Introduction to Quantum Computing: Qubits, Gates, and Algorithms

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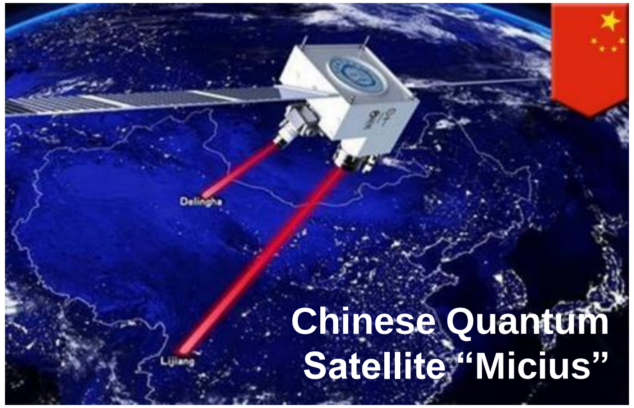
Quantum Information Science and Technology

Quantum Sensing

Improves sensitivity, drift, & spatial resolution

Quantum Networks



Enables distributed quantum states

Quantum Computing

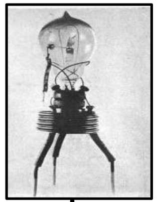
Solves select problems that are intractable with classical computing

Quantum Information Science utilizes a quantum mechanical description of nature to compute, sense, and communicate information in ways unobtainable by means based on a classical description of nature

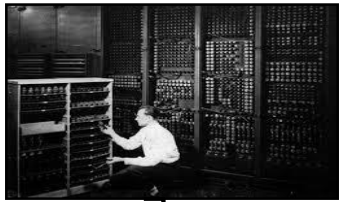
Computing Development Timeline

Classical Computing (Electronic)

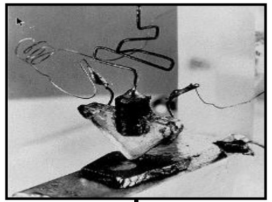
Vacuum tube
(1906)



ENIAC
(1946)

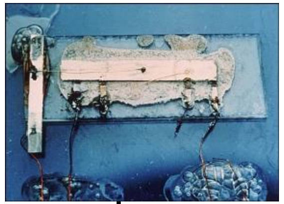


Transistor
(1947)

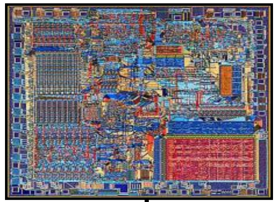


TX-0
(1956)

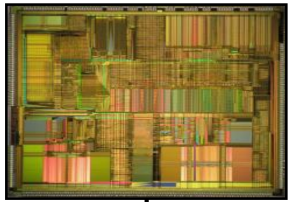
Integrated
circuit
(1958)



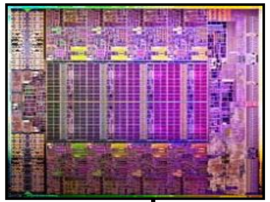
2K transistors
i4004
(1971)



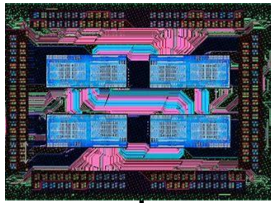
5.5M transistors
Pentium Pro
(1995)



18 cores
5.5B transistors
Xeon Haswell
(2014)



32 cores
19.2B transistors
Epyc GPU
(2017)



Quantum computing is transitioning from scientific curiosity to technical reality.

Advancing from discovery to useful machines takes time & engineering

You must be in the game to play

Quantum Computing

Grover
(1996)

Factor 15
(2002)

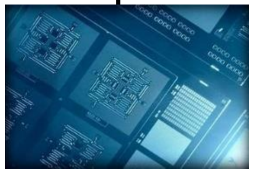
Few-qubit
processors
(2010-2016)

Business
development
(2019-2021)

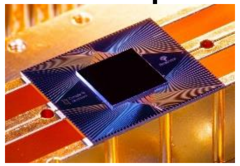
Quantum simulator
proposed
(1981)

Shor's algorithm
& CSS error correction
(1994-95)

Quantum annealing
& adiabatic QC
(1998-2000)



Cloud-based
QCs
(2017)

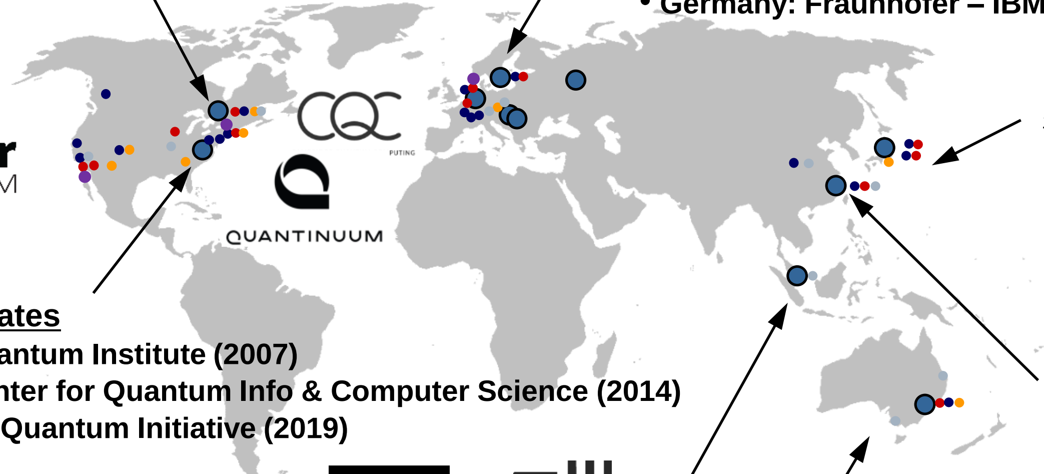


Quantum
Advantage
53-qubit QCs
(2019)

Quantum Worldwide (not exhaustive)

Canada
- Inst. for Quantum Computing (2002)
- Inst. Quantique (2015)

Europe
- Netherlands: QuTech (2014)
- United Kingdom: National Quantum Technologies Program (2014)
- EU: Quantum Flagship (2016)
- Sweden: Wallenberg Center for Quantum Technology (2017)
- Germany: Fraunhofer – IBM alliance (2019)



Japan
- Gate-model and QA
- JST, RIKEN, AIST, NICT

United States
- Joint Quantum Institute (2007)
- Joint Center for Quantum Info & Computer Science (2014)
- National Quantum Initiative (2019)

China
- Key Lab, Quantum Information, CAS (2001)
- Satellite quantum communication (2016)
- Alibaba – CAS cloud computer (2018)

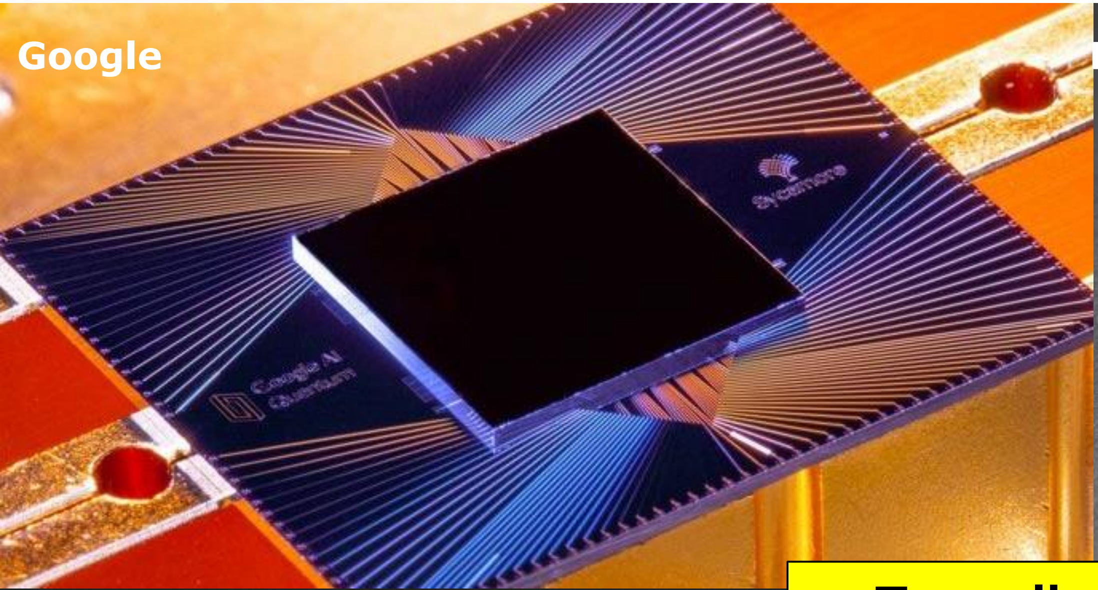
Australia
- ARC Centers of Excellence
  - Center for Quantum Computing Technology (2000)
  - Engineered Quantum Systems (2011)
- CommBank – Telstra – UNSW (2015)

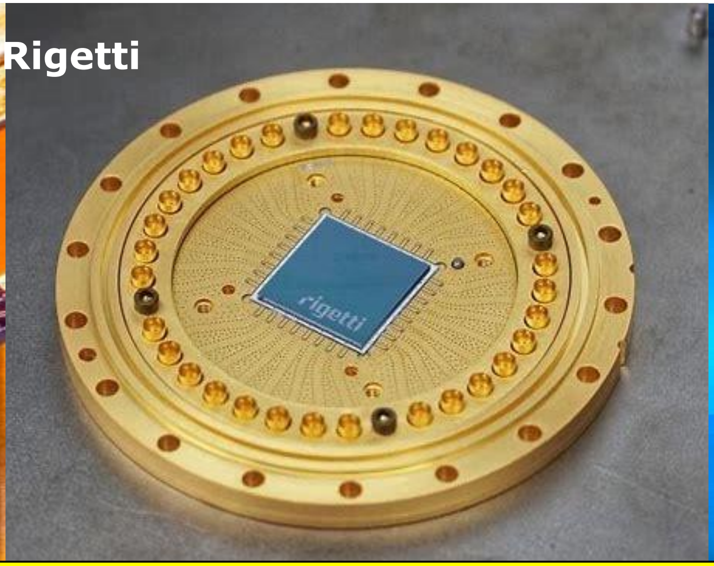
Singapore
- Research Center on Quantum Information Science and Technology (2007)

● Superconducting qubits ● Ion trap qubits ● Semiconducting qubits ● Quantum optics ● NV centers

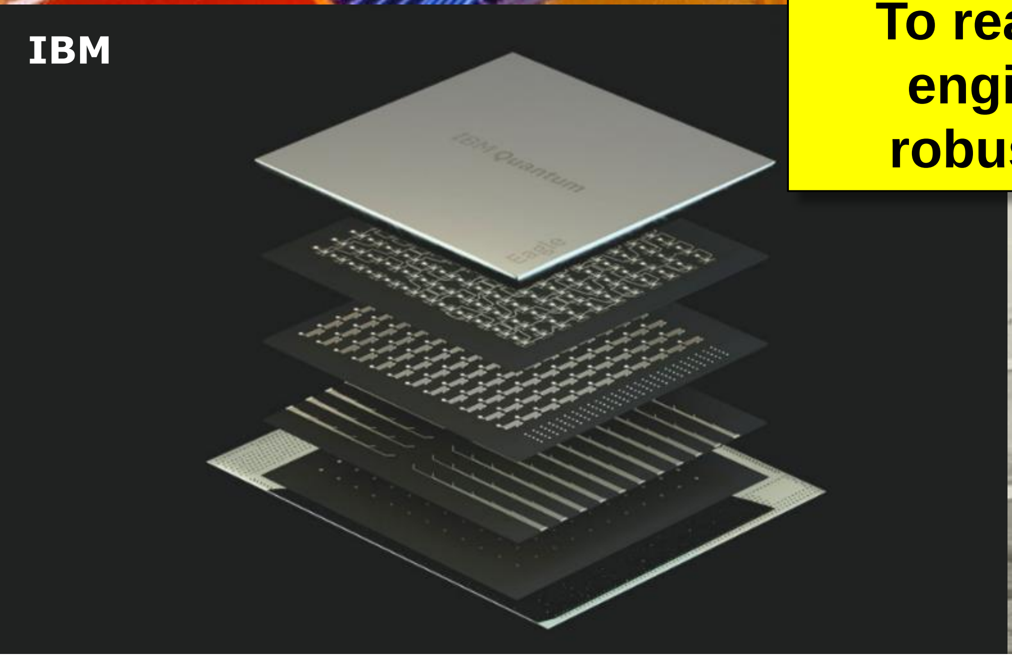
* European Commission

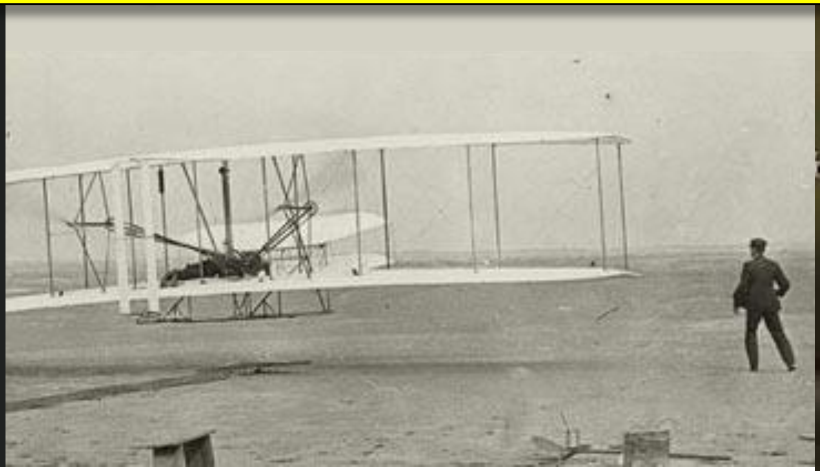
Nascent Commercial Quantum Processors





To realize the promise of QC, we must engineer quantum systems that are robust, reproducible, and extensible.





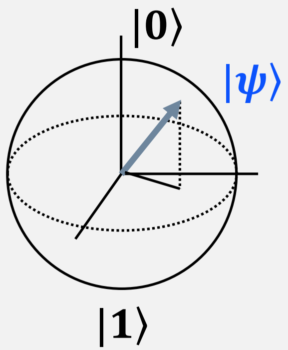
Outline

- Introduction
- Classical and Quantum Bits
- Quantum Gates and Algorithms
- Engineering Quantum Systems

How is a Quantum Computer Different?

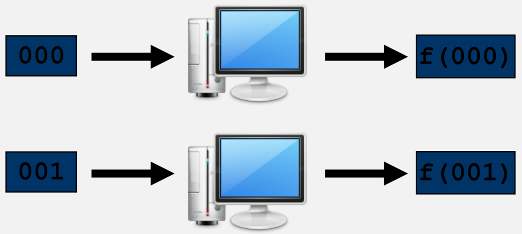
	Classical Computer
Fundamental logic element	"Bit" : classical bit (transistor, spin in magnetic memory, …)
State	0 "Or" 1
Measurement	• *Discrete* states • Deterministic measurement: Ex: Set as 1, measure as 1

How is a Quantum Computer Different?

	Classical Computer	Quantum Computer
Fundamental logic element	**"Bit" : classical bit** **(transistor, spin in magnetic memory, …)**	**"Qubit" : quantum bit** **(any coherent two-level system)**
State	**0 "Or" 1**	 **Superposition:** $\alpha\|0\rangle + \beta\|1\rangle$ $\|0\rangle$ **"And"** $\|1\rangle$ $\|\psi\rangle = \alpha\begin{bmatrix}1\\0\end{bmatrix} + \beta\begin{bmatrix}0\\1\end{bmatrix}$
Measurement	• ***Discrete*** **states** • **Deterministic measurement:** **Ex: Set as 1, measure as 1**	• ***Superposition*** **states** • **Probabilistic measurement:** **Ex: If** $\|\alpha\| = \|\beta\|$**, 50%** $\|0\rangle$**, 50%** $\|1\rangle$

Quantum computers rely on encoding information in a fundamentally different way than classical computers

How is a Quantum Computer Different?

	Classical Computer
Fundamental logic element	**"Bit" : classical bit** **(transistor, spin in magnetic memory, …)**
Computing	• **N bits: One N-bit state** **000, 001, …, 111 (N = 3)** • **Change a bit: new calculation (classical parallelism)** 

How is a Quantum Computer Different?

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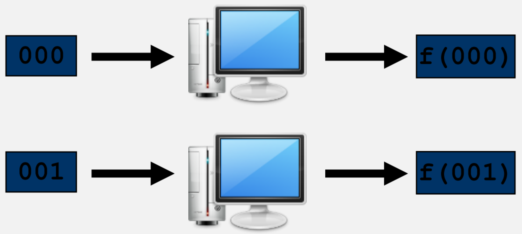
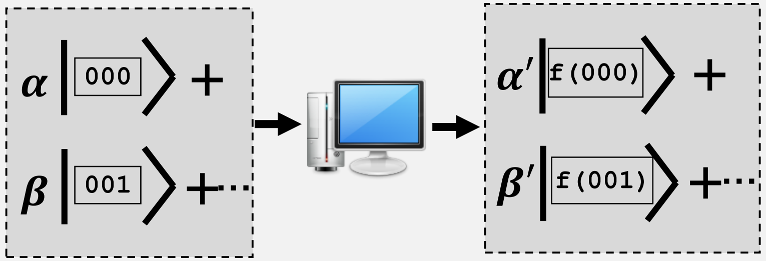
Fundamental logic element	**"Bit" : classical bit** **(transistor, spin in magnetic memory, …)**	**"Qubit" : quantum bit** **(any coherent two-level system)**
Computing	• **N bits: One N-bit state** **000, 001, …, 111 (N = 3)** • **Change a bit: new calculation (classical parallelism)**	• **N qubits: 2^N components to one state** $\alpha\|000\rangle + \beta\|001\rangle + \cdots + \gamma\|111\rangle$ **(N = 3)** • **Quantum parallelism & interference**





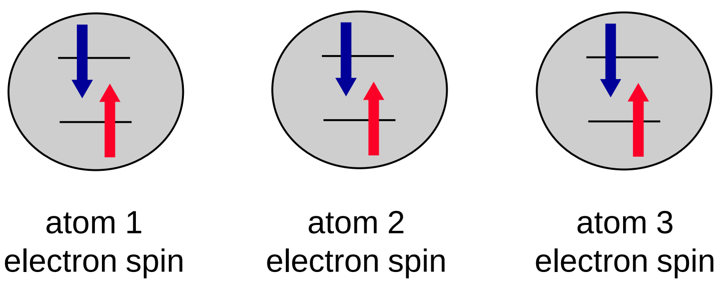
Quantum computers rely on encoding information in a fundamentally different way than classical computers

Classical and Quantum Bits

Three spins

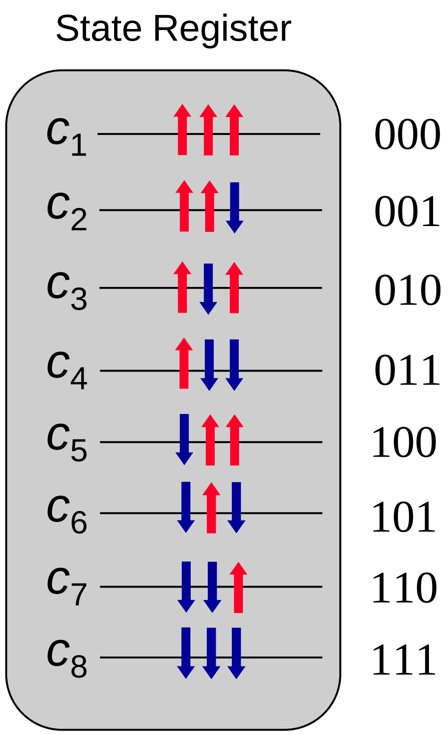
$\downarrow = |1\rangle$

$\uparrow = |0\rangle$



eight (2^N) classical states

(classical parallelism)



single quantum state

$$|\psi\rangle = c_1|\uparrow\uparrow\uparrow\rangle + c_2|\uparrow\uparrow\downarrow\rangle + c_3|\uparrow\downarrow\uparrow\rangle + c_4|\uparrow\downarrow\downarrow\rangle + c_5|\downarrow\uparrow\uparrow\rangle + c_6|\downarrow\uparrow\downarrow\rangle + c_7|\downarrow\downarrow\uparrow\rangle + c_8|\downarrow\downarrow\downarrow\rangle$$

$$= c_1|000\rangle + c_2|001\rangle + c_3|010\rangle + c_4|011\rangle + c_5|100\rangle + c_6|101\rangle + c_7|110\rangle + c_8|111\rangle$$

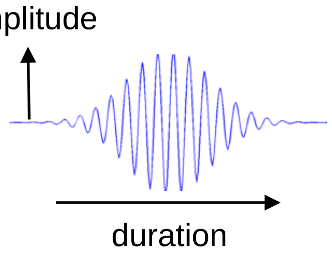
Quantum superposition state: eight complex numbers

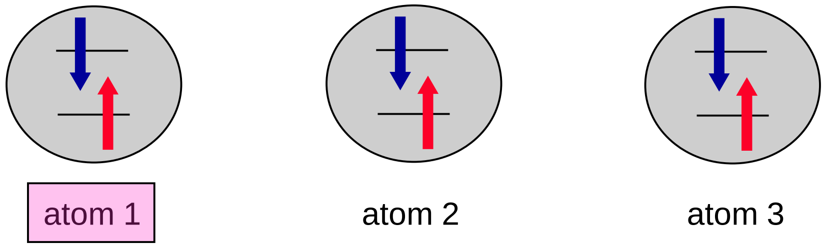
$$2^N \rightarrow 2^3 = 8 \rightarrow \{c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8\}$$

Quantum superposition & gates:
Quantum parallelism
Quantum interference

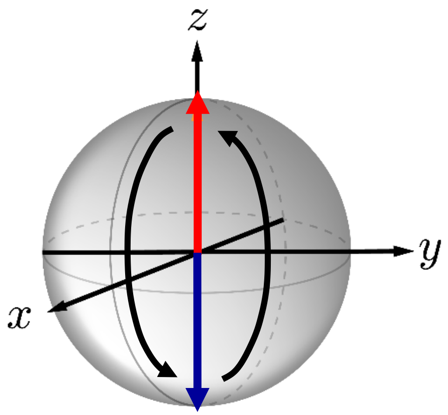
Quantum Parallelism

EM pulse flips spin of atom 1 (π-pulse)

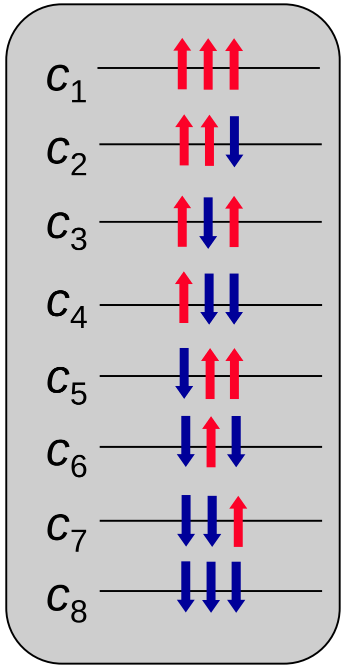




before pulse



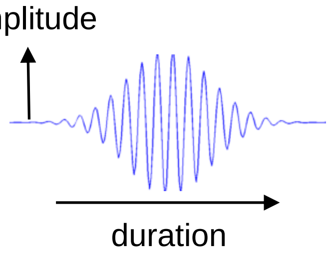
Coefficients are shuttled between states

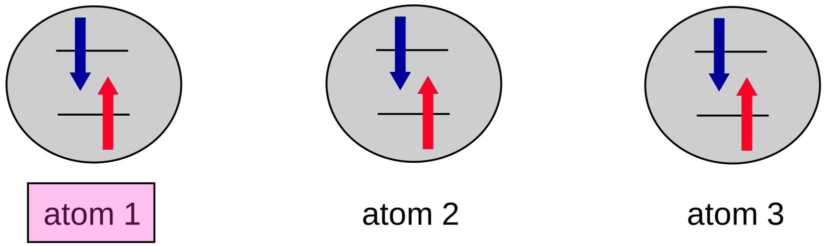


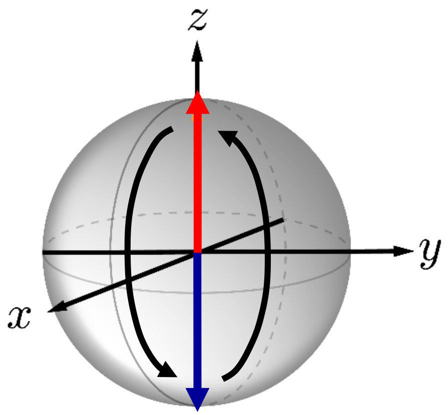
3-spin system

Quantum Parallelism

EM pulse flips spin of atom 1 (π-pulse)







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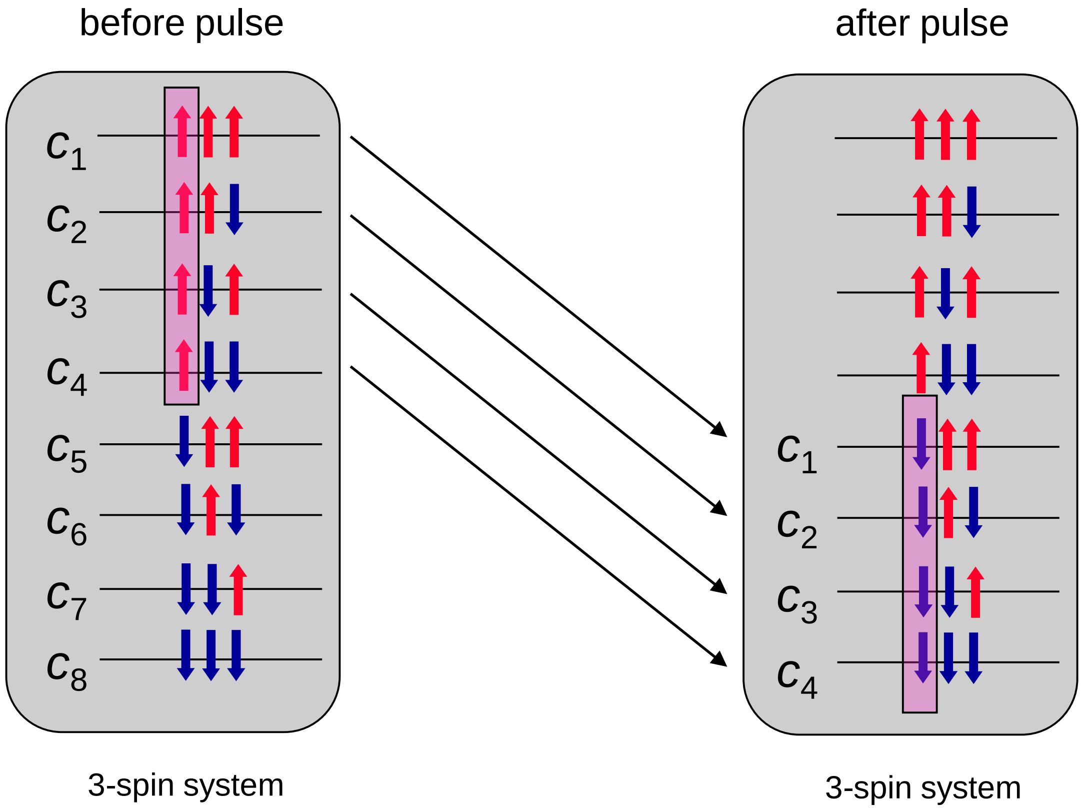
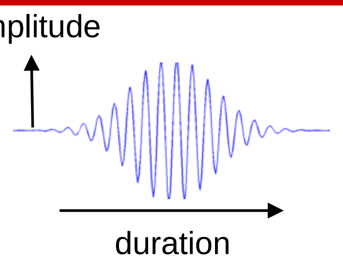
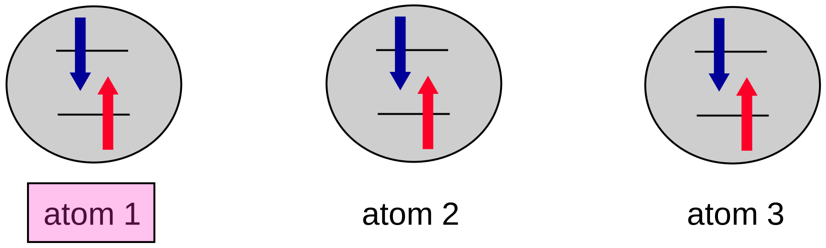
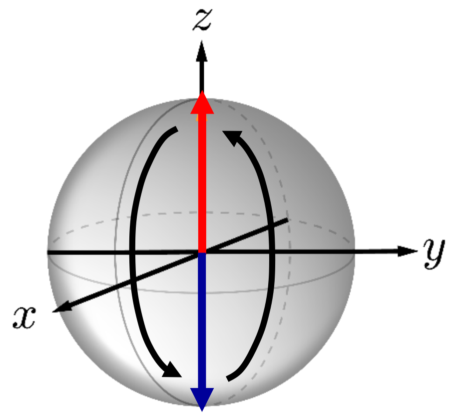


