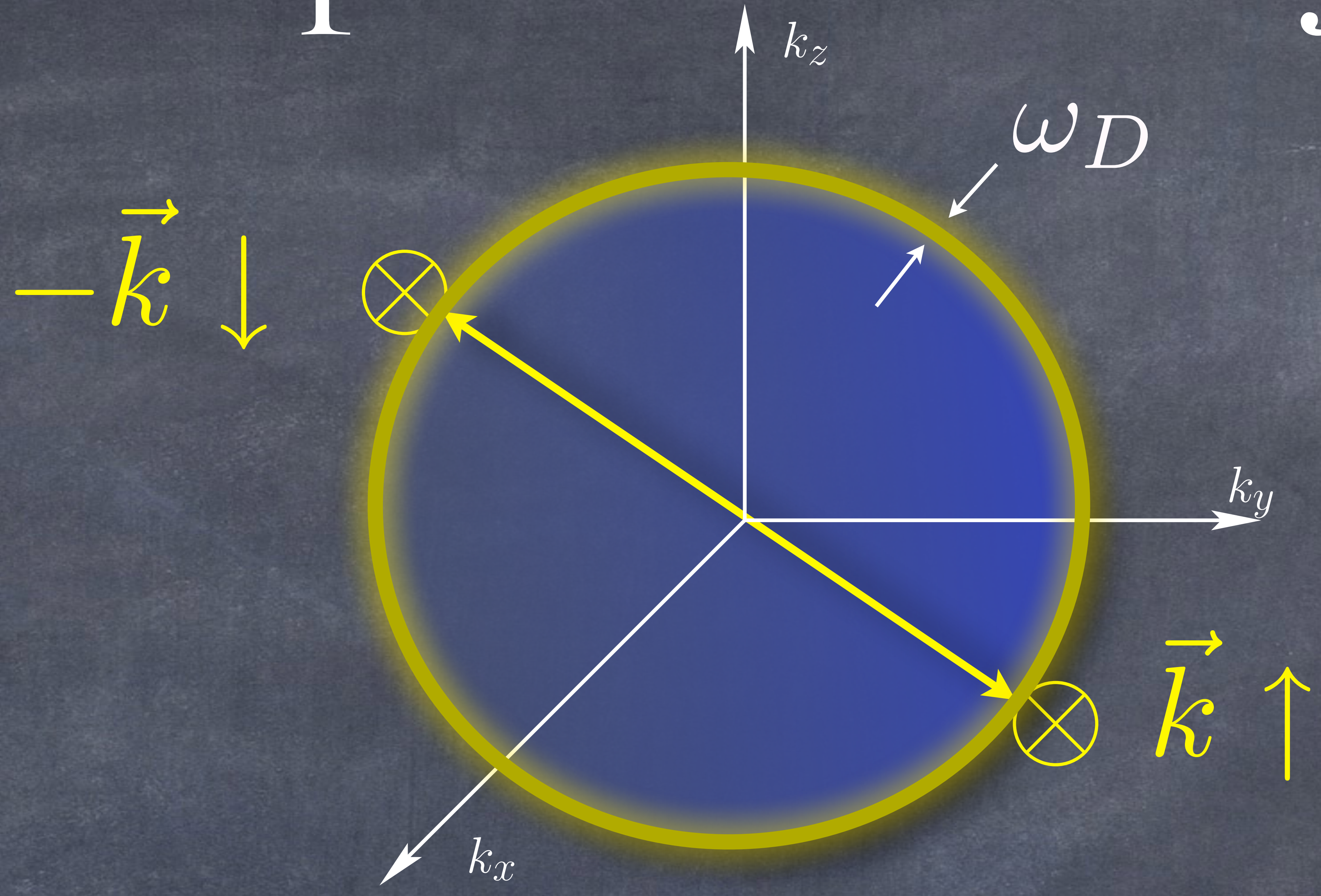
Cooper's Instability



$$2 \times \frac{\hbar^2 k^2}{2m}\varphi_{\vec{k}}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

Cooper's Instability

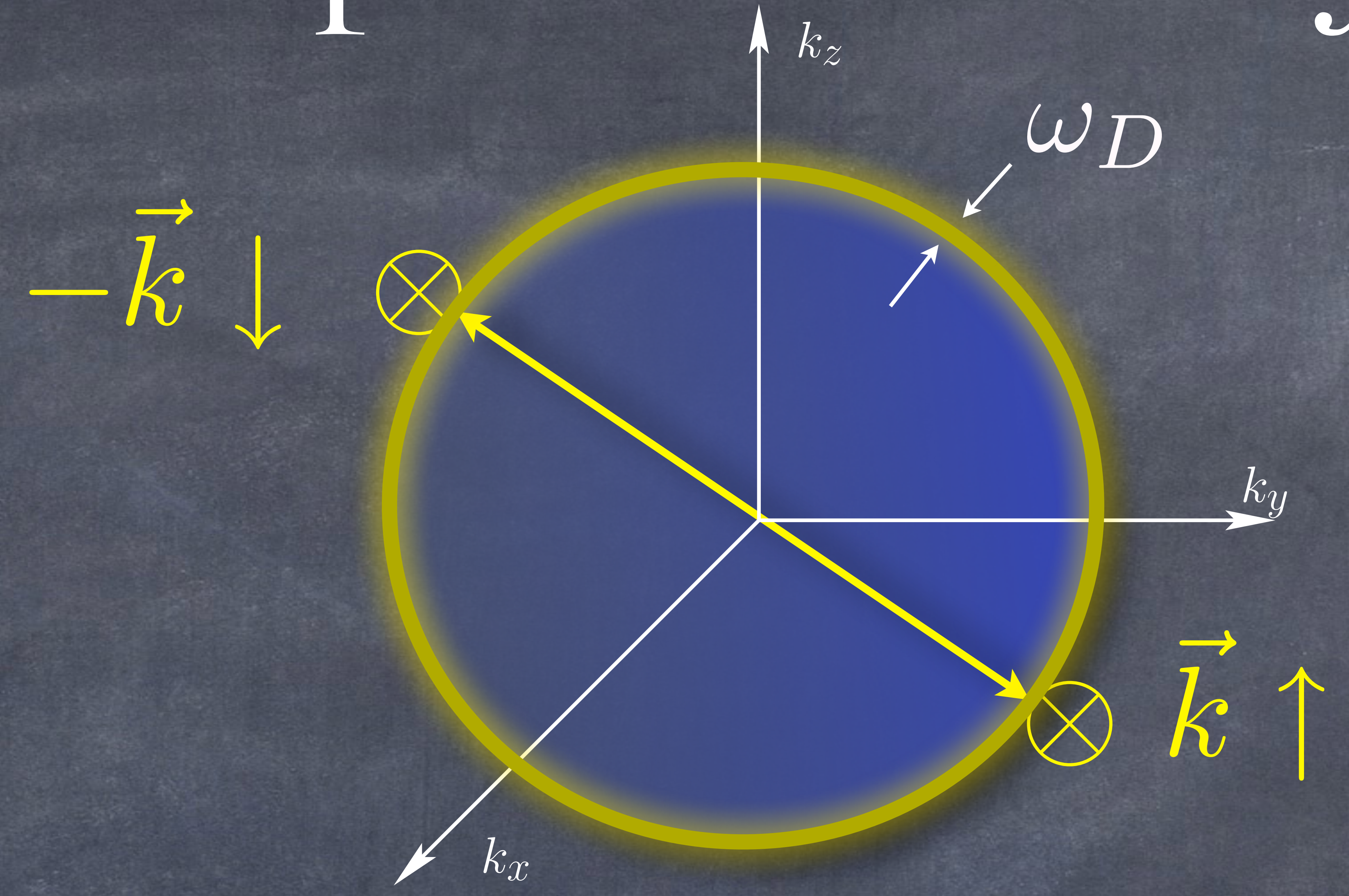


$$2 \times \frac{\hbar^2 k^2}{2m}\varphi_{\vec{k}}$$

$$\boxed{V_{\vec{k},\vec{k}'}\varphi_{\vec{k}'}}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

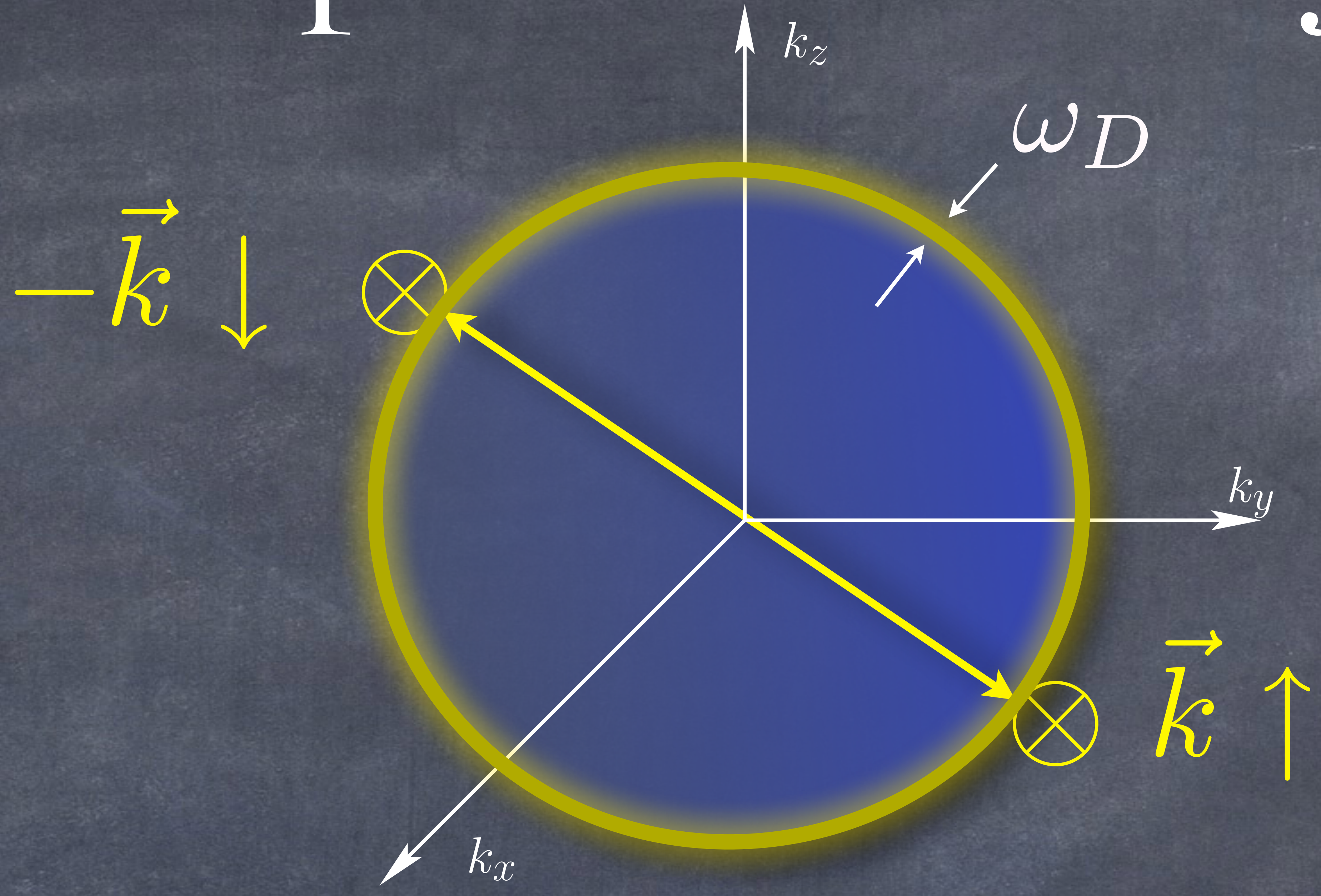
Cooper's Instability



$$2 \times \frac{\hbar^2 k^2}{2m}\varphi_{\vec{k}} - \int \frac{d^3k'}{8\pi^3}\boxed{V_{\vec{k},\vec{k}'}\varphi_{\vec{k}'}}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

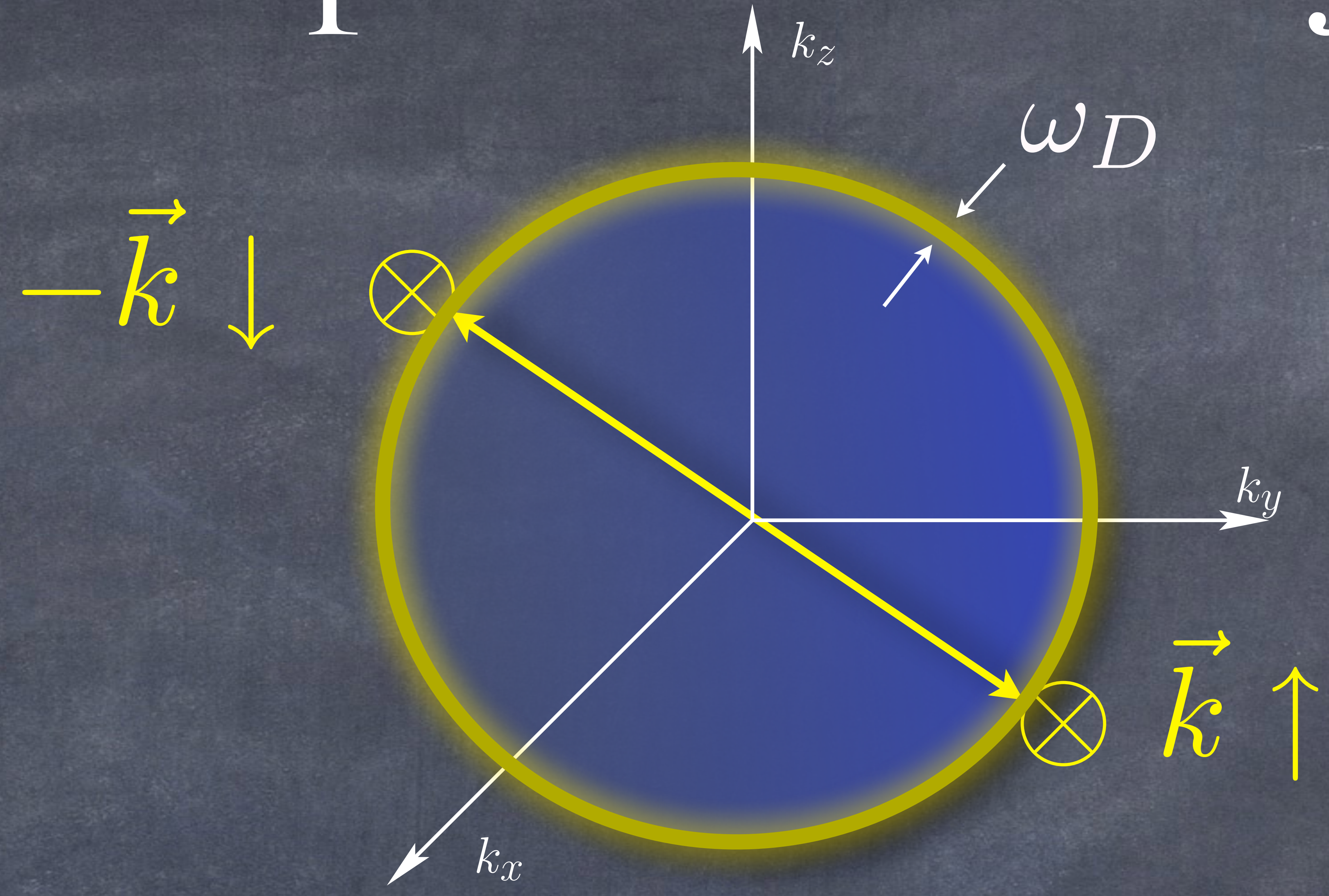
Cooper's Instability



$$2\times\frac{\hbar^2k^2}{2m}\varphi_{\vec{k}}-\int\frac{d^3k'}{8\pi^3}\boxed{V_{\vec{k},\vec{k}'}\varphi_{\vec{k}'}}=\left(2\frac{\hbar^2k_f^2}{2m}+\epsilon\right)\varphi_{\vec{k}}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

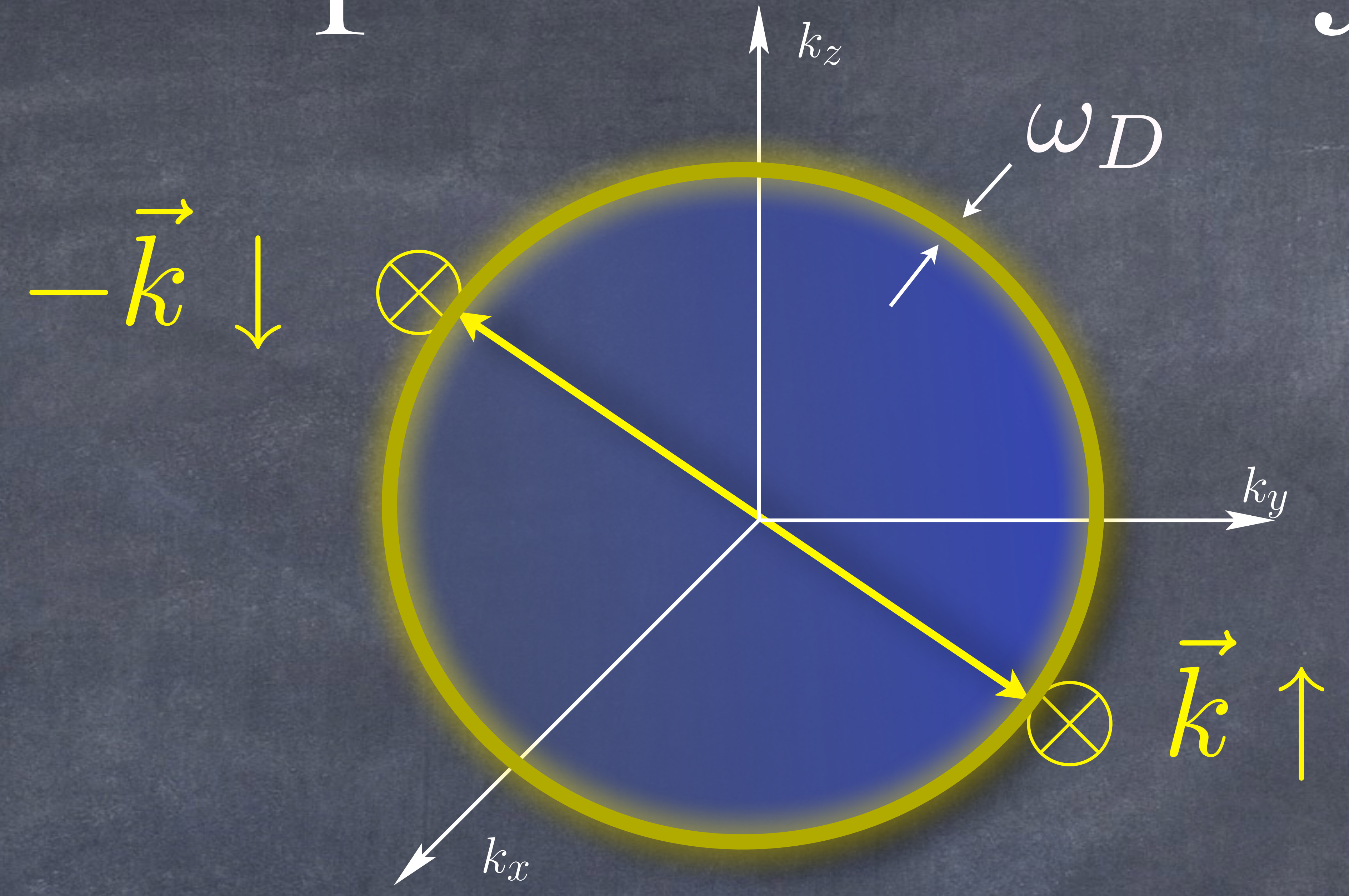
Cooper's Instability



$$2 \times \frac{\hbar^2 k^2}{2m}\varphi_{\vec{k}} - \int \frac{d^3k'}{8\pi^3} \boxed{V_{\vec{k},\vec{k}'}\varphi_{\vec{k}'}} = (2\frac{\hbar^2 k_f^2}{2m} + \epsilon)\varphi_{\vec{k}}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

Cooper's Instability



$$2 \times \frac{\hbar^2 k^2}{2m}\varphi_{\vec{k}} - \int \frac{d^3k'}{8\pi^3} \boxed{V_{\vec{k},\vec{k}'}\varphi_{\vec{k}'}} = (2\frac{\hbar^2 k_f^2}{2m} + \epsilon)\varphi_{\vec{k}}$$

Bound-State of 2 electrons on the Fermi Sea

$$\epsilon_{bs} = -\,\omega_D\, e^{-1/N(0)V}$$

L. N. Cooper, Phys. Rev. 104, 1189 (1956)

$$|BCS\rangle = \left[\sum_k \Phi_k \, a^\dagger_{k\uparrow} \, a^\dagger_{-k\downarrow}\right]^{N/2} |Fermi\rangle$$

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

$$|BCS\rangle = \left[\sum_k \Phi_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right]^{N/2} |Fermi\rangle$$

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

Electron Pairs - charge = 2e

$$|BCS\rangle = \left[\sum_k \Phi_k\, a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right]^{N/2} |Fermi\rangle$$

BCS Condensation: Macroscopic State of Fermion Pairs

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

Electron Pairs - charge = 2e

$$|BCS\rangle = \left[\sum_k \Phi_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right]^{N/2} |Fermi\rangle$$

BCS Condensation: Macroscopic State of Fermion Pairs

- Condensation Energy
- Supercurrents
- Energy Gap for Excitations
- Josephson Effect
- Meissner Effect
- Flux Quantization

Reduced to London & Ginzburg-Landau Theories in appropriate limits

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois
(Received July 8, 1957)

Electron Pairs - charge = 2e

$$|BCS\rangle = \left[\sum_k \Phi_k a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right]^{N/2} |Fermi\rangle$$

BCS Condensation: **Macroscopic State of Fermion Pairs**

- Condensation Energy
- Supercurrents
- Energy Gap for Excitations
- Josephson Effect
- Meissner Effect
- Flux Quantization

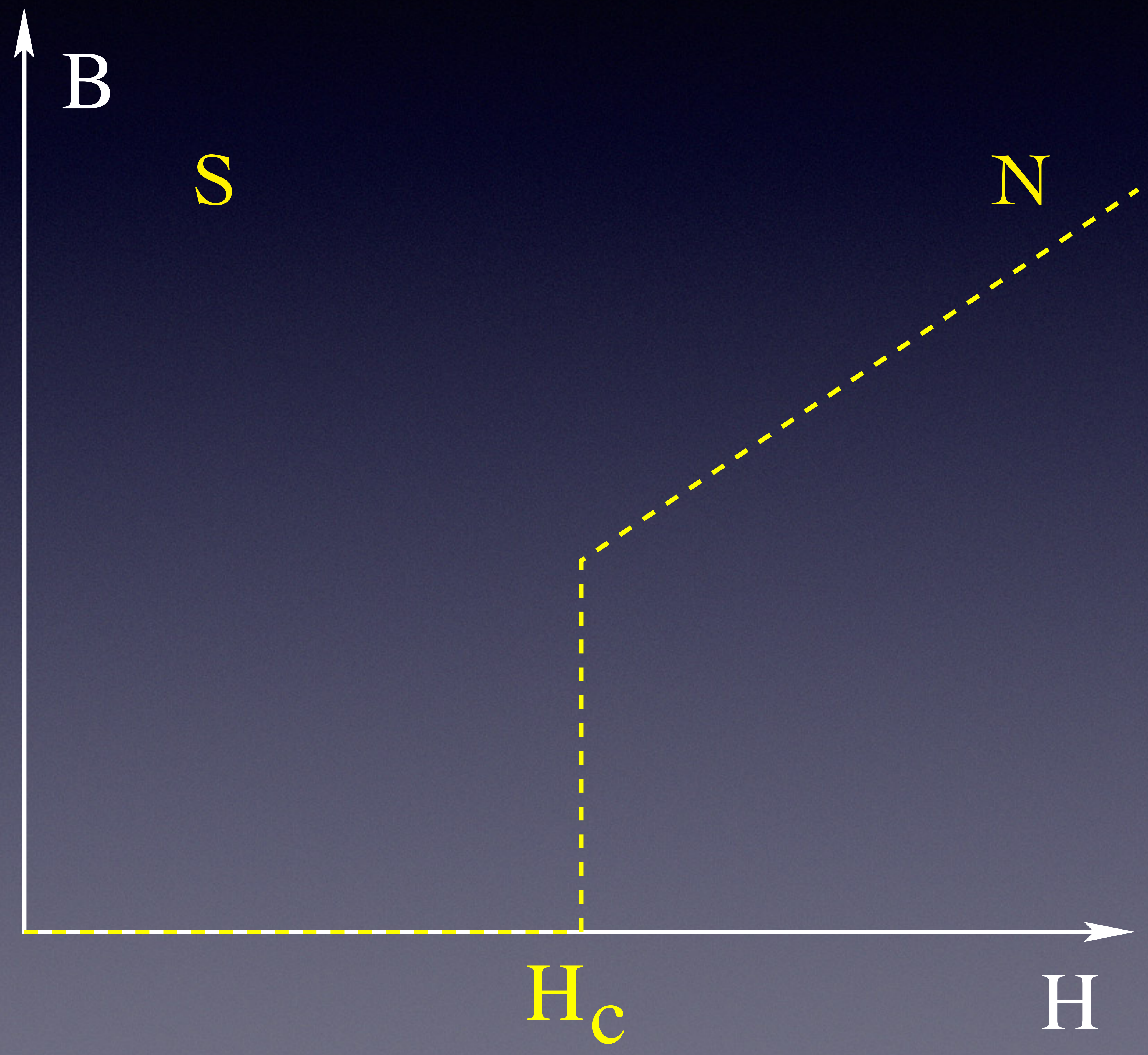
Reduced to London & Ginzburg-Landau Theories in appropriate limits

"It would have been very difficult to have arrived at the theory [of superconductivity] by purely deductive reasoning from the basic equations of quantum mechanics. Even if someone had done so, no one would have believed that such remarkable properties would really occur in nature."

