Qubit measurements: Theory and implementation (Part II)

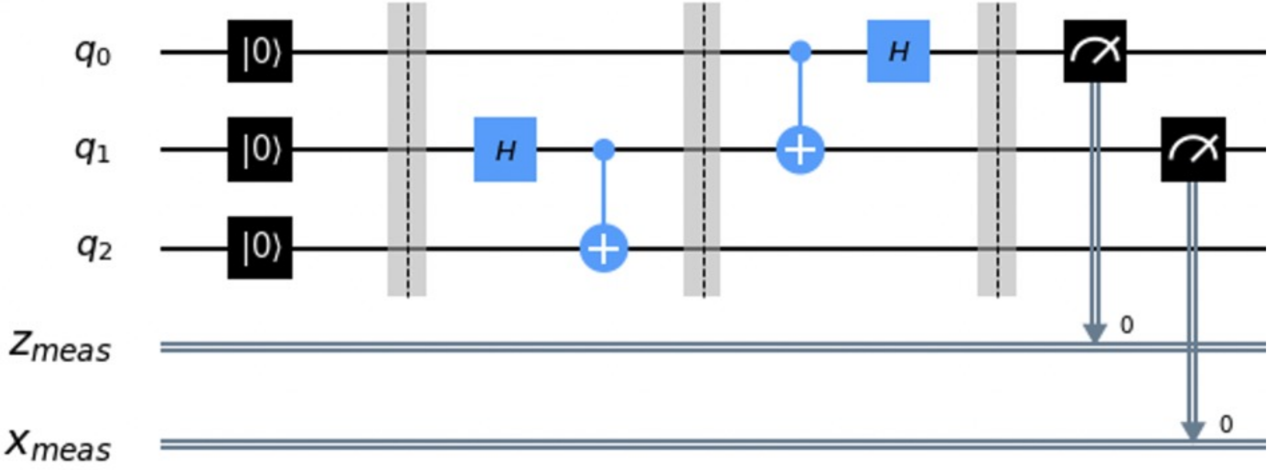
Tanay Roy, Silvia Zorzetti

Quantum algorithms

Reset

Gates

Measurements



Calibration

- Gate
- Noise

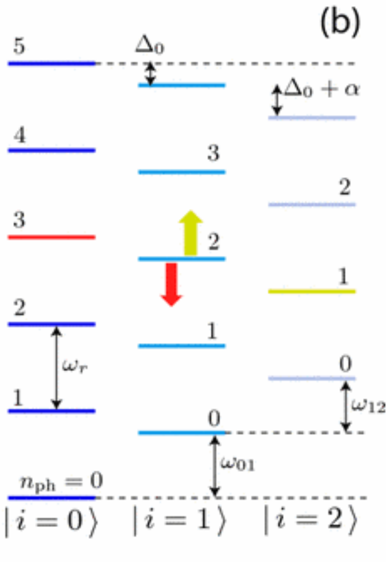
Circuit QED measurements

Circuit QED

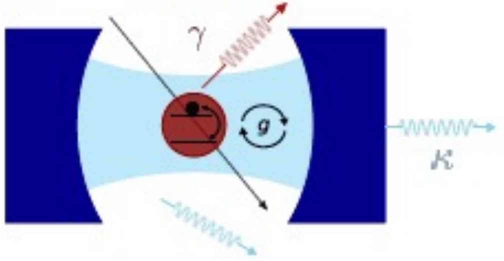
Measurements of a superconducting transmon qubit via a coupled microwave resonator

Dispersive regime: The detuning between the cavity and the qubit (Δ_0) is much larger than the qubit-cavity coupling (g): $\Delta_0 \gg g_{01}$

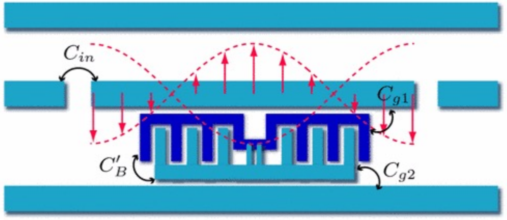
Dispersive interaction: The interaction between the cavity and the qubit is shown via a shift in the coupled qubit-cavity energy levels



The cavity resonator is used as an indirect measurement channel



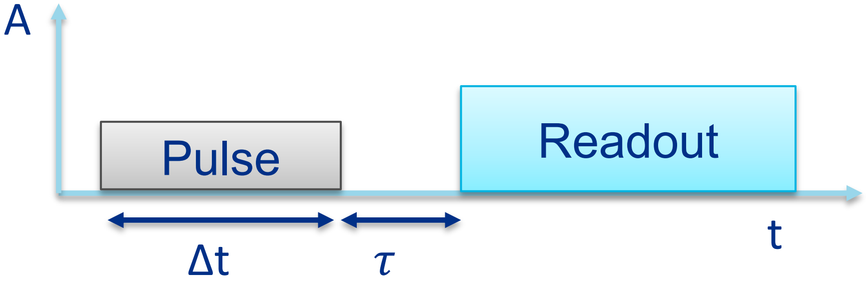
Schematic representation of cavity QED

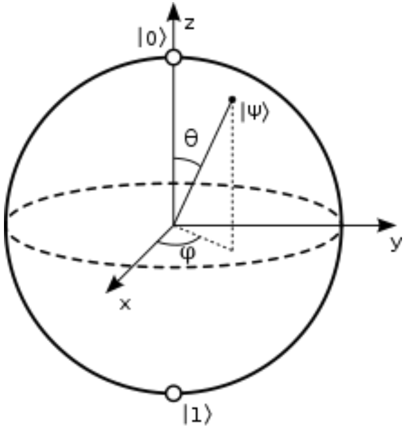


Schematic of 2D on-chip circuit QED, showing a resonator and the coupled transmon

Koch, Jens, et al. "Charge-insensitive qubit design derived from the Cooper pair box." *Physical Review A* 76.4 (2007): 042319.

Encoding a quantum state



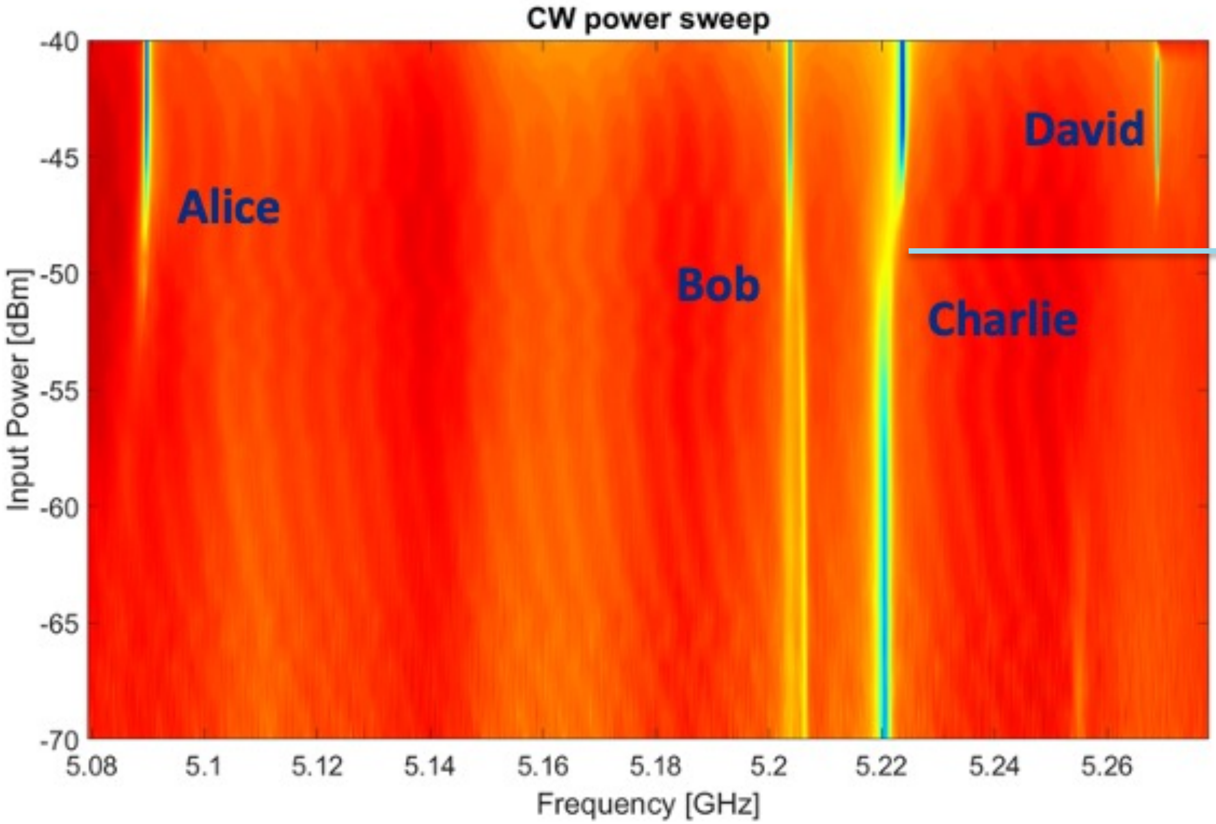


$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\varphi}\sin(\theta/2)|1\rangle$$

- *Δt*: Pulse duration
- τ: Delay
- Readout: Measure the state during the readout

Qubit characterization: find the amplitude and the length of qubit and readout pulses

Qubit characterization



Dispersive effect

Qubit characterization

All these data are necessary to calibrate a qubit

Qubit	f_q (GHz)	f_{max} (GHz)	α_q (MHz)	f_r (GHz)	χ (MHz)	T_1	T_2^*	T_2^E	P(e)
Alice									
Bob									
Charlie									
David									

