Qubit measurements: Theory and implementation (Part I)

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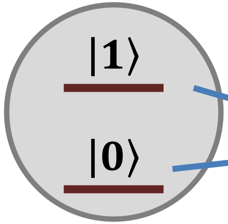
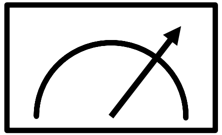
Effect of Qubit Measurement



Basis states



Measure

Classical signal

$|0\rangle : V_0 \longrightarrow 0$

$|1\rangle : V_1 \longrightarrow 1$

Quantum two level systems

Projection

$\alpha|0\rangle + \beta|1\rangle$

$V_0 \quad \mathrm{P}(|\alpha|^2) \longrightarrow |0\rangle$

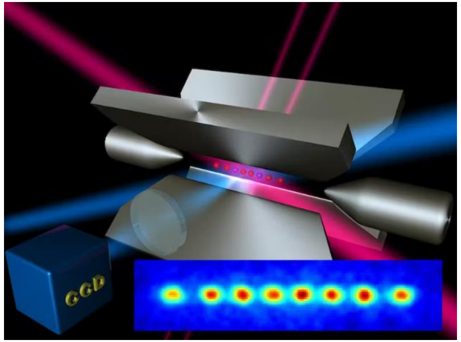
$V_1 \quad \mathrm{P}(|\beta|^2) \longrightarrow |1\rangle$

Measurement postulate

Collapse of wavefunction

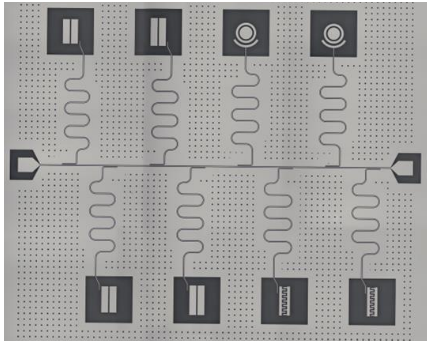
Different Platforms

Trapped ions



laserfocusworld.com

Superconducting circuits



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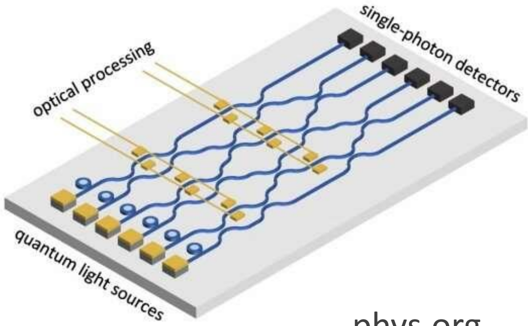
Photonic crystals



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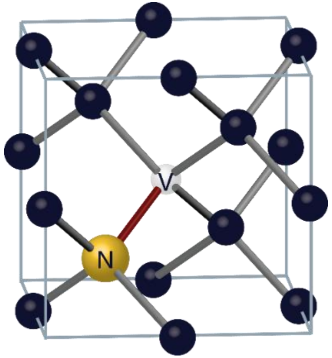
NV centers



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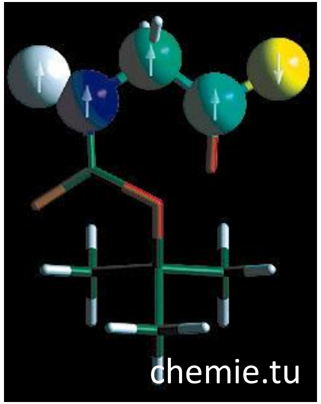
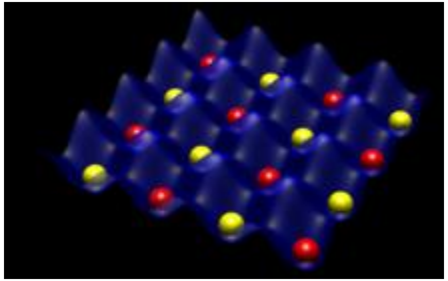
NMR



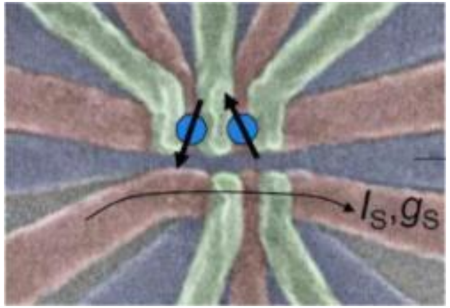
chemie.tu

Neutral atoms



NIST

Quantum dots



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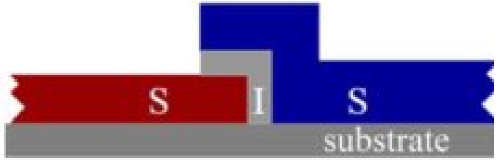
Transmon circuit



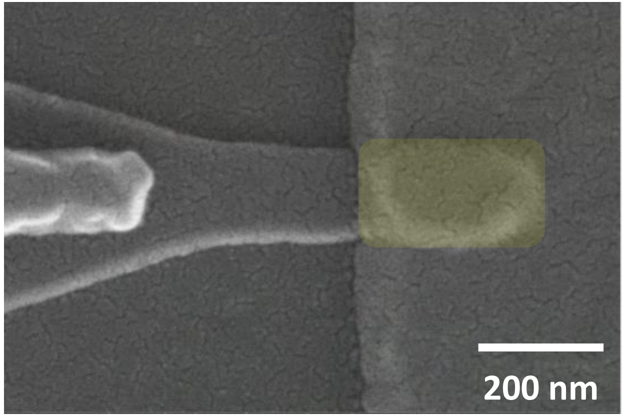
Josephson Junction

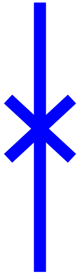
$$I(t) = I_0 \sin \delta(t)$$
$$V(t) = \varphi_0 \dot{\delta}(t)$$