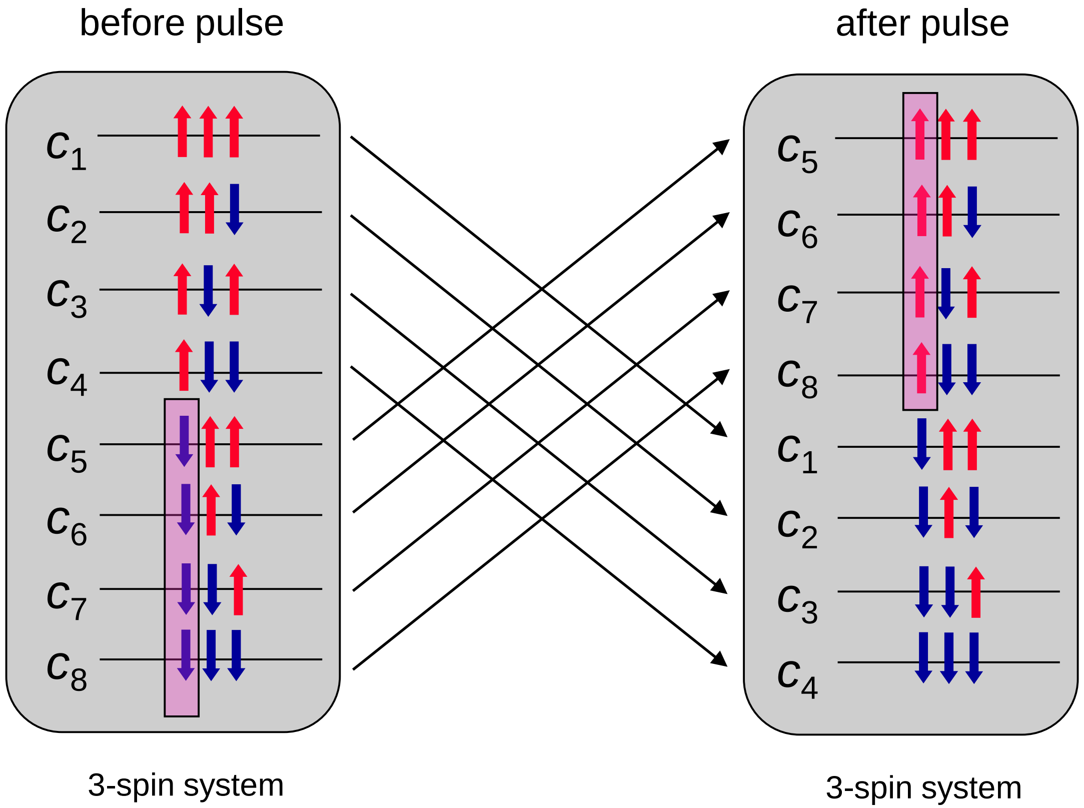
Quantum Parallelism

EM pulse flips spin of atom 1 (π-pulse)







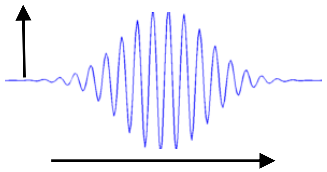


Coefficients are shuttled between states

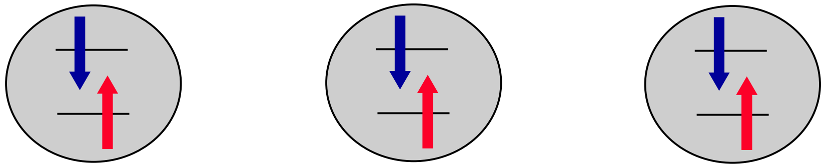
Quantum Parallelism

EM pulse flips spin of atom 1 (π-pulse)

amplitude



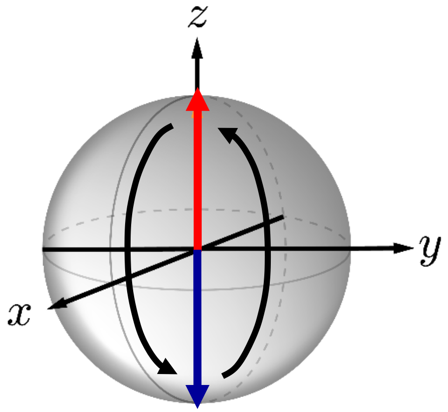
duration



atom 1

atom 2

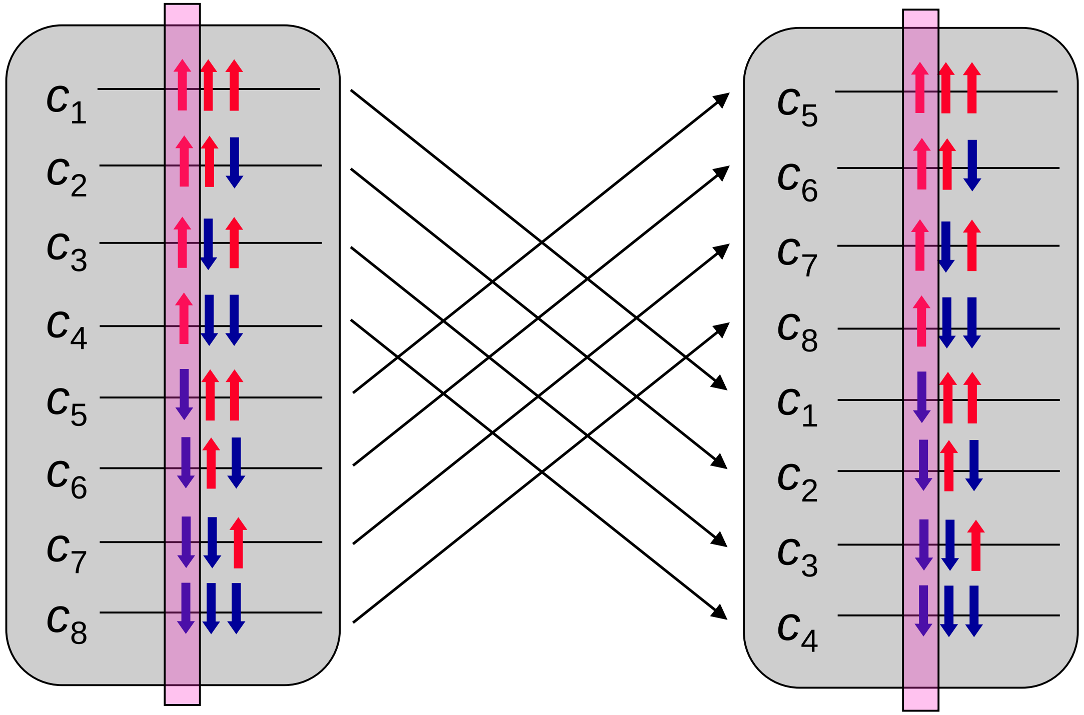
atom 3



Coefficients are shuttled between states

before pulse

after pulse



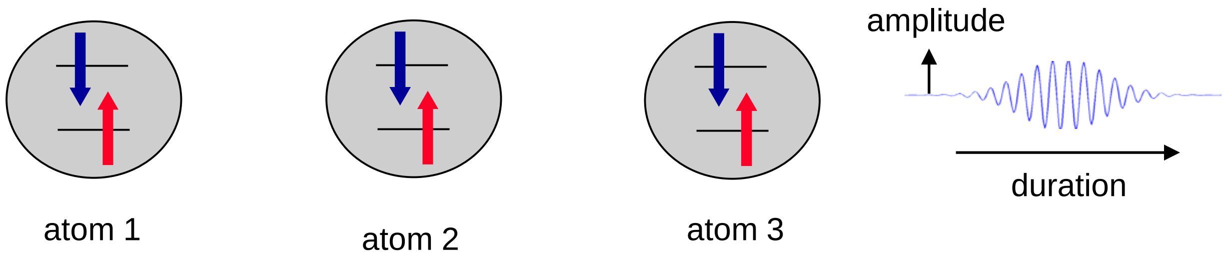
3-spin system

3-spin system

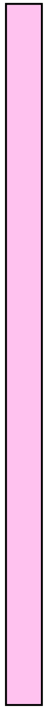
Operates on entire system simultaneously

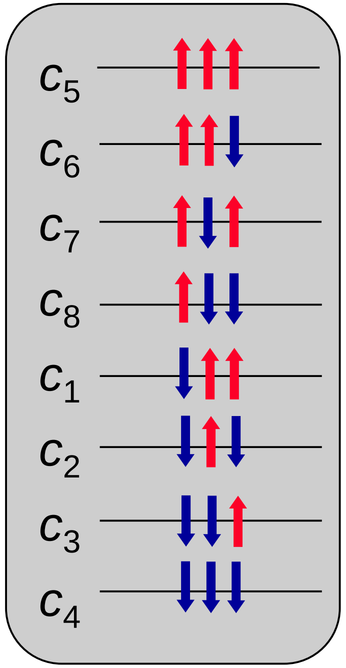
Quantum Parallelism

Quantum Interference



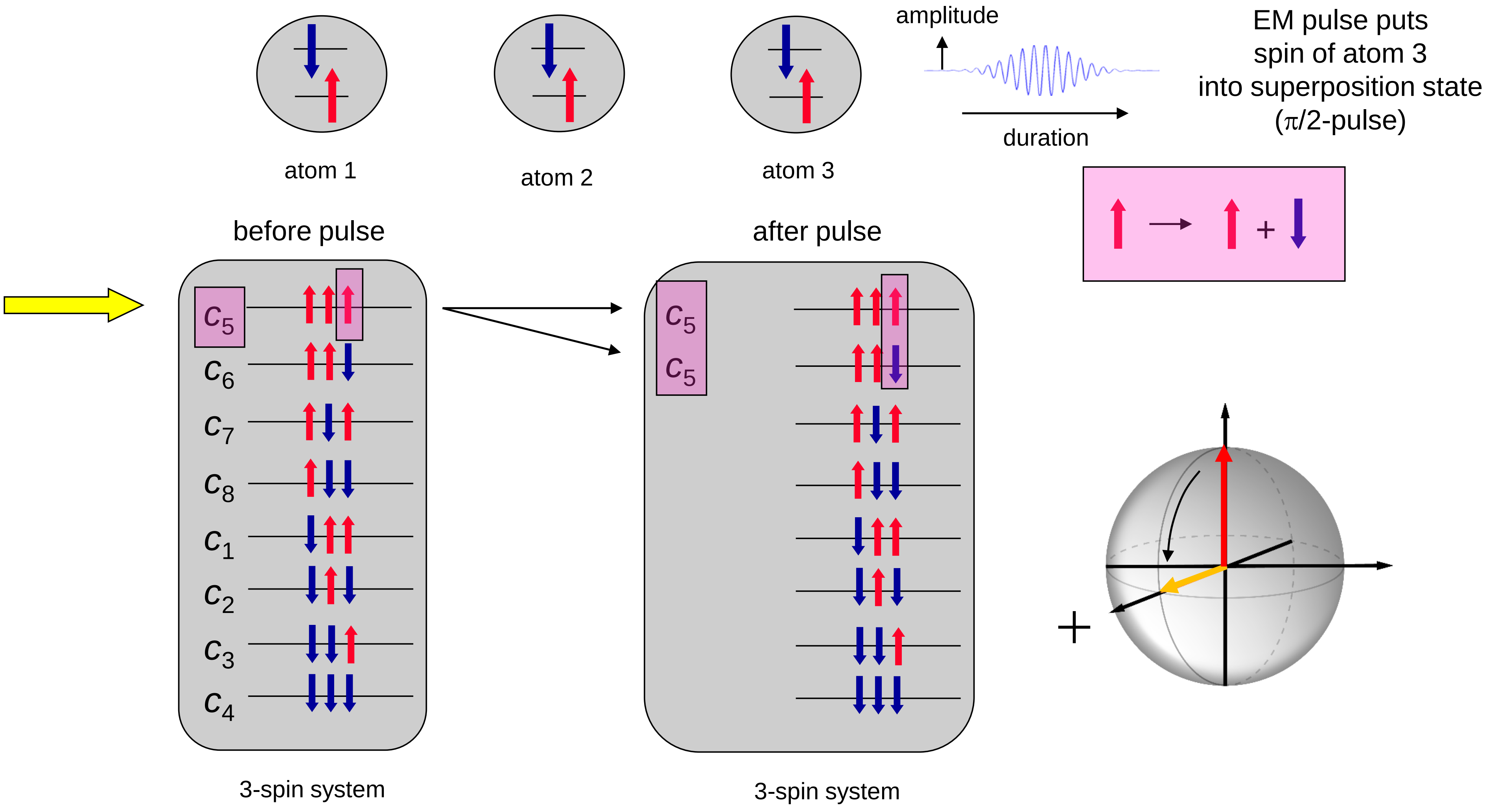
EM pulse puts
spin of atom 3
into superposition state
(π/2-pulse)



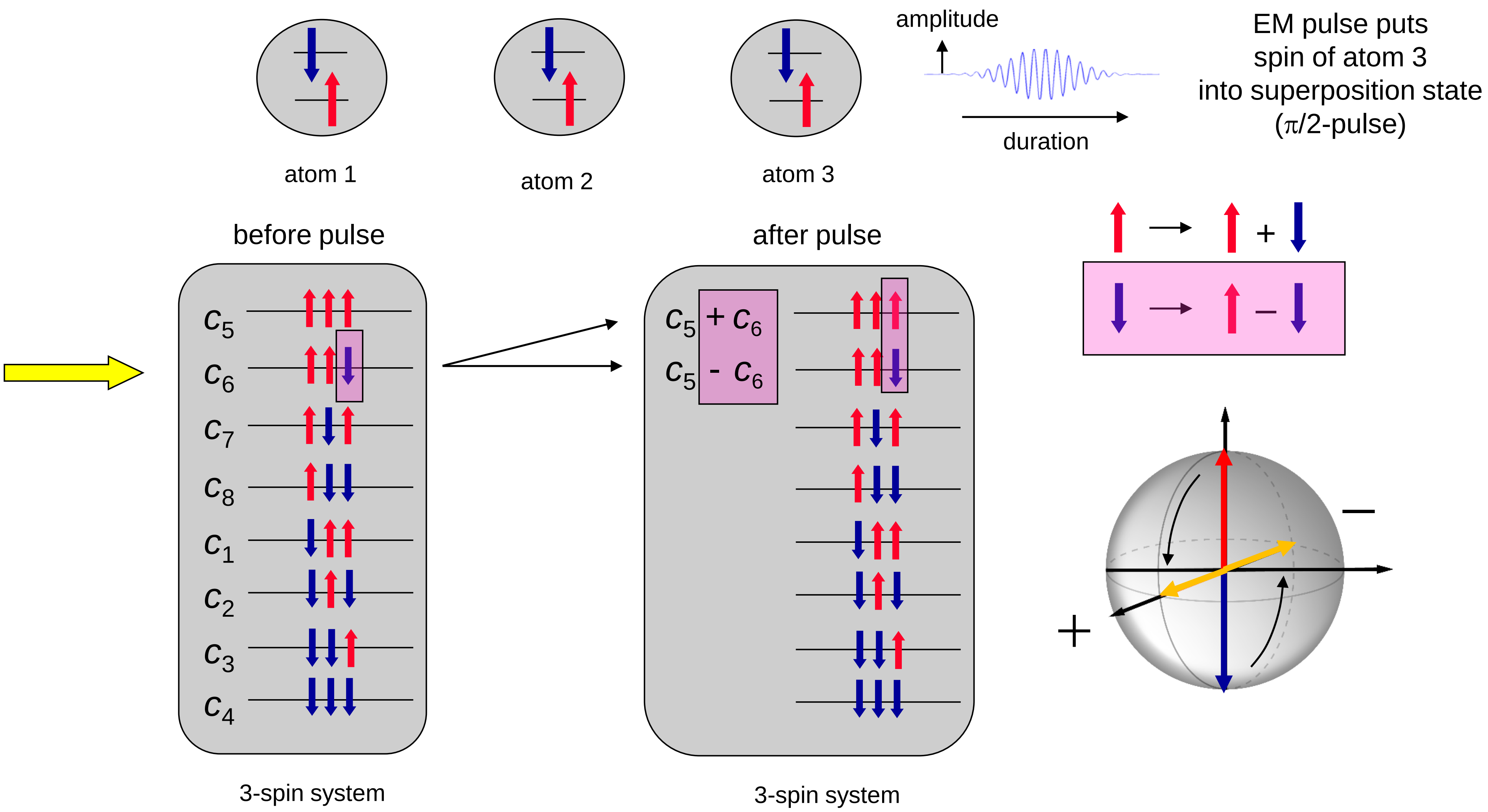


3-spin system

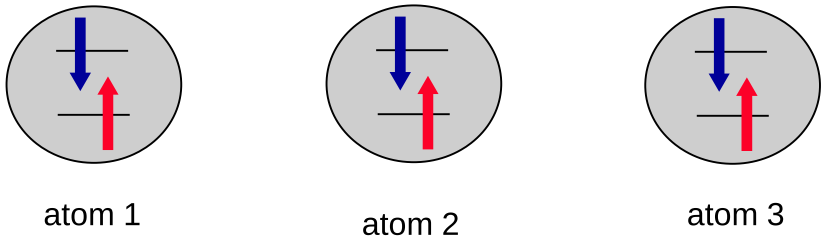
Quantum Interference

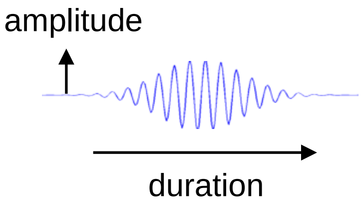


Quantum Interference



Quantum Interference





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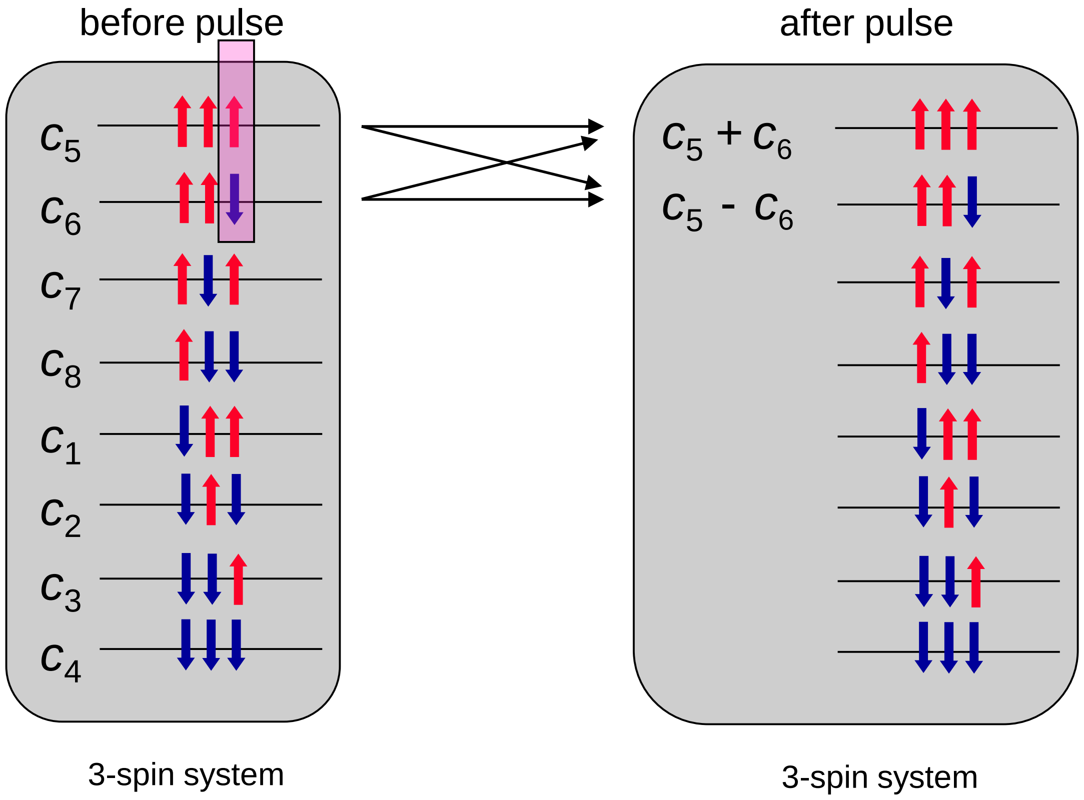


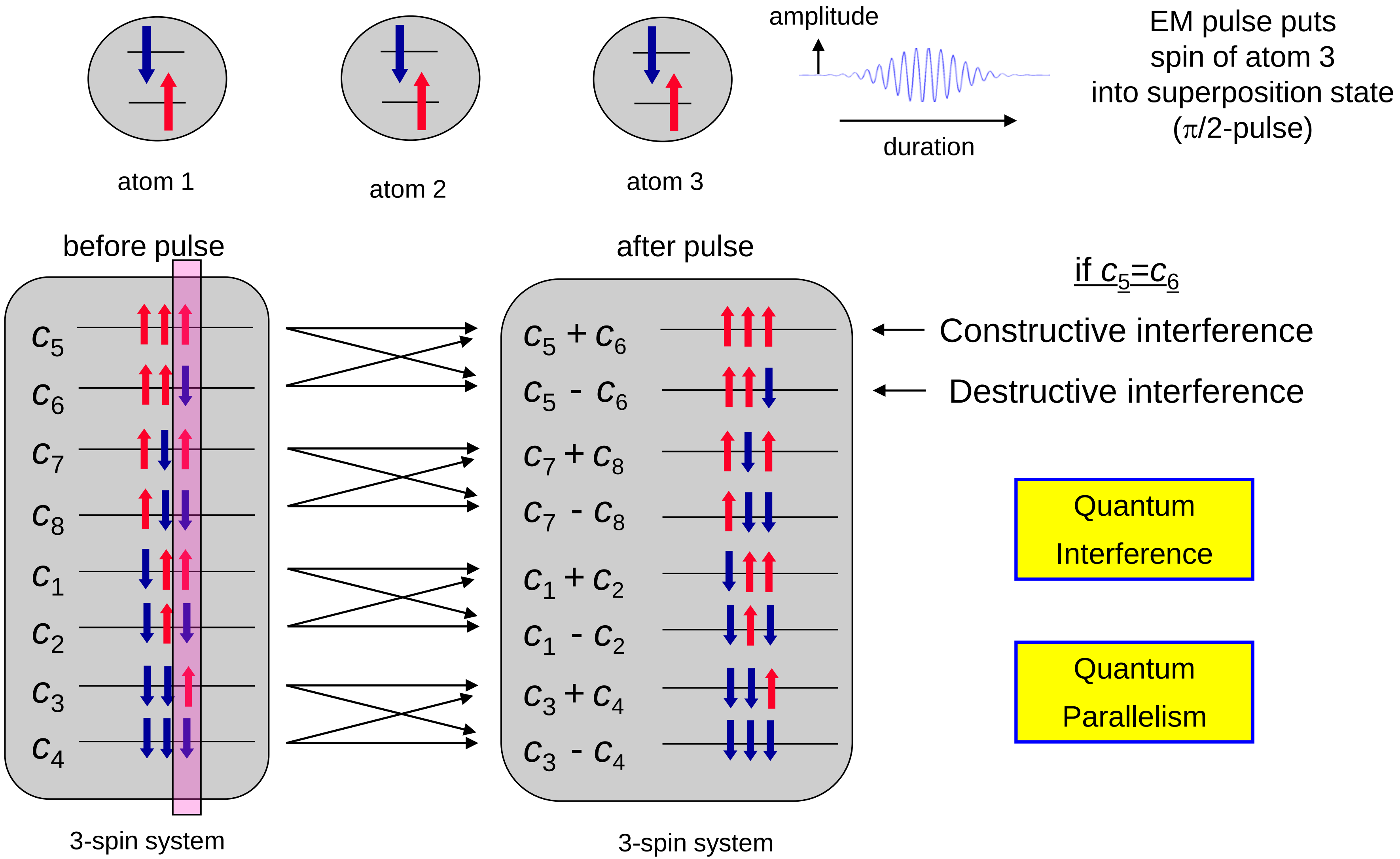
if $c_5 = c_6$

← Constructive interference

← Destructive interference

Quantum Interference

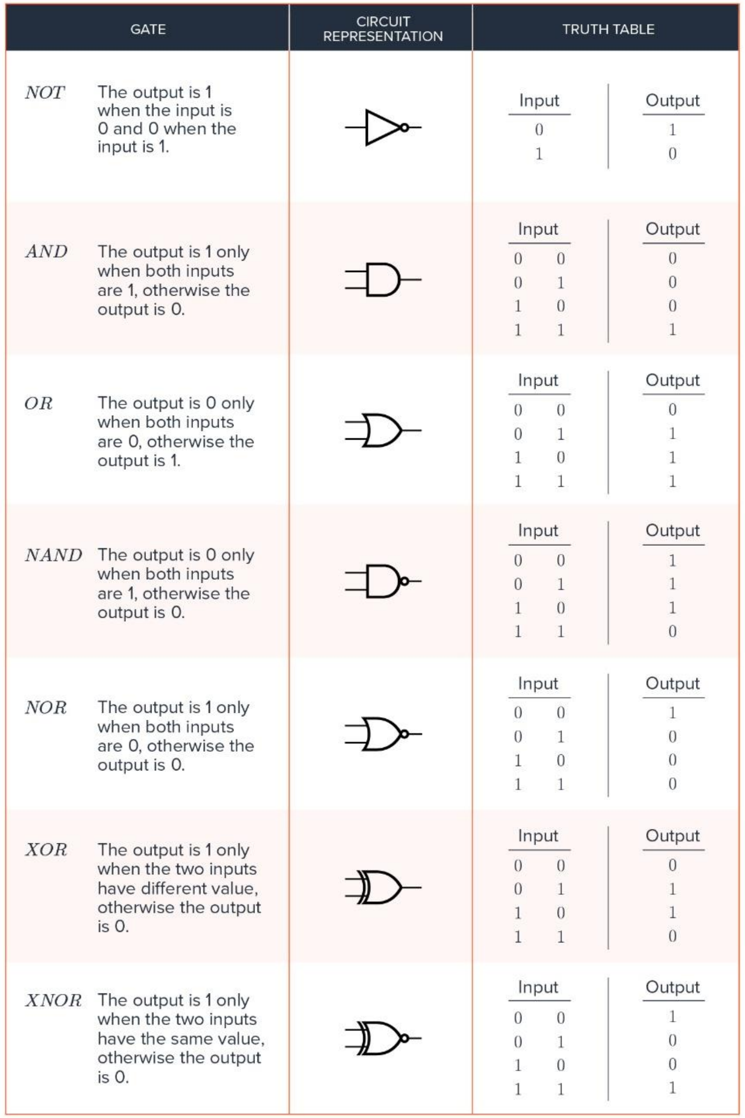
Quantum Interference



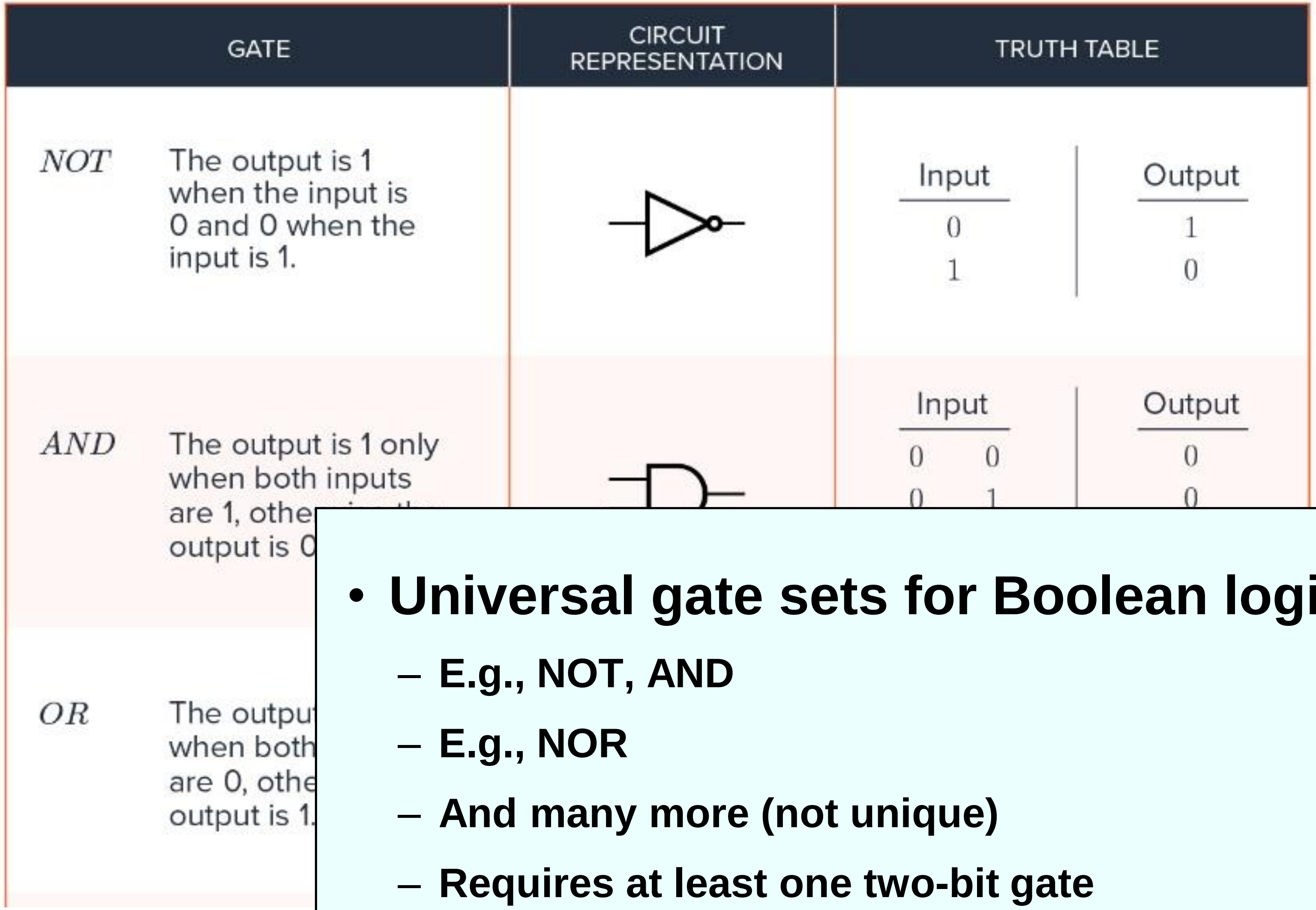
Quantum Interference

Quantum Parallelism

Classical Gates



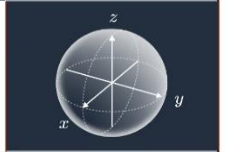
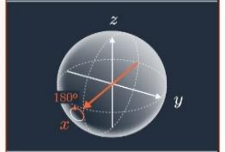
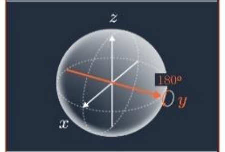
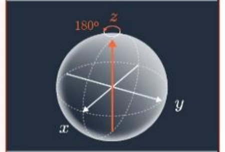
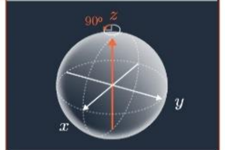
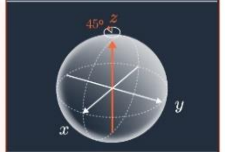
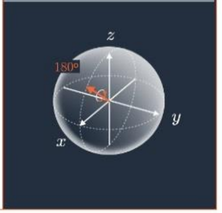
GATE		CIRCUIT REPRESENTATION	TRUTH TABLE	
NOT	The output is 1 when the input is 0 and 0 when the input is 1.		Input 0 1	Output 1 0
AND	The output is 1 only when both inputs are 1, otherwise the output is 0.		Input 0 0 0 1 1 0 1 1	Output 0 0 0 1
OR	The output is 0 only when both inputs are 0, otherwise the output is 1.		Input 0 0 0 1 1 0 1 1	Output 0 1 1 1
NAND	The output is 0 only when both inputs are 1, otherwise the output is 0.		Input 0 0 0 1 1 0 1 1	Output 1 1 1 0
NOR	The output is 1 only when both inputs are 0, otherwise the output is 0.		Input 0 0 0 1 1 0 1 1	Output 1 0 0 0
XOR	The output is 1 only when the two inputs have different value, otherwise the output is 0.		Input 0 0 0 1 1 0 1 1	Output 0 1 1 0
XNOR	The output is 1 only when the two inputs have the same value, otherwise the output is 0.		Input 0 0 0 1 1 0 1 1	Output 1 0 0 1

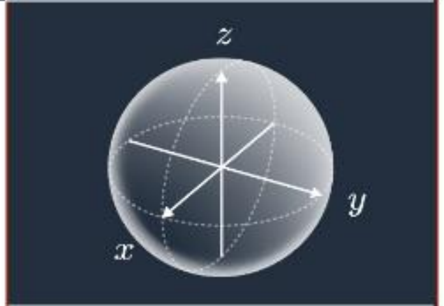
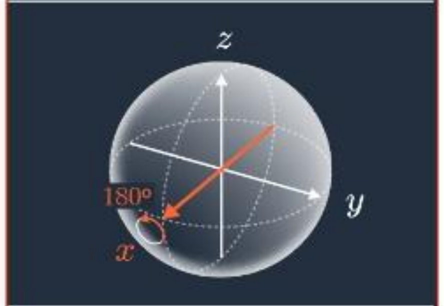
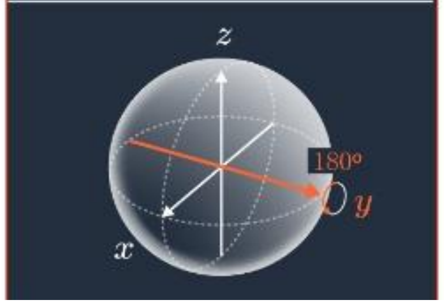


GATE		CIRCUIT REPRESENTATION	TRUTH TABLE	
NOT	The output is 1 when the input is 0 and 0 when the input is 1.		Input 0 1	Output 1 0
AND	The output is 1 only when both inputs are 1, othe… output is 0…		Input 0 0 0 1	Output 0 0
OR	The outpu… when both… are 0, othe… output is 1.			

