SQMS QIS school

Track 2: **Materials characterization**:

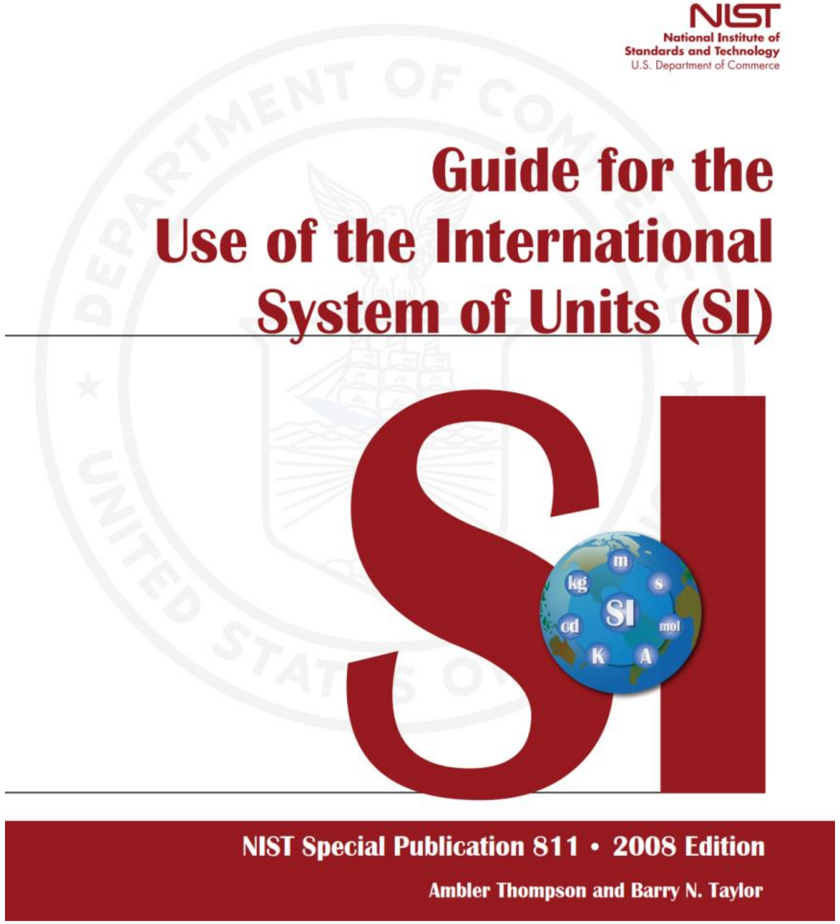
Resistivity, magnetization, magneto-optics, superconducting gap – superfluid density

Ruslan Prozorov

Ames National Laboratory

10 August 2023

Note on units



Guide for the Use of the International System of Units (SI): https://physics.nist.gov/cuu/pdf/sp811.pdf

CGS (Gauss's) units

Units for Magnetic Properties

	Symbol	Gaussian & cgs emu[a]	Conversion Factor, C[b]	SI & rationalized mks[c]
Magnetic flux density, magnetic induction	B	gauss (G)[d]	10^{-4}	tesla (T), Wb/m^2
Magnetic flux	Φ	maxwell (Mx), G·cm^2	10^{-8}	weber (Wb), volt second (V·s)
Magnetic potential difference, magnetomotive force	U, F	gilbert (Gb)	$10/4\pi$	ampere (A)
Magnetic field strength, magnetizing force	H	oersted (Oe),[e] Gb/cm	$10^3/4\pi$	A/m[f]
(Volume) magnetization[g]	M	emu/cm^3 [h]	10^3	A/m
(Volume) magnetization	4πM	G	$10^3/4\pi$	A/m
Magnetic polarization, intensity of magnetization	J, I	emu/cm^3	$4\pi \times 10^{-4}$	T, Wb/m^2 [i]
(Mass) magnetization	σ, M	emu/g	1 $4\pi \times 10^{-7}$	A·m^2/kg Wb·m/kg
Magnetic moment	m	emu, erg/G	10^{-3}	A·m^2, joule per tesla (J/T)
Magnetic dipole moment	j	emu, erg/G	$4\pi \times 10^{-10}$	Wb·m[i]
(Volume) susceptibility	χ, κ	dimensionless, emu/cm^3	4π $(4\pi)^2 \times 10^{-7}$	dimensionless henry per meter (H/m), Wb/(A·m)
(Mass) susceptibility	χ_ρ, κ_ρ	cm^3/g, emu/g	$4\pi \times 10^{-3}$ $(4\pi)^2 \times 10^{-10}$	m^3/kg H·m^2/kg
(Molar) susceptibility	χ_{mol}, κ_{mol}	cm^3/mol, emu/mol	$4\pi \times 10^{-6}$ $(4\pi)^2 \times 10^{-13}$	m^3/mol H·m^2/mol
Permeability	μ	dimensionless	$4\pi \times 10^{-7}$	H/m, Wb/(A·m)
Relative permeability[j]	μ_r	not defined	—	dimensionless
(Volume) energy density, energy product[k]	W	erg/cm^3	10^{-1}	J/m^3
Demagnetization factor	D, N	dimensionless	$1/4\pi$	dimensionless

a. *Gaussian units and cgs emu are the same for magnetic properties. The defining relation is* $B = H + 4\pi M$.
b. *Multiply a number in Gaussian units by C to convert it to SI (e.g.,* $1\ G \times 10^{-4}\ T/G = 10^{-4}\ T$*).*
c. *SI (Système International d'Unités) has been adopted by the National Bureau of Standards. Where two conversion factors are given, the upper one is recognized under, or consistent with, SI and is based on the definition* $B = \mu_0(H + M)$, *where* $\mu_0 = 4\pi \times 10^{-7}$ *H/m. The lower one is not recognized under SI and is based on the definition* $B = \mu_0 H + J$, *where the symbol I is often used in place of J.*
d. *1 gauss =* 10^5 *gamma (γ).*
e. *Both oersted and gauss are expressed as* $cm^{-1/2} \cdot g^{1/2} \cdot s^{-1}$ *in terms of base units.*
f. *A/m was often expressed as "ampere-turn per meter" when used for magnetic field strength.*
g. *Magnetic moment per unit volume*
h. *The designation "emu" is not a unit*
i. *Recognized under SI, even though based on the definition* $B = \mu_0 H + J$. *See footnote c.*
j. $\mu_r = \mu/\mu_0 = 1 + \chi$, *all in SI.* μ_r *is equal to Gaussian* μ.
k. $B \cdot H$ *and* $\mu_0 M \cdot H$ *have SI units J/m*^{3}; $M \cdot H$ *and* $B \cdot H/4\pi$ *have Gaussian units erg/cm*^{3}.

R. B. Goldfarb and F. R. Fickett, U.S. Department of Commerce, National Bureau of Standards, Boulder, Colorado 80303, March 1985
NBS Special Publication 696 for sale by the Superintendent of Documents, U. S. Government Printing Office, Washington, DC 20402

The Drude theory of electric resistance

Electrons are separated from ions. Ions are stationary points in the electron sea. An electron moves between collisions average time τ (collision time). Suppose at $t = 0$ an electron left the last collision, then its velocity at time t is (acceleration is $a = -e\mathbf{E}/m$) is

$$\langle \mathbf{v} \rangle = a\tau = -\frac{e\mathbf{E}}{m}\tau$$

electric current is (charge $-e$):

$$\mathbf{j} = -\langle \mathbf{v} \rangle en$$

where n is the electron density (typically $n\tilde{}1 - 20 \times 10^{22} cm^{-3}$) and e is charge. Therefore,

$$\mathbf{j} = \frac{ne^2\tau}{m}\mathbf{E}$$

Electrical conductivity: $$j = \sigma E \Rightarrow \sigma = \frac{ne^2\tau}{m}$$

Electrical resistivity: $$E = \rho j \Rightarrow \rho = \frac{m}{ne^2\tau}$$

Paul Karl Ludwig Drude
July 12, 1863 – July 5, 1906

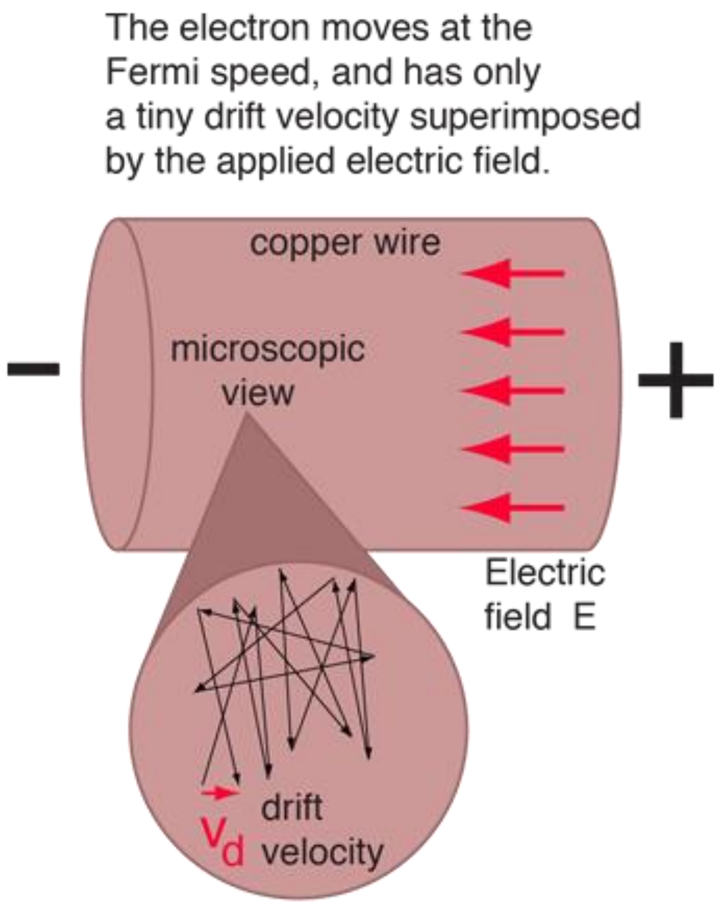
The mean free path and the scattering time

mean-free path is the average distance between the collisions

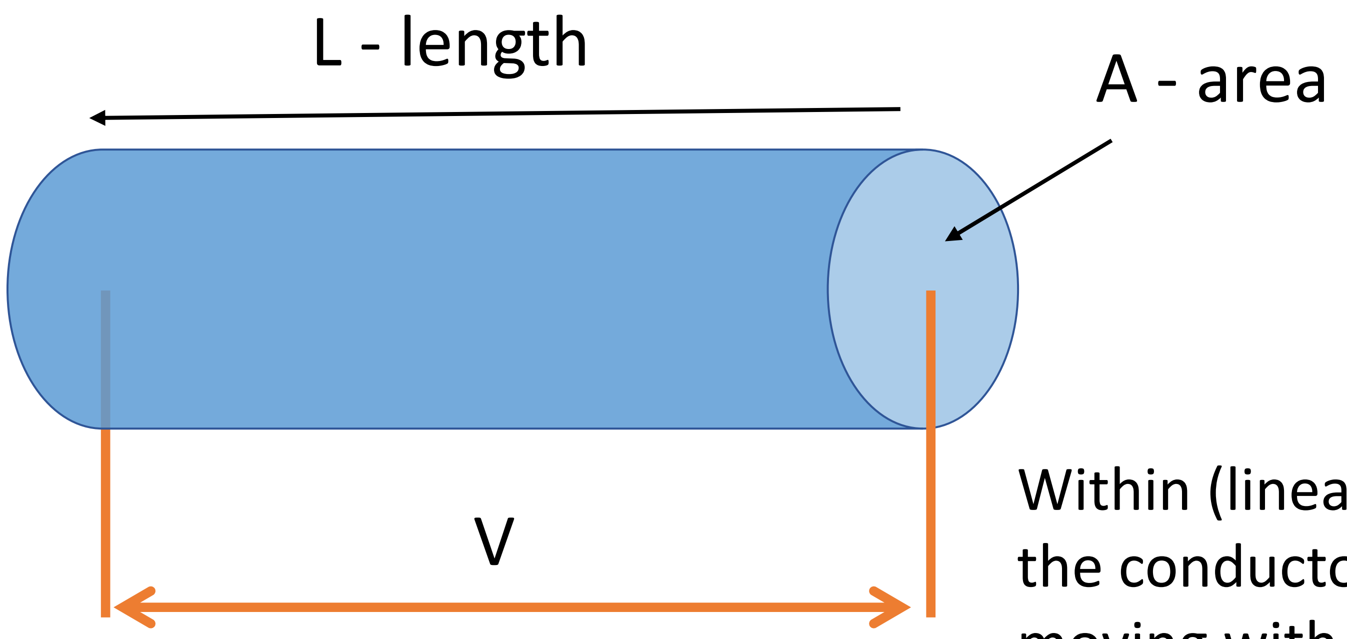
$l = v_F\tau$

scattering time

Fermi (not drift!!!) velocity



Resistance and Resistivity



Ohm's law:

$$V = RI$$

$$E = \rho j$$

Within (linear) Ohm's law, the electric field inside the conductor is constant because charges are moving with constant "terminal" velocity.

$$\left.\begin{array}{r} V = EL = \rho j L = \dfrac{\rho L}{A} I \\ V = RI \end{array}\right\} \Rightarrow R = \rho \frac{L}{A}$$

Resistance **resistivity** **geometric factor**

Fermi surface

minimum volume in p - space:

$$\Delta^3 p_{\min} = \frac{(2\pi\hbar)^3}{V}$$

So, total number of available microscopic states is:

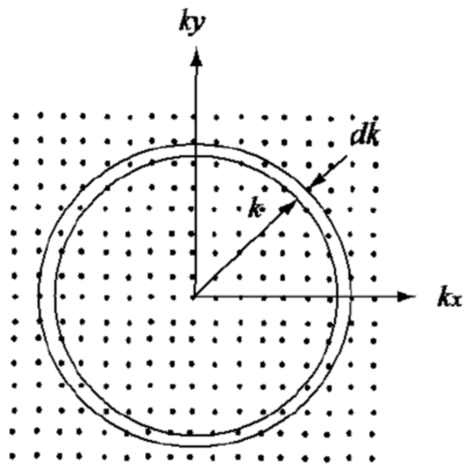
$$N = 2 \times \frac{\frac{4}{3}\pi p_F^3}{\frac{(2\pi\hbar)^3}{V}}$$

where 2 comes from two spin projections. Therefore,

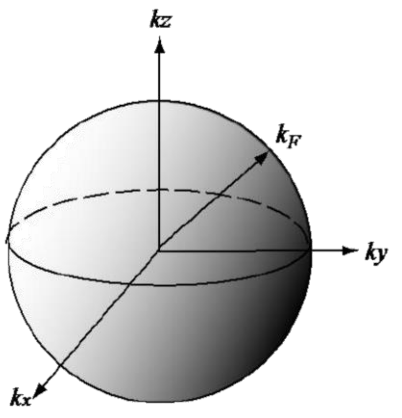
$$p_F = (3\pi^2 n)^{1/3}\hbar$$

corresponding to the kinetic energy:

$$E = \frac{p_F^2}{2m} = \frac{(3\pi^2 n)^{2/3}\hbar^2}{2m}$$



(b)



(a)

$$p_F = \hbar k_F$$

electronic mean free path with the Drude assumption

Paul Drude was wrong, but he was incredibly lucky

The collision time is about $10^{-14} - 10^{-15}$ seconds and it is not a convenient measure. More useful is the average distance between collision (also for finite size effects).

$$l = \tau v_0$$

where v_0 is the average velocity of electrons. Drude used thermal average

$$\frac{mv^2}{2} = \frac{3k_BT}{2}$$

which at room temperature gives

$$v = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3 \times 1.3806568 \times 10^{-23}\,\mathrm{J\,K^{-1}} \times 300\,\mathrm{K}}{9.1093897 \times 10^{-31}\,\mathrm{kg}}} = 1.1679{\times}10^5\,\frac{\mathrm{m}}{\mathrm{s}} = \frac{1.1679 \times 10^5}{2.99792458 \times 10^8} = 4{\times}10^{-4}c$$

this implies mean free path

$$l = (10^{-15} - 10^{-14}) \times 10^5\,\tilde{}\,1 - 10\,\text{Å}$$

so Drude and Co. were happy, because it is of the order of the distance between atoms. The problem is that it implies that v_0 decreases with temperature, which is not the case.

estimate the actual drift (terminal) velocity

Suppose we have a wire of 1 mm in diameter carrying electric current of 1 A.

Estimate the value of v_d?

1 A means that a total of 1 C of charge crosses the area in 1 s.

Each electron carries charge e = -1.6×10^{-19} C and typical concentration of electrons in good metals: $n=5\text{x}10^{28}$ $1/m^3$

$$Q = \frac{\pi d^2}{4} \times v_d \times e \times n \times 1[\mathrm{s}] = 1\ [\mathrm{C}]$$

therefore, $v_d \approx 1.6 \times 10^{-4} \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$ - 10 times slower than a typical snail (1 mm/s)

The summary of velocities:

Typical instantaneous velocity: $v_F \approx 1 \times 10^{+6} \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$ temperature-independent

Typical drift velocity: $v_d \approx 1 \times 10^{-4} \left[\frac{\mathrm{m}}{\mathrm{s}}\right]$

---

a ratio of: $\frac{v_F}{v_d} \approx 10^{+10}$

Thermal velocity (Drude):

$$v_{th} \approx 1 \times 10^{+5} \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \quad @300\ K$$

$$v_{th} \approx 7 \times 10^{+3} \left[\frac{\mathrm{m}}{\mathrm{s}}\right] \quad @1\ K$$

The original Drude assumption fails to explain the T-dependence of conductivity in metals. It increases with the decrease of T!

classical motion under uniform force

Suppose we have a uniform force f (e.g., due to electric or magnetic field). Probability that an electrons collided in the interval dt is proportional to dt/τ. So, the number of electrons that survived interval dt without collision is $n\left(1 - dt/\tau\right)$. Their momentum is

$$p\left(t + dt\right) = \left(1 - dt/\tau\right)\left(p\left(t\right) + f dt + O\left(dt\right)^2\right)$$

(correction due to electrons that have collided is ~$\left(dt/\tau\right) f dt$~$O\left(dt\right)^2$. Therefore

$$\Delta p = p\left(t + dt\right) - p\left(t\right) \approx -\frac{dt}{\tau} p\left(t\right) + f dt$$

or

$$\frac{dp}{dt} = -\frac{p\left(t\right)}{\tau} + f$$

so the classical Newtonian equation get additional drag force: $-\frac{p(t)}{\tau}$.

Drude model is valid and compatible with the quantum theory

$$\sigma=\frac{1}{3}v_F^2 g\left(\mu\right)\tau e^2$$

It must be equal to Drude formula

$$\sigma_{Drude} = \frac{n}{m}\tau e^2$$

Therefore

$$\frac{n}{m} = \frac{1}{3}v_F^2 g\left(\mu\right)$$

and since (for electron, there is a spin factor $2s+1=2$)

$$g\left(\varepsilon\right) = 2\frac{2\pi}{\left(2\pi\hbar\right)^3}\left(2m\right)^{3/2}\varepsilon^{1/2} = 2\frac{4\pi m^2}{h^3}v\left(\varepsilon\right)$$

$$v_F^3 = \frac{3nh^3}{8\pi m^3}$$

$$v_F = \frac{h}{m}\left(\frac{3n}{8\pi}\right)^{1/3}$$

$$\varepsilon_F = \frac{m\left(v_F\right)^2}{2} = \frac{h^2}{2m}\left(\frac{3n}{8\pi}\right)^{2/3}$$

As obtained from the simple theory

Quasiparticles

Quasiparticle spectrum in electron gas is determined by the energy of free particle.
In Fermi liquid interactions between particles play important role.

