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Intro to qubit readout and parametric amplifiers

Florent Lecocq

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John Teufel
Gabe Peterson
Maxime Malnou

NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
U.S. DEPARTMENT OF COMMERCE

University of Colorado Boulder

NIST Boulder
Advanced Microwave Photonics Group

NATIONAL INSTITUTE OF
STANDARDS AND TECHNOLOGY
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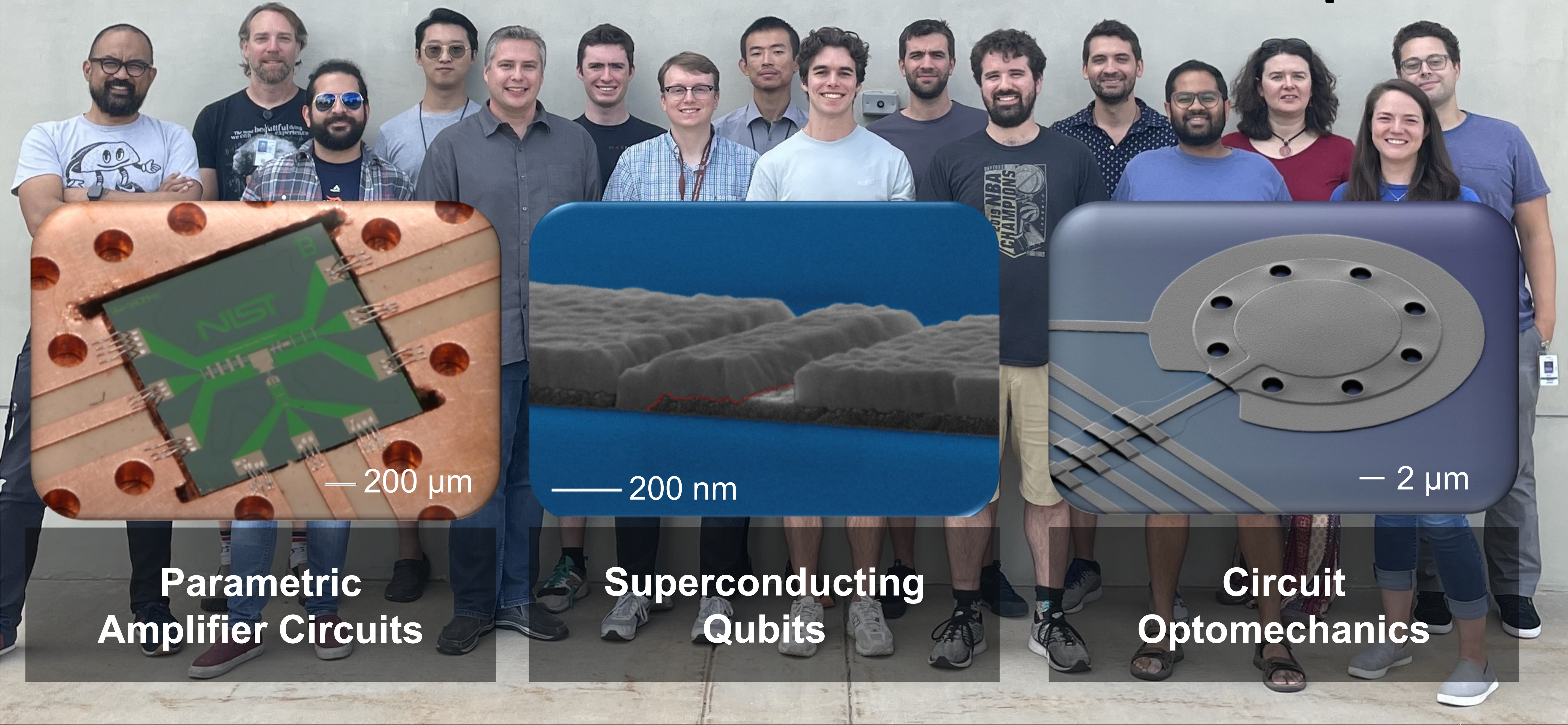


Parametric Amplifier Circuits

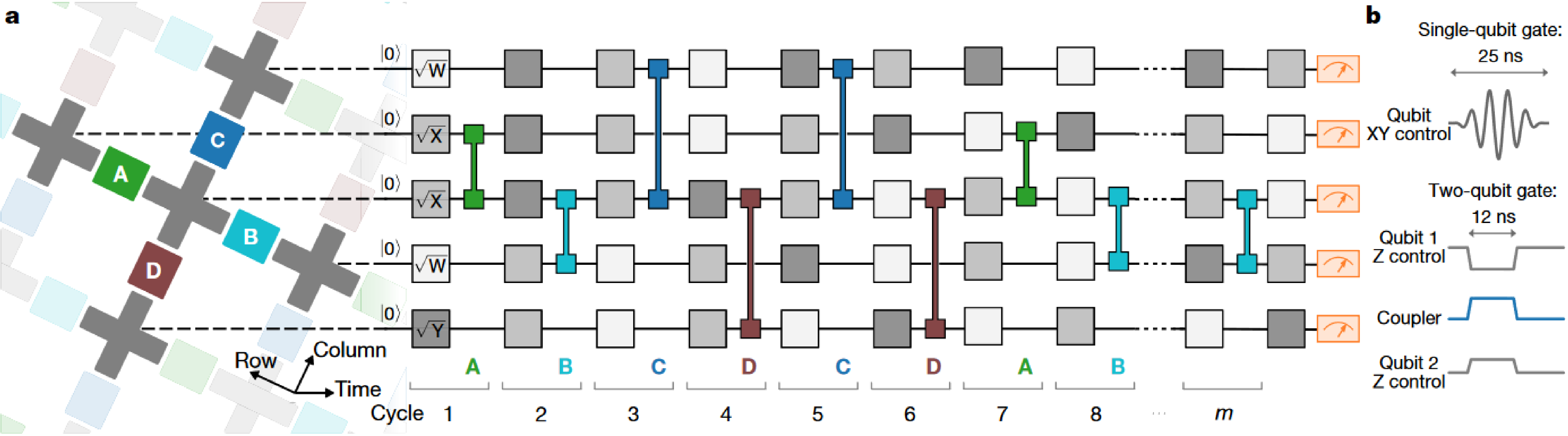
Superconducting Qubits

Circuit Optomechanics

High fidelity qubit measurements

Google
Quantum AI

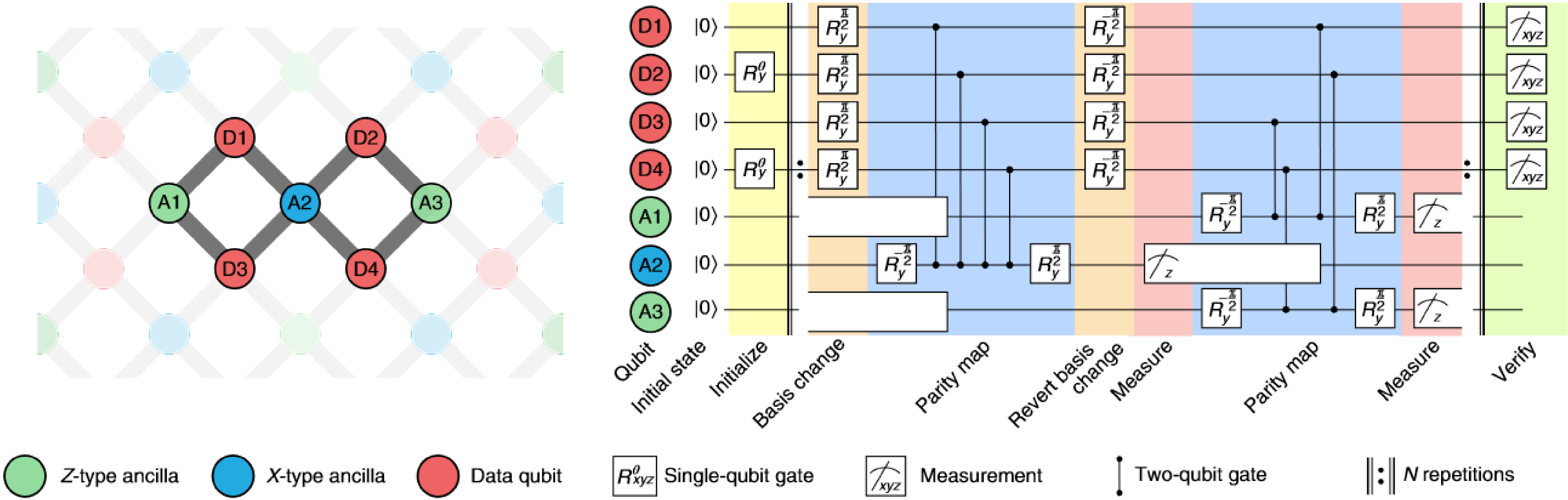
*Arute et al, **Nature** 574 (2019)*



Measurement of the outcome of a quantum algorithm

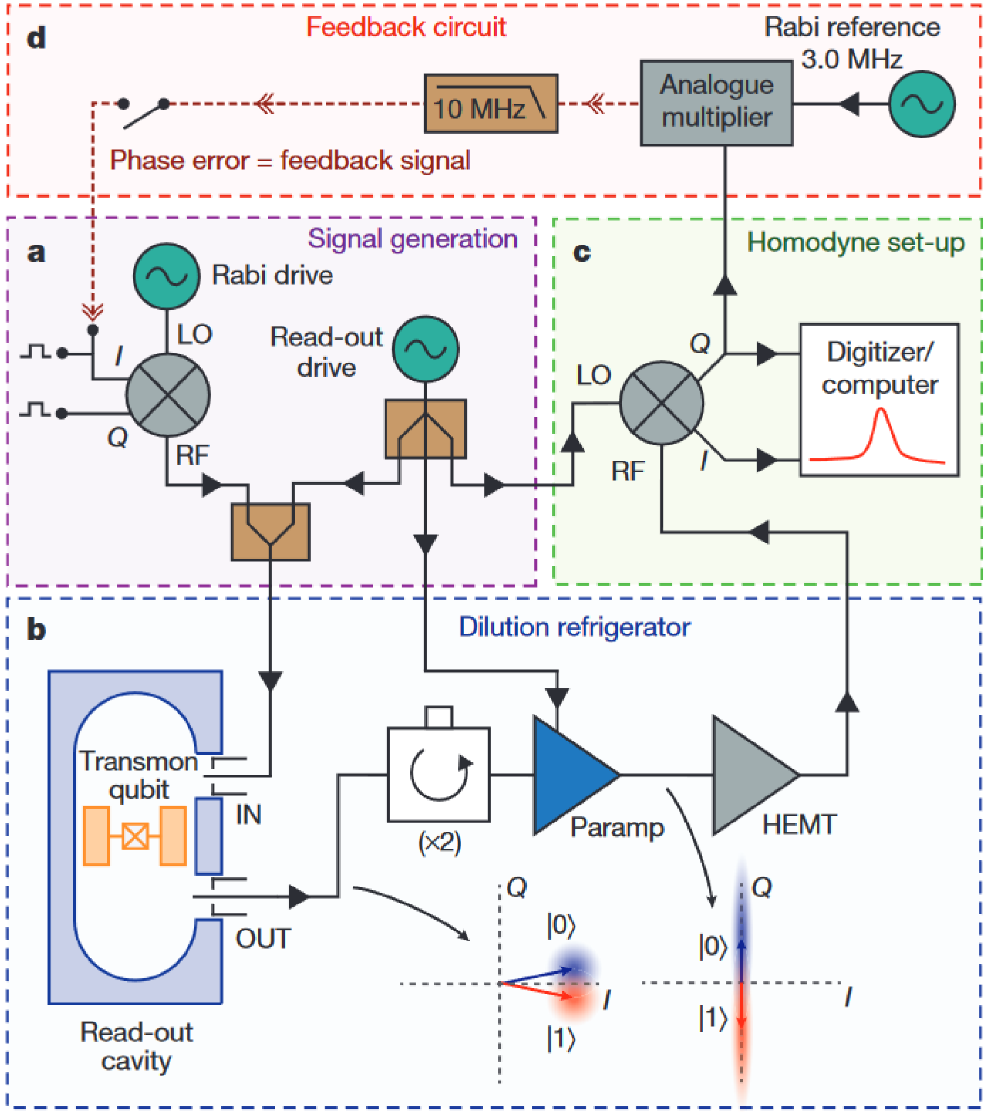
High fidelity qubit measurements

Andersen et al, ***Nat. Phys.*** *16 (2020)*

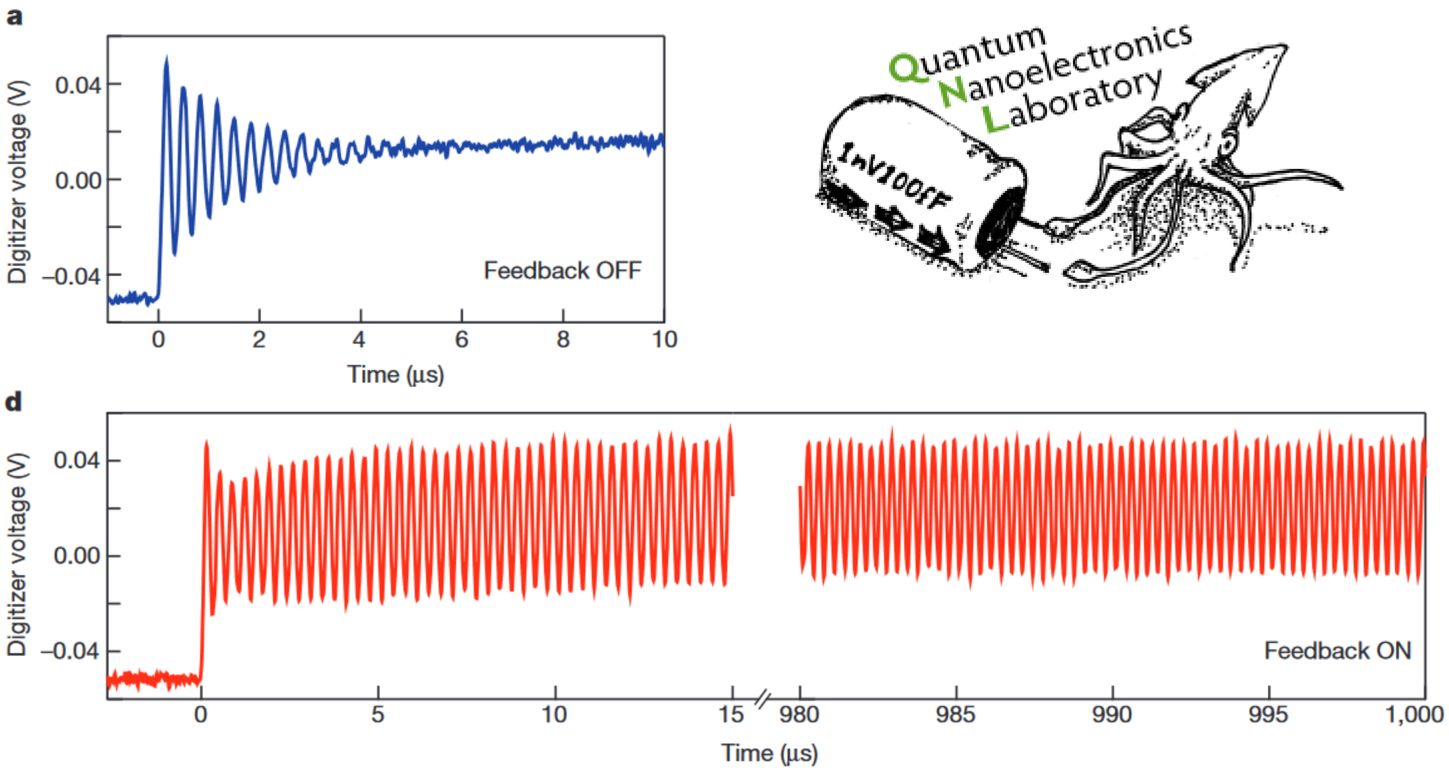


Critical for error correction

Efficient qubit measurement for feedback



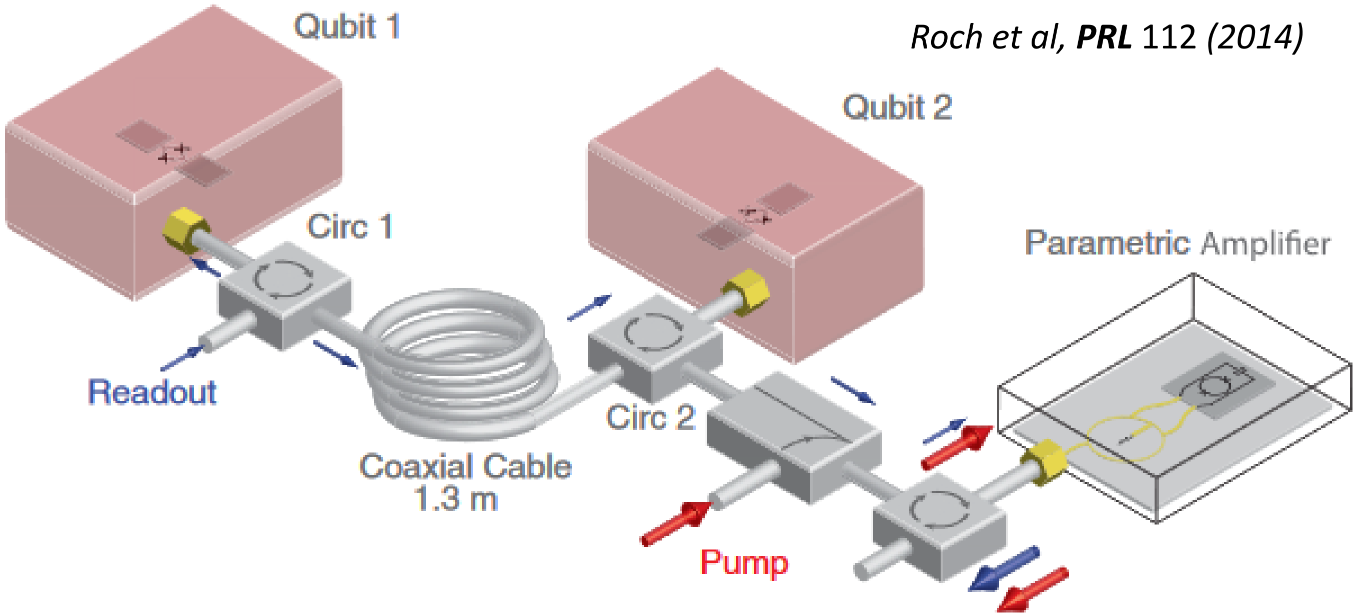
Vijay et al, ***Nature*** 490 *(2012)*



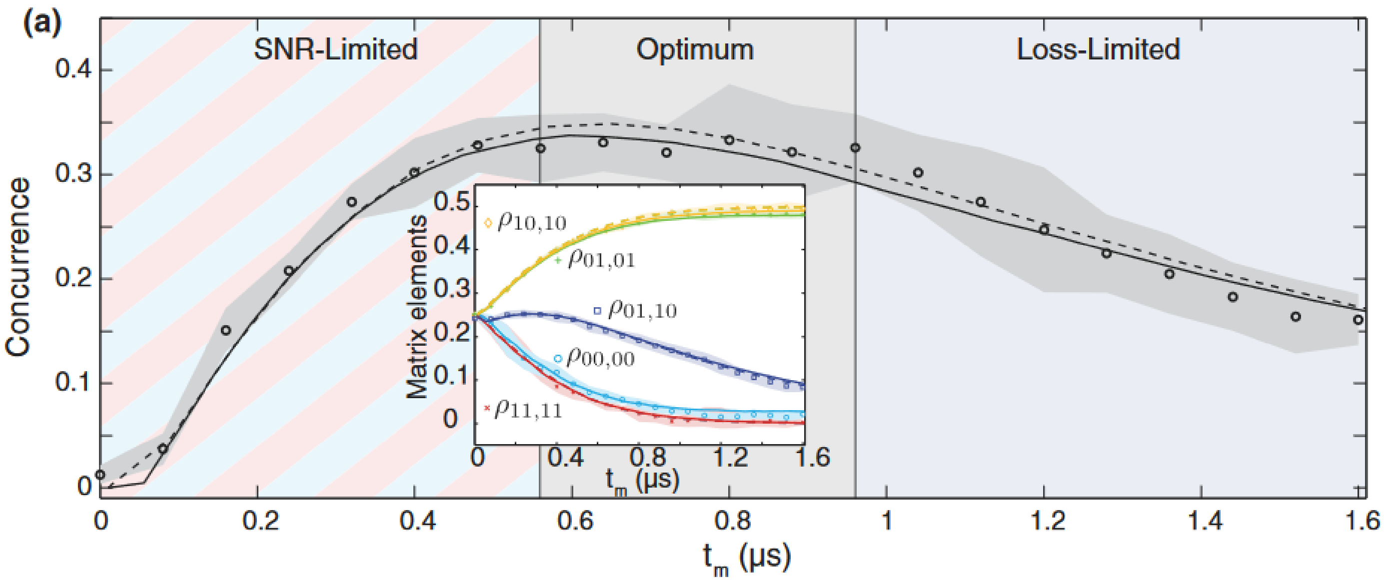
Efficient quantum measurement enables analog feedback