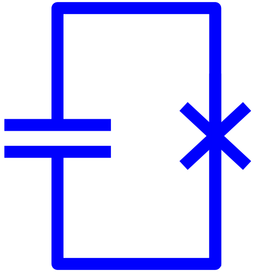
$$\varphi_0 = \hbar/2e$$

Lossless nonlinear inductor

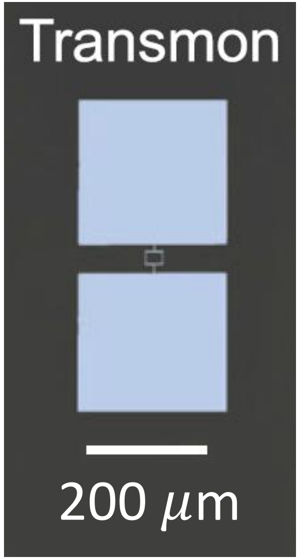
$$L_J(I) = \frac{\varphi_0}{(I_0^2 - I^2)^{1/2}}$$



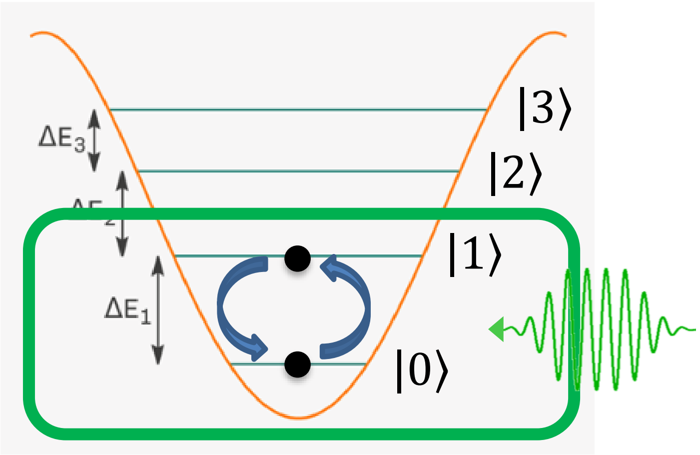




Transmon



Effective Qubit



$$f_{01} \neq f_{12} \neq f_{23}$$

$$\alpha = f_{12} - f_{01}$$

Anharmonicity

100 – 300 MHz

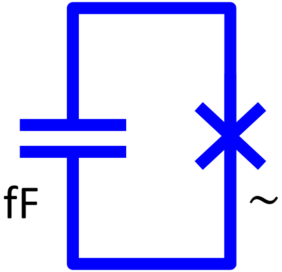
$$f_{01} \approx \frac{1}{2\pi\sqrt{L_J C}}$$