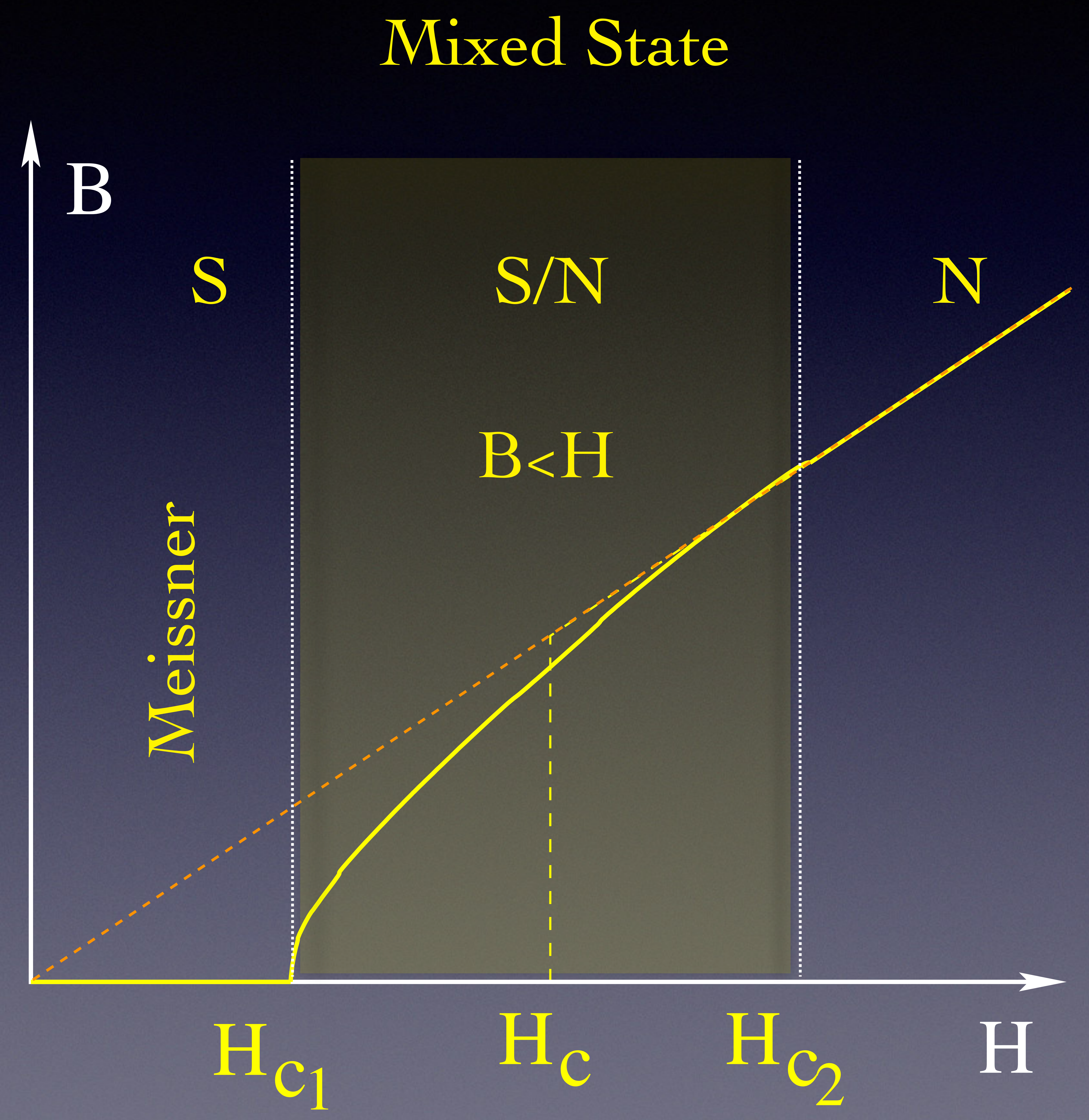
John Bardeen Nobel Lecture, Stockholm, December 11, 1972

Type I Superconductors

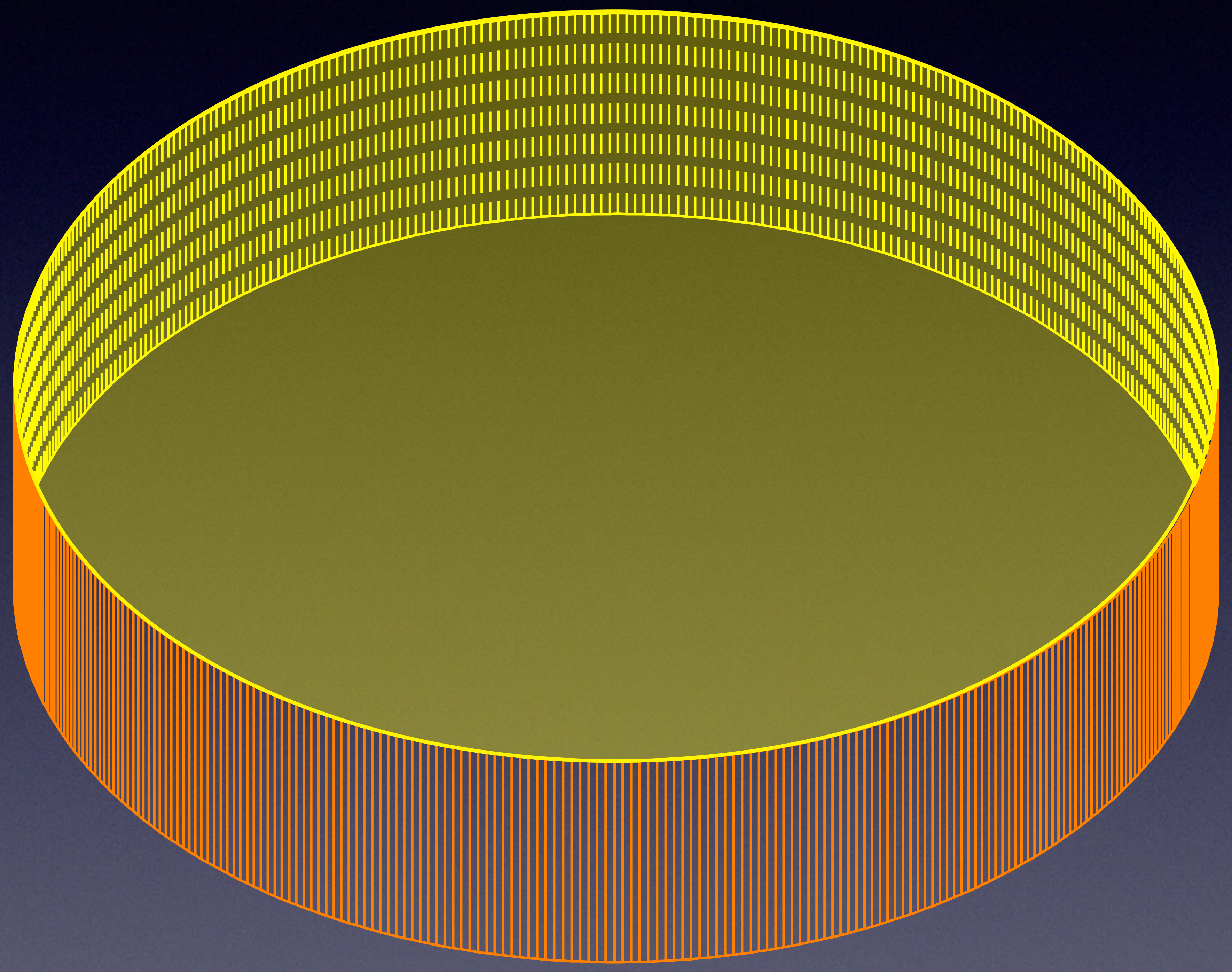
Type I Superconductors



Type II Superconductors

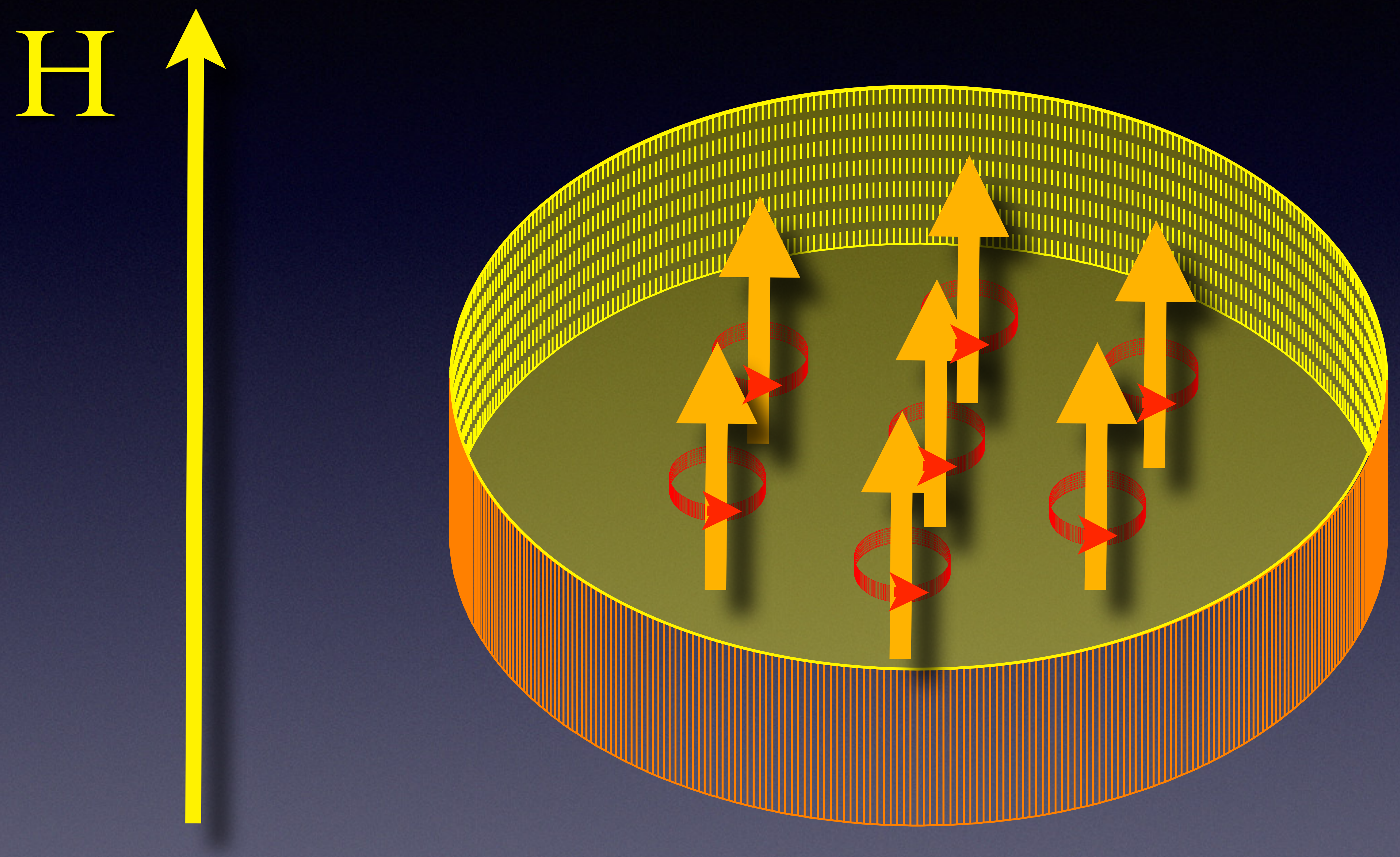


Abrikosov Vortices



A.A. Abrikosov "*On the magnetic properties of … *", Soviet Physics JETP 5, 1174 (1957)

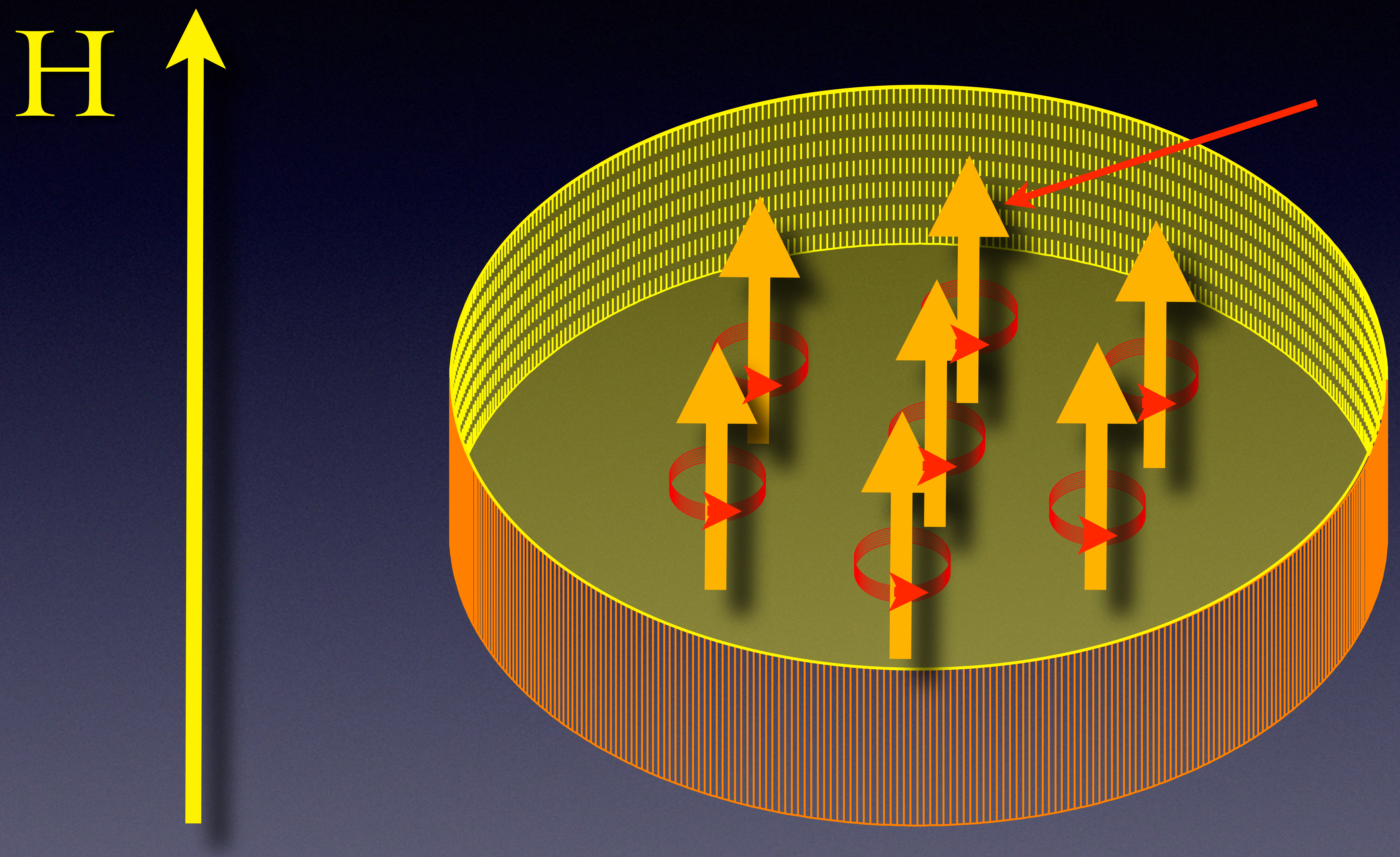
Abrikosov Vortices



$$H_{c_1} < H < H_{c_2}$$

A.A. Abrikosov "*On the magnetic properties of ...* ", Soviet Physics JETP 5, 1174 (1957)

Abrikosov Vortices

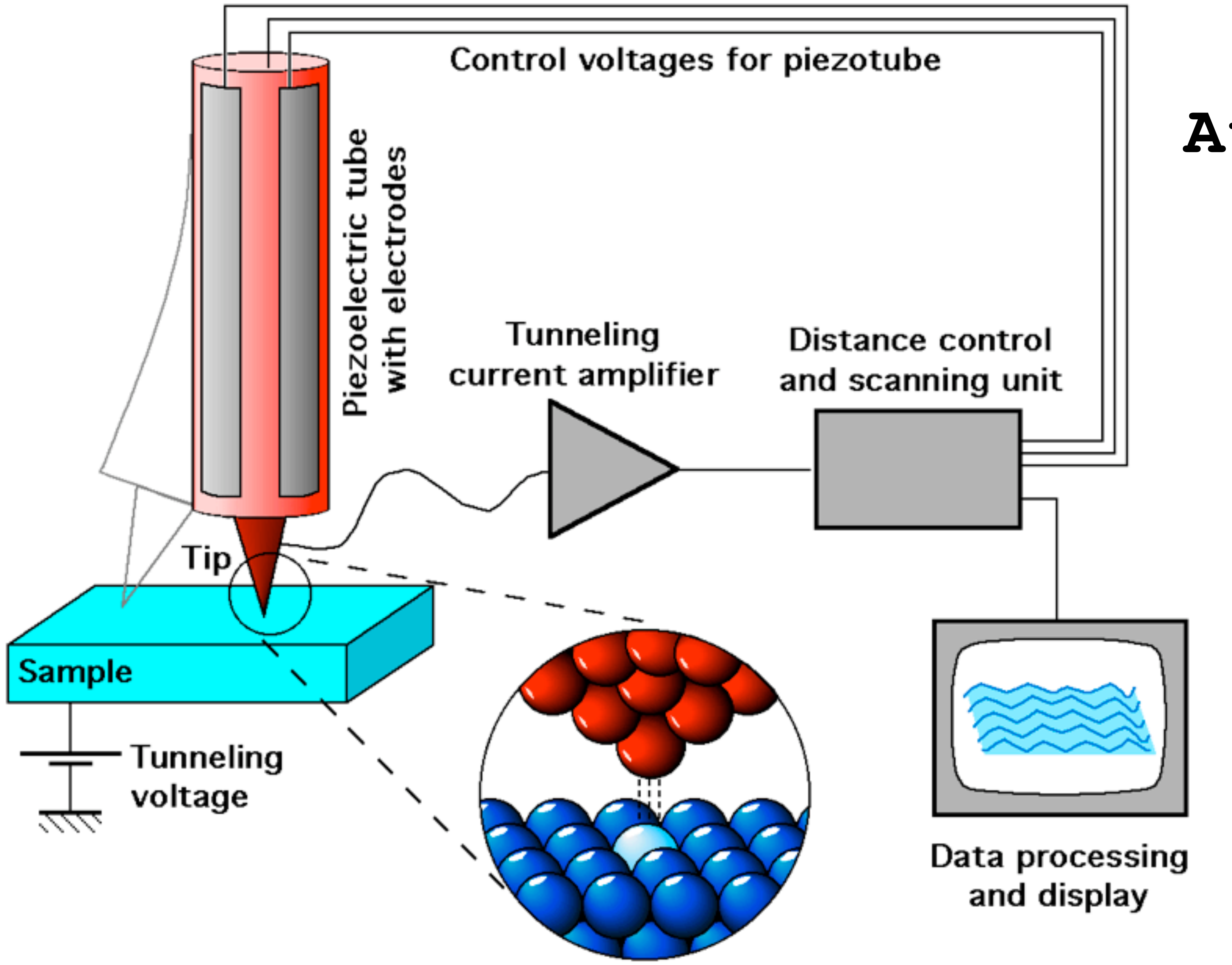


$$\Phi_0 = \frac{hc}{2e}$$

$$H_{c_1} < H < H_{c_2}$$

A.A. Abrikosov "*On the magnetic properties of ...* ", Soviet Physics JETP 5, 1174 (1957)