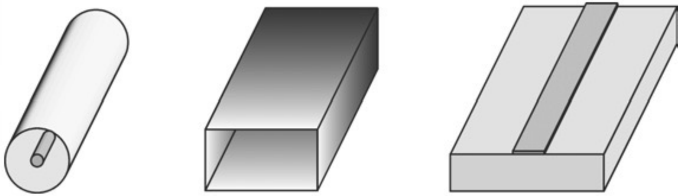
We need to introduce RF measurements

A glimpse of RF

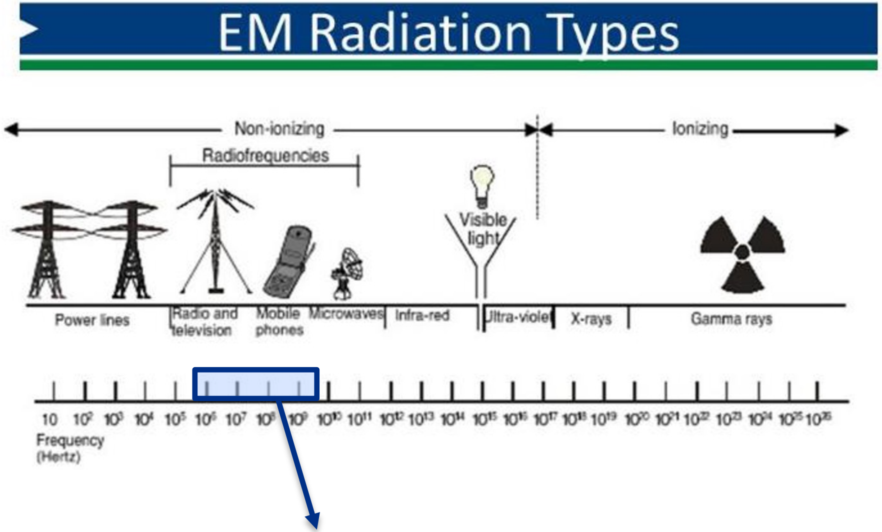
Some of this material is based upon Manfred Wendt’s lectures at CAS and USPAS

Also see Alex Romanenko lecture

Transmission lines



Transmission lines radiating electro-magnetic (EM) fields

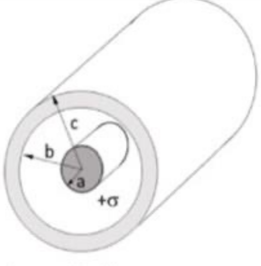


MHz – GHz for many applications in quantum and particle accelerators

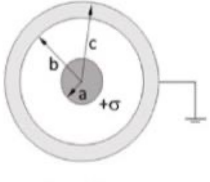
Different types of transmission lines

Transport electromagnetic waves

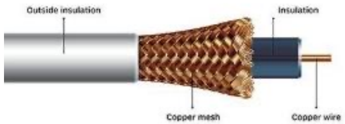
Coaxial cable



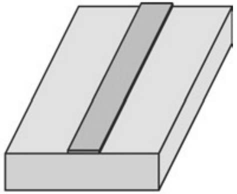
Perspective View

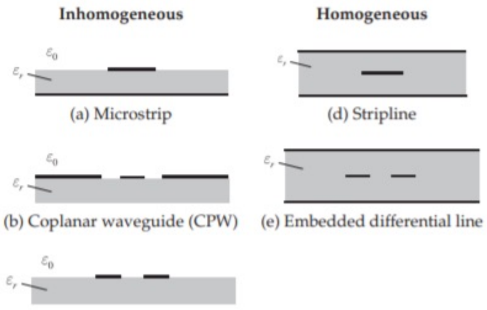


Front View



Planar transmission lines





(a) Microstrip

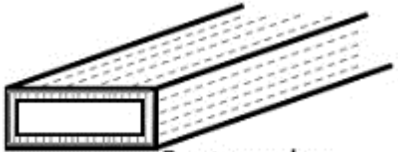
(d) Stripline

(b) Coplanar waveguide (CPW)

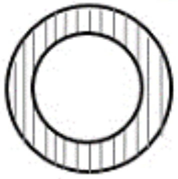
(e) Embedded differential line

(c) Differential line

Waveguides

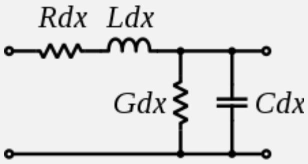


Rectangular waveguide



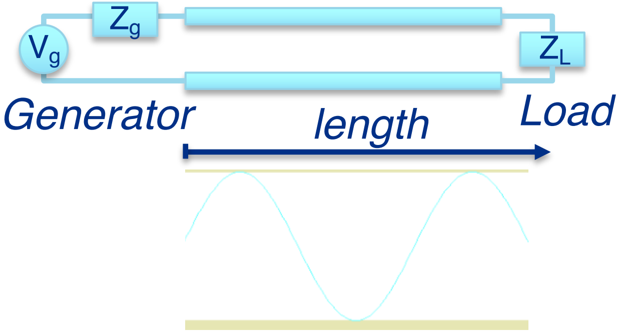
Circular waveguide

Telegrapher's equations

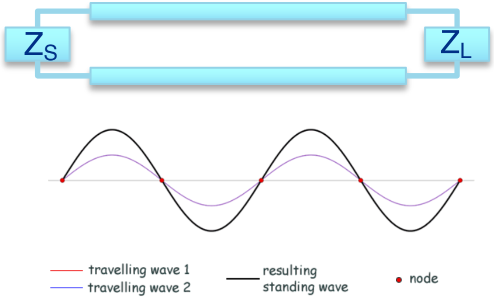


- Characteristic impedance of line $Z_0 = \sqrt{\frac{L}{C}}$
- Velocity Factor $v_f = \frac{1}{\sqrt{\varepsilon_r \mu_r}}$
- Medium properties: C, L, $\boldsymbol{\varepsilon}$, $\boldsymbol{\mu}$

From transmission lines to resonant cavities

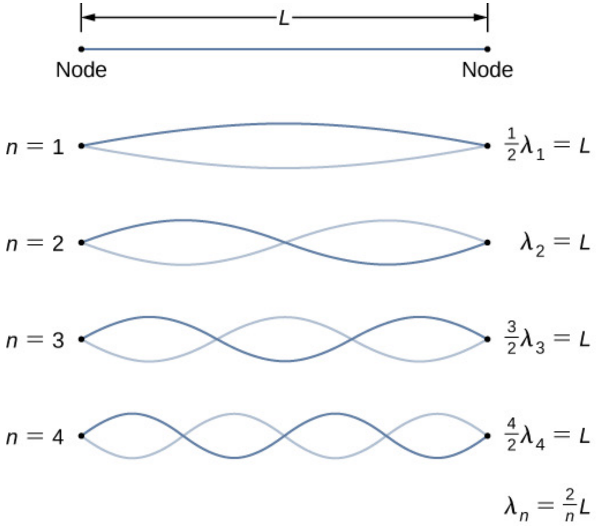


Matched load: The energy is dissipated in the load, leading to zero reflections.

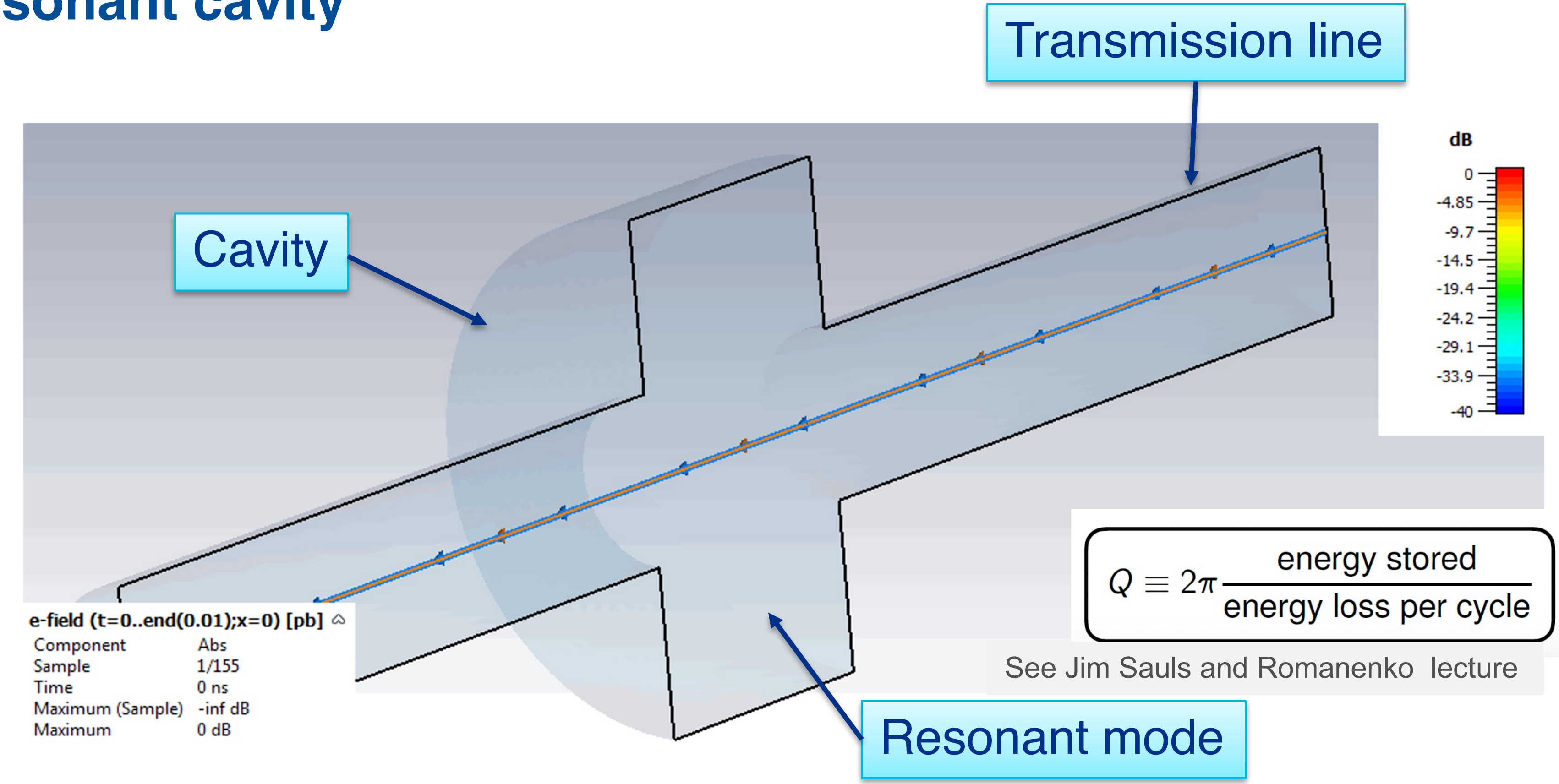


Non-dissipative loads: No energy is dissipated, and the wave is reflected.