Electron gas:

$$\xi_p = \frac{p^2}{2m} - \frac{p_F^2}{2m}$$

$$\xi_a = \frac{p_F^2}{2m} - \frac{p^2}{2m}$$

$$\text{at } |p - p_F| \ll p_F$$

$$\xi_p \approx v(p - p_F)$$

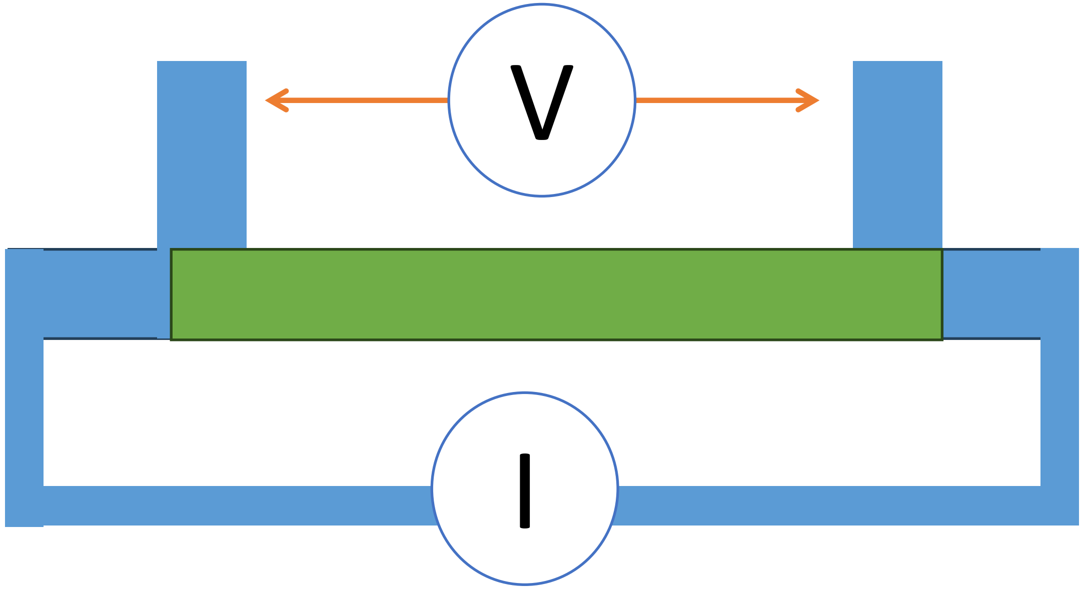
$$\xi_a \approx v(p_F - p)$$

$$v = p_F/m^*$$

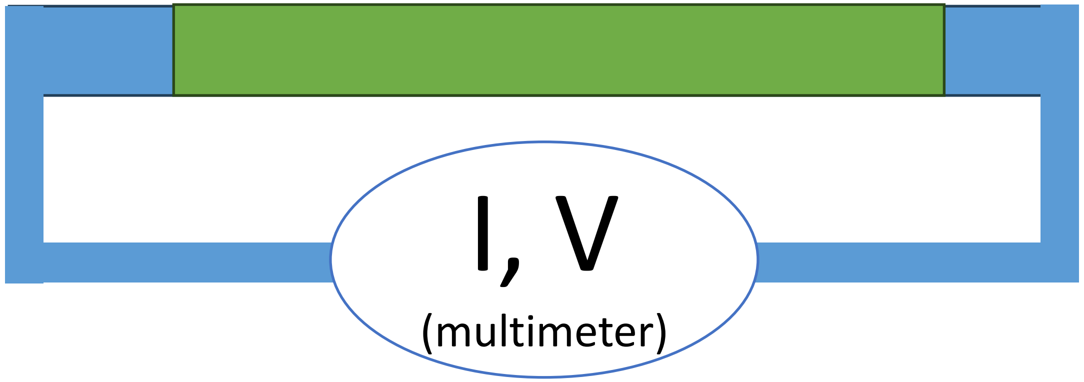
According to the idea expressed by Landau (1956), the interaction of quasiparticles may be introduced as a certain self-consistent field generated by surrounding quasiparticles, which acts on a given quasiparticle. But here the energy of the quasiparticle will evidently depend on the state of other quasiparticles, i.e., in other words, it will be a functional of their distribution function[1].

this new circumstance, in effect, does not prove to be important, with rare exceptions. In some cases it leads to numerical differences but does not alter the order of magnitude of the result. But there are phenomena which occur exclusively due to the dependence of the spectrum on the distribution function. Such phenomena are known as Fermi-liquid effects.

How to measure resistance? 2-probe vs 4-probe



$$R = \frac{V}{I}$$ - sample only



$$R = \frac{V}{I}$$ - sample with leads

In cryogenics with long leads, must use a 4-probe configuration.

The resistivity of a metal. Why is RRR so useful?

Low temperatures (T << Θ_R)

$$\rho(T) = \rho_0 + AT^2 + BT^5$$

impurities e-e e-phonons

High (room) temperatures T >> Θ_R)

$$\rho(T) \sim \alpha T$$ (e-phonons)

$$\alpha \propto 5 \times 10^{-3}\, K^{-1}$$

$$\rho(T) = A\left(\frac{T}{\Theta_R}\right)^n \int_0^{\Theta_R/T} \frac{t^n}{(e^t - 1)(1 - e^{-t})} dt.$$

The Bloch–Grüneisen temperature: $k_B \Theta_R = 2 v_s p_F$

Residual Resistivity Ratio:

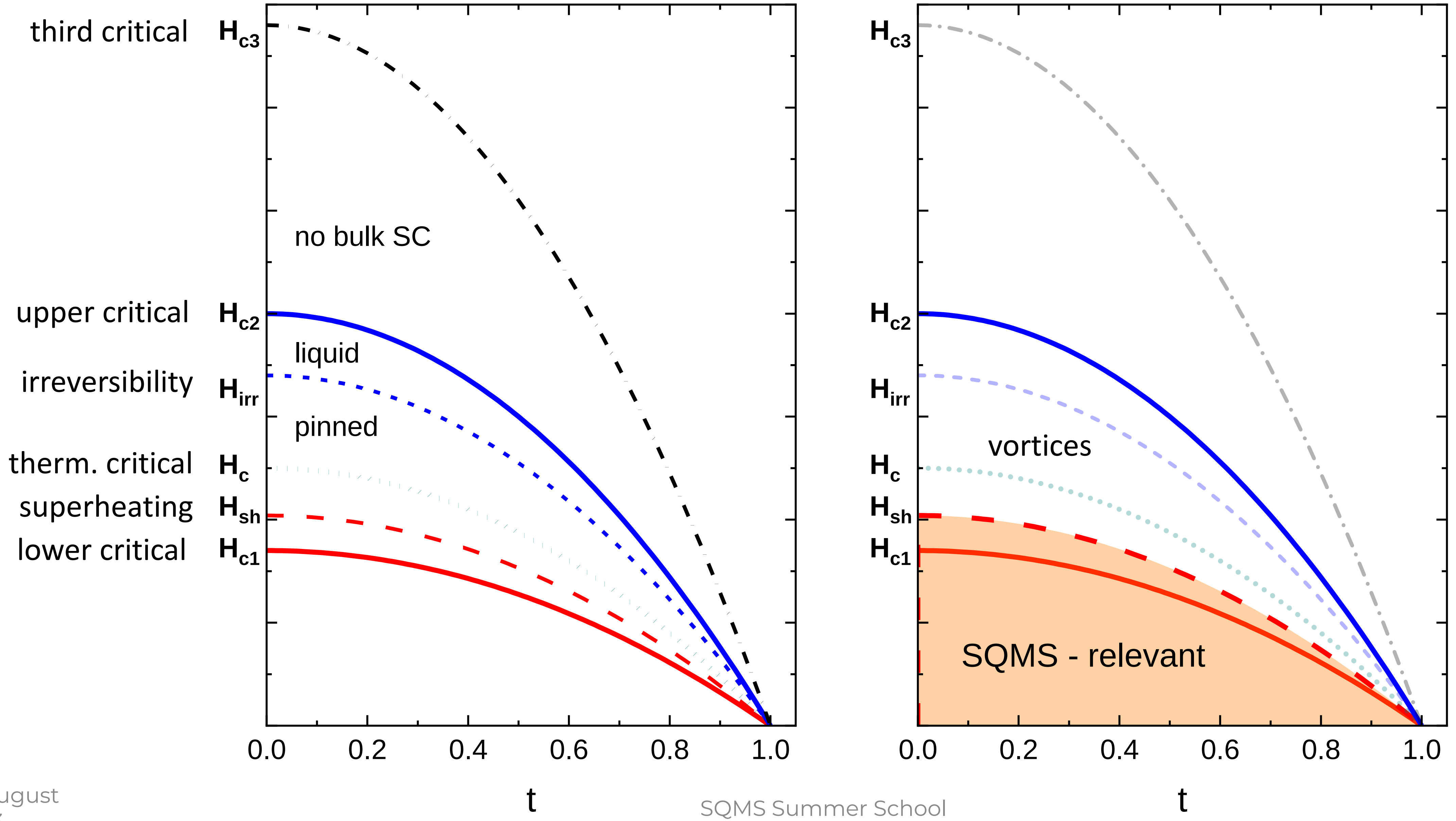
$$\mathrm{RRR} = \frac{R(300\,K)}{R_0} = \frac{\rho(300\,K)}{\rho_0}$$

- Direct estimate of the impurity level
- Does not require a geometric factor
- Problems with granular samples

$$\text{Nb: } \rho(300\,K) \approx 15.2\ \mu\Omega \cdot \text{cm}$$

- Practically regardless of morphology (crystals, films, foils, wires)
- However, RRR, does depend on morphology!

Characteristic magnetic fields of a superconductor



Effects of non-pair-breaking disorder

$$\lambda(0) = \lambda_{\text{clean}}\sqrt{1 + \frac{\xi_0}{\ell}},\ \xi(0) = \sqrt{\xi_0 \ell}$$

$$\kappa_{\text{dirty}}(\ell \ll \xi_0) = \frac{\lambda}{\xi} \approx \frac{\lambda_{\text{clean}}}{\ell} \approx \kappa_{\text{clean}}\Gamma$$

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2} = \frac{\phi_0}{2\pi\xi_0\ell} = \frac{\Gamma}{0.882}\frac{\phi_0}{2\pi\xi_0^2} = \Gamma(1.134 H_{c2,BCS})$$

$$\Gamma = \frac{\hbar}{2\pi T_c \tau} = \frac{\hbar v_F}{2\pi T_c \ell} = 0.882\frac{\xi_0}{\ell}$$

- Dimensionless scattering rate (non-pair-breaking!)

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta_0}$$

- The BCS coherence length

measurements of the slope of H_{c2} at T_c

$$\left.\frac{dH_{c2}}{dT}\right|_{T=T_c} = \frac{2\pi\phi_0}{\hbar^2 v_F^2} T_c \left.\frac{dh}{dt}\right|_{t\to 1} \propto \frac{T_c}{v_F^2}$$

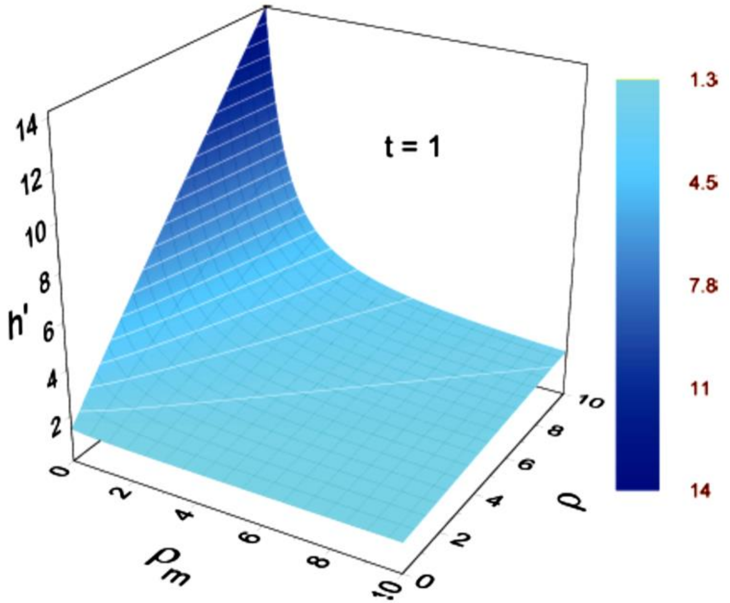
$$\left.\frac{dh}{dt}\right|_{t\to 1} = \frac{3\rho_-^2\left[1-\rho_m\psi'\left(\rho_m+\frac{1}{2}\right)\right]}{\psi\left(\rho_m+\frac{1}{2}\right)-\psi\left(\frac{\rho_+ +1}{2t}\right)+\frac{\rho_-}{2}\psi'\left(\rho_m+\frac{1}{2}\right)}$$

pros:

- convenient low-field measurements
- always in the "orbital" limit

cons:

- irreversibility field is measured instead
- dynamics (relaxation)
- broadness of the transition (criterion)
- fluctuations



The HW ratio:

$$h^*(0) = \frac{H_{c2}(0)}{T_c\left|H'_{c2}(T_c)\right|} = \frac{h(0)}{h'(1)}$$

$$h^*(0) = \begin{cases} \frac{7\zeta(3)e^2}{48e^C} \simeq 0.727 & \text{- clean} \\ \frac{\pi^2 e^{\psi(0,1/2)}}{2} \simeq 0.693 & \text{- dirty potential} \\ 0.5 & \text{- dirty magnetic} \end{cases}$$

$C \simeq 0.577216$ - Euler's constant

$\zeta(3) \simeq 1.202057$ - Riemann zeta function

E. Helfand and N. R. Werthamer, Phys. Rev. **147**, 288 (1966)

Upper critical field

E. Helfand and N. R. Werthamer, Phys. Rev. **147**, 288 (1966)

$$H_{c2}(T=0)=\boldsymbol{\alpha}T_c\left.\frac{dH_{c2}}{dT}\right|_{T\to T_c}$$

$$\boldsymbol{\alpha}=\begin{cases}\dfrac{7\zeta(3)e^2}{48e^C}\approx 0.727 \ - \ \text{clean limit}\\[2ex] \dfrac{\pi^2 e^{\psi\left(0,\frac{1}{2}\right)}}{2}\approx 0.693 \ - \ \text{dirty limit}\\[2ex] 0.5 \ - \ \text{dirty magnetic limit}\end{cases}$$

$C\simeq 0.577216$ - Euler's constant

$\zeta(3)\simeq 1.202057$ - Riemann zeta function

$$\left.\frac{dH_{c2}}{dT}\right|_{T=T_c}=\frac{2\pi\phi_0}{\hbar^2 v_F^2}T_c\left.\frac{dh}{dt}\right|_{t\to 1}\propto\frac{T_c}{v_F^2}$$

$$\left.\frac{dh}{dt}\right|_{t\to 1}=\frac{3\rho_-^2\left[1-\rho_m\psi'\left(\rho_m+\frac{1}{2}\right)\right]}{\psi\left(\rho_m+\frac{1}{2}\right)-\psi\left(\frac{\rho_+ +1}{2t}\right)+\frac{\rho_-}{2}\psi'\left(\rho_m+\frac{1}{2}\right)}$$

Track 2 lab: measure resistivity to estimate H_{c2} and ξ

- Measure resistance R(T) at H=0, H = 1000 Oe and H = 5000 Oe.
- Determine superconducting transition temperature (offset), $T_c(H)$.
- Calculate resistivity, ρ, from the measured resistance, R.
- Calculate RRR – residual resistivity ratio
- Determine the upper critical field, $H_{c2}(0)$, from the measured $T_c(H)$.
- Calculate the coherence length, $\xi(0)$, from $H_{c2}(0)$.

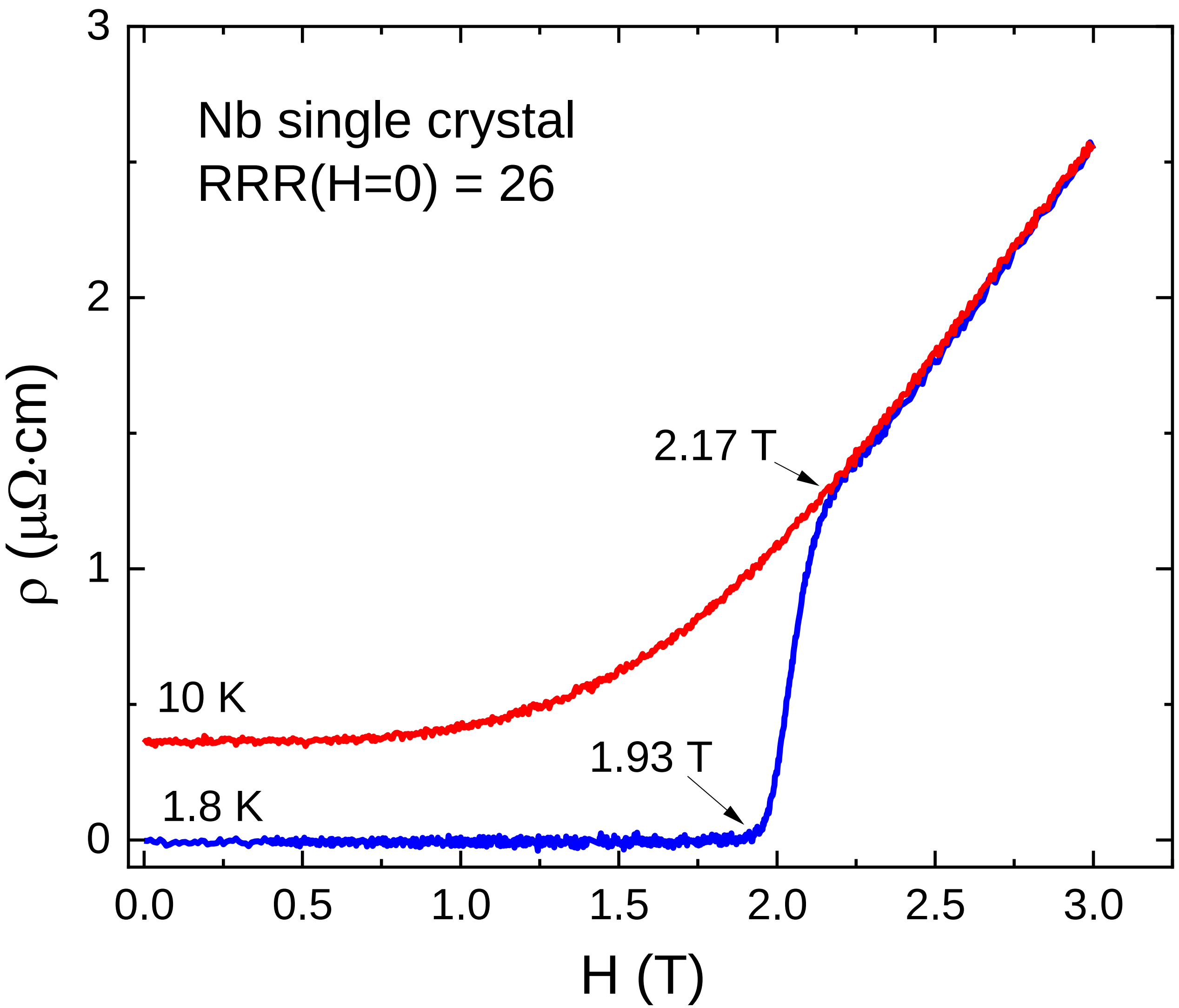
Formula sheet:

$$\rho = \frac{RA}{L}; \ \mathrm{RRR} \approx \frac{R(300\ \mathrm{K})}{R(T_c)}$$

$$H_{c2} = \frac{\phi_0}{2\pi\xi^2}; \quad H_{c2}(0) = 0.7T_c \left.\frac{dH_{c2}}{dT}\right|_{T \to T_c}$$

$$\phi_0 = 2.06783385 \times 10^{-15}\ \text{webers}$$

Current observations:



Magnetic measurements: basic characterization tool for SC

Magnetic field strength, H

$$1\left[\frac{\mathrm{A}}{\mathrm{m}}\right]=\frac{4\pi}{10^3}\,[\mathrm{Oe}]\approx 0.013\,[\mathrm{Oe}],\quad 1\,[\mathrm{Oe}]=\frac{10^3}{4\pi}\left[\frac{\mathrm{A}}{\mathrm{m}}\right]\approx 79.58\left[\frac{\mathrm{A}}{\mathrm{m}}\right]$$

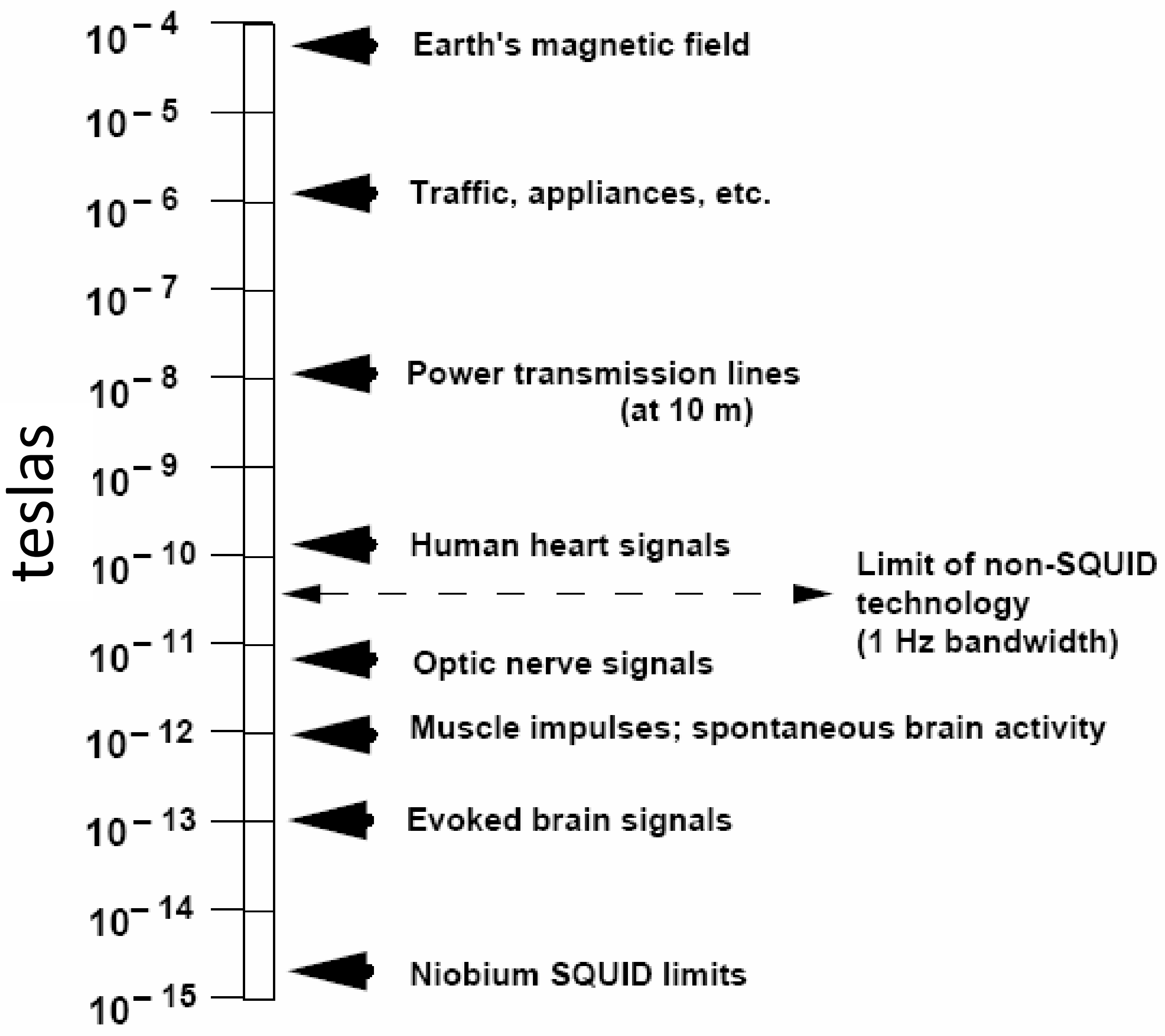
Magnetic induction, B

$$1\,[\mathrm{T}]=10^4\ [\mathrm{G}]$$

Magnetic moment, m

$$1\,[\mathrm{A}\cdot\mathrm{m}^2]=1\left[\frac{\mathrm{J}}{\mathrm{T}}\right]=10^3\left[\frac{\mathrm{erg}}{\mathrm{G}}\right]=10^3\ [\mathrm{emu}]$$

Magnetic Signal Levels



The trivia:

- The strongest field in the universe: magnestar J0243.6+6124 (an ultraluminous pulsar in the Milky Way): **1.6×10^{9} T**
- Surface of a regular neutron star: 10^{11} T
- The **world's highest steady magnetic field** generated by a working magnet is 45.22 T (Steady High Magnetic Field Facility in Hefei, China, August 2022)
- A peak indoor field of 1200 T was generated by the electromagnetic flux-compression (EMFC) technique (Japan, 2018)
- Pulsed Field (LANL) close to 100 T
- Destructibe explosive magnets: 2800 T

- surface of a strong ferromagnet: 2 T
- Maximum magnetic field withstand by a superconductor: BSCCO is a Type-II superconductor. The upper critical field Hc2 in Bi-2212 polycrystalline samples at 4.2 K has been measured as 200 ± 25 T

5×10^{-16} Guinness World Record lowest field (2013, in a hospital!)