Efficient qubit measurement for entanglement

Roch et al, ***PRL*** 112 *(2014)*



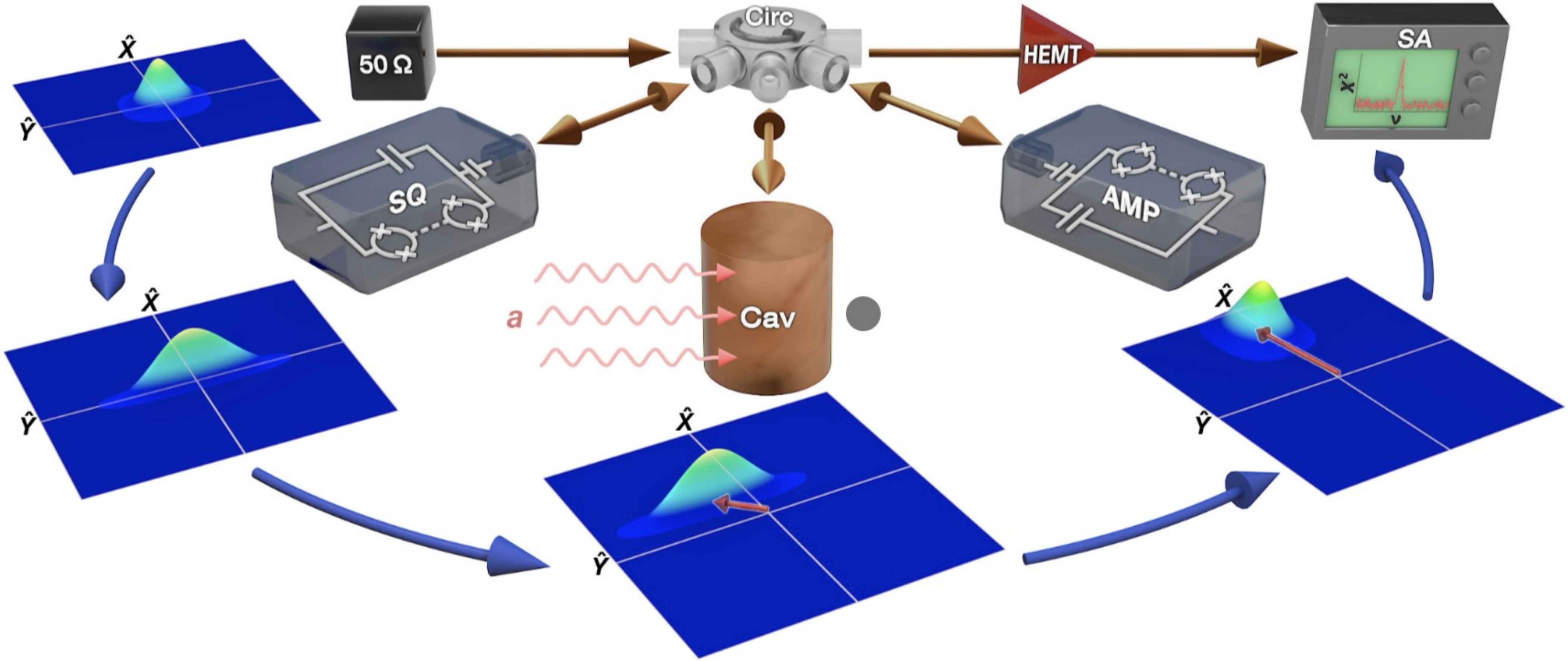
Quantum
Nanoelectronics
Laboratory



Efficient quantum measurement for propagating entanglement

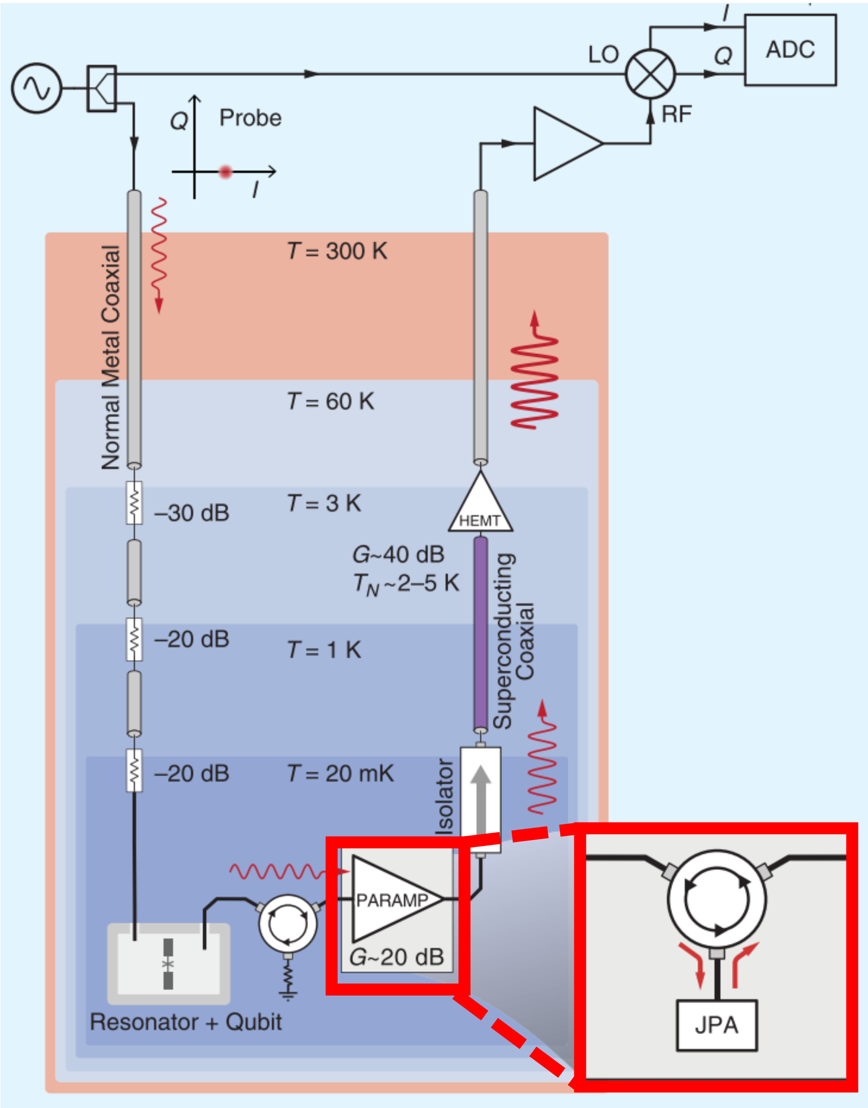
Dark Matter search

Backes et al, ***Nature*** 590 *(2021)*



Efficient quantum measurements as tools for fundamental physics

Parametric amplifiers



*J. Aumentado, **IEEE MW magazine** 21 (2020)*

Parametric amplifiers as an enabling technology

Quantum sensing

Quantum computing

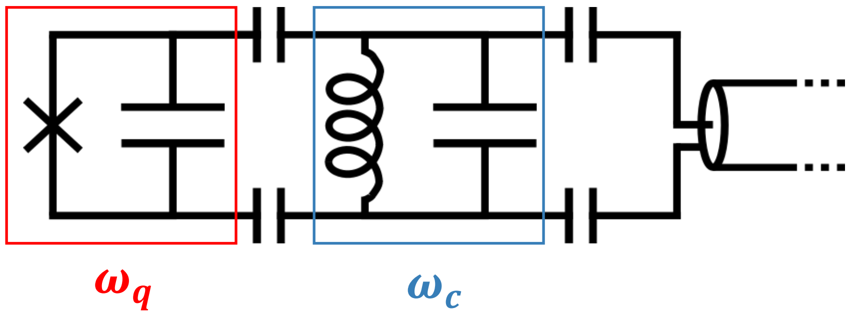
Outline

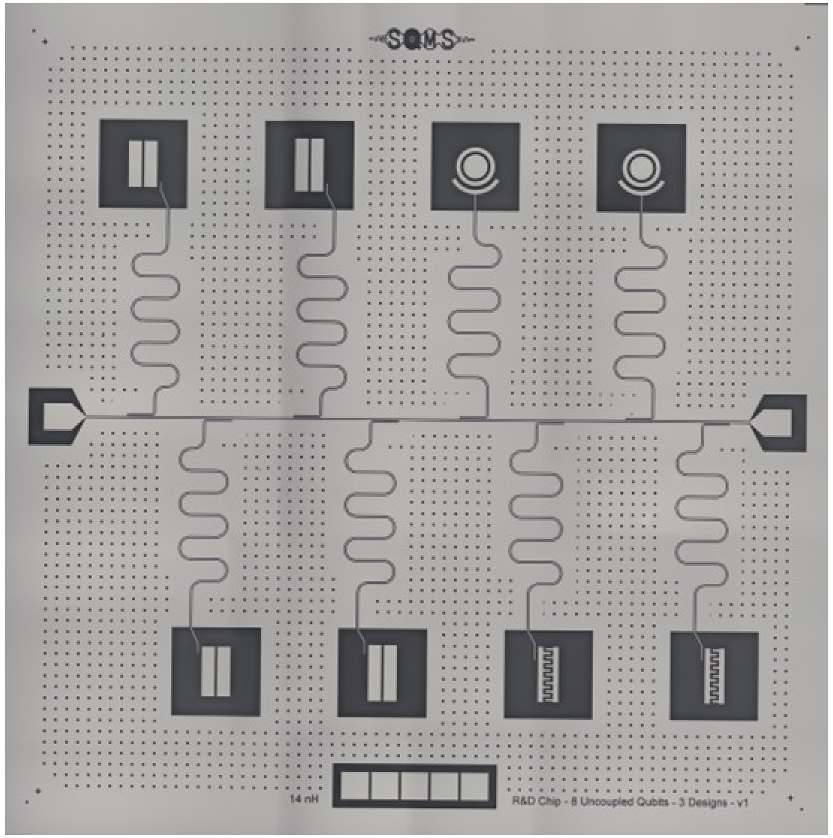
- Dispersive Readout of superconducting qubits

- Intro to parametric amplifiers:
  - Resonant parametric amplifiers
  - Traveling-waves parametric amplifiers

- Future directions

Quantum measurements

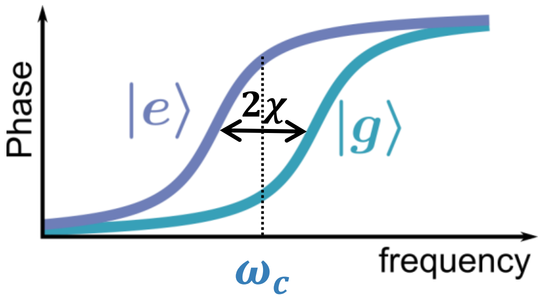




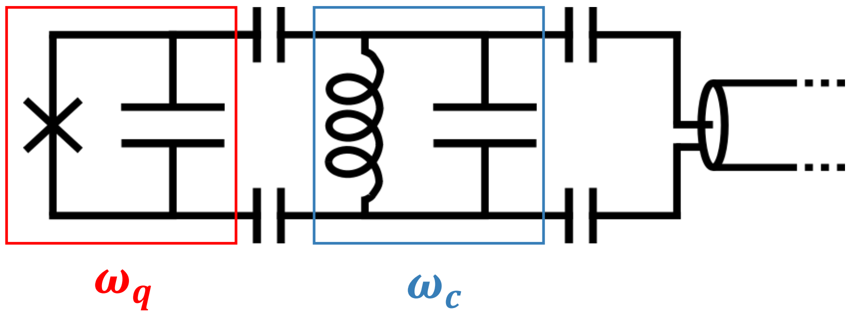
$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_q \hat{\sigma}_z + g(\sigma_+ \hat{a} + \sigma_- \hat{a}^\dagger)$$

$$\Big\downarrow \quad g \ll |\omega_c - \omega_d|$$

$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_q \hat{\sigma}_z - \chi\hat{\sigma}_z \hat{a}^\dagger \hat{a}$$



Quantum measurements



$$\hat{H} = \hbar\omega_c \hat{a}^\dagger \hat{a} + \frac{1}{2}\hbar\omega_q \hat{\sigma}_z - \chi\hat{\sigma}_z \hat{a}^\dagger \hat{a}$$

$$\hat{H} = \hbar\left(\omega_c \underbrace{\pm\chi}_{\text{Dispersive shift}} \underbrace{-\frac{i\kappa}{2}}_{\text{Dissipation}}\right)\hat{a}^\dagger\hat{a} + \underbrace{i\sqrt{\kappa}\left(\hat{a}^\dagger\hat{a}_{in} - \hat{a}\hat{a}_{in}^\dagger\right)}_{\text{External drive}}$$

$$\frac{d\hat{a}}{dt} = -\frac{i}{\hbar}\left[\hat{a},\hat{H}\right] \qquad \left[\hat{a},\hat{a}^\dagger\right] = 1$$

$$\dot{a} = -i(\omega_c \pm \chi)a - \frac{\kappa}{2}a + \sqrt{\kappa}\left(\hat{a}_{in} - \hat{a}_{in}^\dagger\right)$$

Expectation values $a \equiv \langle\hat{a}\rangle$

$$\dot{a} = -i(\omega_c \pm \chi - \omega_d)a - \frac{\kappa}{2}a + \sqrt{\kappa}\hat{a}_{in}$$

$$a \rightarrow a e^{-i\omega_d t}$$
$$a_{in} \rightarrow a_{in} e^{-i\omega_d t}$$

*See also: Gambetta et al **PRA** 77 (2008), Bultink et al **APL** 112 (2018), Touzard et al **PRL** 122 (2019)*

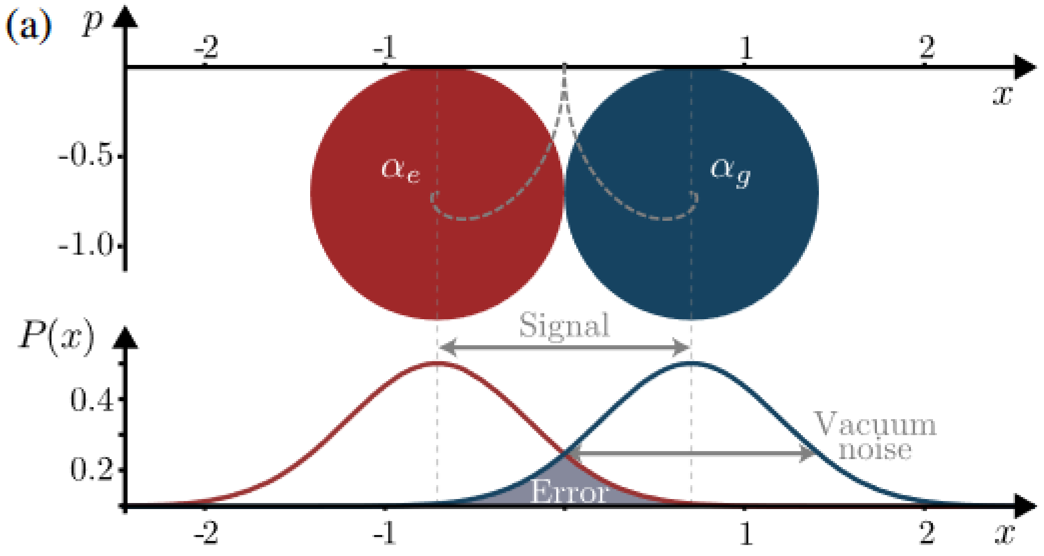
Quantum measurements

$$\dot{a} = -i(\omega_c \pm \chi - \omega_d)a - \frac{\kappa}{2}a + \sqrt{\kappa}\hat{a}_{in}$$

$$\Big\downarrow \; \omega_d = \omega_c$$

$$\begin{cases} \dot{\alpha_g} = -i\chi\alpha_g - \frac{\kappa}{2}\alpha_g + \sqrt{\kappa}\hat{a}_{in} \\ \dot{\alpha_e} = +i\chi\alpha_e - \frac{\kappa}{2}\alpha_e + \sqrt{\kappa}\hat{a}_{in} \end{cases}$$

$$SNR = 2\kappa \int_0^\tau |\alpha_e - \alpha_g|^2 = 2\kappa|\alpha_e - \alpha_g|^2\tau$$



A. Blais et al, ***Rev. Mod. Phys.*** 93 *(2021)*

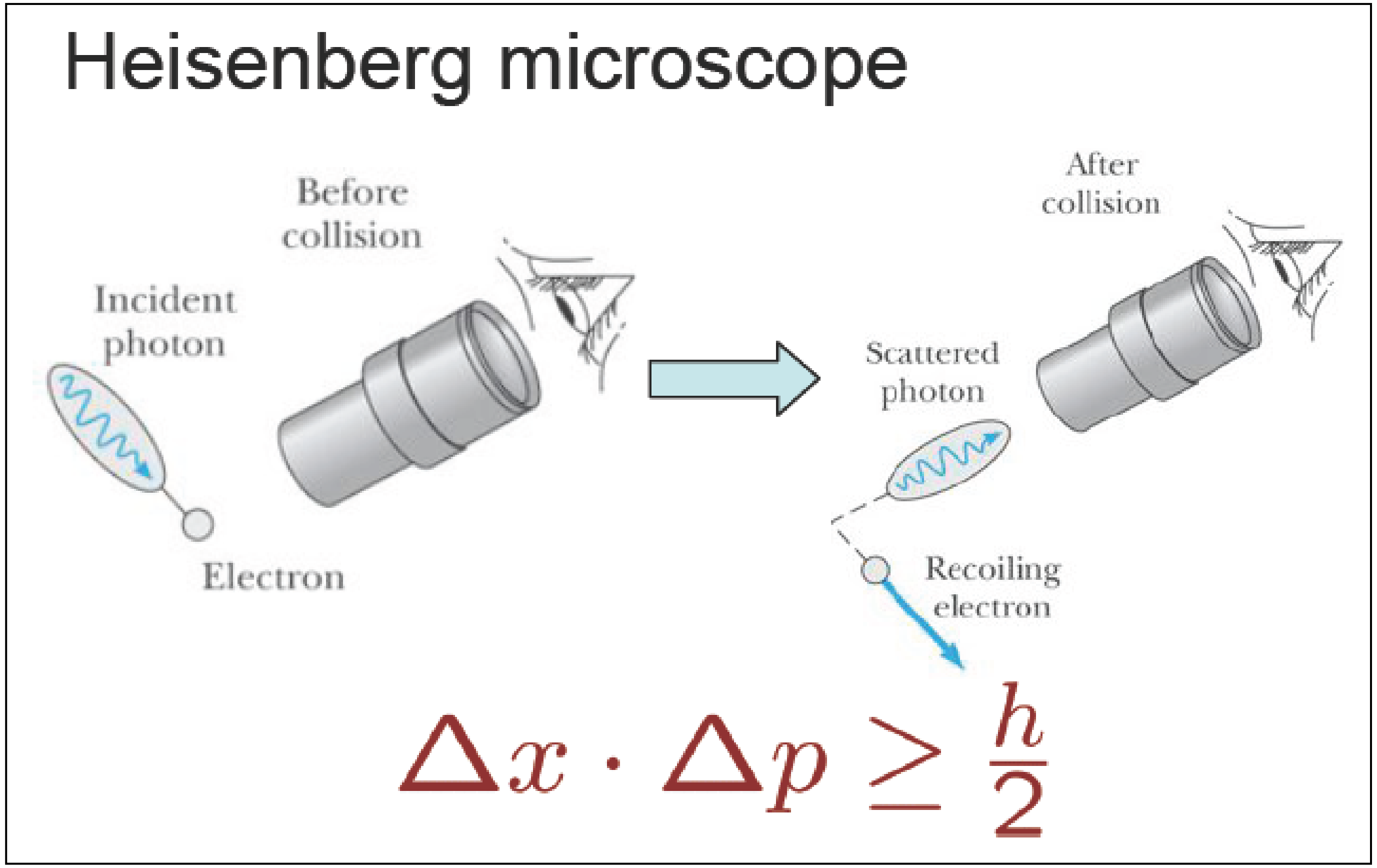
$SNR \ll 1$: weak measurement

$SNR \gg 1$: strong measurement (projective)

$$SNR_{exp} = \frac{(\langle I_e\rangle - \langle I_g\rangle)^2}{\sigma_g^2 + \sigma_e^2} = \eta SNR$$

See also: Gambetta et al ***PRA*** *77 (2008), Bultink et al* ***APL*** *112 (2018), Touzard et al* ***PRL*** *122 (2019)*

The flip side of quantum measurements



Any interaction that is strong enough to acquire information about the system is necessarily strong enough to affect the system

Qubit dephasing rate is proportional to measurement rate

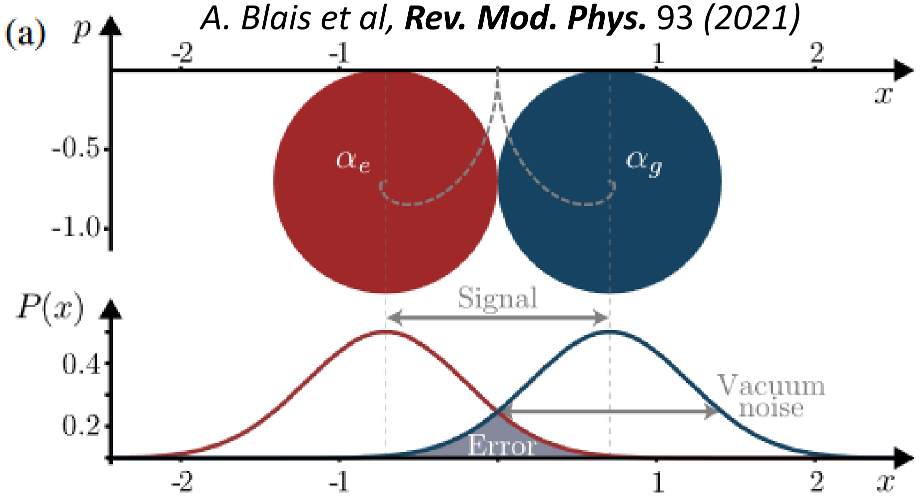
$$\Gamma_d = \frac{\kappa}{2}\left|\alpha_e - \alpha_g\right|^2$$

$$SNR_{exp} = \eta 2\kappa \int_0^{\tau} \left|\alpha_e - \alpha_g\right|^2$$

In an efficient measurement, the measurement rate matches the measurement induced dephasing rate

See also: Gambetta et al ***PRA*** *77 (2008), Bultink et al* ***APL*** *112 (2018), Touzard et al* ***PRL*** *122 (2019)*