~ 5 GHz

$$k_B\, T \ll h\, f_{01}$$

20 mK

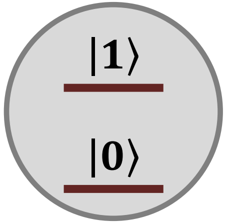
~ 240 mK

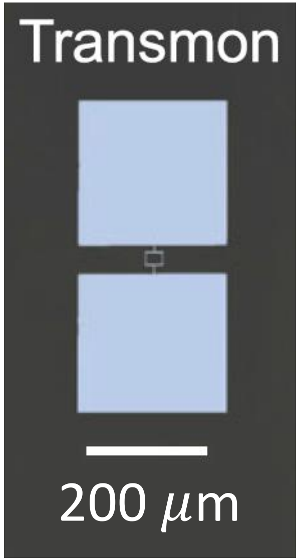


~50 fF

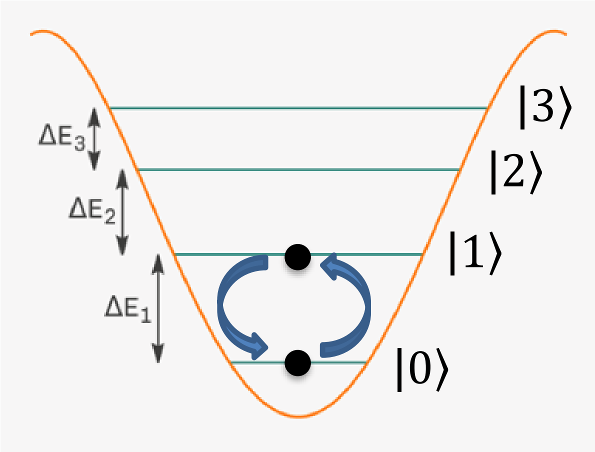
~20 nH

Transmon

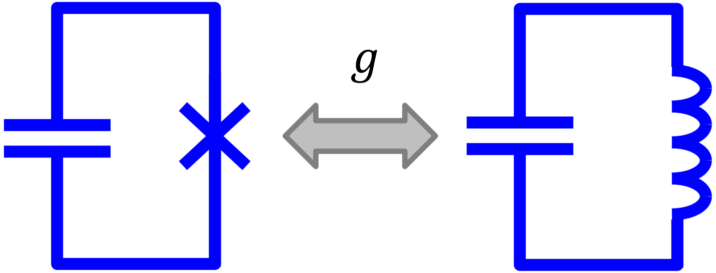




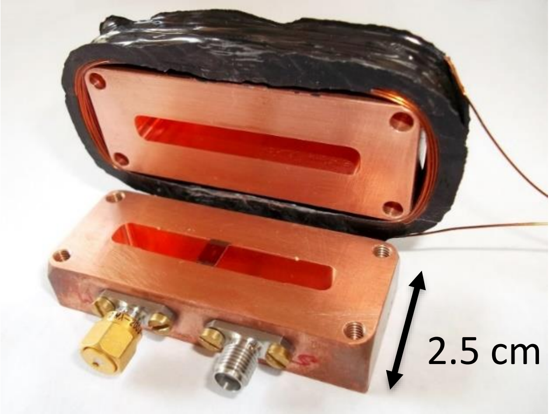
Circuit Quantum Electrodynamics

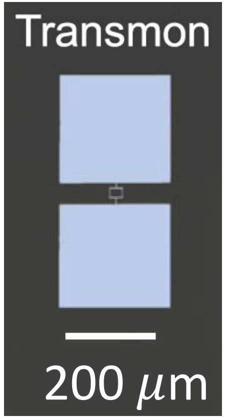


Transmon

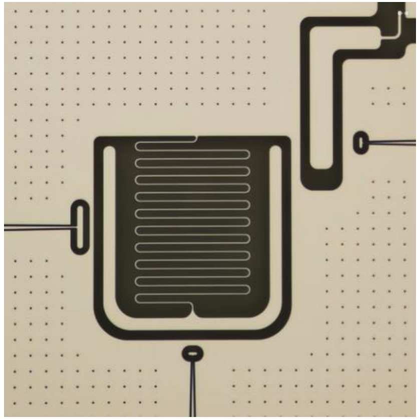


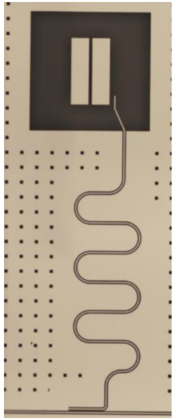
3D



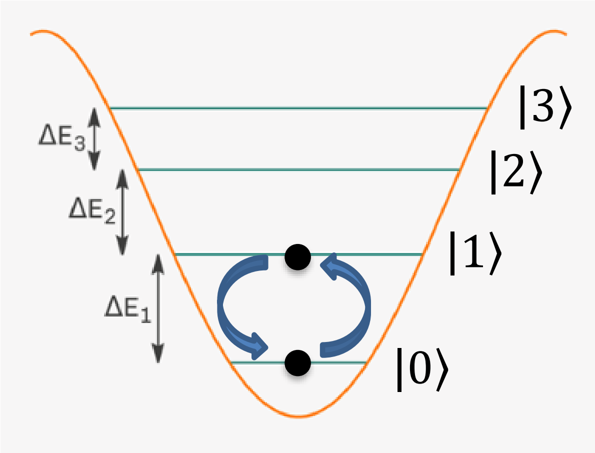


2D





Mathematical Description for Qubits



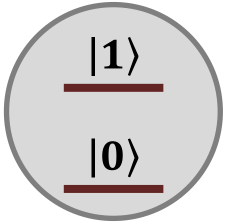
Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

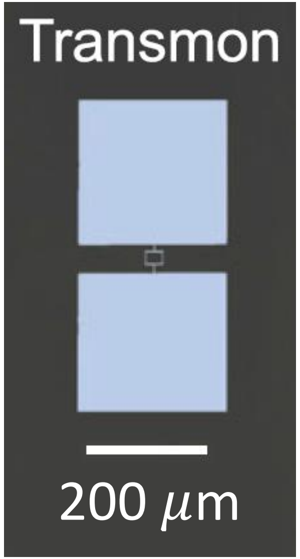
$$\frac{\hbar\omega_q}{2}\sigma_z$$



$$+1: \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$-1: \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

Measurement basis



$$\sigma_+|0\rangle = |1\rangle$$

$$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

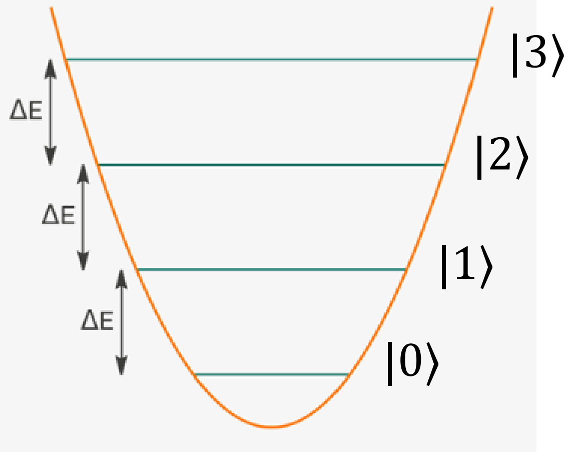
Raising/creation

$$\sigma_-|1\rangle = |0\rangle$$

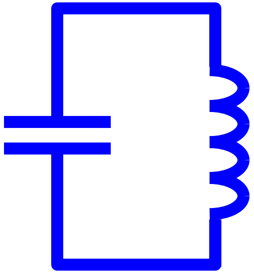
$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Lowering/annihilation

Mathematical Description for HO



Harmonic Oscillator



Bosonic operators

$a^\dagger$ ⇧ $\quad a^\dagger|n\rangle = \sqrt{n+1}\,|n+1\rangle$ — Raising/creation

a ⇩ $\quad a|n\rangle = \sqrt{n}\,|n-1\rangle$ — Lowering/annihilation

$\hat{n} = a^\dagger a$ — Photon number

$$\hat{n}|n\rangle = a^\dagger a|n\rangle = \sqrt{n}\,a^\dagger|n-1\rangle = n\,|n\rangle$$

Jaynes-Cummings Hamiltonian

$$H_{Rabi} = \frac{\omega_q}{2}\sigma_z + \omega_c a^\dagger a + g(a^\dagger + a)\sigma_x$$

$$\sigma_x = \sigma_+ + \sigma_-$$

$$= \frac{\omega_q}{2}\sigma_z + \omega_c a^\dagger a + g\left(a^\dagger\sigma_- + a\sigma_+\right) + g\left(a^\dagger\sigma_+ + a\sigma_-\right)$$

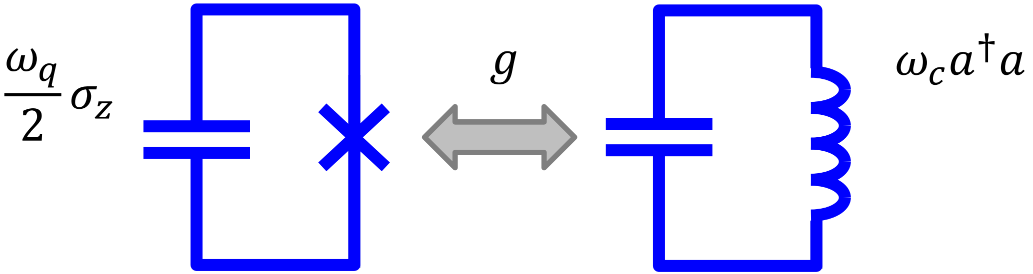
$$\approx \frac{\omega_q}{2}\sigma_z + \omega_c a^\dagger a + \frac{\chi}{2}(a^\dagger a)\sigma_z$$