1 emu is:

- M of a 1 m^2 loop carrying a 1 mA current
- M of a loop of radius 1.78 cm carrying a 1 A current
- Typical permanent magnet (1 mm^3) ~ 1 emu

- M of a neutron star ~ 10^{30} emu
- The Earth's magnetic moment ~ $8x10^{25}$ emu
- An electron spin: μ_B~10^{-20} emu
- Proton and neutron: μ_N~10^{-23} emu

- One Abrikosov vortex (0.1 mm long) ~ 10^{-10} emu
- Change in M due to d-wave gap < 10^{-10} emu/K
- Hard superconductors ~ 0.1 emu

Demagnetization: often underestimated but extremely important

One of those "inconvenient schmutz effects."

PHYSICAL REVIEW APPLIED **10**, 014030 (2018)

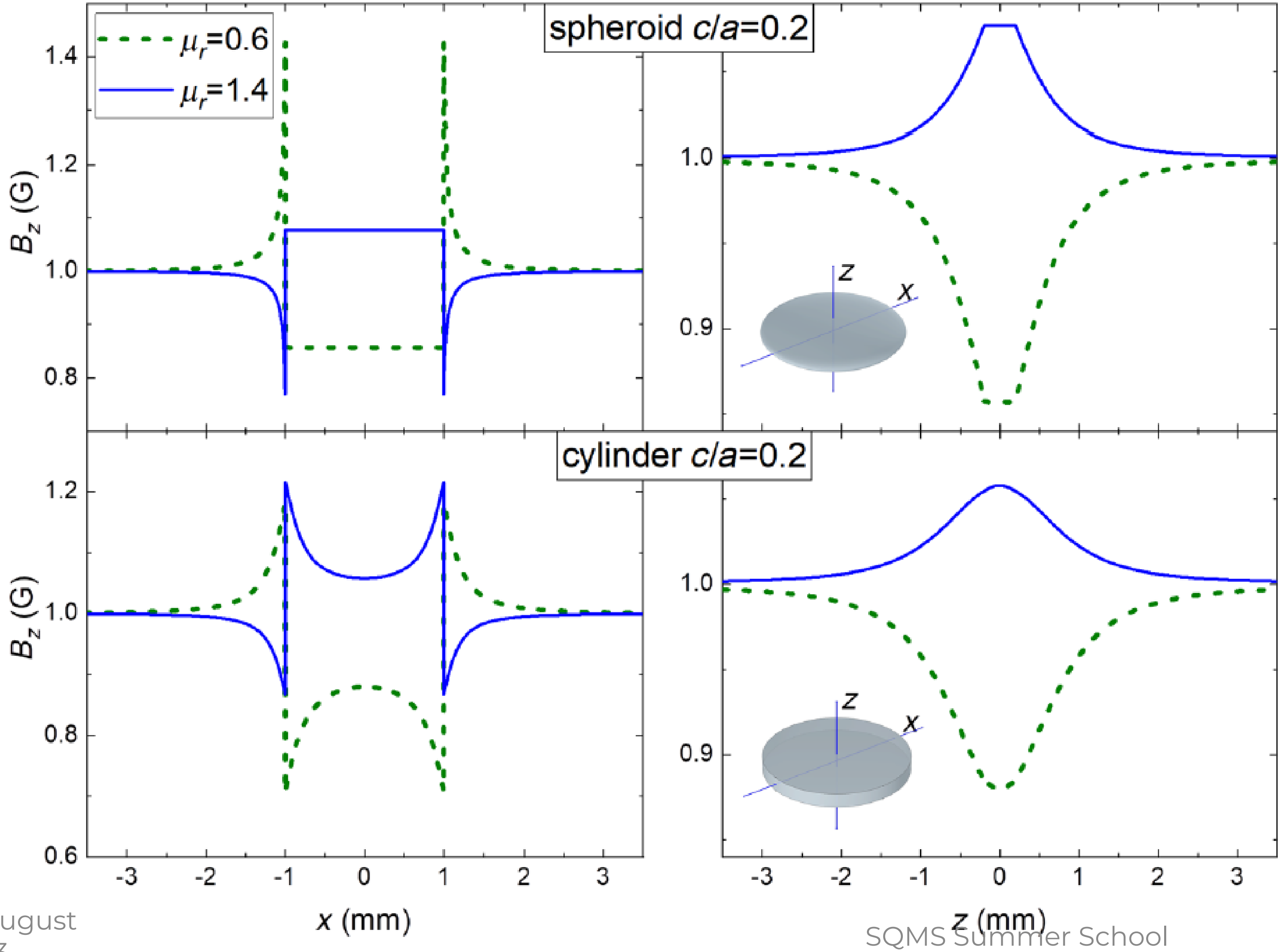


FIG. 2. The B_z component of the magnetic induction across the sample in the x direction (left-hand panels) and in the z direction (right-hand panels) for two values of the relative magnetic permeability, $\mu_r = 0.6$ (diamagnetic, dashed lines) and $\mu_r = 1.4$ (paramagnetic, solid lines). The top panels are for an oblate spheroid and the bottom panels are for a cylinder of the same aspect ratio (see the insets). Note the constant field inside a spheroid and the strongly nonuniform magnetic induction inside a cylinder.

at very low fields:

$$m = \frac{\chi}{1 + N\chi} HV$$

demagnetizing factor

Example: a disc-shaped sample for magnetic measurements

Consider a disc (e.g., cut from a transmon Nb film), 1 mm in diameter and 200 nm thick

Aspect ratio: c/a = 0.0001

If a magnetic field is directed along the plane, neglecting λ,

$$m \simeq -\frac{VH}{4\pi} \approx 1.25 \times 10^{-8} H \text{ emu} (= \text{erg/G})$$

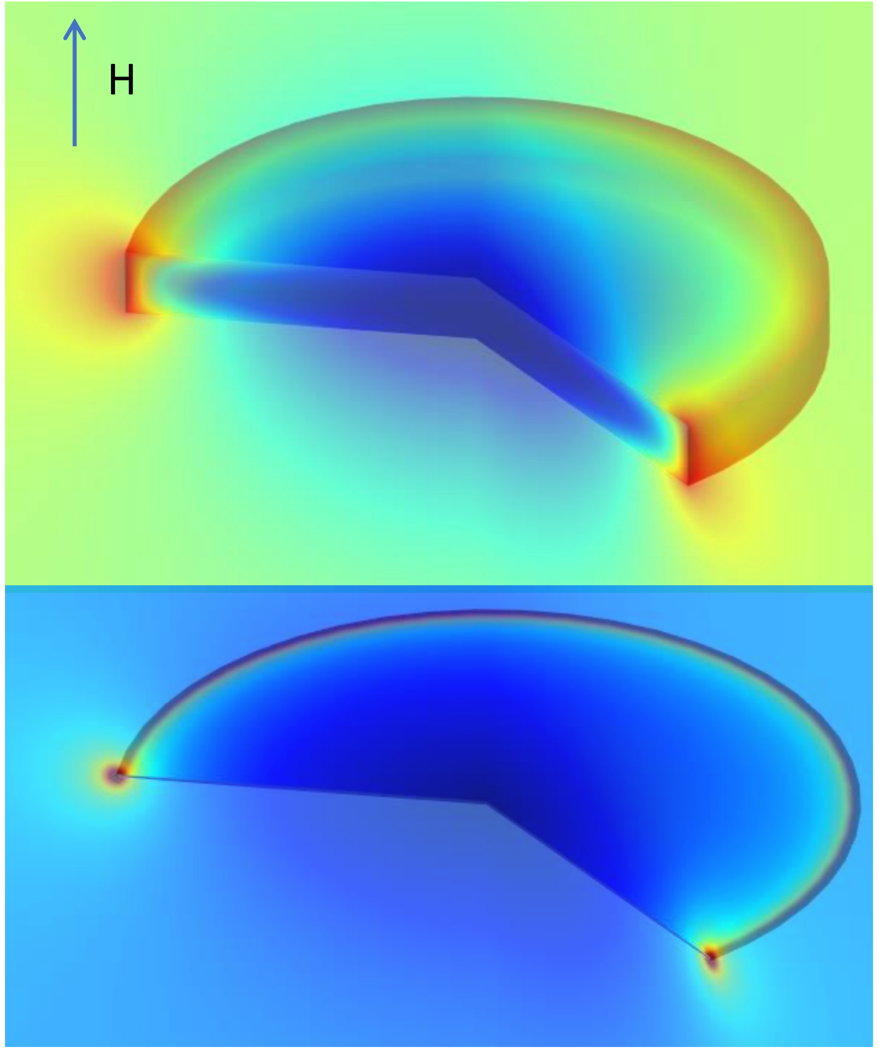
If a magnetic field is directed perpendicular to the plane,

Demag-factor and correction: $\begin{cases} N = 0.99976739012138 \\ \dfrac{1}{1-N} = 4300! \end{cases}$

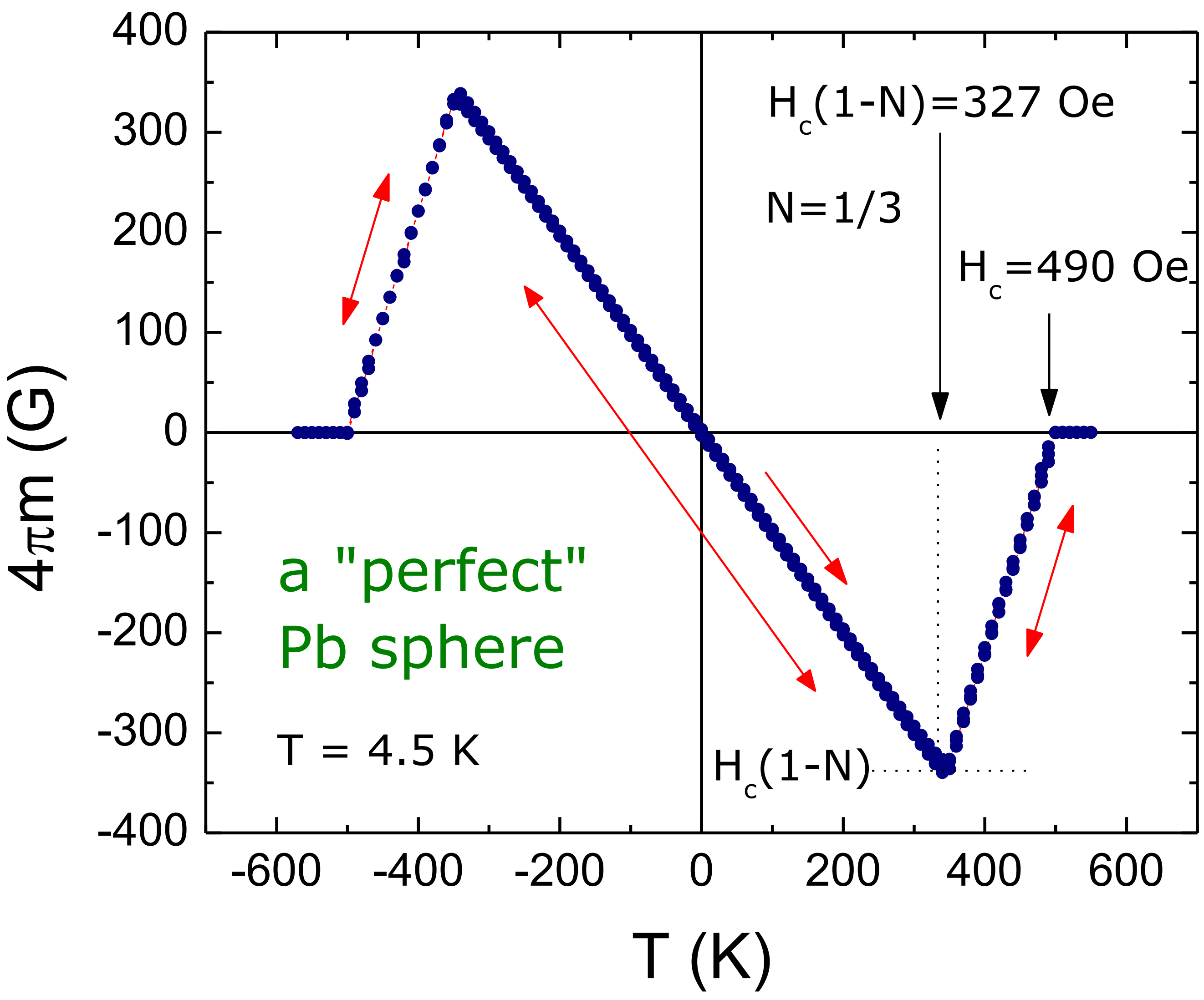
$$m \simeq -\frac{VH}{4\pi(1-n)} \approx 5.37 \times 10^{-5} H \text{ emu} (= \text{erg/G})$$

"typical" (best) sensitivity of a good magnetometer is 10^{-6} emu

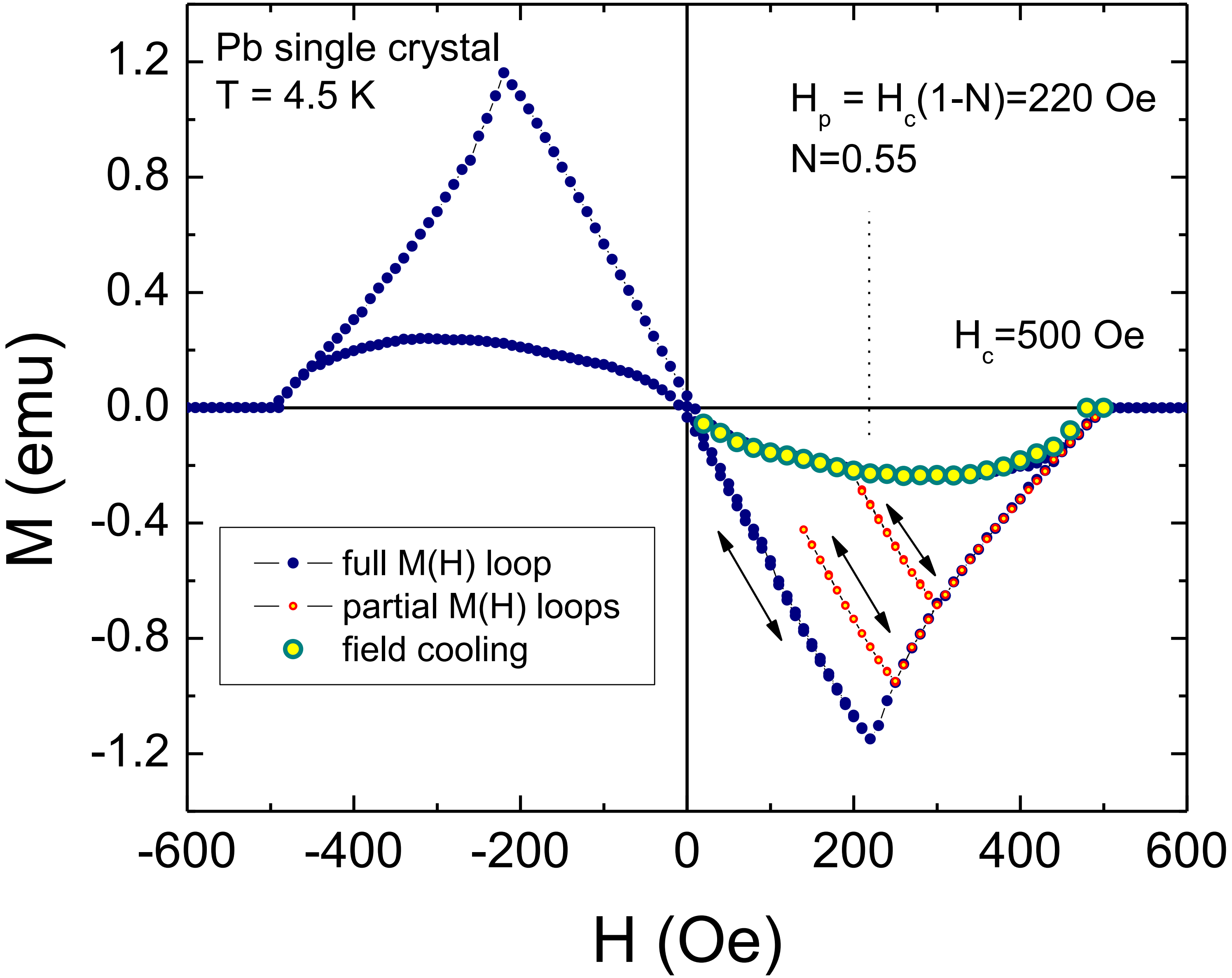
If H = 10 Oe …



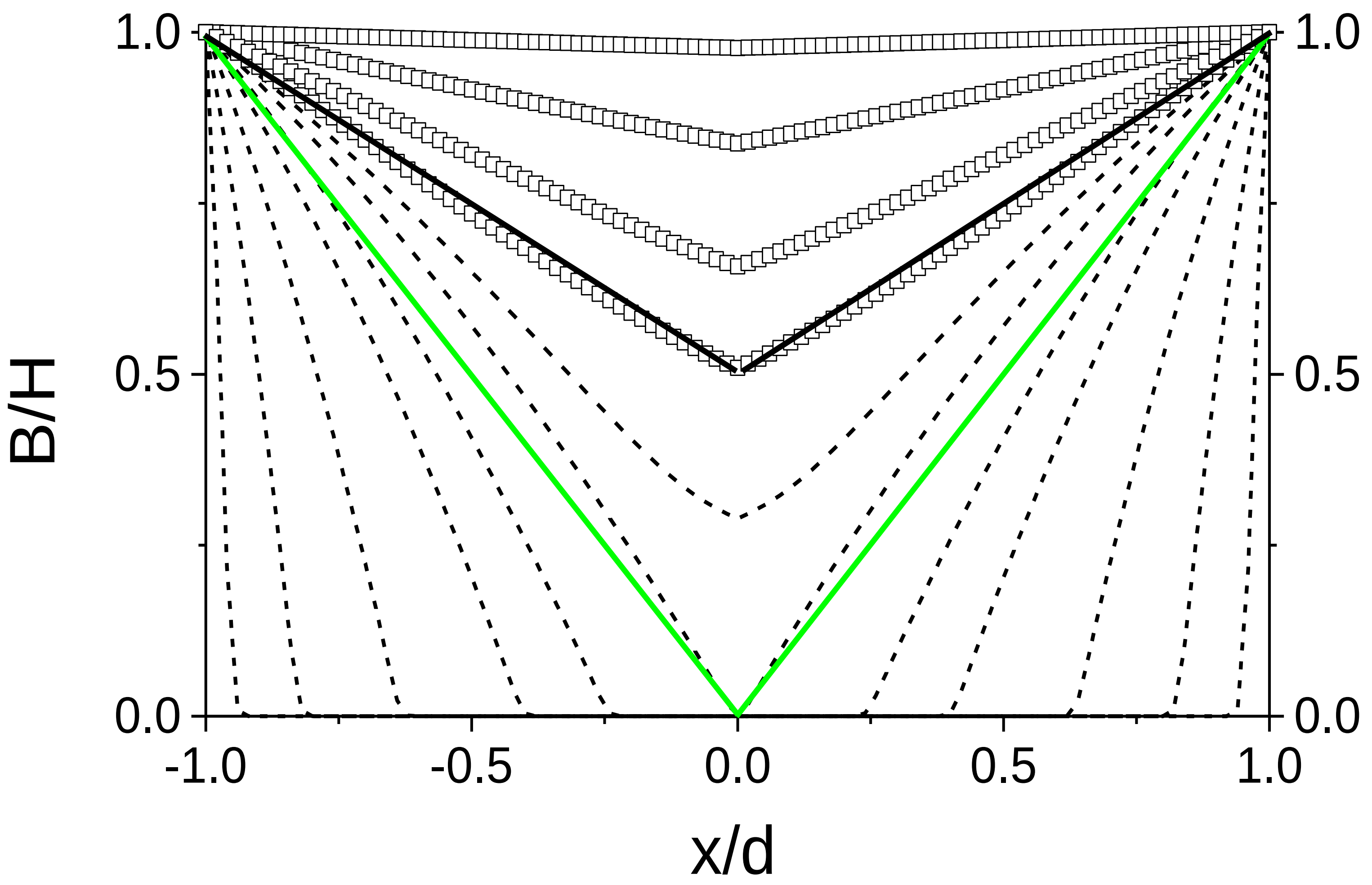
type-I superconductor



hysteresis is a generic feature



m(r) in a superconductor after ZFC+H(pulse)