- **Universal gate sets for Boolean logic**
  - **E.g., NOT, AND**
  - **E.g., NOR**
  - **And many more (not unique)**
  - **Requires at least one two-bit gate**

Single-Qubit Quantum Gates

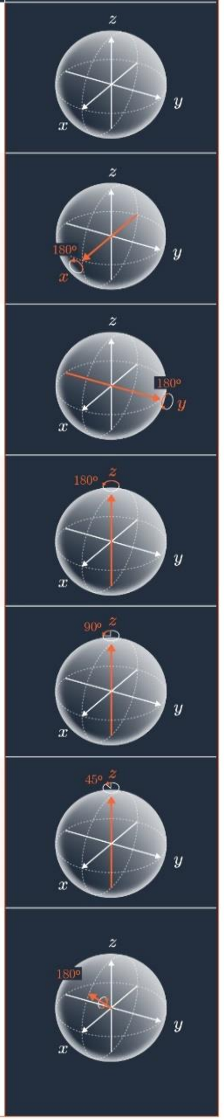
GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
I Identity-gate: no rotation is performed.	I	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|0\rangle$ $\|1\rangle$ / $\|1\rangle$	
X gate: rotates the qubit state by π radians (180º) about the x-axis.	X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|1\rangle$ $\|1\rangle$ / $\|0\rangle$	
Y gate: rotates the qubit state by π radians (180º) about the y-axis.	Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Input / Output $\|0\rangle$ / $i\|1\rangle$ $\|1\rangle$ / $-i\|0\rangle$	
Z gate: rotates the qubit state by π radians (180º) about the z-axis.	Z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|0\rangle$ $\|1\rangle$ / $-\|1\rangle$	
S gate: rotates the qubit state by $\frac{\pi}{2}$ radians (90º) about the z-axis.	S	$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|0\rangle$ $\|1\rangle$ / $e^{i\frac{\pi}{2}}\|1\rangle$	
T gate: rotates the qubit state by $\frac{\pi}{4}$ radians (45º) about the z-axis.	T	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|0\rangle$ $\|1\rangle$ / $e^{i\frac{\pi}{4}}\|1\rangle$	
H gate: rotates the qubit state by π radians (180º) about an axis diagonal in the x-z plane. This is equivalent to an X-gate followed by a $\frac{\pi}{2}$ rotation about the y-axis.	H	$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$ $\|1\rangle$ / $\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$	

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE	BLOCH SPHERE
I Identity-gate: no rotation is performed.	I	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|0\rangle$ $\|1\rangle$ / $\|1\rangle$	
X gate: rotates the qubit state by π radians (180º) about the x-axis.	X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Input / Output $\|0\rangle$ / $\|1\rangle$ $\|1\rangle$ / $\|0\rangle$	
Y gate: rotates the qubit state by π radians (180º) about the y-axis.	Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Input / Output $\|0\rangle$ / $i\|1\rangle$ $\|1\rangle$ / $-i\|0\rangle$	

Two-Qubit Quantum Gates

GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE Input	TRUTH TABLE Output
I Identity-gate: no rotation is performed.	I	$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\|0\rangle$ $\|1\rangle$
X gate: rotates the qubit state by π radians ($180°$) about the x-axis.	X	$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\|1\rangle$ $\|0\rangle$
Y gate: rotates the qubit state by π radians ($180°$) about the y-axis.	Y	$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$i\|1\rangle$ $-i\|0\rangle$
Z gate: rotates the qubit state by π radians ($180°$) about the z-axis.	Z	$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\|0\rangle$ $-\|1\rangle$
S gate: rotates the qubit state by $\frac{\pi}{2}$ radians ($90°$) about the z-axis.	S	$S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{2}} \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\|0\rangle$ $e^{i\frac{\pi}{2}}\|1\rangle$
T gate: rotates the qubit state by $\frac{\pi}{4}$ radians ($45°$) about the z-axis.	T	$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\|0\rangle$ $e^{i\frac{\pi}{4}}\|1\rangle$
H gate: rotates the qubit state by π radians ($180°$) about an axis diagonal in the x-z plane. This is equivalent to an X-gate followed by a $\frac{\pi}{2}$ rotation about the y-axis.	H	$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\|0\rangle$ $\|1\rangle$	$\frac{\|0\rangle + \|1\rangle}{\sqrt{2}}$ $\frac{\|0\rangle - \|1\rangle}{\sqrt{2}}$

BLOCH SPHERE

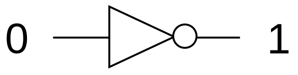


GATE	CIRCUIT REPRESENTATION	MATRIX REPRESENTATION	TRUTH TABLE Input	TRUTH TABLE Output
Controlled-NOT gate: apply an X-gate to the target qubit if the control qubit is in state $\|1\rangle$		$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$	$\|00\rangle$ $\|01\rangle$ $\|10\rangle$ $\|11\rangle$	$\|00\rangle$ $\|01\rangle$ $\|11\rangle$ $\|10\rangle$
Controlled-phase gate: apply a Z-gate to the target qubit if the control qubit is in state $\|1\rangle$	Z	$cZ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$	$\|00\rangle$ $\|01\rangle$ $\|10\rangle$ $\|11\rangle$	$\|00\rangle$ $\|01\rangle$ $\|10\rangle$ $-\|11\rangle$

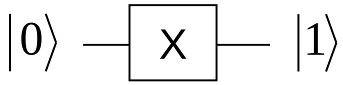
- **Universal gate sets for quantum logic**
  - **E.g., H, S, T, CNOT**
  - **And many more (not unique)**
  - **Requires at least one two-qubit entangling gate**

Single-Qubit Gate Example

Classical NOT-gate

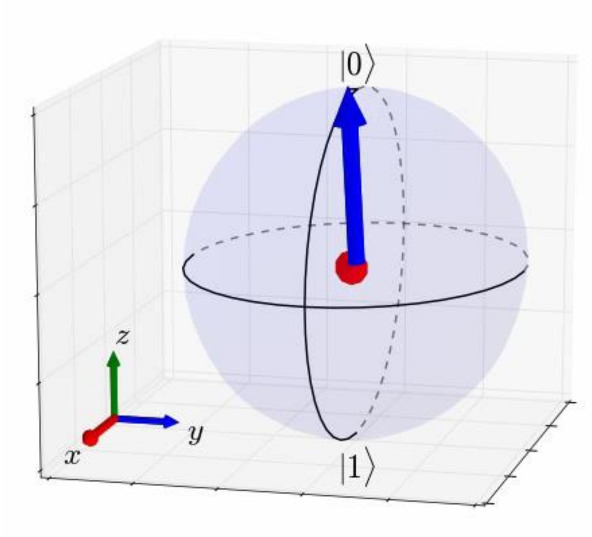


Quantum NOT-gate example: X-gate

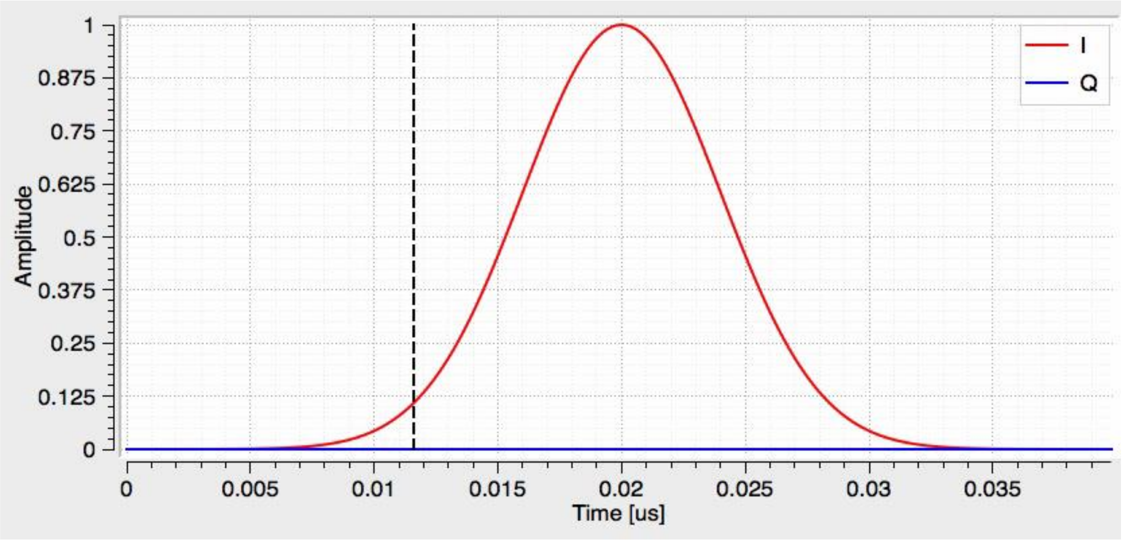


X-gate: π-pulse around x-axis

Bloch Sphere



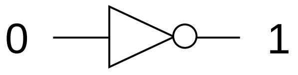
Driving Field (envelope only)



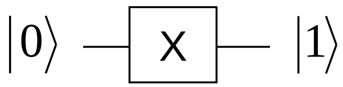
X-gate applied to qubit along +Z: $|0\rangle \rightarrow |1\rangle$

Single-Qubit Gate Example

Classical NOT-gate

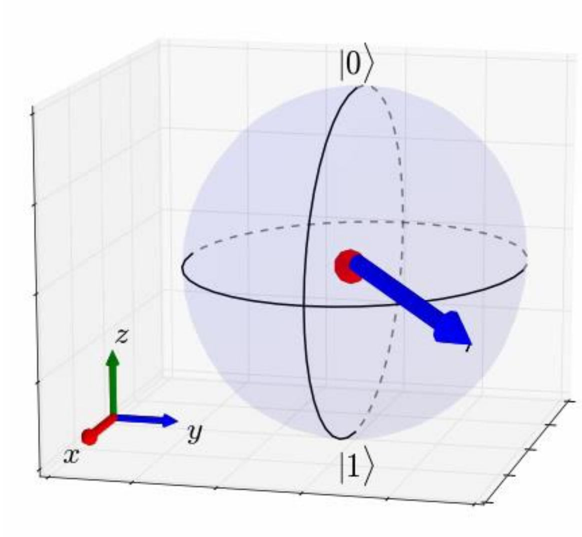


Quantum NOT-gate: X-gate

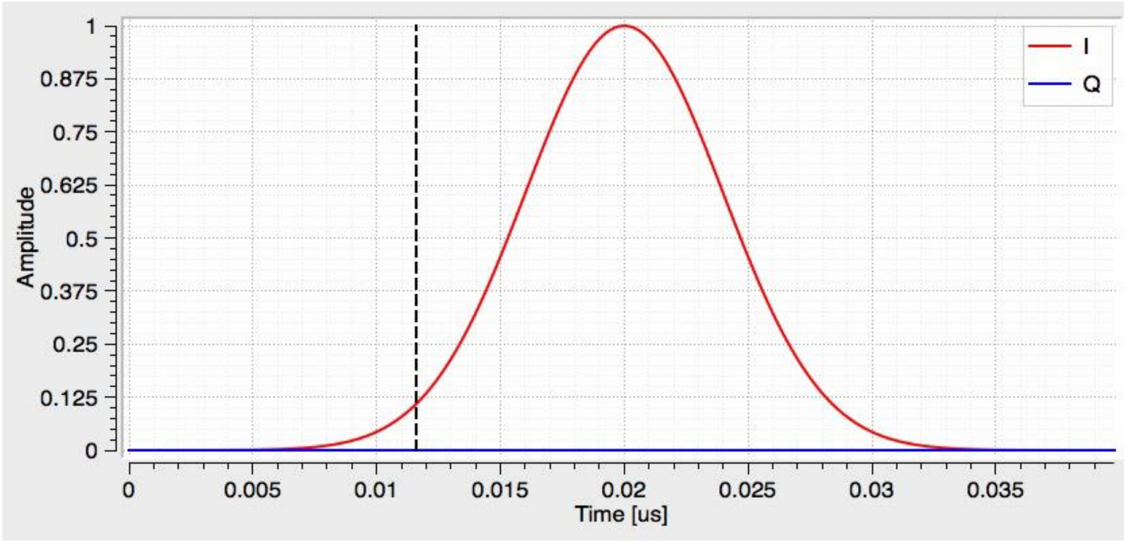


X-gate: π-pulse around x-axis

Bloch Sphere



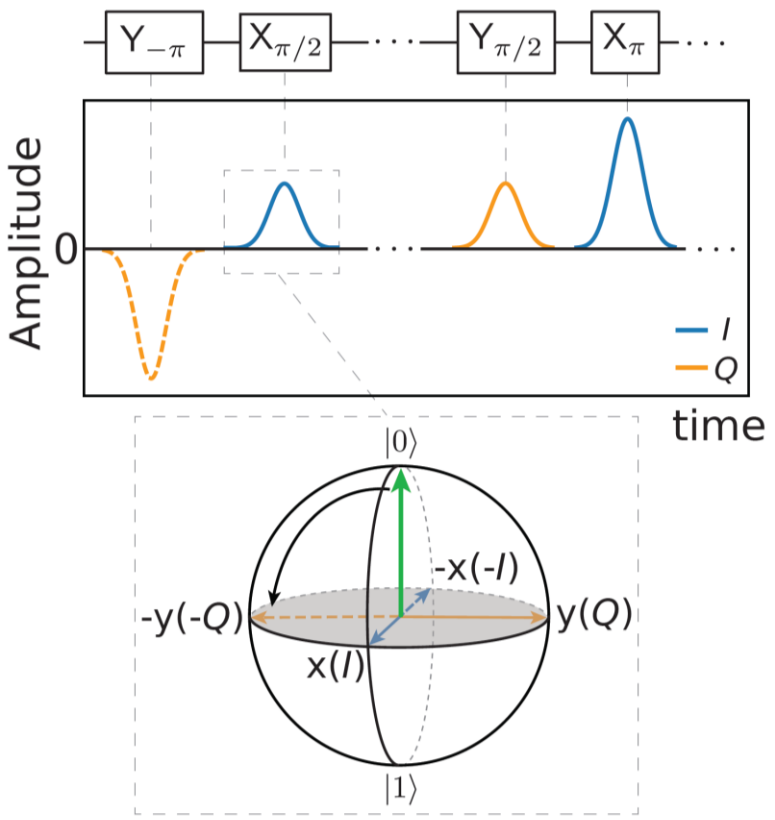
Driving Field (envelope only)



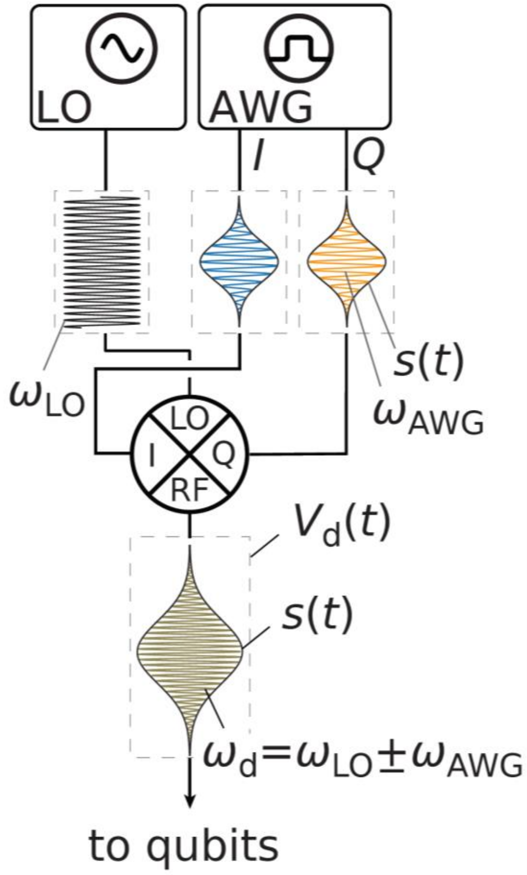
X-gate applied to arbitrary qubit state: $\alpha|0\rangle + \beta|1\rangle \rightarrow \beta|0\rangle + \alpha|1\rangle$

Microwave Pulse Control

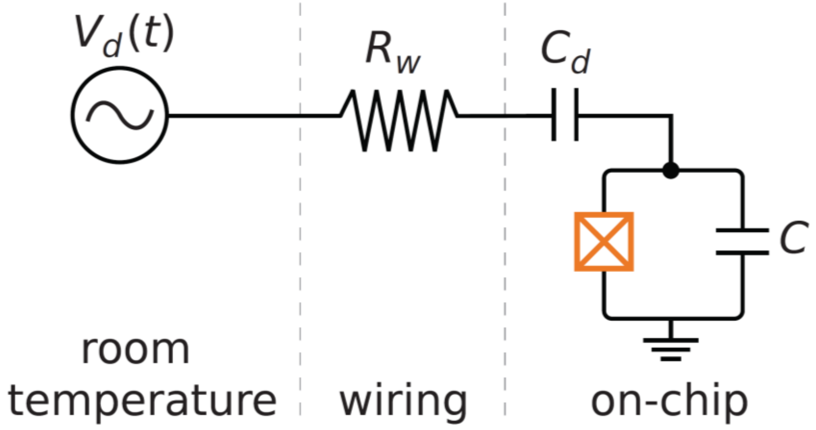
Gate Sequence



I-Q Mixing



Application

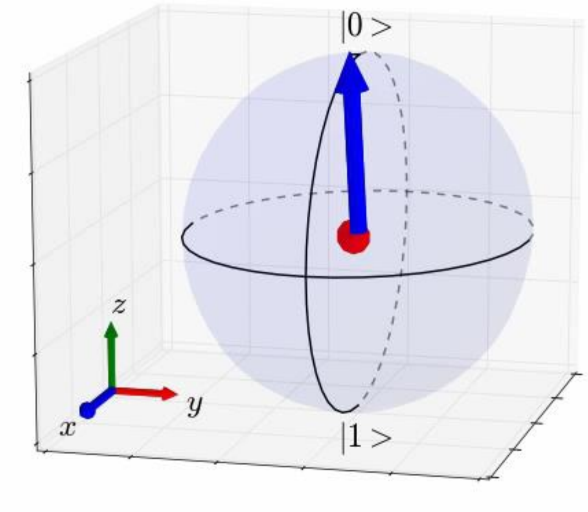


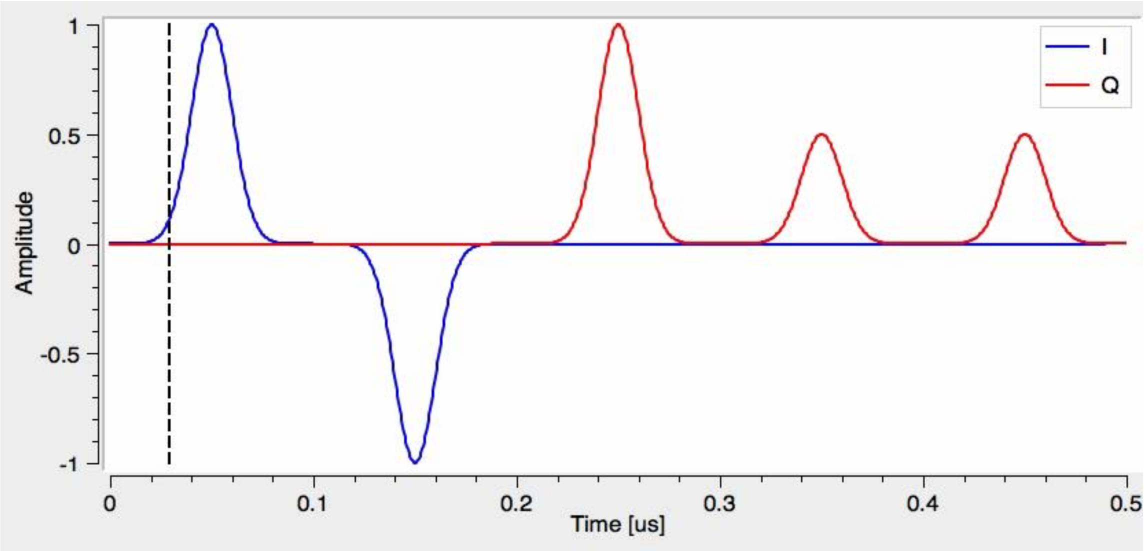
Control applied via capacitive or inductive coupling of a microwave pulse to the qubit

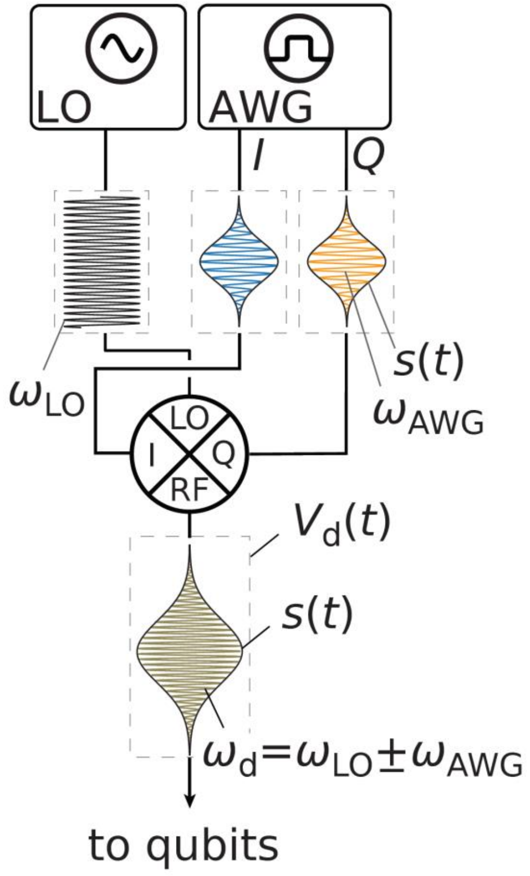
Microwave Pulse Control

X and Y Rotations on the Bloch Sphere

I-Q Mixing







I: in-phase (0^0) → x axis

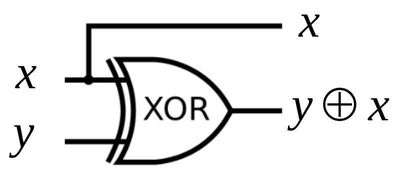
Q: quadrature (90^0) → y axis

Two-Qubit Gate Example

Classical XOR-gate

"control" bit

"target" bit

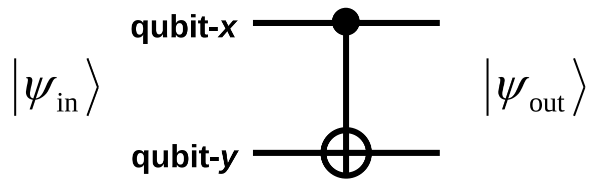


input x	input y	output x	output $y \oplus x$
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Quantum CNOT-gate

"control" qubit

"target" qubit



input $|x\rangle_x$	input $|y\rangle_y$	output $|x\rangle_x$	output $|y \oplus x\rangle_y$
$|0\rangle_x$	$|0\rangle_y$	$|0\rangle_x$	$|0\rangle_y$
$|0\rangle_x$	$|1\rangle_y$	$|0\rangle_x$	$|1\rangle_y$
$|1\rangle_x$	$|0\rangle_y$	$|1\rangle_x$	$|1\rangle_y$
$|1\rangle_x$	$|1\rangle_y$	$|1\rangle_x$	$|0\rangle_y$

Rotation of qubit-y depends on the state of qubit-x

For example:

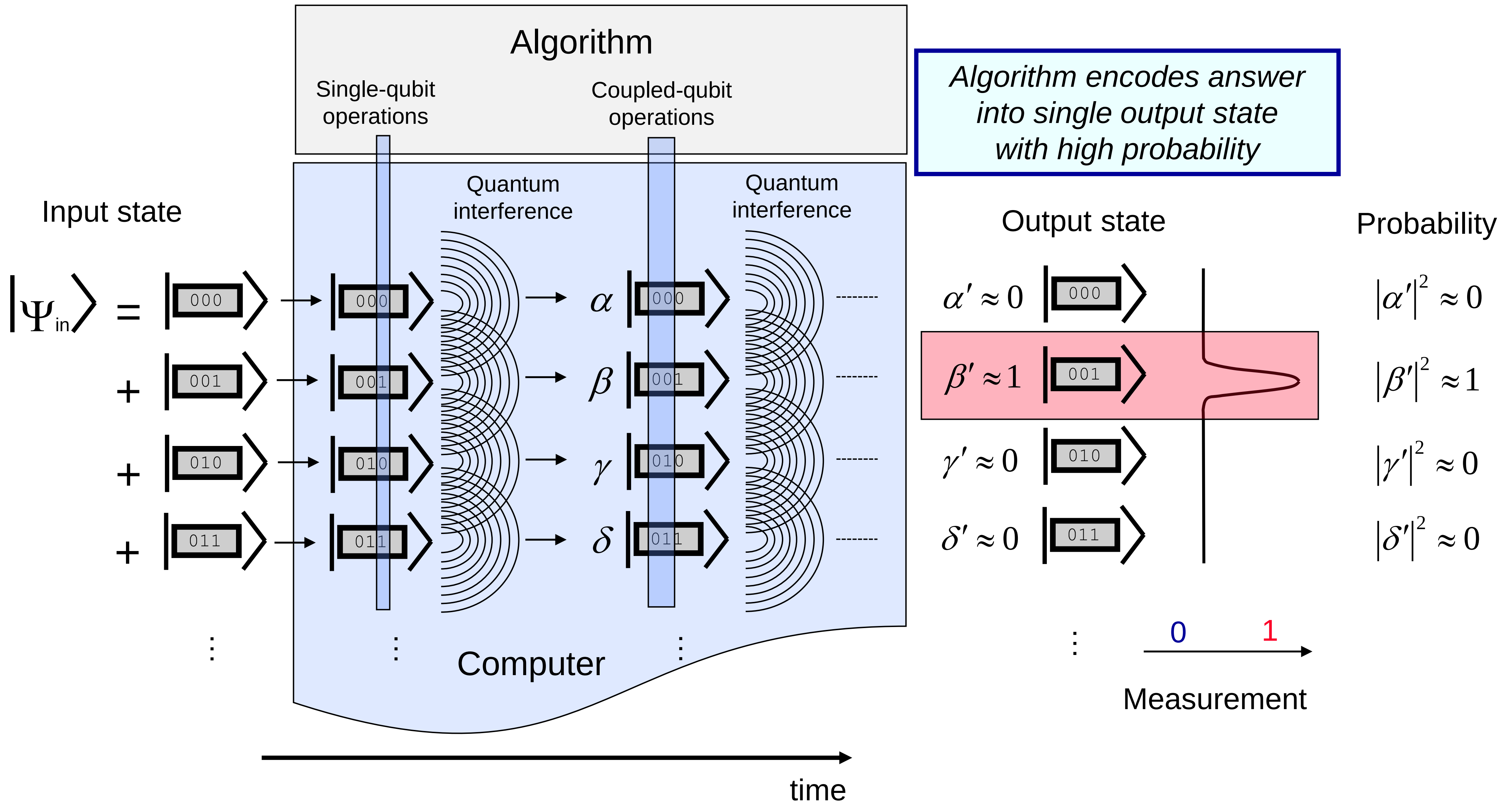
$$|\psi_{\text{in}}\rangle \propto \left(|0\rangle + |1\rangle\right)_{\text{x}} |0\rangle_{\text{y}}$$

$$|\psi_{\text{out}}\rangle \propto |0\rangle_{\text{x}} |0\rangle_{\text{y}} + |1\rangle_{\text{x}} |1\rangle_{\text{y}} \neq (\cdots)_x (\cdots)_y$$

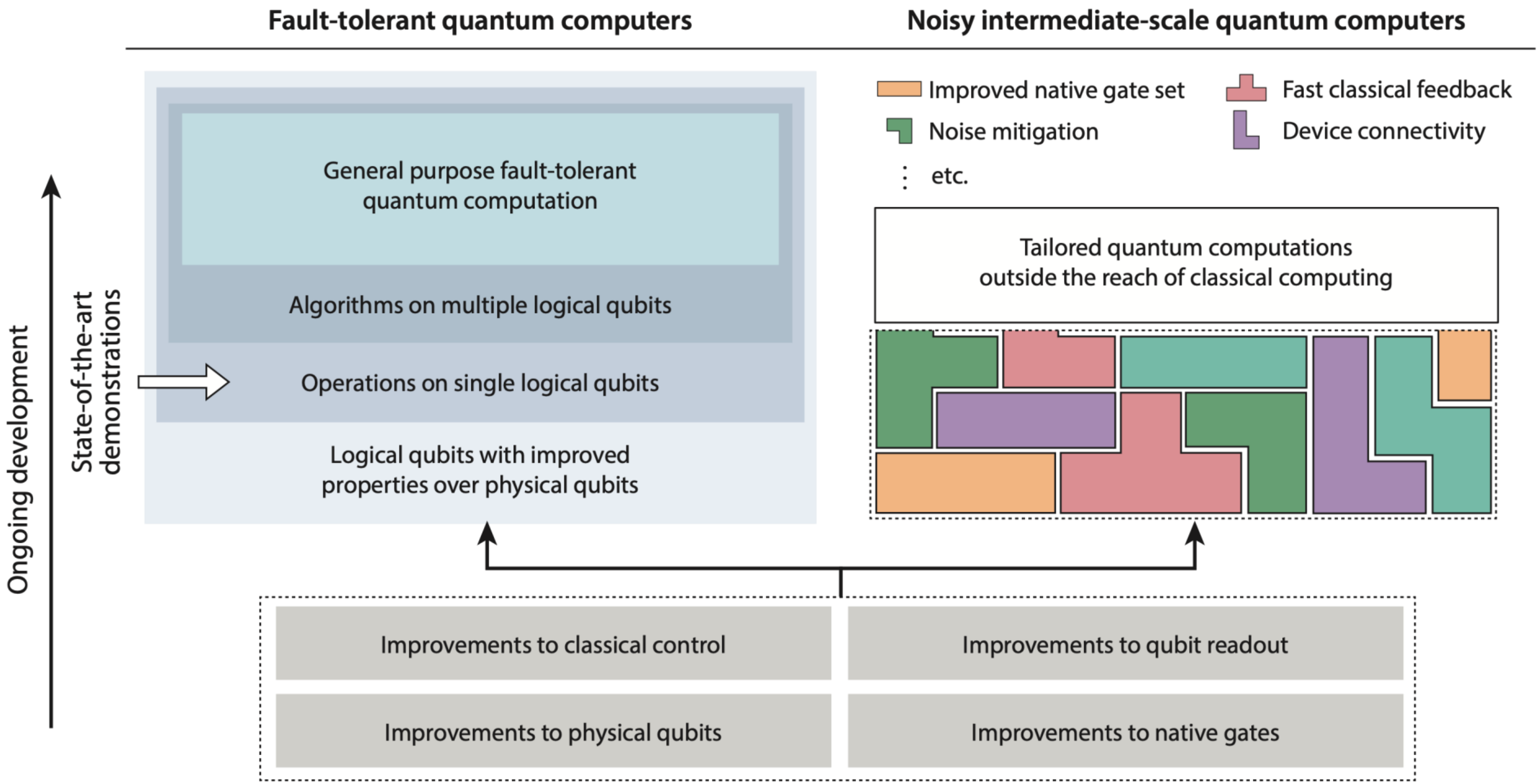
Results in an *entangled state* (cannot be factored)

Universal gate-model quantum computation is achievable with a small set of single and two-qubit gates.

Quantum Algorithm

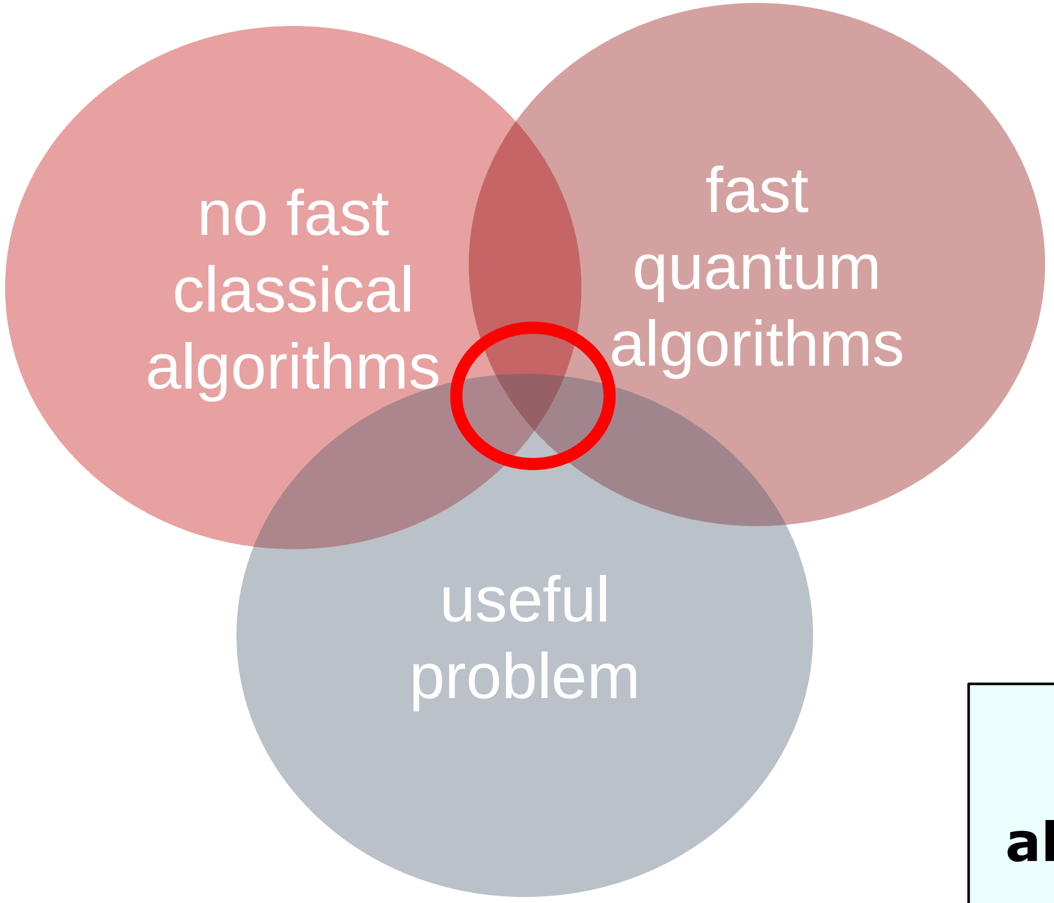


Paths to Applications



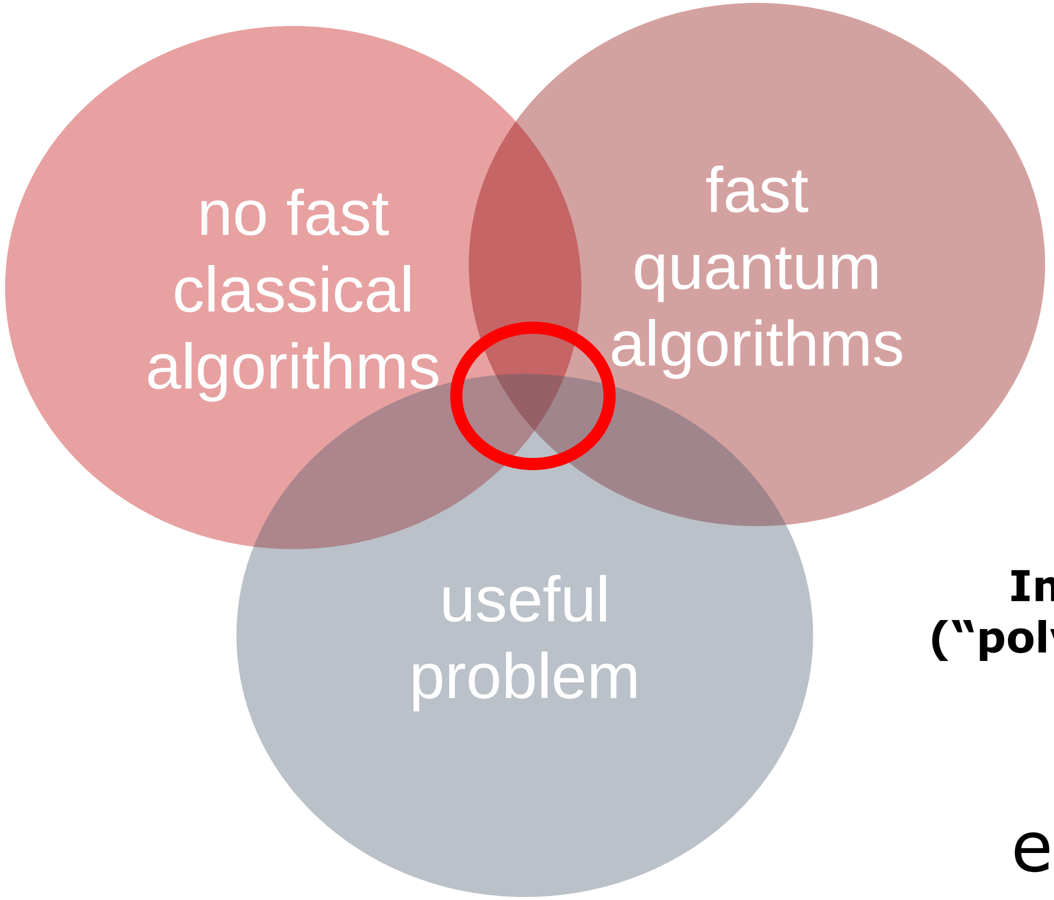
M. Kjaergaard, WDO, et al., Annual Reviews of CMP 11, 369-395 (2020)

Commercial Quantum Advantage



Small region where useful quantum algorithms exist (as we know them today)

Types of Quantum Advantage



Two Types of Quantum Advantage

- System size,
- Time to solution,
- Other resources

$$\propto A(N)\exp(\beta N)$$

Improve the prefactor ("polynomial improvement")

Reduce exp. to polynomial ("exponential improvement")

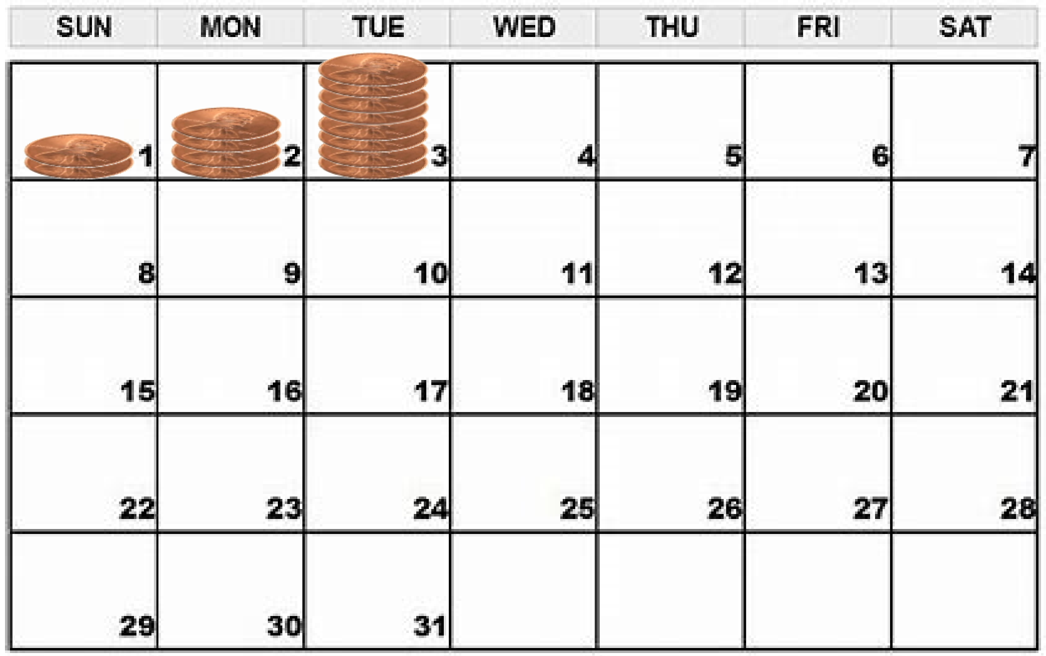
e.g., $N \rightarrow N^{1/2}$

e.g., $2^N \rightarrow N^3$

Exponential Growth

Exponential Growth:
Doubling Pennies Every Day for 1 Month

2^0 = 1 penny



2^1 = 2 pennies
2^2 = 4 pennies
2^3 = 8 pennies

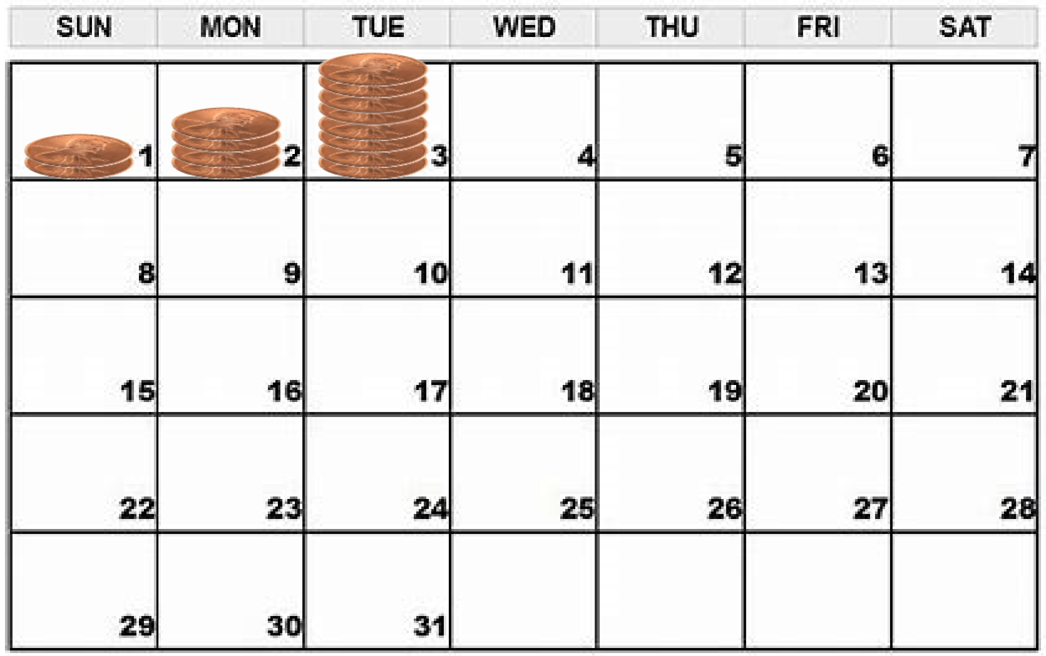
⋮

After 31 days, would you take the pennies or $10M?

Exponential Growth

Exponential Growth:
Doubling Pennies Every Day for 1 Month

2^0 = 1 penny



2^1 = 2 pennies
2^2 = 4 pennies
2^3 = 8 pennies

⋮

2^{31} = 2,147,483,648 pennies > $21M !!

Exponential Power

- **Simulating quantum computers (QCs) on classical computers**

Qubits	Size of simulator
30	laptop

Exponential Power

- **Simulating quantum computers (QCs) on classical computers**

Qubits	Size of simulator
30	laptop
50	supercomputer

Exponential Power

- **Simulating quantum computers (QCs) on classical computers**

Qubits	Size of simulator
30	laptop
50	supercomputer
80	all computers on Earth

Exponential Power

- **Simulating quantum computers (QCs) on classical computers**

Qubits	Size of simulator
30	laptop
50	supercomputer
80	all computers on Earth
160	all Si atoms in Earth

Exponential Power

- **Simulating quantum computers (QCs) on classical computers**

Qubits	Size of simulator
30	laptop
50	supercomputer
80	all computers on Earth
160	all Si atoms in Earth
300	> all atoms than in known universe

Digital Quantum Algorithms

Algorithm	Classical Time	Quantum Time	Speedup	Limitation
Simulation[1] (quantum chemistry)	2^N (for N atoms)	N^c	Exp. in space, polynomial in time	Mapping problem to qubits
Factoring[2] (+ related number theoretic)	2^N (for N digits)	N^3	Exponential	Classical runtime limit unproven
Linear systems[3] (Ax=b)	2^N (for N digits)	~N	Exponential	Strict conditions, e.g. sparse matrix
Optimization[4]	2^N	?	?	Empirical
Search[5] (unsorted / unstructured data)	N	$\sqrt{N}$	Polynomial ($\sqrt{N}$)	Data loading


Anand Natarjan
EECS


Ike Chuang
EECS


Seth Lloyd[1,3]
Mech. Eng.


Peter Shor[2]
Math


Aram Harrow[3]
Physics


Eddie Farhi[4]
Physics, Google


Michael Sipser[4]
Math

DiVincenzo Criteria

Minimum requirements for the physical implementation of a quantum computer

D1: Robust, reproducible qubit technology

D2: Initialization

D3: Measurement

D4: Universal set of gates

D5: Coherence & fidelity

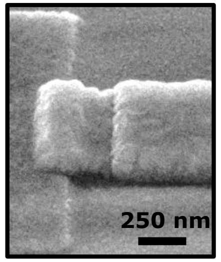
Dedicated Superconducting Qubit Fab

- **200-mm wafers & 50-mm wafers**
- **Qubits and classical digital electronics**
- **Deposition, dry etch, PECVD, CMP**
- **Unique facility worldwide**

WDO, et al., 2006 - 2016

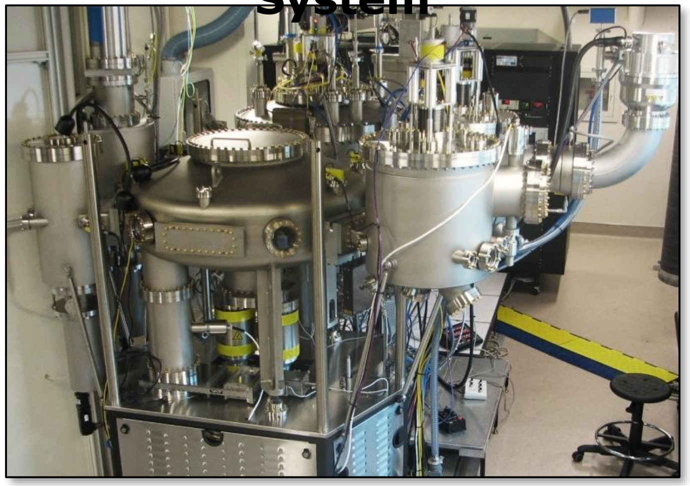
Custom Plassys Evaporator

Electron Beam Lithography



MIT-LL Raith EBPG5200 routinely patterns <150 nm Josephson junctions

Veeco Gen-200 MBE System



DiVincenzo Criteria

Minimum requirements for the physical implementation of a quantum computer

D1: Robust, reproducible qubit technology

D2: Initialization

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D4: Universal set of gates

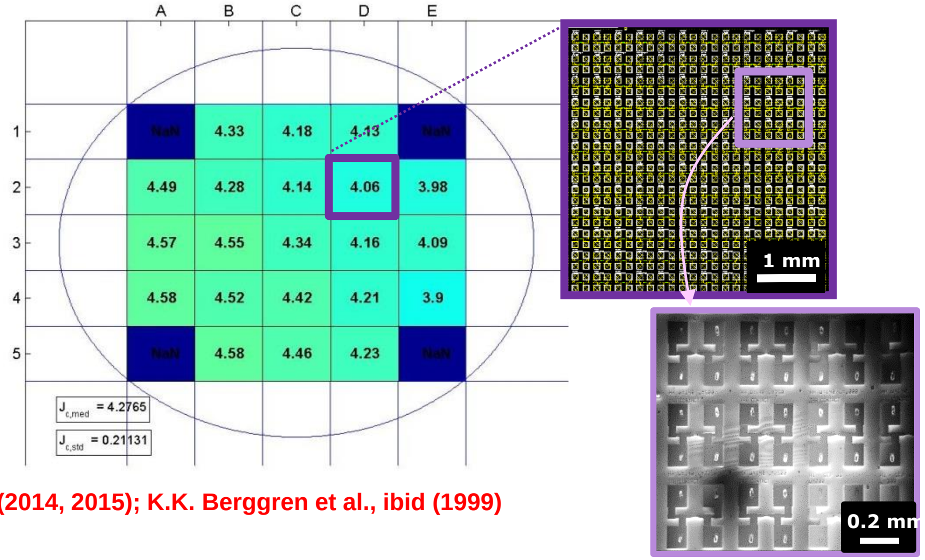
D5: Coherence & fidelity

Fabrication Process Monitoring

- **Data-driven process development**
- **>1000-10,000 test structures (50-200 mm wafers)**
- **JJs, lines, combs & snakes, contacts, crossovers, chains, …**
- **Automated testing and analysis**

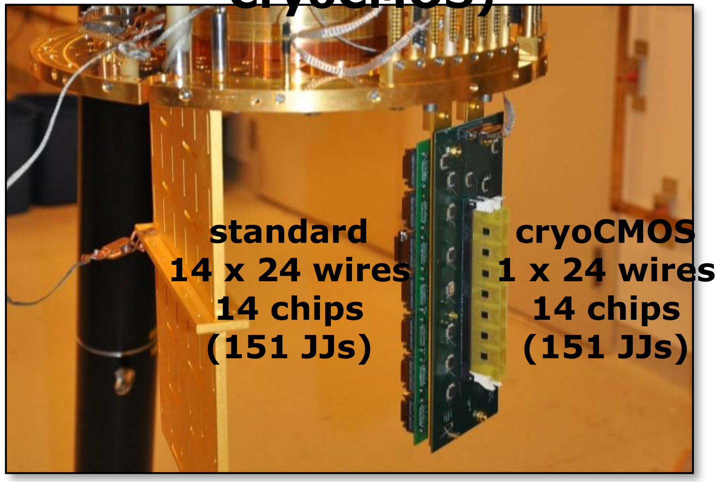
Room-Temp Probe Station

Cross-Wafer Variation Maps



S. Tolpygo, …, WDO, IEEE Appl. Supercond. (2014, 2015); K.K. Berggren et al., ibid (1999)

Cryogenic Testing (w/ CryoCMOS)



WDO, et al., unpublished (2006)

DiVincenzo Criteria

Minimum requirements for the physical implementation of a quantum computer

D1: Robust, reproducible qubit technology

D2: Initialization

D3: Measurement

D4: Universal set of gates

D5: Coherence & fidelity

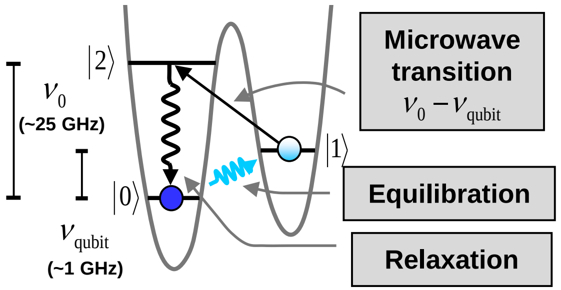
High-fidelity (99.9%) state initialization

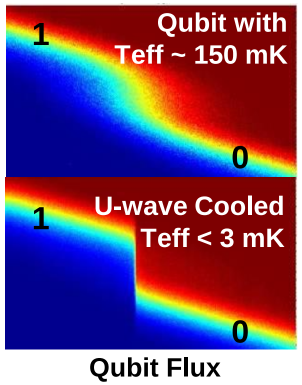
- **Microwave cooling (active)**
- **Cryogenic engineering (passive)**
- **Active measurement-based feedback**

Microwave Cooling

WDO et al., Science 310, 1653 (2005); Science 310, 1589 (2006); Nature 455, 51 (2008)

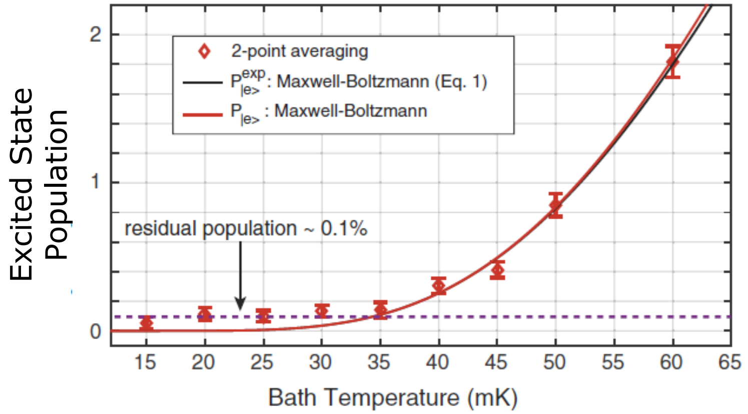
Superconducting qubit ("artificial atom") cooled 100x below refrigerator temperature





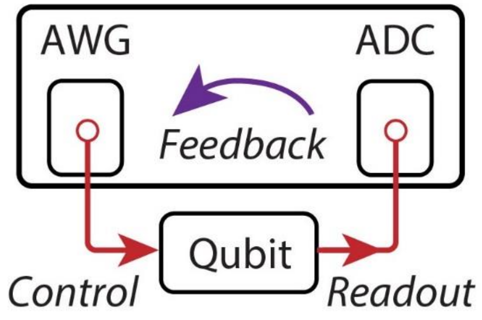
Cryogenic Engineering

X. Jin, …, WDO, PRL 114, 240501 (2015)



Active Feedback

A. Greene, …, WDO, APS MM (2018)



DiVincenzo Criteria

Minimum requirements for the physical implementation of a quantum computer

D1: Robust, reproducible qubit technology

D2: Initialization

D3: Measurement

D4: Universal set of gates

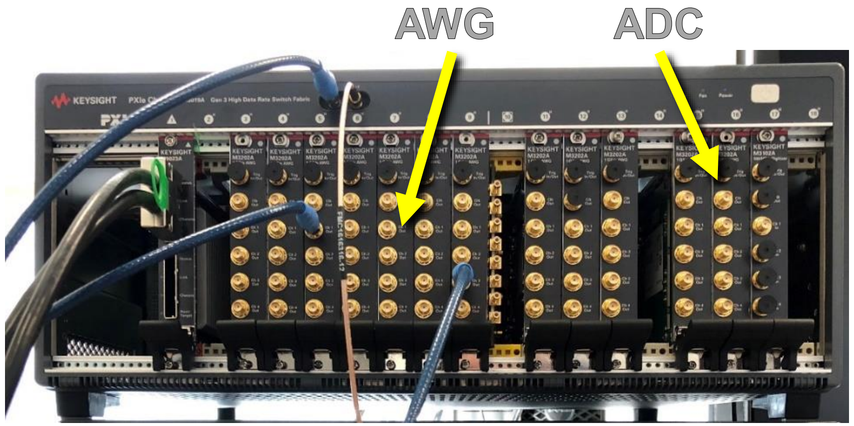
D5: Coherence & fidelity

High-fidelity (99%) measurement

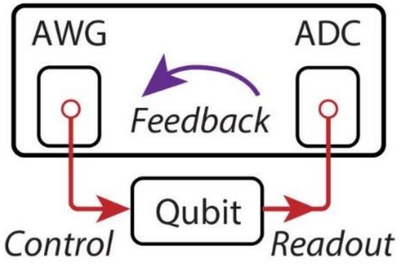
- **Control electronics and software**
- **Syndrome measurement and feedback**
- **Error detection and correction**