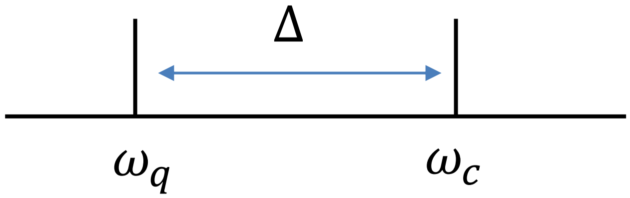
$$\Delta = \omega_q - \omega_c, g \ll \Delta$$

$$\chi = 2g^2/\Delta$$

Dispersive approximation

$$= \frac{\omega_q}{2}\sigma_z + \left(\omega_c + \frac{\chi}{2}\sigma_z\right)a^\dagger a$$



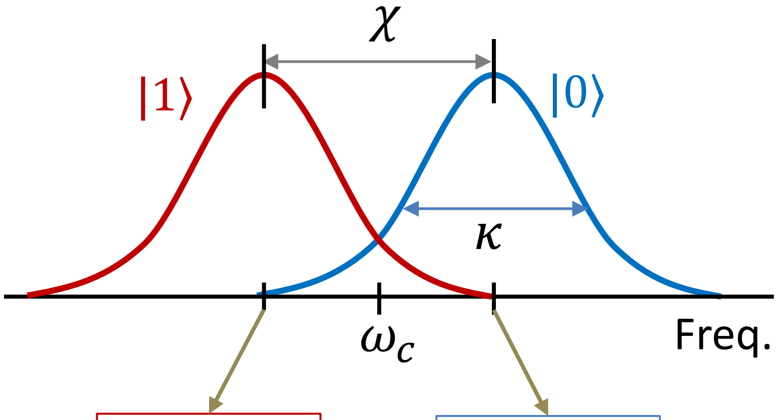


Qubit-dependent Cavity Frequency

$$H = \frac{\omega_q}{2}\sigma_z + \left(\omega_c + \frac{\chi}{2}\sigma_z\right) a^\dagger a$$

$$\omega_c'(|0\rangle_q) = \omega_c + \chi/2$$

$$\omega_c'(|1\rangle_q) = \omega_c - \chi/2$$

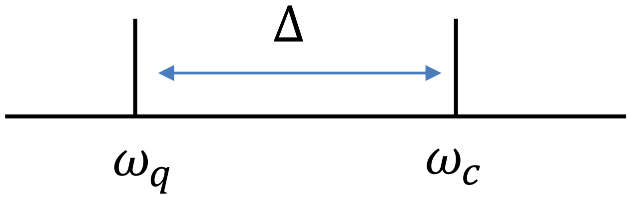


$$\omega_c - \frac{\chi}{2}$$

$$\omega_c + \frac{\chi}{2}$$

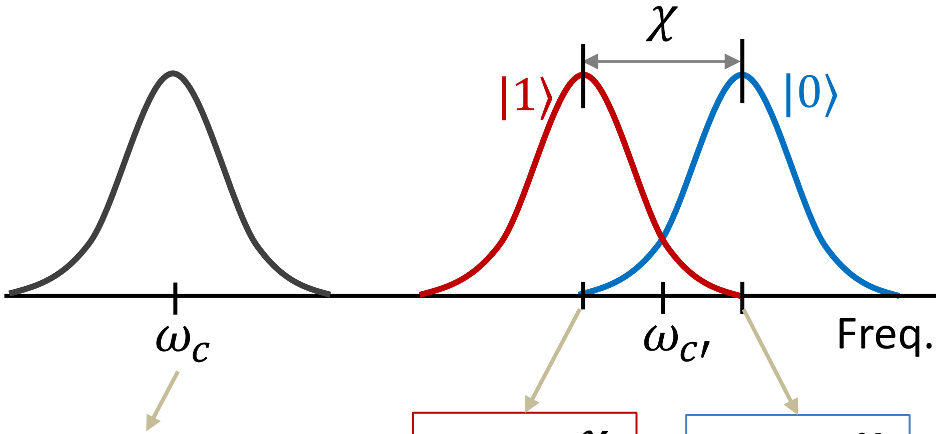
$\chi \geq \kappa$

Resolvable



Transmon-dependent Cavity Frequency

$$H = \frac{\omega_q}{2}\sigma_z + \left(\omega_c + \frac{g^2}{\Delta} + \frac{\chi}{2}\sigma_z\right)a^\dagger a$$



$\omega_{c'} - \frac{\chi}{2}$

$\omega_{c'} + \frac{\chi}{2}$

High power

Low power

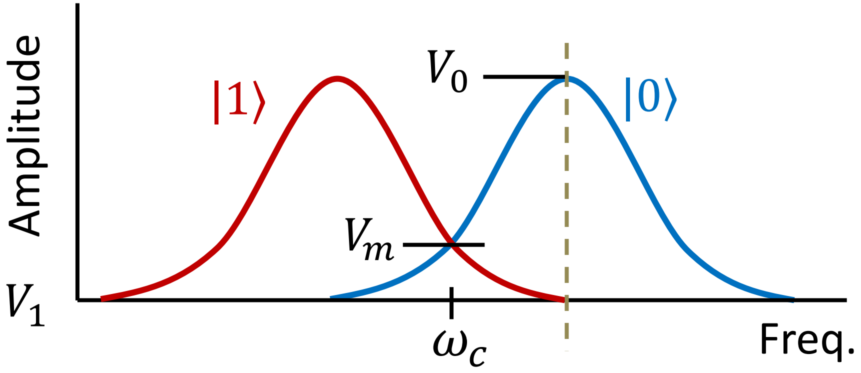
$$\omega_c'(|0\rangle_q) = \omega_c + \frac{g^2}{\Delta} + \frac{\chi}{2}$$