Josephson effect

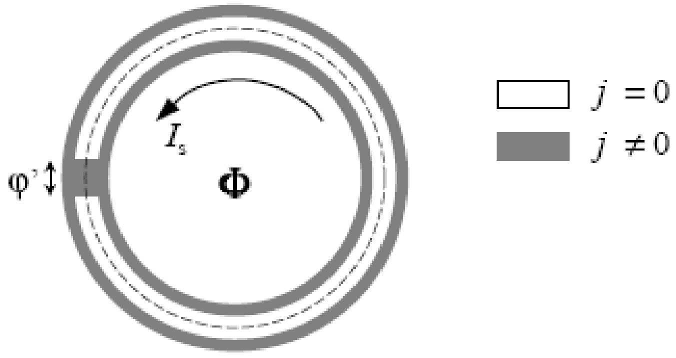


Fig. 2.7 : Sketch of superconducting ring interrupted by a weak link. The dashed line is the integration path.

$$\varphi = \theta_2 - \theta_1$$

The previous phase was not gauge invariant. The gauge invariant phase is obtained considering gauge invariant gradient

$$\frac{\hbar}{i}\nabla - \frac{2e}{c}\mathbf{A}$$

and therefore,

$$\gamma = \Delta\varphi - \frac{2\pi}{\Phi_0}\int_L \mathbf{A}d\mathbf{l}$$

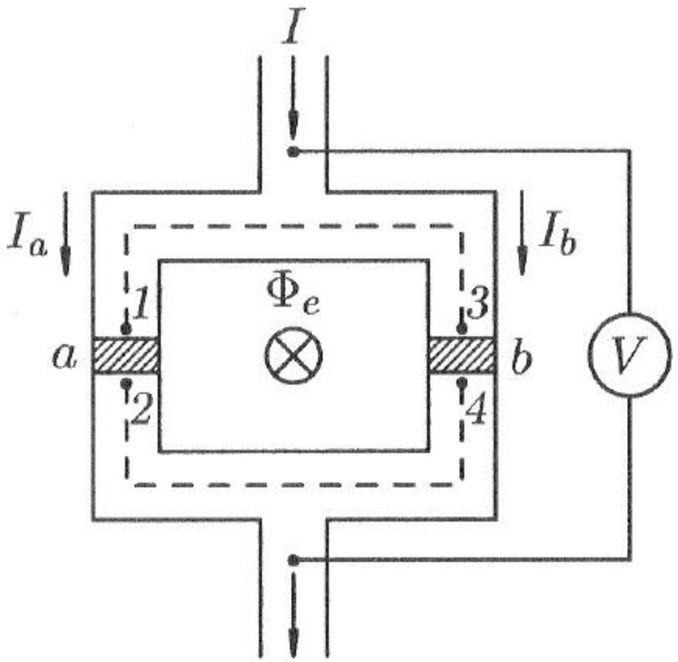
and

$$I_s = I_c \sin(\gamma)$$

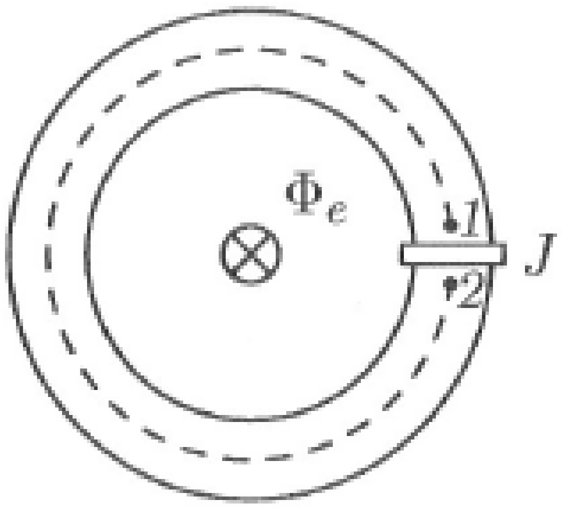
Superconducting Quantum Interference Device (SQUID)

flux-voltage convertors

DC SQUID



AC (RF) SQUID



Josephson junction

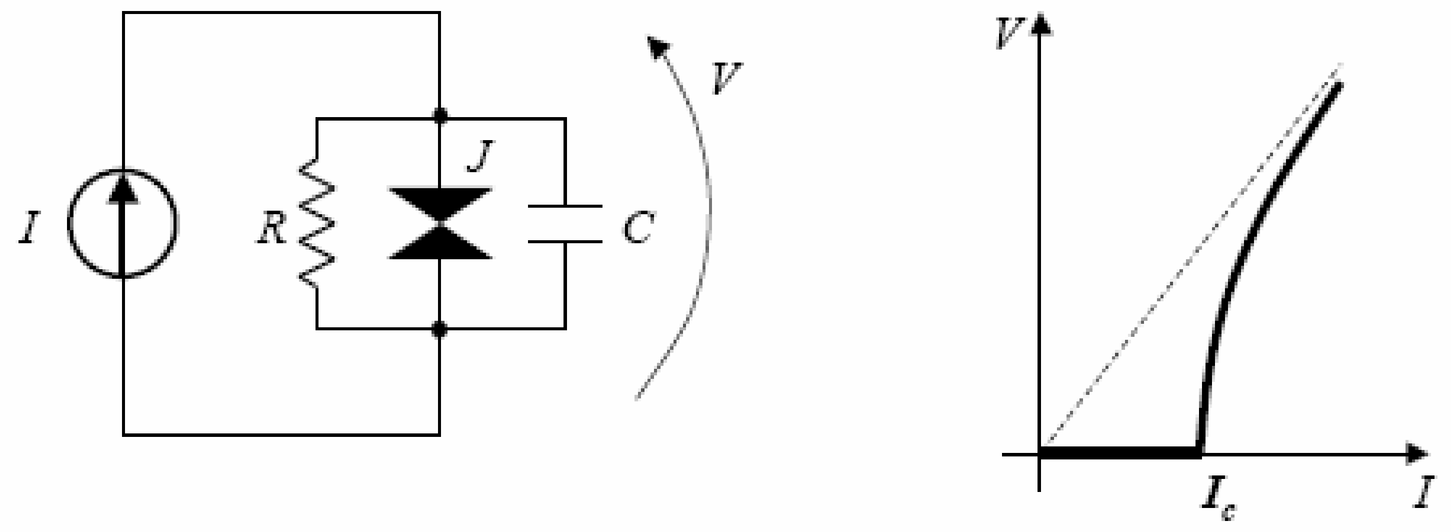
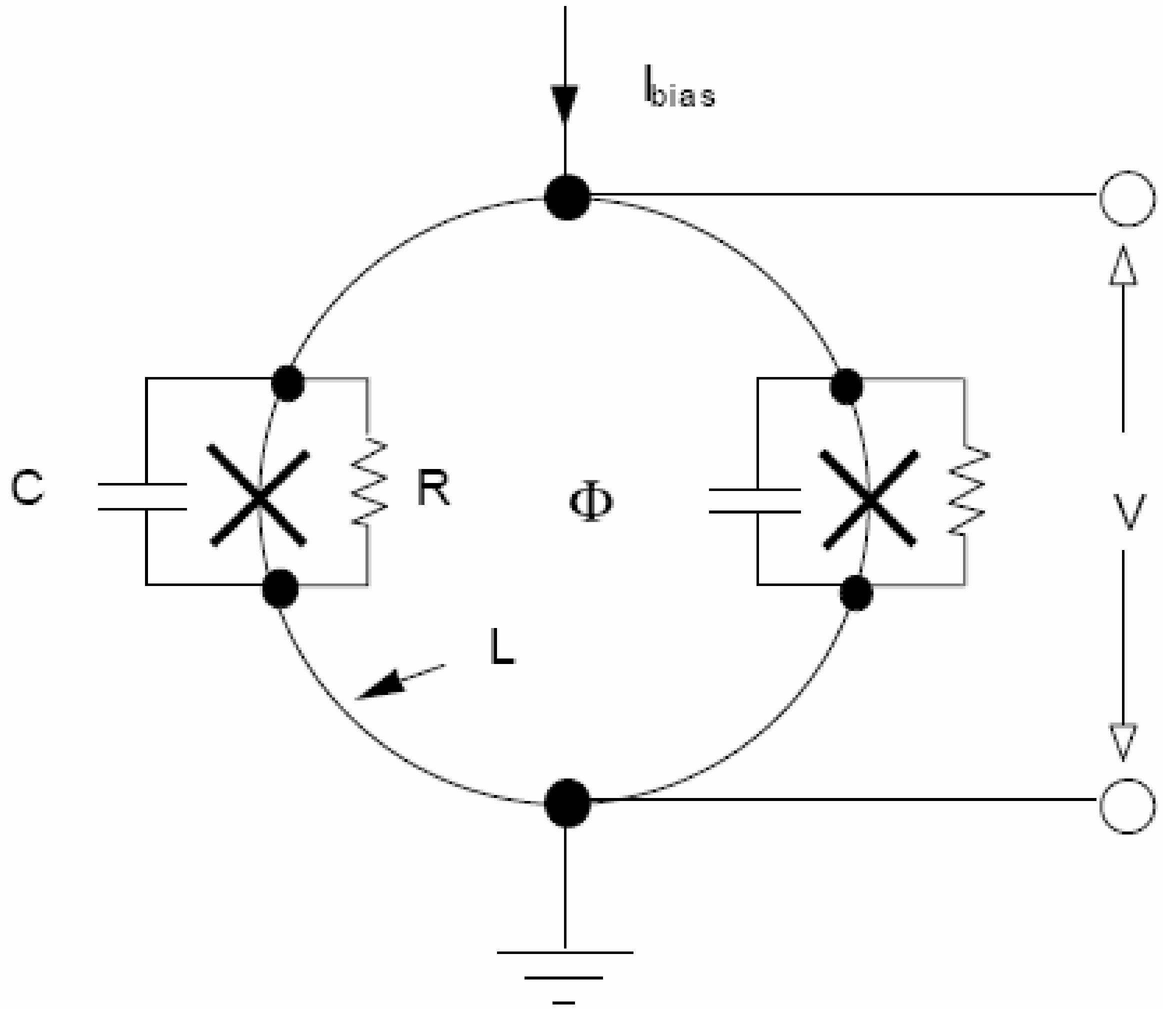


Fig. 2.8 : Model of current-biased Josephson junction (J) with dissipation (R) and capacitance (C), and the relative V − I characteristic.

A more complete model of a DC SQUID



DC SQUID and SQUID magnetometry

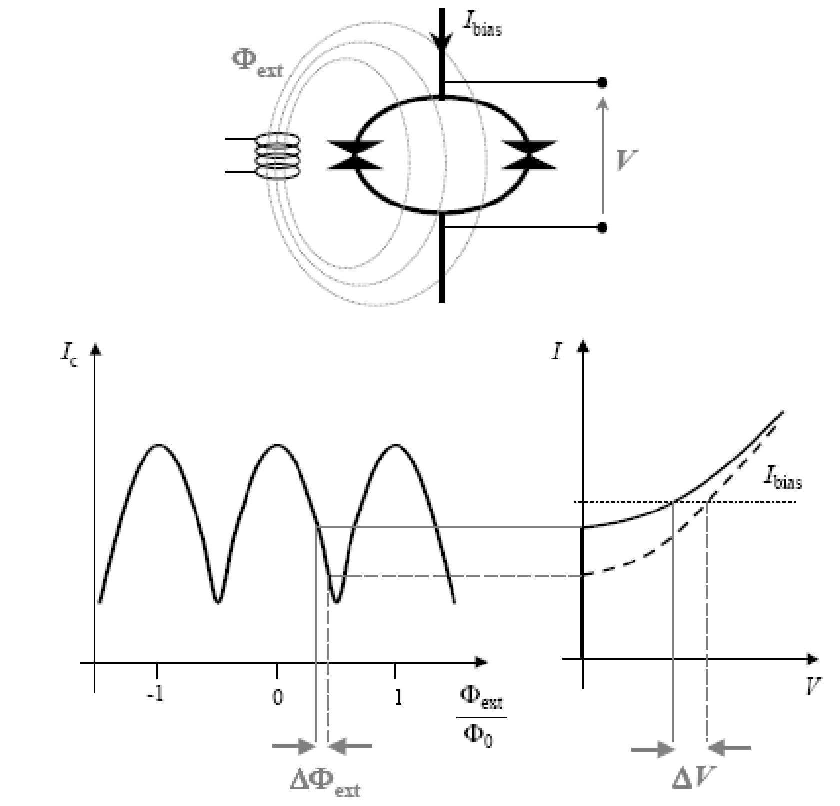


Fig. 2.9 : Working principle of a dc-SQUID as flux-to-voltage converter.

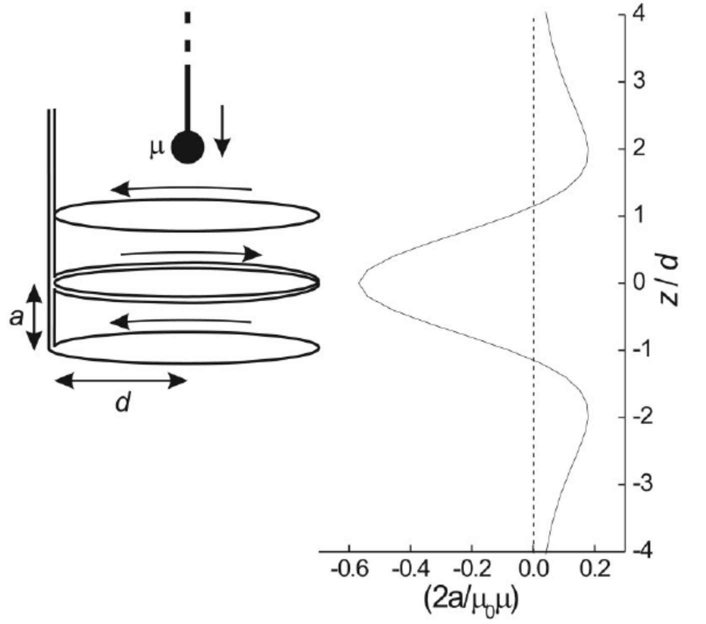


FIG. 3. A second-order gradiometer in Helmoltz geometry ($a=2d$) and the magnetic flux induced by a dipole moving along z. Notice that the side peaks in $\Phi(z)$ do not coincide with the positions of the external coils. The factor a in the x-axis scale is obtained by using Eq. (1).

$$\Phi_{\text{loop}}(z) = \frac{1}{a}\left[1 + \frac{z_0^2}{a^2}\left(\frac{z}{z_0} - 1\right)^2\right]^{-3/2}. \qquad (1)$$

Characterization by magnetization

PHYSICAL REVIEW B 74, 220511(R) (2006)

Collapse of the critical state in superconducting niobium

Ruslan Prozorov,[1,*] Daniel V. Shantsev,[2,3] and Roman G. Mints[4]

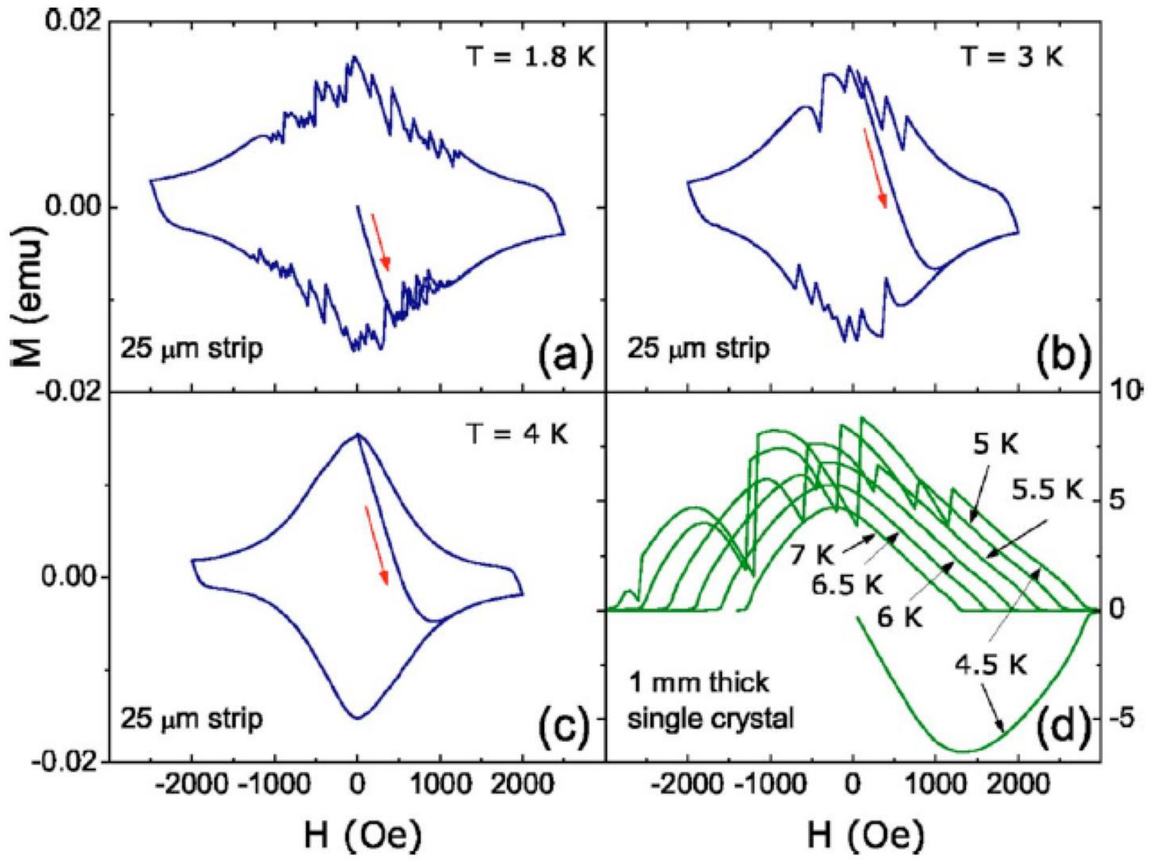


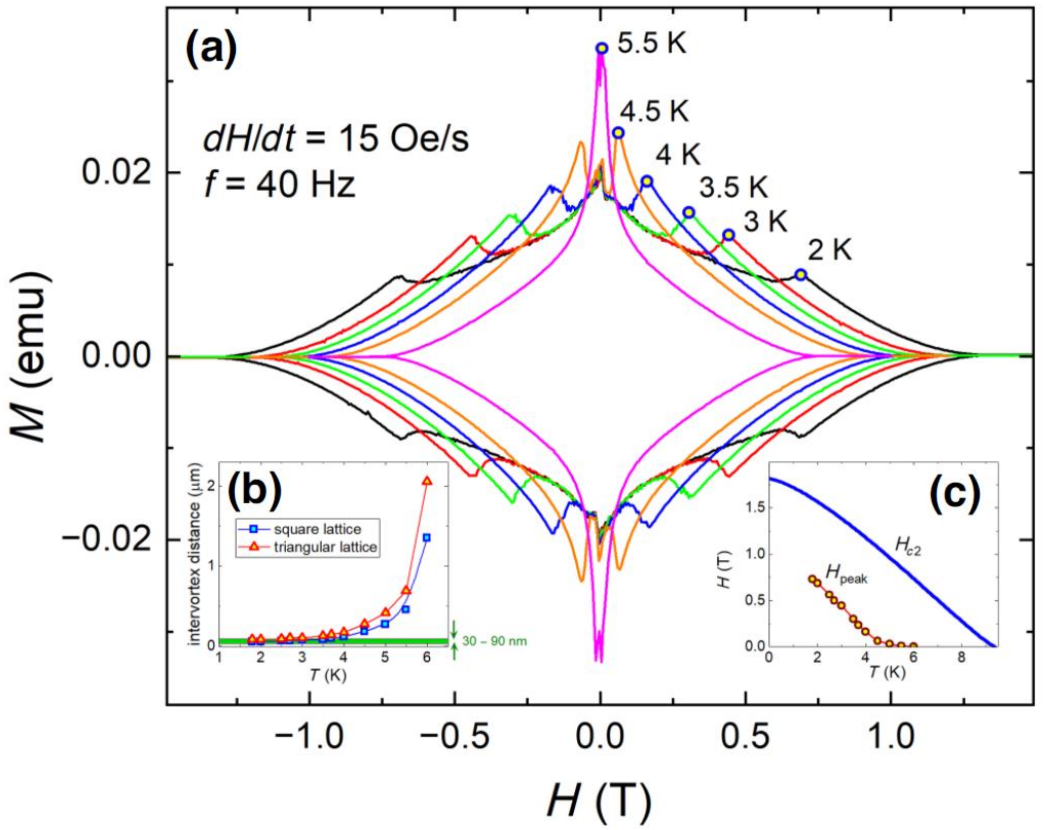
FIG. 1. (Color online) Magnetization loops measured in 25-μm-Nb foil at different temperatures, (a) 1.8 K, (b) 3 K, (c) 4 K, and, for comparison, (d) several *M* (*H*) curves measured from 4 to −4 kOe in large Nb single crystal. For (b) and (c) curves start from remanent magnetization, see text for details.