Amplifiers for high fidelity readout



A. Blais et al, ***Rev. Mod. Phys.*** 93 *(2021)*

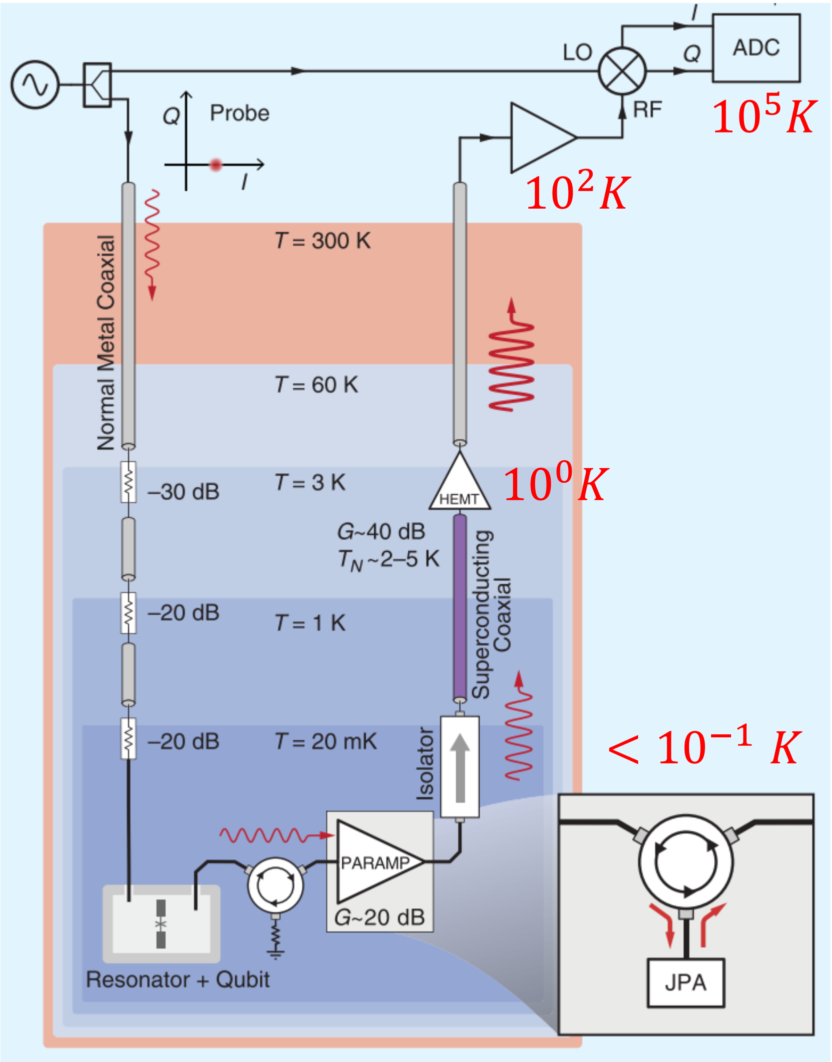
$$\boldsymbol{F = 1 - P(e|g) - P(g|e) = erf(\sqrt{\mathrm{SNR}/2})}$$

$$\boldsymbol{\xrightarrow[\mathrm{SNR=10}]{} F = 99.9}$$

- Typical readout power is limited: $|\alpha|^2 = \frac{2}{\kappa}\frac{P}{\hbar\omega} = 10$ leads to $P \approx -130dBm$ @ $6GHz, \kappa = 2\chi = 2\pi \times 1MHz$
- Linear measurement are sensitive to microwave vacuum noise: $PSD_{vac} = \frac{\hbar\omega}{2} = -207\ dBm/Hz$
- Ideally, $SNR = \frac{1}{\kappa}\frac{P}{PSD_{vac}} = |\alpha|^2$
- Room temperature instruments have more noise: $PSD_{instr} = -146\ dBm/Hz$ leading to $SNR = 1$ in 25 ms $\gg \mathrm{T}_1$

High fidelity readout requires amplification

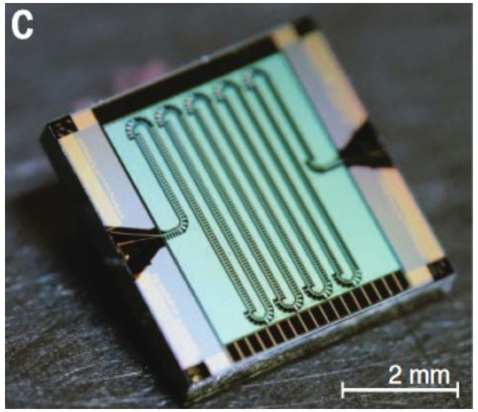
Typical measurement chain



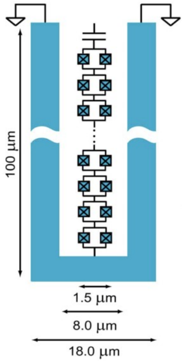
*J. Aumentado, **IEEE MW magazine** 21 (2020)*

- Both loss and amplification degrade SNR: $\eta_{sys} = \frac{SNR_{out}}{SNR_{in}} < 1$
- Commercial HEMT amplifiers: $\eta_{sys} < 5\%$
- Parametric amplifiers: $\eta_{sys} \sim 20 - 50\%$

Low Noise Factory

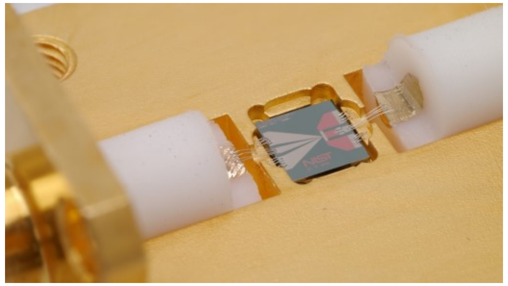


MIT-LL



Lehnert Lab

$F = 99\%\ in\ 100ns$



NIST

An old idea

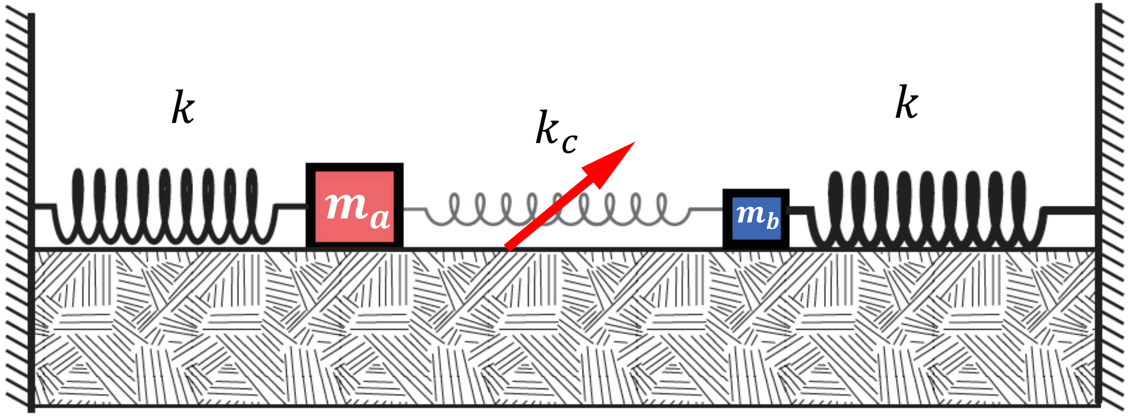
New amplifier battles "noise"

Four-stage junction diode amplifier was developed at Bell Telephone Laboratories by Rudolf Engelbrecht for military applications. Operates on the "varactor" principle, utilizing the variable capacitance of diodes. With 400-mc. signal, the gain is 10 db. over the 100-mc. band.

The tremendous possibilities of semiconductor science are again illustrated by a recent development from Bell Telephone Laboratories. The development began with research which Bell Laboratories scientists were conducting for the U. S. Army Signal Corps. The objective was to reduce the "noise" in UHF and microwave receivers and thus increase their ability to pick up weak signals.

Radio-Electronic Engineering mag. Vol. 29, No. 11 *(1958)*

Parametric coupling



$$H_I = k_c(x_a - x_b)^2 \approx k_c x_a x_b$$

$$x_a = a + a^\dagger$$

$$x_b = b + b^\dagger$$

$$H_I \approx k_c(a + a^\dagger)(b + b^\dagger)$$

$a \propto e^{-i\omega_a t}$
$b \propto e^{-i\omega_b t}$ $\longrightarrow$ $\langle k_c x_a x_b \rangle = 0$

Weak residual dispersive coupling

$k_c \propto |k_c| e^{\pm i\omega_p t + \phi}$ $\longrightarrow$ $\langle k_c x_a x_b \rangle \neq 0$

Net coupling
Proportional to modulation strength

If $\omega_p = \omega_b - \omega_a$

$$H_I \propto k_c a b^\dagger + \boxed{k_c^*} a^\dagger b$$

Frequency Conversion (FC)

If $\omega_p = \omega_b + \omega_a$

$$H_I \propto k_c a b + \boxed{k_c^*} a^\dagger b^\dagger$$

Parametric Amplification (PA)

Coupled mode equations

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} - 2\hbar g(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

$$\begin{cases} \dfrac{d\hat{a}}{dt} = -\dfrac{i}{\hbar}\left[\hat{a}, \hat{H}\right] = -i\omega_a \hat{a} + ig(\hat{b} + \hat{b}^\dagger) \\ \dfrac{d\hat{b}}{dt} = -\dfrac{i}{\hbar}\left[\hat{b}, \hat{H}\right] = -i\omega_b \hat{b} + ig(\hat{a} + \hat{a}^\dagger) \end{cases}$$

$$\Big\downarrow$$

$$\begin{cases} \dot{a} = ig\left(be^{i[\omega_a - \omega_b \pm \omega_p]t} + b^* e^{i[\omega_a + \omega_b \pm \omega_p]t}\right) \\ \dot{b} = ig\left(ae^{i[\omega_b - \omega_a \pm \omega_p]t} + a^* e^{i[\omega_a + \omega_b \pm \omega_p]t}\right) \end{cases}$$

$a \equiv \langle \hat{a} \rangle$ and $b \equiv \langle \hat{b} \rangle$

$$2g \rightarrow 2g\cos(\omega_p t) = g(e^{i\omega_p t} + e^{-i\omega_p t})$$

$$a \rightarrow ae^{-i\omega_a t}$$

$$b \rightarrow be^{-i\omega_b t}$$

Coupled mode equations

$$\begin{cases} \dot{a} = ig\left(b e^{i[\omega_a-\omega_b\pm\omega_p]t} + b^* e^{i[\omega_a+\omega_b\pm\omega_p]t}\right) \\ \dot{b} = ig\left(a e^{i[\omega_b-\omega_a\pm\omega_p]t} + a^* e^{i[\omega_a+\omega_b\pm\omega_p]t}\right) \end{cases}$$

Case 1: $\omega_p = \omega_a - \omega_b$

$$\begin{cases} \dot{a} = ig\left(b + b^* \xcancel{e^{2i\omega_b t}} + \cdots\right) \\ \dot{b} = ig\left(a + a^* \xcancel{e^{2i\omega_b t}} + \cdots\right) \end{cases}$$

$$\begin{cases} \dot{a} = ig\mathrm{b} \\ \dot{b} = iga \end{cases}$$

Case 2: $\omega_p = \omega_a + \omega_b$

$$\begin{cases} \dot{a} = ig\left(b^* + b \xcancel{e^{-2i\omega_b t}} + \cdots\right) \\ \dot{b} = ig\left(a^* + a \xcancel{e^{2i\omega_b t}} + \cdots\right) \end{cases}$$

$$\begin{cases} \dot{a} = igb^* \\ \dot{b} = iga^* \end{cases}$$

Coupled mode equations

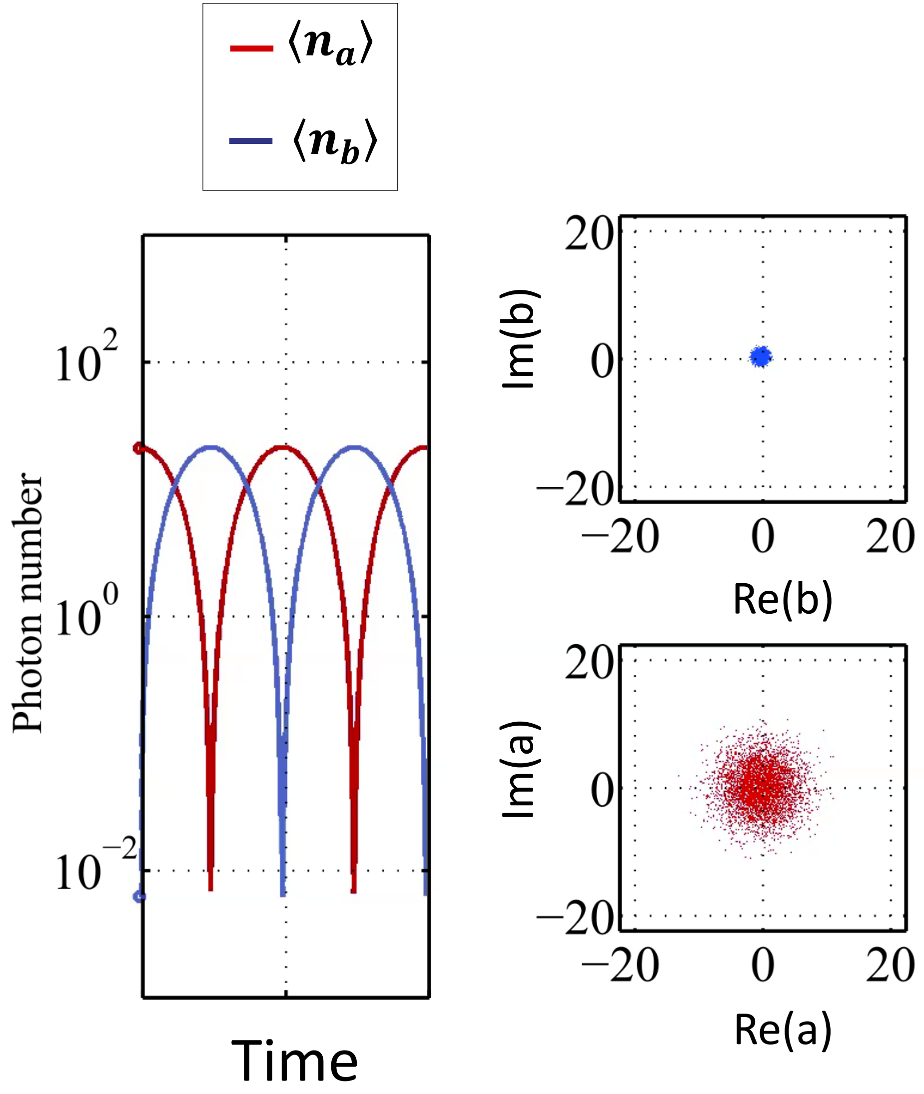
<u>Case 1:</u> $\omega_p = \omega_a - \omega_b$

$$\begin{cases} \dot{a} = ig\text{b} \\ \dot{b} = iga \end{cases} \qquad \begin{cases} \ddot{a} = -g^2\text{a} \\ \ddot{b} = -g^2\text{b} \end{cases}$$

$$\begin{cases} a(t) = a(0)\cos(gt) + b(0)\sin(gt) \\ b(t) = b(0)\cos(gt) + a(0)\sin(gt) \end{cases}$$

Energy exchange between two modes

Unlike Jaynes-Cummings: Swap is independent of the states being transferred!



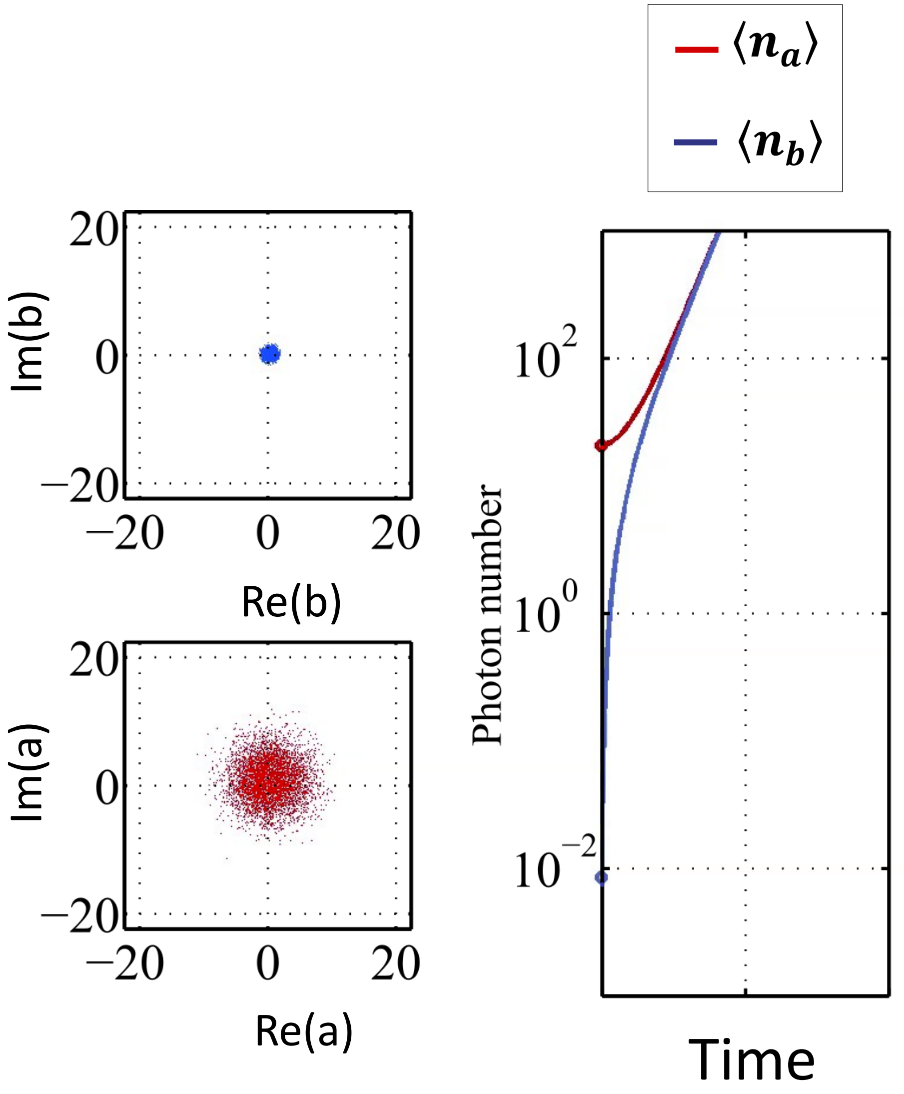
Coupled mode equations

Case 2: $\omega_p = \omega_a + \omega_b$

$$\begin{cases} \dot{a} = igb^* \\ \dot{b} = iga^* \end{cases} \qquad \begin{cases} \ddot{a} = g^2\mathrm{a} \\ \ddot{b} = g^2\mathrm{b} \end{cases}$$

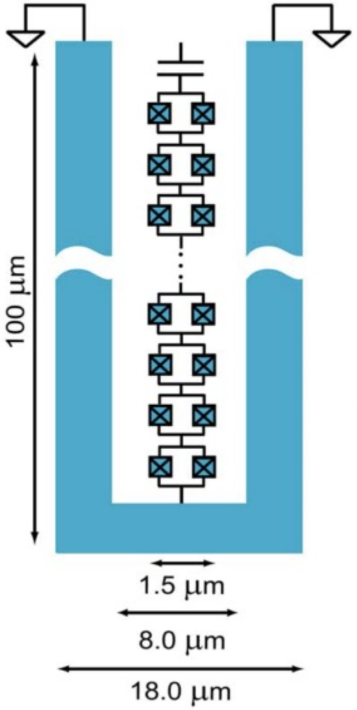
$$\begin{cases} a(t) = a(0)\cosh(gt) + b^*(0)\sinh(gt) \\ b(t) = b(0)\cosh(gt) + a^*(0)\sinh(gt) \end{cases}$$

Exponential growth with time leads to gain



Parametric interactions in superconducting circuits

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} - 2\hbar \boldsymbol{g(t)} (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$



*M. Castellanos-Beltran, **APL** 91 (2007)*

$$\mathrm{L(I)} = \frac{\varphi_0}{I_c\sqrt{1 - I^2/I_c^2}}$$

$$U = \frac{1}{2} L(I) I^2$$

Josephson junction at zero dc current

$$L(I) = L_0[1 + \xi I^2]$$

Current at signal frequency $I_a \sim \hat{a} + \hat{a}^\dagger$

Current at idler frequency $I_b \sim \hat{b} + \hat{b}^\dagger$

Current at pump frequency I_p

Total current $I = I_a + I_b + I_p$

$$U = \frac{1}{2} L_0 \left[1 + \xi (I_a + I_b + I_p)^2\right] (I_a + I_b + I_p)^2$$

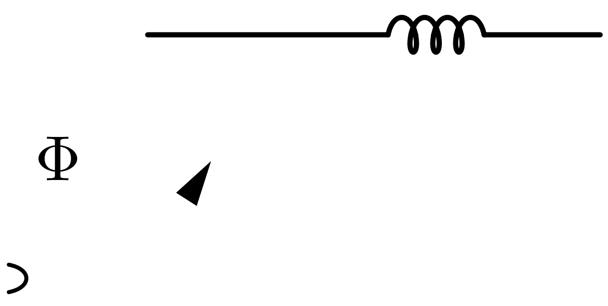
$$U = \frac{1}{2} L_0 I_a^2 + \frac{1}{2} L_0 I_b^2 + \frac{1}{2} L_0 \boldsymbol{\xi I_p^2} I_a I_b + \cdots$$

$$2\omega_p = \omega_a + \omega_b$$

4 waves mixing

Parametric interactions in superconducting circuits

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} - 2\hbar \boldsymbol{g(t)} (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$



$$U = \frac{1}{2} L(\Phi) I^2 \qquad L(\Phi) = L_0[1 + \xi\Phi^2 + \cdots] \text{ around } \Phi = 0$$