$$\omega_c'(|1\rangle_q) = \omega_c + \frac{g^2}{\Delta} - \frac{\chi}{2}$$

$$\omega_{c'} - \omega_c = \frac{g^2}{\Delta}$$

$$\chi = \frac{2g^2}{\Delta}\frac{\alpha}{\Delta + \alpha}$$

Qubit-dependent Cavity Response



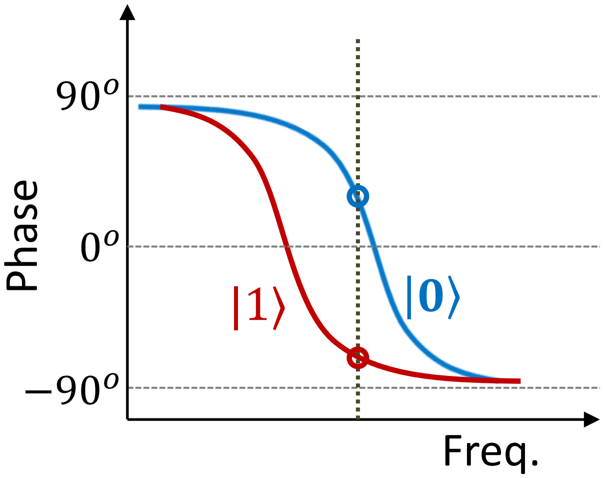
Probe at $\omega_c + \chi/2$

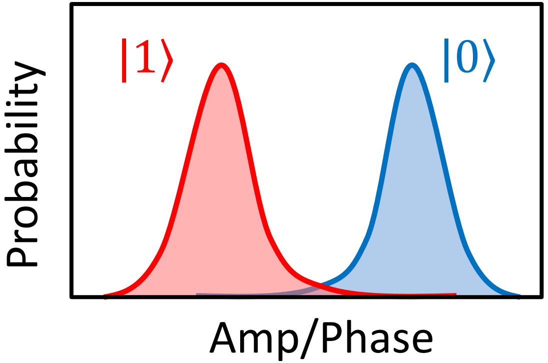
$|0\rangle: V_0$
$|1\rangle: V_1$

Probe at ω_c

$|0\rangle: V_m$
$|1\rangle: V_m$

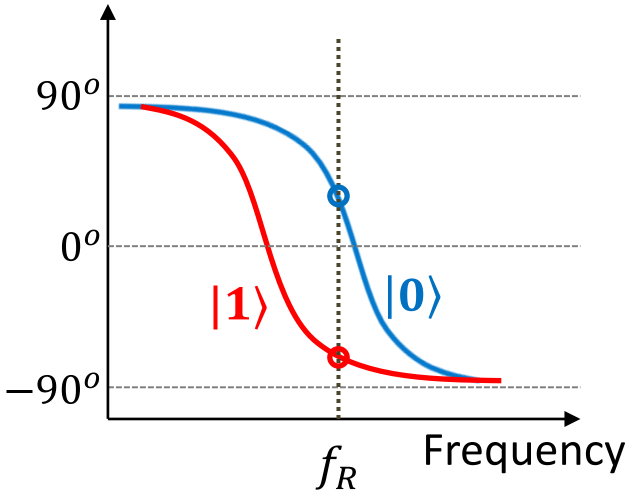
Can't distinguish

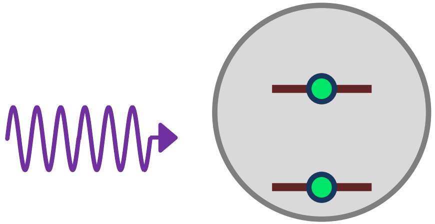




$|0\rangle: \phi_0$
$|1\rangle: \phi_1$

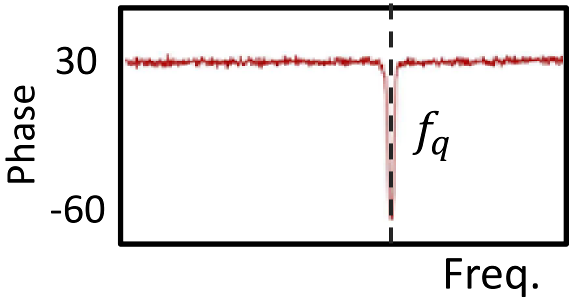
Qubit Spectroscopy





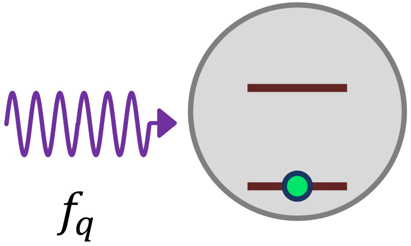
Two-tone spectroscopy

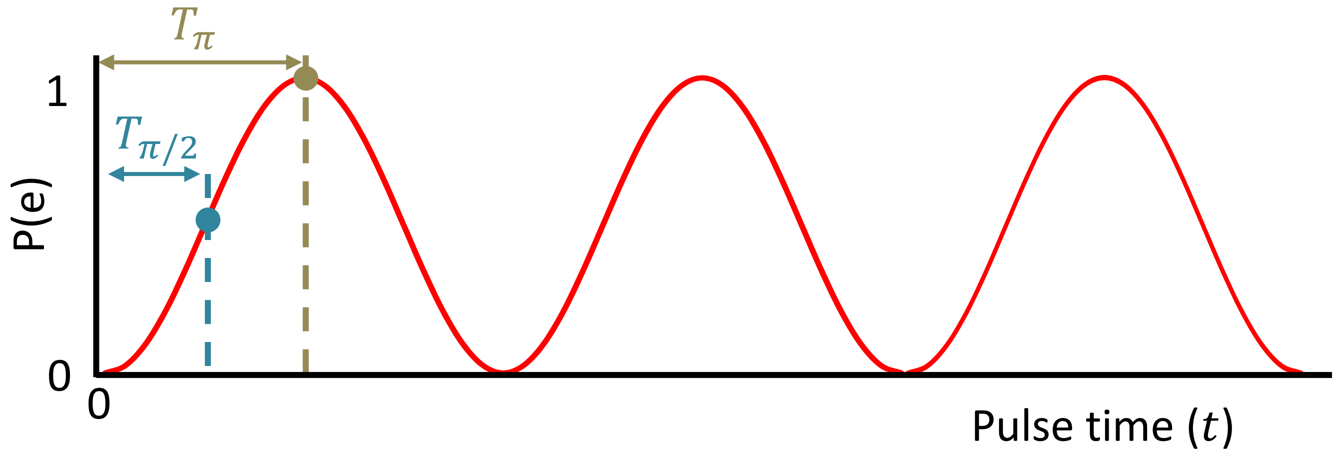
Probe at f_R + Sweep 2nd tone around f_q