Resonant modes



Resonant cavity



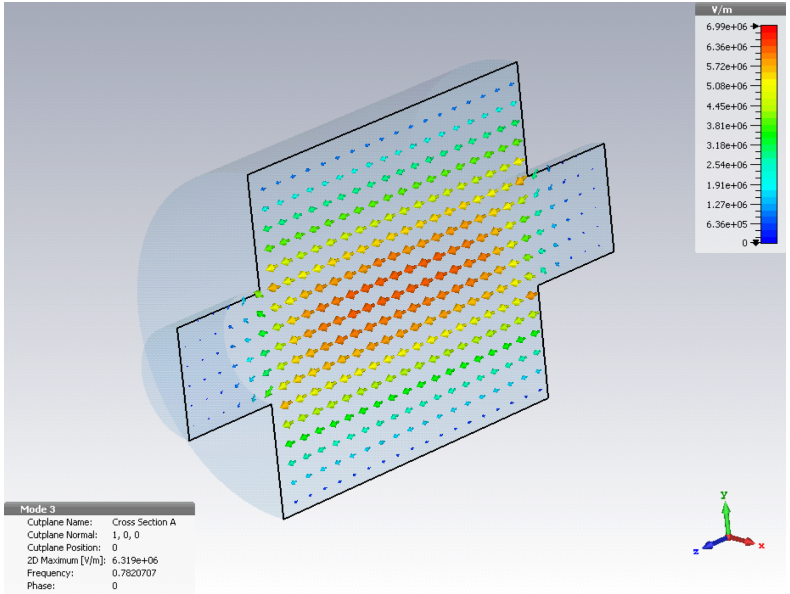
```
e-field (t=0..end(0.01);x=0) [pb]
Component          Abs
Sample             1/155
Time               0 ns
Maximum (Sample)   -inf dB
Maximum            0 dB
```

$$Q \equiv 2\pi \frac{\text{energy stored}}{\text{energy loss per cycle}}$$

See Jim Sauls and Romanenko lecture

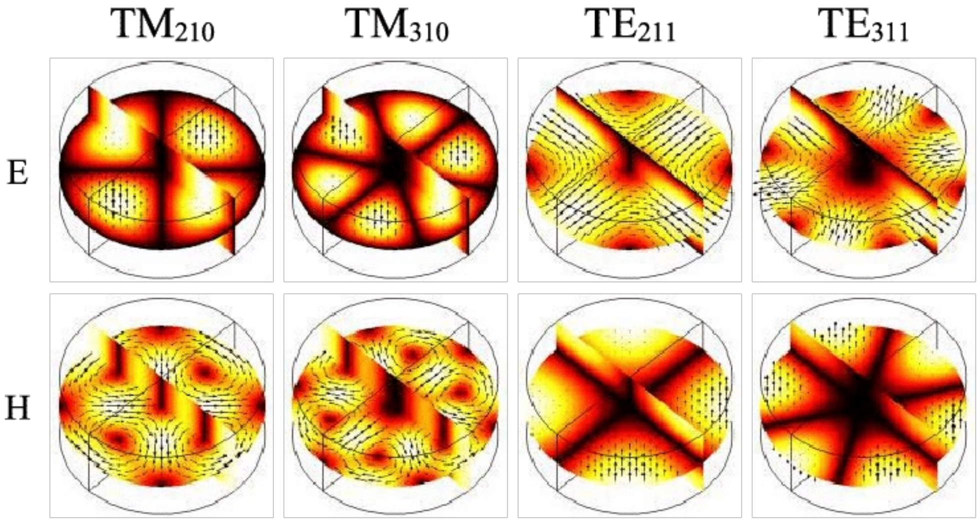
Resonant modes

3D cylindrical cavity



Mode 3

Cutplane Name:	Cross Section A
Cutplane Normal:	1, 0, 0
Cutplane Position:	0
2D Maximum [V/m]:	6.319e+06
Frequency:	0.7820707
Phase:	0



RF measurements

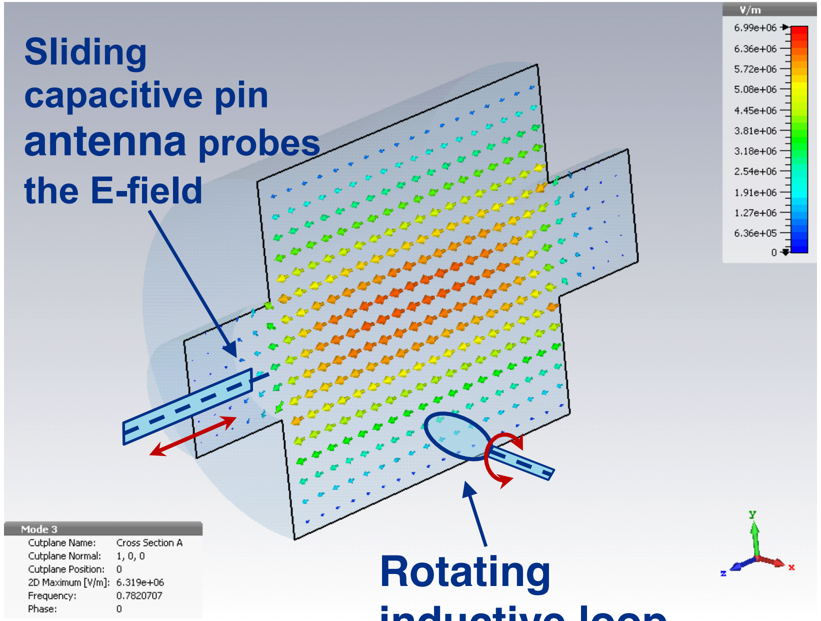
Resonators and cavities excite RF modes at different frequencies

Antennas are needed to probe the E-field and measure the

- E-field on the z-axis using a capacitive coupling pin
- H-field with inductive loop antenna probes

Antenna probes

Resonant cavity



```
Mode 3
  Cutplane Name:        Cross Section A
  Cutplane Normal:      1, 0, 0
  Cutplane Position:    0
  2D Maximum [V/m]:     6.319e+06
  Frequency:            0.7820707
  Phase:                0
```

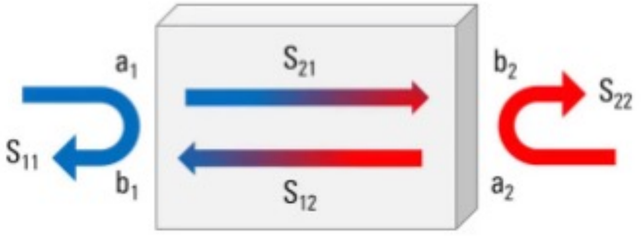
Sliding capacitive pin antenna probes the E-field

Rotating inductive loop antenna probes the H-field

Scattering parameters

Scattering parameters to describe the input-output relationships between ports in an electrical system

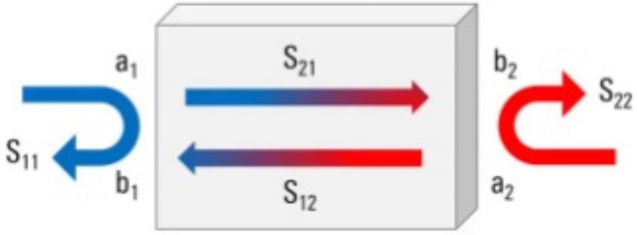
- **S11** is the input port voltage reflection coefficient
- **S12** is the reverse voltage gain
- **S21** is the forward voltage gain
- **S22** is the output port voltage reflection coefficient



$S_{11} = b_1/a_1$
$S_{21} = b_2/a_1$
$S_{12} = b_1/a_2$

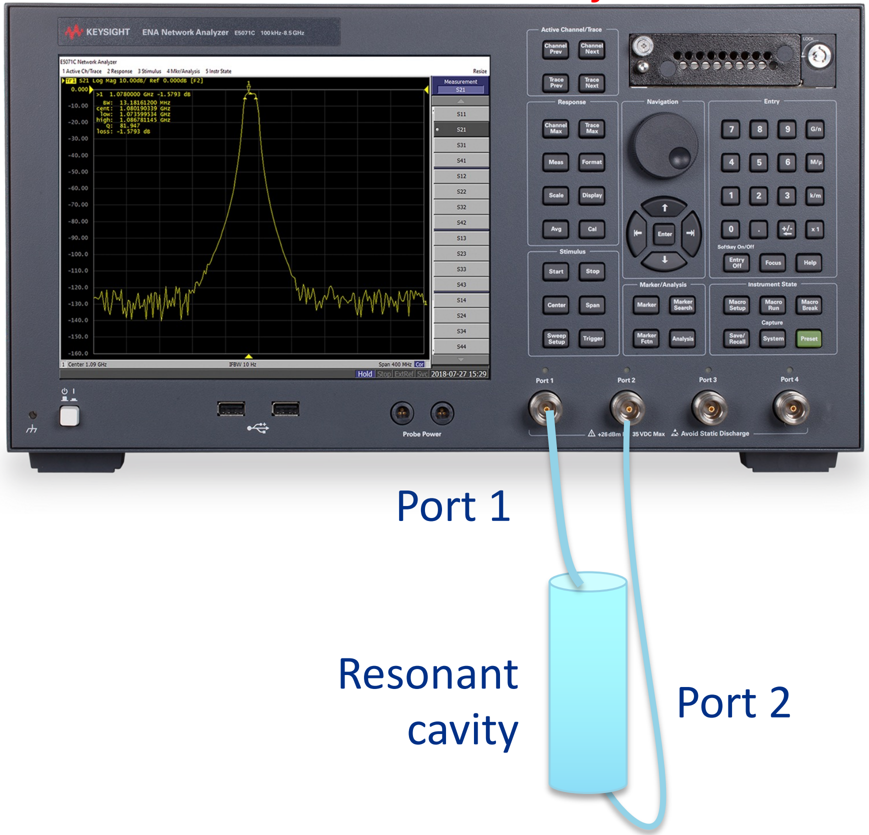
Scattering parameters

The S-parameters matrix of an RF network is acquired by measurement characterization with a vector network analyzer (VNA)



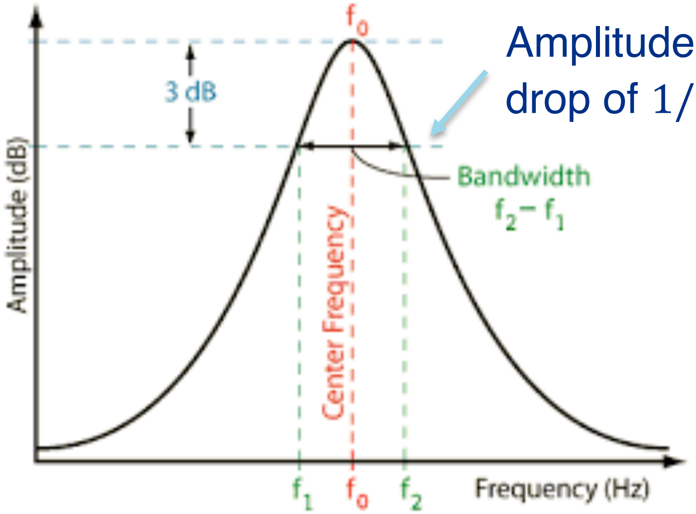
$$S_{11} = b_1/a_1$$
$$S_{21} = b_2/a_1$$
$$S_{12} = b_1/a_2$$