Current at signal frequency $I_a \sim \hat{a} + \hat{a}^\dagger$

Current at idler frequency $I_b \sim \hat{b} + \hat{b}^\dagger$

Flux at pump frequency Φ_p

*F. Lecocq, **PR Applied** 7 (2017)*

$$U = \frac{1}{2} L_0 \left[1 + \xi{\Phi_p}^2\right](I_a + I_b)^2$$

$$\mathrm{L}(\Phi) = \frac{L_J}{\cos\left(\pi \frac{\Phi}{\Phi_0}\right)}$$

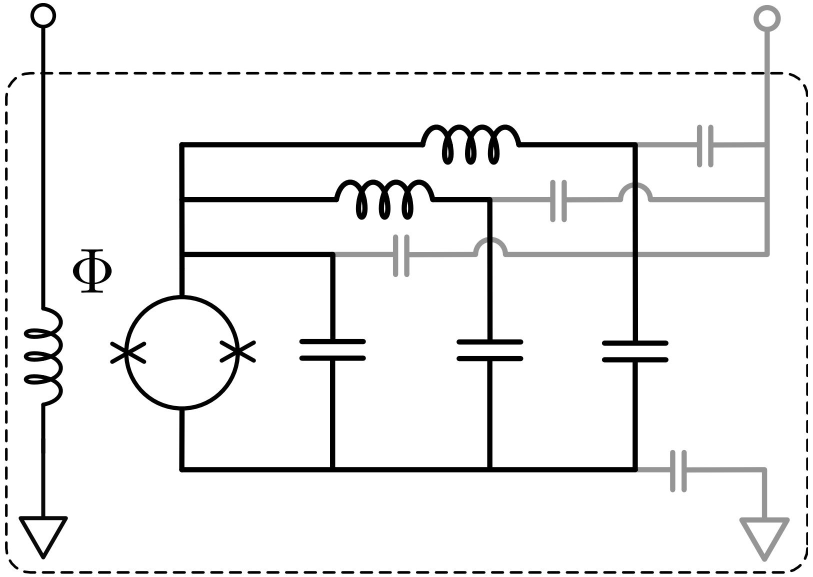
$$U = \frac{1}{2} L_0 I_a^2 + \frac{1}{2} L_0 I_b^2 + \frac{1}{2} L_0 \boldsymbol{\xi \Phi_p^2} I_a I_b + \cdots$$

$$2\omega_p = \omega_a + \omega_b$$

4 waves mixing

Parametric interactions in superconducting circuits

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} - 2\hbar \boldsymbol{g(t)} (\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$



*F. Lecocq, **PR Applied** 7 (2017)*

$$L(\Phi) = \frac{L_J}{\cos\left(\pi \frac{\Phi}{\Phi_0}\right)}$$

$$U = \frac{1}{2} L(\Phi) I^2 \qquad L(\Phi) = L_0[1 + \epsilon\Phi + \cdots] \text{ around } \Phi = \Phi_0/4$$

Current at signal frequency $I_a \sim \hat{a} + \hat{a}^\dagger$
Current at idler frequency $I_b \sim \hat{b} + \hat{b}^\dagger$
Flux at pump frequency Φ_p

$$U = \frac{1}{2} L_0 [1 + \epsilon\Phi_p](I_a + I_b)^2$$

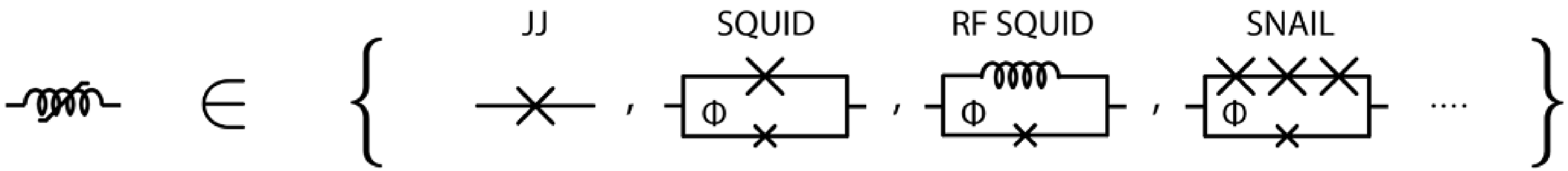
$$U = \frac{1}{2} L_0 I_a^2 + \frac{1}{2} L_0 I_b^2 + \frac{1}{2} L_0 \boldsymbol{\epsilon\Phi_p} I_a I_b + \cdots$$

$$\omega_p = \omega_a + \omega_b$$

3 waves mixing

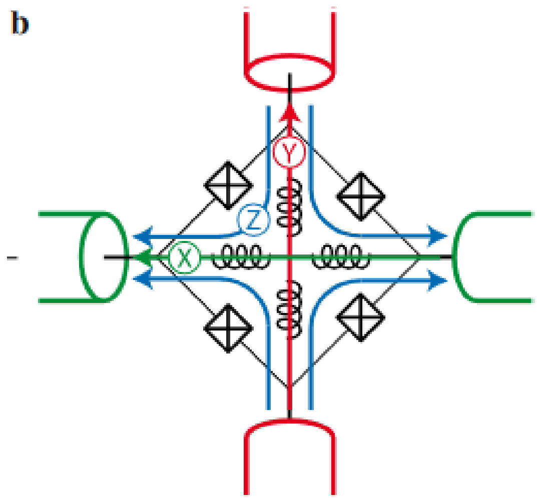
Parametric interactions in superconducting circuits

$$\hat{H} = \hbar\omega_a \hat{a}^\dagger \hat{a} + \hbar\omega_b \hat{b}^\dagger \hat{b} - 2\hbar \boldsymbol{g(t)}(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$



*V. V. Sivak, **PRapplied** (2020)*

Many other options (ATS, kinetic inductance, etc…)

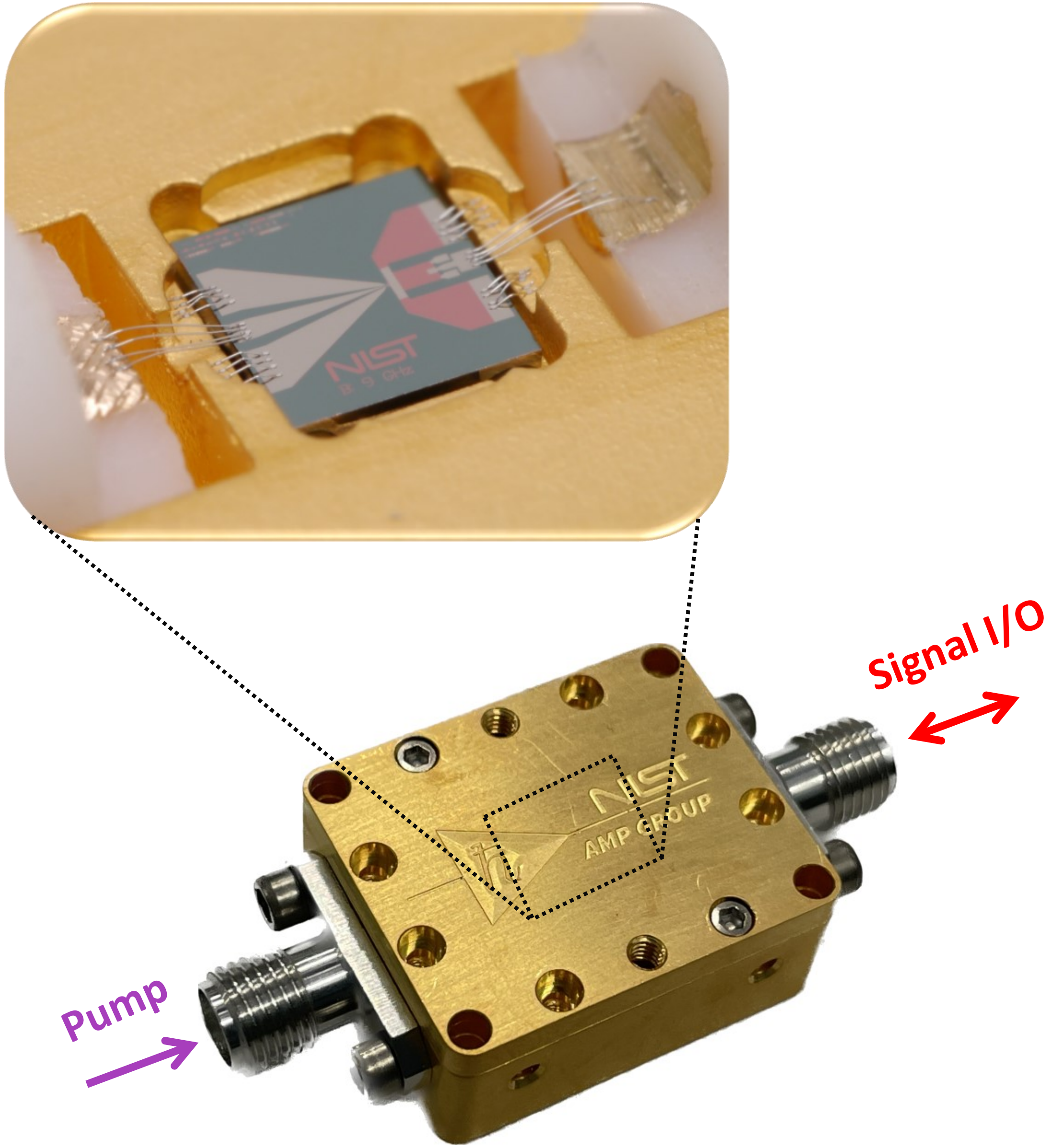


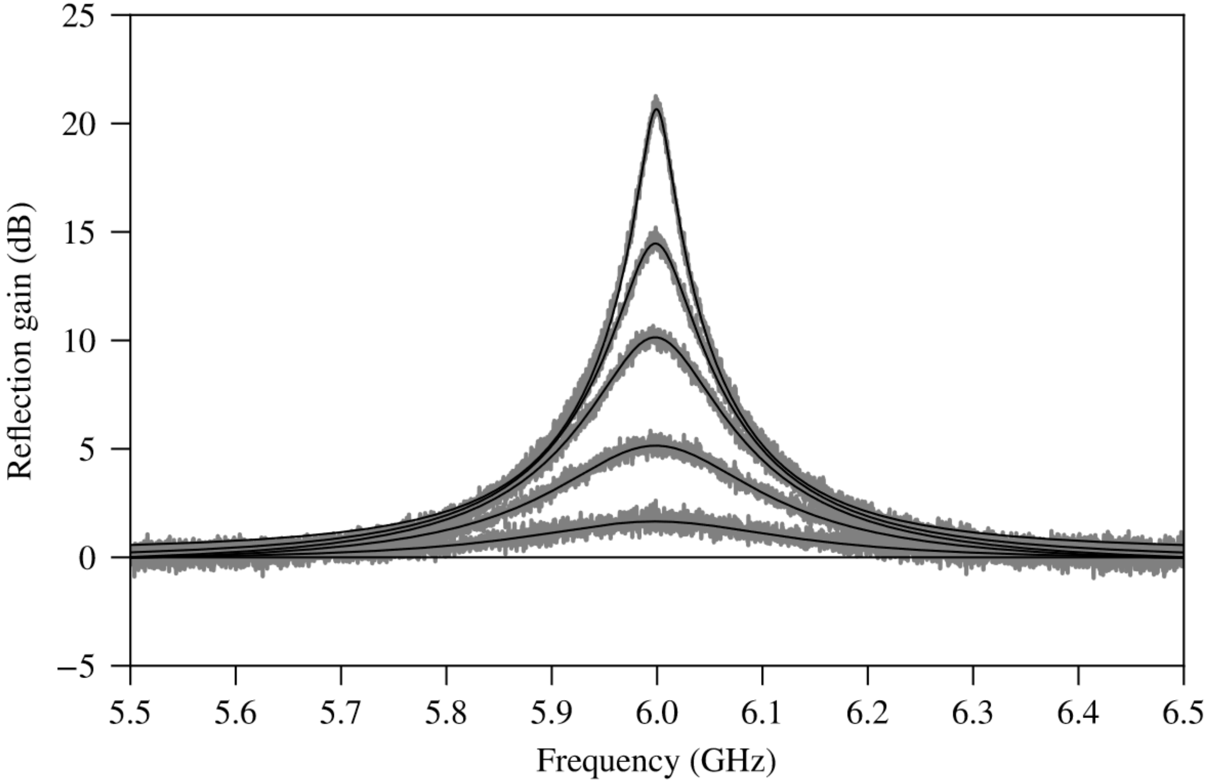
*N. Roch, **PRL*** 108 *(2012)*

Outline

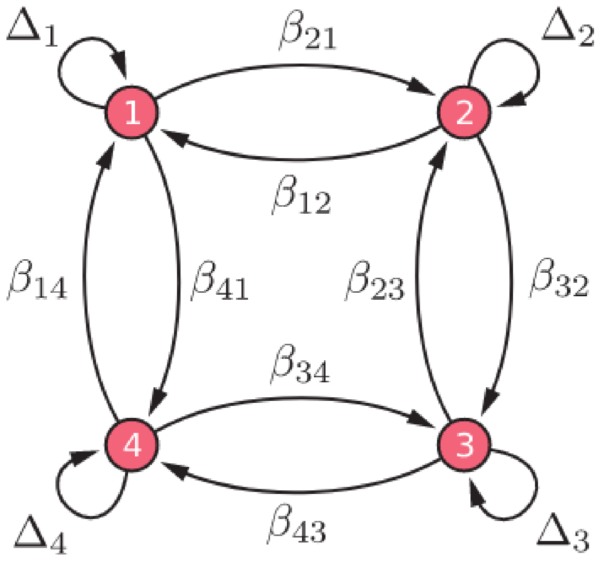
- Dispersive Readout of superconducting qubits
- Intro to parametric amplifiers:
  - Resonant parametric amplifiers
  - Traveling-waves parametric amplifiers
- Future directions

Resonant Parametric Amplifiers





Graph Theory



J. Aumentado

L. Ranzani

G. Peterson

References:

Ranzani and Aumentado, New J. Phys. **17**, 023024 (2015) GLOBAL NORMALIZATION

F. Lecocq *at al*, Phys. Rev. Applied **7**, 024028 (2017)
F. Lecocq *et al*, Phys. Rev. Applied **13**, 044005 (2020) NORMALIZATION BY MODE

G. Peterson's thesis, [Parametric Coupling between Microwaves and Motion in Qantum Circuits]() (2020) NO NORMALIZATION

Driven and damped harmonic oscillator

Harmonic oscillator with angular frequency ω_a and loss rate κ_a

$$\hat{H}_a = \hbar\omega_a \hat{a}^\dagger \hat{a} \rightarrow \hat{H}_a = \hbar\left(\omega_a - \frac{i\kappa_a}{2}\right)\hat{a}^\dagger \hat{a}$$

Coupled to N port with rates $\kappa_{a,j}$ with $\kappa_a = \sum_j^N \kappa_{a,j}$

$$\hat{H}_a = \hbar\left(\omega_a - \frac{i\kappa_a}{2}\right)\hat{a}^\dagger \hat{a} + i\hbar\sum_j^N \sqrt{\kappa_{a,j}}\left(\hat{a}^\dagger \hat{a}_{in,j} - \hat{a}\hat{a}_{in,j}^\dagger\right)$$

Let's just consider a single external drive term $\hat{a}_{in}$ with coupling rate κ_a^{ext}, an internal loss rate $\kappa_a^{int}$and no internal drive term

$$\frac{d\hat{a}}{dt} = -\frac{i}{\hbar}\left[\hat{a}, \hat{H}\right]$$

Driven and damped harmonic oscillator

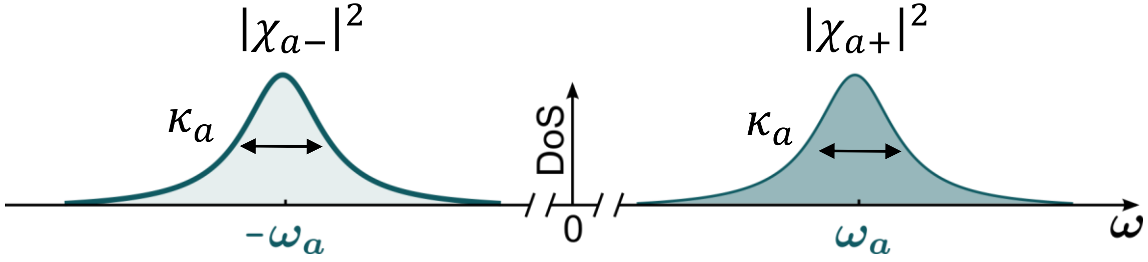
$$\begin{cases} \dfrac{da}{dt} = i\left(-\omega_a + \dfrac{i\kappa_a}{2}\right) a + \sqrt{\kappa_a^{ext}}\, a_{in} \\ \dfrac{da^*}{dt} = i\left(\omega_a + \dfrac{i\kappa_a}{2}\right) a^* + \sqrt{\kappa_a^{ext}}\, a_{in}^* \end{cases}$$

$$\begin{cases} a[\omega] = i\chi_{a+}[\omega]\sqrt{\kappa_a^{ext}}\, a_{in} \\ a^*[\omega] = i\chi_{a-}[\omega]\sqrt{\kappa_a^{ext}}\, a_{in}^* \end{cases}$$

Expectation values $a \equiv \langle \hat{a} \rangle$

Fourrier transform $a[\omega] = \int dt\, e^{i\omega t} a$

$$\chi_{a\pm} = \left[\omega \mp \omega_a + \frac{i\kappa_a}{2}\right]^{-1}$$

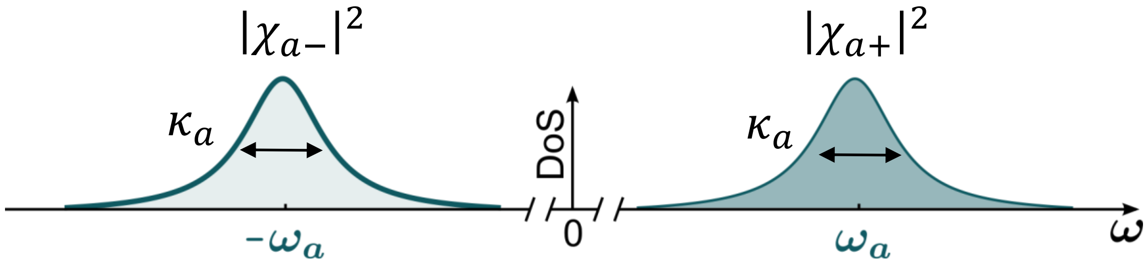


Input-output formalism: $a_{in} + a_{out} = \sqrt{\kappa_a^{ext}}\, a$

$$S_{aa}(\omega) = \frac{a_{out}}{a_{in}} = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$

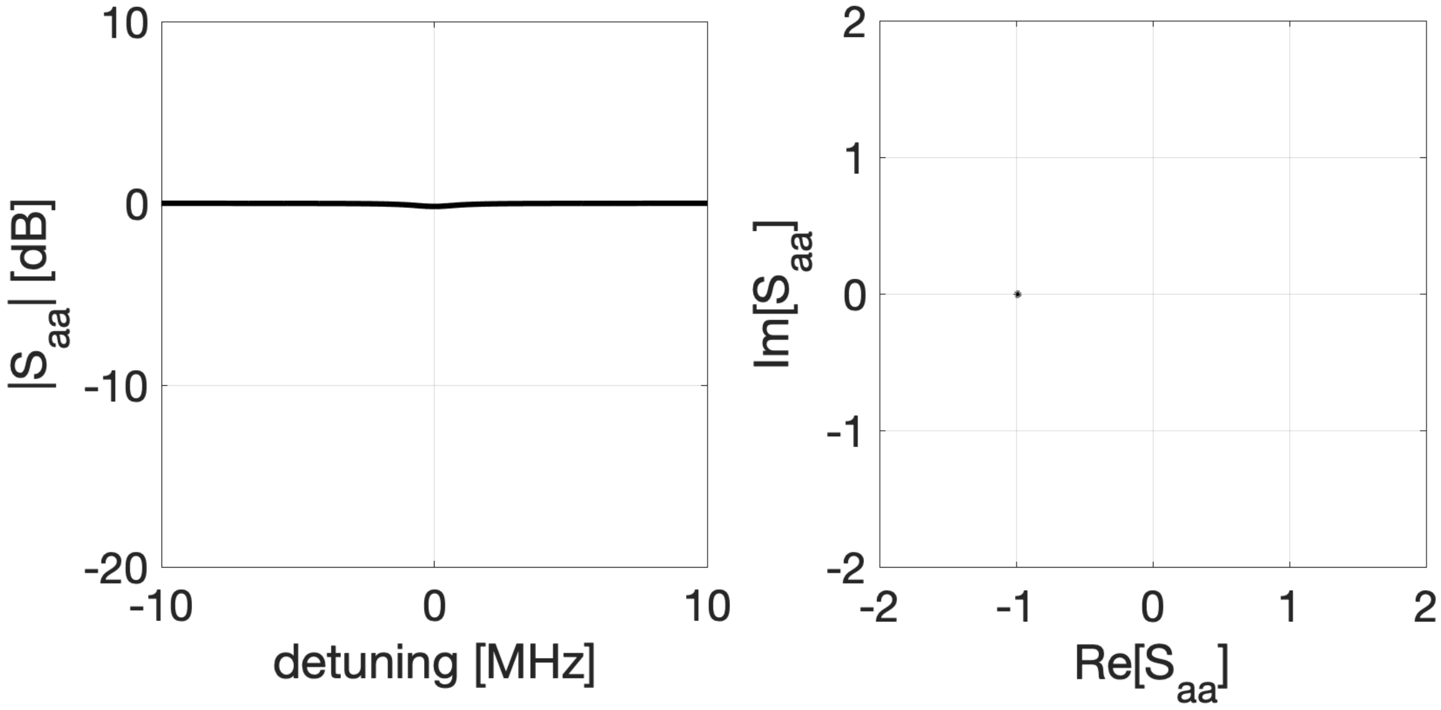
$$\kappa_a^{ext} + \kappa_a^{int} = \kappa_a$$