Scanning Tunneling Microscope



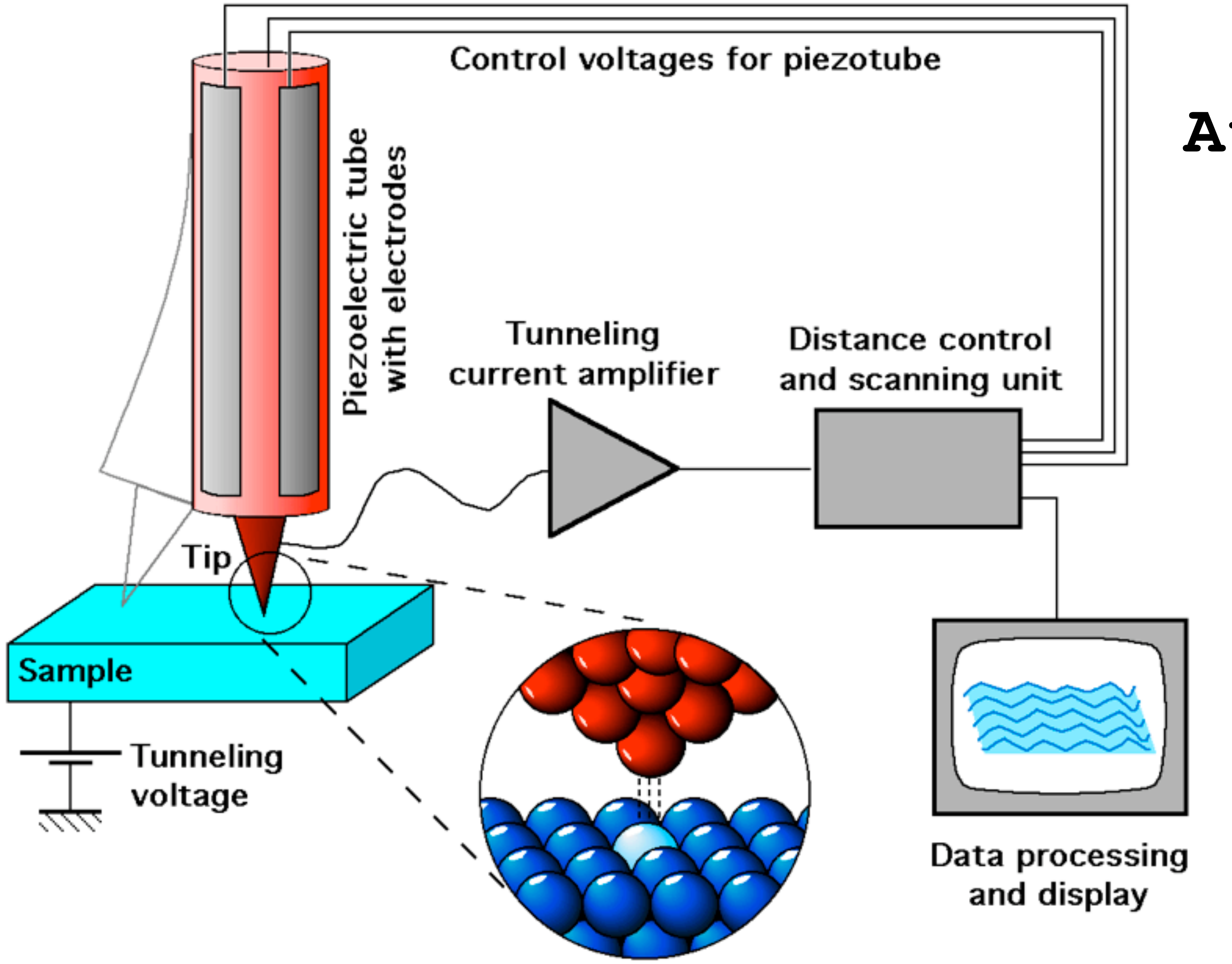
Atomic Scale Positioning

x y z
directions

Tunnel Current

G. Binnig & H. Rohrer (1986) IBM J. Res. Develop. 30, 355

Scanning Tunneling Microscope



Tunnel Current

Atomic Scale Positioning

x y z
directions

Local Spectroscopy

$$\boxed{\frac{dI}{dV} \propto N(E = eV, x, y)}$$

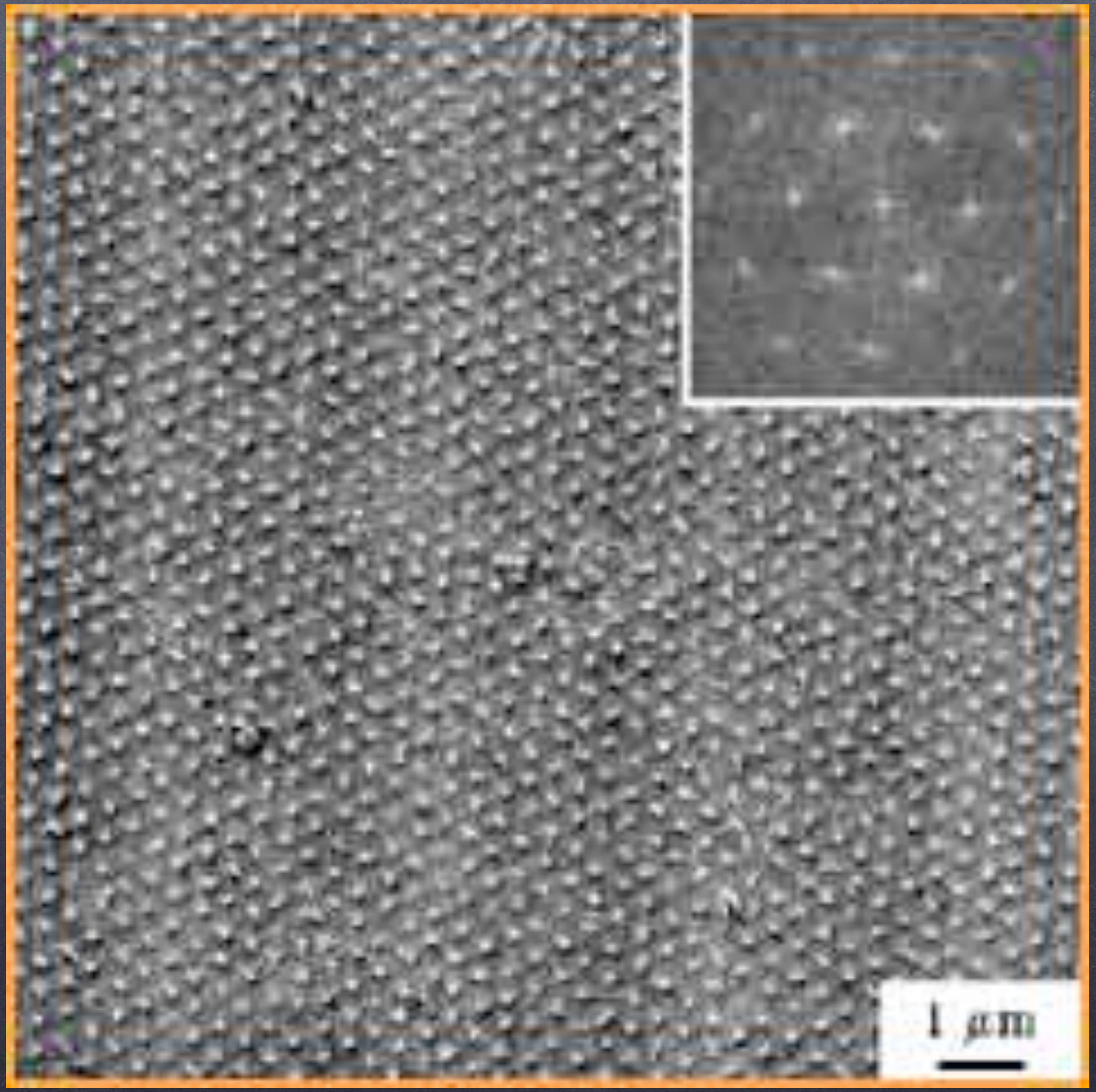
G. Binnig & H. Rohrer (1986) IBM J. Res. Develop. 30, 355

Abrikosov Vortex Lattices

Abrikosov Vortex Lattices

Magnetic Decoration

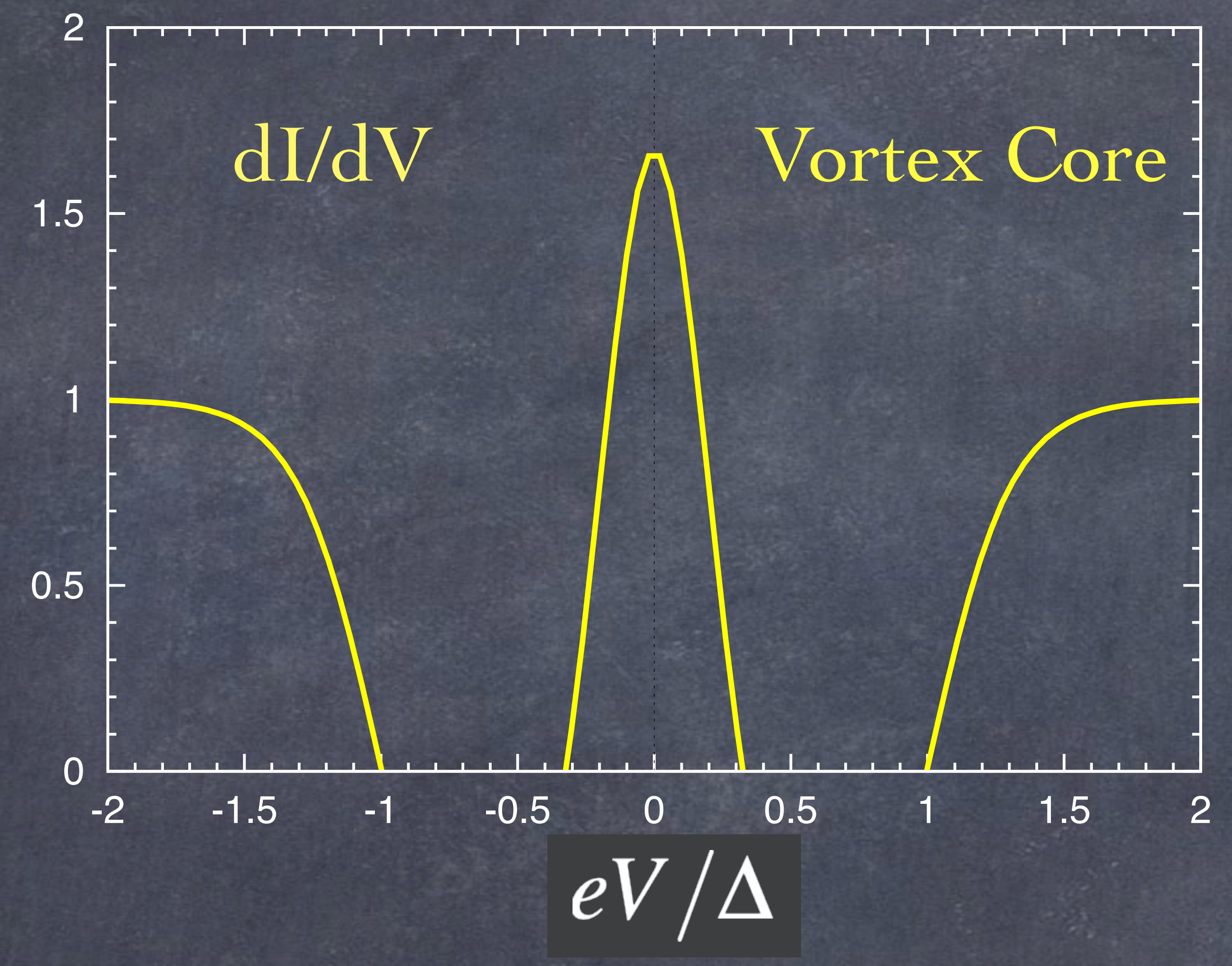
MgB_2 crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



Abrikosov Vortex Lattices

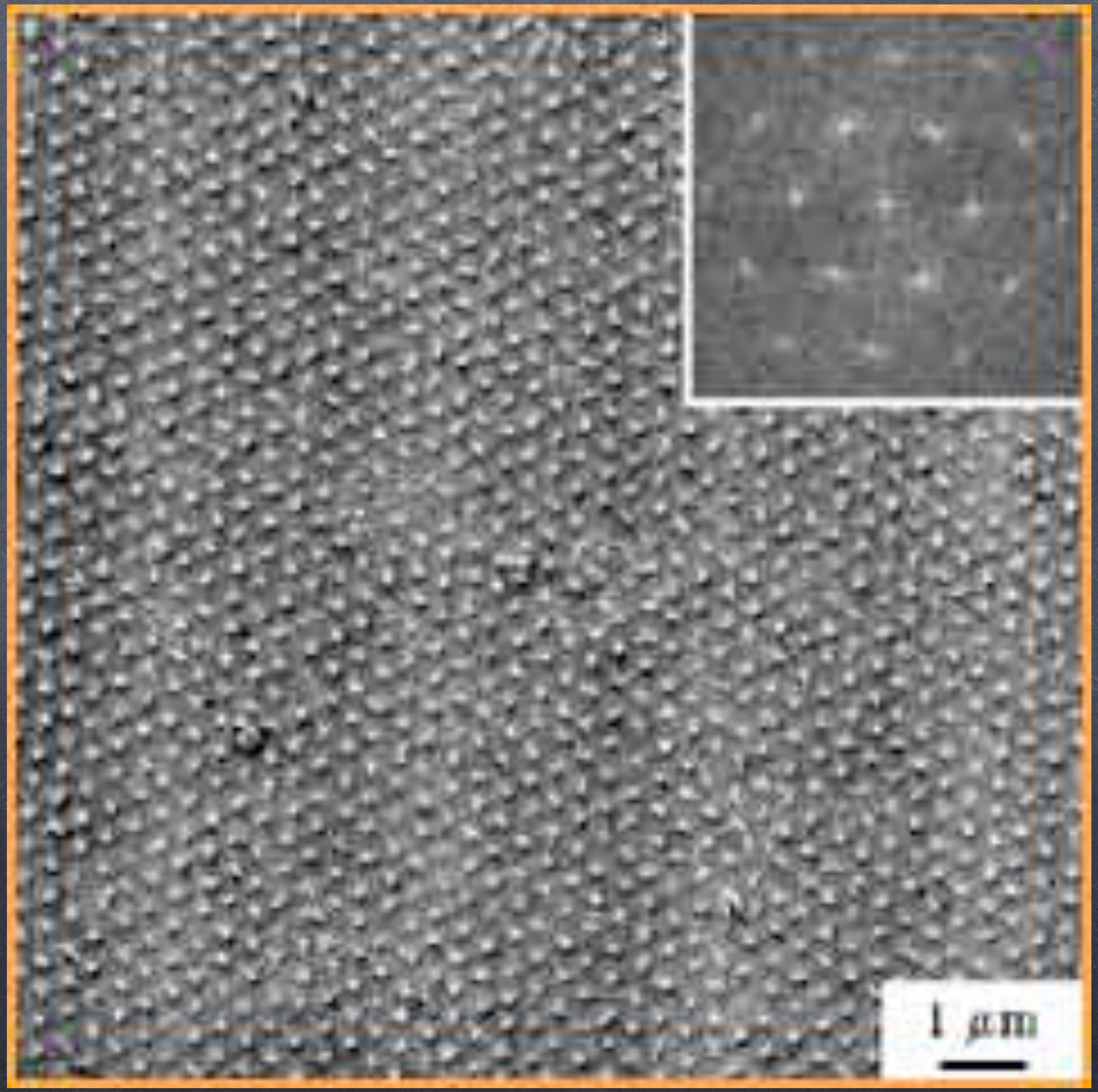
Scanning Tunneling Microscope Spectroscopy

$NbSe_2$ Tc=5K, Hess et al. PRL (1989)



Magnetic Decoration

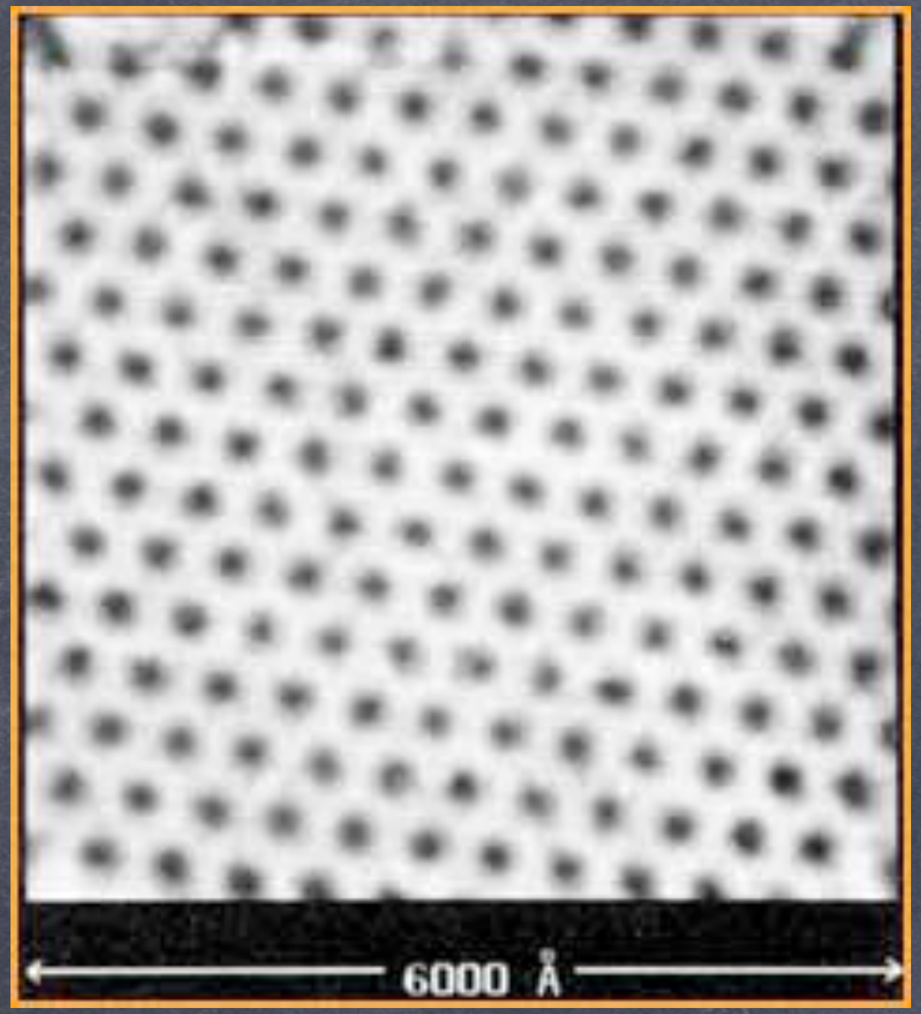
MgB_2 crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



Abrikosov Vortex Lattices

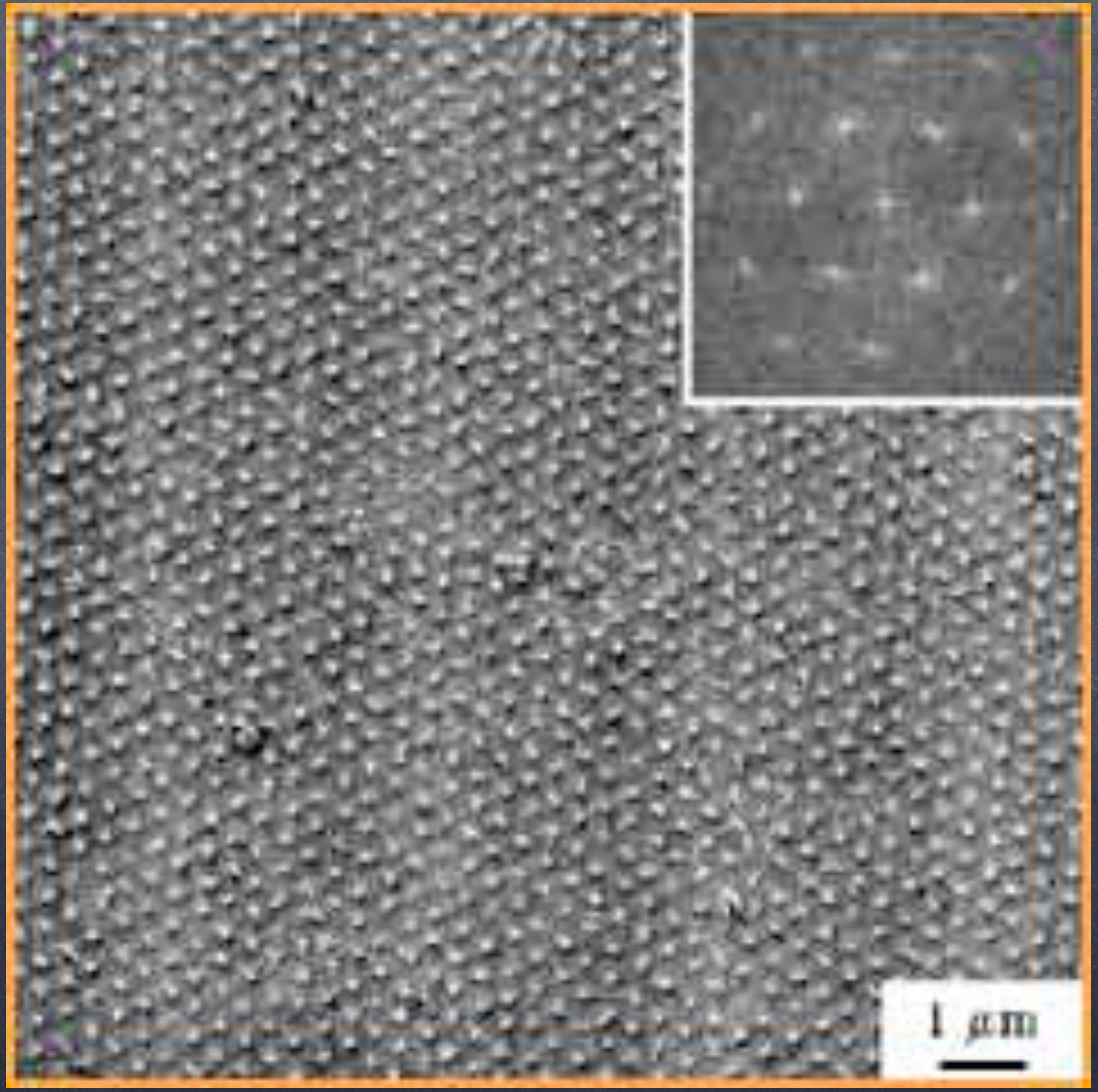
Scanning Tunneling
Microscope Spectroscopy

NbSe$_2$ Tc=5K, Hess et al. PRL (1989)