Vector Network Analyzer



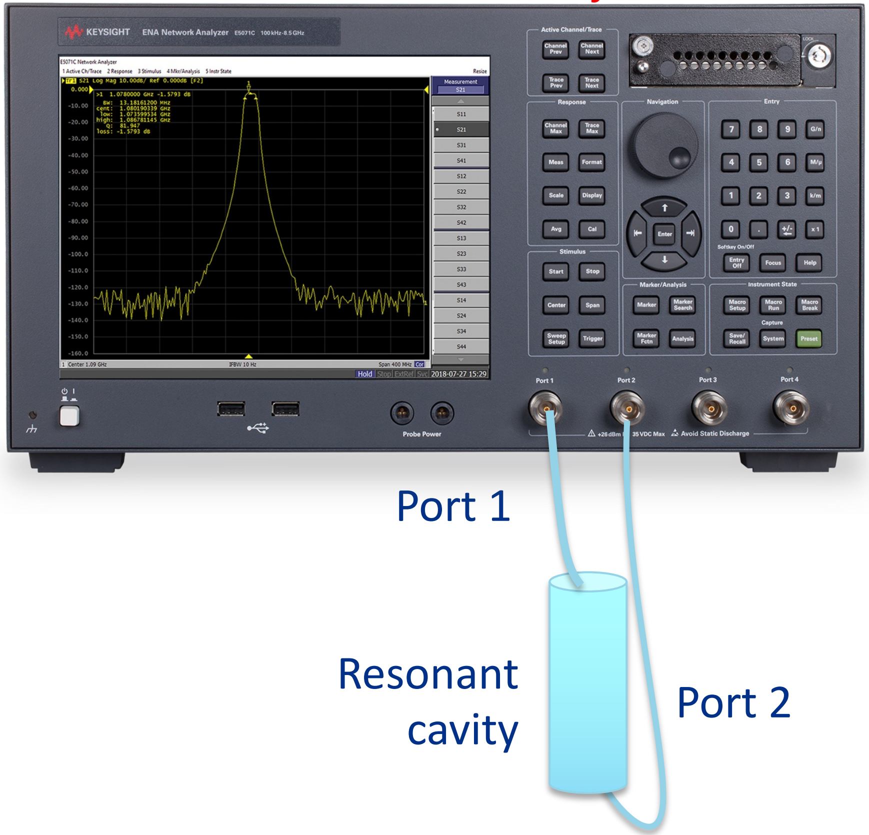
Scattering parameters

Quality factor: $Q = \dfrac{f_0}{BW}$



Amplitude drop of $1/\sqrt{2}$

Vector Network Analyzer



Port 1

Resonant cavity

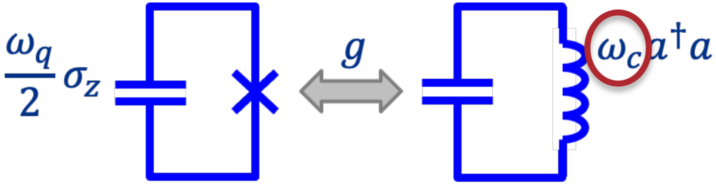
Port 2

… back to **Circuit QED measurements**

Resonator Spectroscopy (1)

This is the first measurements to be performed when characterizing a superconducting qubit

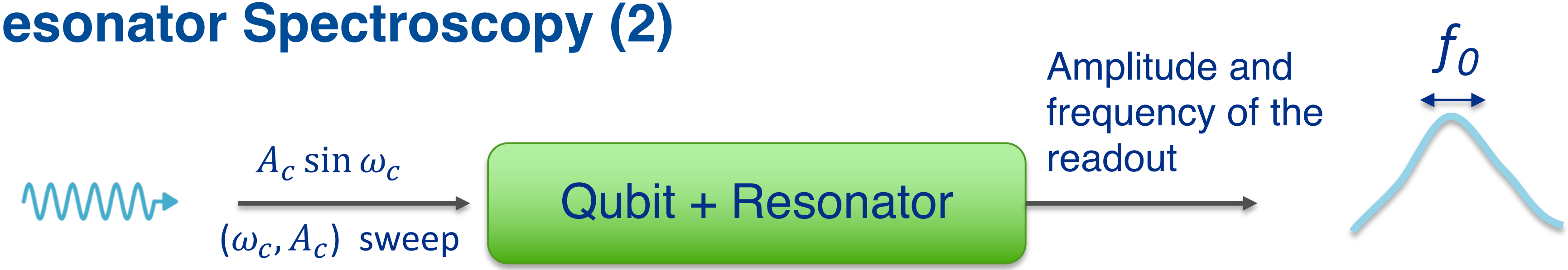
Goal: Find the frequency of the cavity/resonator and verify that the qubit is 'alive'

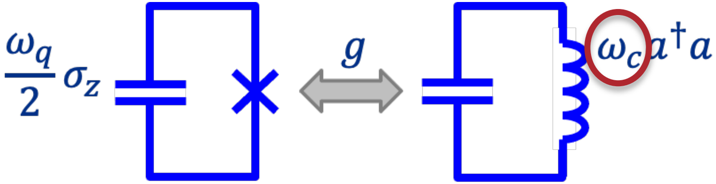


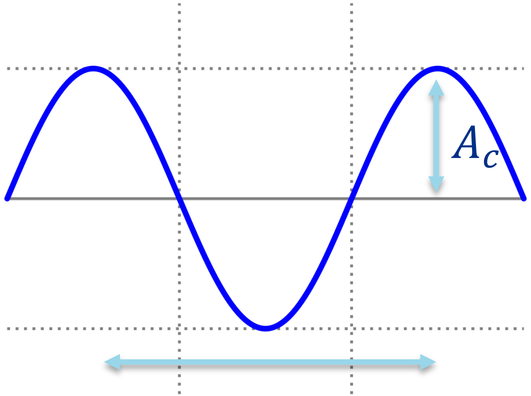
What to do: Perform a frequency sweep at 'high' power and then reduce the power

What to observe: The quantum effect of the dispersive shift

Resonator Spectroscopy (2)







$$period = \frac{c}{\omega_c/2\pi}$$

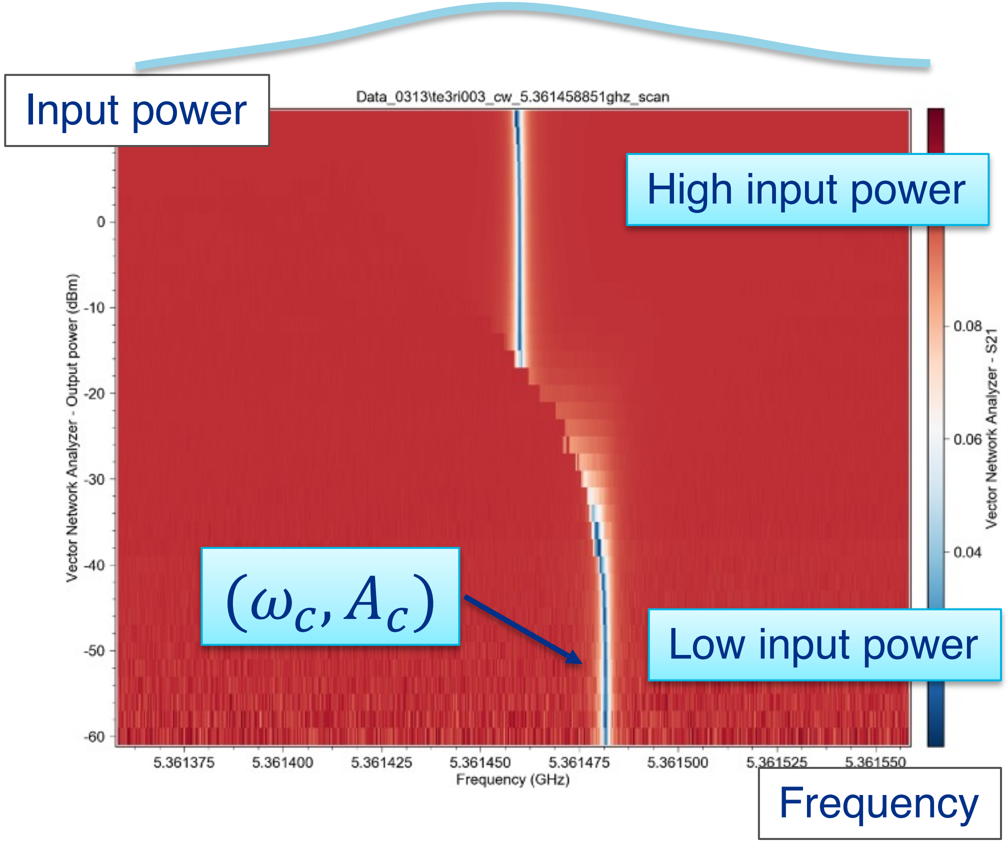
Resonator Spectroscopy (3)

At high power: The system behaves classically, and it is possible to observe the 'bare' frequency of the cavity.

- The eigenvalues are simply the same as for the cavity and qubit when not interacting.

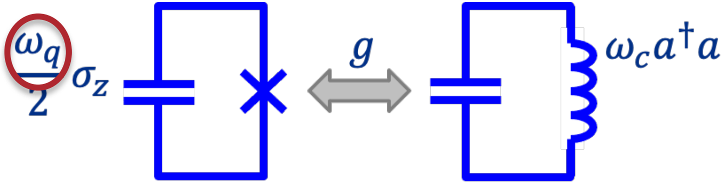
At low applied power: The resonator population is on the order of a few photons, and it is possible to observe the quantum effects of the dispersive shift.