Control electronics, software, and quantum-limited amplifiers high-fidelity measurement of error syndromes

Engineering Quantum Systems Group and IARPA QEO Program (2015-2021)



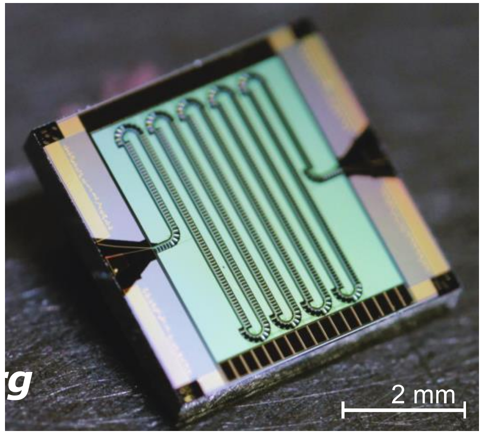
KEYSIGHT TECHNOLOGIES



Gustavsson, Krantz, Hover, and WDO

http://quantum.labbe[...]g

Macklin, WDO, et al., Science (2015)



DiVincenzo Criteria

Minimum requirements for the physical implementation of a quantum computer

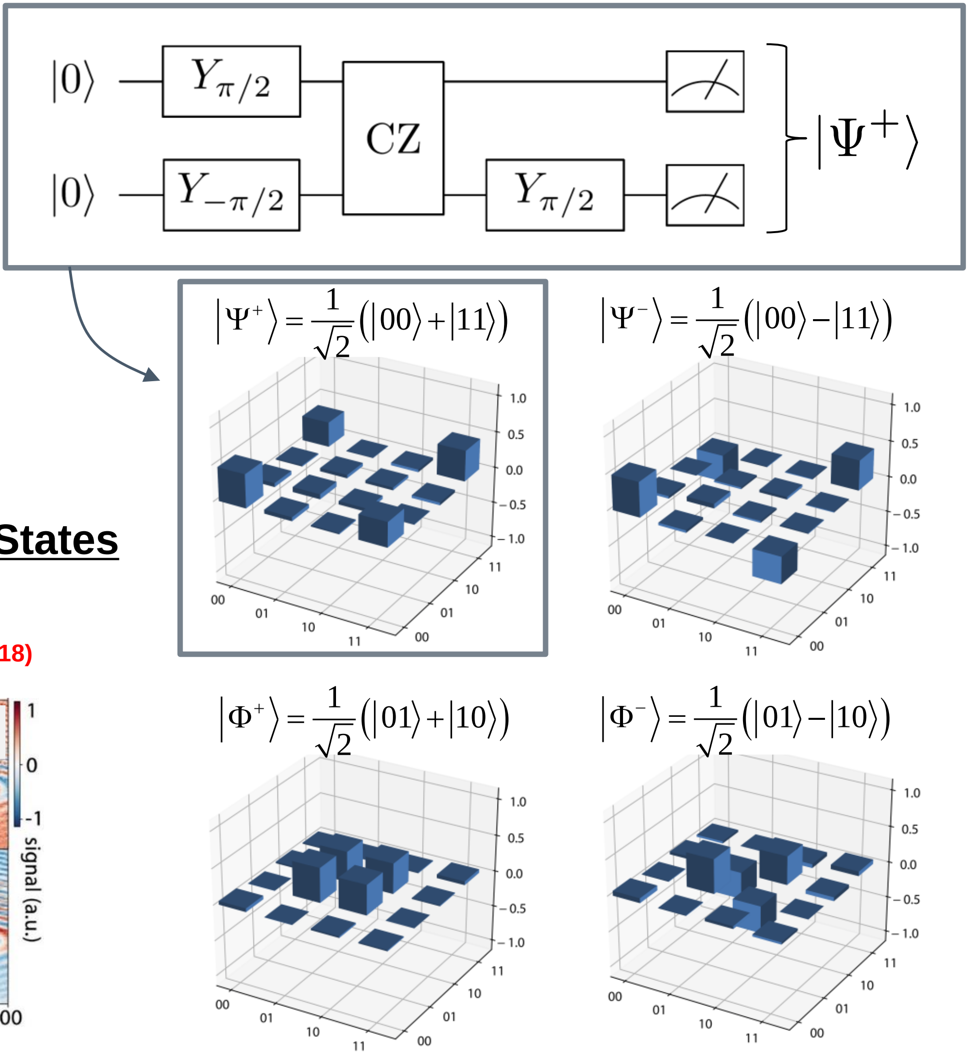
D1: Robust, reproducible qubit technology

D2: Initialization

D3: Measurement

D4: Universal set of gates

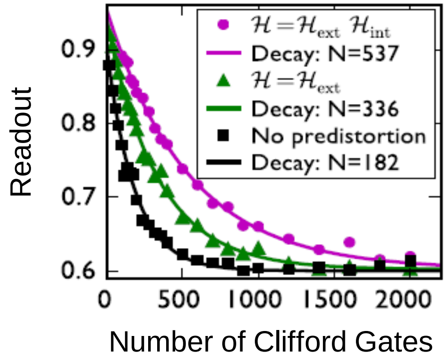
D5: Coherence & fidelity



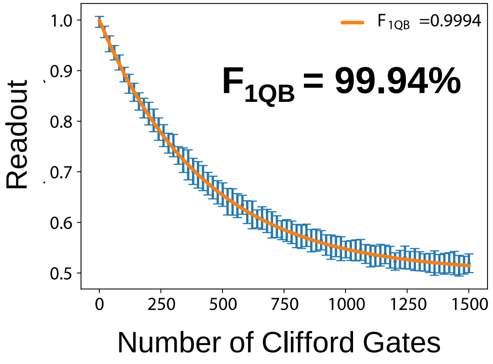
<u>Single-Qubit Gates</u>

<u>Two-Qubit Gates & Bell States</u>

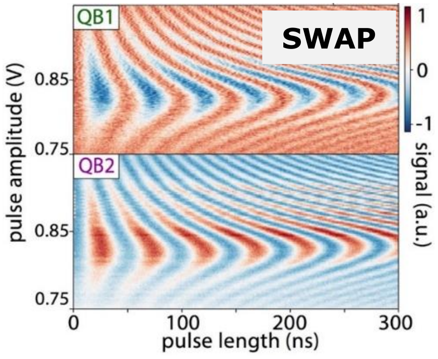
S. Gustavsson, ..., WDO, PRL 110, 040502 (2013)



EQuS Team, ..., WDO (2018)

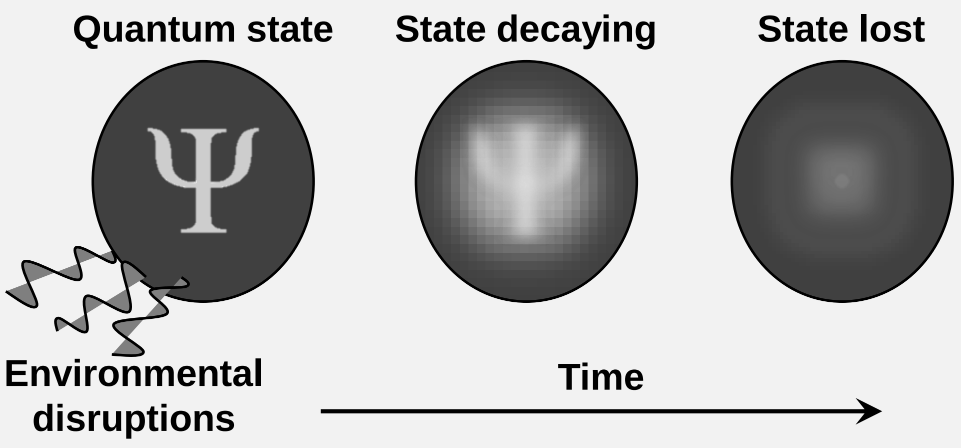


EQuS Team, ..., WDO (2016-2018)



Coherence Time and Gate Time

Coherence time t_{coh}: The qubit's lifetime



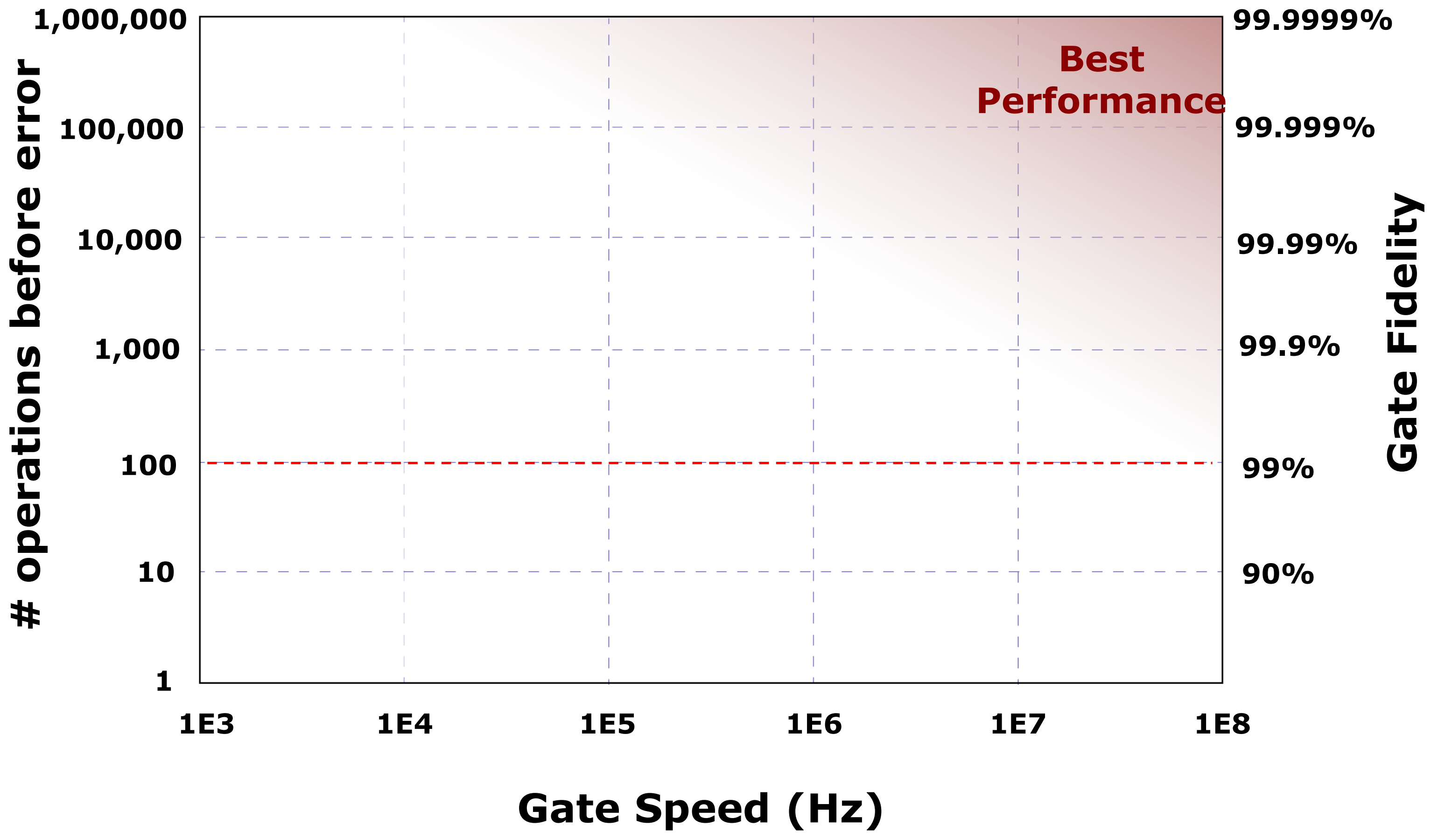
Gate time t_{gate}: Time required for a single gate operation

Figure of Merit * : # of gates per coherence time = t_{coh}/t_{gate}

(* Rigorous metric: gate & readout fidelity)

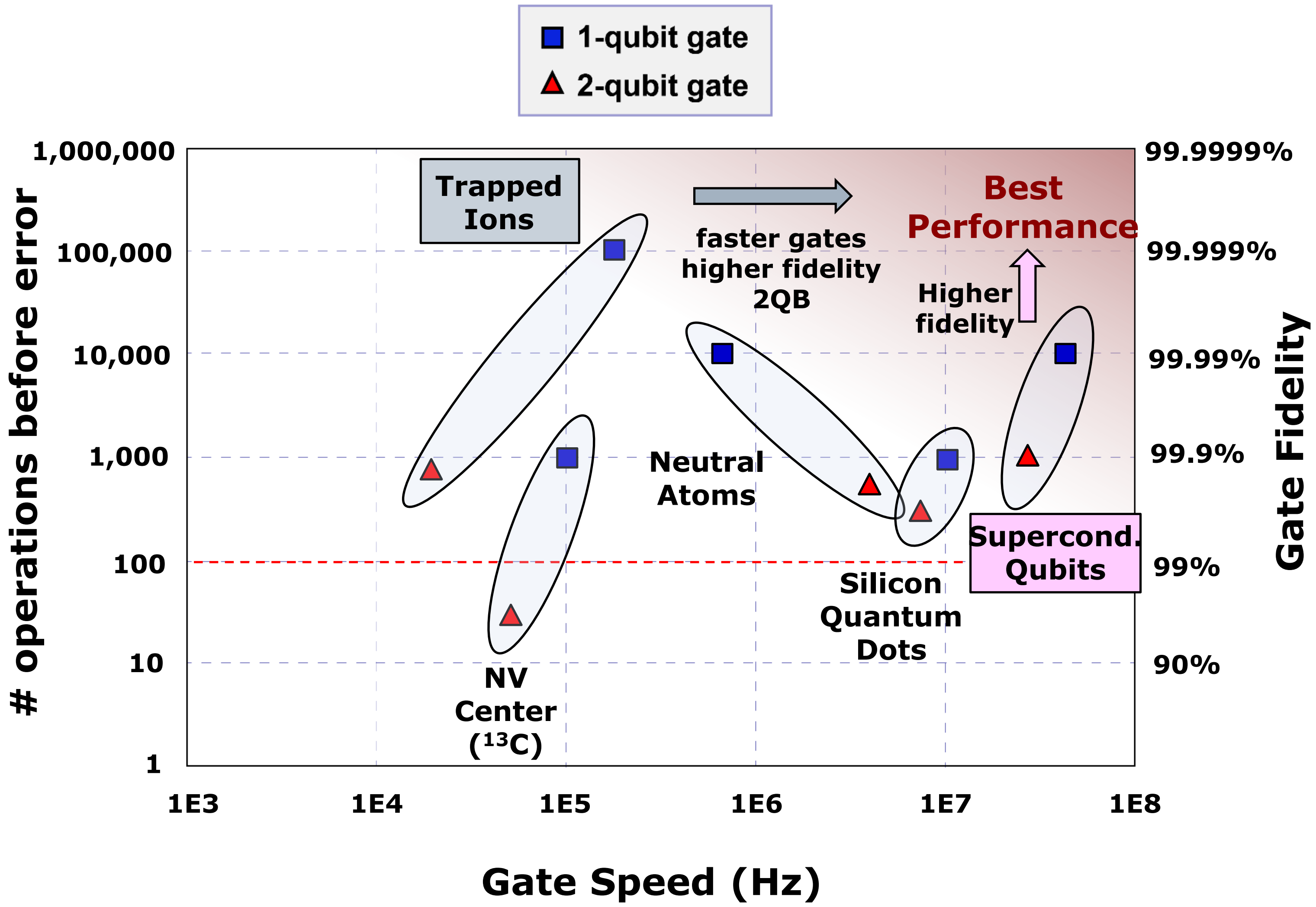
Long coherence times are not sufficient, it's the number of gates before an error

Qubit Modalities



Thanks to: P. Cappellaro, J. Chiaverini, D. Englund, T. Ladd, A. Morello, J. Petta, M. Saffman, J. Sage, D. Bluvstein

Qubit Modalities



MIT Campus **MIT Lincoln Lab**

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LL, RLE

Will Oliver
EECS, Phys., LL

Kevin O'Brien
EECS

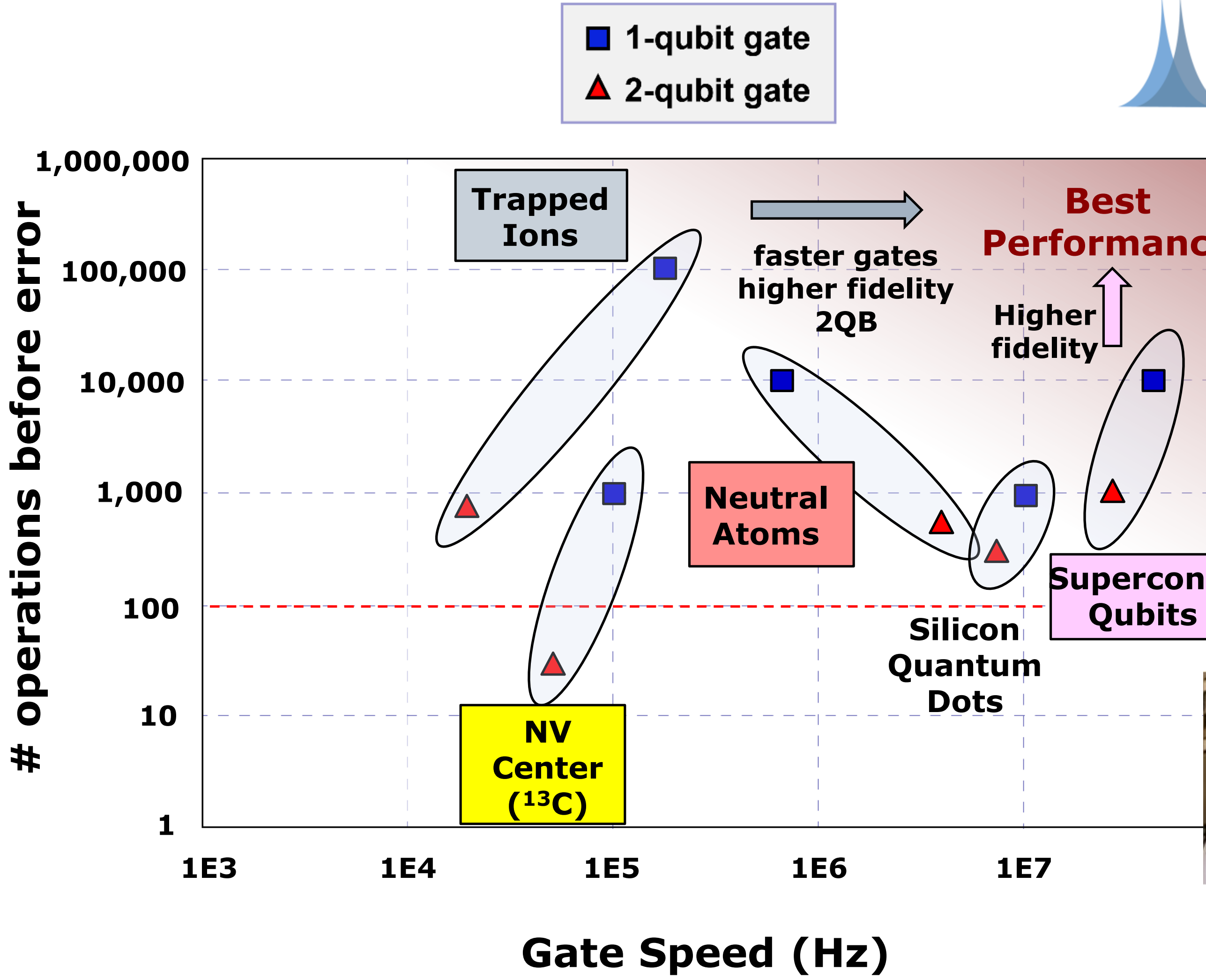
Terry Orlando
EECS

Jamie Kerman
LL

<u>and large teams at MIT & LL</u>

Thanks to: P. Cappellaro, J. Chiaverini, D. Englund, T. Ladd, A. Morello, J. Petta, M. Saffman, J. Sage, D. Bluvstein

Qubit Modalities



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Danielle Braje
QuIIN

Many outstanding research efforts in quantum at MIT campus and Lincoln Lab. Plus many collaborators not shown.

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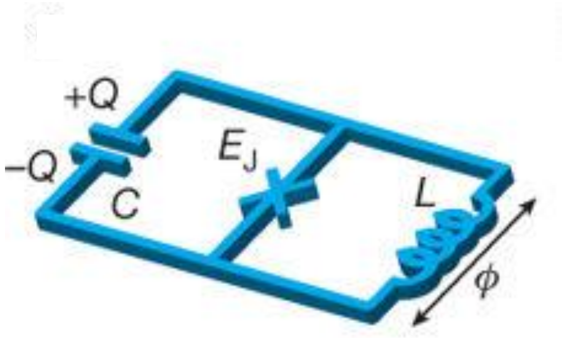
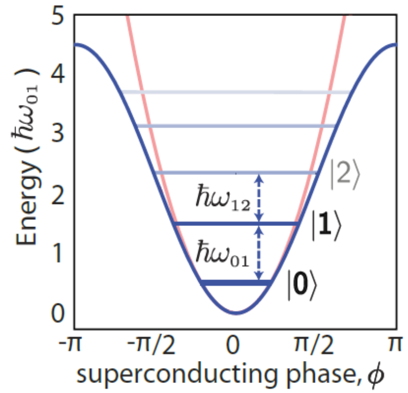
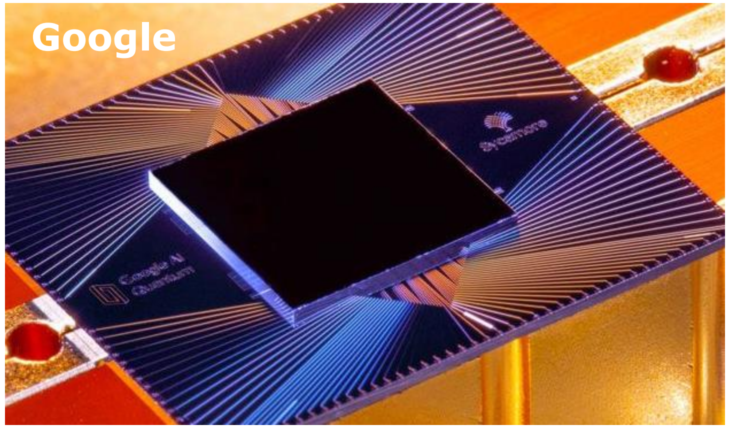
Artificial Atom: Superconducting Qubits

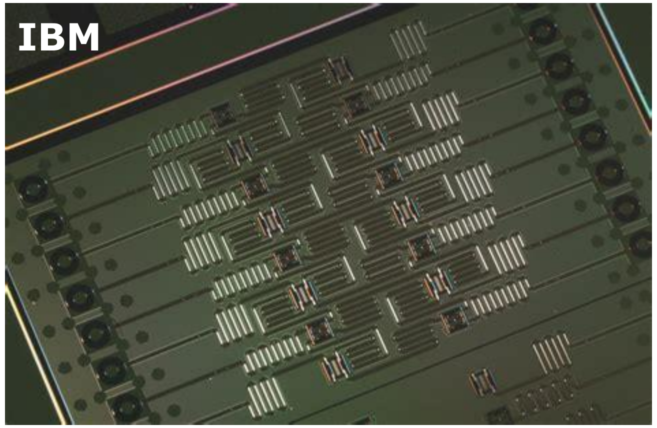
Electrical Circuit -- Anharmonic Oscillator

- Qubit: superconducting circuit
  - Phase, flux, or charge
- Coherence times: ~ 100 us
- Fidelity and operation times
  - 1 QB: 99.99% in 10 ns
  - 2 QB: 99.9% in 40 ns
  - Readout: 99.0% in 200 ns
- Clock rate: ~ 25 MHz
- Largest algorithm: 53 qubits
- Companies:
  - AWS, Google, IBM, QCI, Rigetti, …
  - Annealing: D-Wave





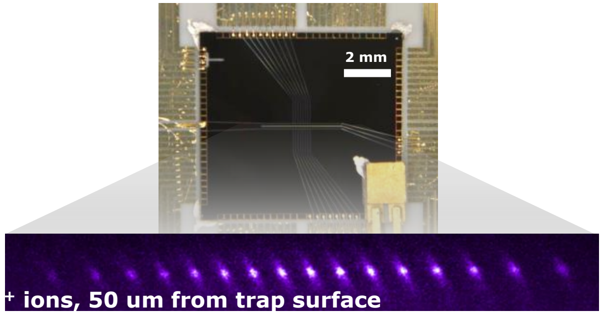




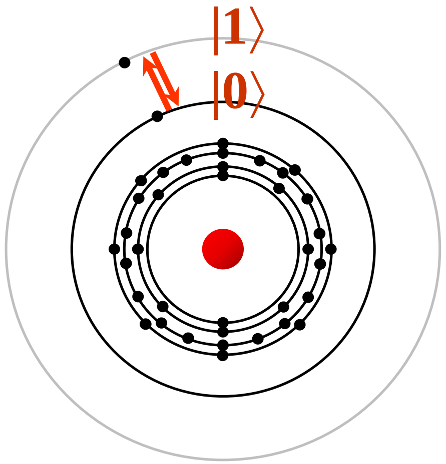
Atomic State: Trapped Ion Qubits

- Qubit: energy levels of an ionized atom
  - Ca+, Sr+, Be+
  - Optical or microwave transitions
- Coherence times: 10 s
- Fidelity and operation times
  - 1 QB: 99.999% in 5 us
  - 2 QB: 99.900% in 50 us
  - Readout: 99.990% in 30 us
- Clock rate: ~ 20 kHz
- Largest algorithm: 30 qubits
- Companies: Honeywell, Ion-Q, AQT, Universal Quantum, …

Trapped Ions in Surface Trap



Energy Levels

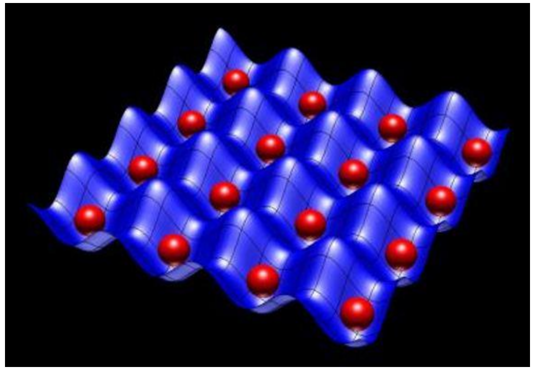


Sr^{+} ion

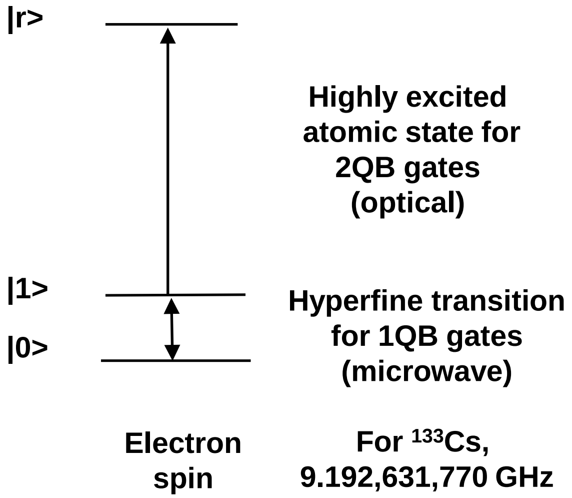
Atomic State: Neutral Atoms

- Qubit: energy levels of a neutral atom
  - Rb, Cs, Ho trapped in an optical lattice
  - Optical and microwave fields
- Coherence times: 1 s
- Fidelity and operation times
  - 1 QB: 99% in 3 us
  - 2 QB: >99% in 300 us
  - Readout: 99.90% in >3 *milliseconds*
- Clock rate: 10 kHz
- Largest lattices: 100-300 qubits
- Companies: Atom Computing, ColdQuanta, Pasqual, QuEra

Neutral Atoms in Optical Lattice



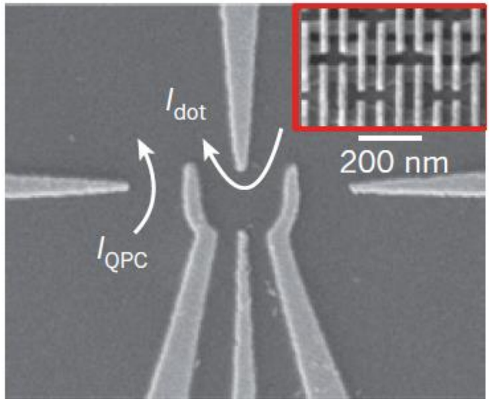
Energy Levels



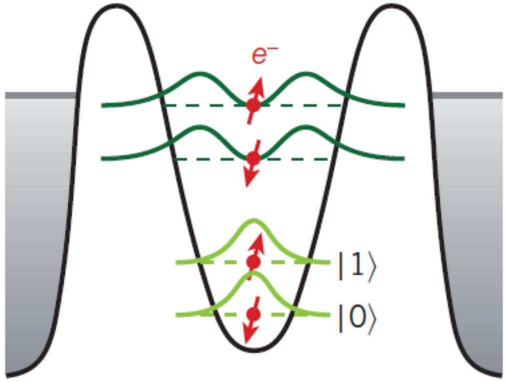
Electron Spin: SiGe Quantum Dots

- Qubit: electron spin
  - Quantum dots in SiGe 2DEGs
  - RF and baseband pulsing
  - Double-dot, triple-dot, CMOS dot
- Coherence times: 400 us
- Fidelity and operation times
  - 1 QB: 99.5% in 100 ns
  - 2 QB: >99% in 200 ns
  - Readout: 99% in 1 us
- Clock rate: 5 MHz
- Companies: HRL, Intel