Driven and damped harmonic oscillator

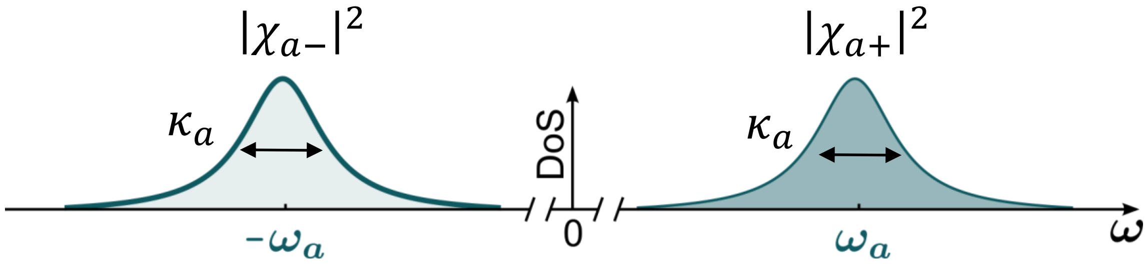


$$\kappa_a^{ext} \ll \kappa_a^{int}$$

$$S_{aa}(\omega) = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$

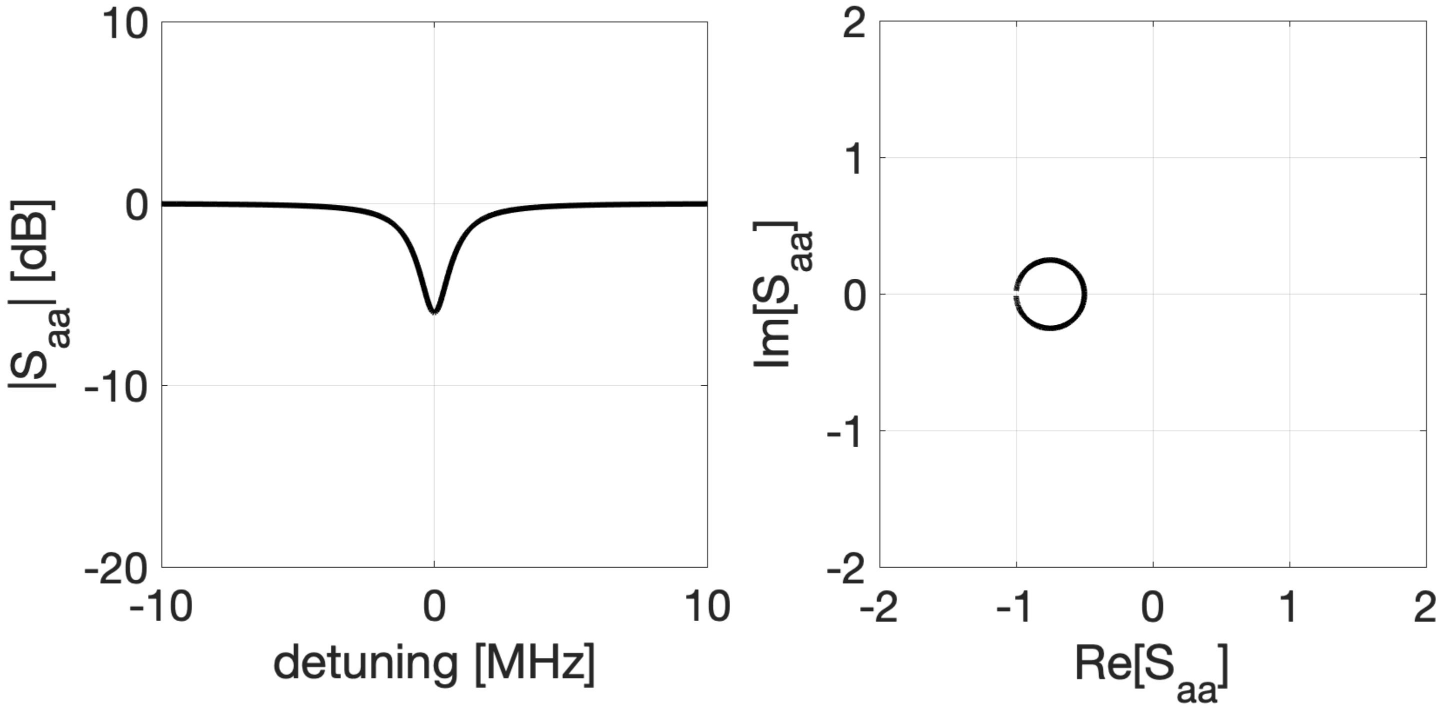


Driven and damped harmonic oscillator

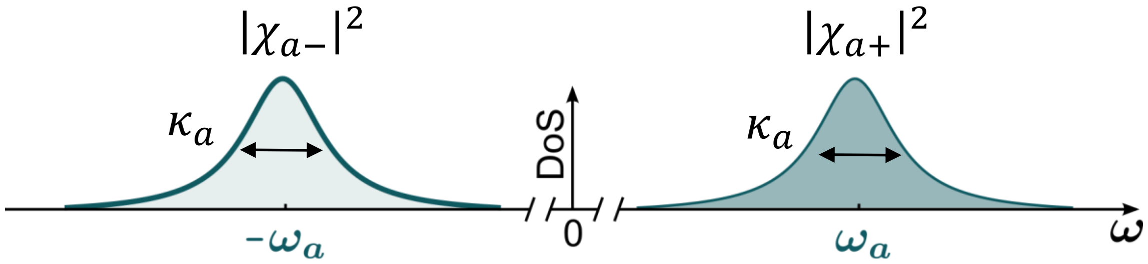


$$\kappa_a^{ext} < \kappa_a^{int}$$

$$S_{aa}(\omega) = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$

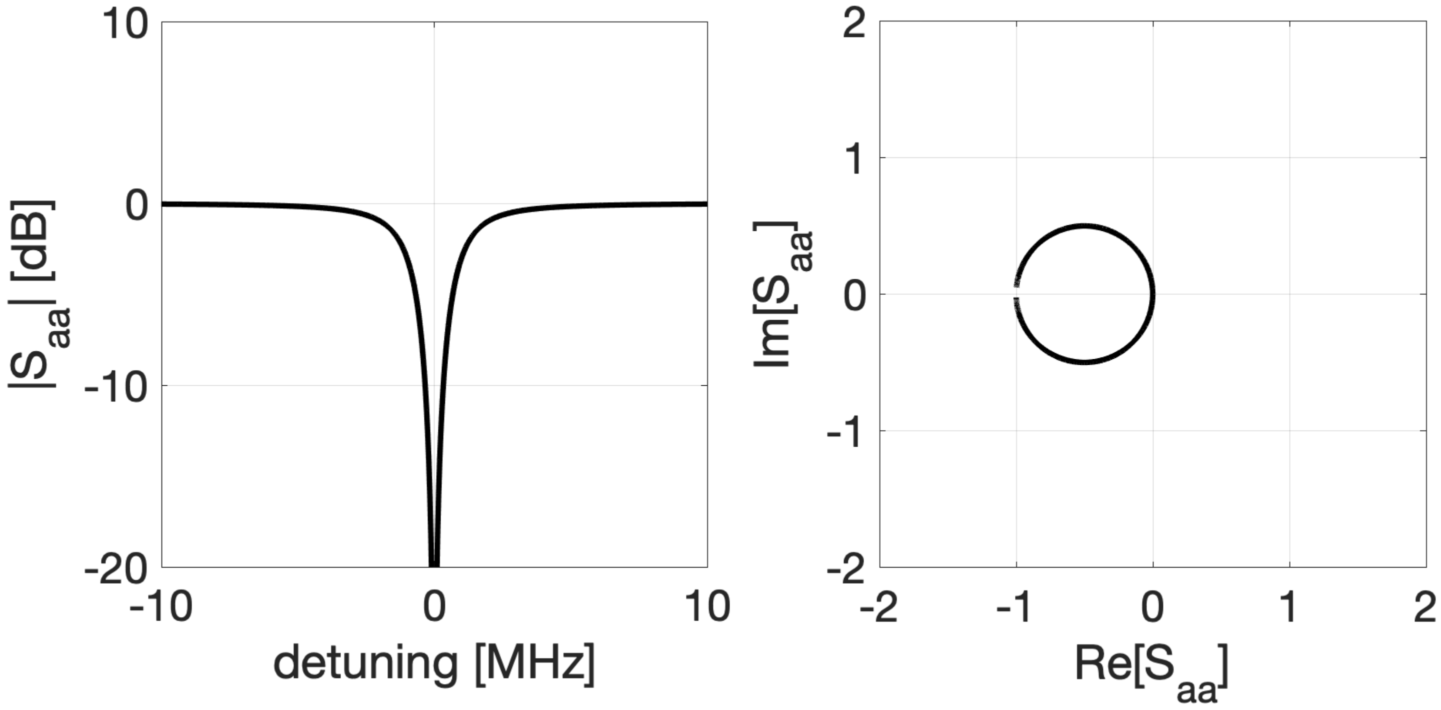


Driven and damped harmonic oscillator

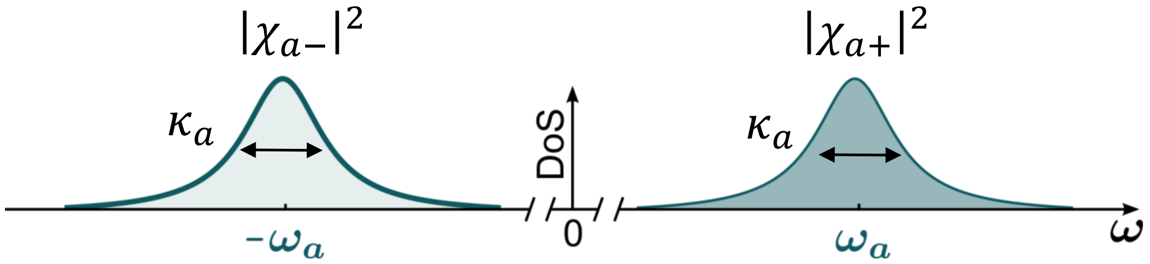


$$S_{aa}(\omega) = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$

$$\kappa_a^{ext} = \kappa_a^{int}$$

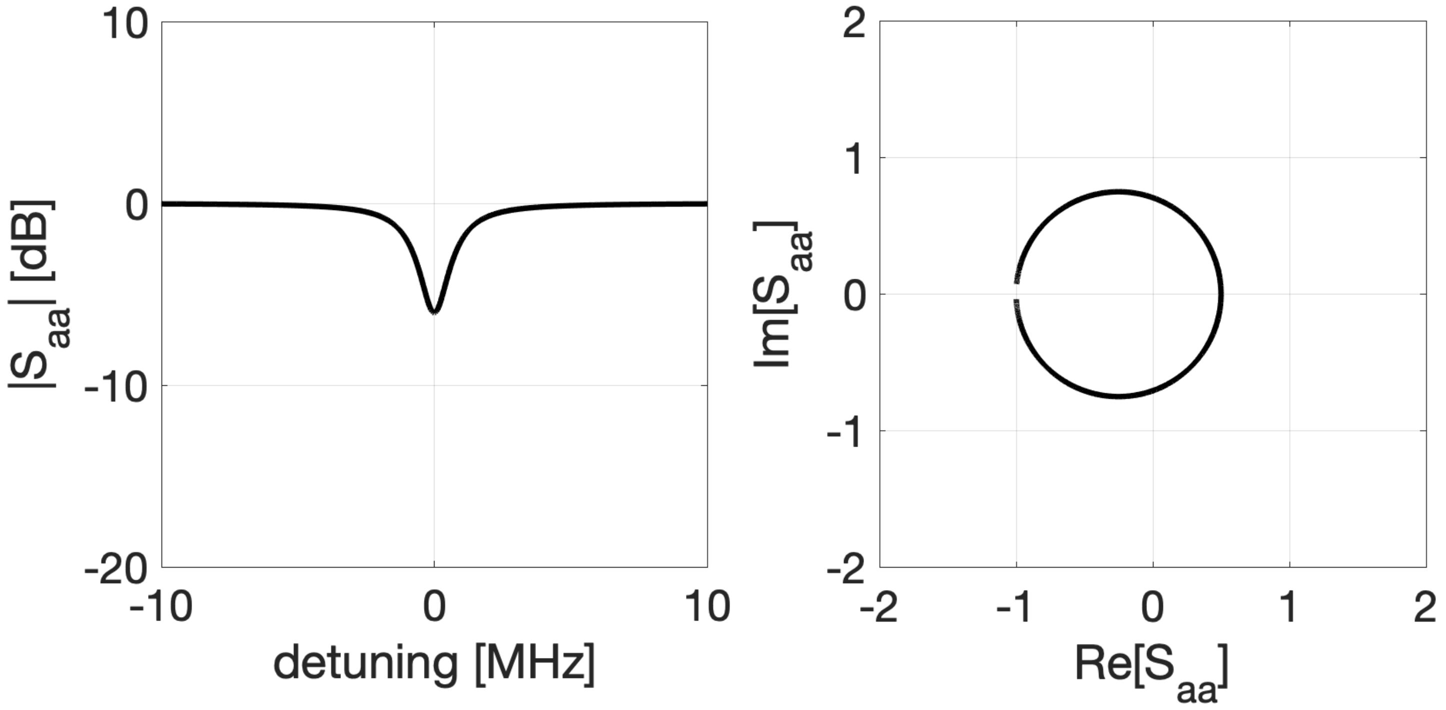


Driven and damped harmonic oscillator

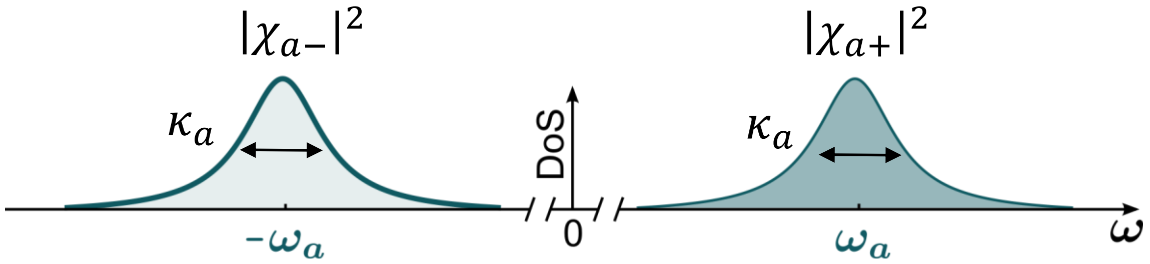


$$S_{aa}(\omega) = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$

$\kappa_a^{ext} > \kappa_a^{int}$

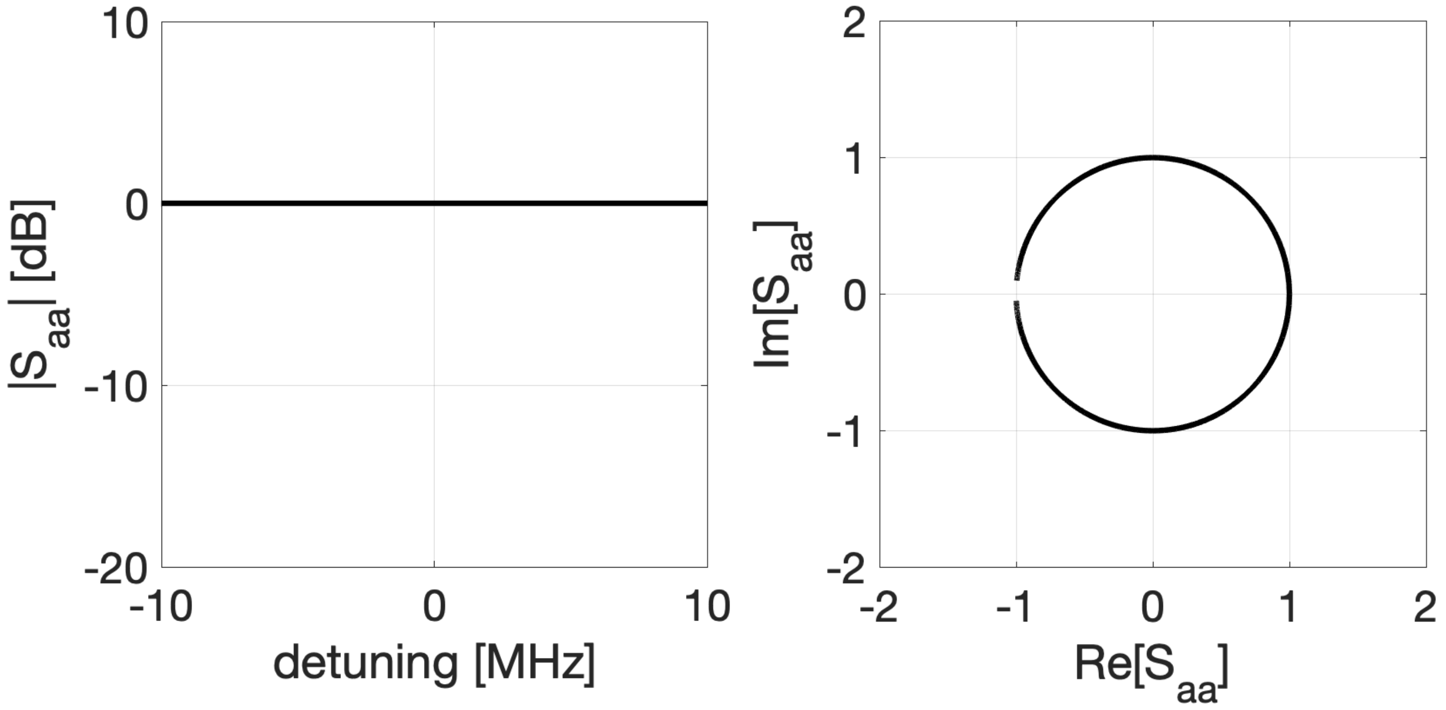


Driven and damped harmonic oscillator



$\kappa_a^{ext} \gg \kappa_a^{int}$

$$S_{aa}(\omega) = \frac{2\kappa_a^{ext}}{\kappa_a + 2i(\omega - \omega_a)} - 1$$



Parametric amplifier

Lecocq, at al **Phys. Rev. Applied** 7 (2017)

$$\frac{\hat{H}_a}{\hbar} = \left(\omega_a - \frac{i\kappa_a}{2}\right)\hat{a}^\dagger\hat{a} + \sqrt{\kappa_a^{ext}}\left(\hat{a}^\dagger\hat{a}_{in} - \hat{a}\hat{a}_{in}^\dagger\right) + \left(\omega_b - \frac{i\kappa_b}{2}\right)\hat{b}^\dagger\hat{b} + \sqrt{\kappa_b^{ext}}\left(\hat{b}^\dagger\hat{b}_{in} - \hat{b}\hat{b}_{in}^\dagger\right) - 2\mathrm{g(t)}(\hat{a} + \hat{a}^\dagger)(\hat{b} + \hat{b}^\dagger)$$

$$\begin{cases} \dfrac{d\hat{a}}{dt} = -\dfrac{i}{\hbar}[\hat{a}, \hat{H}] \\ \dfrac{d\hat{b}^\dagger}{dt} = -\dfrac{i}{\hbar}[\hat{b}^\dagger, \hat{H}] \end{cases}$$

$a \equiv \langle\hat{a}\rangle$ and $b \equiv \langle\hat{b}\rangle$

$$2g(t) = ge^{-i\omega_p t} + c.c$$

$$a \rightarrow ae^{-i\omega_a^s t}$$

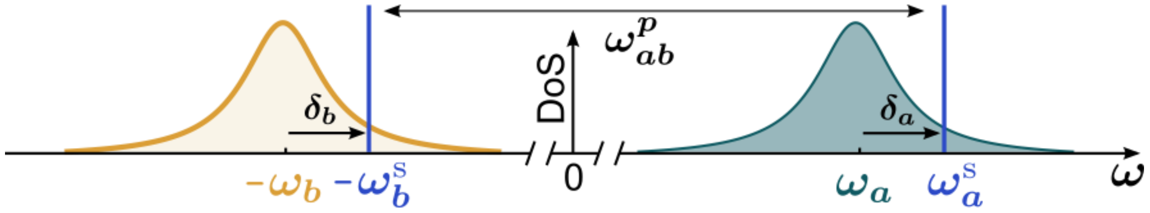
$$b \rightarrow be^{-i\omega_b^s t}$$

RWS

$$\begin{cases} a + g\chi_{a+}b^* = i\chi_{a+}\sqrt{\kappa_a^{ext}}a_{in} \\ b^* - g^*\chi_{b-}a = i\chi_{a-}\sqrt{\kappa_b^{ext}}b_{in}^* \end{cases}$$

Parametric amplifier

Lecocq, at al **Phys. Rev. Applied** 7 (2017)

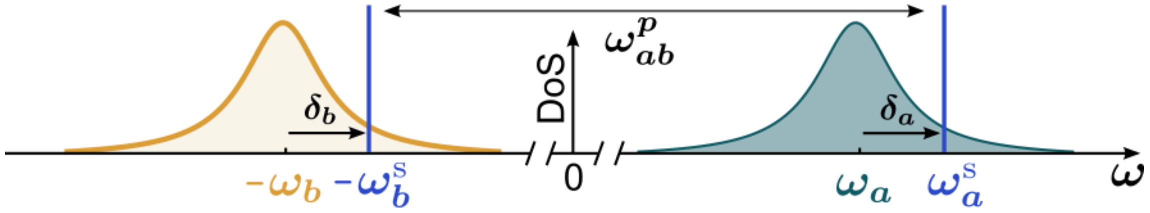


$$C = \frac{4g^2}{\kappa_a \kappa_b}$$

$$\begin{cases} a + g\chi_{a+}b^* = i\chi_{a+}\sqrt{\kappa_a^{ext}}a_{in} \\ b^* - g^*\chi_{b-}a = i\chi_{a-}\sqrt{\kappa_b^{ext}}b_{in}^* \end{cases} \xrightarrow[\kappa_{a,b}^{ext} = \kappa_{a,b}]{\omega_{a,b}^{s} = \omega_{a,b}} \begin{pmatrix} a_{out} \\ b_{out}^{\dagger} \end{pmatrix} = \begin{pmatrix} \frac{1+C}{1-C} & \frac{2i\sqrt{C}}{1-C}e^{i\Phi} \\ -\frac{2i\sqrt{C}}{1-C}e^{-i\Phi} & \frac{1+C}{1-C} \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^{\dagger} \end{pmatrix}$$