- **Bare states:** no more eigenstates for the system
- **Dressed states:** hybridization of the cavity photon and the qubit excitation

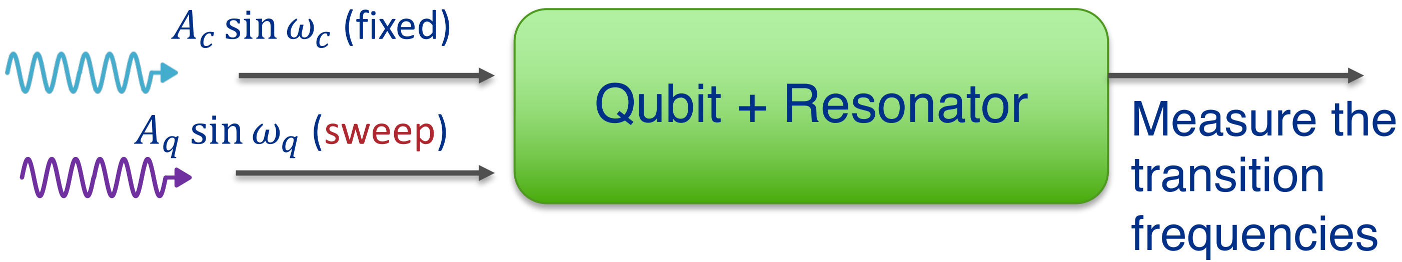


Qubit Spectroscopy (1)

Goal: Find the transition frequencies of the qubit



What to do: Apply two tones: resonator + sweep of qubit frequency



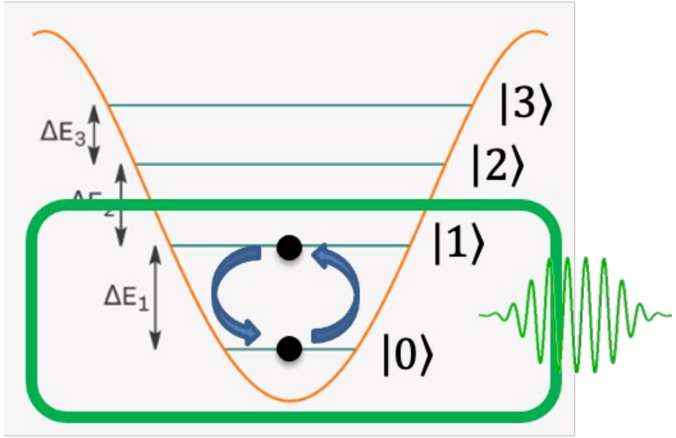
$A_c \sin \omega_c$ (fixed)

$A_q \sin \omega_q$ (sweep)

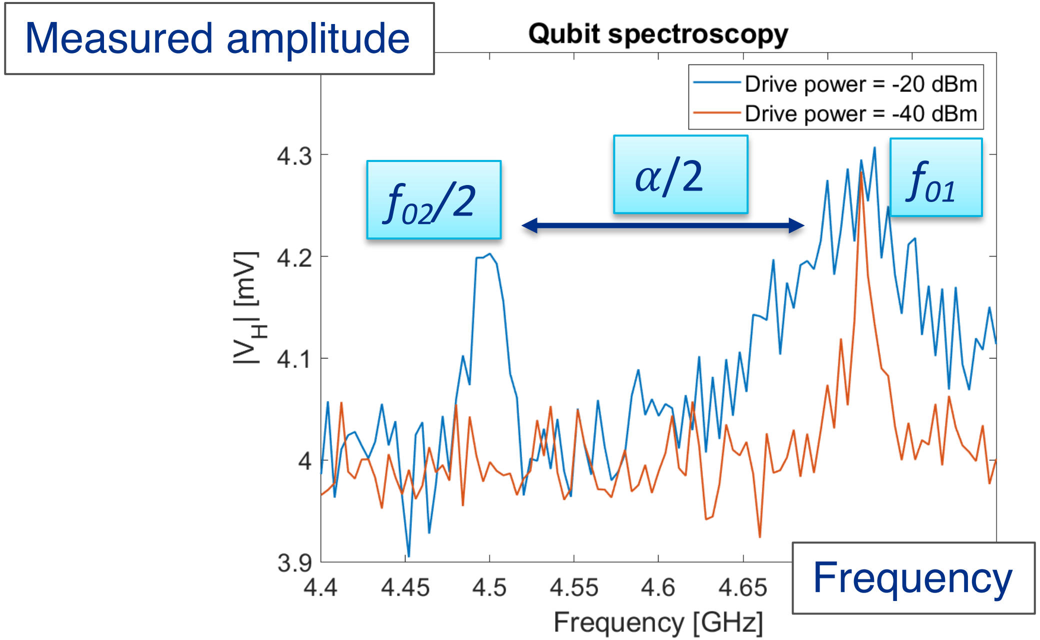
Qubit + Resonator

Measure the transition frequencies

Qubit Spectroscopy (2)



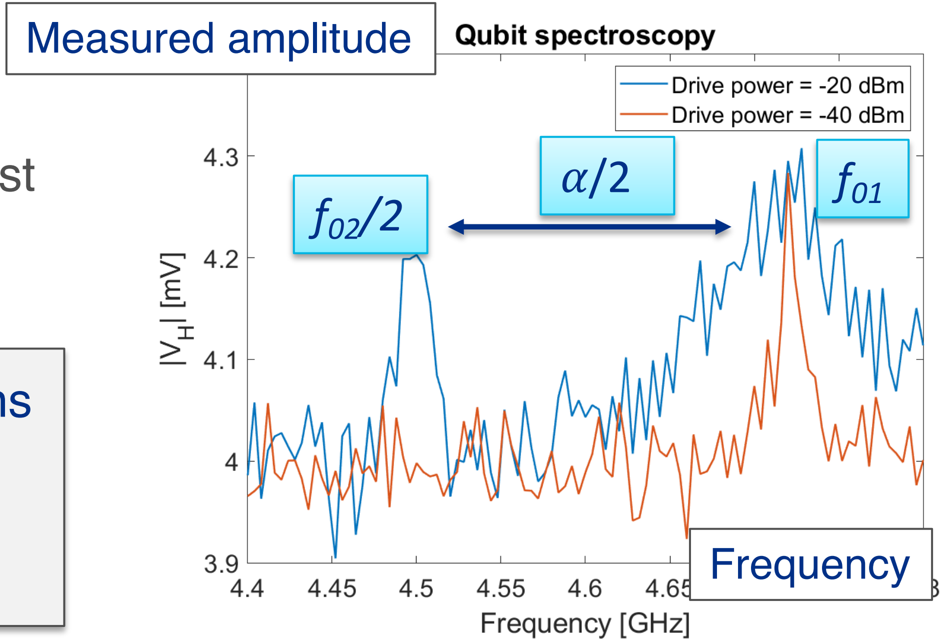
$f_{01} \neq f_{12} \neq f_{23}$


Measured amplitude

Frequency

Qubit Spectroscopy (3)

When the drive signal approaches f01 the qubit transition is driven and the first excited state becomes populated.



Anharmonicity and other transitions

$$\frac{\alpha}{2\pi} = \frac{\omega_{12} - \omega_{01}}{2\pi} = -E_C$$

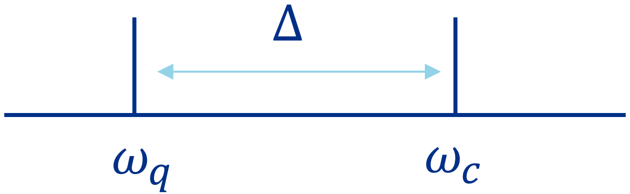
Dispersive shift and photon number counting

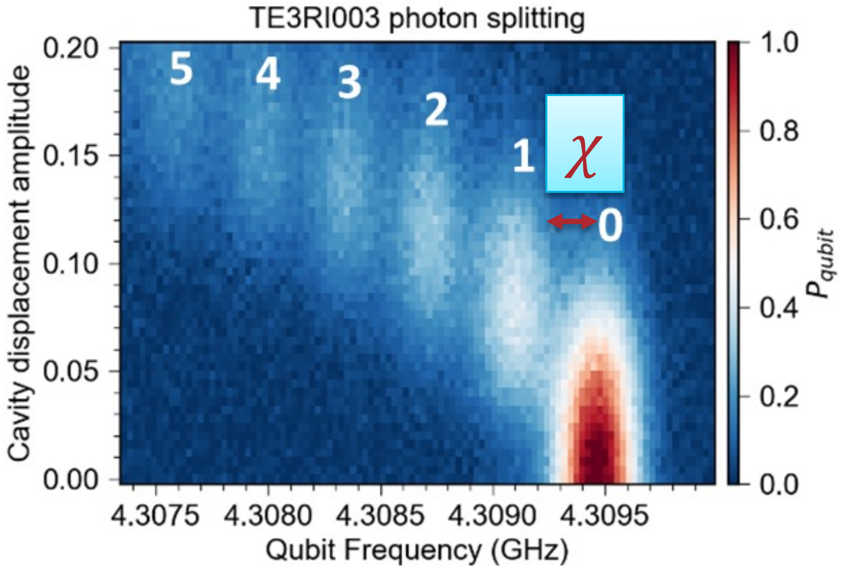
This effected is observed through a shift in the resonator frequency by χ

Evaluate the coupling **g** with dispersive approximation

$$\boxed{\Delta = \omega_q - \omega_c}$$

$$\boxed{g \ll \Delta, \chi = 2g^2/\Delta}$$





Qubit characterization

We need to make sure to have all these data

Qubit	f_q (GHz)	f_{max} (GHz)	α_q (MHz)	f_r (GHz)	χ (MHz)	T_1	T_2^*	T_2^E	P(e)
Alice	✅		✅	✅	✅	Rabi			
Bob									
Charlie									
David									

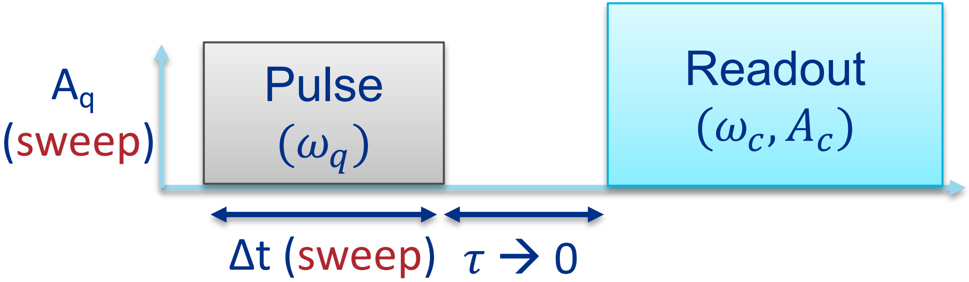
Rabi oscillation (1)

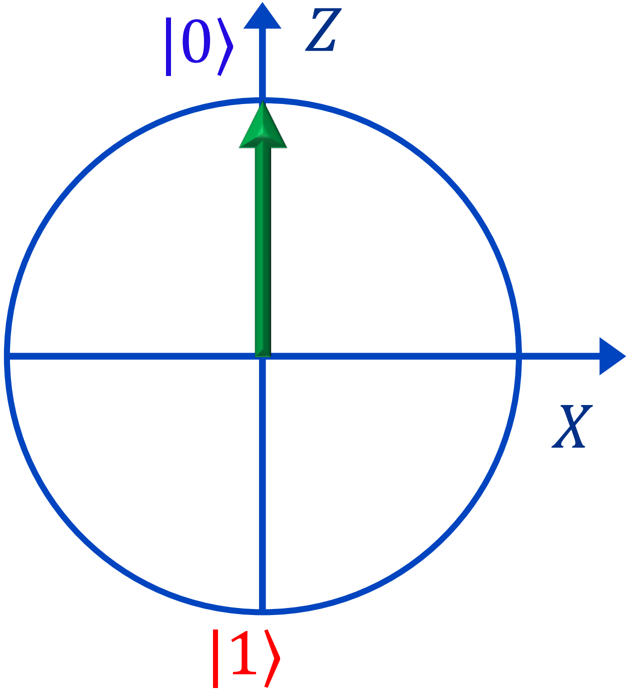
This is a fundamental measurement to calibrate pulses (length and amplitude)