Don't we want to know what happens during these magnetization jumps?

(later…)

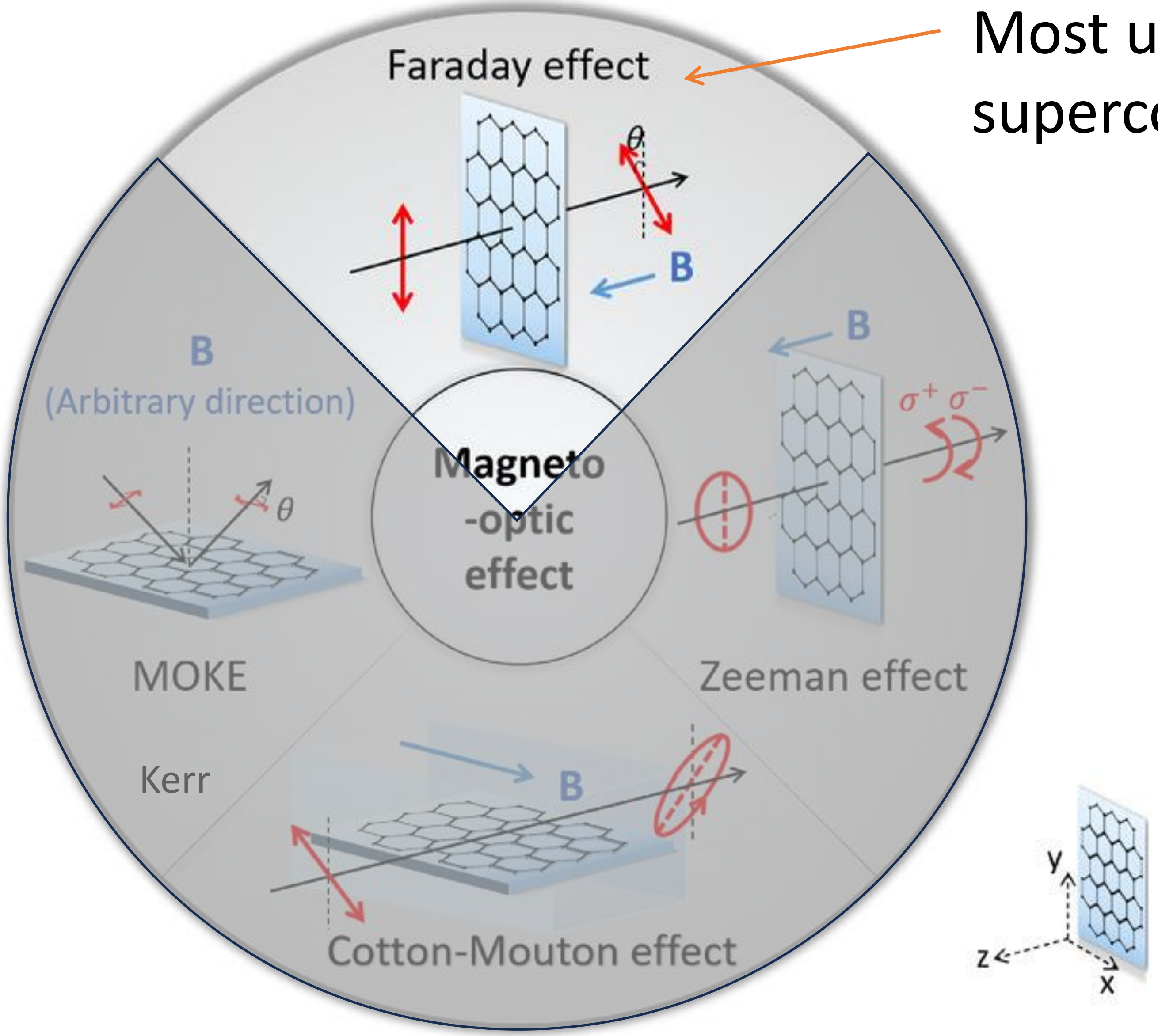
M(H) loop measured in a thin film (SQMS material)

PHYS. REV. APPLIED **0**, XXXXXX (2023)



An example from just – accepted paper. This is magnetization in a Nb thin film. What are all these features?

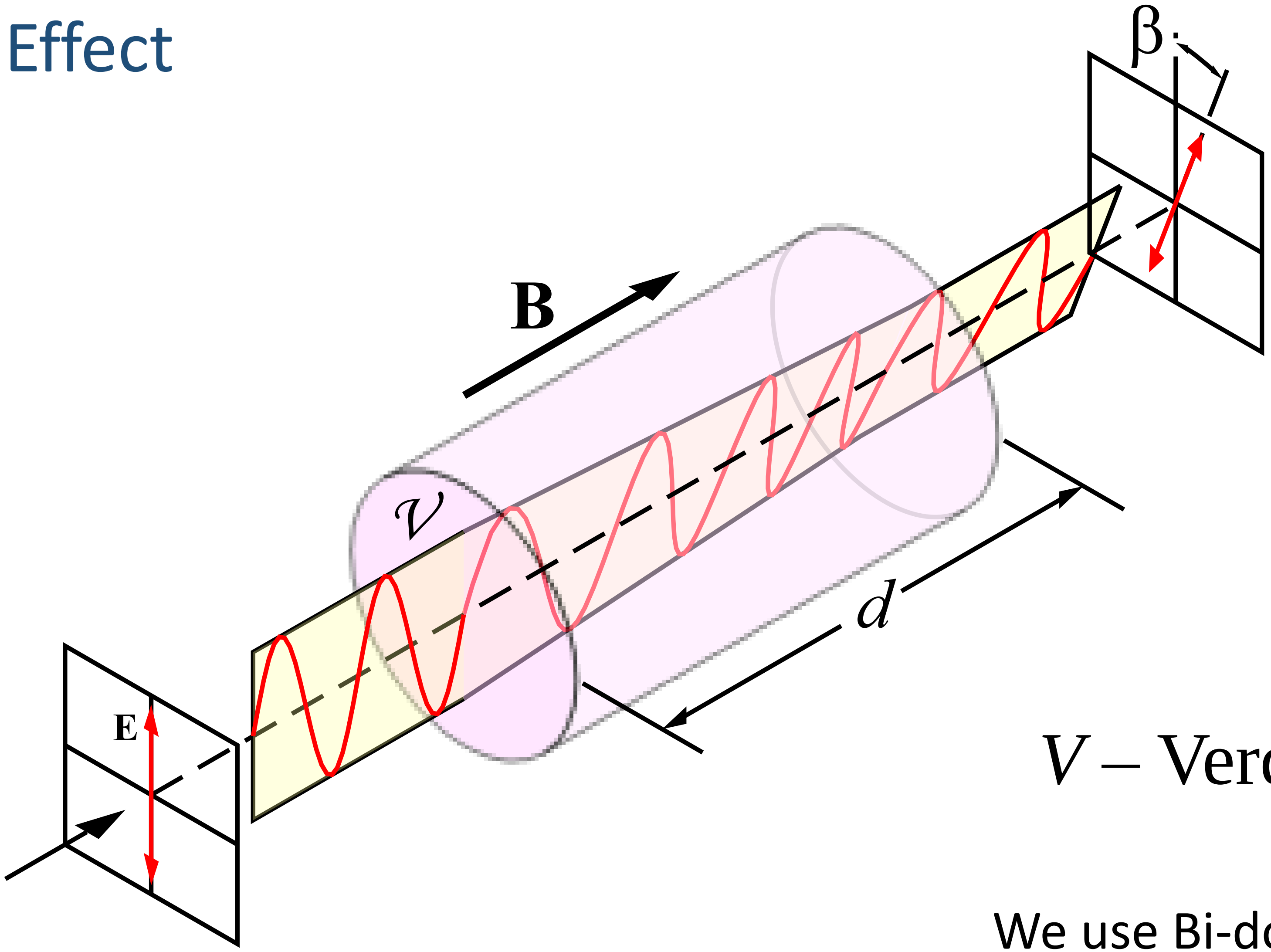
Visualization of the magnetic fields: magneto-optics



Most useful to study superconductors

Source: Nano Select 1(3), 298 (2020)

Faraday Effect



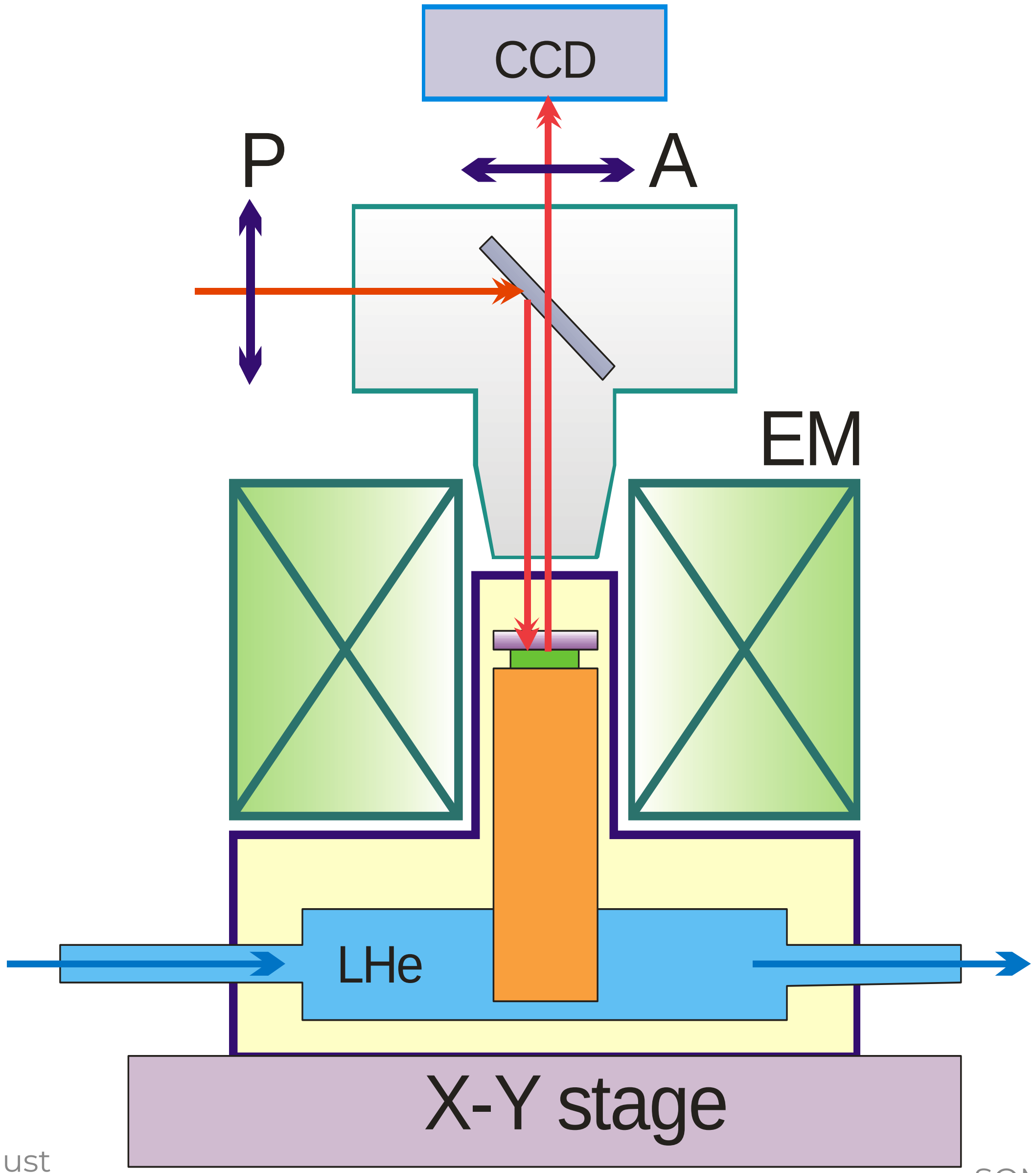
$$\beta = VBd$$

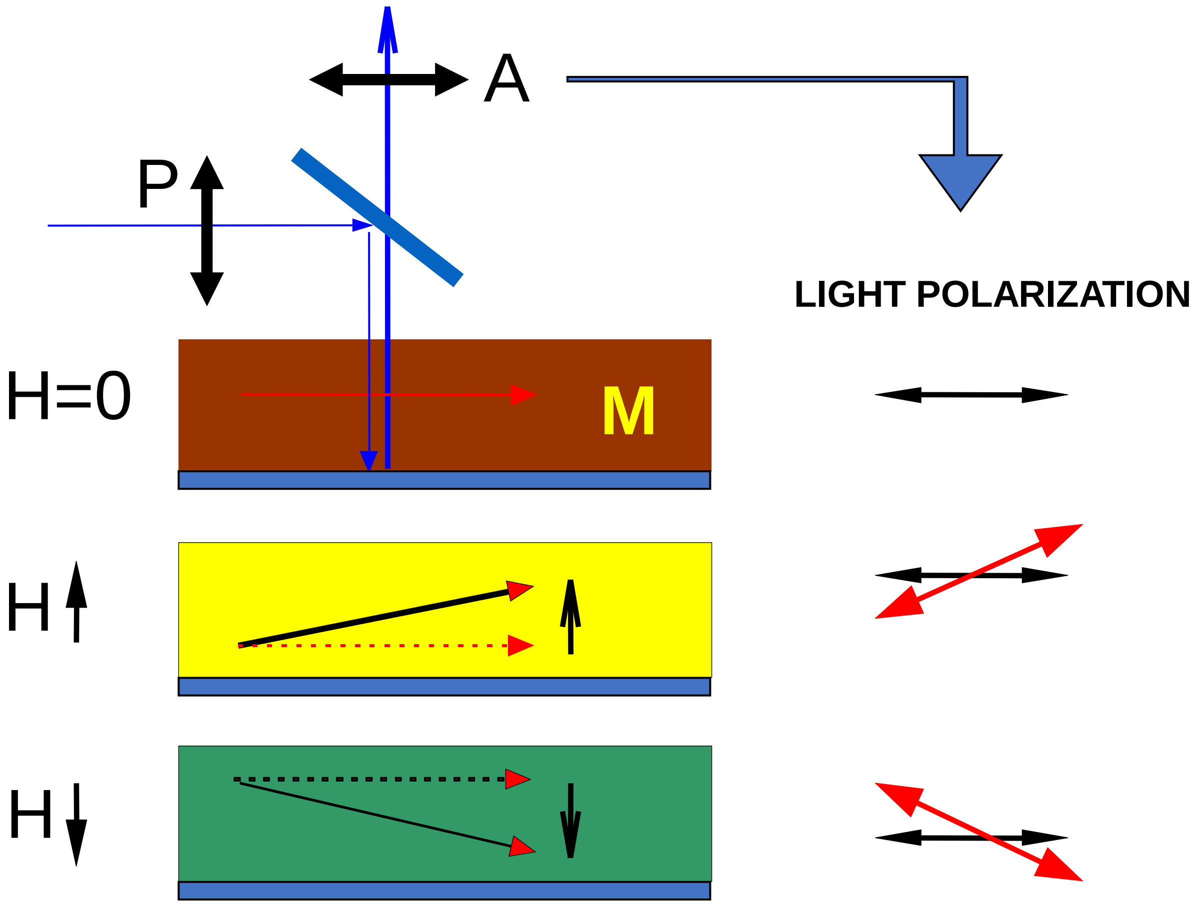
V – Verdet constant

We use Bi-doped Fe garnets

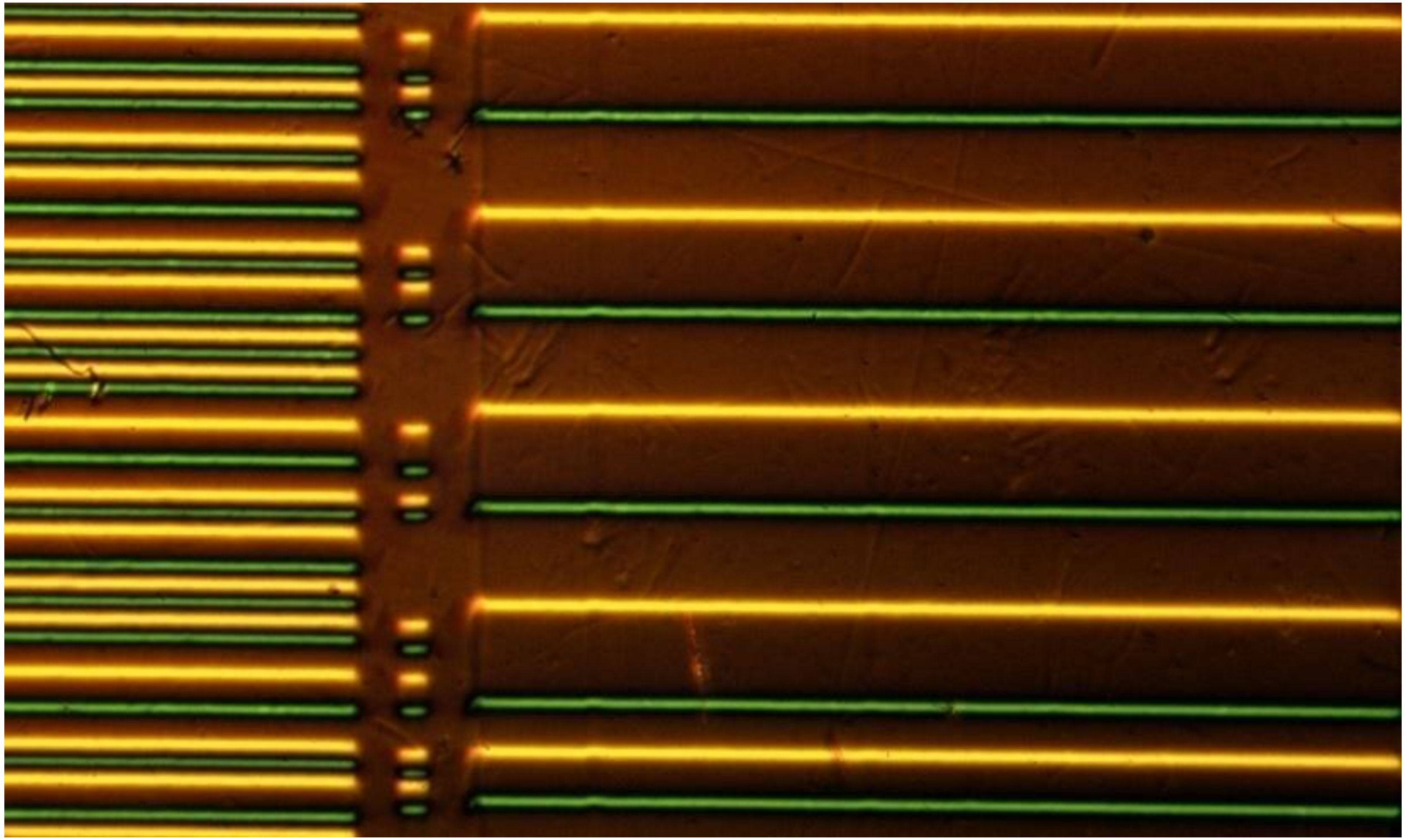
https://en.wikipedia.org/wiki/Faraday_rotator#/media/File:Faraday-effect.svg

Magneto-optical setup





example: barcode of a credit card



Characterization by magnetization

PHYSICAL REVIEW B **74**, 220511(R) (2006)

Collapse of the critical state in superconducting niobium

Ruslan Prozorov,[1,*] Daniel V. Shantsev,[2,3] and Roman G. Mints[4]