SiGe Quantum Dots



Energy Levels

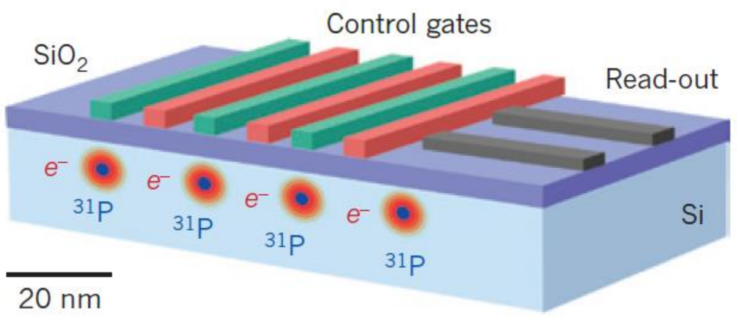


Nature 479, 345 (2011)

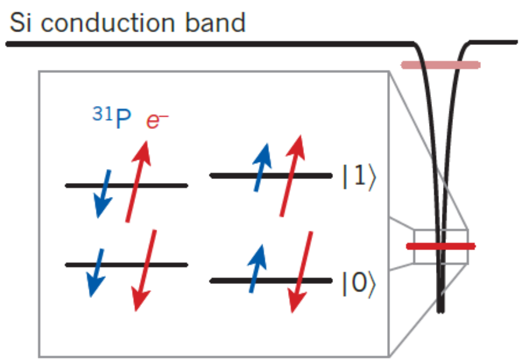
Electron Spin: Phosphorus-Doped Silicon

- Qubit: electron spin (nuclear spins)
  - Phosphorus donor in silicon
  - Microwave pulses
- Coherence times: 100 ms (1 s)
- Fidelity and operation times
  - 1 QB: 99.5% in 200 ns (99.99% in 100 us)
  - 2 QB: ~ 90% in 1-100 ns
  - Readout: 95.0% in 1 ms (99.9% in 50 ms)
- Clock rate: *TBD*
- Companies: SQC (Silicon Quantum Computing)

Phosphorous-Doped Silicon



Energy Levels

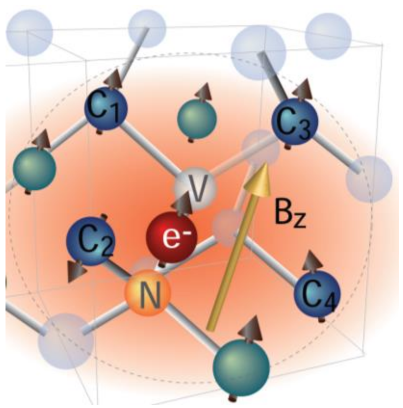


Nature 479, 345 (2011)

Electron and Nuclear Spins: NV Centers

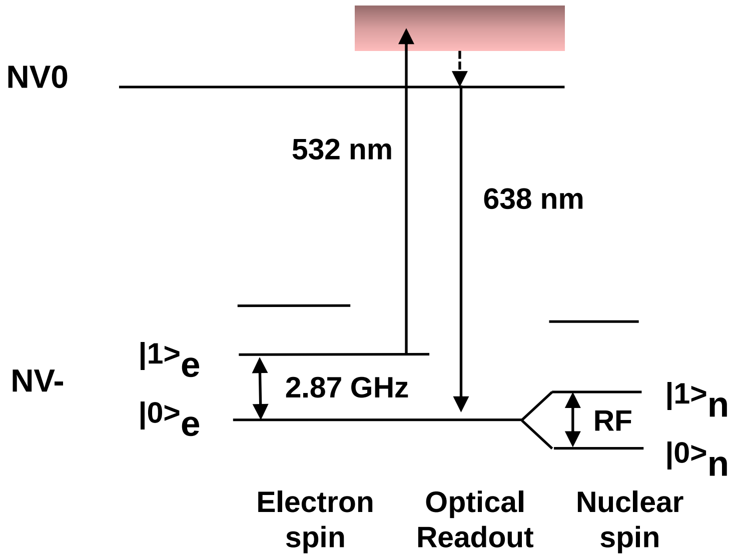
- Qubit: electron or nuclear spin
  - Nitrogen vacancy electron (NV-)
  - Nitrogen or carbon nuclear spins
  - Other defects may be used
- Coherence times: 20 ms
- Fidelity and operation times
  - 1 QB: 99.5% in 10 us
  - 2 QB: >90% in 25 us
  - Readout: 94.0% in 50 us
- Clock rate: 40 kHz
- Companies: N/A (mostly sensing applications)

Diamond with Nitrogen Vacancy



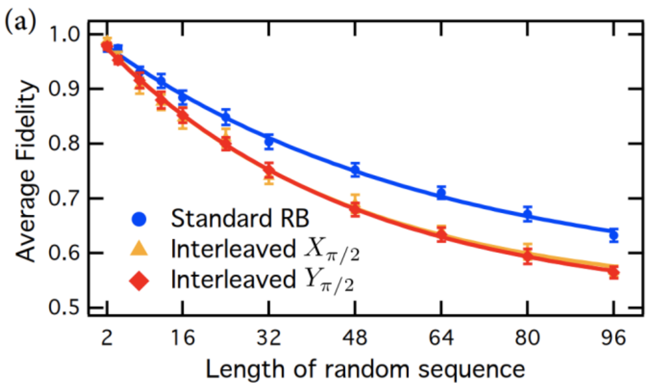
(note: redraw and have all carbon atoms be blue with a nuclear spin. Do not label C1…C4, just put an "n" inside one. Put an "e" inside the electron instead of e-. Label B rather than Bz

Energy Levels



Benchmarking Methods

Randomized Benchmarking



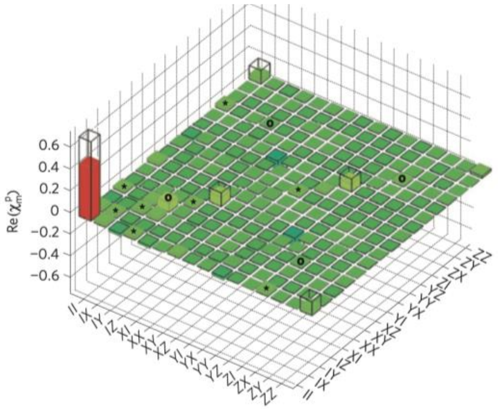
Pros

- Simple and efficient procedure to obtain fidelity
- Current 'gold standard'

Cons

- Time dependent errors may alter decay curve

Quantum Process Tomography



Pros

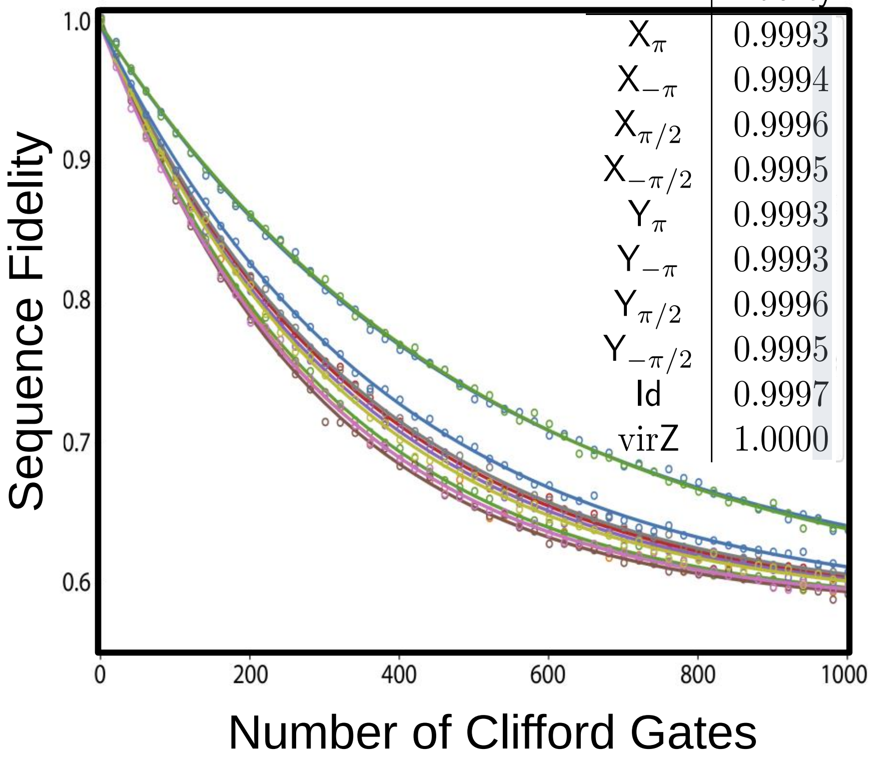
- Exact reconstruction of any quantum process

Cons

- Exponential resource requirement (3 qubits is the borderline)
- Cannot separate gate errors from SPAM errors

Gate Fidelities

Single-Qubit Gate Fidelity > 0.999



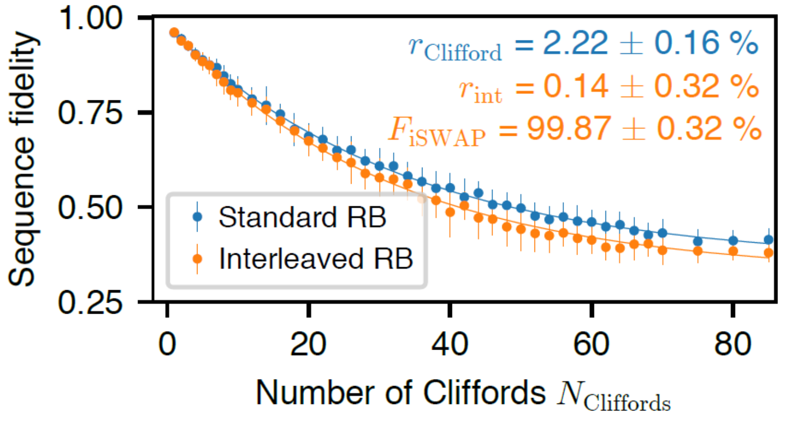
	Fidelity
X_{π}	0.9993
$X_{-\pi}$	0.9994
$X_{\pi/2}$	0.9996
$X_{-\pi/2}$	0.9995
Y_{π}	0.9993
$Y_{-\pi}$	0.9993
$Y_{\pi/2}$	0.9996
$Y_{-\pi/2}$	0.9995
Id	0.9997
virZ	1.0000

Two-Qubit Gate Fidelity > 0.995

iSWAP gate

$$U_{\mathrm{iSWAP}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

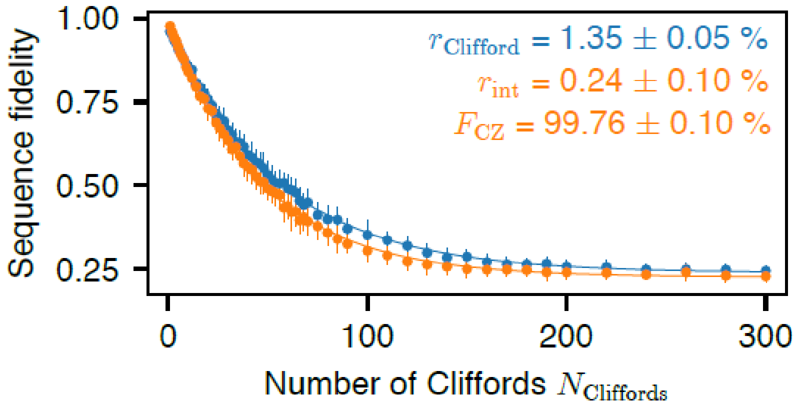
iSWAP Fidelity: 99.87 ± 0.32 %



Controlled-Z (CZ) gate

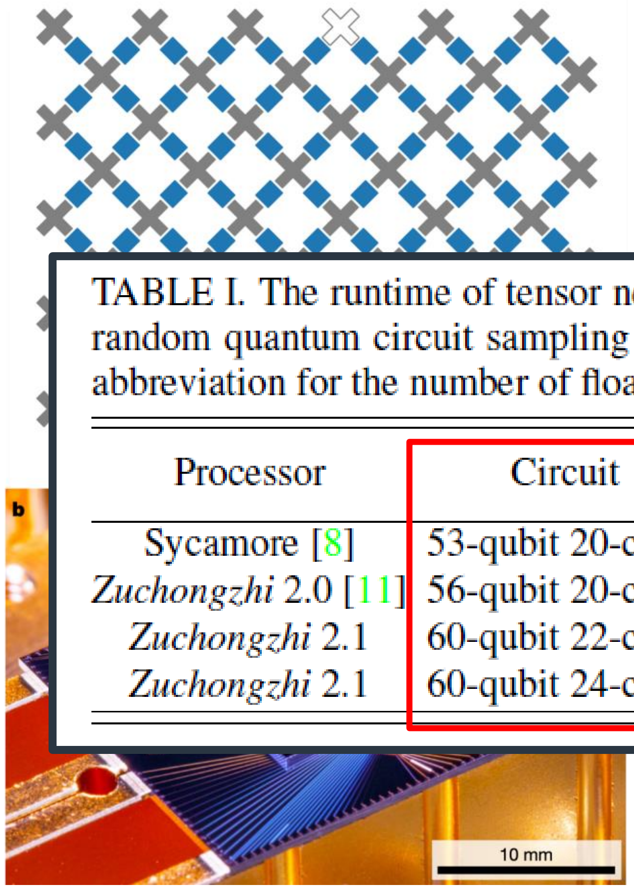
$$U_{\mathrm{CZ}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

CZ Fidelity: 99.76 ± 0.10 %



F. Yan *et al.* *PRApplied* **10**, 054062 (2018): Theory
Y. Sung *et al.* arXiv:2011.01261 (2020)**:** Experiment

Quantum Advantage Demonstrations



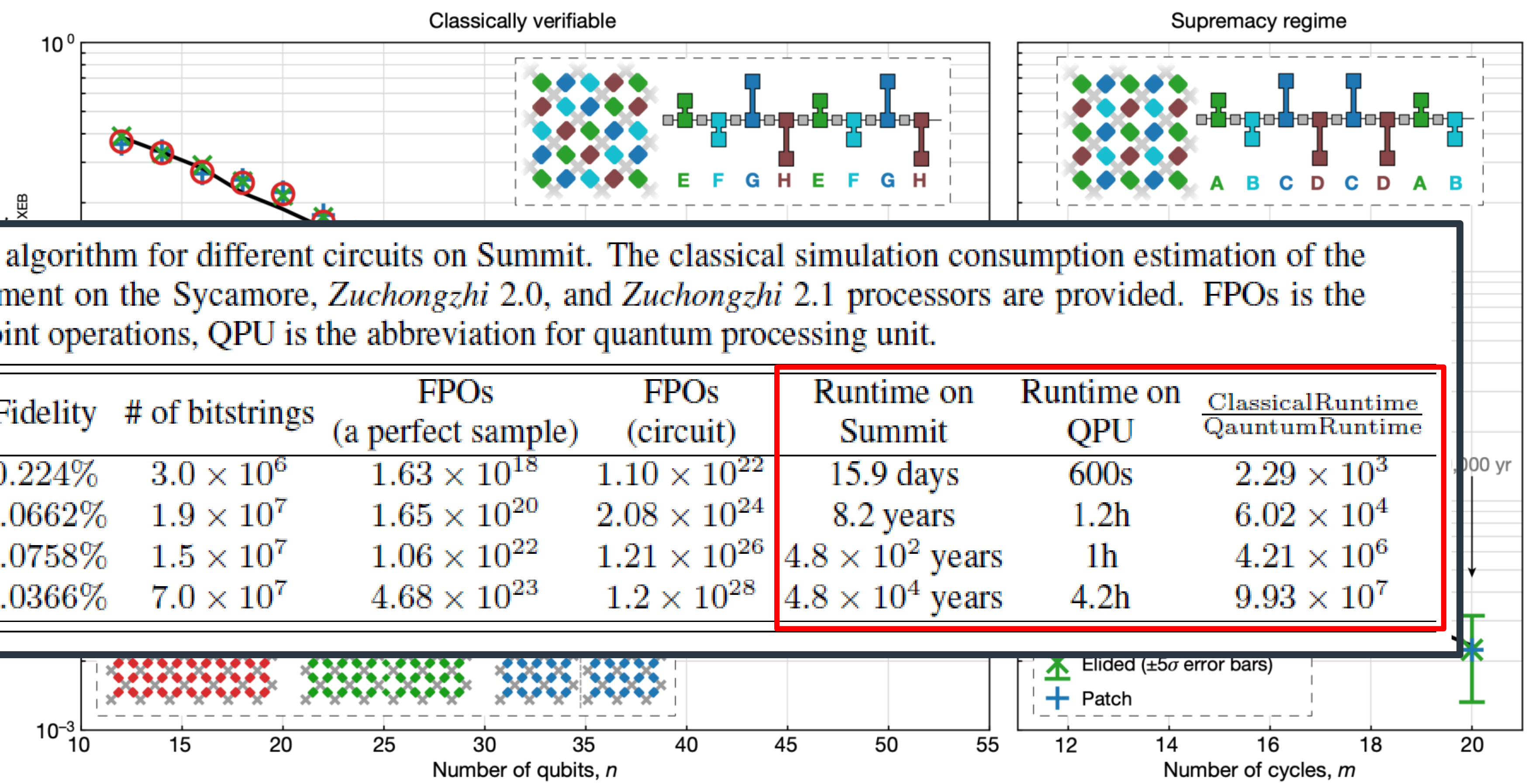


TABLE I. The runtime of tensor network algorithm for different circuits on Summit. The classical simulation consumption estimation of the random quantum circuit sampling experiment on the Sycamore, *Zuchongzhi* 2.0, and *Zuchongzhi* 2.1 processors are provided. FPOs is the abbreviation for the number of floating point operations, QPU is the abbreviation for quantum processing unit.

Processor	Circuit	Fidelity	# of bitstrings	FPOs (a perfect sample)	FPOs (circuit)	Runtime on Summit	Runtime on QPU	$\frac{\text{ClassicalRuntime}}{\text{QauntumRuntime}}$
Sycamore [8]	53-qubit 20-cycle	0.224%	3.0×10^6	1.63×10^{18}	1.10×10^{22}	15.9 days	600s	2.29×10^3
Zuchongzhi 2.0 [11]	56-qubit 20-cycle	0.0662%	1.9×10^7	1.65×10^{20}	2.08×10^{24}	8.2 years	1.2h	6.02×10^4
Zuchongzhi 2.1	60-qubit 22-cycle	0.0758%	1.5×10^7	1.06×10^{22}	1.21×10^{26}	4.8×10^2 years	1h	4.21×10^6
Zuchongzhi 2.1	60-qubit 24-cycle	0.0366%	7.0×10^7	4.68×10^{23}	1.2×10^{28}	4.8×10^4 years	4.2h	9.93×10^7

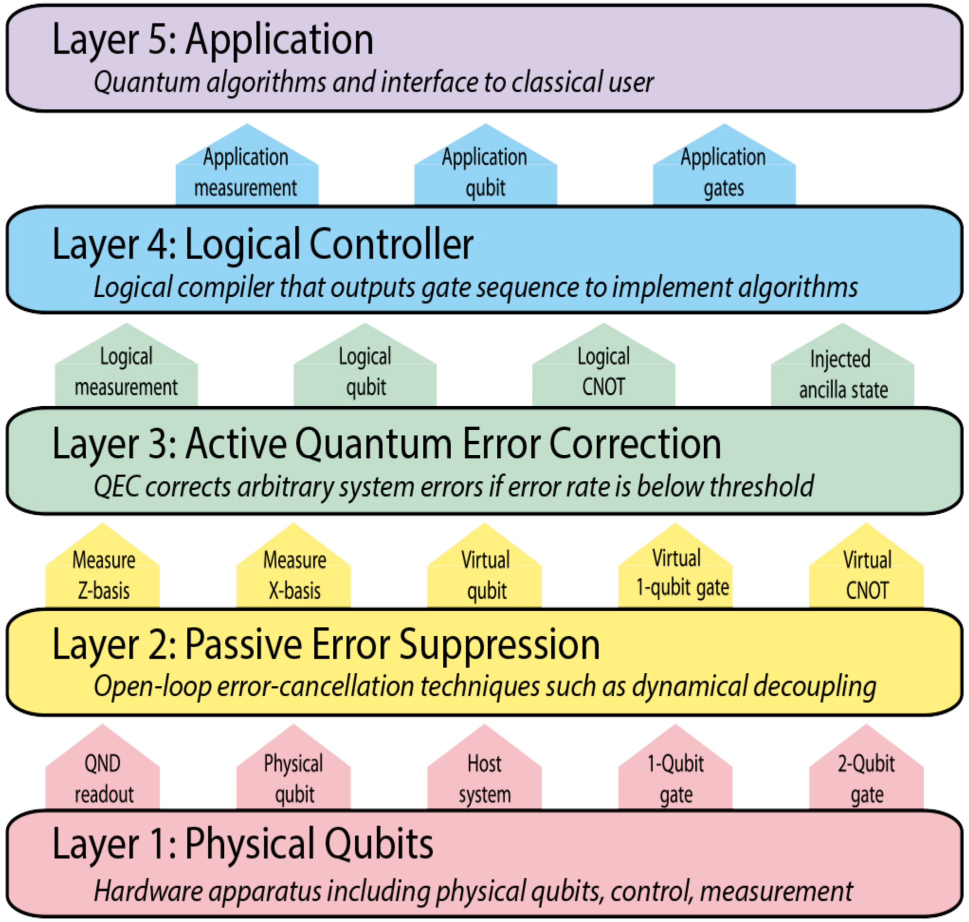
The Google Quantum AI team demonstrated
a calculation in ~200s with one chip, 53 superconducting qubits, drawing around 100 kW of power

On the Summit supercomputer (Oak Ridge National Laboratory),
it would take several days, with all 40,000 CPUs & GPUs, 10^{17} transistors & memory, and 100's MW of power

Google AI, Nature 505, 574 (2019); USTC, arXiv:2109.03494 (2021)

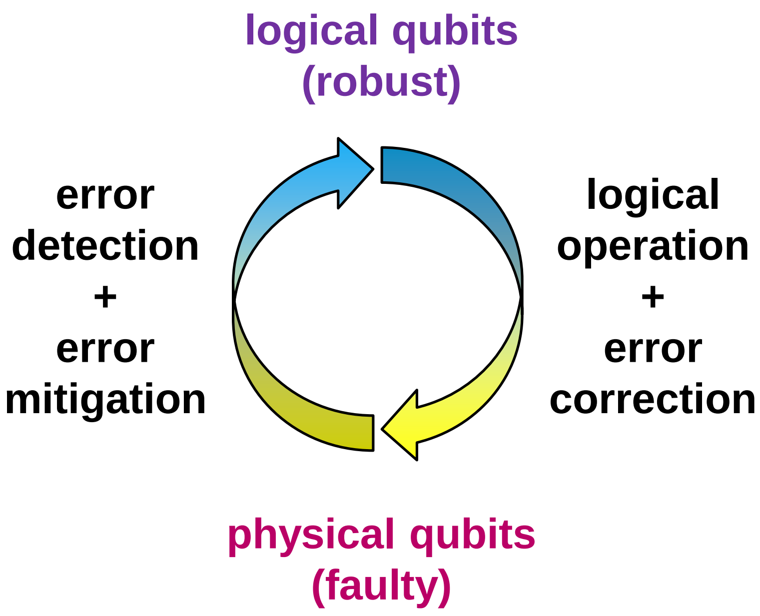
Architectural Layers of a QIP

Layered Architecture

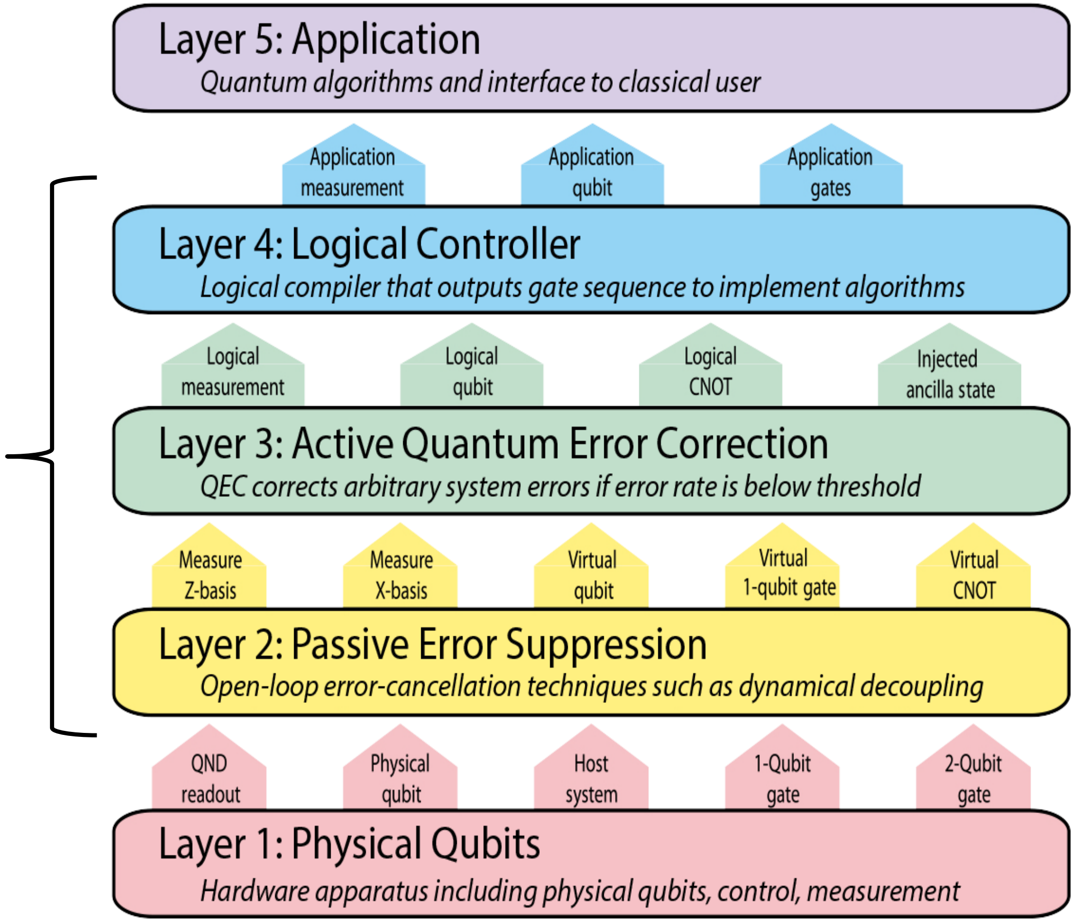


N.C. Jones PRX 2, 031007 (2012)

Architectural Layers of a QIP



Layered Architecture



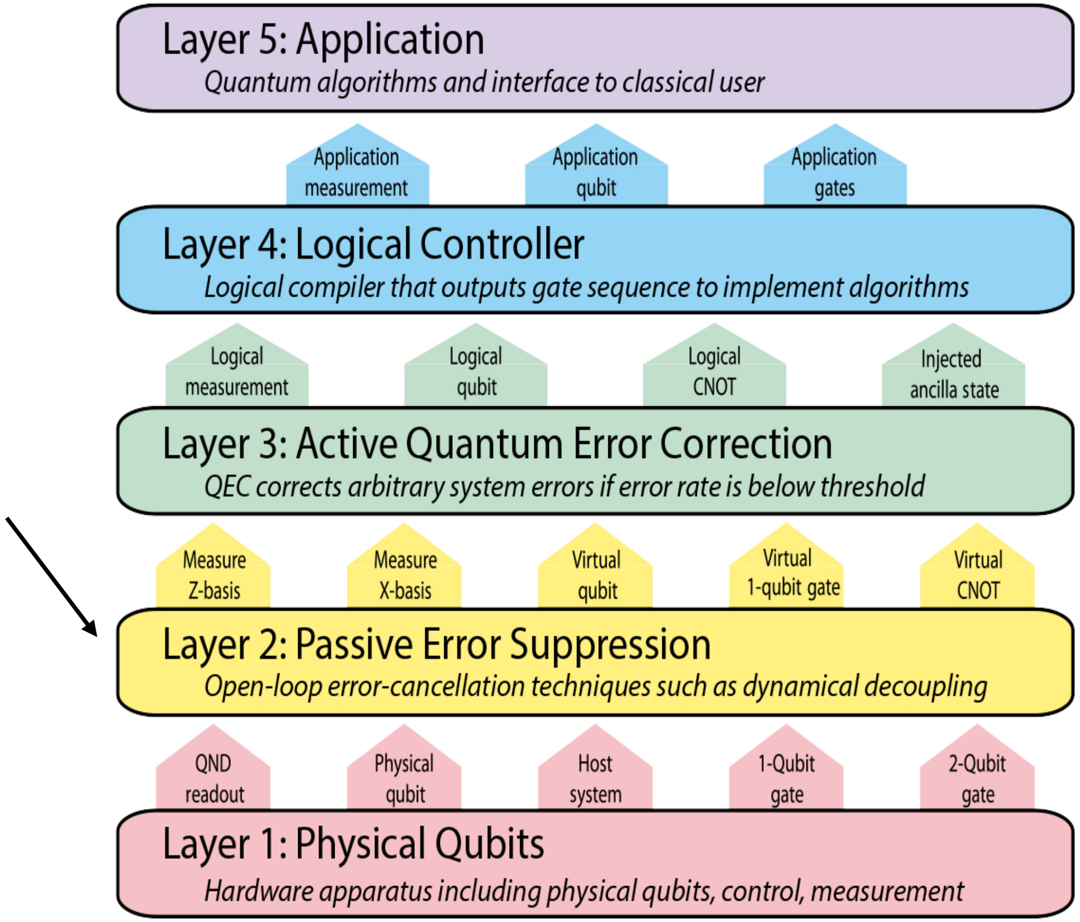
N.C. Jones PRX 2, 031007 (2012)