Goal: Pulses calibration

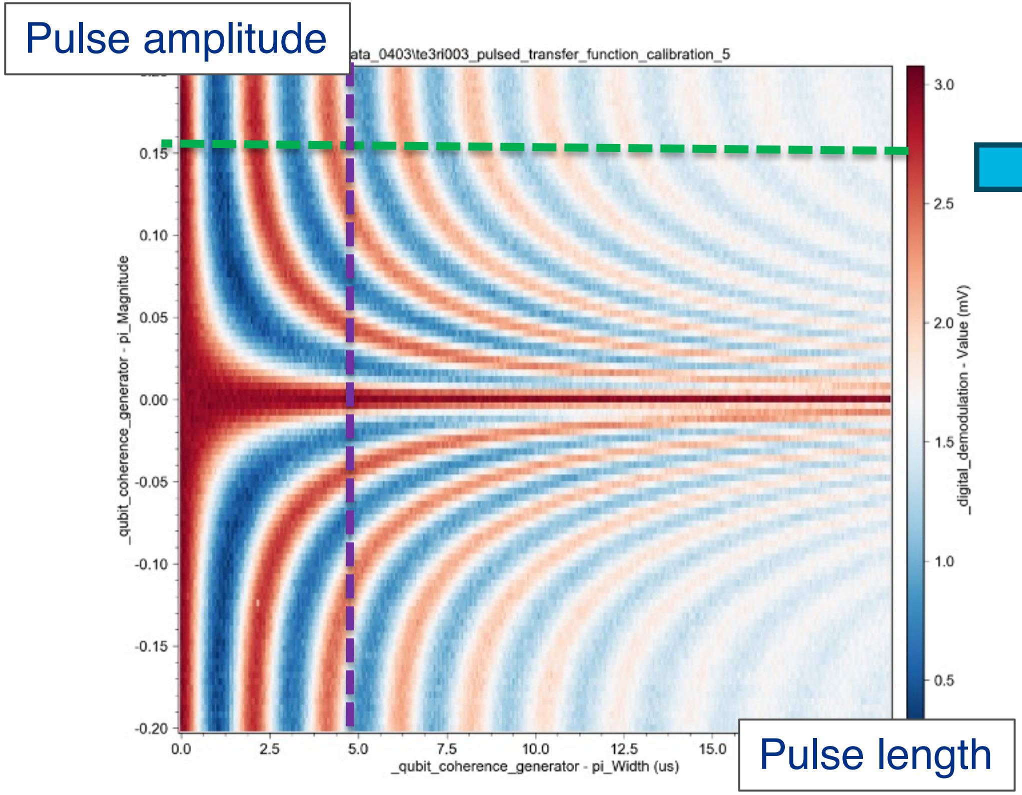
What to do: Sweep amplitude and pulse length of the qubit - In other words: vary the energy

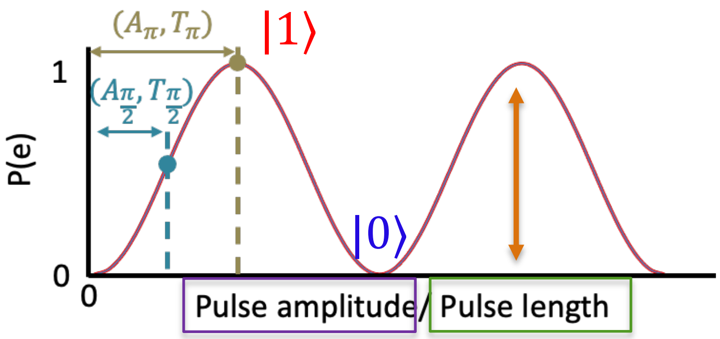
What to observe: Rotations around the Bloch sphere

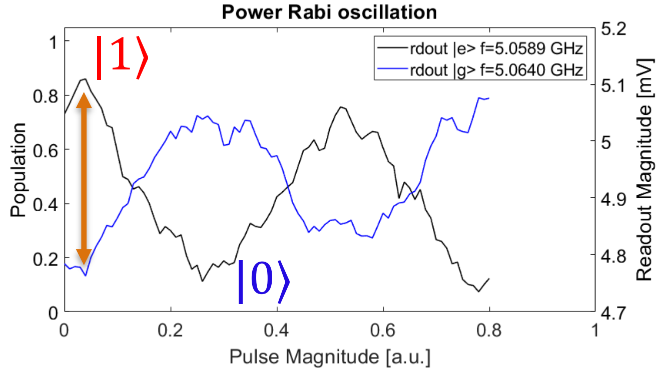




Rabi oscillation (3)

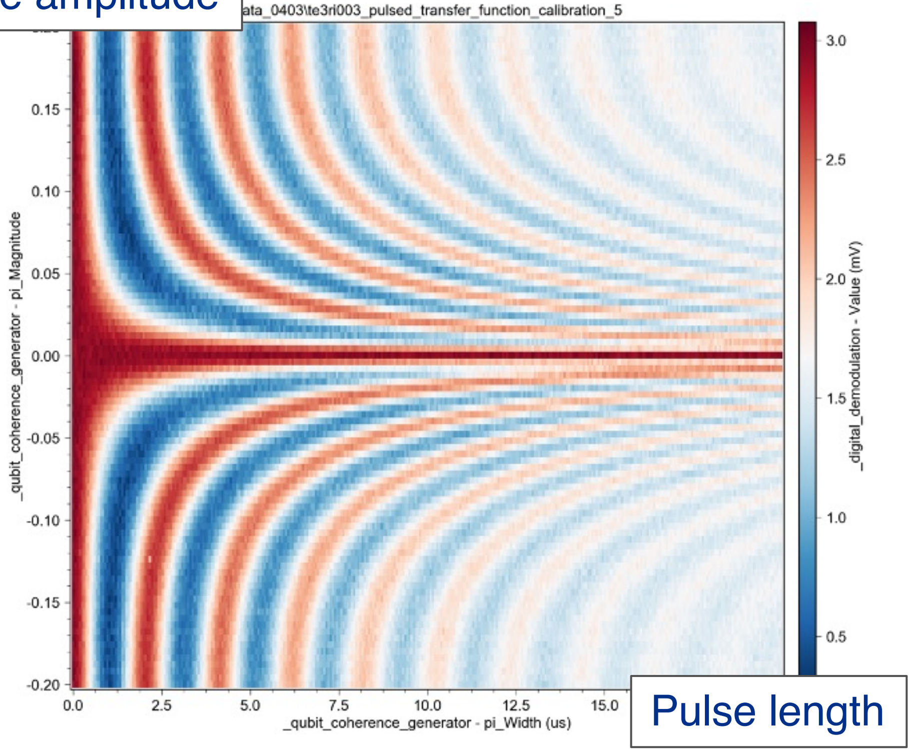




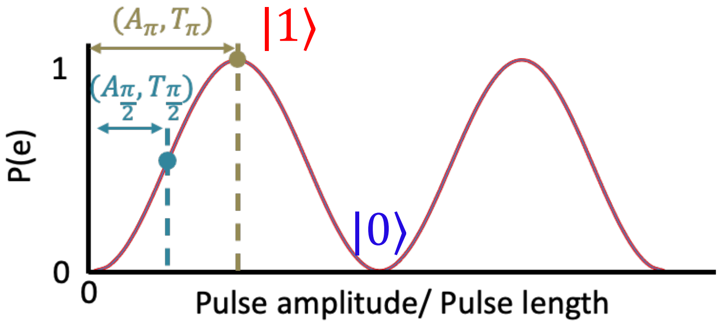


Rabi oscillation (4)

Pulse amplitude



Pulse length

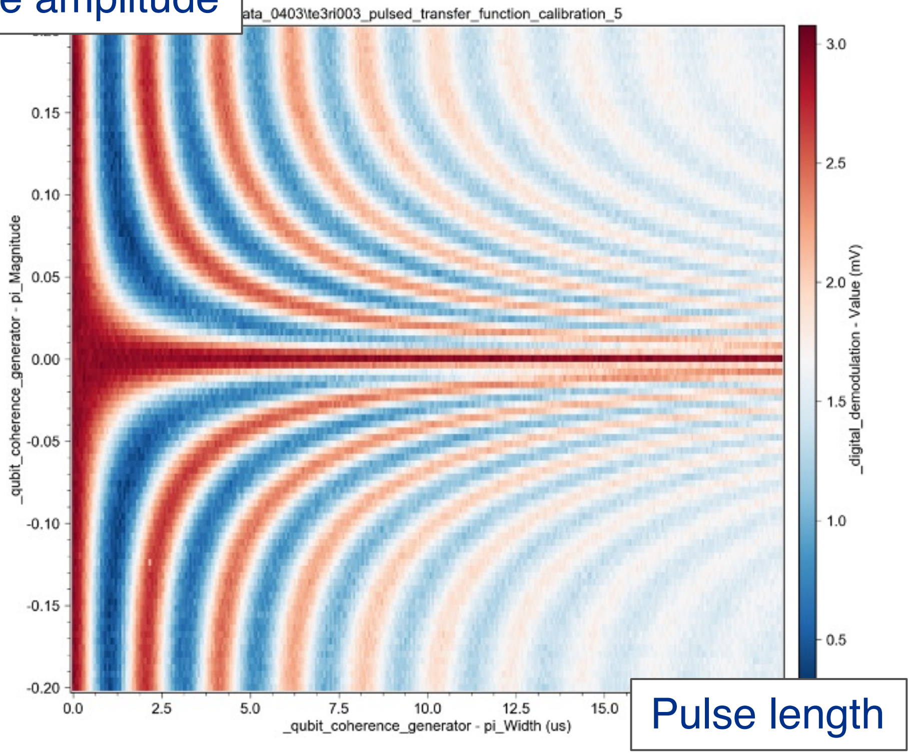


Probability that the qubit initially in the ground state is in the first excited state after applying a pulse of length *Δt*

$$|p_1|^2 = \sin^2(\Omega_{Rabi}\,\Delta t/2)$$

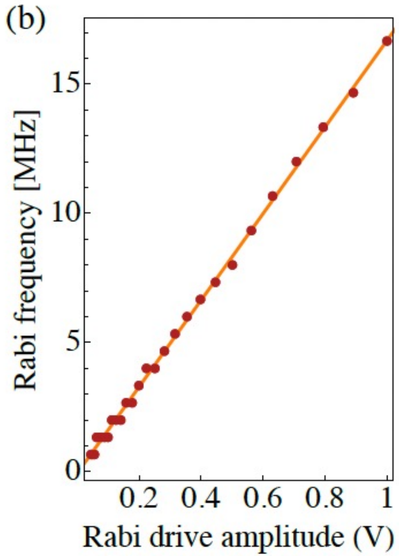
Rabi oscillation (5)