Magnetic Decoration

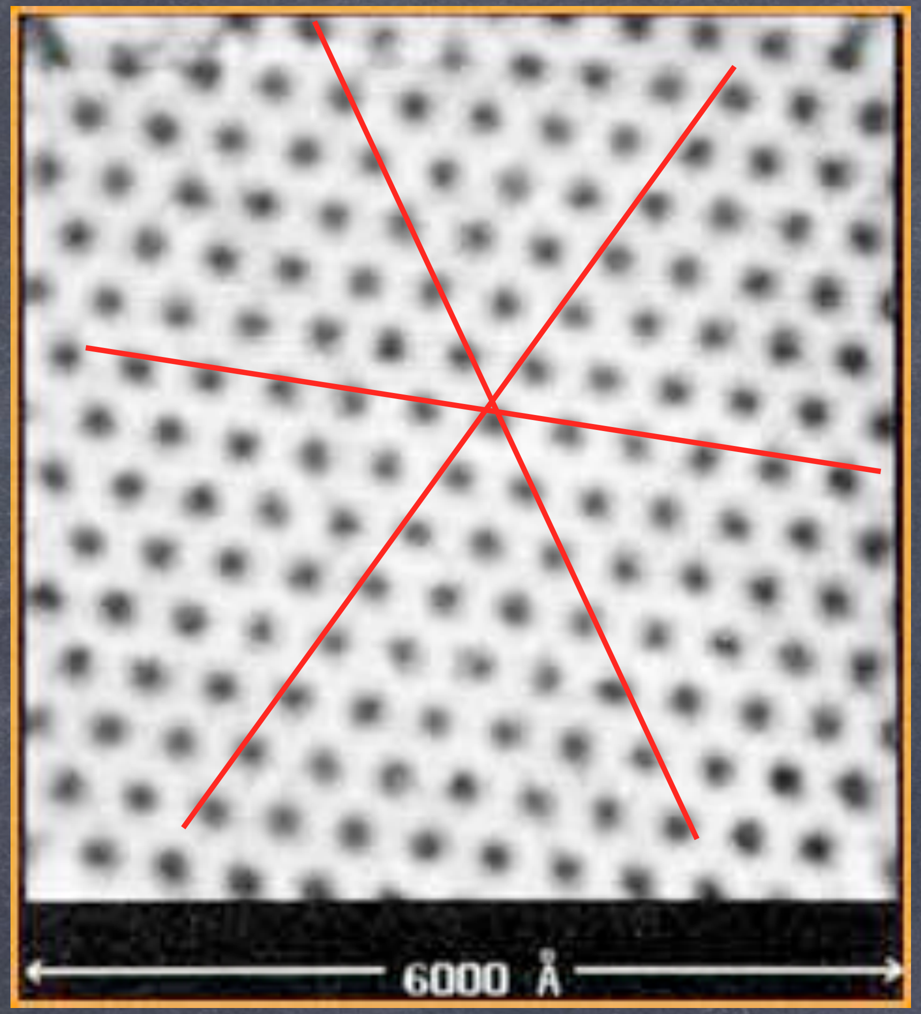
MgB$_2$ crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



Abrikosov Vortex Lattices

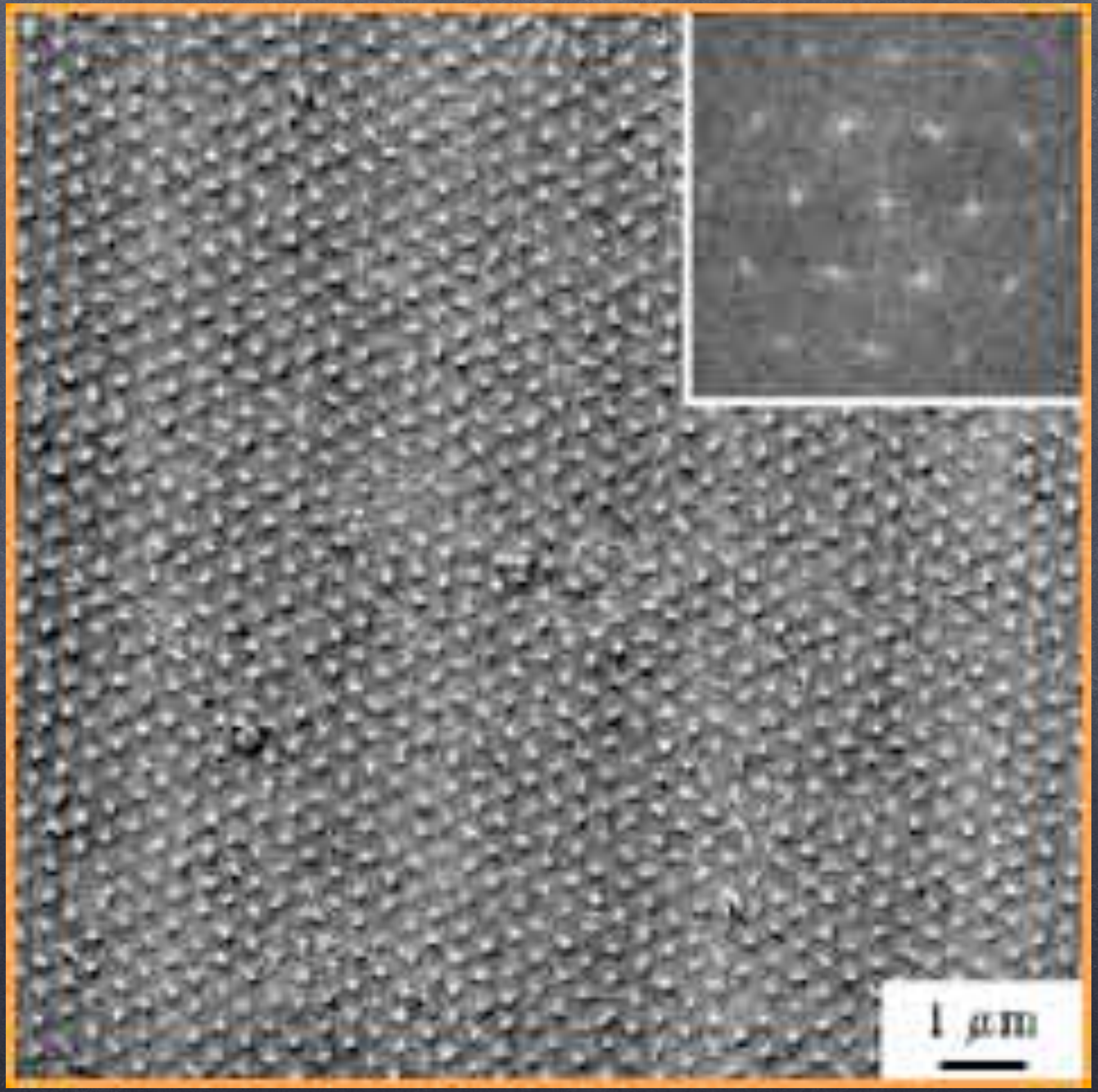
Scanning Tunneling Microscope Spectroscopy

$NbSe_2$ Tc=5K, Hess et al. PRL (1989)



Magnetic Decoration

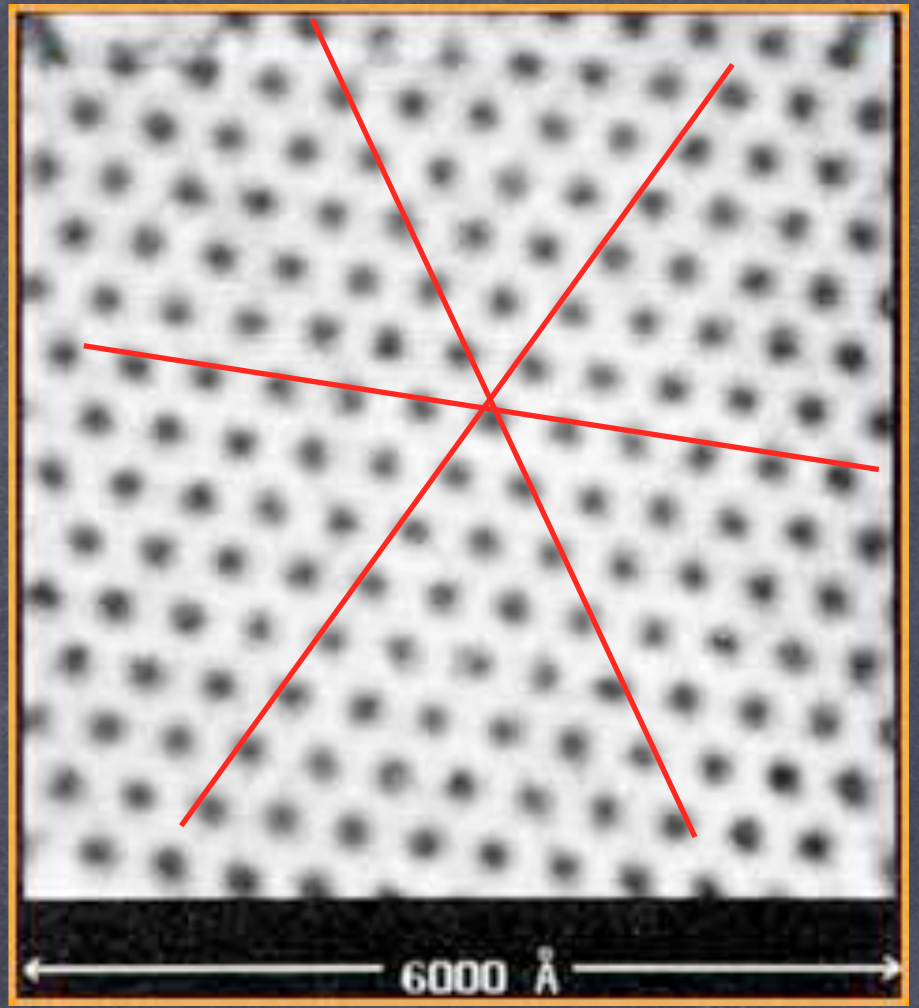
MgB_2 crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



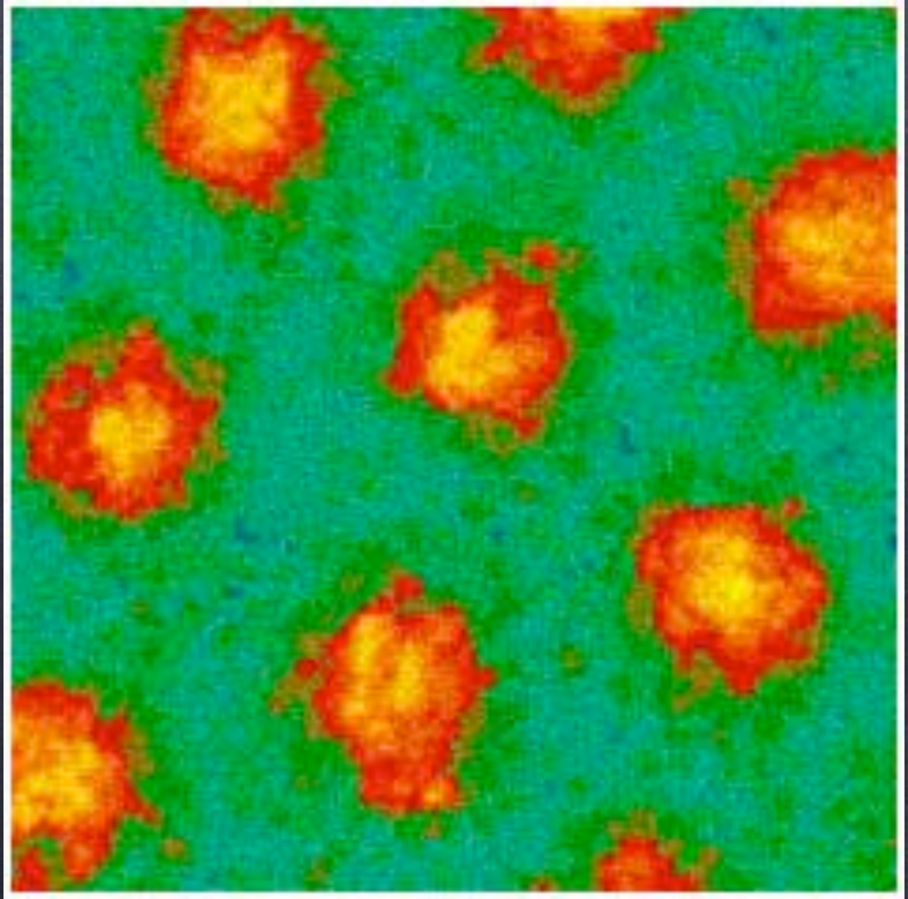
Abrikosov Vortex Lattices

Scanning Tunneling
Microscope Spectroscopy

$NbSe_2$ Tc=5K, Hess et al. PRL (1989)

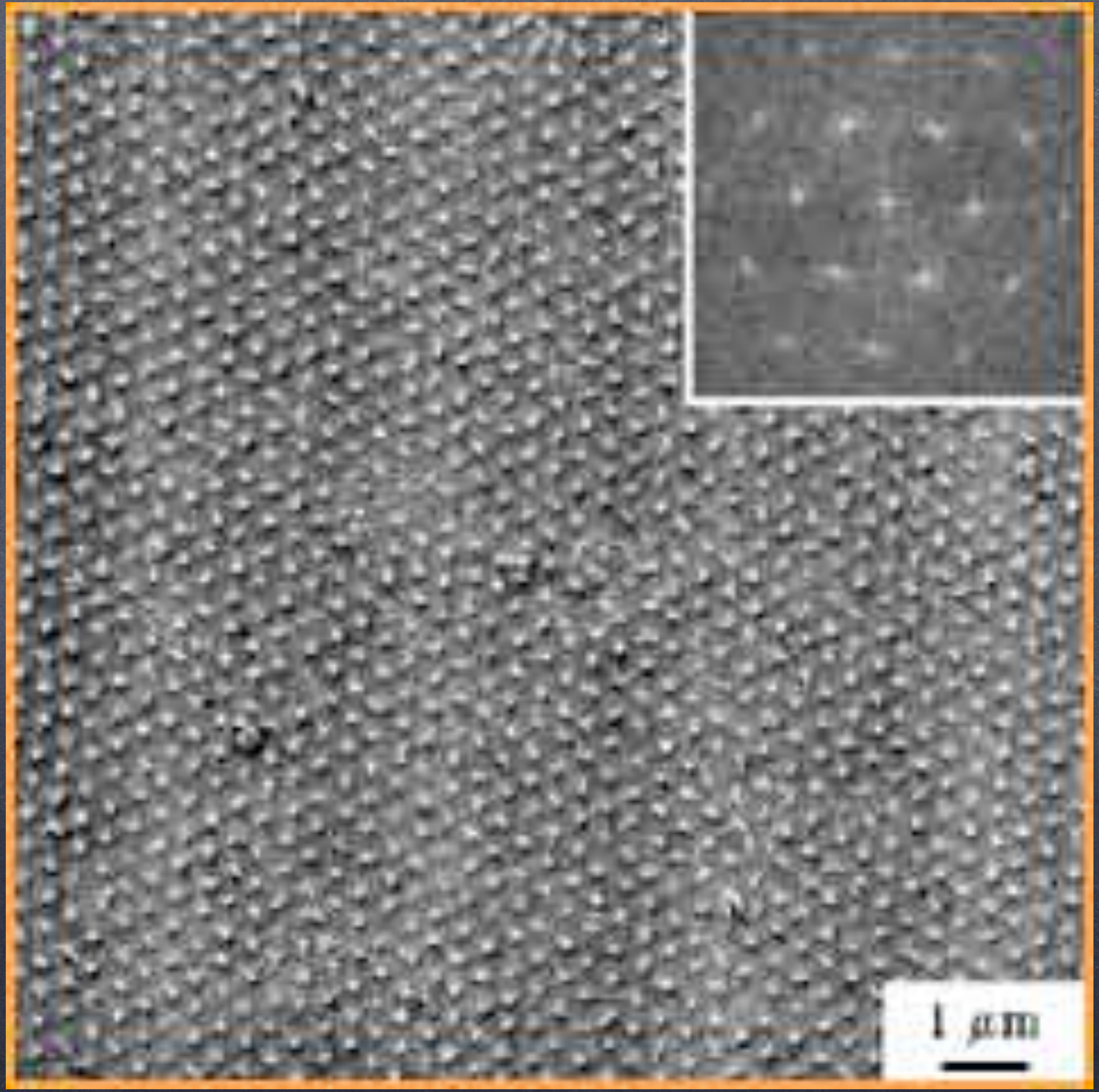


Small Angle
Neutron Diffraction
By the Magnetic field of
Flux Lines



Magnetic Decoration

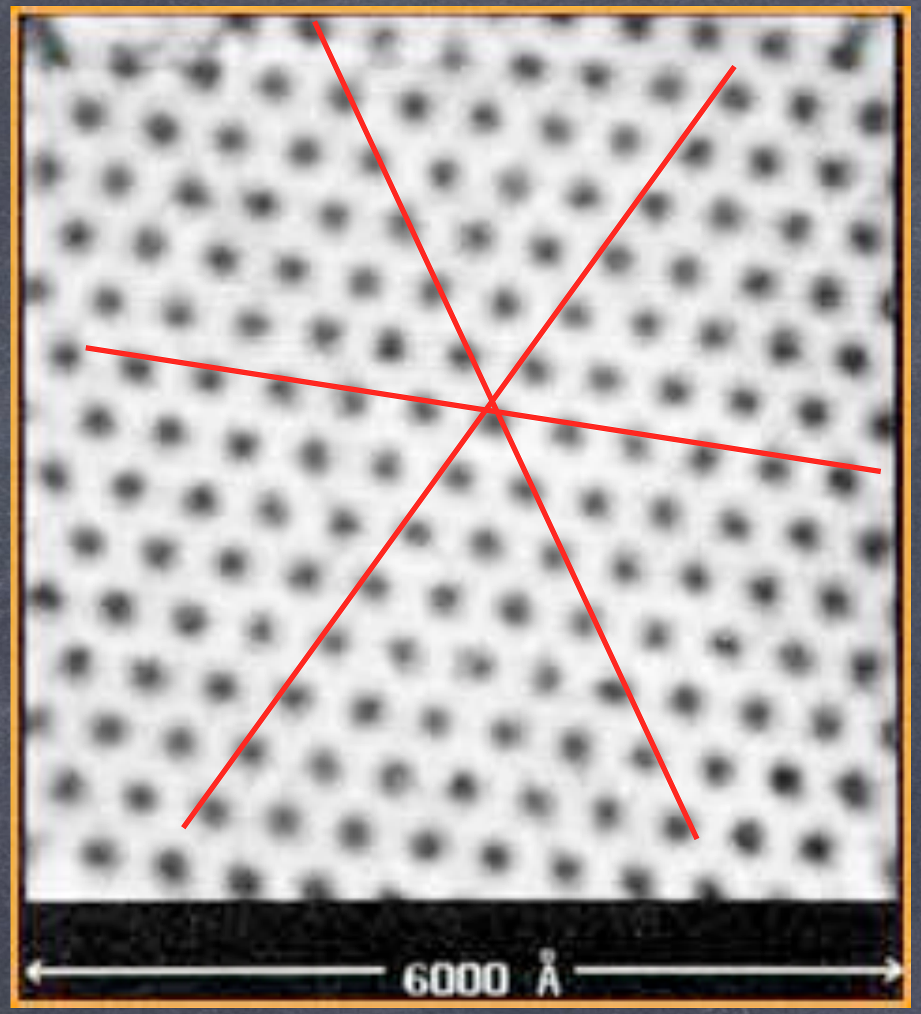
MgB_2 crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



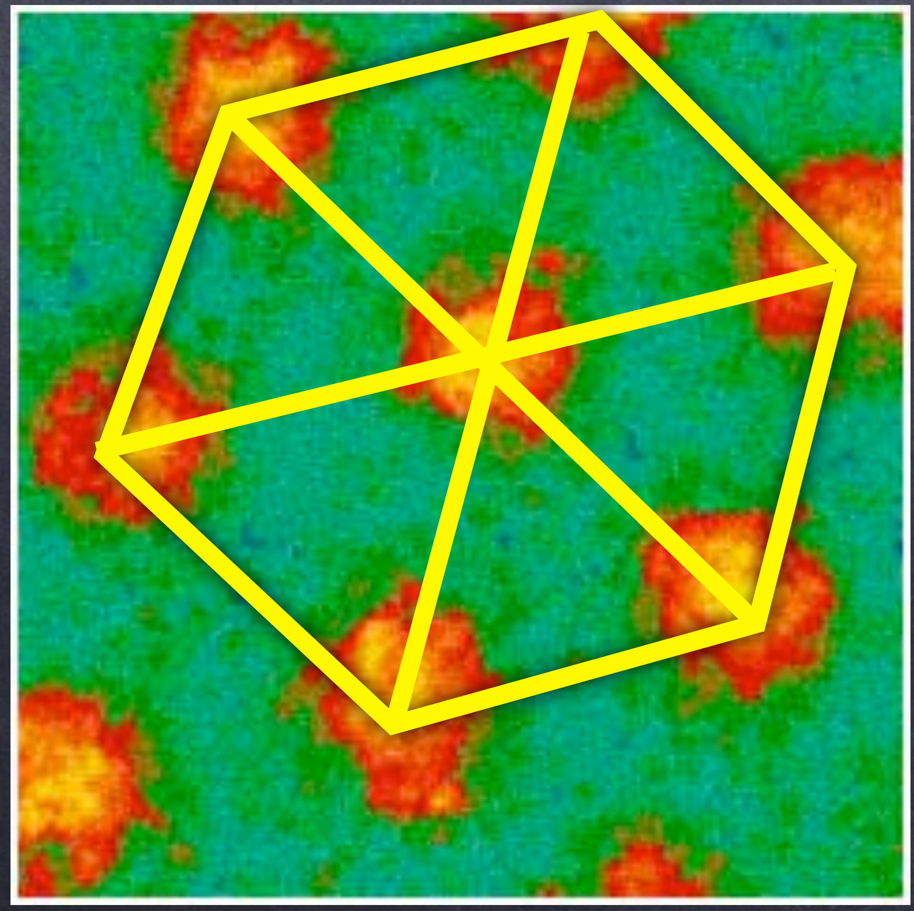
Abrikosov Vortex Lattices

Scanning Tunneling Microscope Spectroscopy

$NbSe_2$ Tc=5K, Hess et al. PRL (1989)



