Problem Sheet - Cryogenics for Superconducting Quantum Information Science

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1 Dilution Refrigerator Performance

During the lecture, we defined the heat uplift at the mixing chamber of a dilution refrigerator at temperature T_{mc} with a Helium-3 flow rate $\dot{n}_3$ as

$$\dot{Q} = \dot{n}_3(96T_{mc}^2 - 12T_N^2), \tag{1}$$

where T_N is the temperature of the helium exiting the last heat exchanger. Assuming that the helium entering the mixing chamber is 20 % higher than the mixing chamber temperature, what Helium-3 flow rate is required to remove 50 μW of heat at a mixing chamber temperature of 30 mK?

2 Heat Exchange Area and Kapitza Resistance

During the lecture, we discussed the importance of heat exchanger surface area in the transfer of heat, especially at low temperatures. The thermal boundary resistance between helium and other materials, also known as the Kaptiza resistance, R_K in units of K/W at a temperature T is given by

$$R_K = \frac{a}{A}T^{-3}, \tag{2}$$

where A is the heat exchanger area in square meters and a is a constant determined from experiment that is material dependant. For Helium-3, $a \approxeq$ 0.05 K^{4}m^2/W, and for helium mixtures $a \approxeq$ 0.02 K^{4}m^2/W.

a. For a mixing chamber temperature of T =100 mK, calculate the required heat exchanger surface area to transfer 100 μW of heat with a ΔT of 1 mK

b. For the same heat exchanger and heat input, what would the temperature gradient between the helium mixture and the solid material be at T =25 mK

3 Thermal Conductivity at Low Temperatures

The amount of heat flow, Q, conducted through a solid material with a temperature gradient is given by

$$Q = A/l \int_{T_1}^{T_2} \kappa(T) dT \tag{3}$$

where A is the cross-sectional area of the solid and l is the length between two temperatures, T_1 and T_2.

Calculate

a. The conducted heat through a solid rod of diameter 25 mm and length 250 mm between a temperature of 1 K and 100 mK if the rod were composed of PTFE, for which $\kappa(T) = 3T^2$ mW/m K

b. The cross-sectional area needed to keep the heat flow from 2 K below 5 μW, assuming the same length and cold end temperature as above