Ω_{Rabi} scales linearly as a function of amplitude



Pulse amplitude

Pulse length



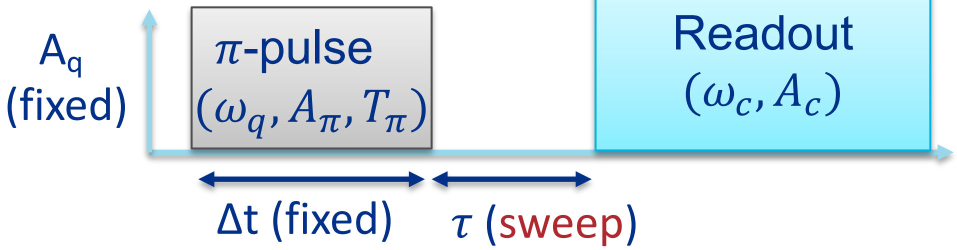
Experiments on Multi-Level Superconducting Qubits and Coaxial Circuit QED, Michael J. Peterer,

T1 experiment (1)

This is a figure of merit of the qubit, which also defines the computational complexity in quantum computing, e.g., how long a quantum algorithm could be running

Goal: Measure the qubit relaxation time

What to do: Apply π-pulse pulse to excite the transition $|0\rangle \rightarrow |1\rangle$, wait a time τ and observe the decay



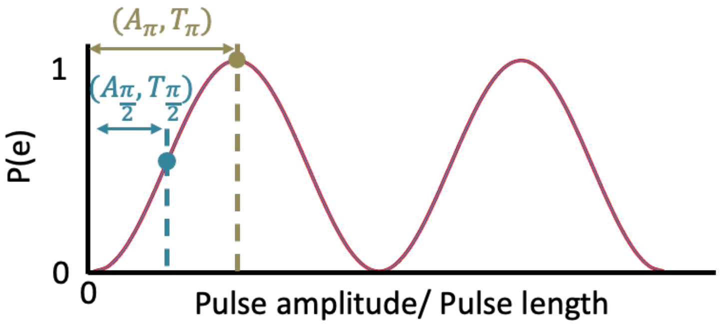
A_q (fixed)

π-pulse (ω_q, A_π, T_π)

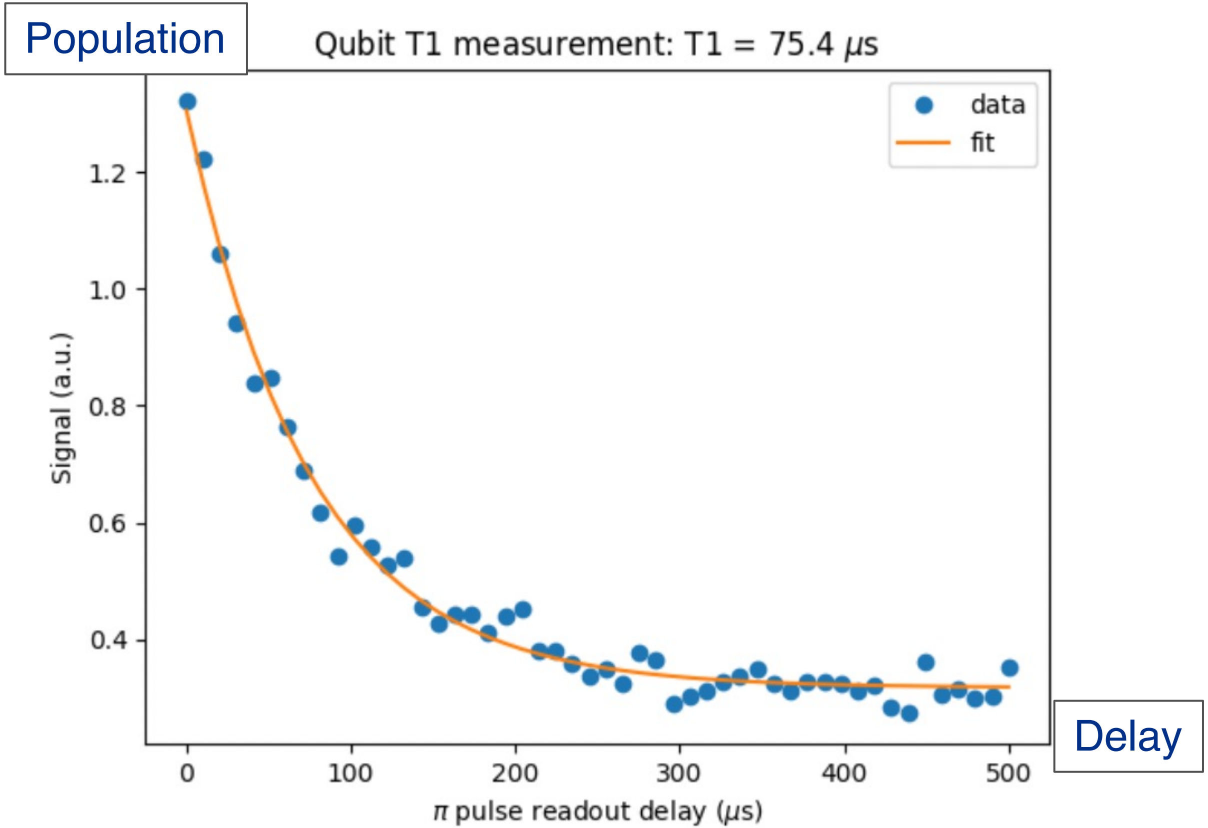
Readout (ω_c, A_c)

Δt (fixed)

τ (sweep)



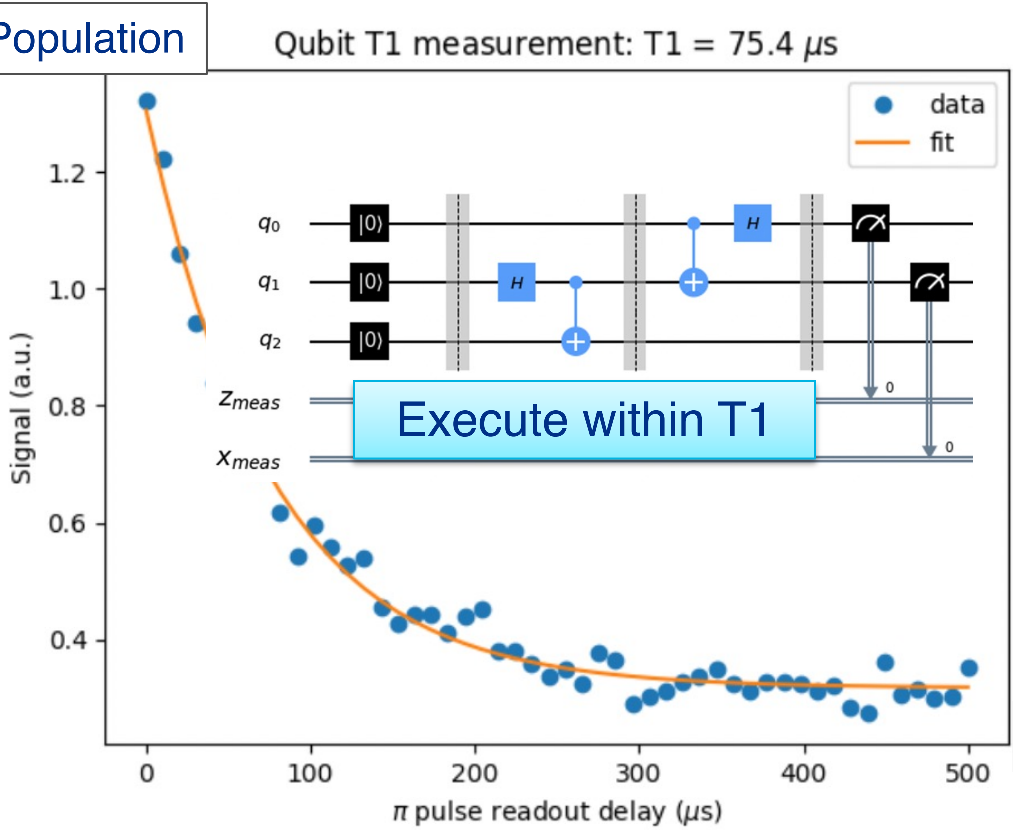
T1 experiment (2)



Fit and measure T_1

$$P(e) = e^{-t/T_1}$$

T1 experiment (2)



Population

Execute within T1

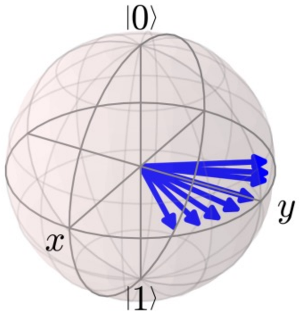
Delay

Fit and measure T_1

$$P(e) = e^{-t/T_1}$$

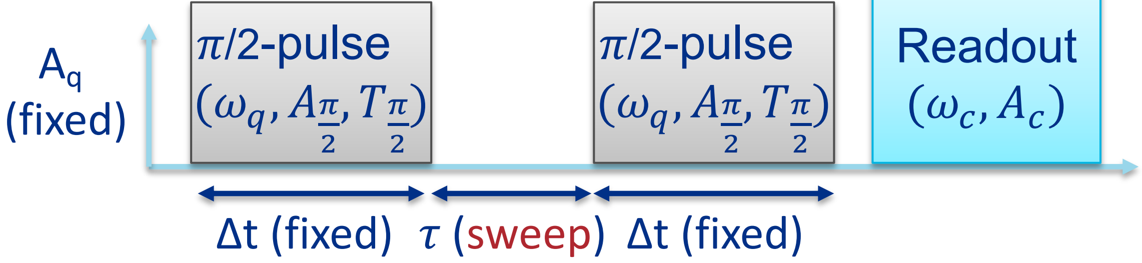
T2 experiment (1)

This is a figure of merit of the qubit. With this measurement we investigate the charge-noise induced decoherence.