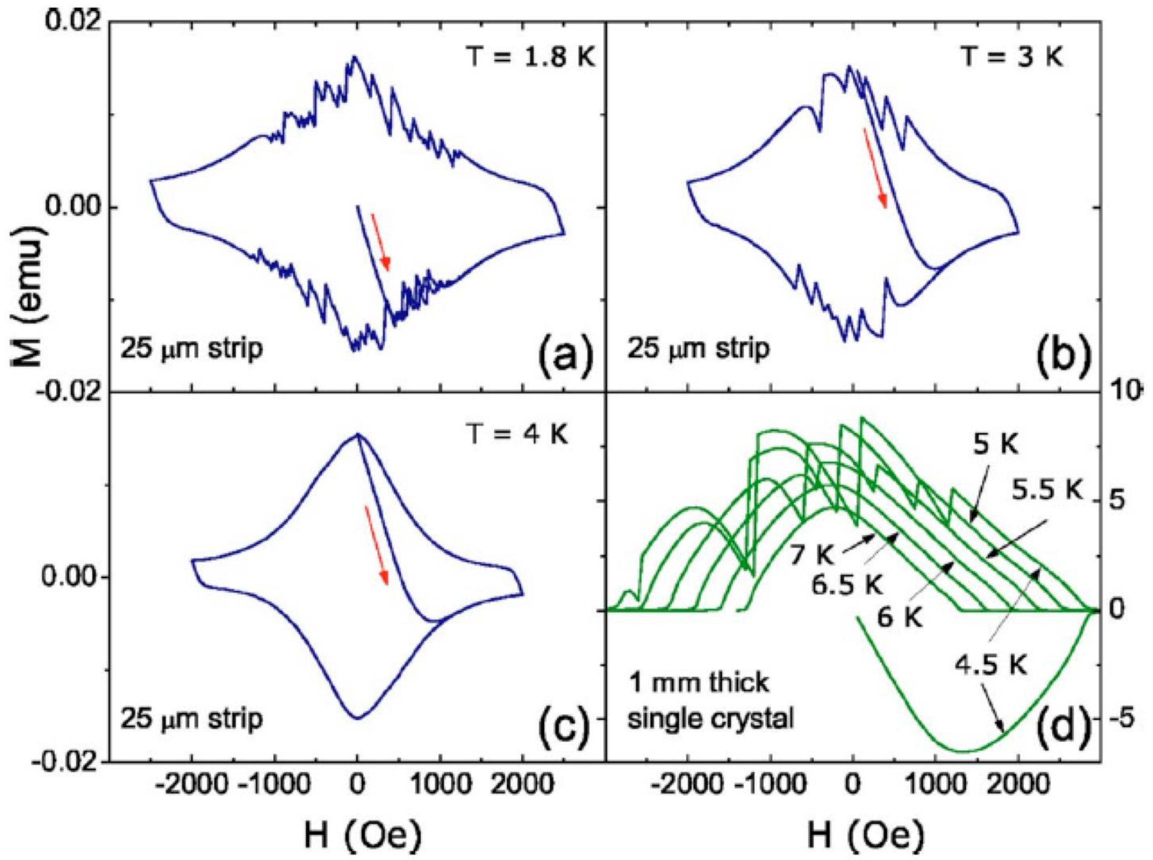
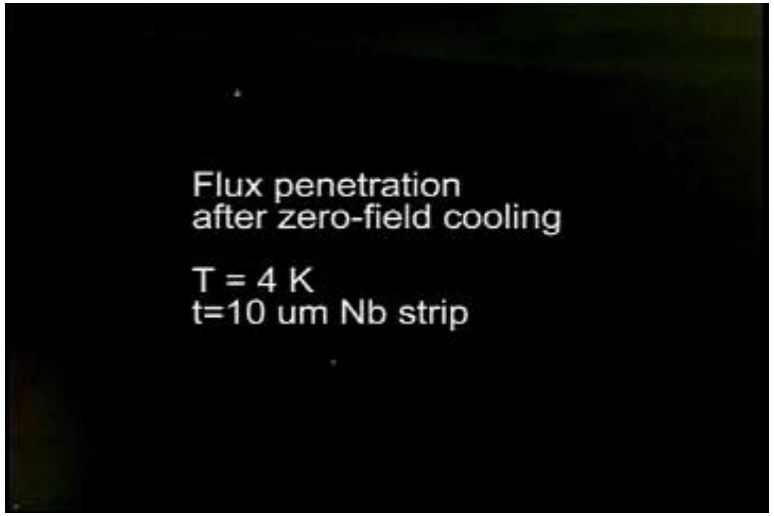
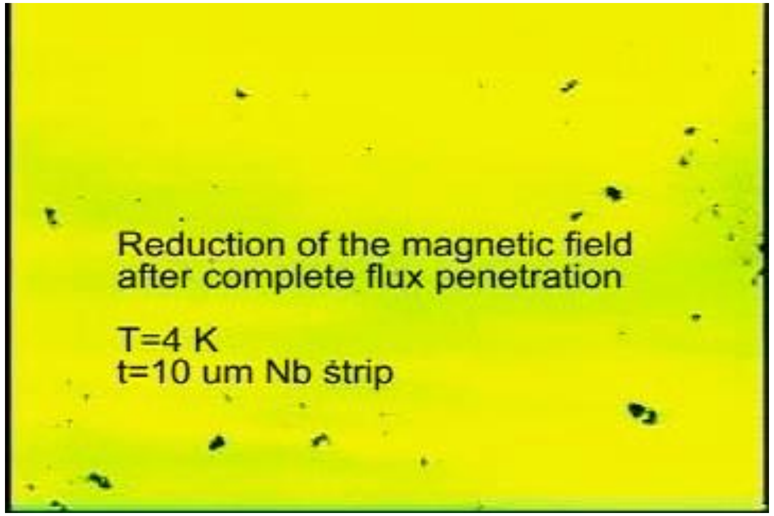


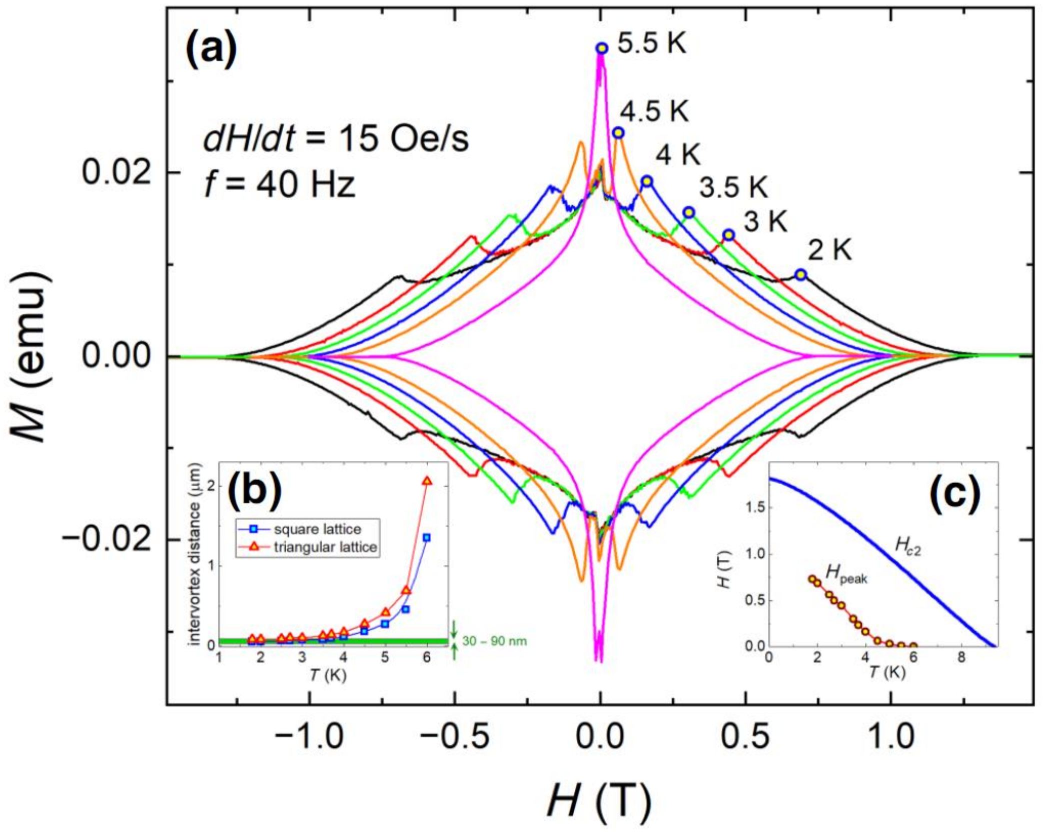
FIG. 1. (Color online) Magnetization loops measured in 25-μm-Nb foil at different temperatures, (a) 1.8 K, (b) 3 K, (c) 4 K, and, for comparison, (d) several M (H) curves measured from 4 to −4 kOe in large Nb single crystal. For (b) and (c) curves start from remanent magnetization, see text for details.

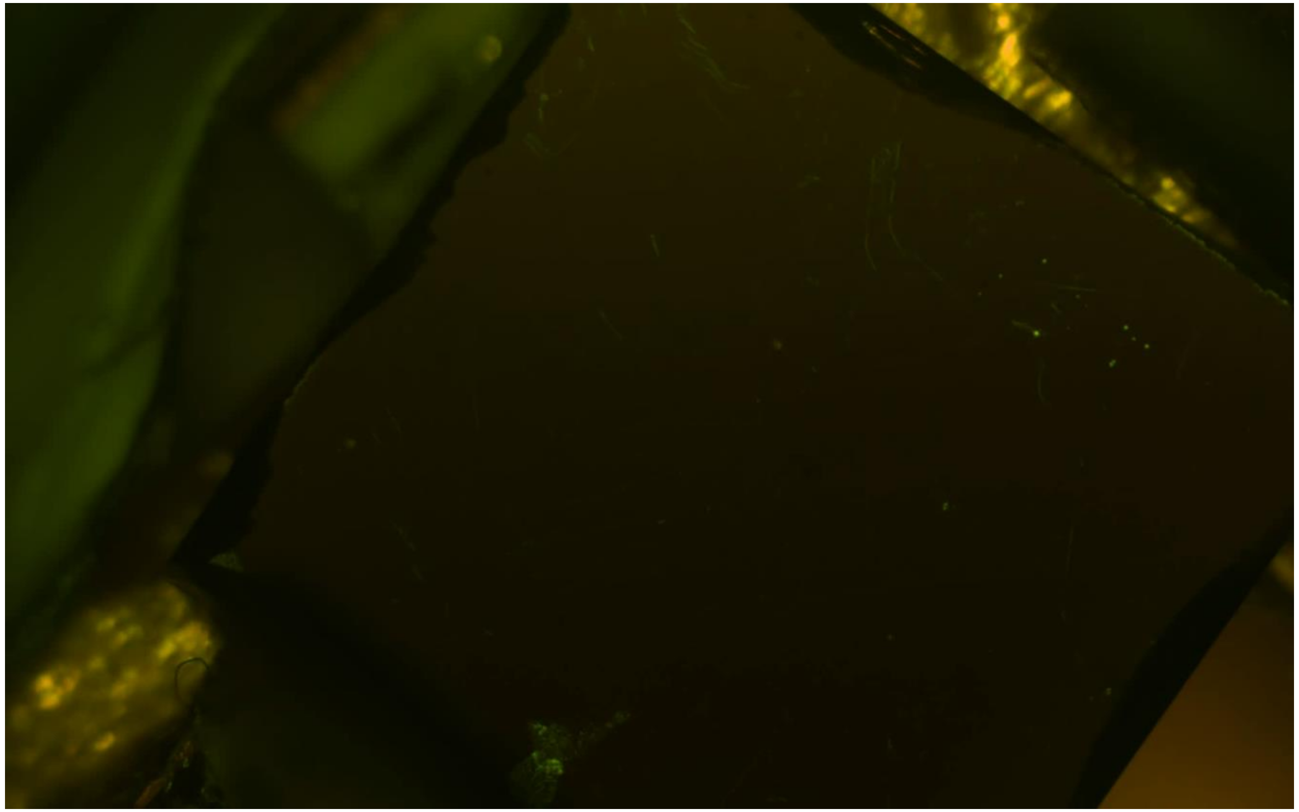


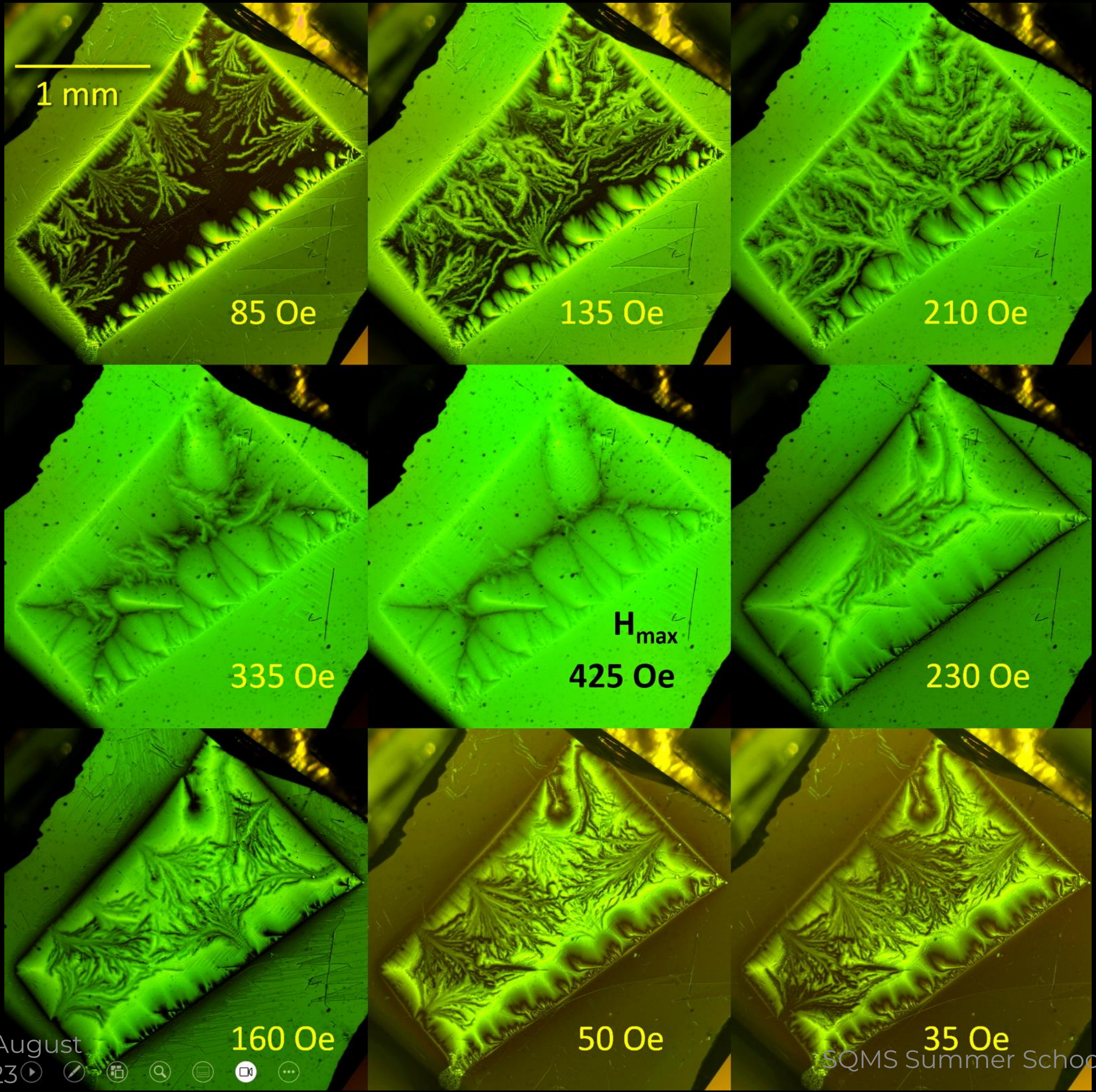


M(H) loop measured in a thin film (SQMS material)

PHYS. REV. APPLIED **0**, XXXXXX (2023)