Basic Characterization of a Qubit

π pulse ⇦ **Rabi Oscillation** ⇨ $\pi/2$ pulse





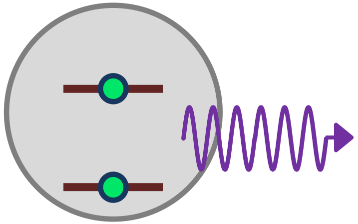
Basic Characterization of a Qubit

Rabi Oscillation

π pulse → T_1: Relaxation

$\pi/2$ pulse → T_2: Dephasing

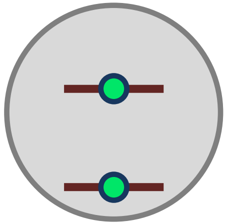
Decoherence



$\alpha|0\rangle + \beta|1\rangle$

↓

$|0\rangle$



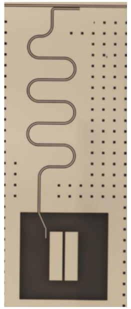
$|0\rangle + e^{i\phi}|1\rangle$

↓

Incoherent mix of $|0\rangle$ and $|1\rangle$

Signal Demodulation

$A \sin \omega_d t$



$$S = A' \sin(\omega_d t + \phi)$$

$$= A' \cos\phi \sin(\omega_d t) + A' \sin\phi \cos(\omega_d t)$$

Quadrature-phase signal (Q)

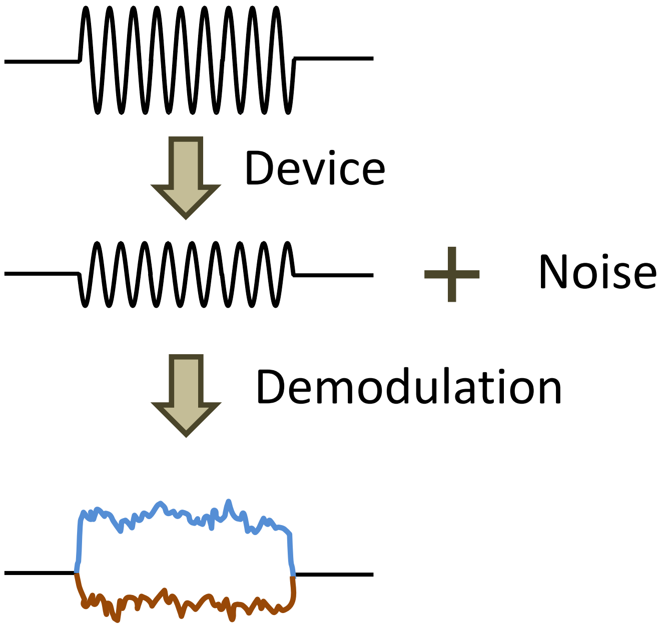
In-phase signal (I)

Demodulation

$$S \cdot 2\cos\omega_d t = A'[\sin\phi \cos(2\omega_d t) + \cos\phi \sin(2\omega_d t) + \sin\phi] \Rightarrow A' \sin\phi$$

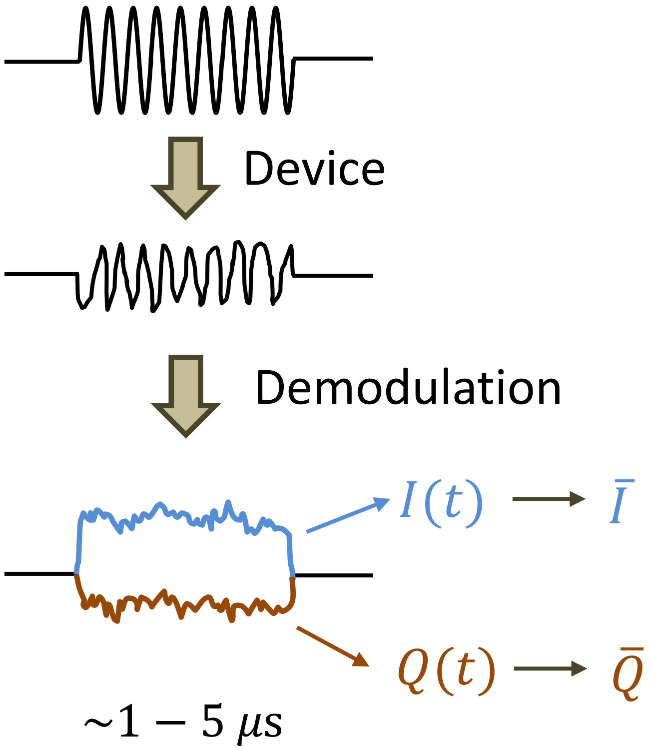
$$S \cdot 2\sin\omega_d t = A'[\sin\phi \sin(2\omega_d t) - \cos\phi \cos(2\omega_d t) + \cos\phi] \Rightarrow A' \cos\phi$$