Goal: Measure the phase decoherence (dephasing)

What to do: Pulse sequence for the Ramsey experiment



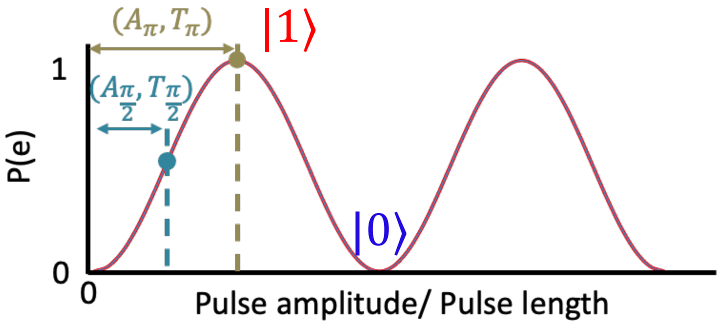
A_q (fixed)

$\pi/2$-pulse $(\omega_q, A_{\frac{\pi}{2}}, T_{\frac{\pi}{2}})$

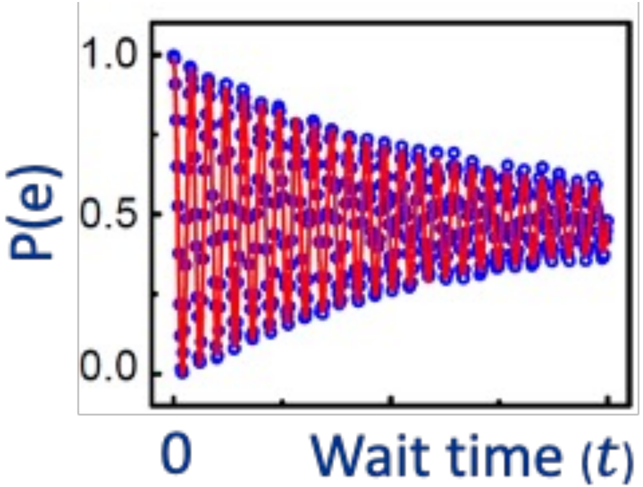
$\pi/2$-pulse $(\omega_q, A_{\frac{\pi}{2}}, T_{\frac{\pi}{2}})$

Readout (ω_c, A_c)

Δt (fixed) τ (sweep) Δt (fixed)



T2 experiment (2)

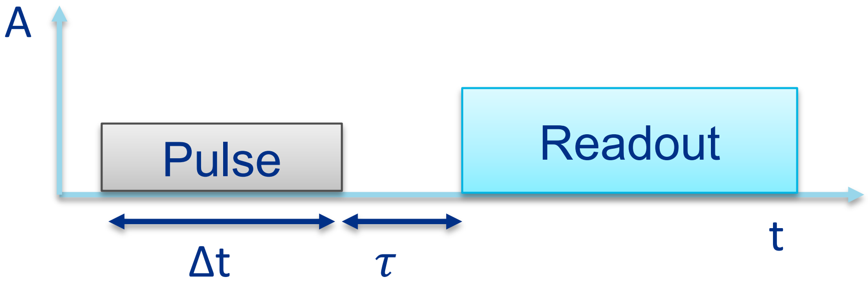


Fit and measure T_2

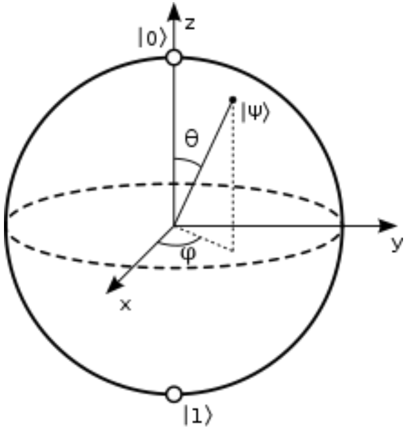
P(e) = $e^{-t/T_2}\sin(\delta\omega t)$

T_2: Ramsey time const.

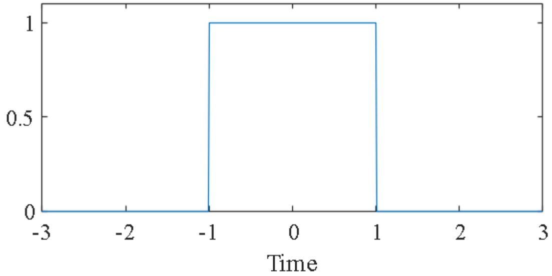
$$\frac{1}{T_\phi} = \frac{1}{T_2} - \frac{1}{2T_1}$$

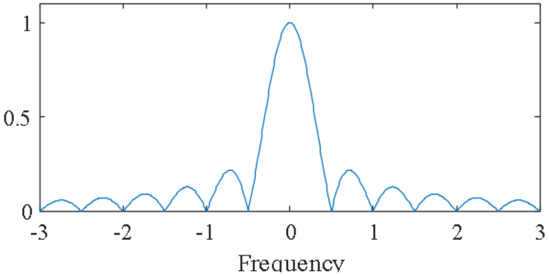


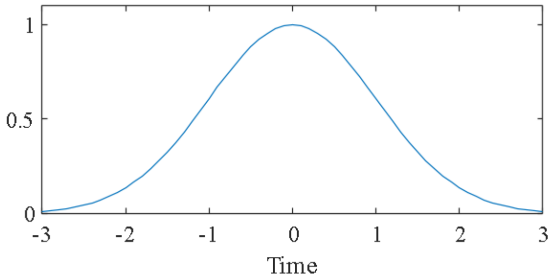
Pulses and Readout optimization

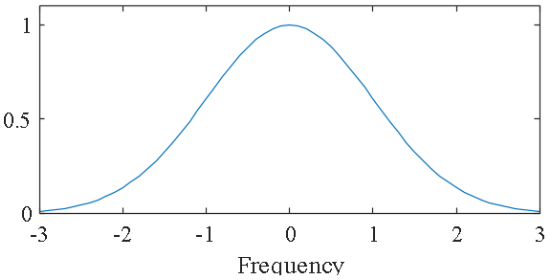


Pulses in time and frequency domain

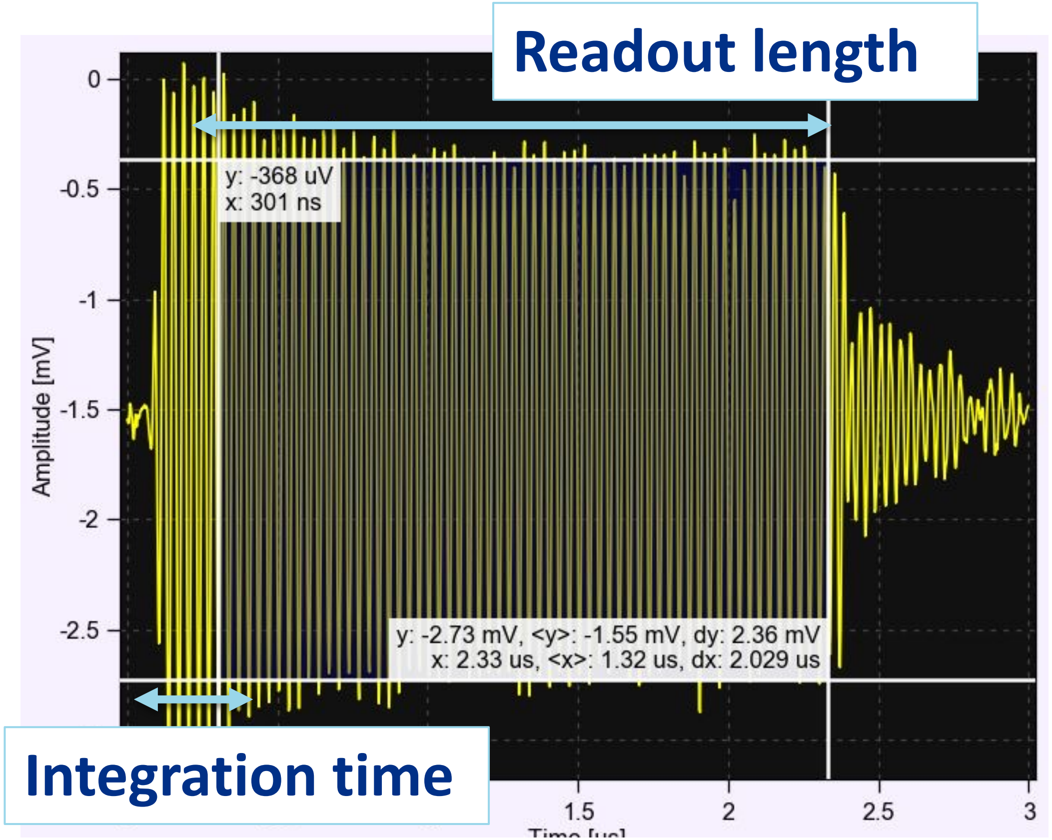

Rectangular pulse of width T


$\mathrm{sinc}(\nu T) = \sin \pi\nu T / \pi\nu T$


Gaussian with rms of σ_T


Gaussian with rms of $1/2\pi\sigma_T$

Readout optimization



Readout length

Integration time

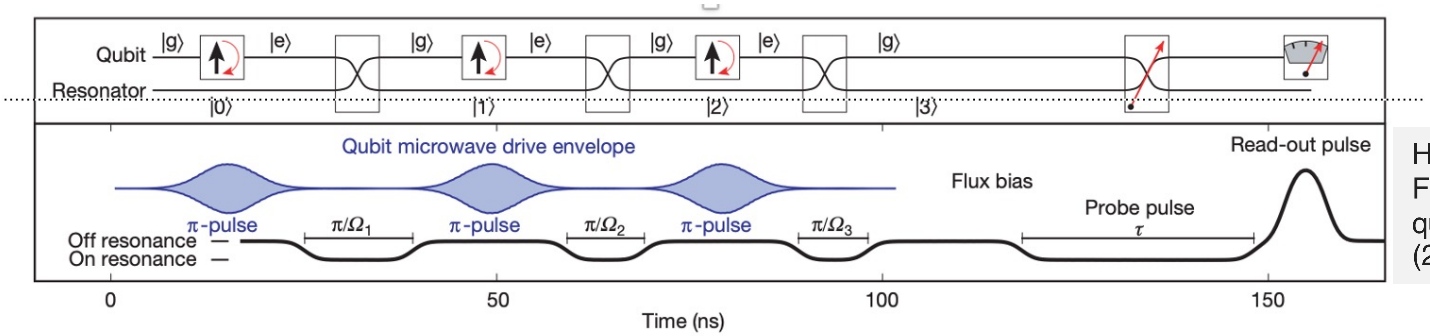
Summary and Outlook

We have covered basic characterization of a single superconducting qubit

- More complex characterization methods include randomized benchmarking and gate optimization

Outlook

- Characterization of multi-qubit devices and QPUs also include evaluation of gate fidelity
- Long coherence time devices lead to quDit encoding



Hofheinz, Max, et al. "Generation of Fock states in a superconducting quantum circuit." *Nature* 454.7202 (2008): 310-314.

Thank you!