Architectural Layers of a QIP

Engineered Error Mitigation: Dynamical Decoupling

Eg. Lacrosse Cradling

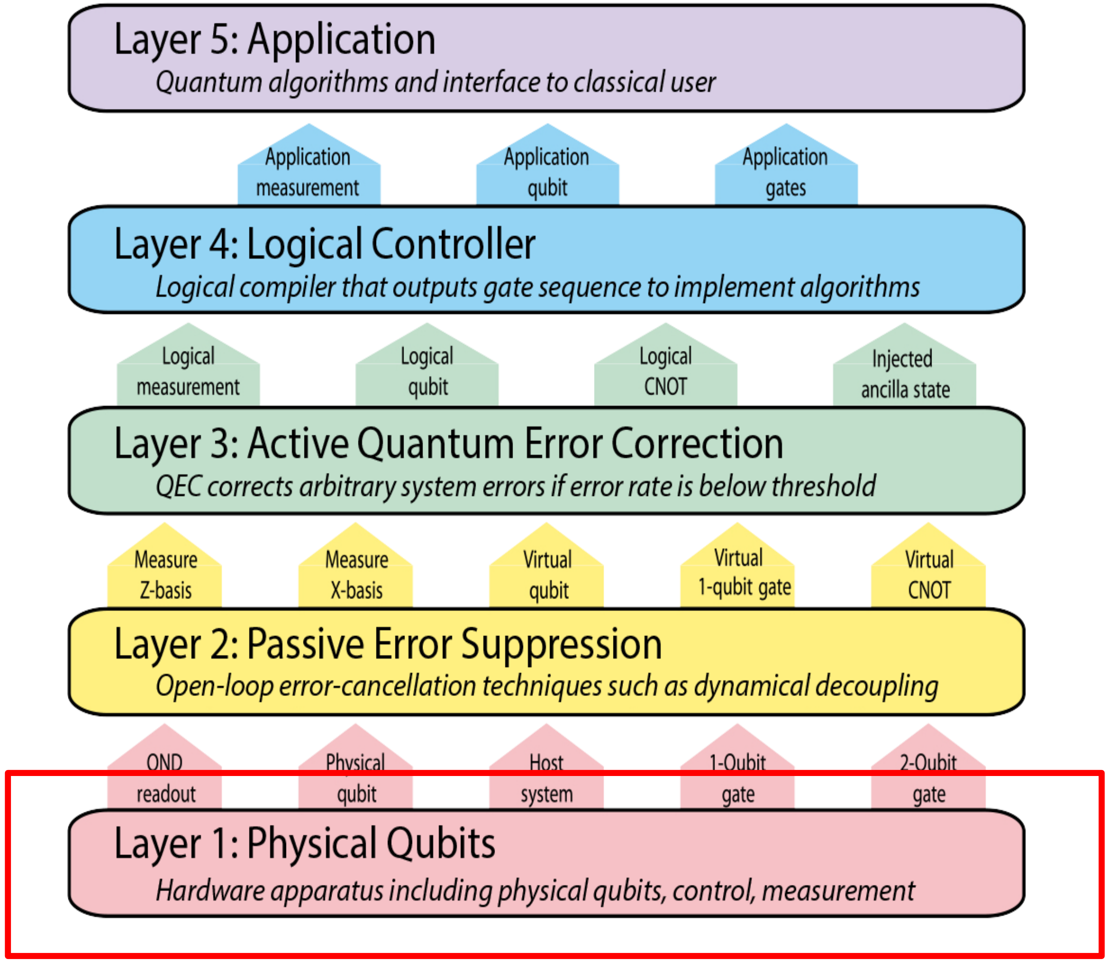
Layered Architecture



N.C. Jones PRX 2, 031007 (2012)

Lacrosse in the Presence of Noise

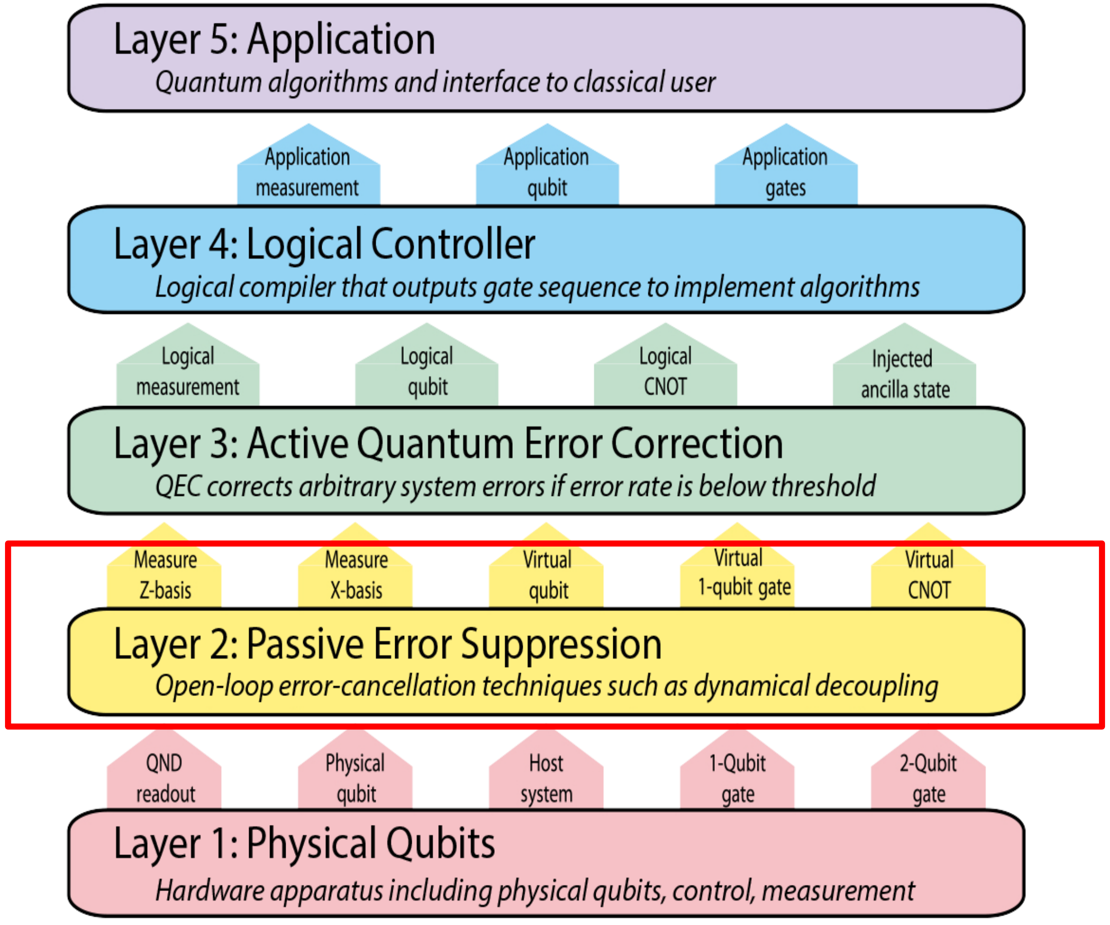
Layered Architecture



N.C. Jones PRX 2, 031007 (2012)

Dynamical Decoupling from Running "Noise"

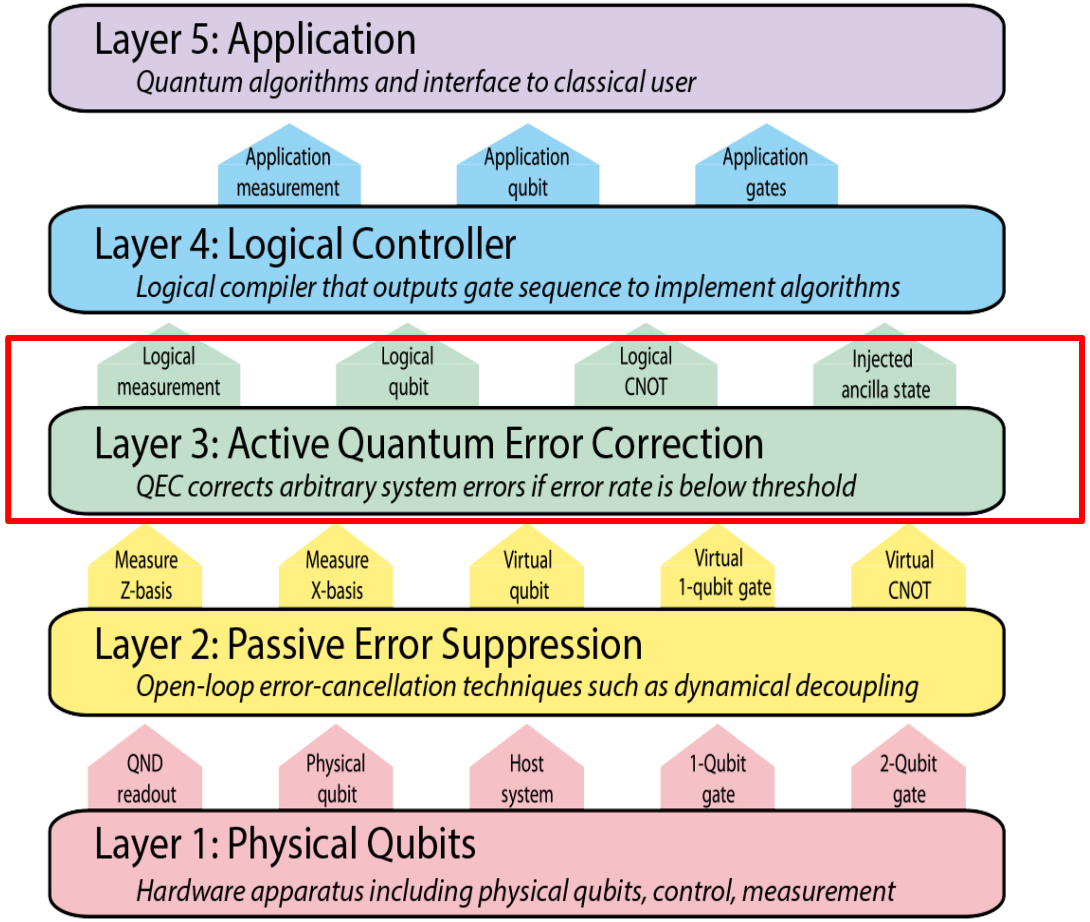
Layered Architecture



N.C. Jones PRX 2, 031007 (2012)

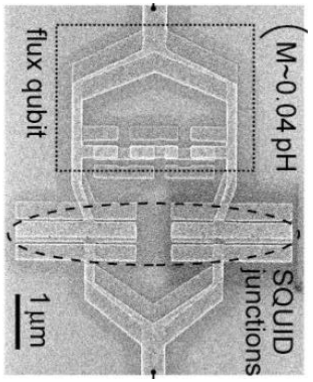
"Active Error Correction" in Lacrosse

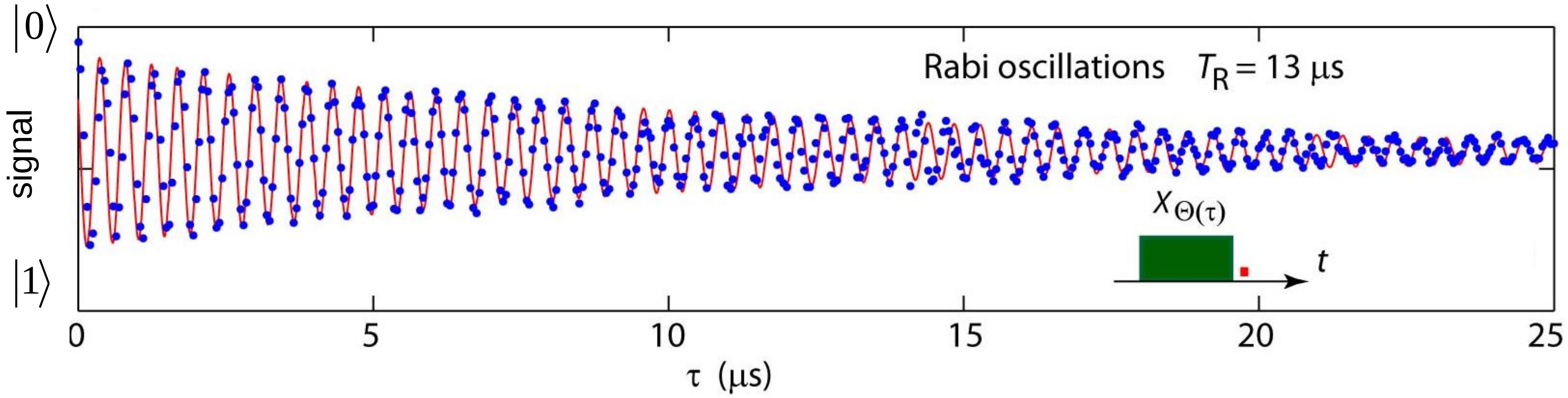
Layered Architecture



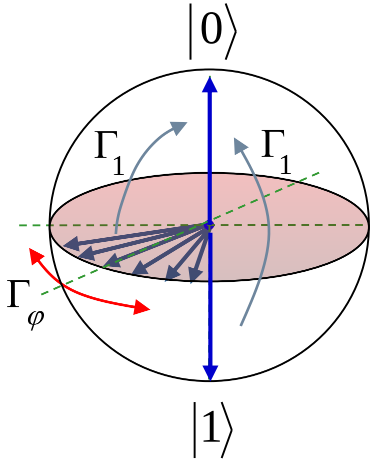
N.C. Jones PRX 2, 031007 (2012)

Coherence Times





- **Relaxation rate:** $\Gamma_1 = 1/T_1$
- **Decoherence rate:** $\Gamma_2 = \frac{1}{T_2} = \frac{1}{2T_1} + \frac{1}{T_\varphi}$
- **Dephasing rate:** $\Gamma_\varphi = 1/T_\varphi$



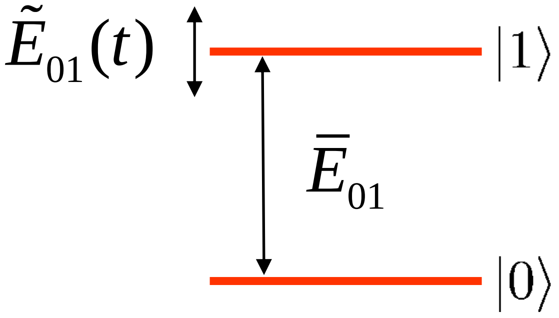
$T_1 = 12\ \mu s$

$T_2 = 2.5\ \mu s$

Can we improve the dephasing time?

J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

Qubit Dephasing and Filter Function



Free evolution of the phase

$$\alpha|0\rangle + \beta|1\rangle \Rightarrow \alpha|0\rangle + \beta e^{i\varphi(t)}|1\rangle$$

$$\varphi(t) = \bar{\varphi}(t) + \tilde{\varphi}(t) \qquad \bar{\varphi}(t) = \frac{\bar{E}_{01}}{\hbar}t$$

dephasing

$$\langle \exp i\tilde{\varphi}(t)\rangle = \left\langle \exp\left(\frac{i}{\hbar}\int_0^\tau dt\tilde{E}_{01}(t)\right)\right\rangle$$

$$= \exp\left[-\frac{\tau^2}{2\hbar^2}\left(\frac{\partial E_{01}}{\partial\lambda}\right)^2 \int d\omega\, S_\lambda(\omega)\, g_N(\omega t)\right]$$

for Gaussian-distributed fluctuations

sensitivity of qubit energy to fluctuations λ

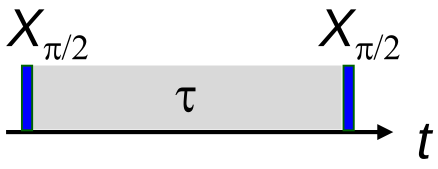
strength (variance) of fluctuations

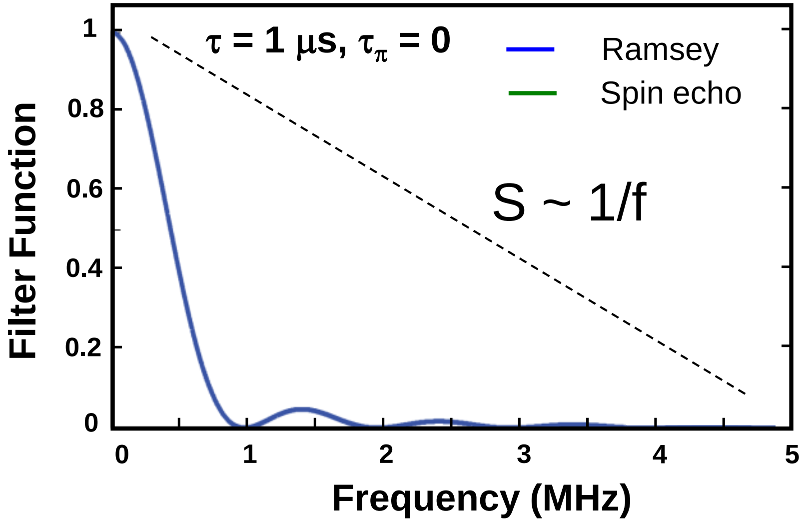
Filter function shapes noise

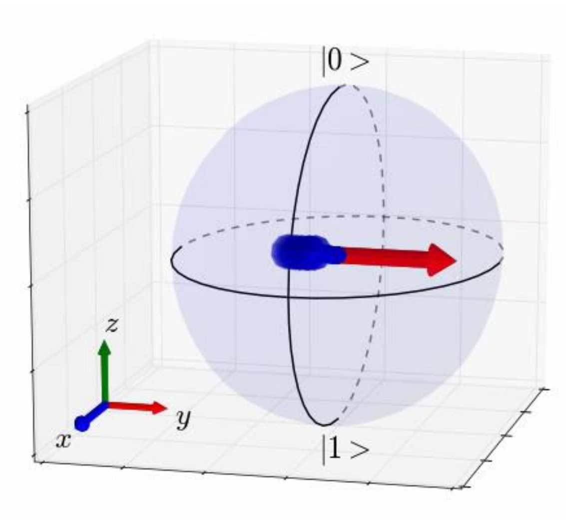
Engineered filter function depends on pulse sequence and windows the PSD $S_\lambda(\omega)$

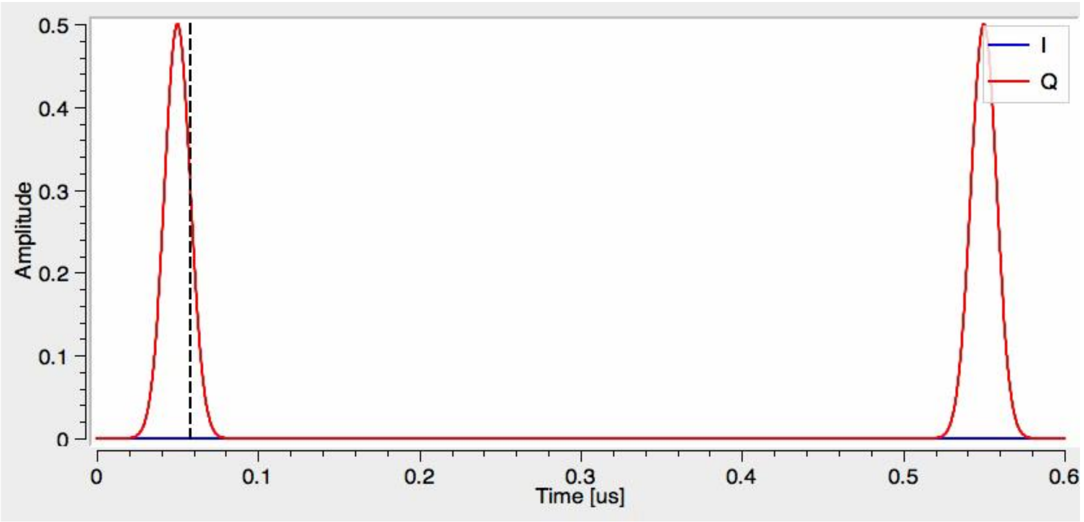
J. Bylander, …, WDO, Nature Physics (2011), Martinis et al., PRB (2003), Ithier et al., PRB (2005); Yoshihara et al., PRL (2006), Cywinski et al. PRB (2008)

Dynamical Decoupling:
Noise Shaping Filters





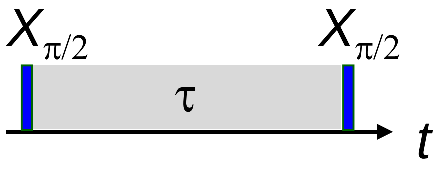




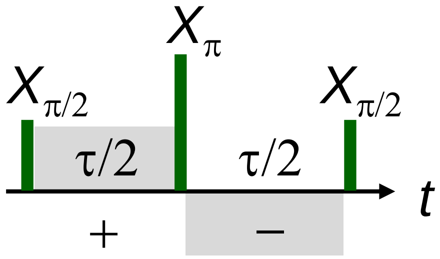
J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

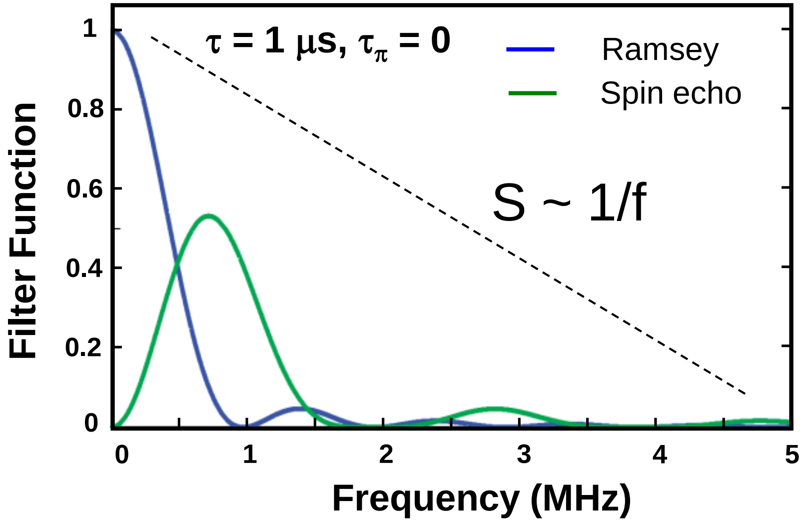
Dynamical Decoupling:
Noise Shaping Filters with 1 π-pulse

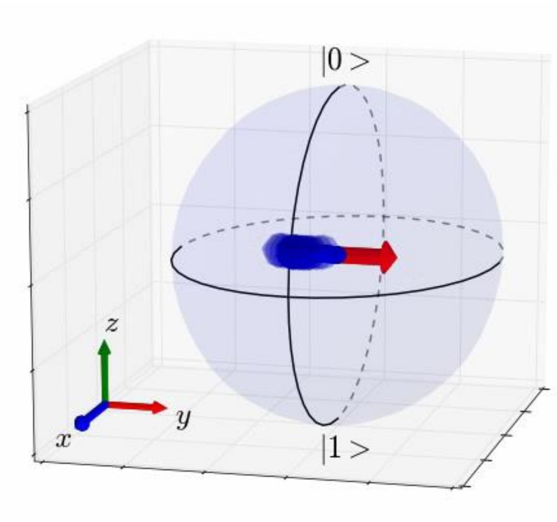
NO Dynam. Decoup.
(Ramsey, N=0)

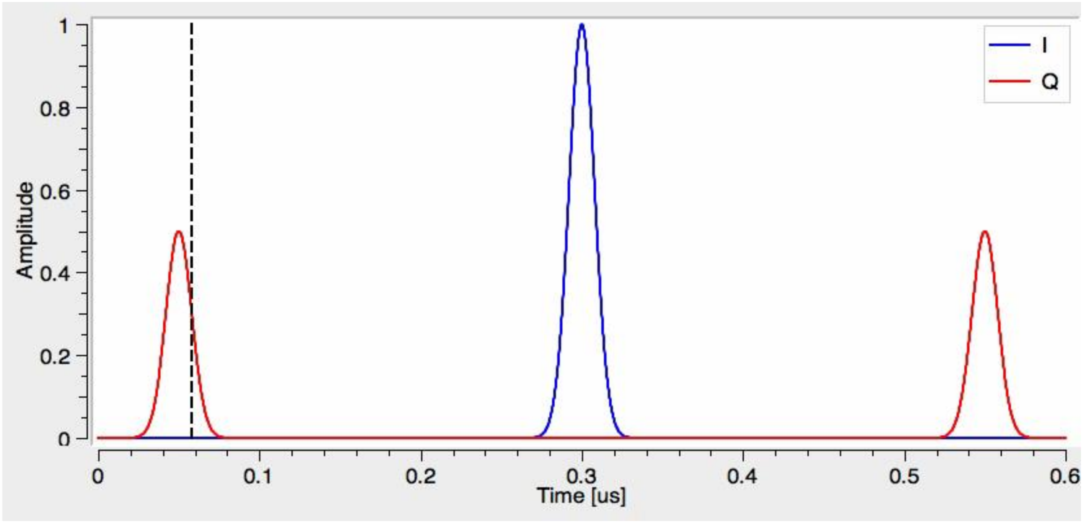


WITH Dynam. Decoup.
(spin echo, N=1)









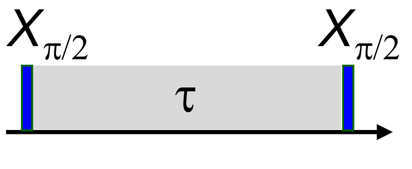
J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

Dynamical Decoupling:

Noise Shaping Filters with 2 π-pulses

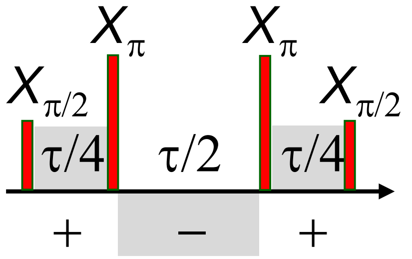
NO Dynam. Decoup.

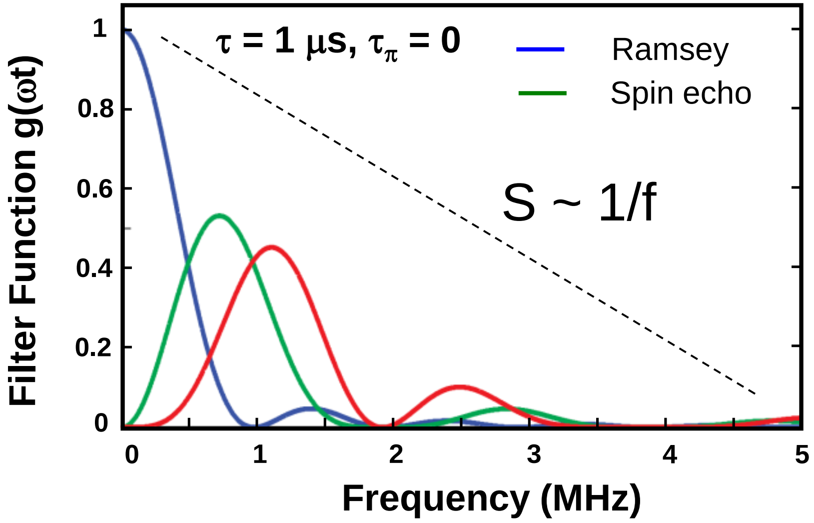
(Ramsey, N=0)

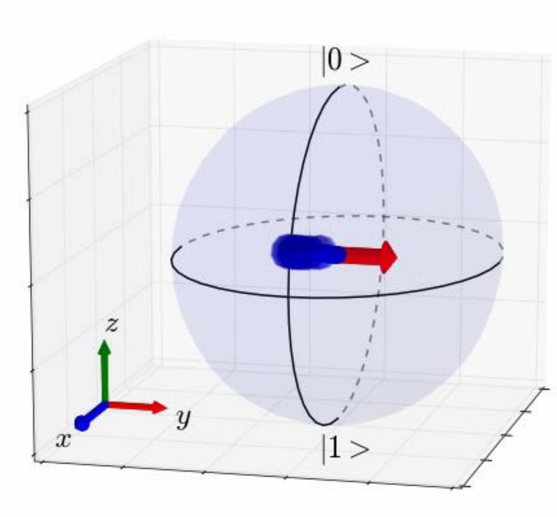


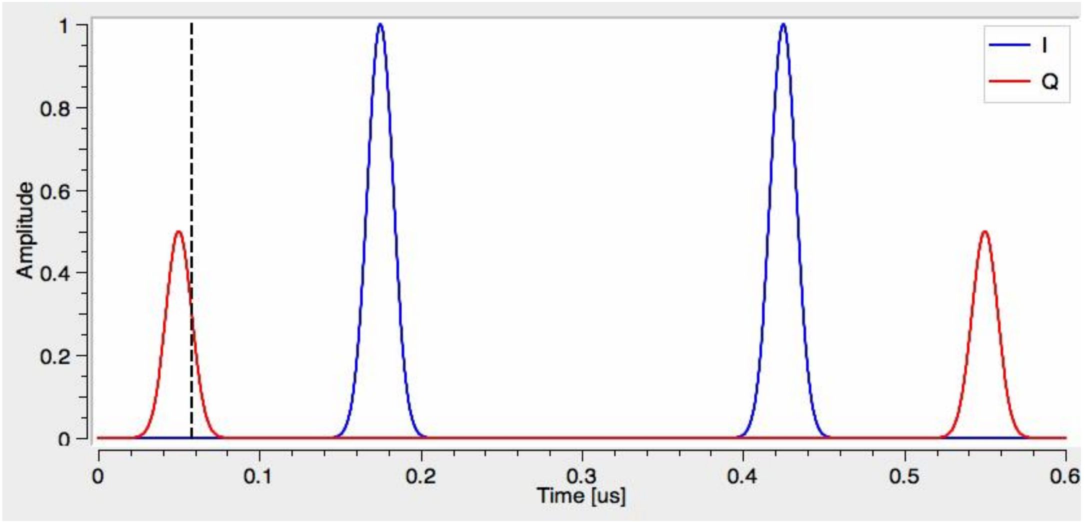
WITH Dynam. Decoup.