I-Q Plot

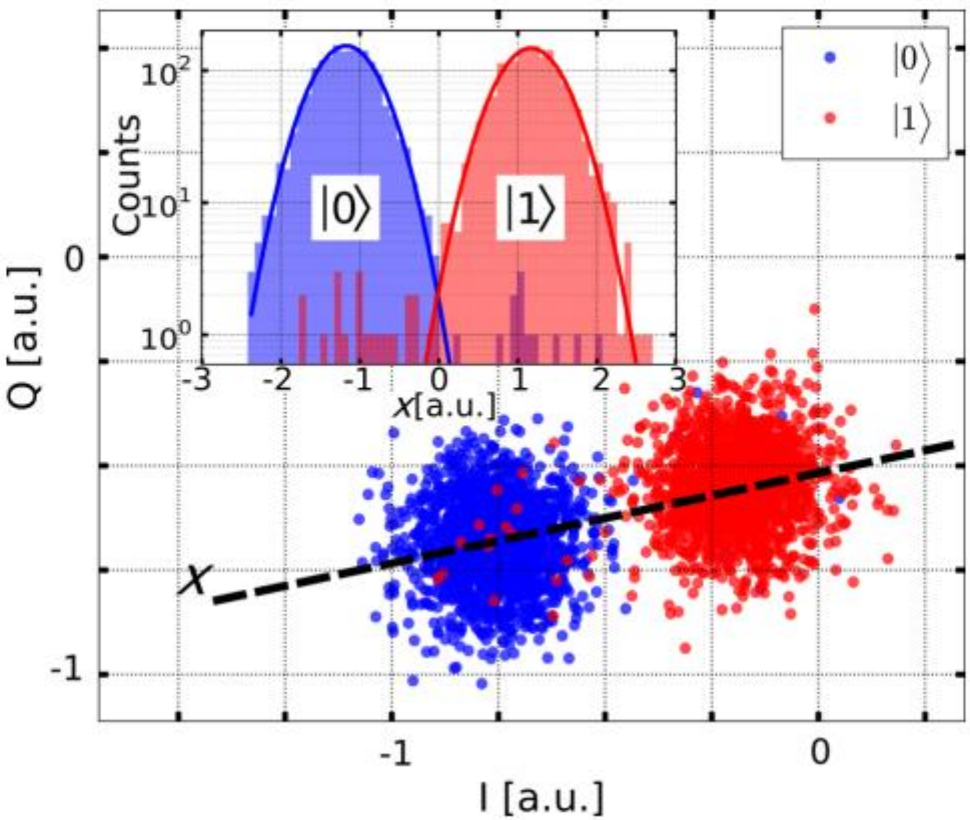


PRL 112, 190504 (2014)

I-Q Plot

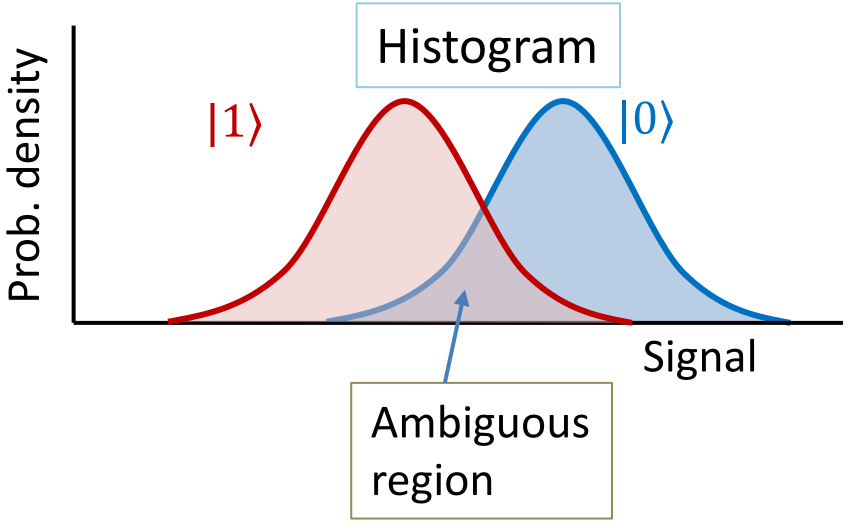


Readout pulse



PRL 112, 190504 (2014)

Measurement Error



Prepared state / Measured state

Measured \ Prepared	$\|0\rangle$	$\|1\rangle$
$\|0\rangle$	96%	10%
$\|1\rangle$	4%	90%

Error = Area of the overlap region/2 = 0.07

$$\begin{bmatrix} 0.96 & 0.10 \\ 0.04 & 0.90 \end{bmatrix} \begin{bmatrix} 100 \\ 0 \end{bmatrix} = \begin{bmatrix} 96 \\ 4 \end{bmatrix}$$

Fidelity: $\mathcal{F}$ = 1 – overlapping area/2 = 0.93

$$\begin{bmatrix} 0.96 & 0.10 \\ 0.04 & 0.90 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \end{bmatrix} = \begin{bmatrix} 10 \\ 90 \end{bmatrix}$$

Fixing Measurement Error

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \Rightarrow \begin{bmatrix} 50 \\ 50 \end{bmatrix}$$

Error matrix
Confusion matrix

$$\begin{bmatrix} 0.96 & 0.10 \\ 0.04 & 0.90 \end{bmatrix} \begin{bmatrix} 50 \\ 50 \end{bmatrix} = \begin{bmatrix} 53 \\ 47 \end{bmatrix}$$

$$\overleftrightarrow{M} \times \vec{n}_{ideal} = \vec{n}_{expt}$$

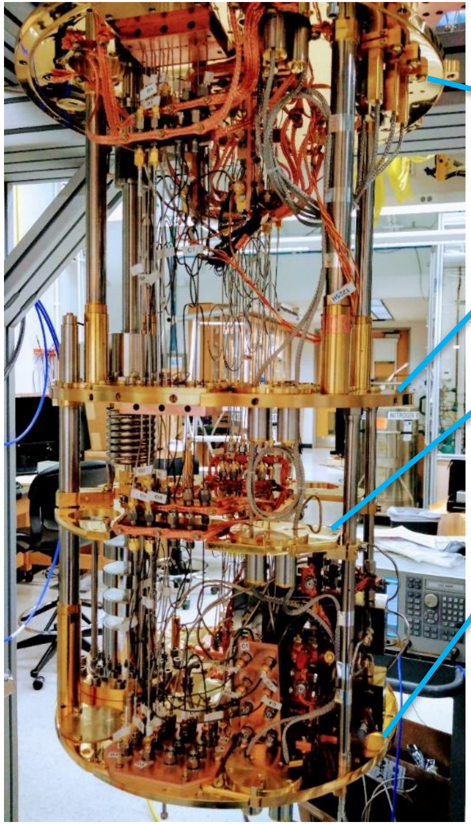
$$\vec{n}_{ideal} = \overleftrightarrow{M}^{-1} \times \vec{n}_{expt}$$