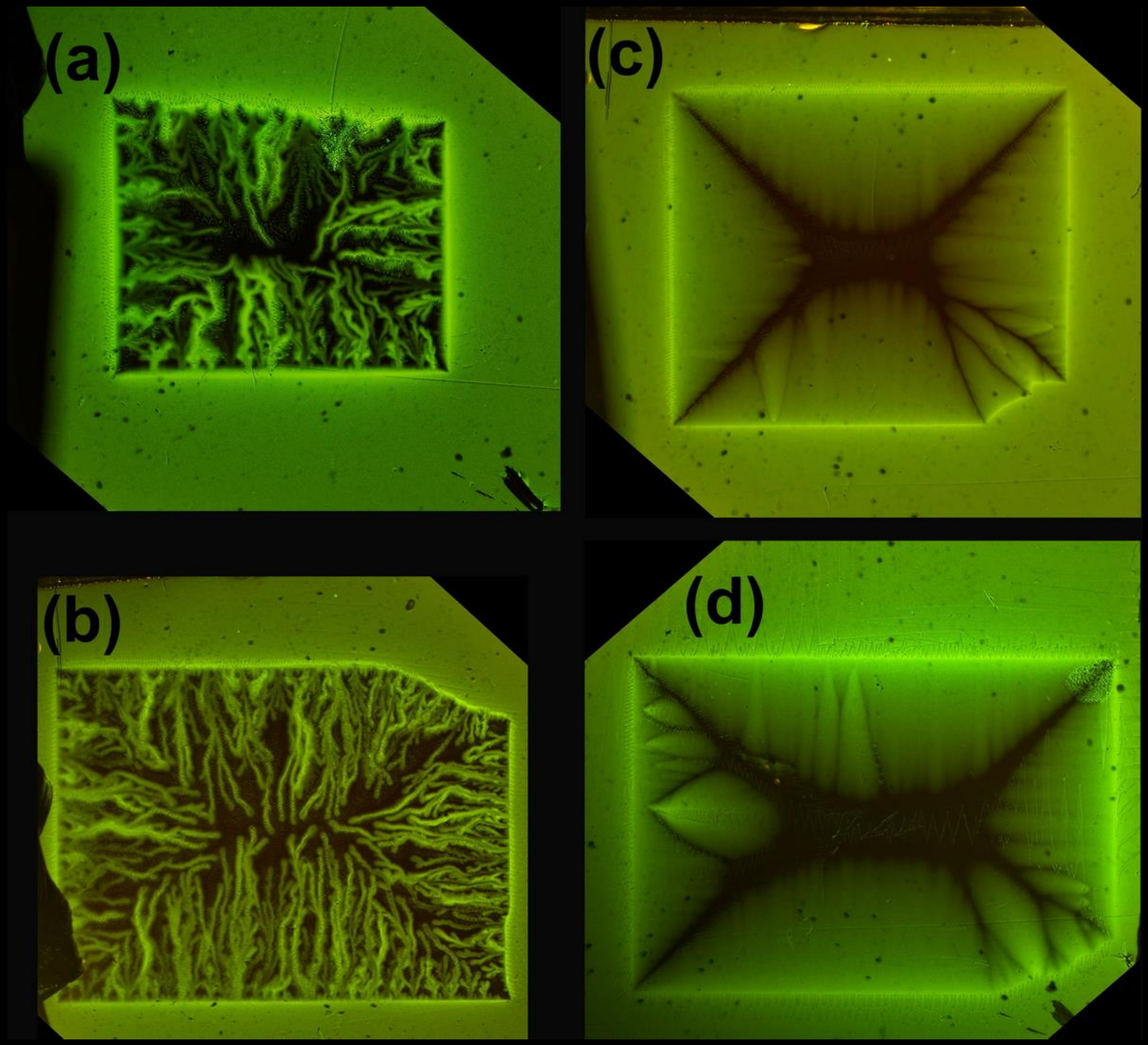




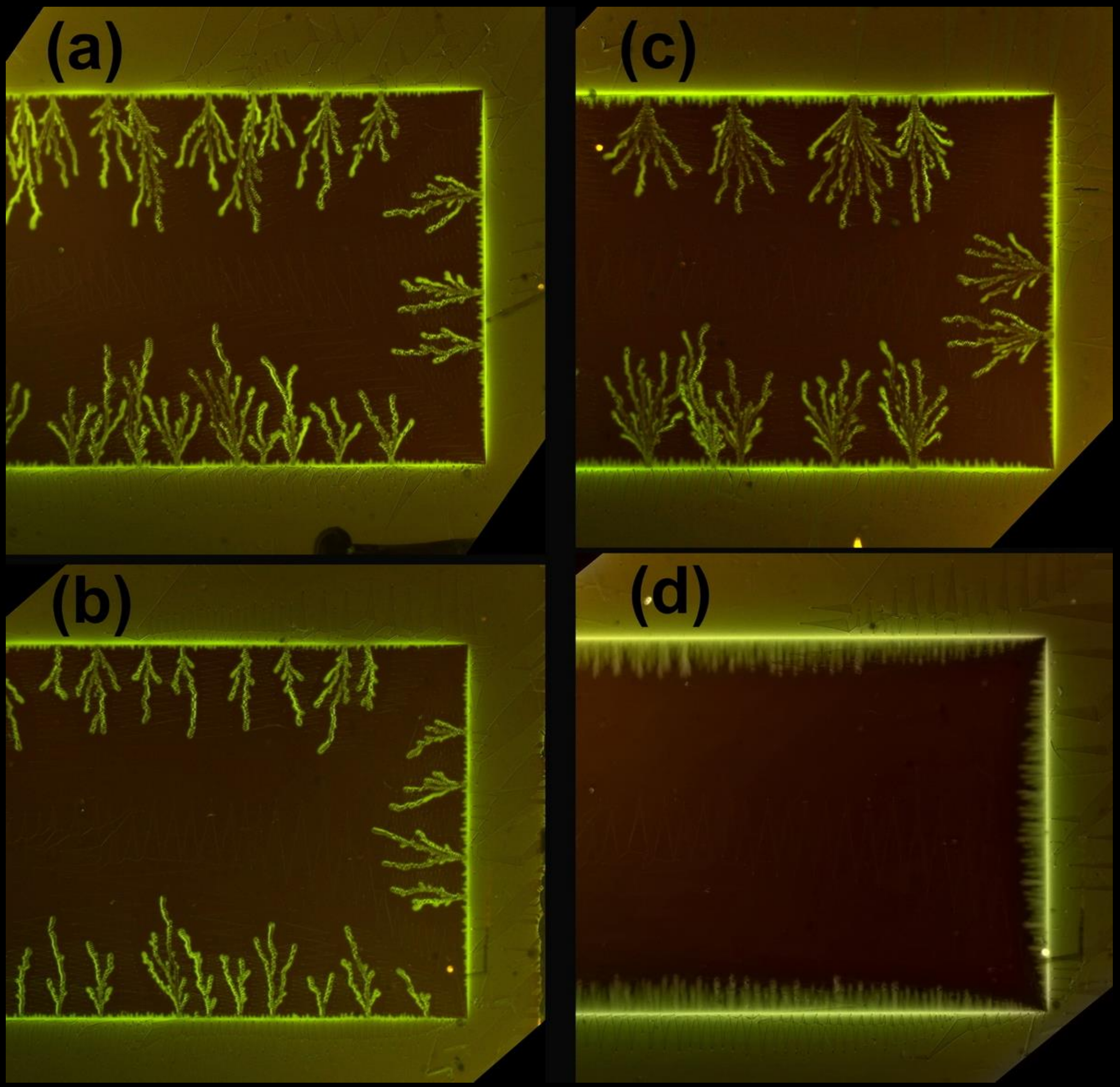


M(H) loop - visualized

Magnetic flux in niobium films prepared by different methods



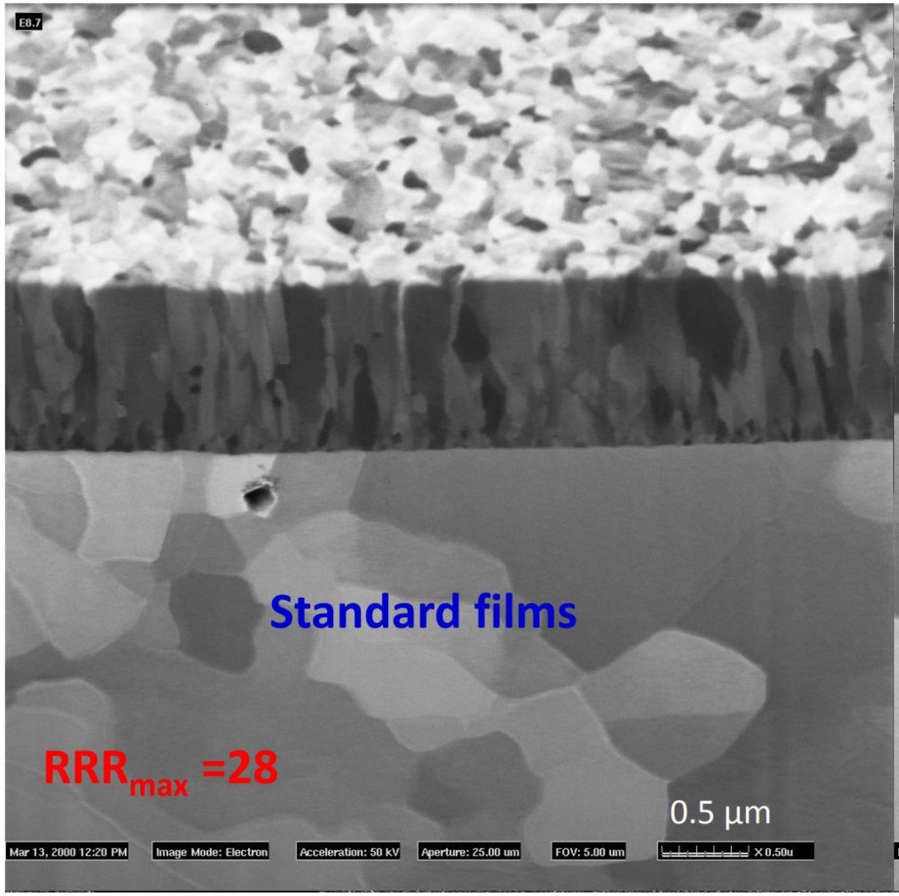
5 K, 130 Oe

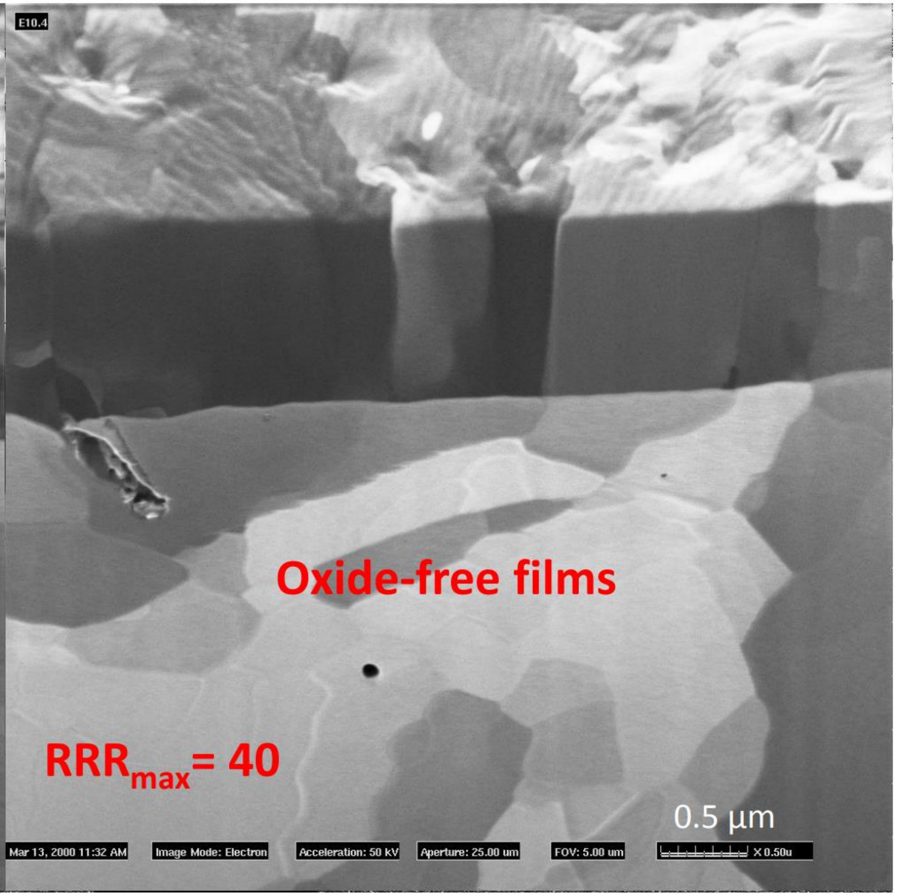


4 K, 40 Oe

Nb/Cu Sputtered Films:

Film structure – FIB cross sections





Columnar grains, size ~ 100 nm
In plan diffraction pattern: powder diagram
(110) fiber texture ⊥ substrate plane

Equi-axed grains, size ~ 1-5µm
In plan diffraction pattern: zone axis [110]
Heteroepitaxy
Nb (110) //Cu(010) , Nb (110) //Cu(111),Nb (100) //Cu(110)

Courtesy: P. Jacob - EMPA

On RRR=R(300)/R(T_c)

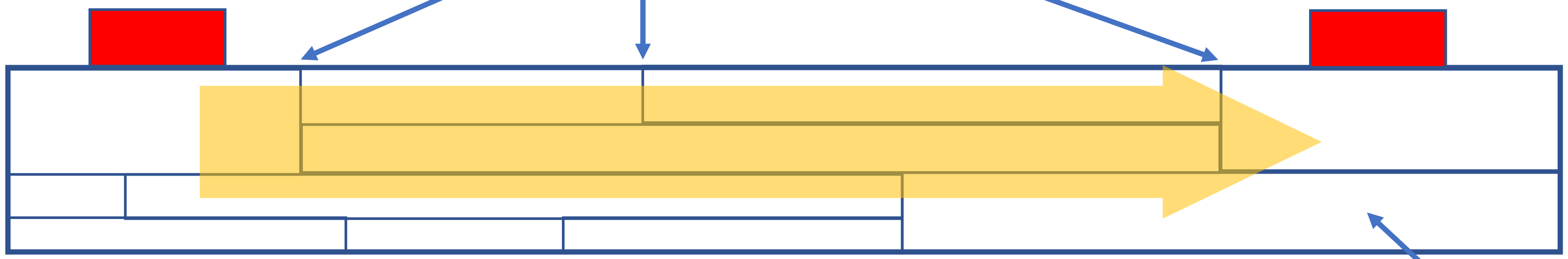
In Nb, there is a huge range of experimentally observed RRR, from about 3 to 90,000

A. Koethe and J. I. Moench, Preparation of Ultra High Purity Niobium, Materials Transactions, JIM **41**, 1 (2000).

In Rigetti thin films, RRR = 5. However, we measure Tc=9.3 K. WHY?
According to ZUS, T_c with such RRR should be below 4 K.

[1]M. Zarea, H. Ueki, and J. A. Sauls, Effects of Anisotropy and Disorder on the Superconducting Properties of Niobium, arXiv:2201.07403 (2022).

grain/structural domain boundaries do not affect bulk Tc!

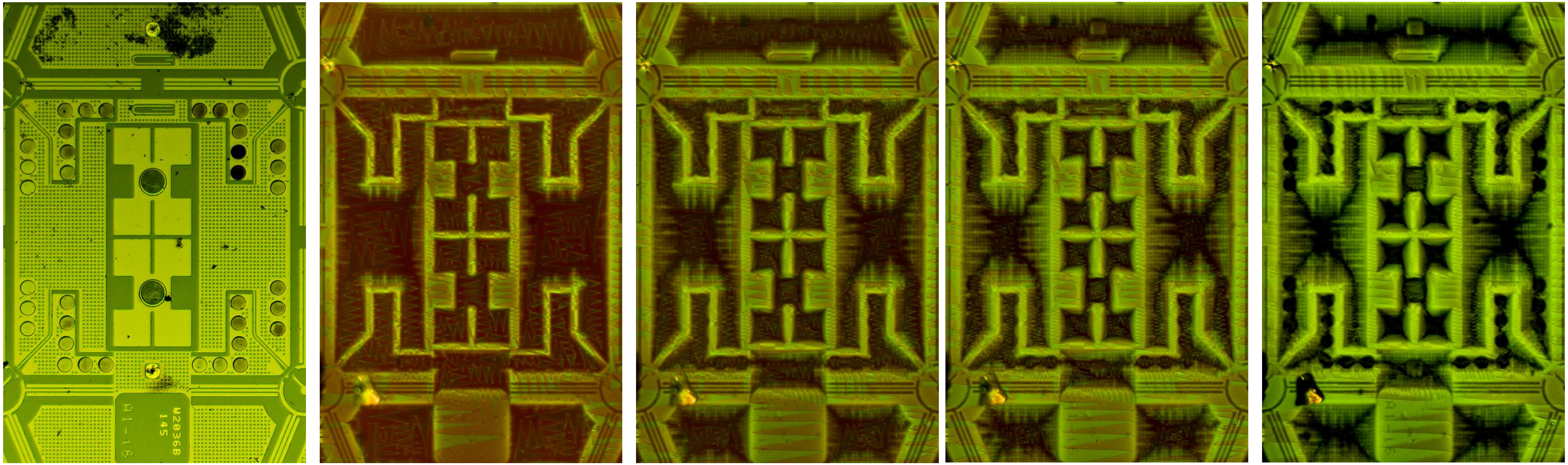


- **300 K – dominant – e-phonon scattering**
- **just above T_c – dominant GB**
- **Just below T_c, GB become SC - proximity**

parallel AND serial connection

bulk T_c is determined by scattering inside grains

A part of a superconducting transmon at 5 K



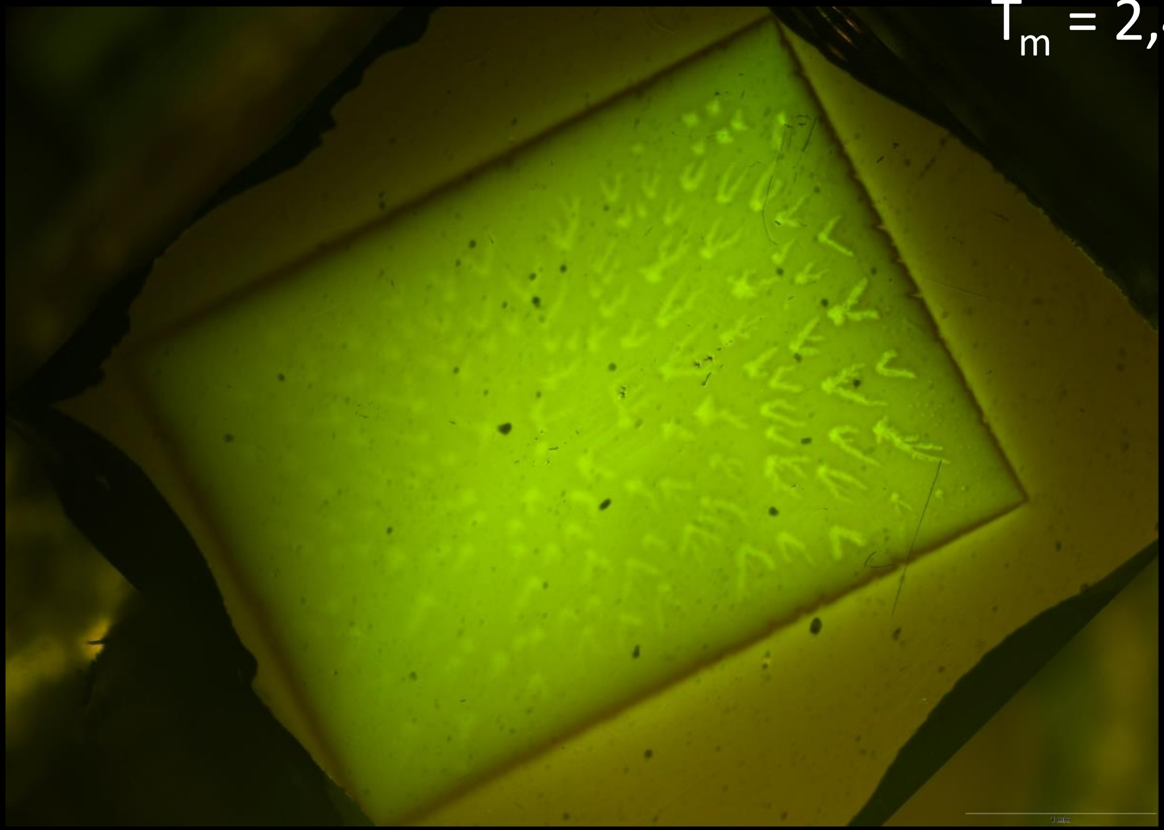
0.5 V   0.8 V   1.5 V   2.5 V

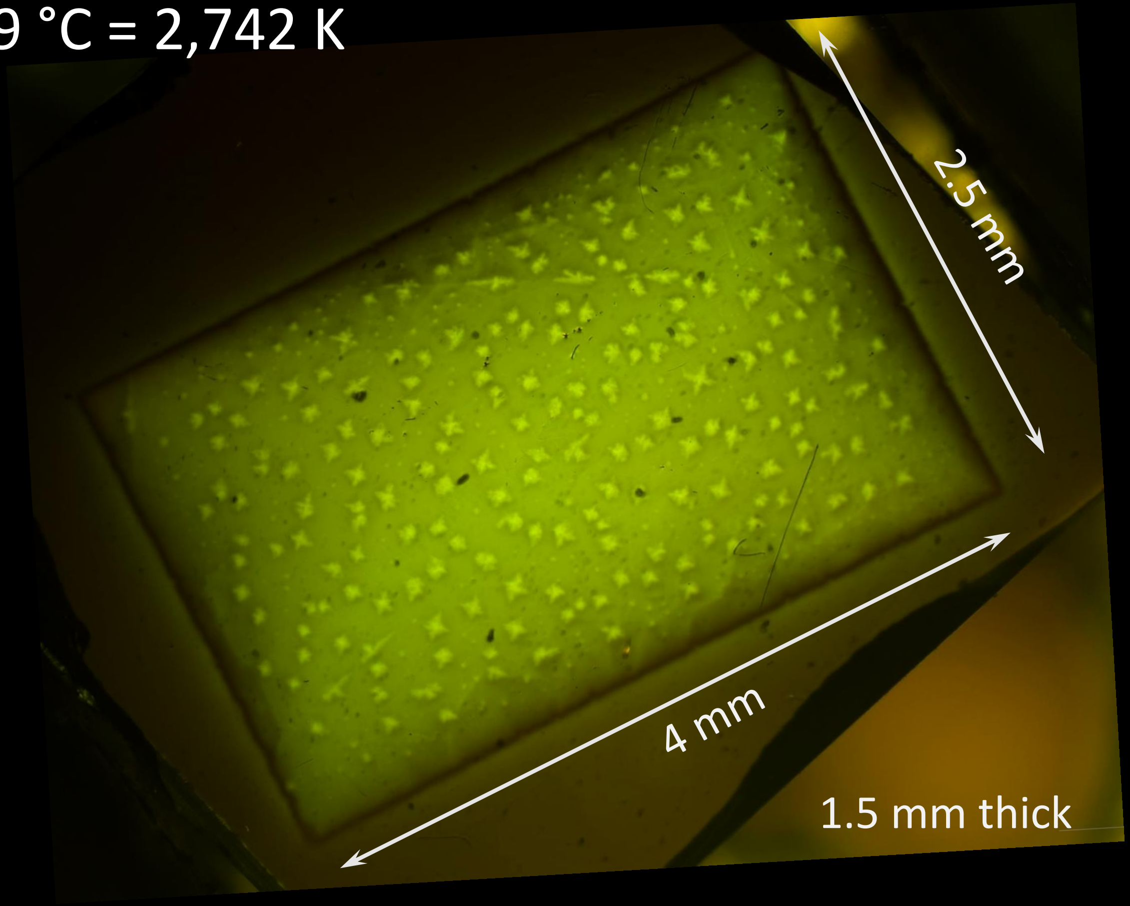
Single crystalline (100) sample after annealing

Sample 1: Before Near-Melting (NM) Annealing

Sample 2: Before 800 C Annealing

T_m = 2,469 °C = 2,742 K

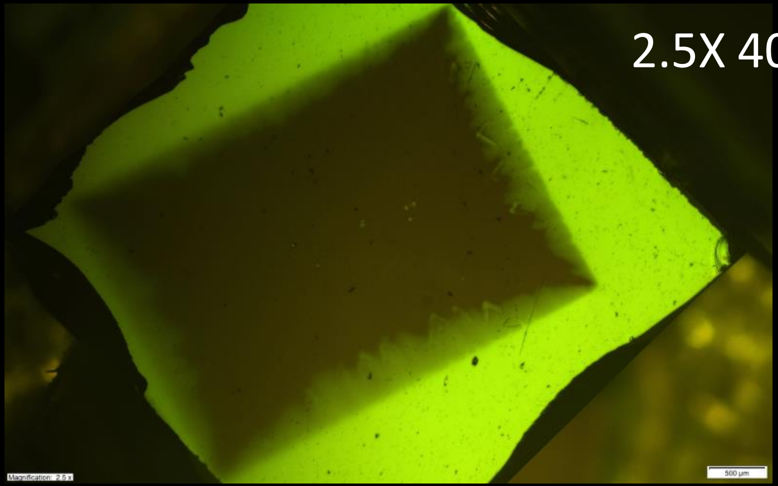




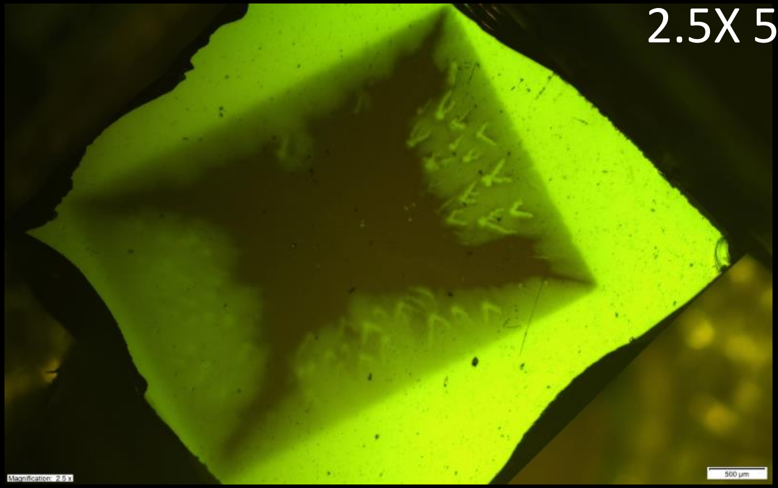
Trapped magnetic flux at 5 K after cooling in 600 Oe and setting the field to zero

Magnetic flux penetration at 8 K after ZFC

Sample 1: Before NM Annealing

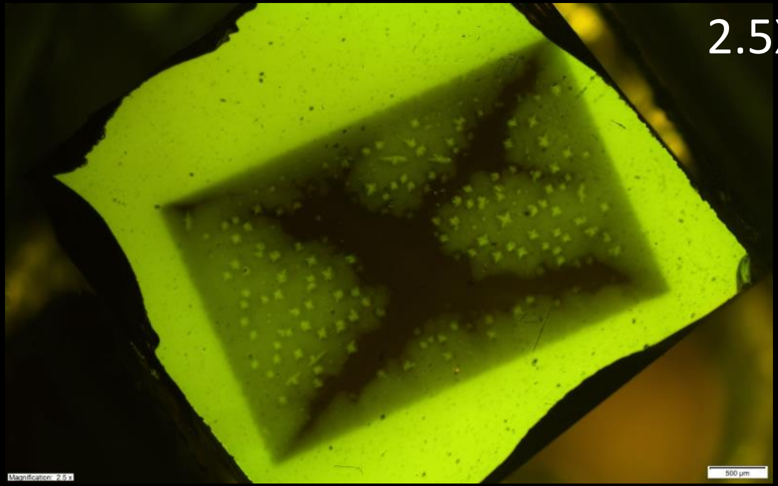


2.5X 400 Oe

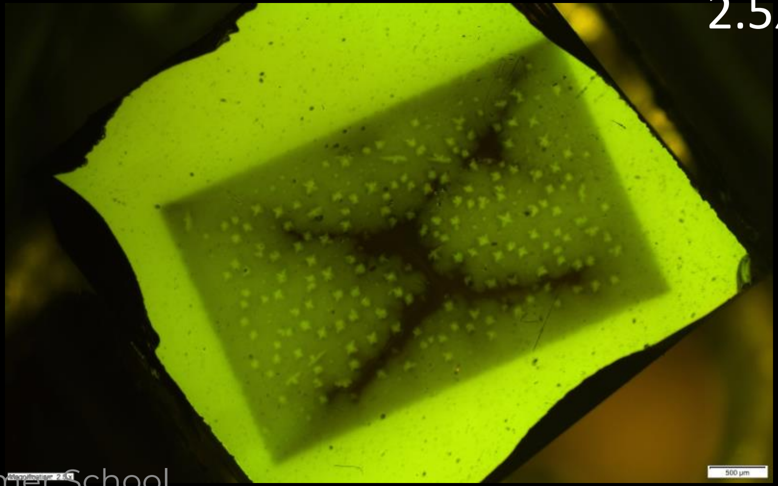


2.5X 500 Oe

Sample 2: Before 800 C Annealing



2.5X 400 Oe



2.5X 450 Oe

Quasiparticle spectroscopy using London penetration depth

Superfluid response to a magnetic field: $\mathbf{j} = -\mathbb{R}\mathbf{A}$

j – supercurrent

A – vector potential

$\mathbb{R}$ - symmetric response tensor

$$\mathbb{R} = \frac{e^2}{4\pi^3\hbar c}\oint_{FS} dS_\kappa \left[\frac{(\mathbf{v}_F \otimes \mathbf{v}_F)}{v_F}\left(1 + 2\int_0^\infty \frac{\partial f(E)}{\partial E}\frac{N(E)}{N(0)}dE\right)\right]$$