Parametric amplifier

Lecocq, at al **Phys. Rev. Applied** 7 (2017)



$$C = \frac{4g^2}{\kappa_a \kappa_b}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^{\dagger} \end{pmatrix} = \begin{pmatrix} \frac{1+C}{1-C} & \frac{2i\sqrt{C}}{1-C} e^{i\Phi} \\ -\frac{2i\sqrt{C}}{1-C} e^{-i\Phi} & \frac{1+C}{1-C} \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^{\dagger} \end{pmatrix}$$

$C = 0.5 \rightarrow \sqrt{G} \approx 3$

$C = 0.9 \rightarrow \sqrt{G} \approx 20$

$C = 0.99 \rightarrow \sqrt{G} \approx 200$

$C = 0$

$C \longrightarrow 1$

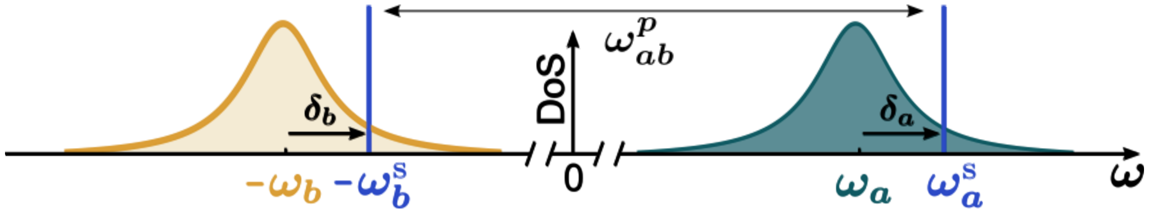
$$\sqrt{G} = \frac{1+C}{1-C} \approx \frac{2}{1-C}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^{\dagger} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^{\dagger} \end{pmatrix}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^{\dagger} \end{pmatrix} \approx \begin{pmatrix} \sqrt{G} & \sqrt{G-1} \\ \sqrt{G-1} & \sqrt{G} \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^{\dagger} \end{pmatrix}$$

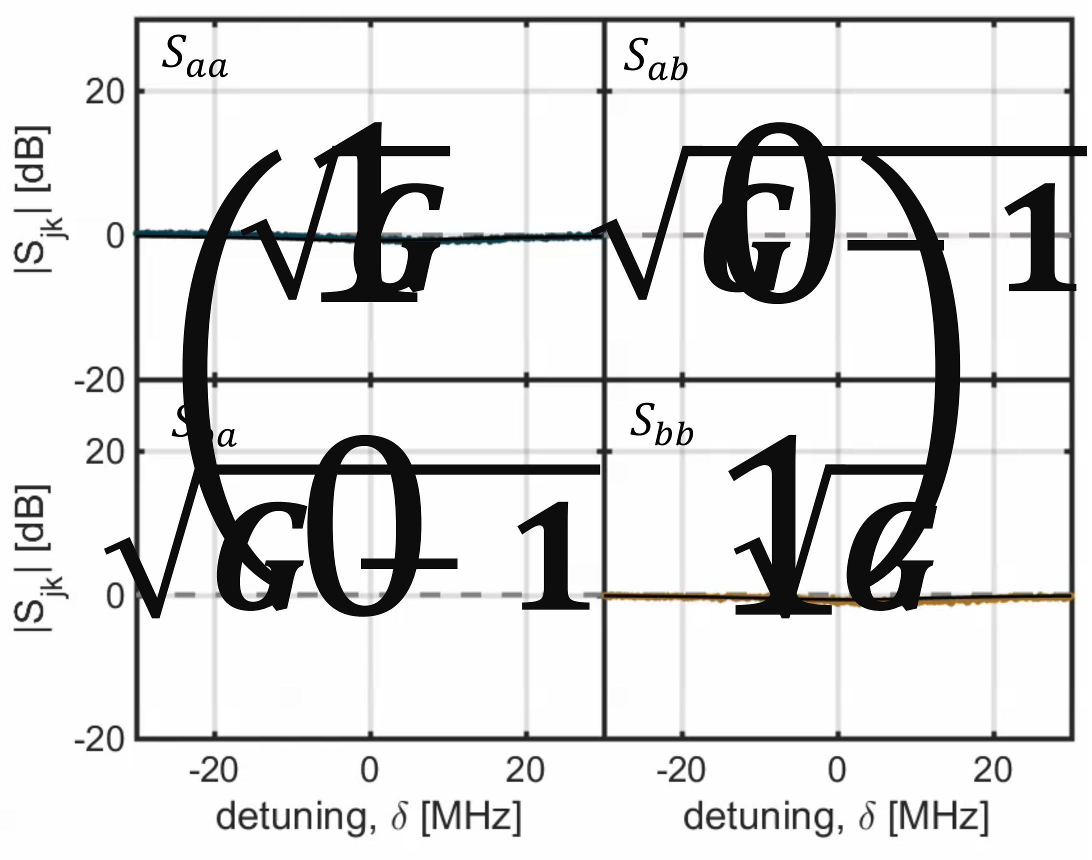
Parametric amplifier

Lecocq, at al **Phys. Rev. Applied** 7 (2017)



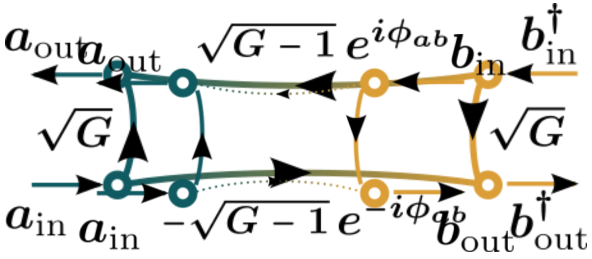
$$\omega_{ab} = \omega_b + \omega_a$$

$$g_{ab} < \sqrt{\kappa_a \kappa_b}$$



$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{G} & \sqrt{G-1} \\ \sqrt{G-1} & \sqrt{G} \end{pmatrix}$$

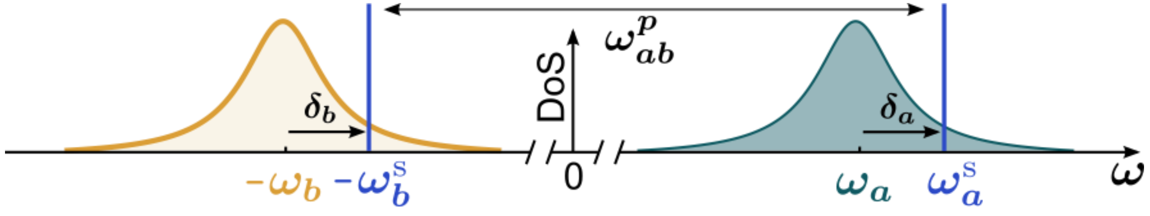


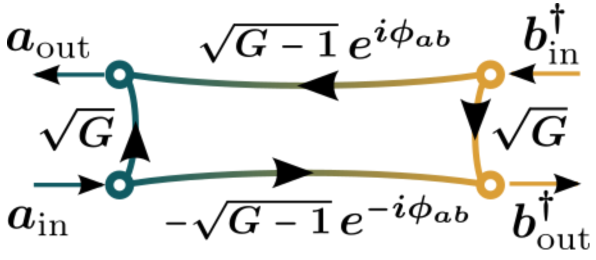
Parametric amplifier

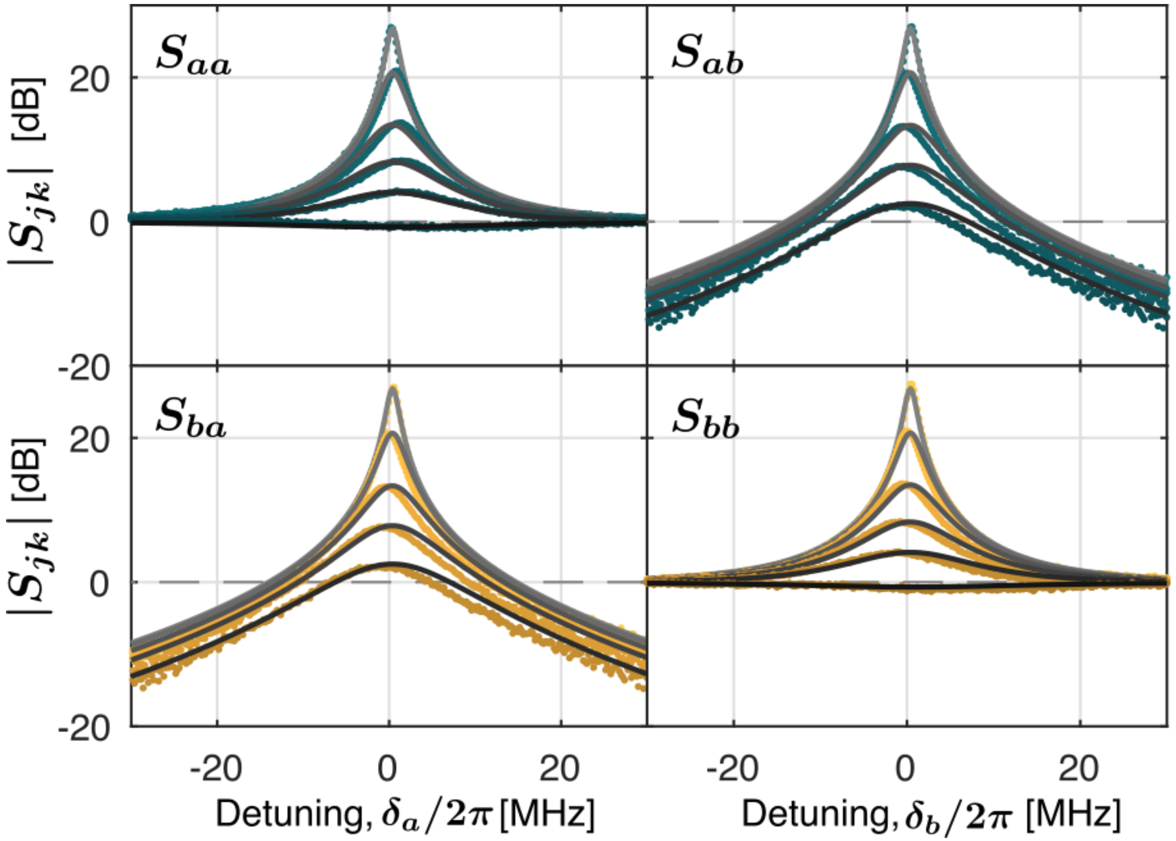
Lecocq, at al **Phys. Rev. Applied** 7 (2017)

$$\omega_{ab} = \omega_b + \omega_a$$

$$g_{ab} < \sqrt{\kappa_a \kappa_b}$$

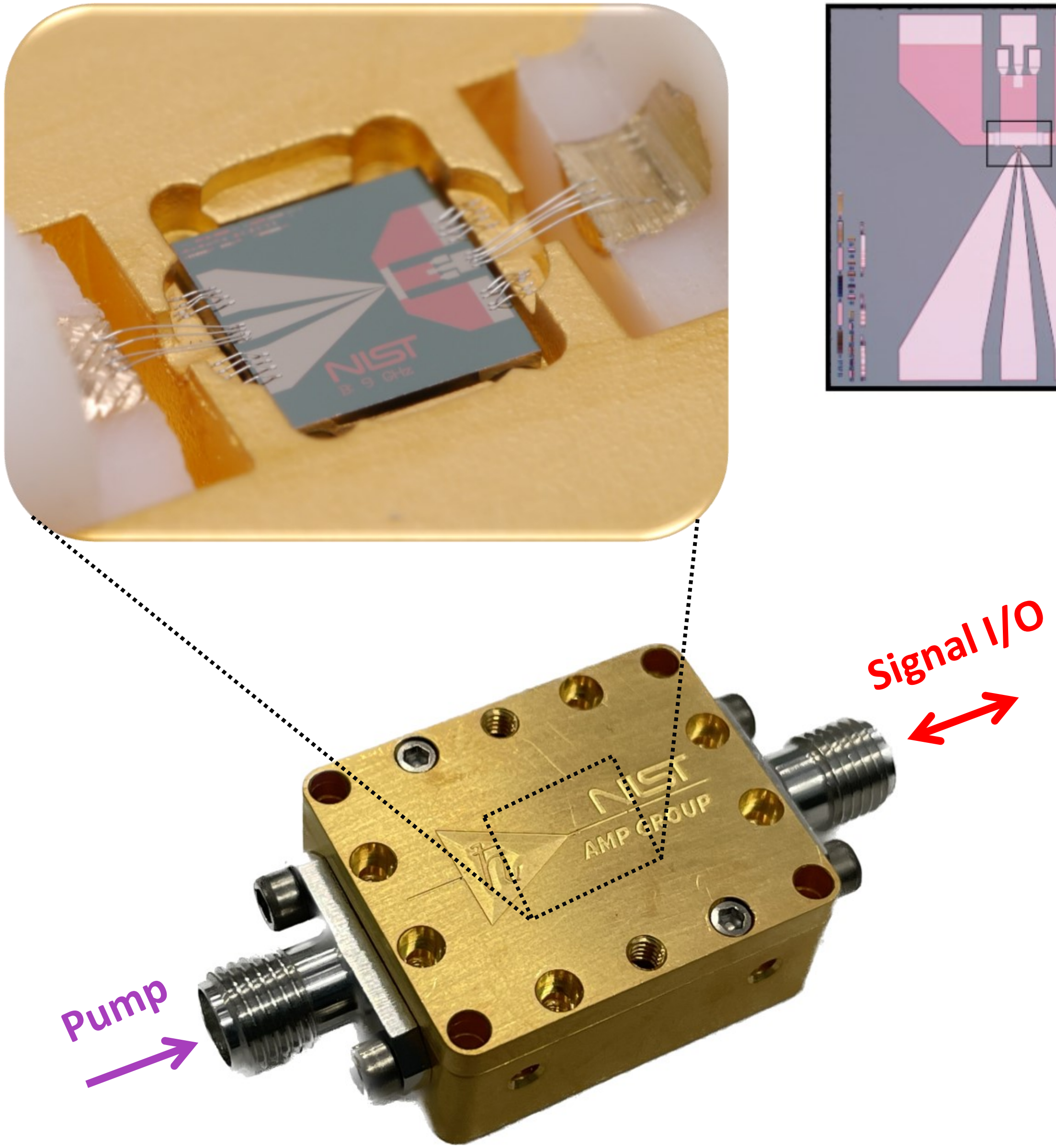


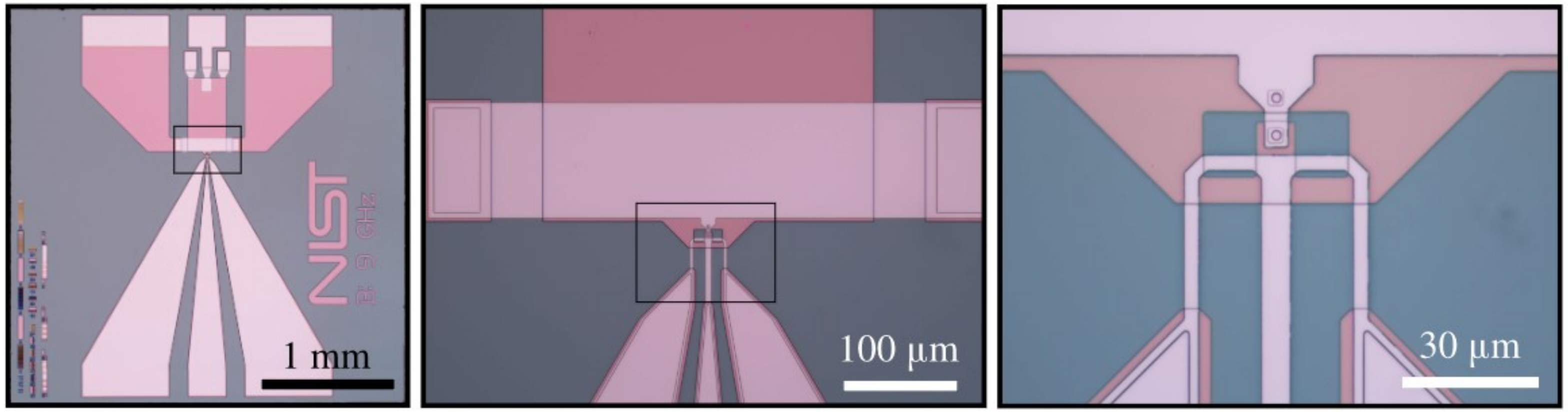


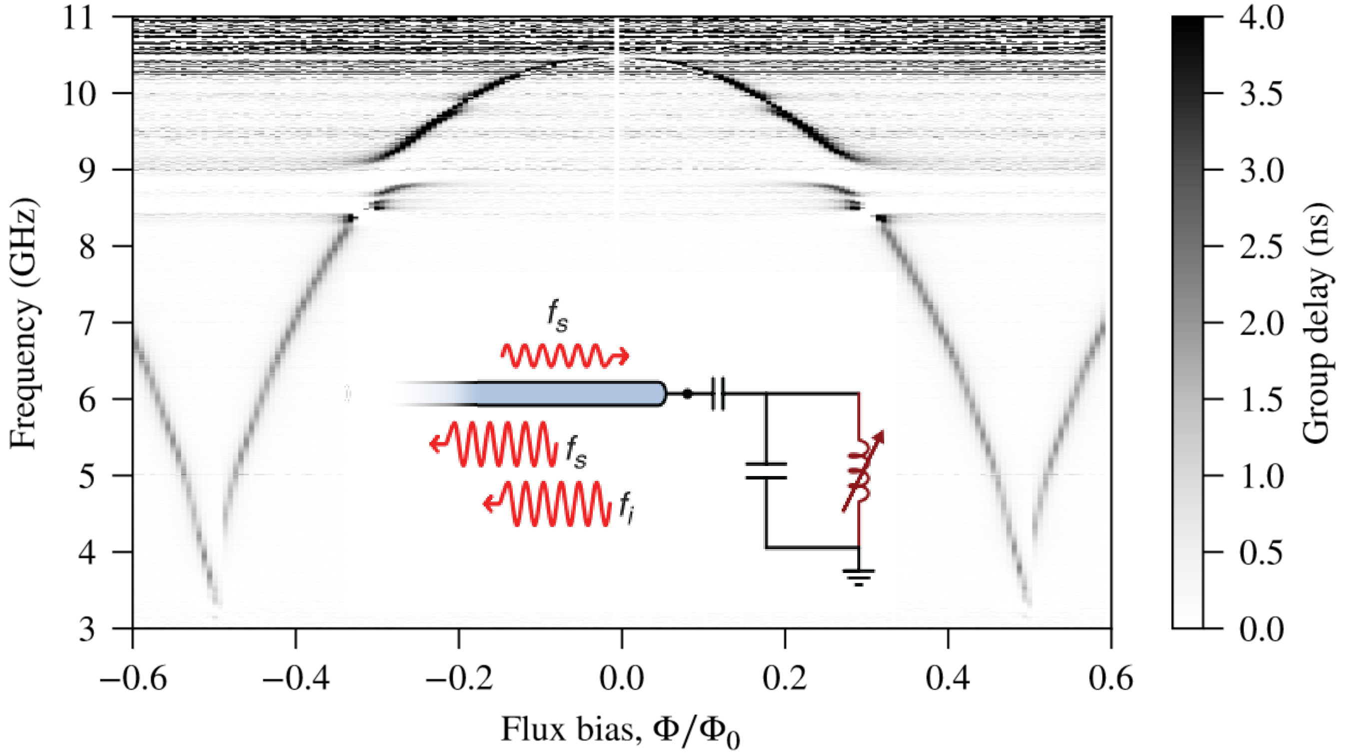


$$S = \begin{pmatrix} \sqrt{G} & \sqrt{G-1} \\ \sqrt{G-1} & \sqrt{G} \end{pmatrix}$$