Small Angle Neutron Diffraction By the Magnetic field of Flux Lines

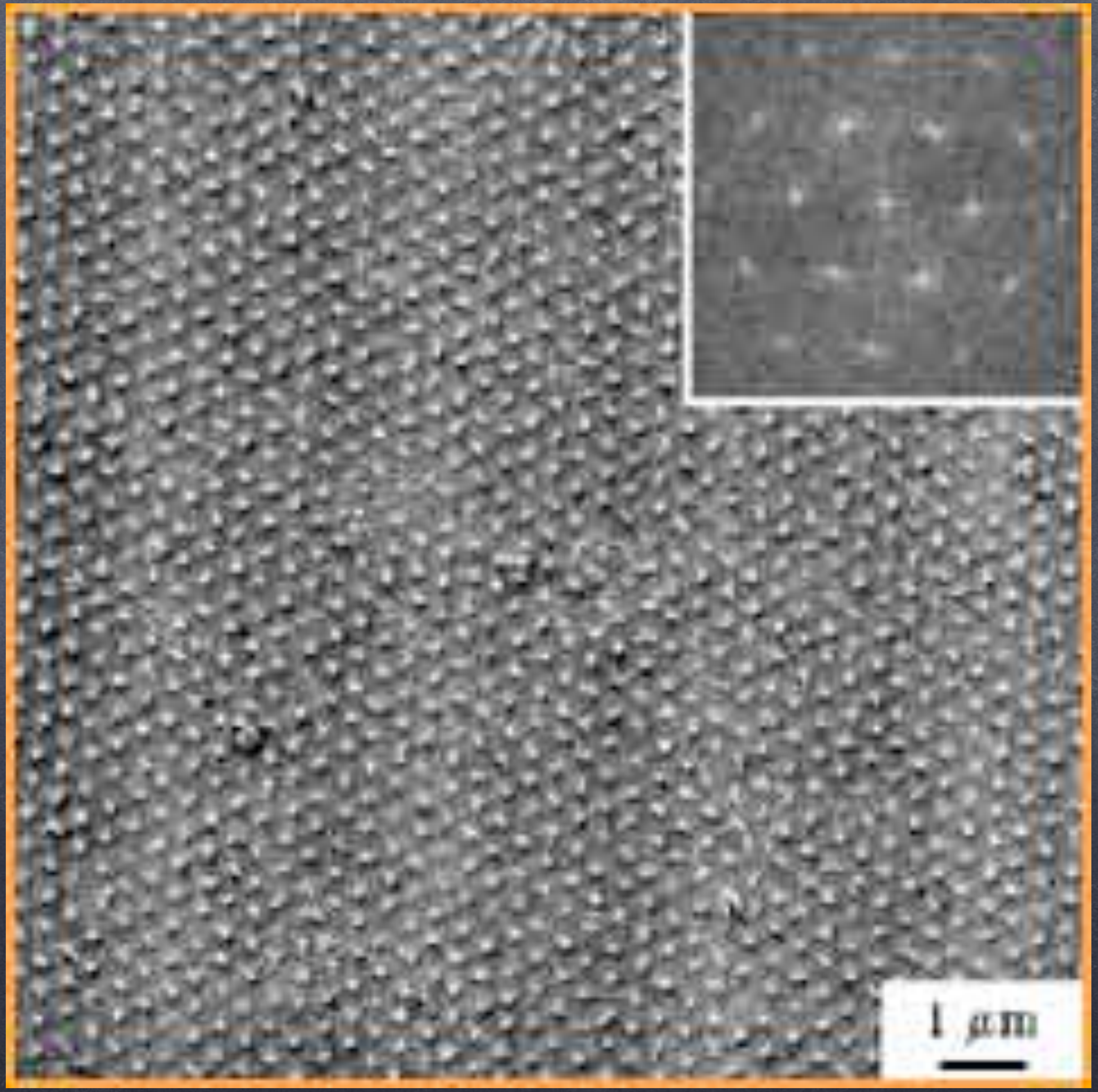


Other Imaging Methods

MgB_2 T=2K, B=2 kG M. R. Eskildsen et al. PRL (2002)

Magnetic Decoration

MgB_2 crystal, 200G
L. Ya. Vinnikov et al.
Phys. Rev. (2003)



Abrikosov Vortices are Often Pinned to Impurities or Structural Defects in the Superconductor ⇝ Disordered and complex Flux pattern

Abrikosov Vortices are Often Pinned to Impurities or Structural Defects in the Superconductor ⇝ Disordered and complex Flux pattern

Levitation by High-*Tc* superconductors

A. B. Riise, et al.

Physical Review B 60, 9855 (1998)

Abrikosov Vortices are Often Pinned to Impurities or Structural Defects in the Superconductor ⇝ Disordered and complex Flux pattern

Levitation by High-*Tc* superconductors

A. B. Riise, et al.

Physical Review B 60, 9855 (1998)

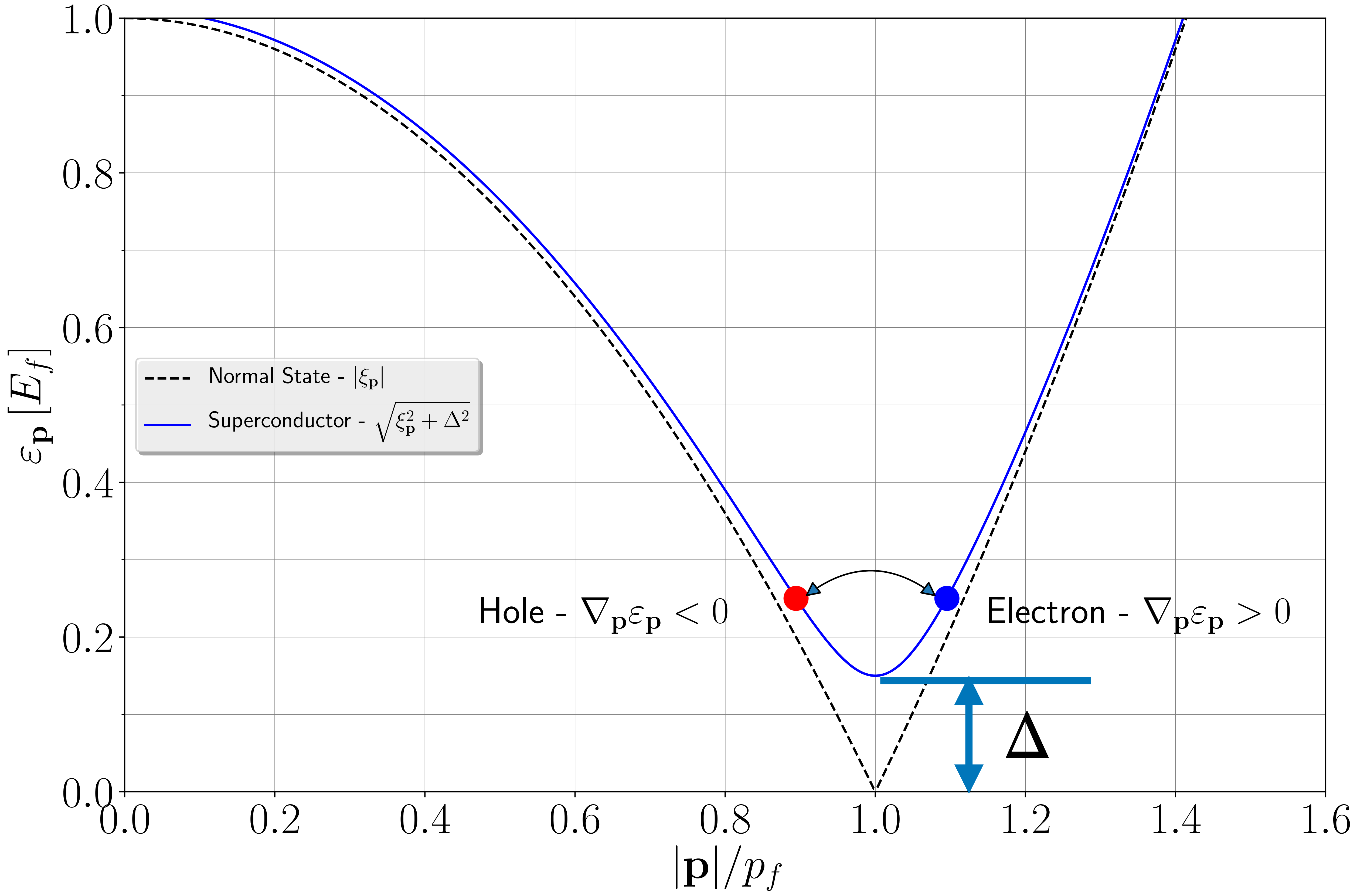
- ❖ NdFeB - *Magnet*
- ❖ YBCO -Superconductor
- ❖ Flux Pinning
- ❖ Flux Creep on warming

The Energy Gap of BCS Superconductors

Normal metal $\xi_{\mathbf{p}} = \frac{|\mathbf{p}|^2}{2m} - \frac{p_f^2}{2m} \approx v_f(|\mathbf{p}| - p_f)$

Superconductor $E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$

Two types of Excitations

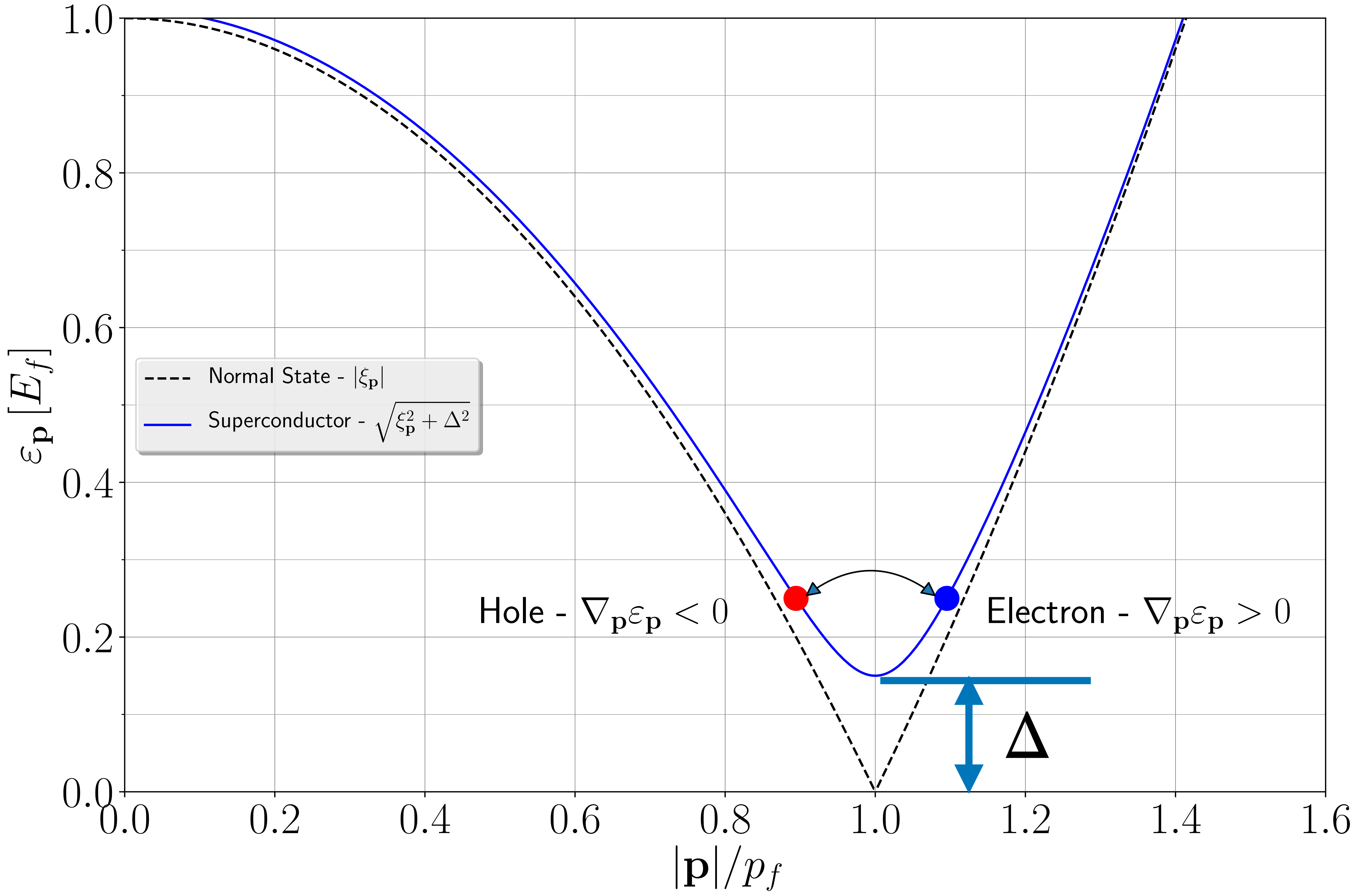


The Energy Gap of BCS Superconductors

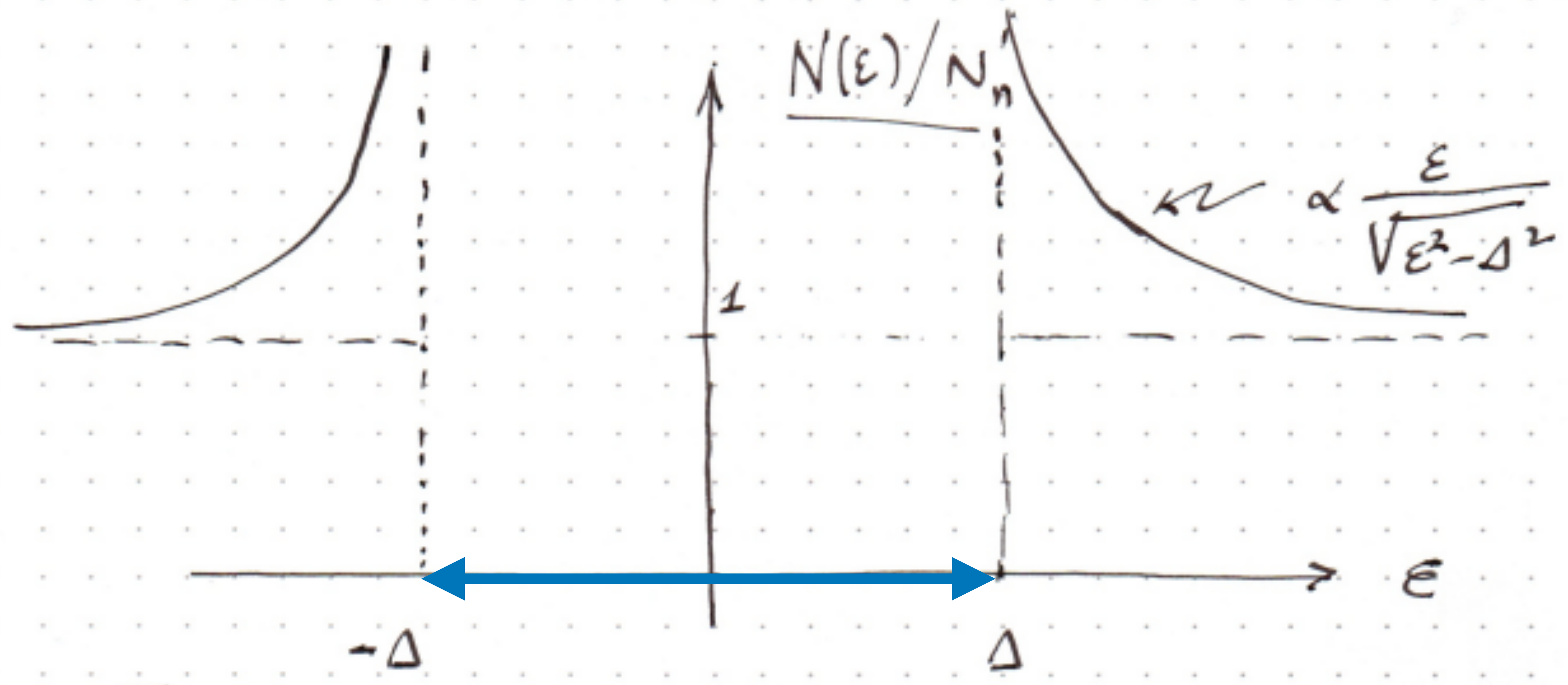
Normal metal $\quad \xi_{\mathbf{p}} = \frac{|\mathbf{p}|^2}{2m} - \frac{p_f^2}{2m} \approx v_f(|\mathbf{p}| - p_f)$

Superconductor $\quad E_{\mathbf{p}} = \sqrt{\xi_{\mathbf{p}}^2 + \Delta^2}$

Two types of Excitations



Number of States per unit Energy in $(\varepsilon, \varepsilon + d\varepsilon)$



BCS Quasiparticle Density of States

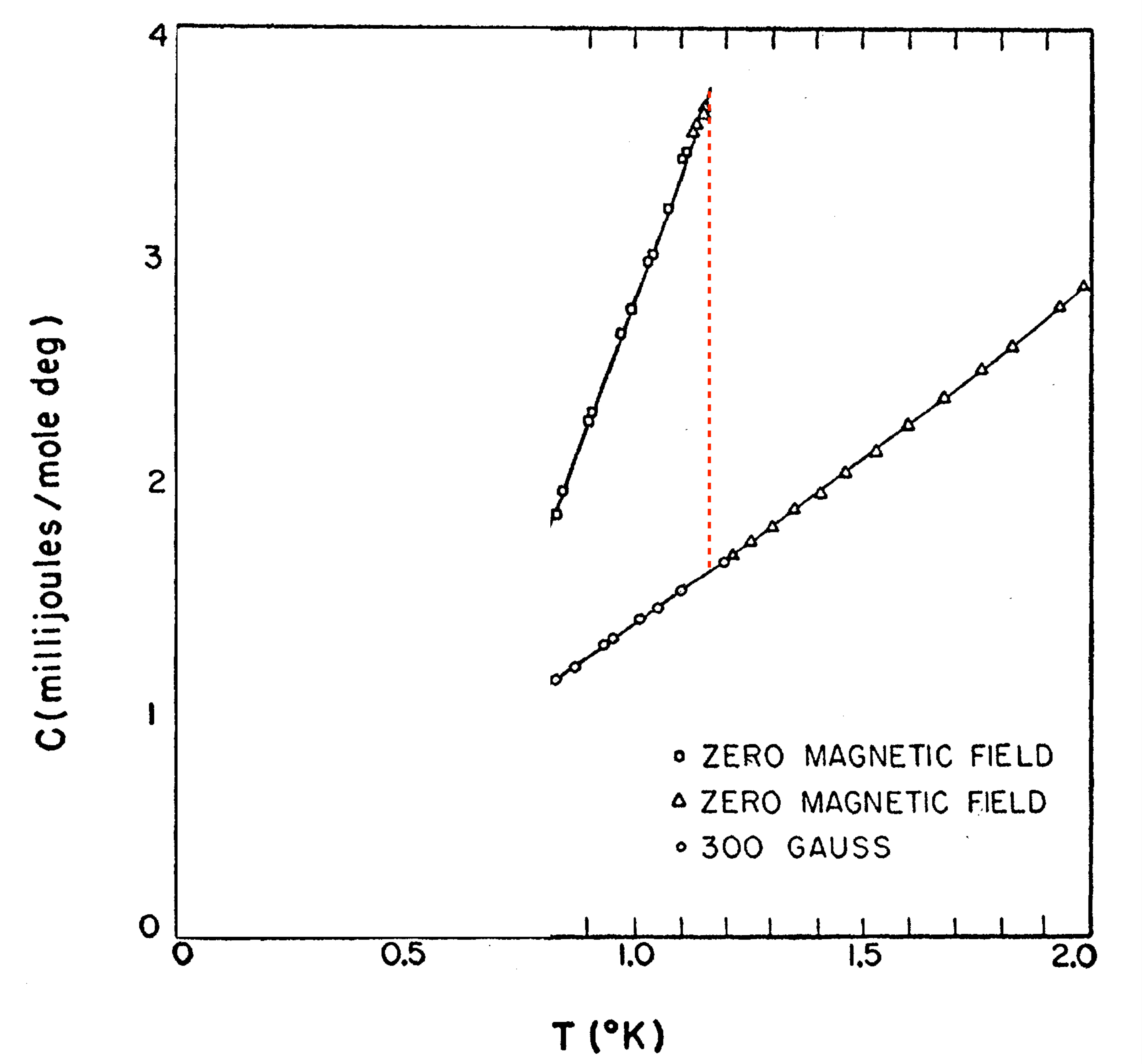
- $N_n = \frac{3}{2}\, n/E_F$ = Normal Metal DOS @ E_F.
- Δ = Excitation Gap for Quasiparticles/Q-holes