(CPMG, N=2)







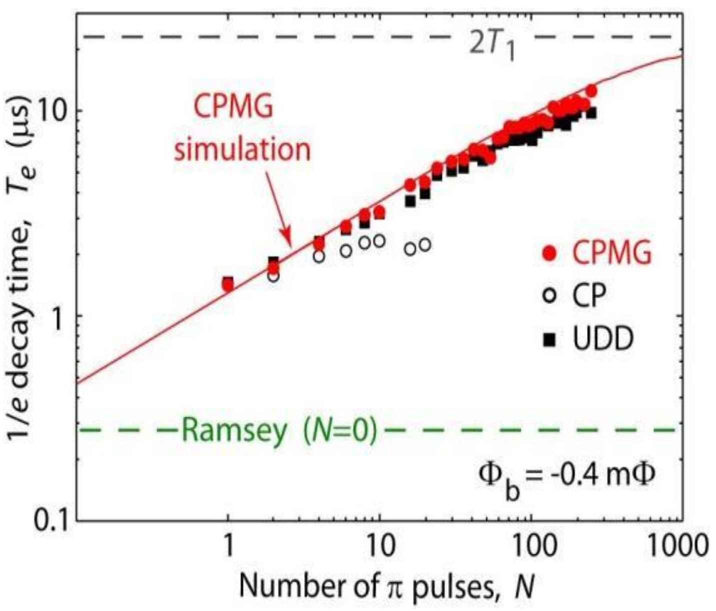


J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

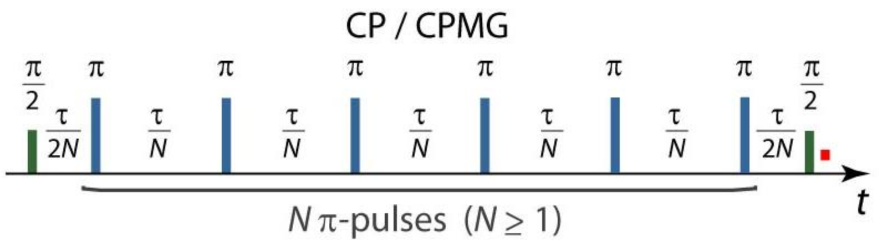
Dynamical Decoupling:

Noise Shaping Filters with N π-pulses

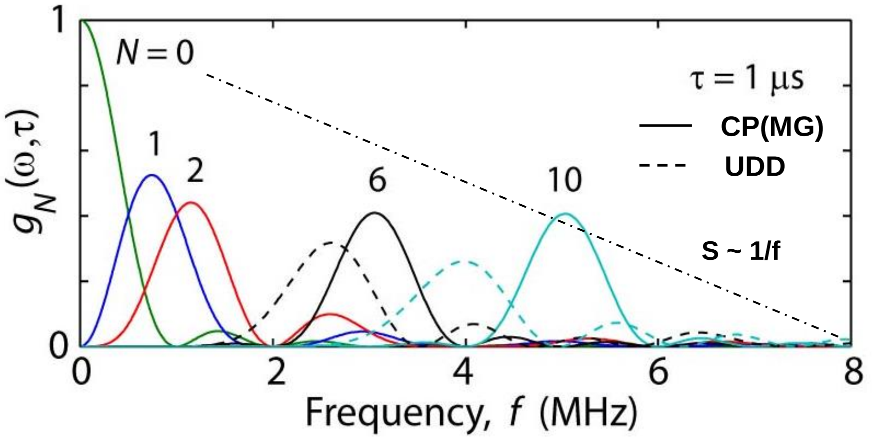
Engineered Error Mitigation:
Dynamical Decoupling
(improves the physical qubit error rate)



Carr – Purcell (– Meiboom – Gill) Sequence



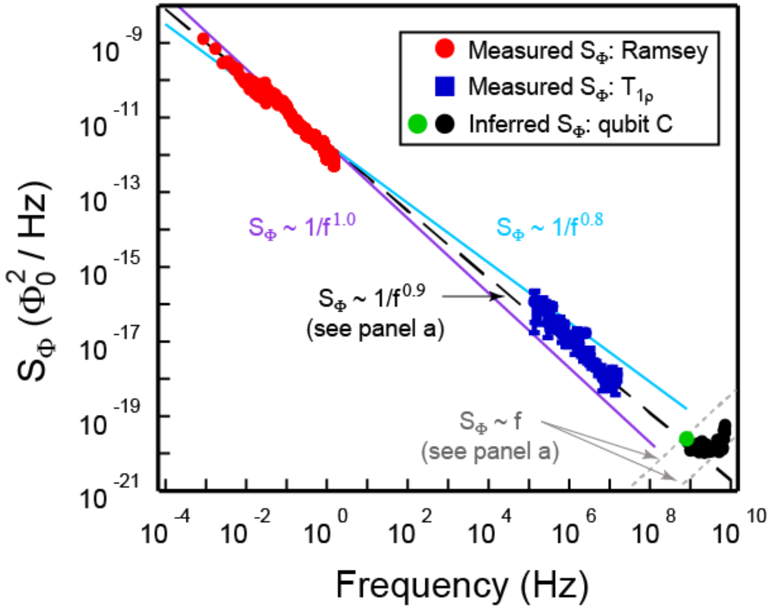
Noise-Shaping Filter Functions



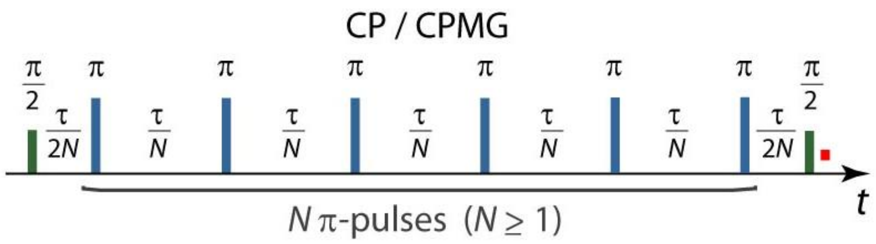
J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

Noise Spectroscopy

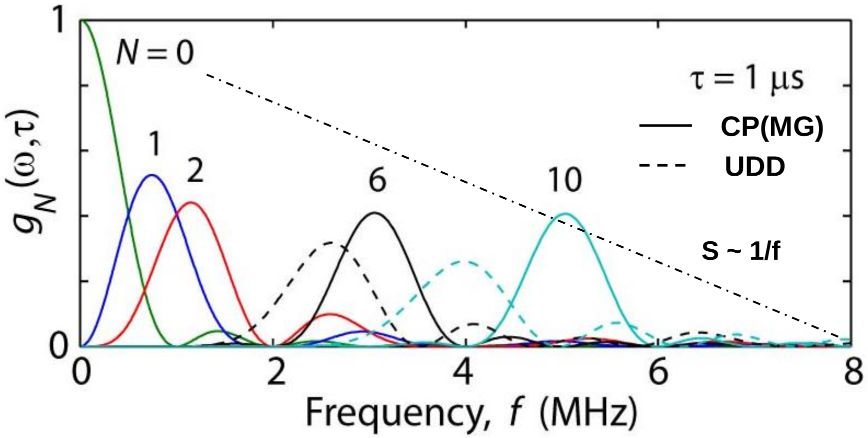
Qubit Noise Spectroscopy
Filter Engineering & Optimal Control



Carr – Purcell (– Meiboom – Gill) Sequence



Noise-Shaping Filter Functions

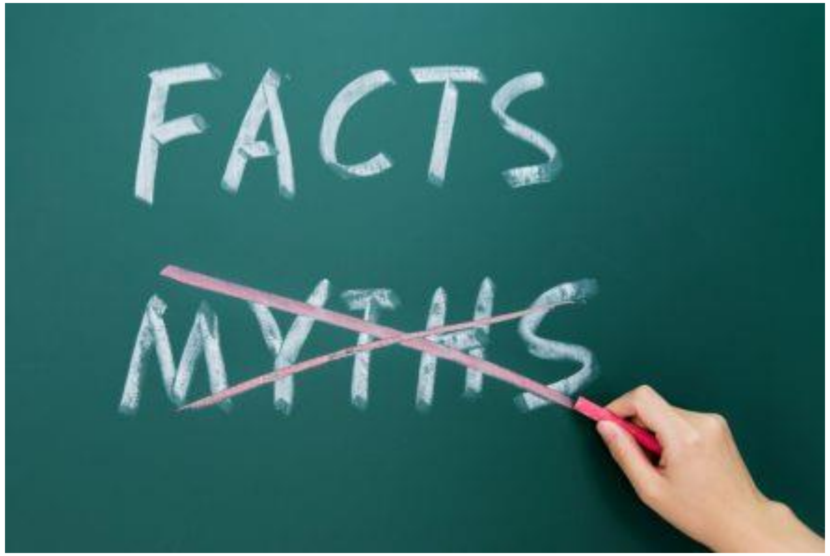


Y. Sung, …, WDO, Nature Communications 10, 3715 (2019)
F. Yan, …, WDO, Nature Communications 7, 12964 (2016)
F. Yan , …, WDO, Nature Communications 4, 2337 (2013)

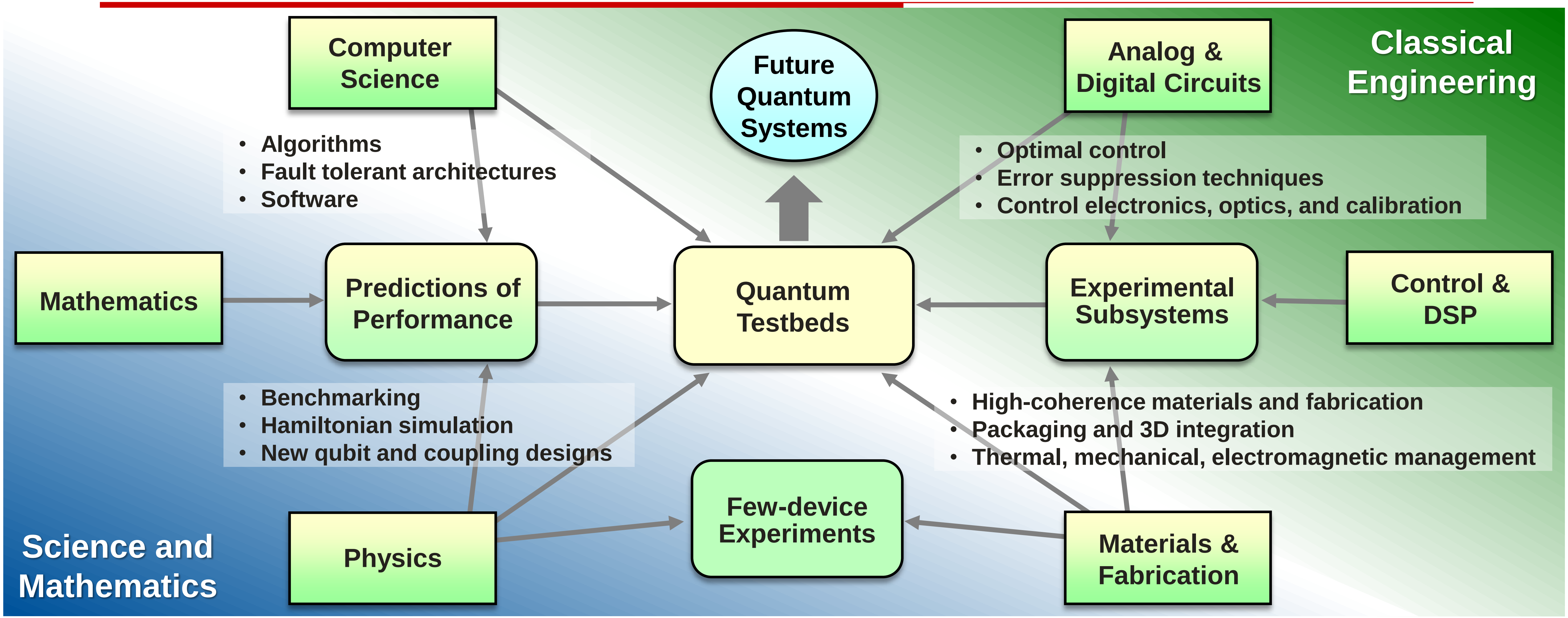
J. Bylander, …, WDO, Nature Physics 7, 565 (2011)

Dispelling Myths About QC

- Quantum computers will not replace classical computers
- Quantum computers will not break encryption soon
  - RSA 2048-bit keys: around 4000 error corrected qubits
  - Bitcoin encryption: around 2300 error corrected qubits
- However, one should not wait until a quantum computer can break RSA to switch to post-quantum encryption



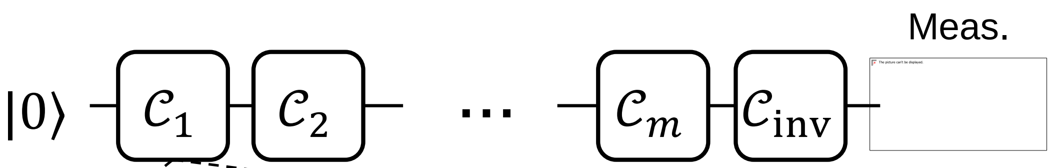
Quantum Engineering



Quantum Engineering is the bridge connecting science, mathematics, and classical engineering

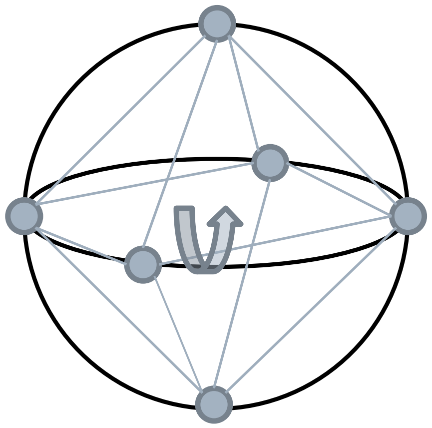
Randomized Benchmarking

– Single-qubit randomized benchmarking



Clifford gate $\mathcal{C}$

- 1QB Clifford: rotate between octahedral points on the Bloch Sphere.
- More generally, normalizer of the Pauli group $\{I, X, Y, Z\}$.



[1] J. Emerson *et al. J. Opt. B* 7, S347 (2005)
[2] E. Knill *et al. Phys. Rev. A.* 77, 012307 (2008)
[3] E. Megesan *et al. Phys. Rev. Lett.* **106**, 180504 (2011)

- Goal: estimate the average error rates of quantum gates.
- (Clifford-based) Randomized Benchmarking [1,2,3]
  - Initialize qubits at the ground state.
  - Apply m randomly chosen Clifford gates ($\mathcal{C}_1$, $\mathcal{C}_2$, …, $\mathcal{C}_m$).
  - At the end, apply the inverse gate s.t. the entire operation = Identity.
  - Measure the survival probability of the ground state (= “sequence fidelity” F_{seq}).
    - ✓ In the absence of error → $\boldsymbol{F_{seq} = 1}$.
    - ✓ In the presence of error → $\boldsymbol{F_{seq} < 1}$.
- Twirling over Cliffords → Depolarization of the gate error [1,2,3]

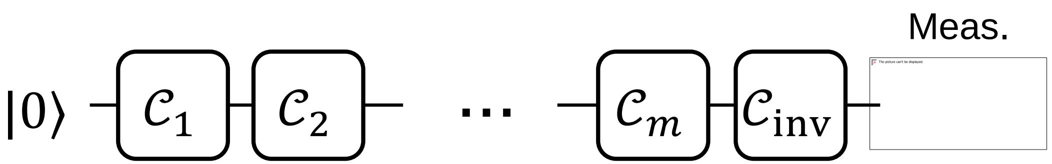
$$\boldsymbol{\rho \rightarrow p\rho + \frac{(1-p)}{2^n} I} \quad (n\text{: \# of qubits})$$

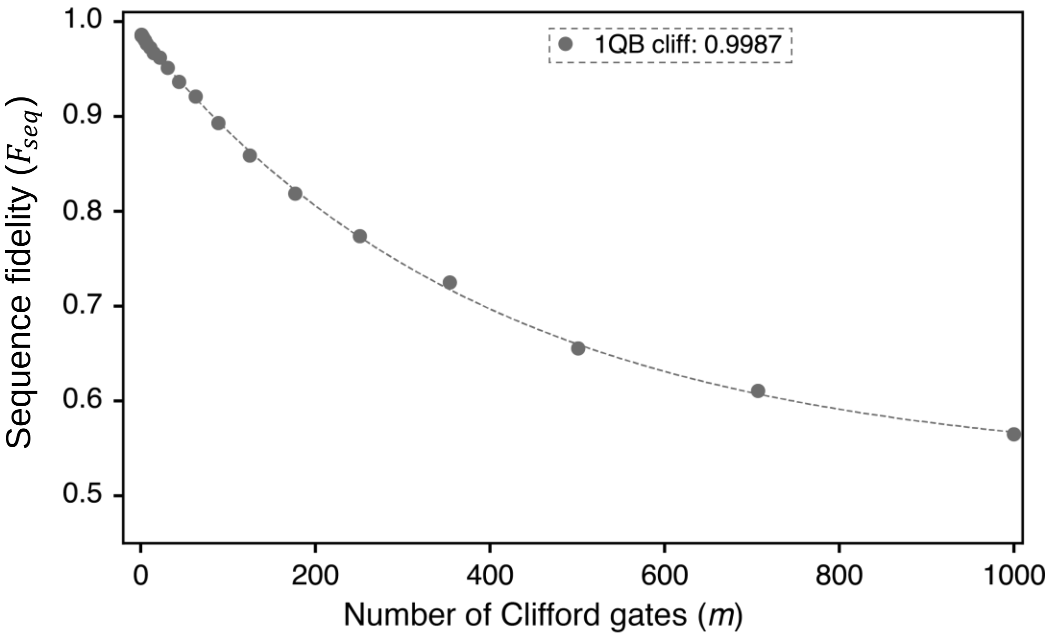
  - F_{seq} will decay exponentially as $\boldsymbol{F_{seq} = Ap^m + B}$.
  - The average error rate per Clifford r_{Cliiford} is related to p as

$$\boldsymbol{r_{\text{Clifford}} = (1-p) \times \frac{2^n - 1}{2^n}}$$

Randomized Benchmarking

– Single-qubit randomized benchmarking





- Goal: estimate the average error rates of quantum gates.

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    - ✓ In the absence of error → $\boldsymbol{F_{seq} = 1}$.
    - ✓ In the presence of error → $\boldsymbol{F_{seq} < 1}$.

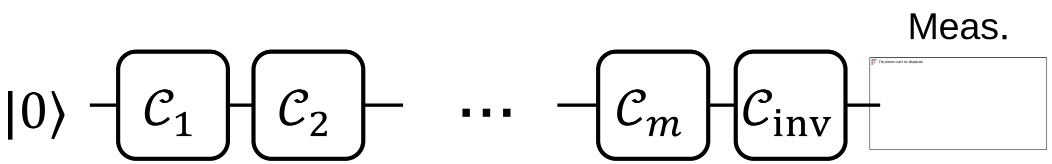
- Measurement of the avg. error rate per 1QB Clifford
  - Fit F_{seq} with exponential ($f(x) = Ae^{Bx} + C$).
  - Extract depolarizing rate p, where $p = e^B$.
  - A, C: absorbs the SPAM error.
  - The average error rate per 1QB Clifford gate r,

$$r_{\mathrm{Clifford}} = (1-p) \times \frac{\mathbf{2^n - 1}}{\mathbf{2^n}} = \frac{\mathbf{1 - p}}{\mathbf{2}}$$

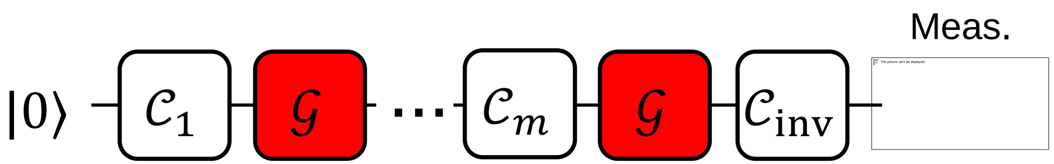
(n: # of qubits)

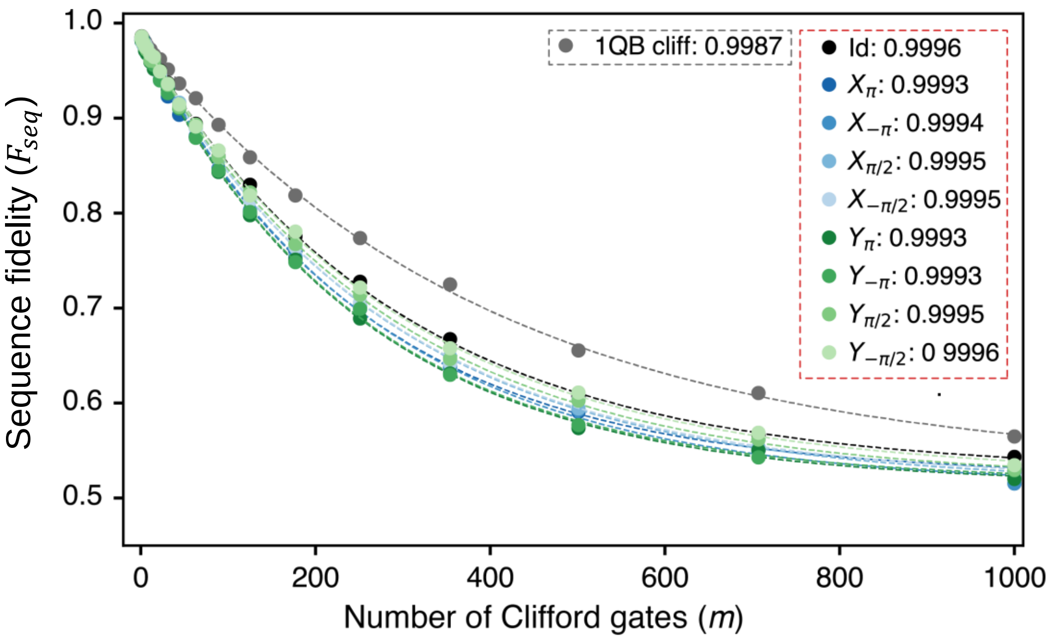
Interleaved Randomized Benchmarking

– **Reference** (1QB) randomized benchmarking ($F_{\text{seq, ref}}$)



– **Interleaved** (1QB) randomized benchmarking ($F_{\text{seq, int}}$)





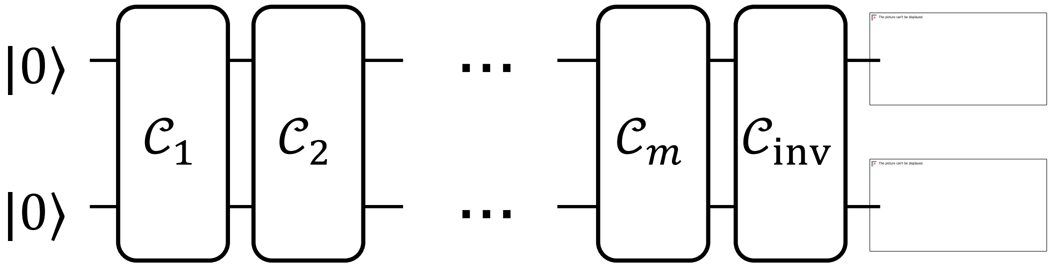
- Interleaved Randomized Benchmarking [1,2]
  - Interleave gate of interest $\mathcal{G}$ at every Clifford ($\mathcal{C}_1$, $\mathcal{C}_2$, …, $\mathcal{C}_m$).
  - Compare it to the reference RB to extract the error rate of $\mathcal{G}$.
  - $F_{\text{seq, ref}} = A{p_{\text{ref}}}^m + B$ (reference curve)
  - $F_{\text{seq, int}} = A'(p_{\text{ref}}p_g)^m + B' \equiv A'p_{\text{int}}^m + B'$ (interleaved curve)
- The average error rate per interleaved gate r_{int},

$$r_{\text{int}} = (1 - p_g) \times \frac{2^n - 1}{2^n} = \left(1 - \frac{p_{\text{int}}}{p_{\text{ref}}}\right) \times \frac{2^n - 1}{2^n}$$

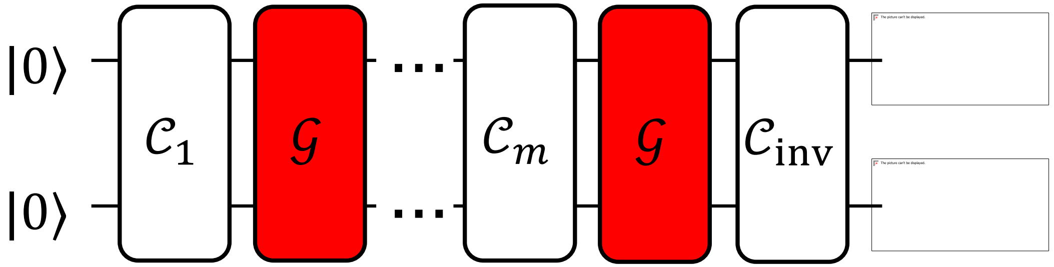
[1] E. Megesan *et al. Phys. Rev. Lett.* **109**, 080505 (2012)
[2] A. D. Corcoles *et al. Phys. Rev. A.* **87**, 030301 (2013)

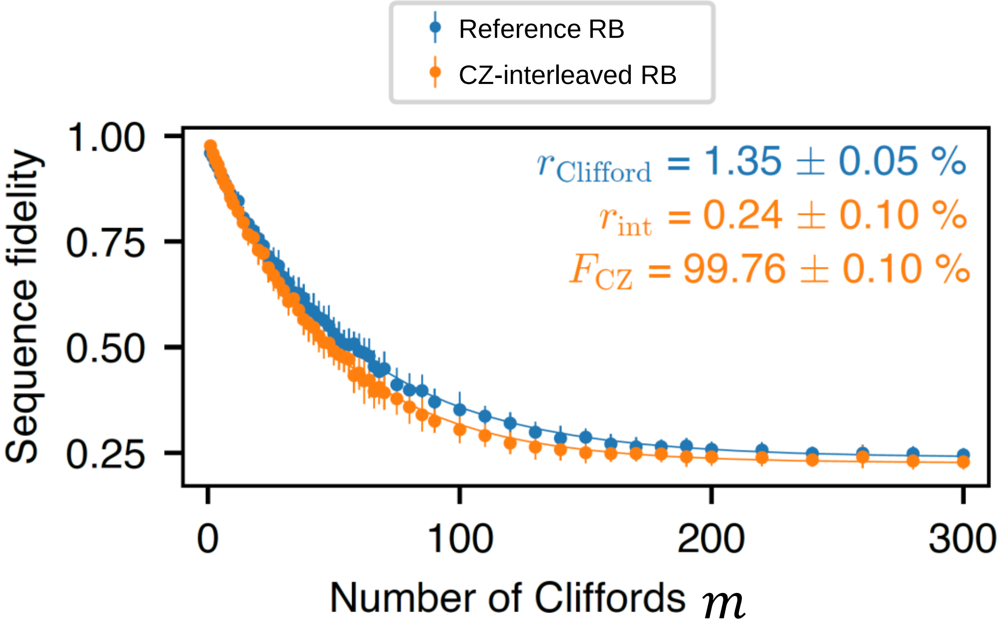
Two-Qubit Randomized Benchmarking

– **Reference** (2QB) randomized benchmarking ($F_{\text{seq, ref}}$)



– **Interleaved** (2QB) randomized benchmarking ($F_{\text{seq, int}}$)





- Measurement of the avg. error rate per interleaved gate
  - Sequence fidelity F_{seq} = the survival probability of $|00\rangle$.
  - $F_{\text{seq, ref}} = A{p_{\text{ref}}}^{m} + B$ (reference curve)
  - $F_{\text{seq, int}} = A' p_{\text{int}}^{m} + B'$ (interleaved curve)
  - The average error rate per interleaved gate (CZ) r_{int},

$$r_{\text{int}} = \left(1 - \frac{p_{\text{int}}}{p_{\text{ref}}}\right) \times \frac{2^n - 1}{2^n} = \left(1 - \frac{p_{\text{int}}}{p_{\text{ref}}}\right) \times \frac{3}{4}$$

  - **Avg. CZ fidelity** $F_{CZ} = 1 - r_{int}$