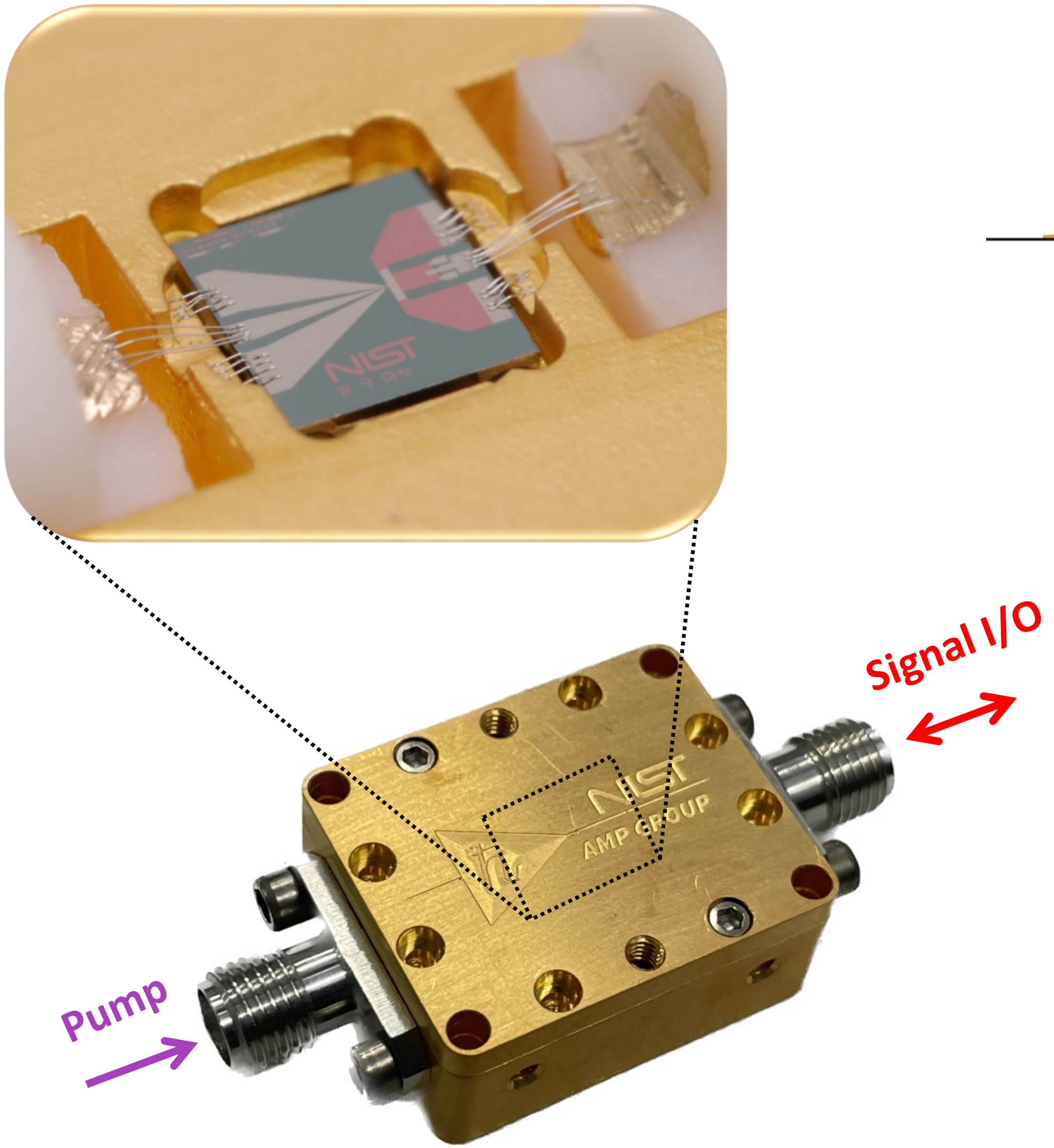
Josephson Parametric Amplifiers

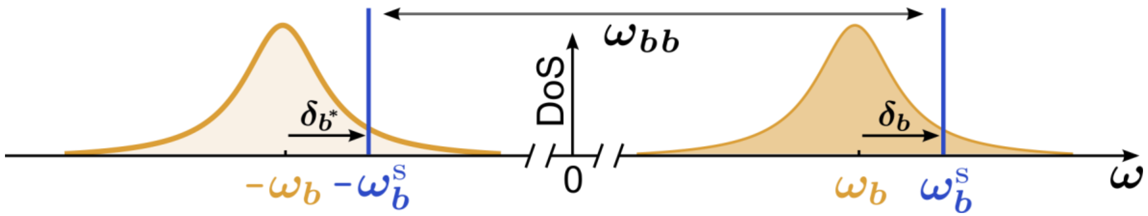


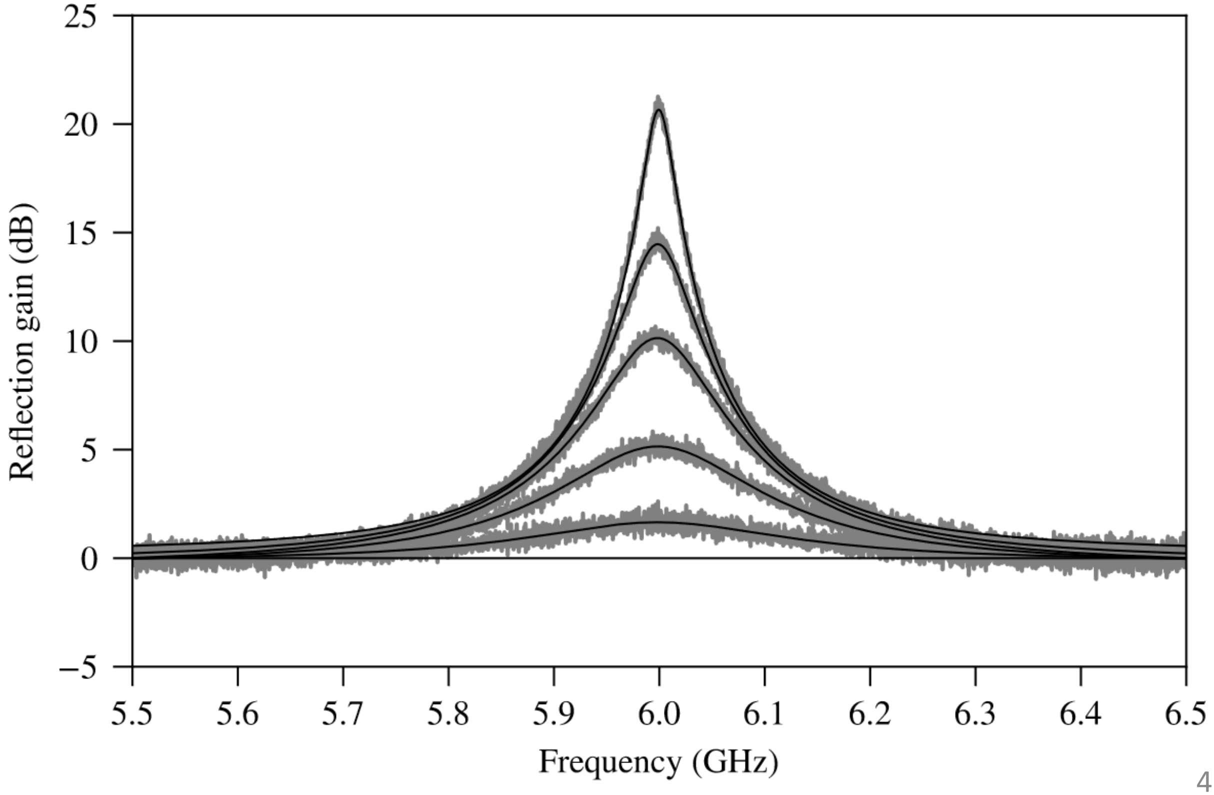




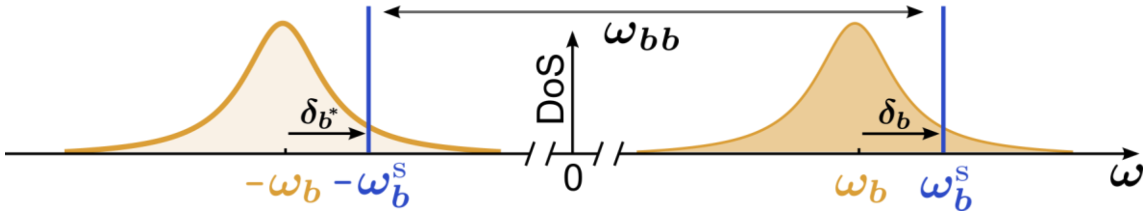
Josephson Parametric Amplifiers







Phase sensitivity



When $\omega_s = \omega_i = \frac{\omega_p}{2}$, phase sensitivity becomes obvious

$$X = \frac{1}{\sqrt{2}}(a + a^\dagger)$$

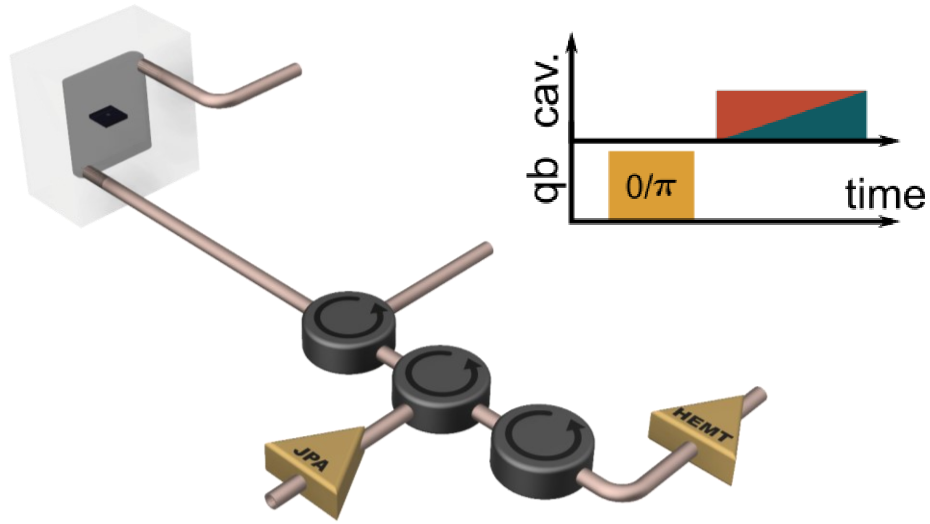
$$P = \frac{i}{\sqrt{2}}(a^\dagger - a)$$

$$\begin{pmatrix} a_{out} \\ a_{out}^\dagger \end{pmatrix} = \begin{pmatrix} \sqrt{G} & \sqrt{G-1} \\ \sqrt{G-1} & \sqrt{G} \end{pmatrix} \begin{pmatrix} a_{in} \\ a_{in}^\dagger \end{pmatrix}$$

$$\begin{pmatrix} X_{out} \\ P_{out} \end{pmatrix} = \begin{pmatrix} 2\sqrt{G} & 0 \\ 0 & \frac{1}{2\sqrt{G}} \end{pmatrix} \begin{pmatrix} X_{in} \\ P_{in} \end{pmatrix}$$

Every parametric amplifier is phase-sensitive, in the right linear combination of signal and idler.

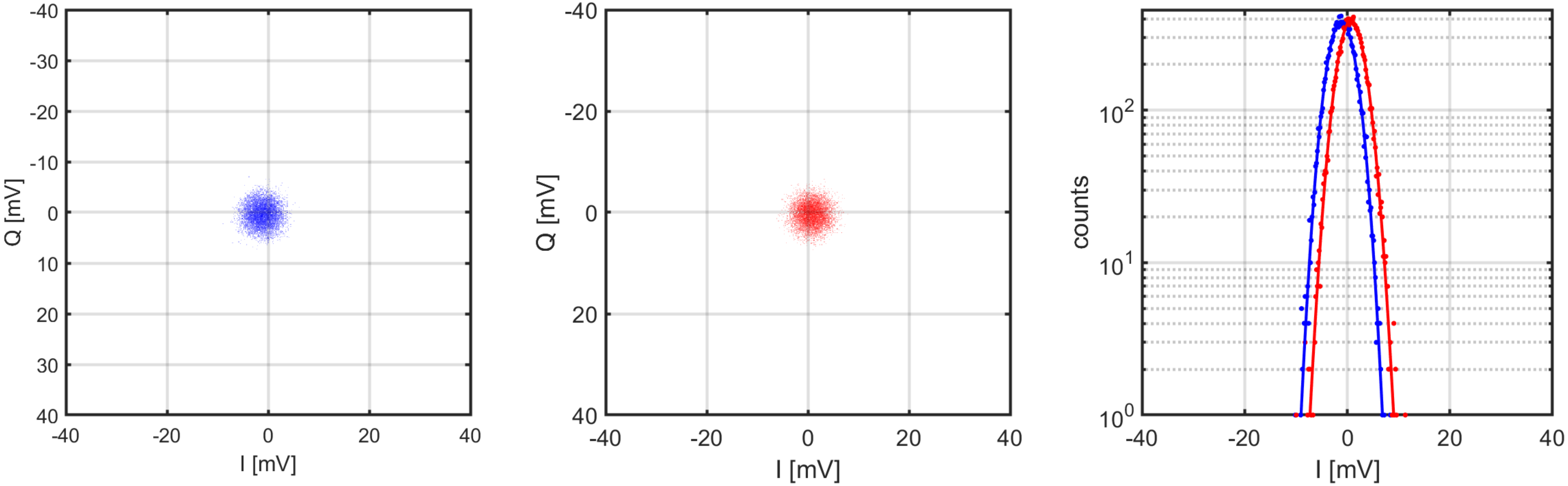
Measurement fidelity with a parametric amplifier



1. Prepare qubit in g or e
2. Drive cavity, acquire voltage $V(t) = |V(t)|e^{-i\omega_d t}$
3. Multiply voltage by $\cos(\omega_d t)$ (or $\sin(\omega_d t)$)
4. Integrate voltage over $\tau = 1\mu s$ to get I (or Q).
5. Repeat 10^4 times.

$\sqrt{G} = 25$

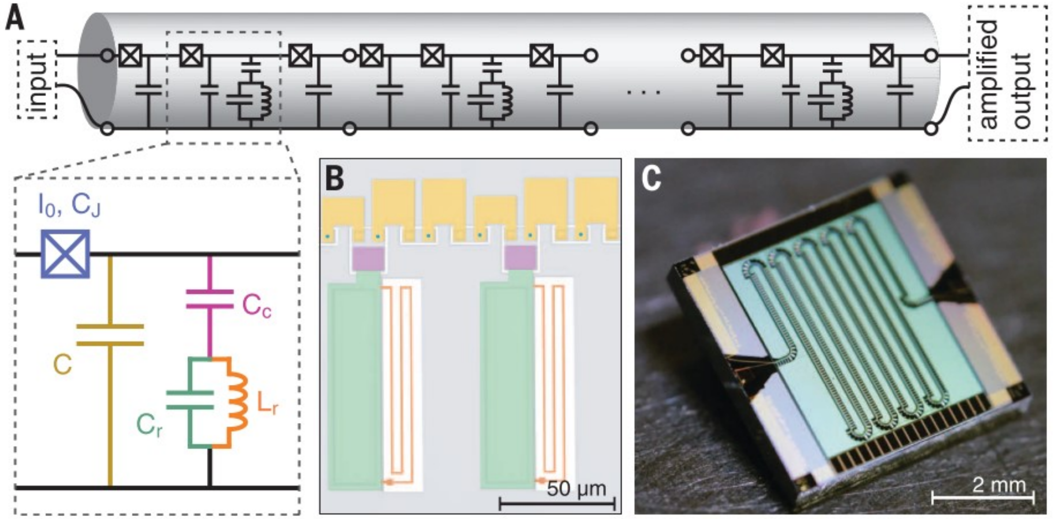
$F = 97\%$



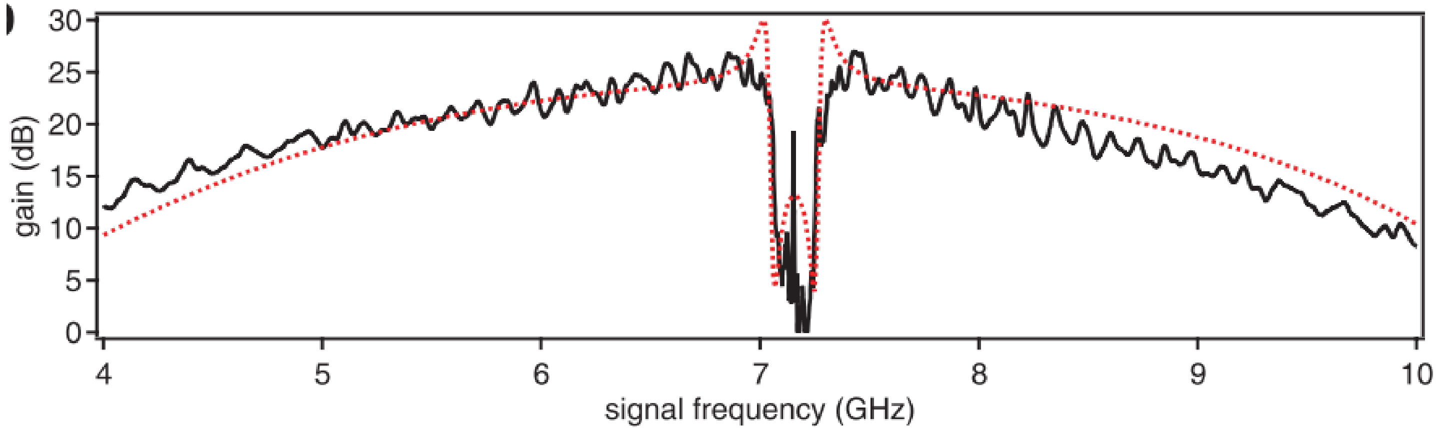
Outline

- Dispersive Readout of superconducting qubits
- Intro to parametric amplifiers:
  - Resonant parametric amplifiers
  - Traveling-waves parametric amplifiers
- Future directions

Traveling wave amplifiers

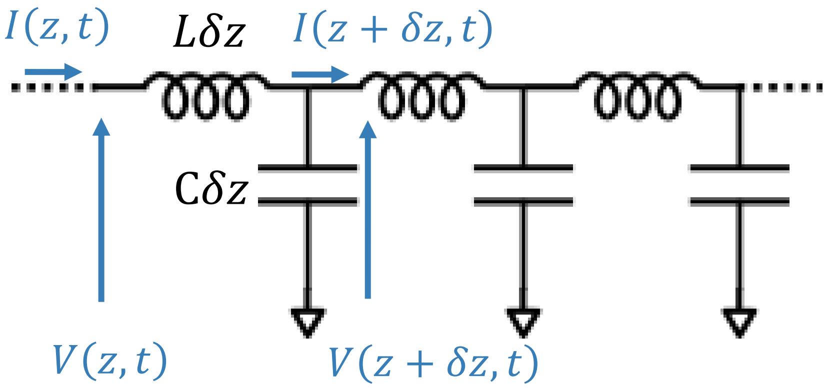


*Macklin, … , Siddiqi, **Science*** 350 *(2015)*



Also: *Planat, **PRX** 10 (2020), Ranadive, **Nat. Com.** 13 (2021), Esposito, **Applied. Phys Lett.** 119 (2021)* and *Ranzani, **PRapplied** (2021)*

Primer to TWPA: telegrapher equations



L: inductance per unit length

C: capacitance per unit length

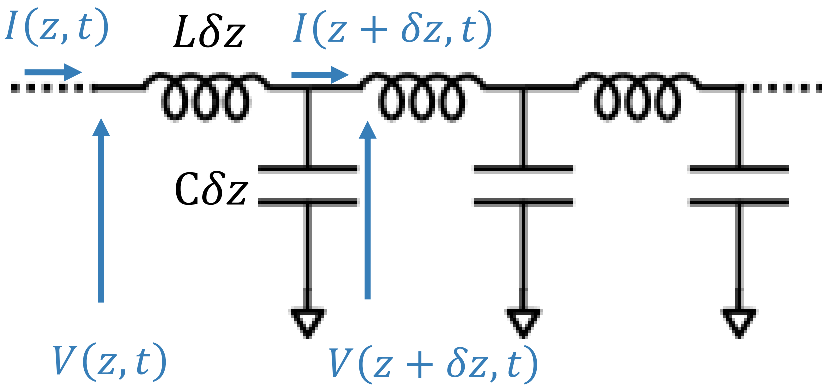
$$\begin{cases} V(z,t) - V(z+\delta z,t) = L\delta z \dfrac{\partial I(z,t)}{\partial t} \\ I(z,t) - I(z+\delta z,t) = C\delta z \dfrac{\partial V(z+\delta z,t)}{\partial t} \end{cases}$$

$$\begin{cases} -\dfrac{\partial V(z+\delta z,t)}{\partial z} = L\dfrac{\partial I(z,t)}{\partial t} \\ -\dfrac{\partial I(z+\delta z,t)}{\partial z} = C\dfrac{\partial V(z+\delta z,t)}{\partial t} \end{cases}$$

So-called telegrapher equations

$$\begin{cases} -\dfrac{\partial V(z,t)}{\partial z} = L\dfrac{\partial I(z,t)}{\partial t} \\ -\dfrac{\partial I(z,t)}{\partial z} = C\dfrac{\partial V(z,t)}{\partial t} \end{cases}$$

Primer to TWPA: telegrapher equations



$$\begin{cases} -\dfrac{\partial V(z,t)}{\partial z} = L\dfrac{\partial I(z,t)}{\partial t} \\ -\dfrac{\partial I(z,t)}{\partial z} = C\dfrac{\partial V(z,t)}{\partial t} \end{cases}$$

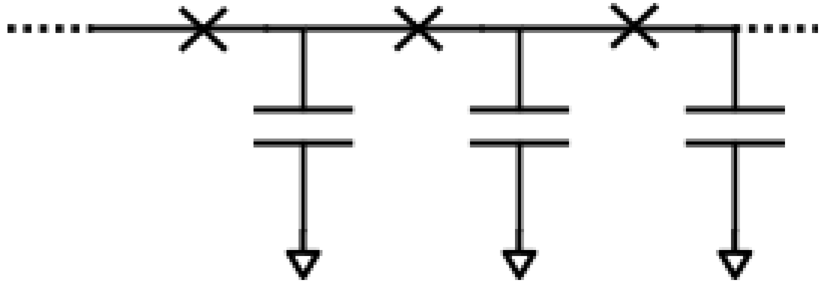
$$\begin{cases} \dfrac{\partial^2 V(z,t)}{\partial z^2} - LC\dfrac{\partial^2 V(z,t)}{\partial t^2} = 0 \\ \dfrac{\partial^2 I(z,t)}{\partial z^2} - LC\dfrac{\partial^2 I(z,t)}{\partial t^2} = 0 \end{cases}$$

$$\begin{cases} V(z,t) = V^+ e^{i[\omega t - kz]} + V^- e^{-i[\omega t - kz]} \\ I(z,t) = \dfrac{V^+}{Z_0} e^{i[\omega t - kz]} - \dfrac{V^-}{Z_0} e^{-i[\omega t - kz]} \end{cases}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$v_p = \frac{\omega}{k} = \frac{1}{\sqrt{LC}}$$