$\Delta(Nb) \cong 2\,\mathrm{meV}$

$\Delta(Al) \cong 0.2\,\mathrm{meV}$

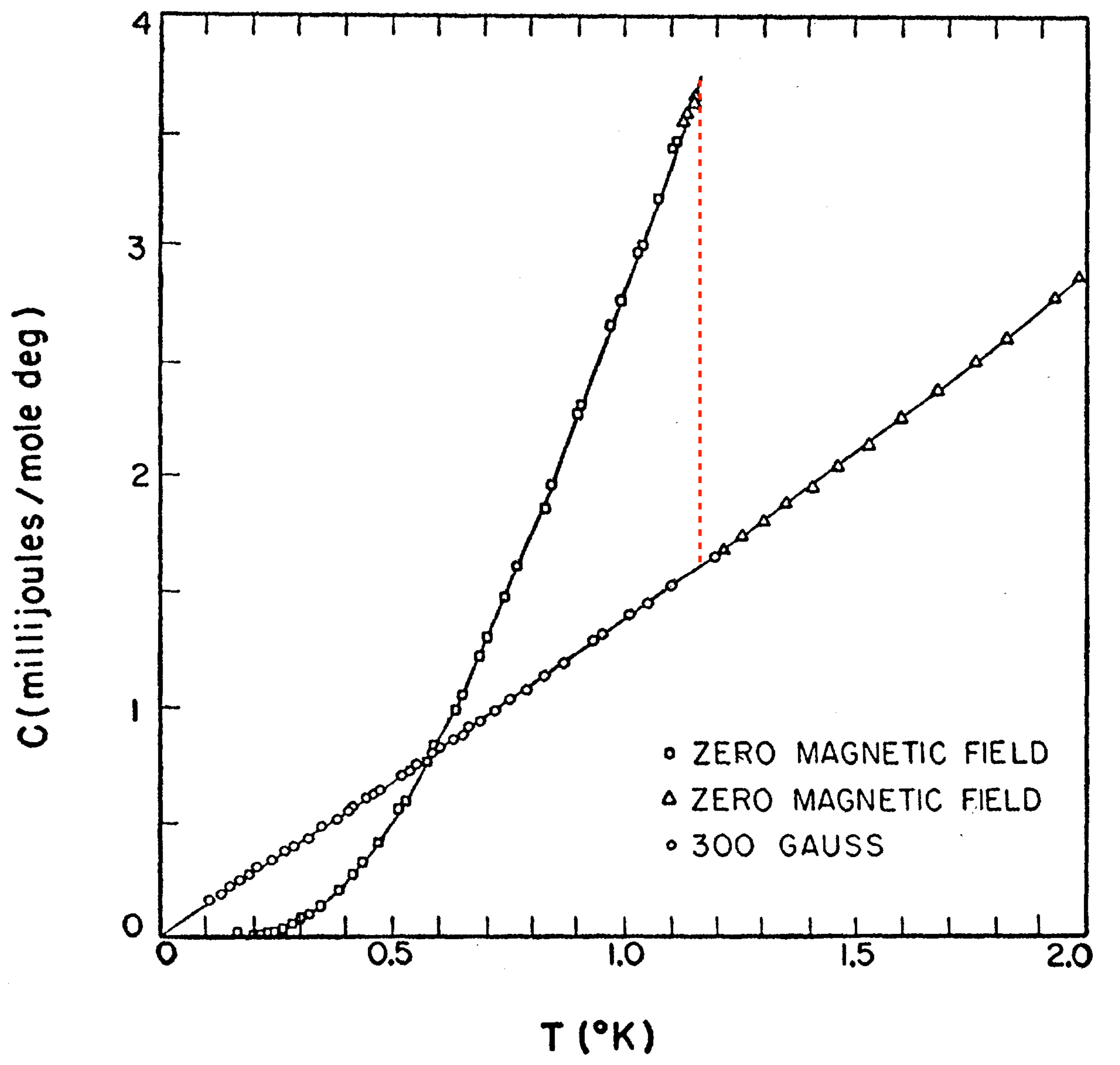
Thermodynamics - Heat Capacity

Heat Capacity of Al, Phys. Rev. 114, 676 (1959), N.E. Phillips



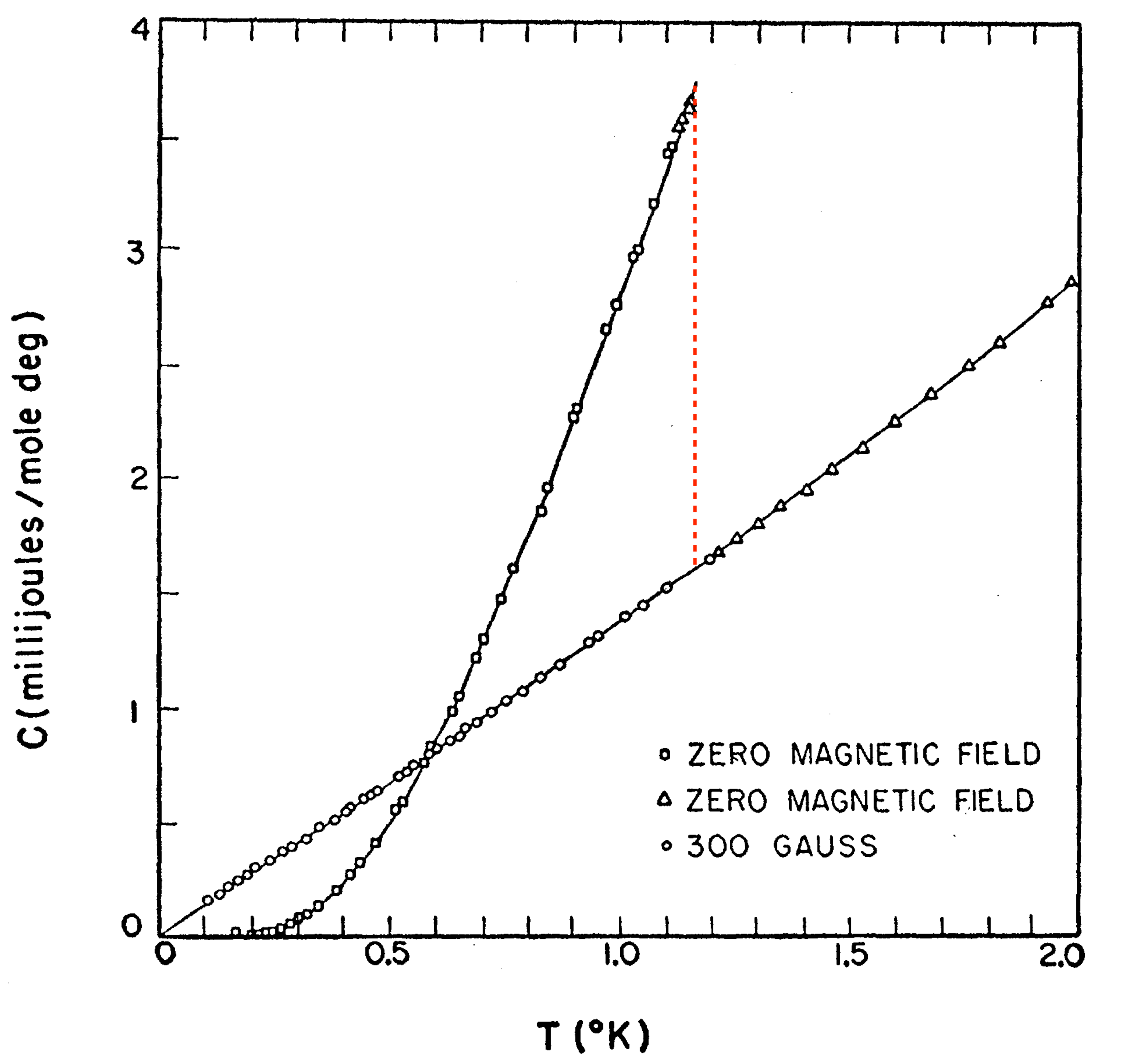
Thermodynamics - Heat Capacity

Heat Capacity of Al, Phys. Rev. 114, 676 (1959), N.E. Phillips



Thermodynamics - Heat Capacity

Heat Capacity of Al, Phys. Rev. 114, 676 (1959), N.E. Phillips



- Electronic Entropy: *Fermions* with $N(E) \approx N(E_f)$
- $T > T_c = 1.16K$: $C = \gamma T$
- 2nd Order Transition: $\Delta C/\gamma T_c \approx 1.6$
- $T < 0.5K$: $C \propto e^{-\Delta/k_B T}$?

Evidence of an Energy Gap for un-bound electrons in Superconductors

Rev. Mod. Phys. 30, 1109 (1958), M. Biondi et al.

Phys. Rev. 122, 1101 (1961), I. Gaiver et al.

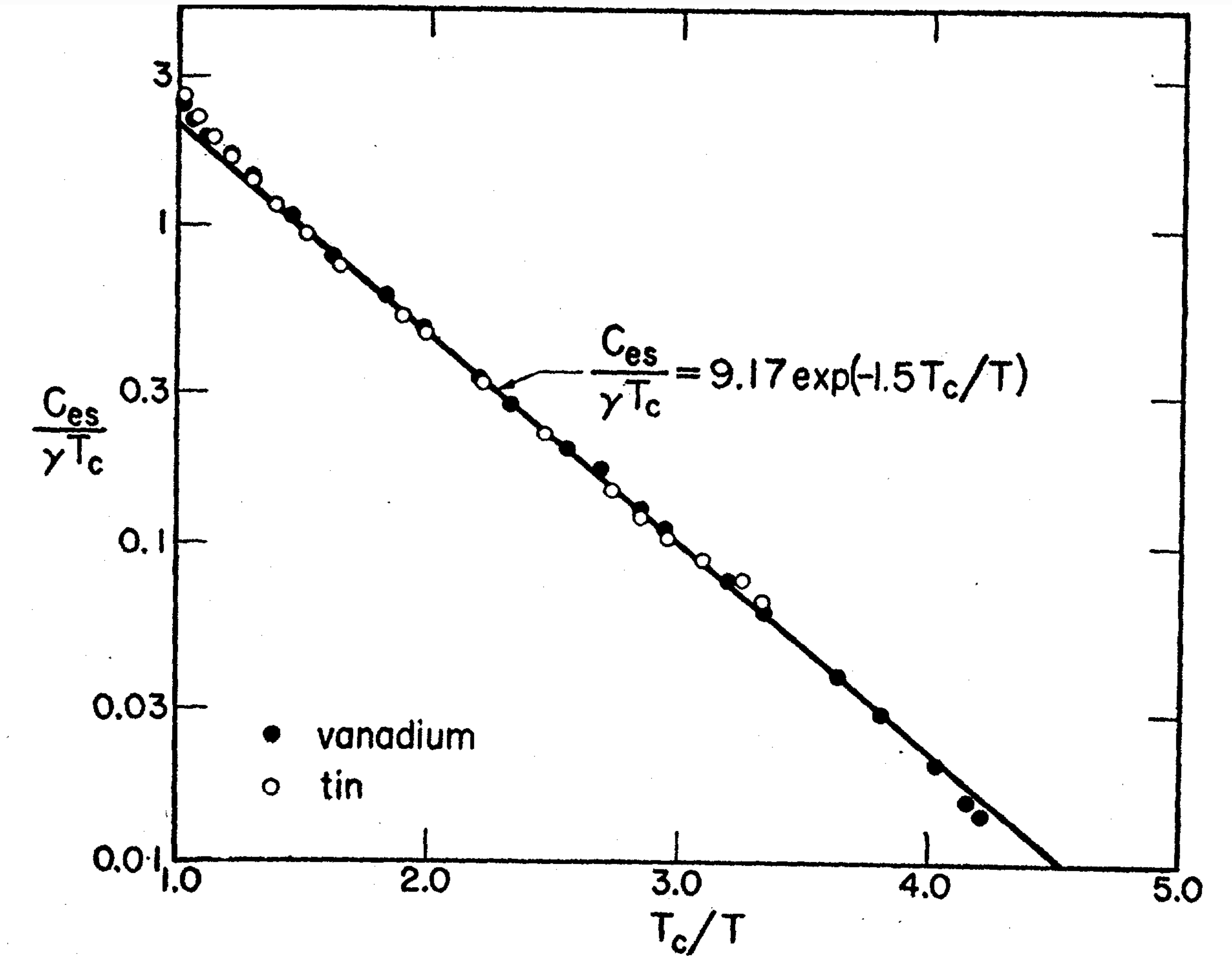
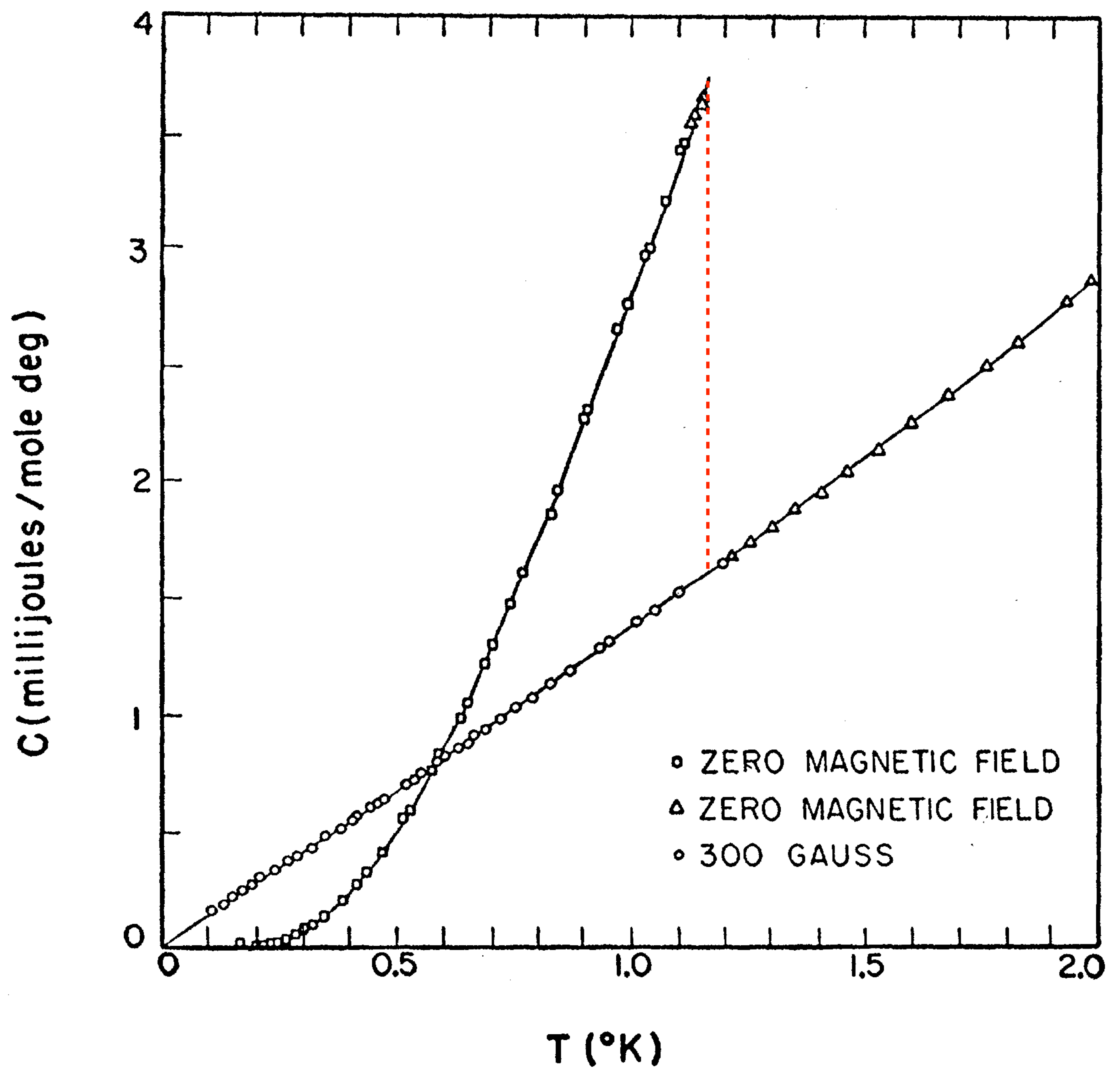


FIG. 6. Reduced electronic specific heat in the superconducting state for vanadium and tin.

Thermodynamics - Heat Capacity

Heat Capacity of Al, Phys. Rev. 114, 676 (1959), N.E. Phillips



- Electronic Entropy: *Fermions* with $N(E) \approx N(E_f)$
- $T > T_c = 1.16K$: $C = \gamma T$
- 2nd Order Transition: $\Delta C/\gamma T_c \approx 1.6$
- $T < 0.5K$: $C \propto e^{-\Delta/k_B T}$?

Evidence of an Energy Gap for un-bound electrons in Superconductors

Rev. Mod. Phys. 30, 1109 (1958), M. Biondi et al.

Phys. Rev. 122, 1101 (1961), I. Gaiver et al.

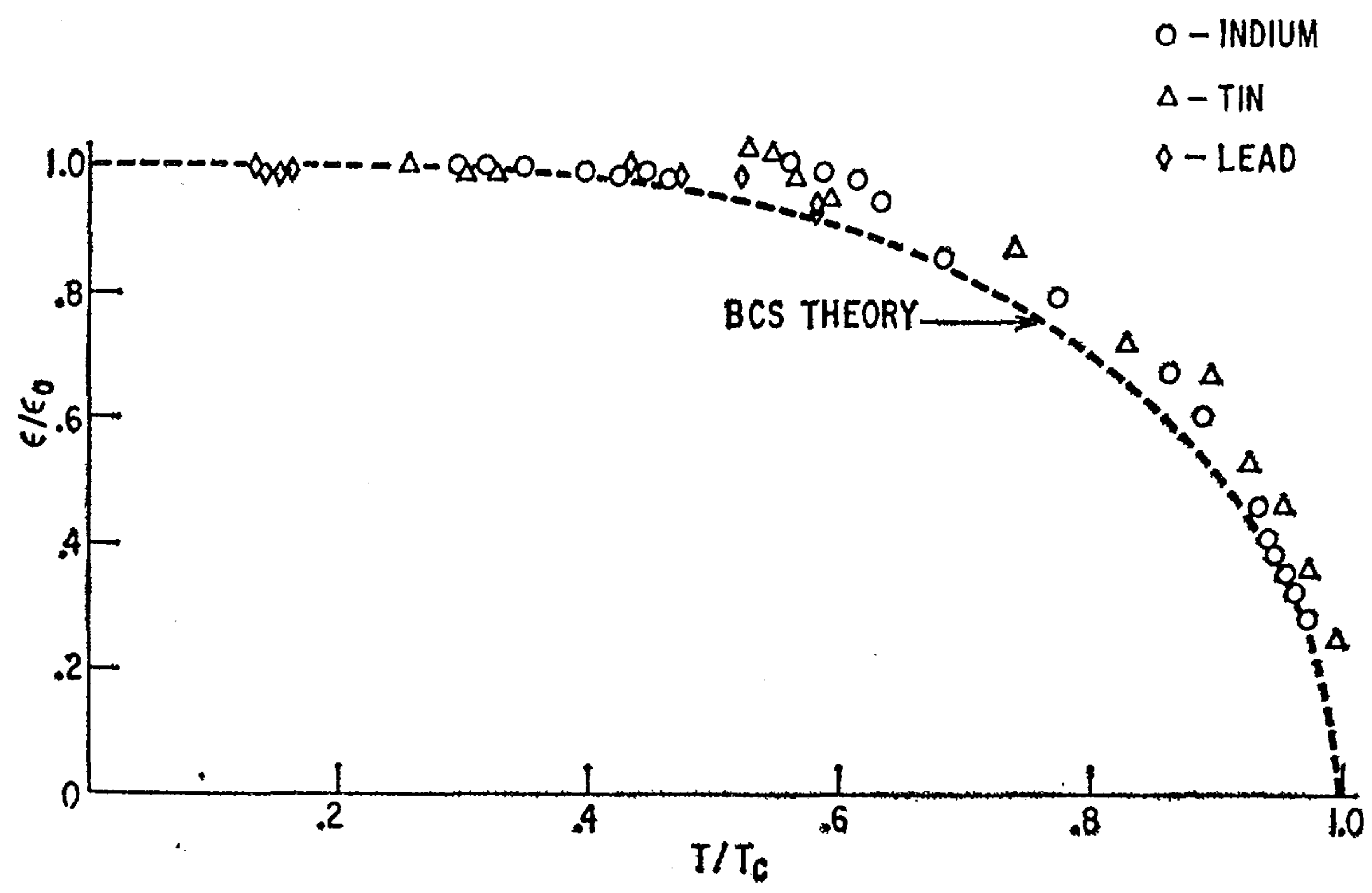


FIG. 11. The energy gap of Pb, Sn, and In films as a function of reduced temperature, compared with the Bardeen-Cooper-Schrieffer theory.

- Energy Gap: $\Delta \approx 1.5 - 1.8 k_B T_c$

Heat Capacity Jumps & Energy Gaps for Elemental SCs

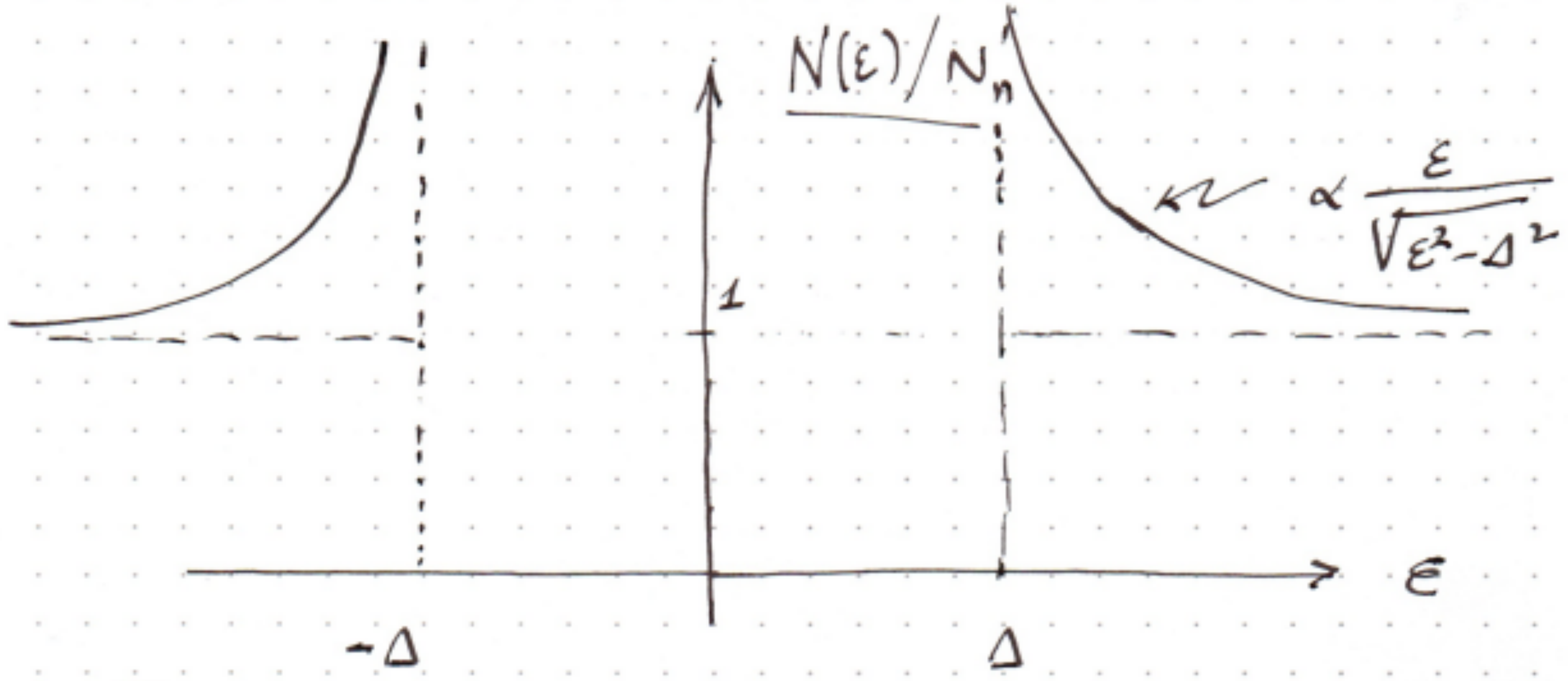
Element	$2\Delta/k_BT$	$\Delta C/\gamma T_c$	Element	$2\Delta/k_BT$	$\Delta C/\gamma T_c$
Al	2.5–4.2	1.3–1.6	Pb	4.0–4.4	2.7
Cd	3.2–3.4	1.3–1.4	Sn	2.8–4.0	1.6
Ga	3.5	1.4	Ta	3.5–3.7	1.6
Hg	4.0–4.6	2.4	Tl	3.6–3.9	1.5
In	3.4–3.7	1.7	V	3.4–3.5	1.5
La	1.7–3.2	1.5	Zn	3.2–3.4	1.2–1.3
Nb	3.6–3.8	1.9–2.0			
BCS	3.53	1.43			

Table: Source: M. Marder, Condensed Matter Physics, Chapter 27, Wiley, 2010

Importance of understanding and controlling the excitation spectrum for superconducting quantum processors and sensor development

Quasiparticle Excitations – BCS Spectrum (Density of States)

Quasiparticle Excitations – BCS Spectrum (Density of States)

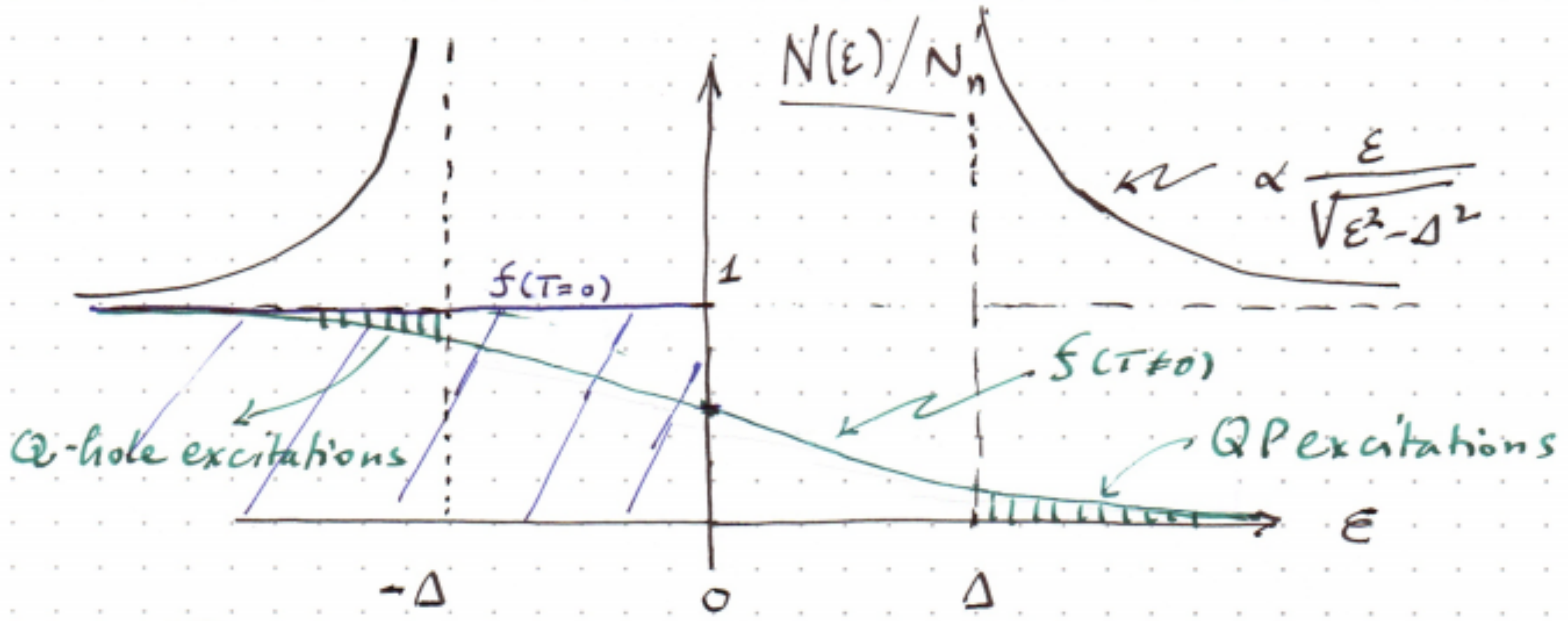


BCS Quasiparticle Density of States

- $N_n = \frac{3}{2}\, n/E_F$ = Normal Metal DOS @ E_F.
- Δ = Excitation Gap for Quasiparticles / Q-holes