Magnetic penetration depth: $\lambda_{ii}^2 = \frac{c}{4\pi\mathbb{R}_{ii}}$ with the effective mass: $m_{ii} = \frac{n_{ii}e^2}{c\mathbb{R}_{ii}}$

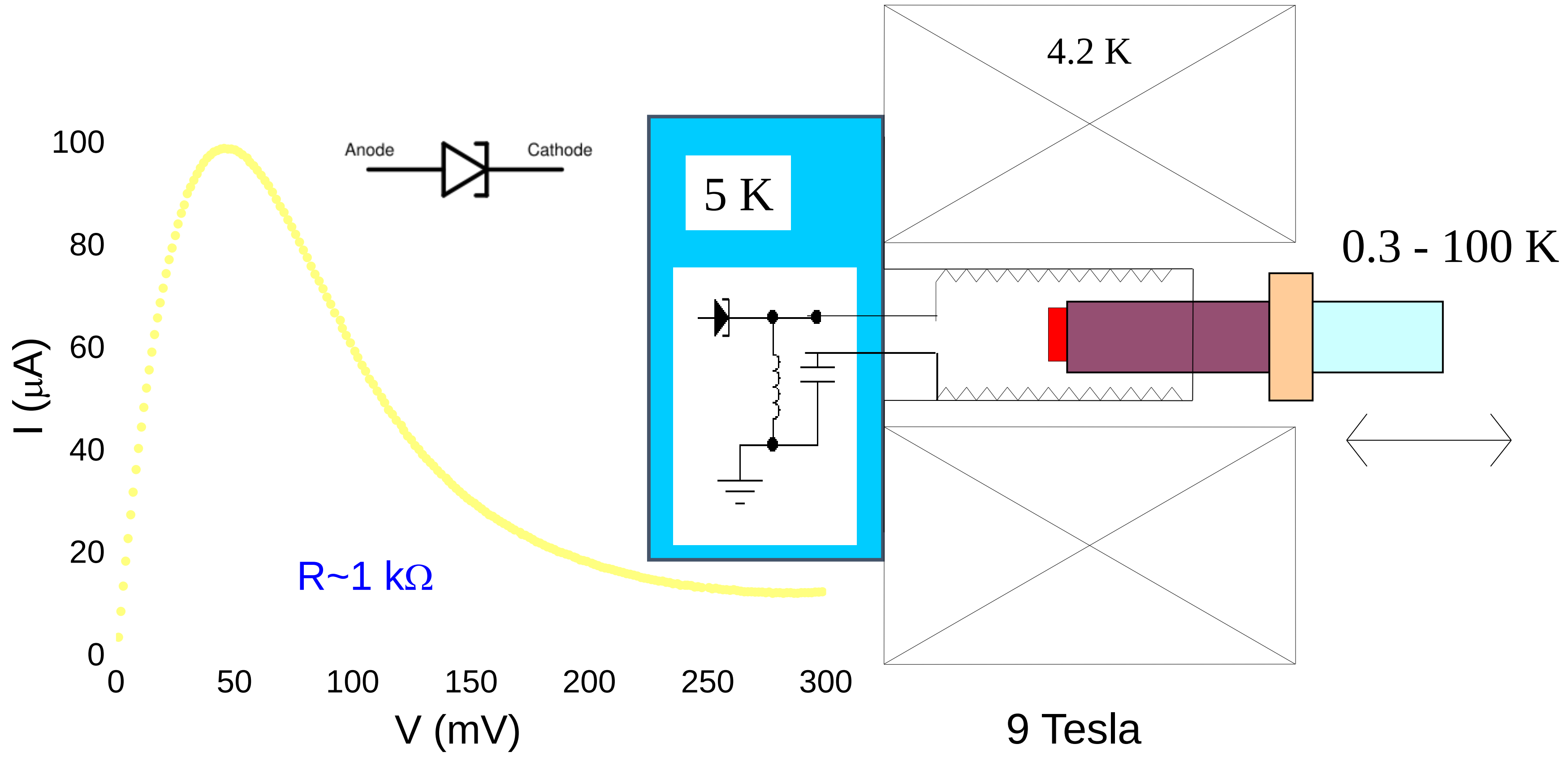
gives London-like penetration depth: $\lambda_{ii}^2 = \frac{m_{ii}c^2}{4\pi n_{ii}e^2}$

---

example For a spherical Fermi surface and **j** || **a**, the normalized superfluid density:

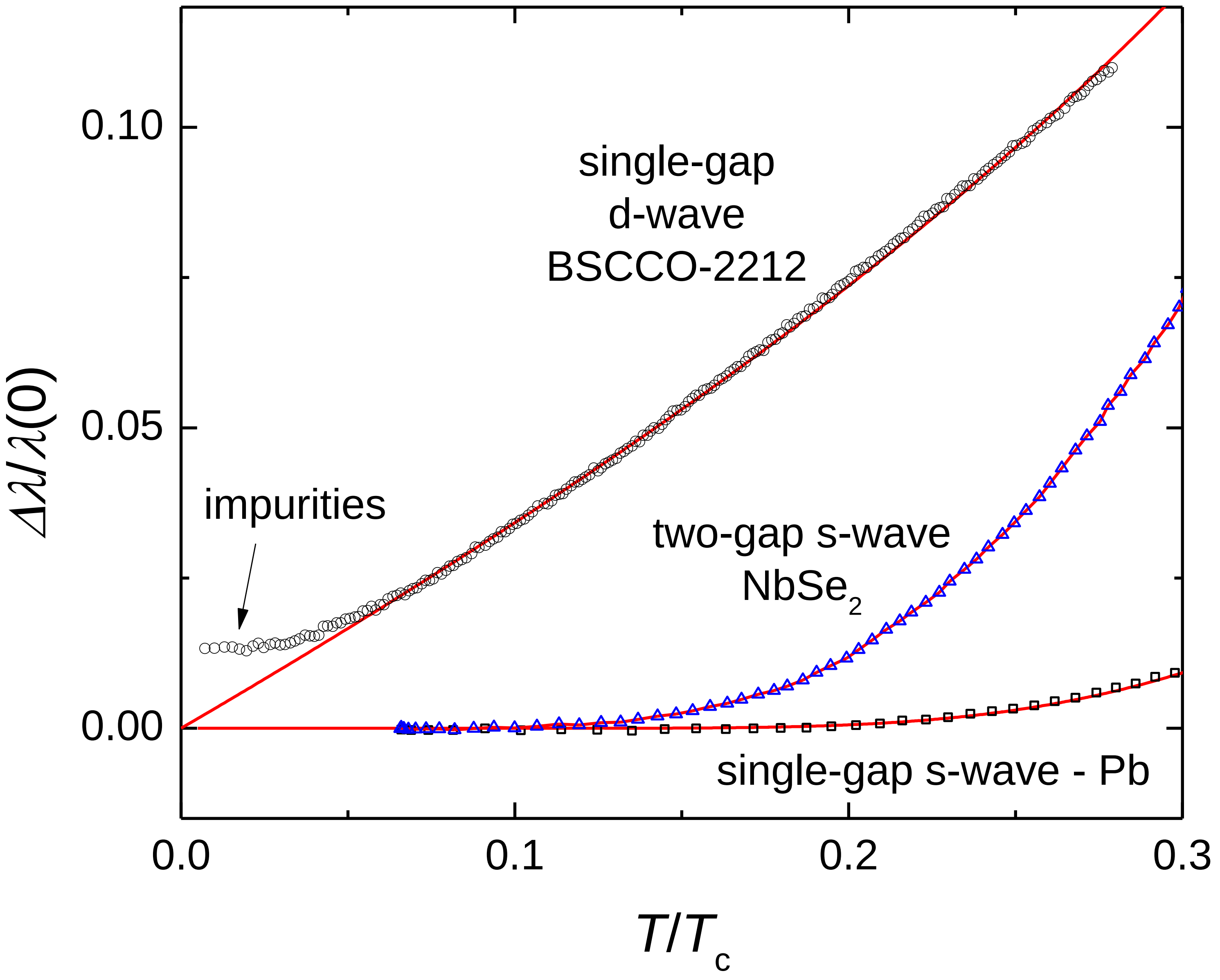
$$\rho_{aa} = \frac{n_s(T)}{n_s(0)} = \left[\frac{\lambda_a(0)}{\lambda_a(T)}\right]^2 = 1 - \frac{3}{4\pi T}\int_0^1 (1-z^2)dz \int_0^{2\pi} \cos^2(\phi)\, d\phi \int_0^\infty \cosh^{-2}\left(\frac{\sqrt{\varepsilon^2 + \Delta^2(\phi,z)}}{2T}\right) d\varepsilon$$

self-resonating circuit: tunnel-diode resonator

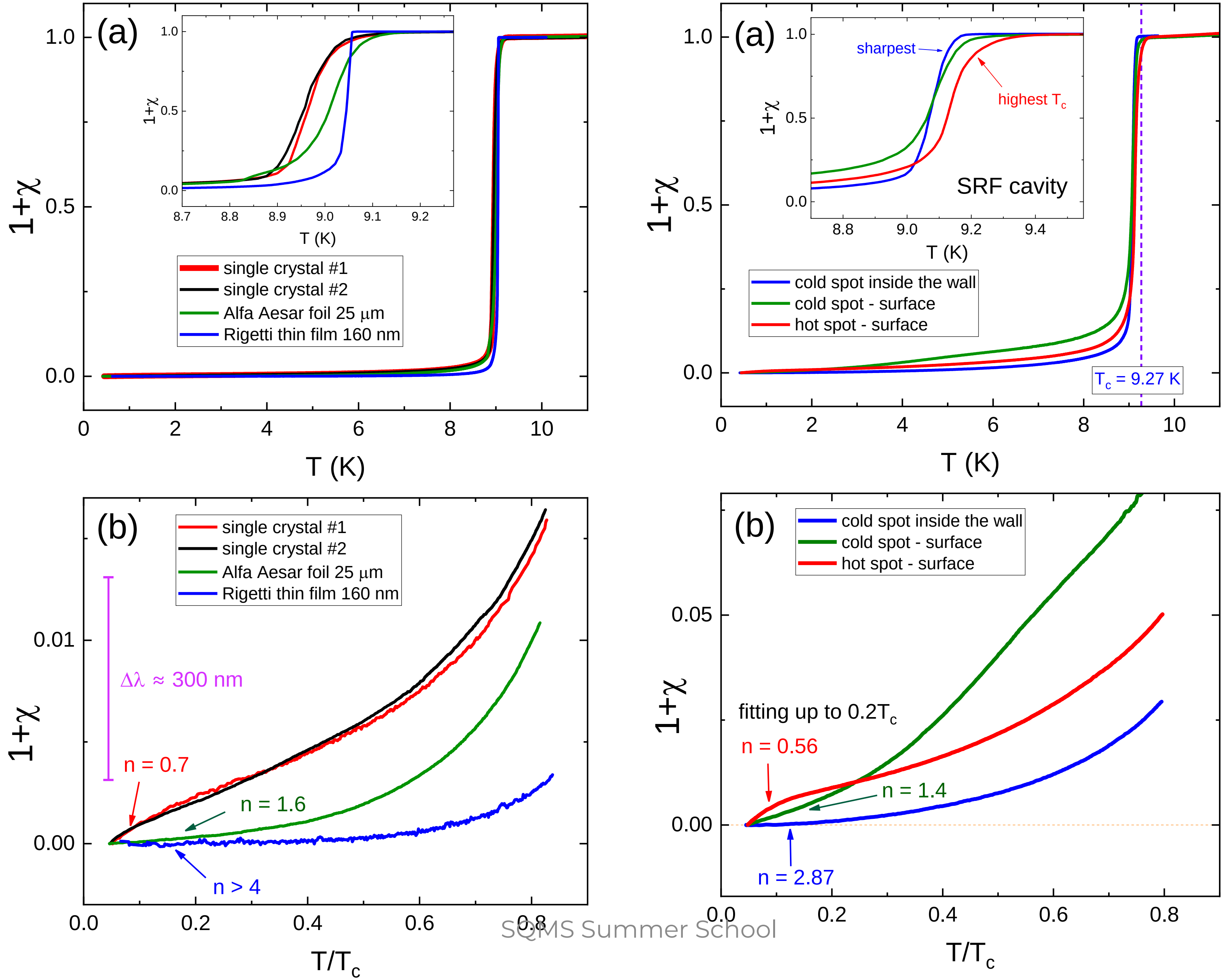


$$2\pi f = \frac{1}{\sqrt{L_0 C_0}} \neq \frac{1}{\sqrt{LC}}$$

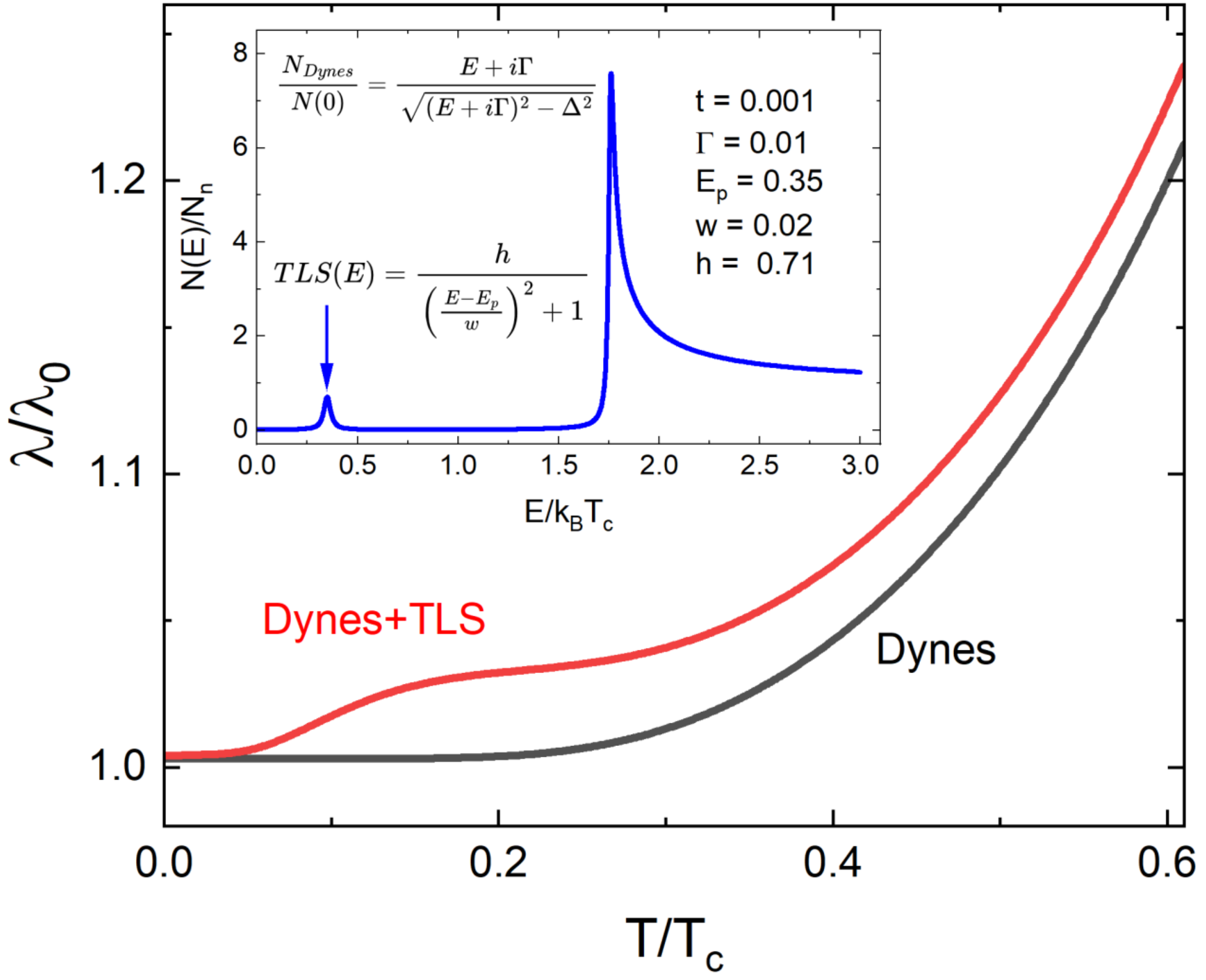
London penetration depth: classical behavior

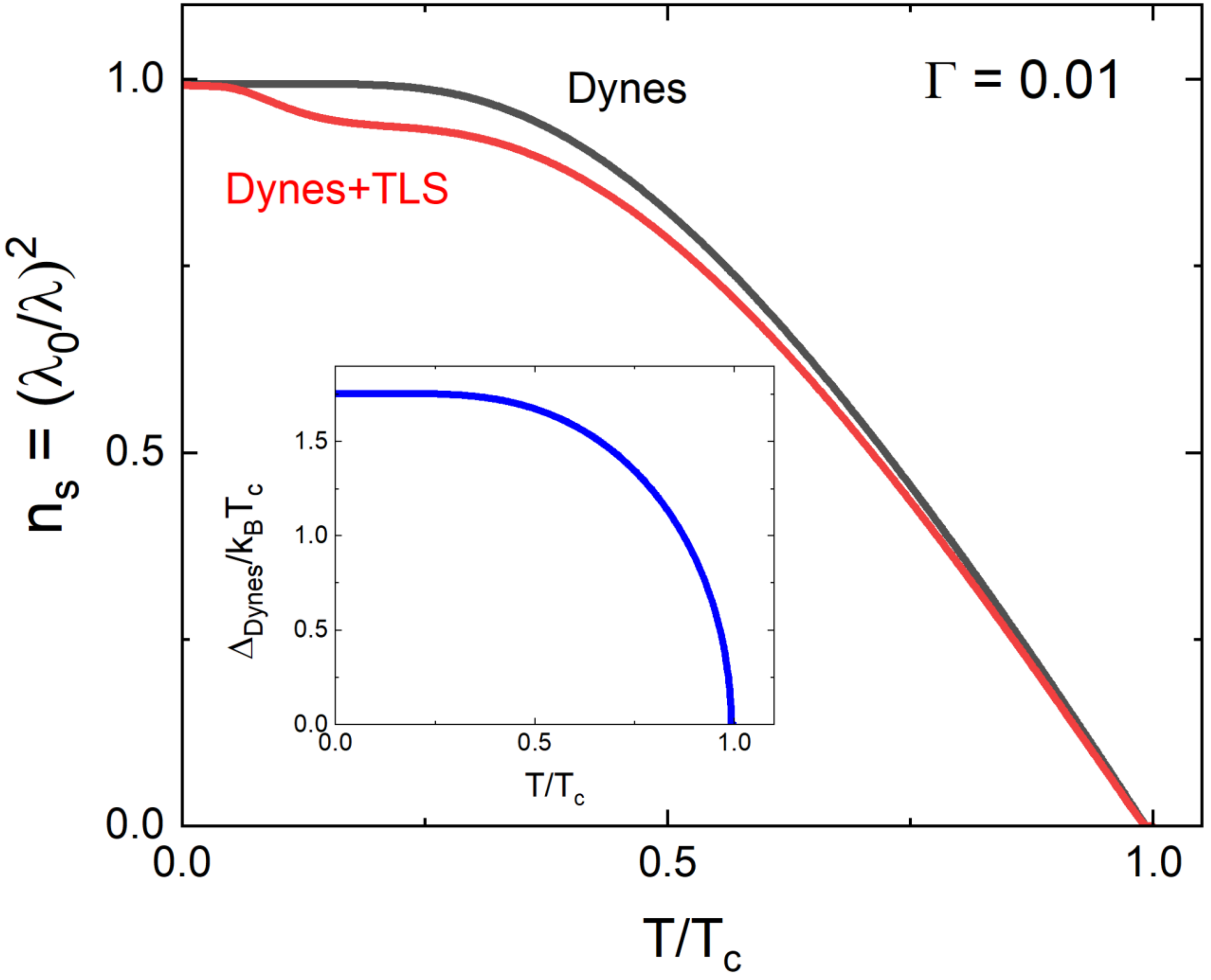


London Penetration Depth in Nb of different morphologies



identification of the in-gap states, e.g. TLS





Conclusions

- Real materials are the bottleneck of quantum technologies. There is a significant lag between desired and available.
- Existing materials characterization techniques must be extended, improved, and adapted to address specific challenges associated with QIS demands.
- Novel techniques must be developed.
- Quantum sensing will play a significant role in QIS technologies.