Measurement error mitigation

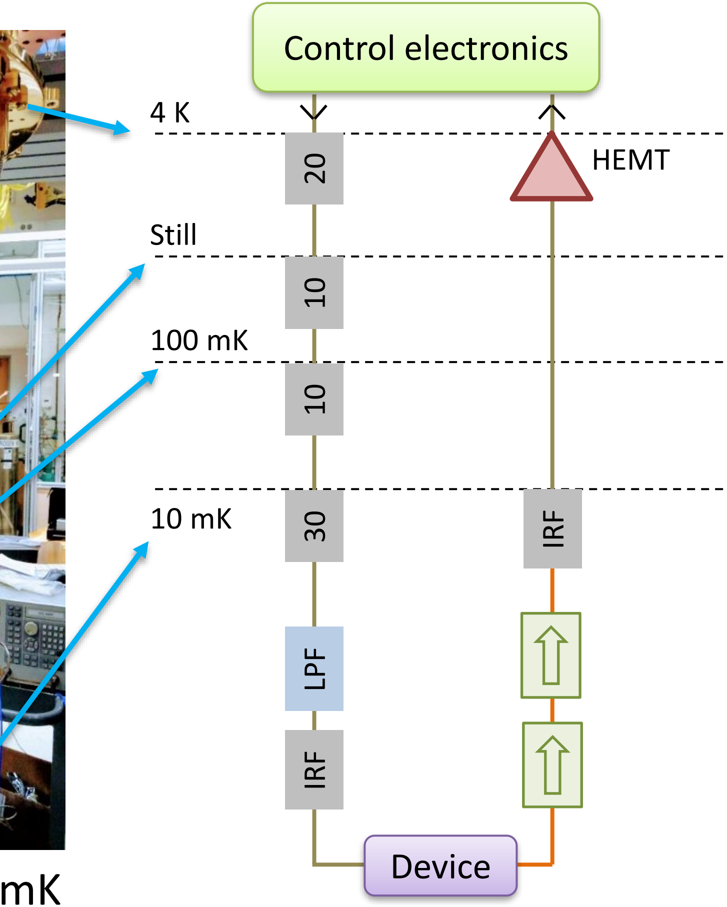
Prepared state

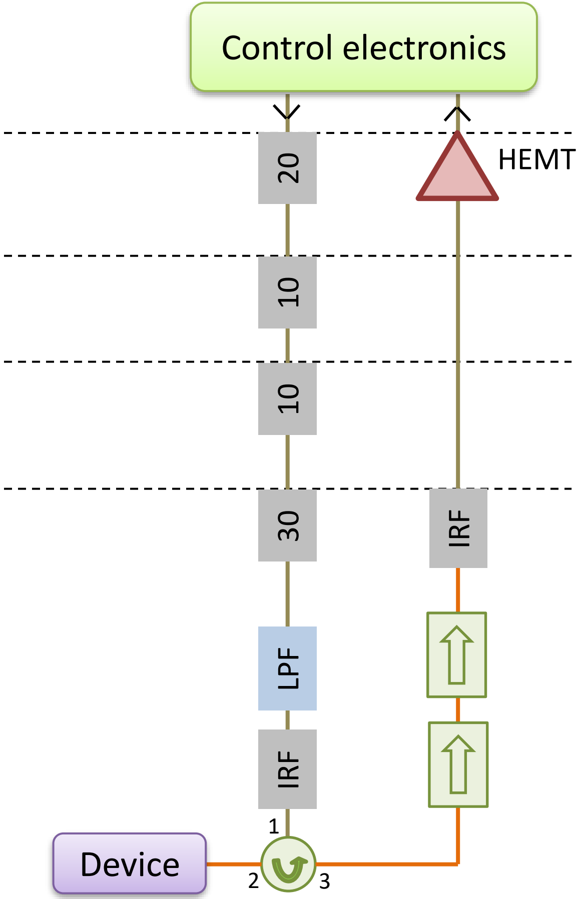
Measured state	$|0\rangle$	$|1\rangle$
$|0\rangle$	96%	10%
$|1\rangle$	4%	90%

Measurement Chain



Dilution fridge ~ 10 mK

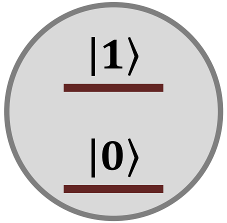


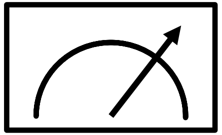


SQMS SUPERCONDUCTING QUANTUM MATERIALS & SYSTEMS CENTER

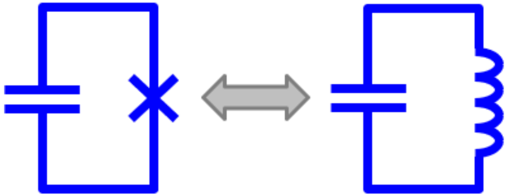
Summary of Part I

- ➢ Meaning of qubit measurement

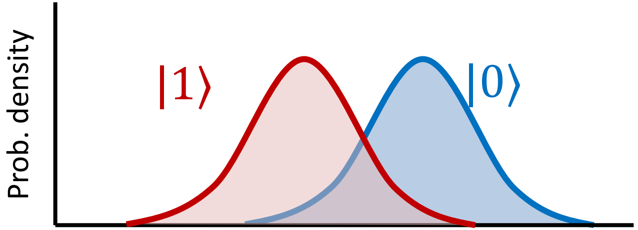




- ➢ Circuit QED architecture



- ➢ Jaynes-Cummings Hamiltonian
- ➢ I-Q plot, histogram



- ➢ Measurement error mitigation