$\Delta(Nb) \cong 2\,meV$

$\Delta(Al) \cong 0.2\,meV$



- Thermal Excitations

$$f(\varepsilon) = \frac{1}{e^{\varepsilon/k_BT}+1}$$

→ BCS Excitation Gap : Δ/k_BT

$10^{-2}\,K°$

Typical DR

$N_{QP} \propto \sqrt{k_BT/\Delta}\; e^{-\Delta/k_BT}$ → Expected Negligible @

Non-Equilibrium Quasiparticle Excitations in QIS devices

PHYSICAL REVIEW LETTERS **121,** 157701 (2018)

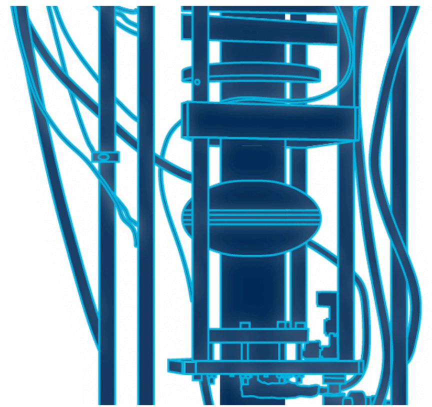
Hot Nonequilibrium Quasiparticles in Transmon Qubits

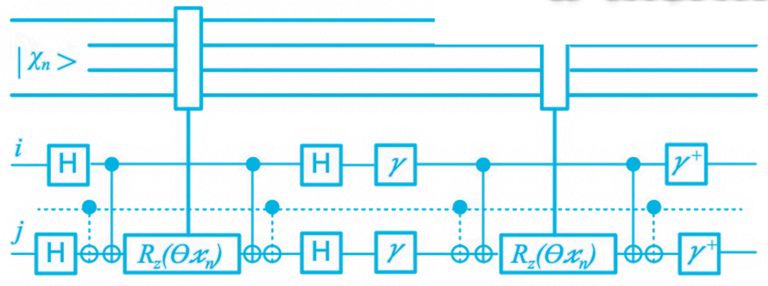
K. Serniak,[1,*] M. Hays,[1] G. de Lange,[1,2] S. Diamond,[1] S. Shankar,[1] L. D. Burkhart,[1]
L. Frunzio,[1] M. Houzet,[3] and M. H. Devoret[1,†]
[1]*Department of Applied Physics, Yale University, New Haven, Connecticut 06520, USA*
[2]*QuTech and Kavli Institute of Nanoscience, Delft University of Technology, 2600 GA Delft, Netherlands*
[3]*Univ. Grenoble Alpes, CEA, INAC-Pheliqs, F-38000 Grenoble, France*

(Received 2 April 2018; revised manuscript received 27 July 2018; published 10 October 2018)

Nonequilibrium quasiparticle excitations degrade the performance of a variety of superconducting circuits. Understanding the energy distribution of these quasiparticles will yield insight into their generation mechanisms, the limitations they impose on superconducting devices, and how to efficiently mitigate quasiparticle-induced qubit decoherence. To probe this energy distribution, we systematically correlate qubit relaxation and excitation with charge-parity switches in an offset-charge-sensitive transmon qubit, and find that quasiparticle-induced excitation events are the dominant mechanism behind the residual excited-state population in our samples. By itself, the observed quasiparticle distribution would limit T_1 to $\approx$200 μs, which indicates that quasiparticle loss in our devices is on equal footing with all other loss mechanisms. Furthermore, the measured rate of quasiparticle-induced excitation events is greater than that of relaxation events, which signifies that the quasiparticles are more energetic than would be predicted from a thermal distribution describing their apparent density.

$$x_{QP}^{neq} \approx 10^{-7} \gg x_{QP}^{thermal}$$





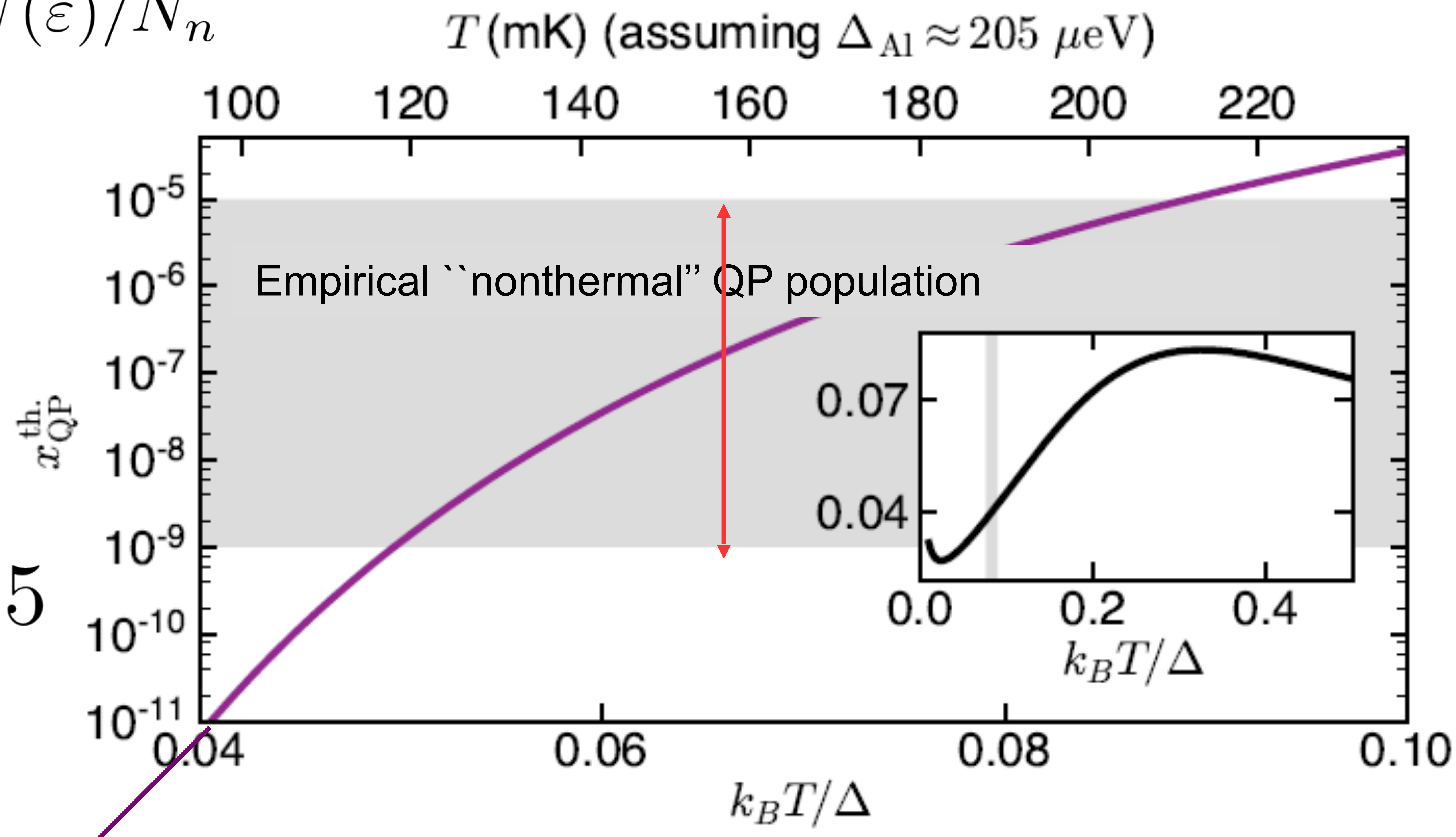
SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

Quasiparticle Excitations – BCS theory vs Observation

Quasiparticle Excitations – BCS theory vs Observation

Thermal population of Quasiparticles

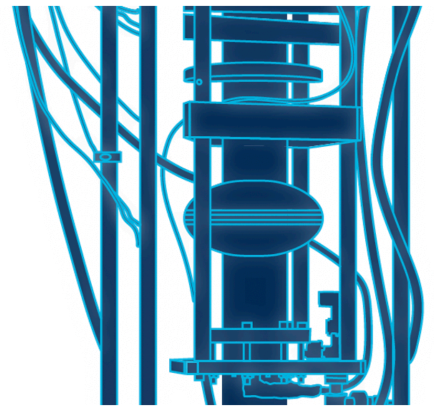
$$x_{QP}^{thermal} = \int_0^\infty d\varepsilon\, f(\varepsilon)\, N(\varepsilon)/N_n$$

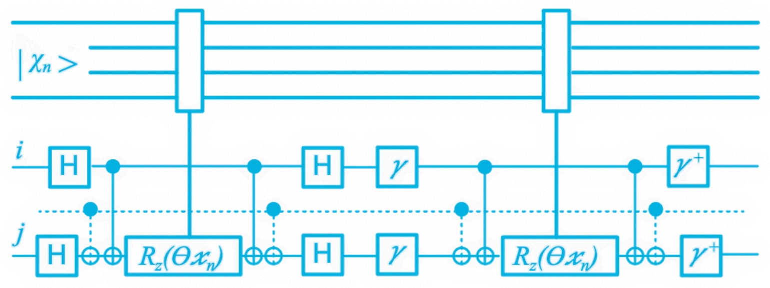


@ T = 10 mK

$$k_BT/\Delta_{Al} \approx 0.025$$

Kyle Serniak, PhD thesis (Yale), 2019

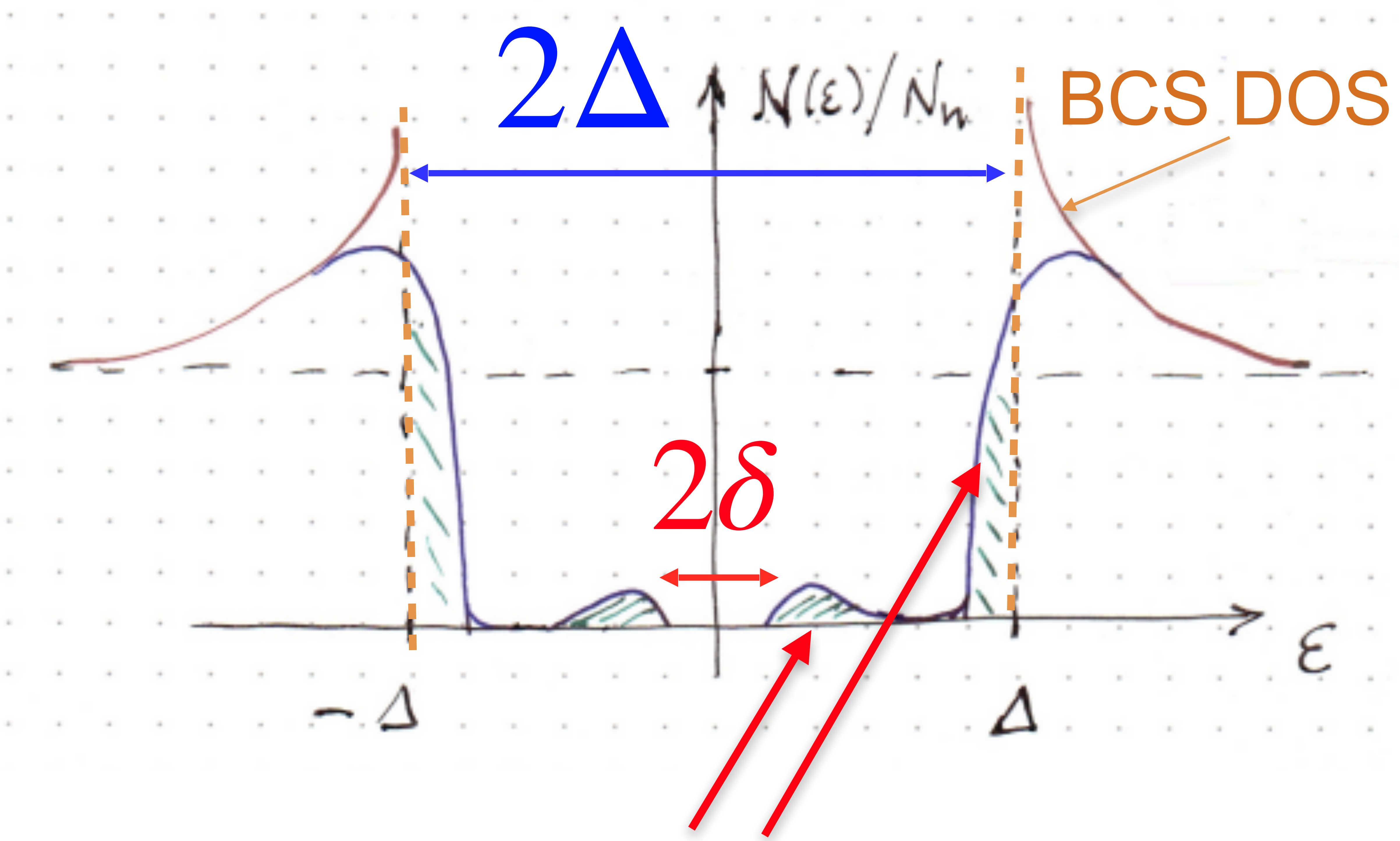




SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

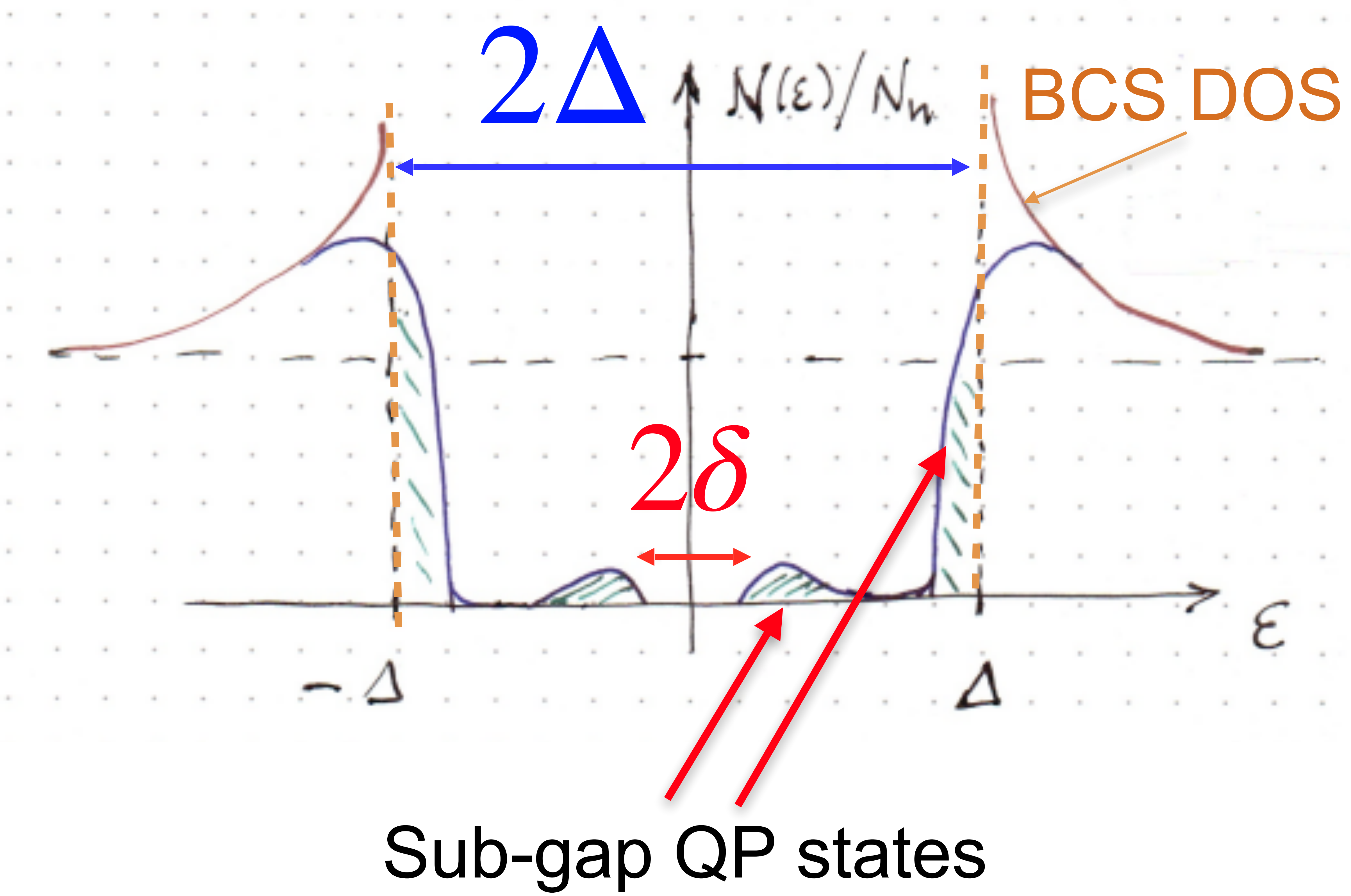
Quasiparticle Excitations (QPs) – Sources and Generation

Quasiparticle Excitations (QPs) – Sources and Generation



Sub-gap QP states

Quasiparticle Excitations (QPs) – Sources and Generation



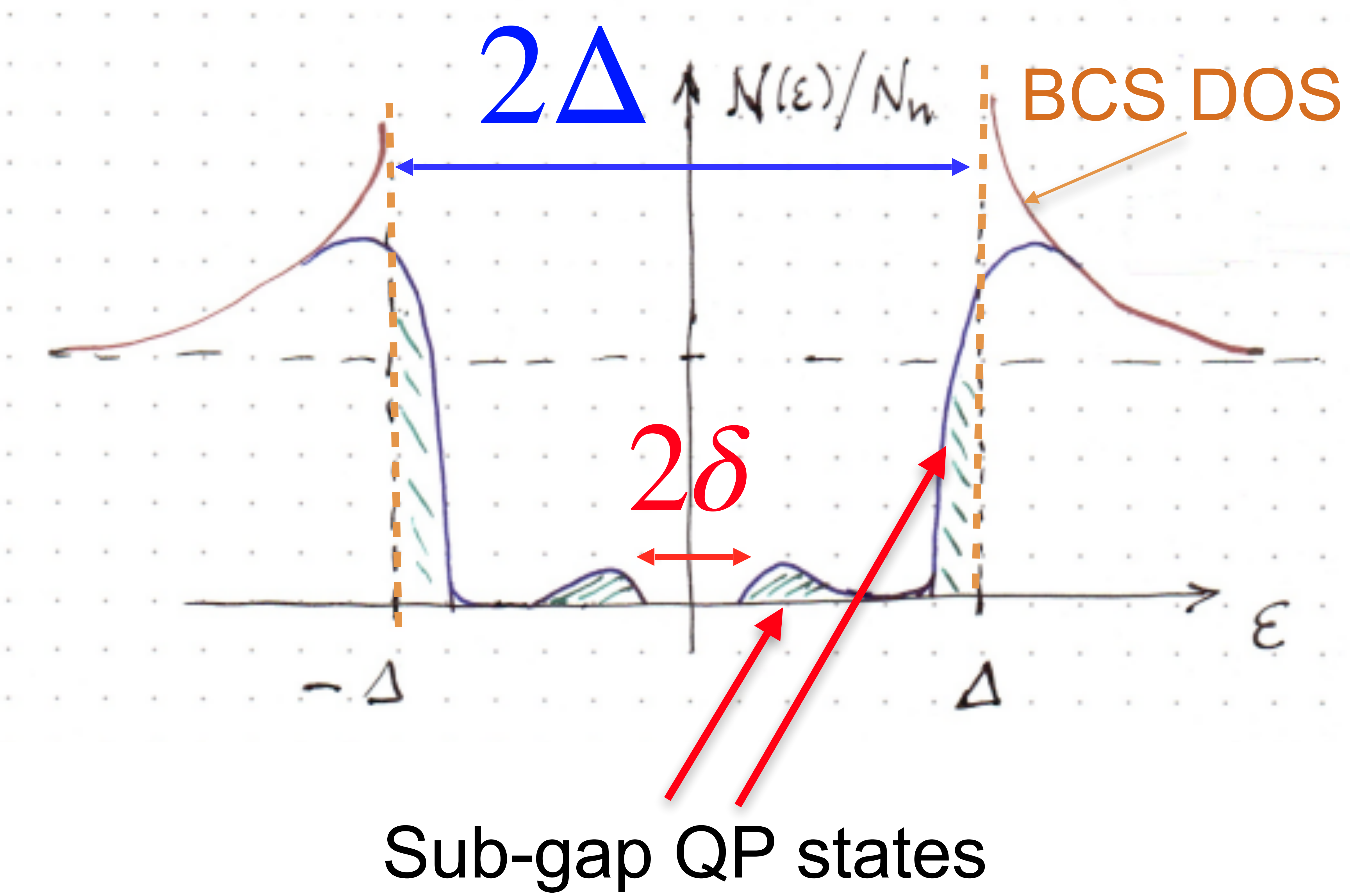
Cooper Pair breaking Mechanisms

- Impurity scattering & $\Delta(\mathbf{p})$
- Inhomogeneous $\Delta(\mathbf{r})$

$\Downarrow$

- Andreev Bound States

Quasiparticle Excitations (QPs) – Sources and Generation



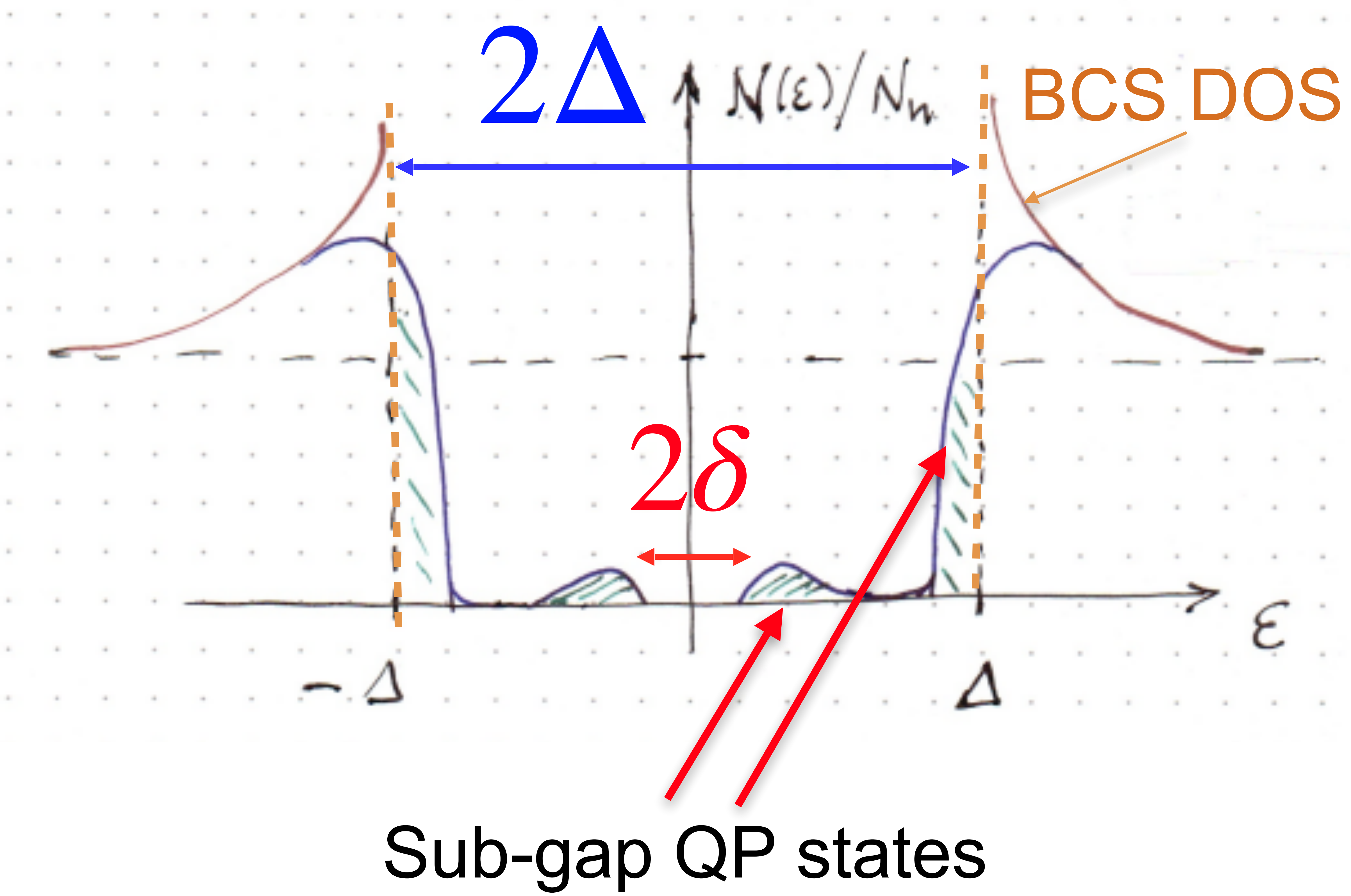
Cooper Pair breaking Mechanisms

- Impurity scattering & $\Delta(\mathbf{p})$
- Inhomogeneous $\Delta(\mathbf{r})$

$$\Downarrow$$

- Andreev Bound States
- Magnetic impurities

Quasiparticle Excitations (QPs) – Sources and Generation



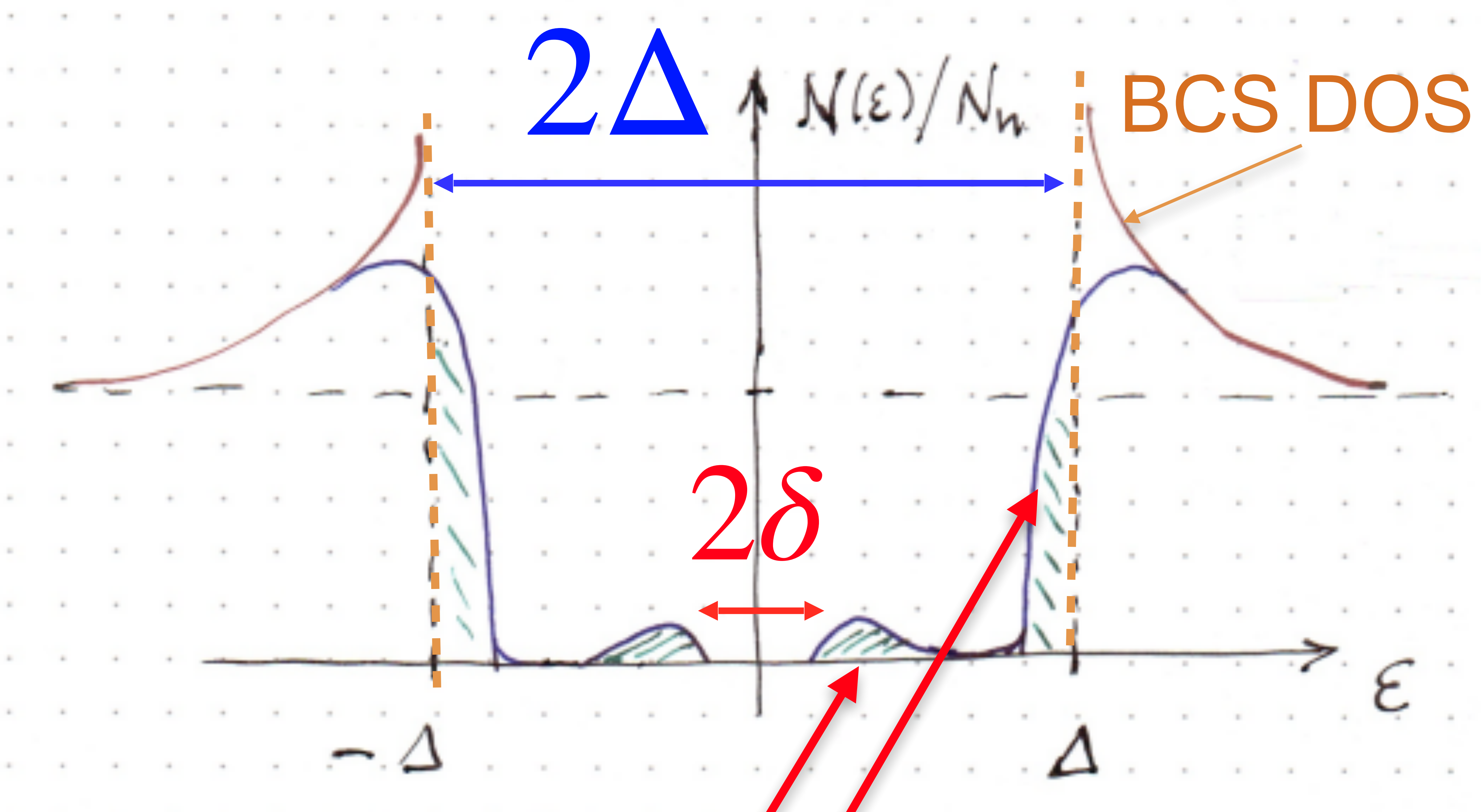
Cooper Pair breaking Mechanisms

- Impurity scattering & $\Delta(\mathbf{p})$
- Inhomogeneous $\Delta(\mathbf{r})$

$\Downarrow$

- Andreev Bound States
- Magnetic impurities
- Dynamical Impurities (TLS†)

Quasiparticle Excitations (QPs) – Sources and Generation



Sub-gap QP states

Cooper Pair breaking Mechanisms

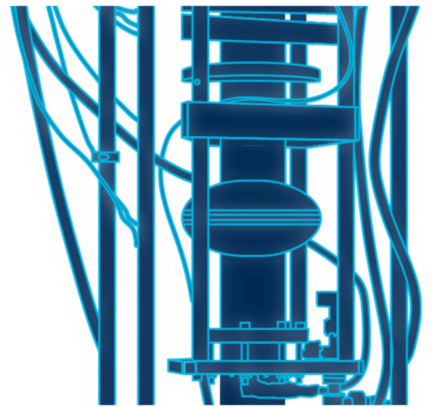
- Impurity scattering & $\Delta(\mathbf{p})$
- Inhomogeneous $\Delta(\mathbf{r})$

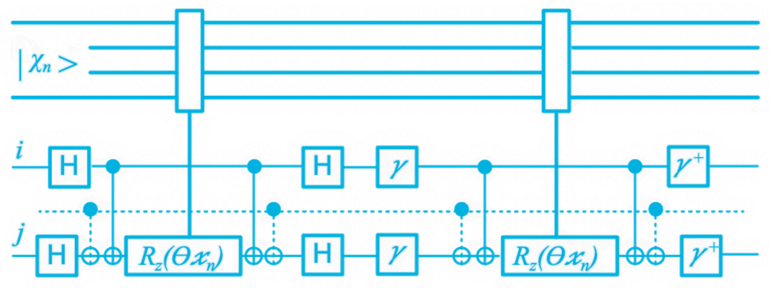
$$\Downarrow$$

- Andreev Bound States
- Magnetic impurities
- Dynamical Impurities (TLS†)

  O, N, C, OH, NH …

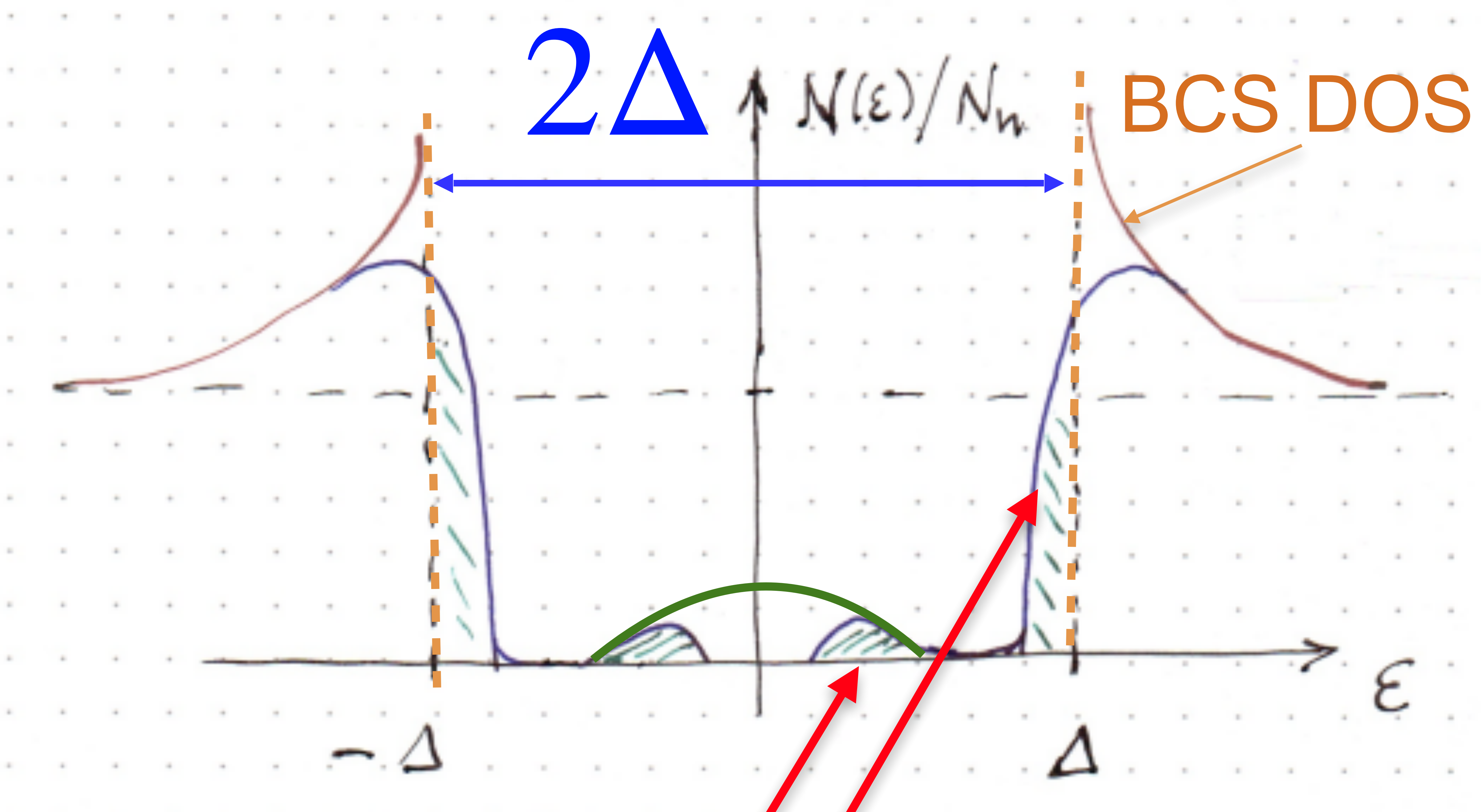
†impurity that tunnels between nearby sites





SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

Quasiparticle Excitations (QPs) – Sources and Generation



Sub-gap QP states

Cooper Pair breaking Mechanisms

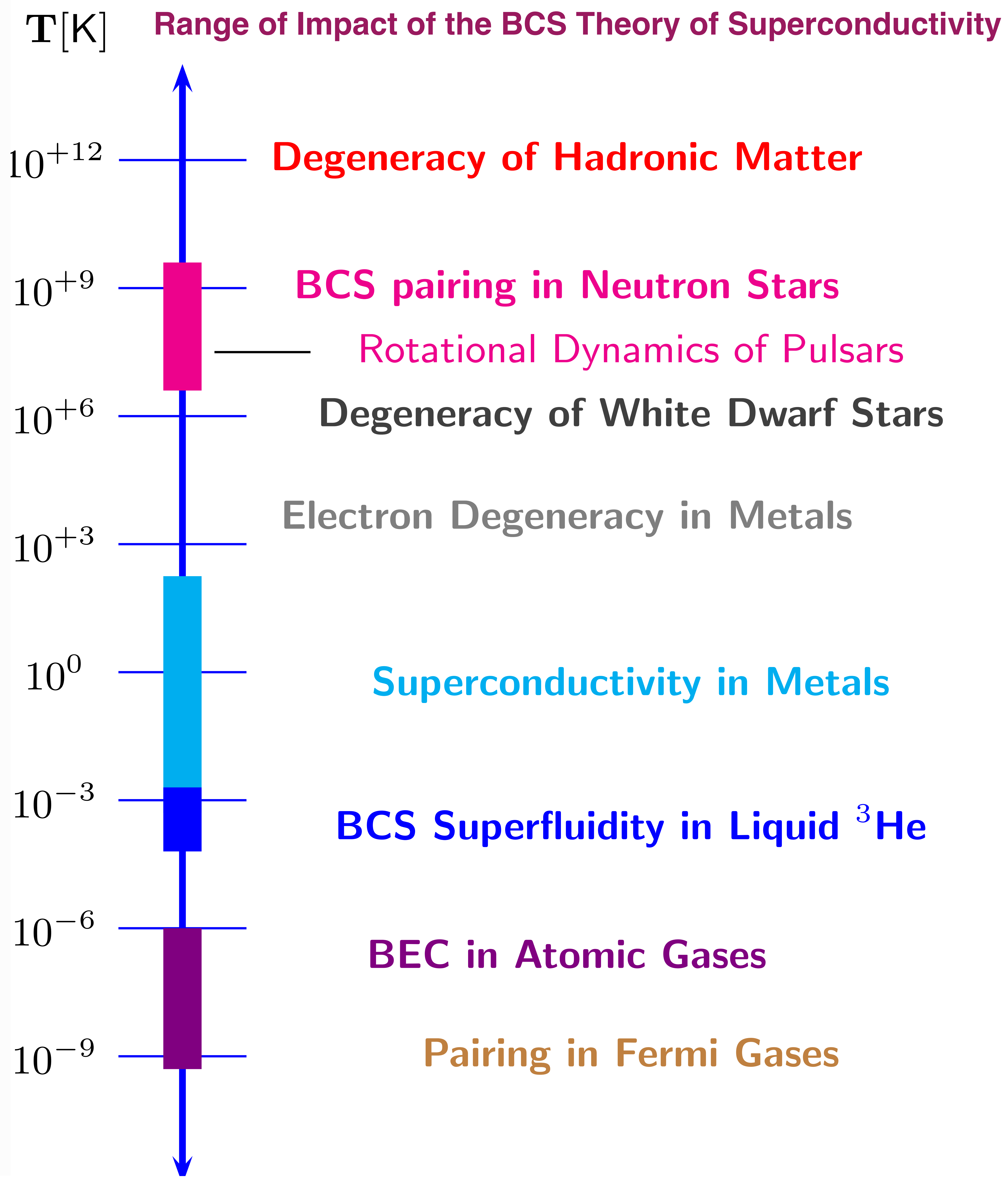
- Impurity scattering & $\Delta(\mathbf{p})$
- Inhomogeneous $\Delta(\mathbf{r})$

$\Downarrow$

- Andreev Bound States
- Magnetic impurities
- Dynamical Impurities (TLS†)
  O, N, C, OH, NH …
- Multi-photon (nonlinear μ-wave excitation)
- Radioactivity (INFN)
- Nonequilibrium QP generation

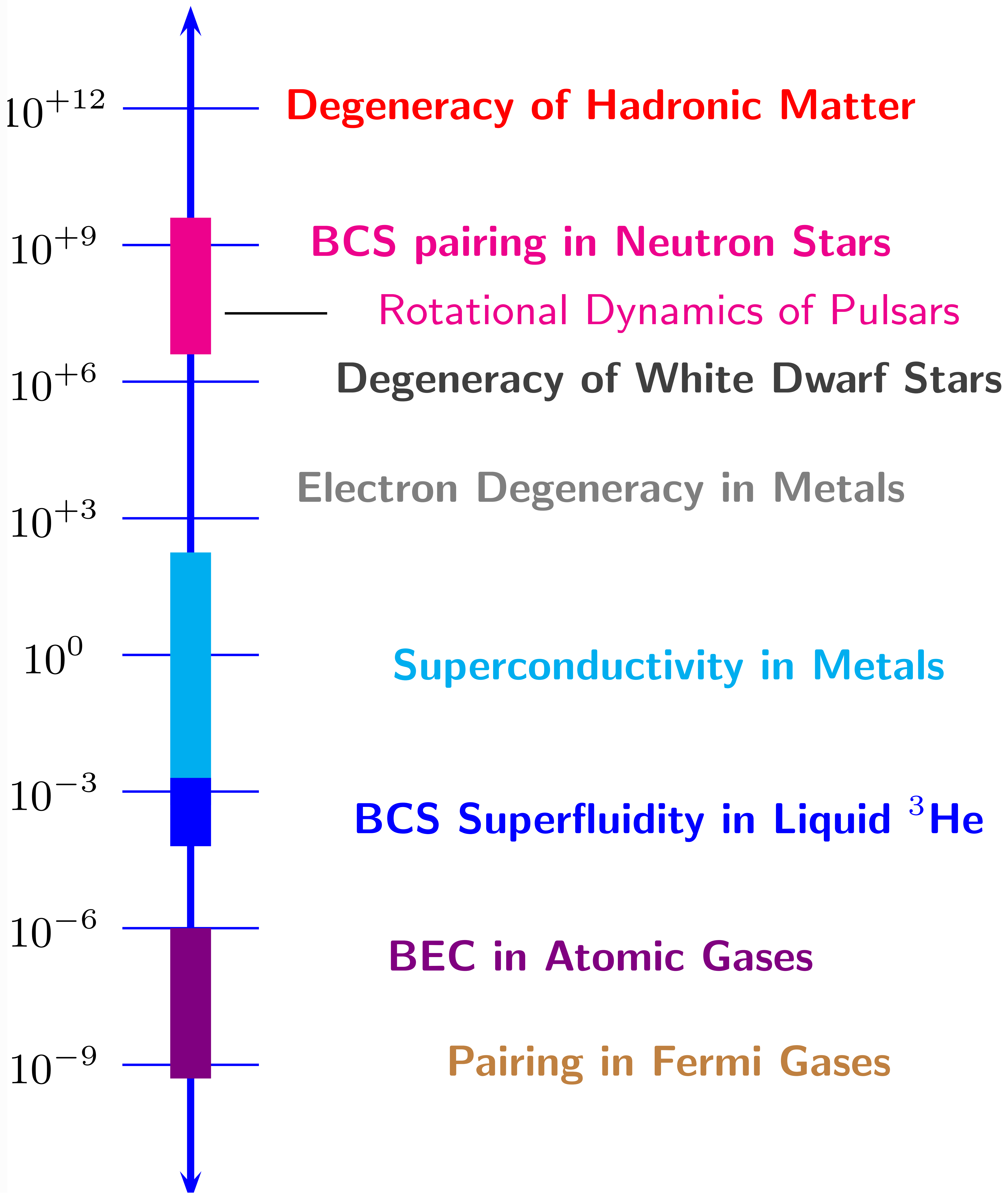
†impurity that tunnels between nearby sites

Range of Impact of the BCS Theory of Superconductivity



T[K]
Range of Impact of the BCS Theory of Superconductivity
10+12
Degeneracy of Hadronic Matter
10+9
BCS pairing in Neutron Stars
Rotational Dynamics of Pulsars
10+6
Degeneracy of White Dwarf Stars
Electron Degeneracy in Metals
10+3
10^0
Superconductivity in Metals
10−3
BCS Superfluidity in Liquid 3He
10−6
BEC in Atomic Gases
10−9
Pairing in Fermi Gases

Range of Impact of the BCS Theory of Superconductivity



Key Discoveries

1908 Helium is liquified

1911 Superconductivity is discovered in Hg
1933 Diamagnetism - Meissner Effect
1935 London's Theory
1950 Ginzburg-Landau Theory

1956 Copper Instability
1957 BCS Pairing Theory
1957 Landau Fermi Liquid Theory
1957 Abrikosov's Theory of Type II SC
1958 Pairing in Nuclei and Nuclear Matter
1959 Gauge-Invariant Pairing Theory
1959 Field Theory formulation of BCS Theory
1961 Theory of Spin-Triplet Pairing
1962 Josephson Effect

1967 Pulsars discovered - Hewish & Bell
1969 Pulsar Glitches observed in Vela
1980 Superfluid hydrodynamics of NS

1972 Discovery of Triplet, P-wave Superfluid ^{3}He
1979 Discovery of Heavy Electron Superconductors
1982 Exotic Pairing in U-based Heavy Fermions
1986 High T_c CuO Superconductivity
1994 Exotic Pairing discovered in Sr_2RuO_4
1995 D-wave Pairing identified in $YBa_2Cu_3O_{7\text{-}x}$
2001 Co-existent Ferromagnetism & Superconductivty
2008 Fe-based Superconductors discovered

1995 Discovery of BEC in cold atomic ^{87}Rb
1998 Degeneracy of Cold Fermionic Gases: ^{6}Li
2007 BEC-BCS Condensation in ^{6}Li, ^{40}K

2008 *Topological* Superfluids & Superconductors