Primer to TWPA: telegrapher equations



$$L(I) = L_0[1 + \epsilon I + \xi I^2]$$

Singe JJ at zero current $\epsilon = 0$

$$\begin{cases} -\dfrac{\partial V}{\partial z} = L(I)\dfrac{\partial I}{\partial t} \\ -\dfrac{\partial I}{\partial z} = C\dfrac{\partial V}{\partial t} \end{cases}$$

$$I = I_p(z)e^{i[k_p z - \omega_p t]} + I_s(z)e^{i[k_s z - \omega_s t]} + I_i(z)e^{i[k_i z - \omega_i t]} + c.c.$$

$$2\omega_p = \omega_s + \omega_i$$

$$|I_p| \gg |I_s|, |I_i|$$

$$I_p(z) = I_p(0)e^{ik_p \chi z}$$

$$\chi \approx \frac{I_p}{I_c}$$

$$\begin{cases} \dfrac{\partial I_s(z)}{\partial z} = i\chi \mathrm{k}_s I_i^*(z)e^{i\Delta\beta z} \\ \dfrac{\partial I_i(z)}{\partial z} = i\chi k_i I_s^*(z)e^{i\Delta\beta z} \end{cases}$$

$$\Delta\beta \approx \Delta k - 2\chi k_p$$

$$\Delta k = 2k_p - k_s - k_i$$

Primer to TWPA: telegrapher equations

$$\begin{cases} \dfrac{\partial I_s(z)}{\partial z} = i\chi \mathrm{k}_s I_i^*(z) e^{i\Delta\beta z} \\ \dfrac{\partial I_i(z)}{\partial z} = i\chi k_i I_s^*(z) e^{i\Delta\beta z} \end{cases}$$

$$\Delta\beta \approx \Delta k - 2\chi k_p$$

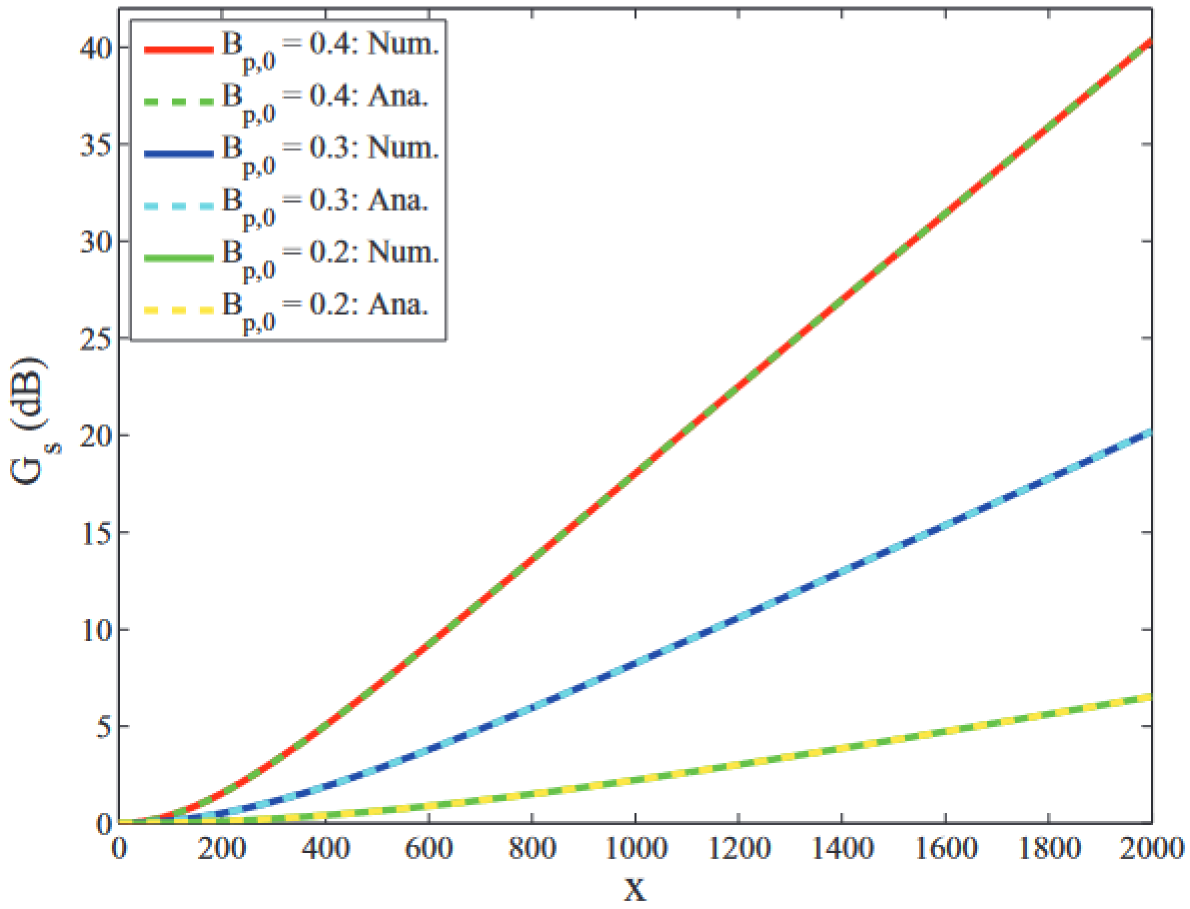
$$\Delta k = 2k_p - k_s - k_i$$

<u>If $\Delta\beta \approx 0$ (phase matching condition)</u>

$$\begin{cases} I_s(z) = I_s(0)\cosh(gz) + I_i(0)\sinh(gz) \\ I_i(t) = I_i(0)\cosh(gz) + I_s(0)\sinh(gz) \end{cases}$$

With g $\approx \chi\sqrt{k_s k_i}$ $\qquad \chi \approx \dfrac{I_p}{I_c}$

Exponential growth with <u>position</u>

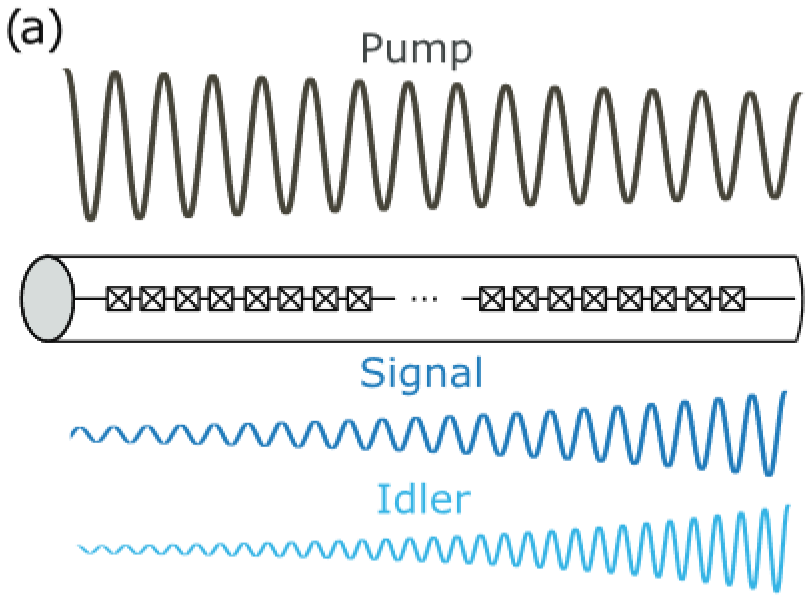


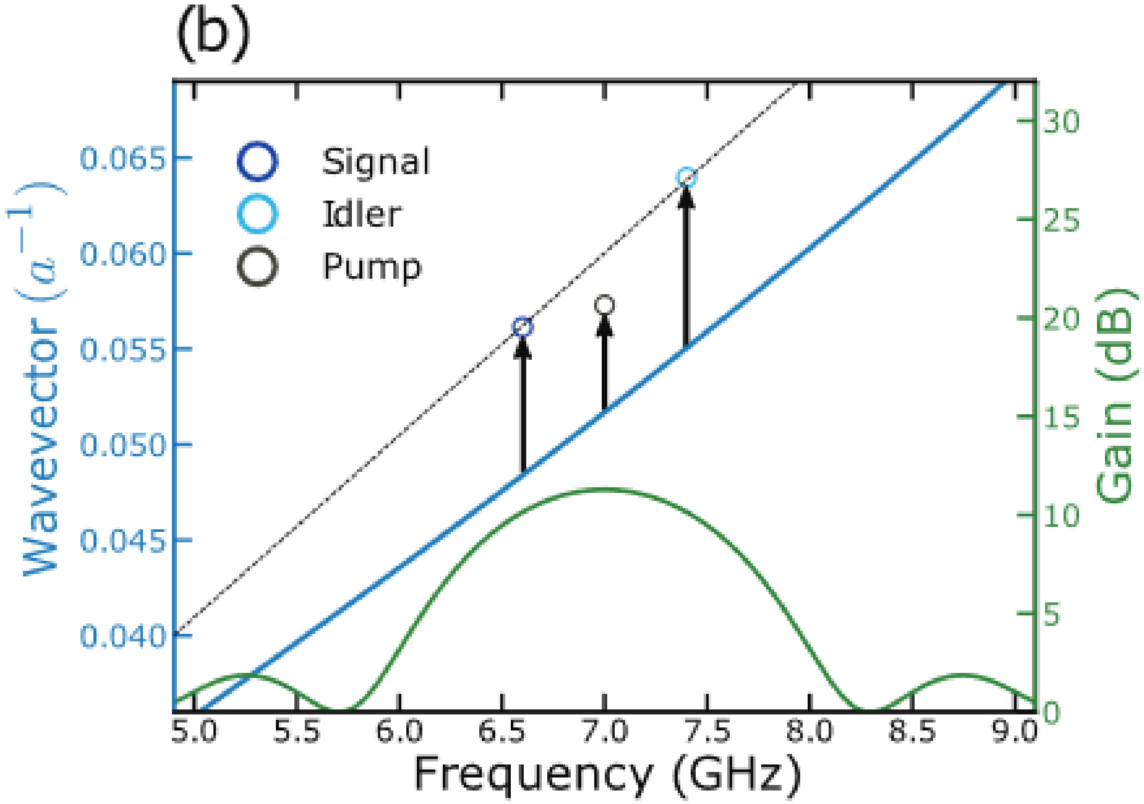
*Yaakobi, **PRB** 87 (2013)*

Primer to TWPA: phase matching and dispersion engineering

$$\Delta\beta \approx \Delta k - 2\chi k_p$$

$$\Delta k = 2k_p - k_s - k_i$$

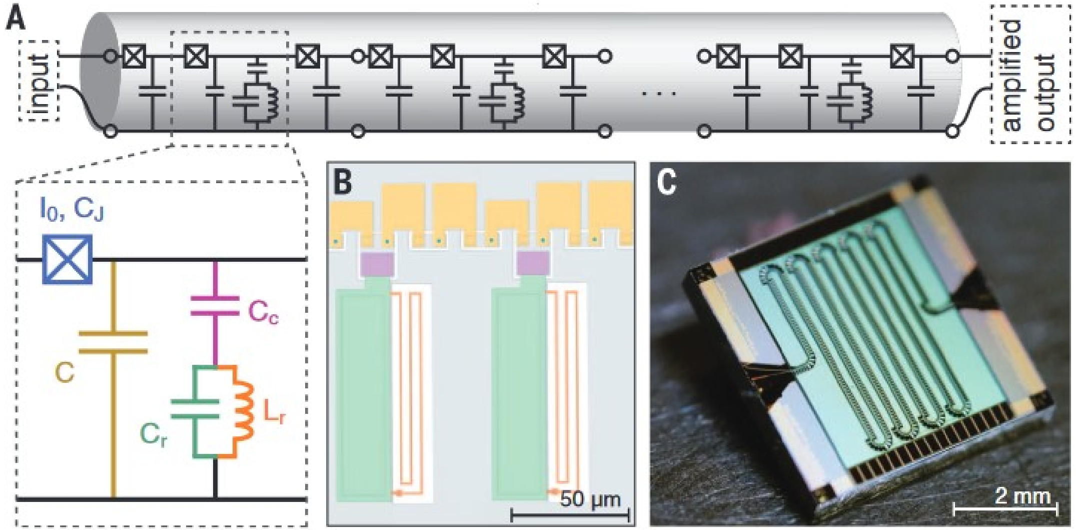




Esposito, ***Applied. Phys Lett.*** *119 (2021)*

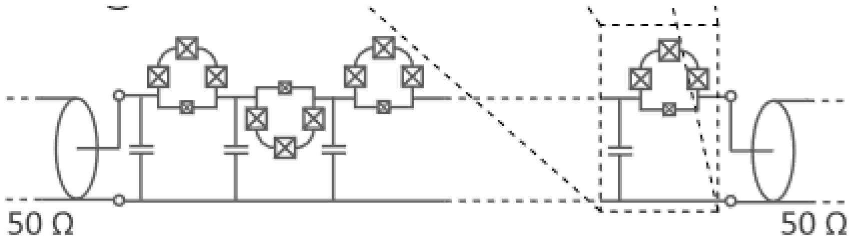
Dispersion engineering for 4WM

Resonant phase matching



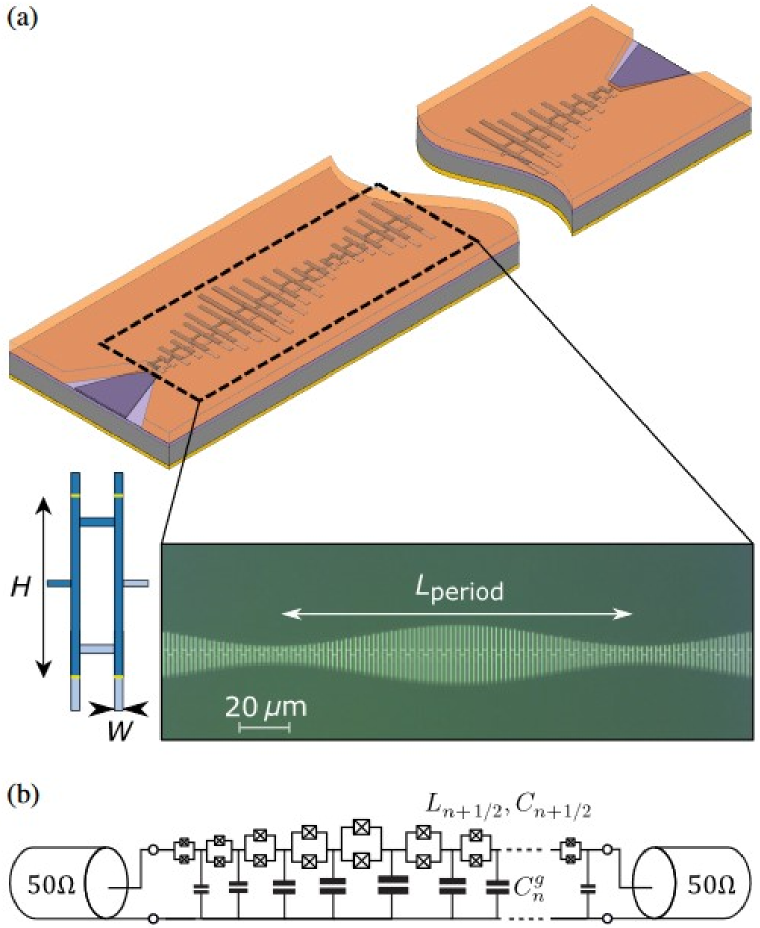
Macklin, ***Science*** 350 *(2015)*

Reverse Kerr (SNAILs)



Ranadive, ***Nat. Com.*** *13 (2021)*

Periodic loading

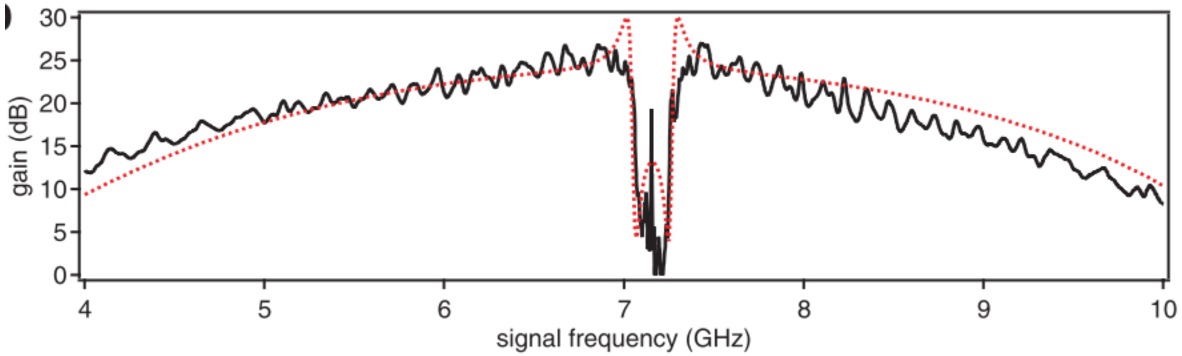


Planat, ***PRX*** *10 (2020)*

Also: *Esposito*, ***Applied. Phys Lett.*** *119 (2021)* and *Kow*, ***arXiv*** *2201.04660 (2022)*

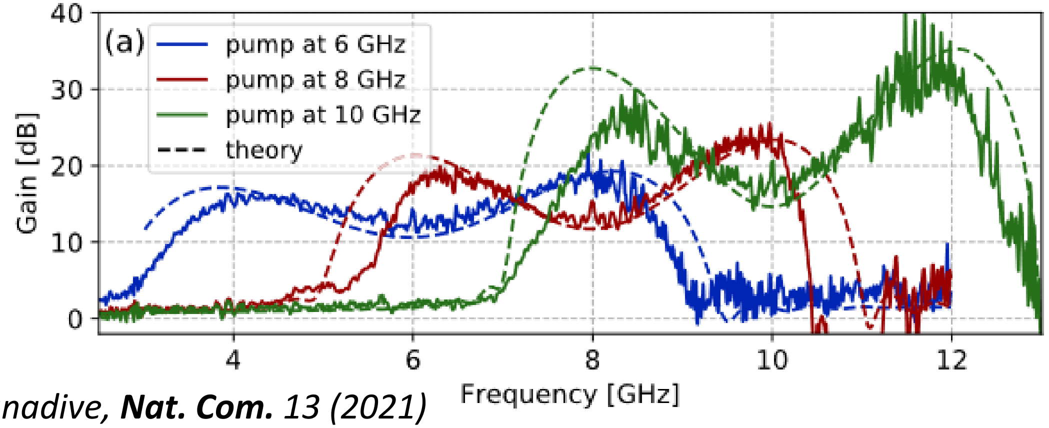
Dispersion engineering for 4WM

Resonant phase matching



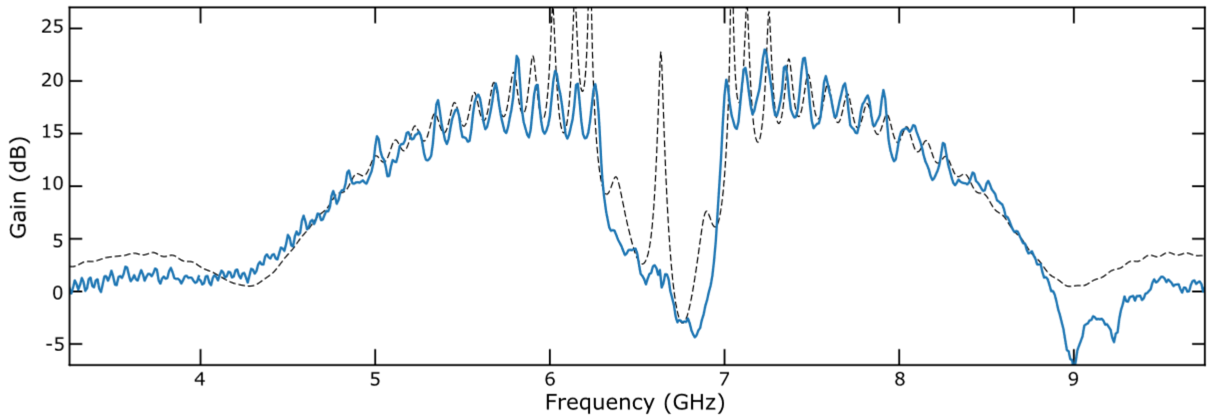
*Macklin, **Science** 350 (2015)*

Reverse Kerr (SNAILs)



*Ranadive, **Nat. Com.** 13 (2021)*

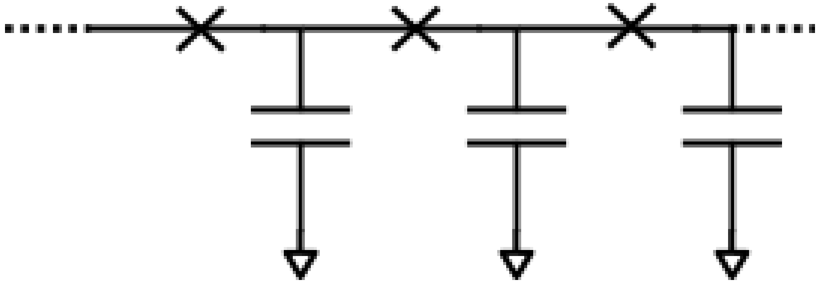
Periodic loading



*Planat, **PRX** 10 (2020)*

Also: *Esposito, **Applied. Phys Lett.** 119 (2021)* and *Kow, **arXiv** 2201.04660 (2022)*

Dispersion engineering for 3WM



$$\mathrm{L(I)} = \frac{\varphi_0}{I_c\sqrt{1-{I^2}/{I_c^2}}} = L_0[1+\epsilon I + \xi I^2 + \cdots]$$

Perfect 3WM for $\xi = 0$ (somewhat achieved for dc-biased JJ)

- No dispersion of the pump with its amplitude
- Everything would be phase match in linear transmission line
- Bad because other processes would arise (pump/signal harmonics)
- In practice, exploit parasitic and intentional dispersion engineering

*Esposito, **Applied. Phys Lett.** 119 (2021), Malnou, **PRXQ** 2 (2021) and Peng, **PRXQ** 3 (2023)*

Lingo

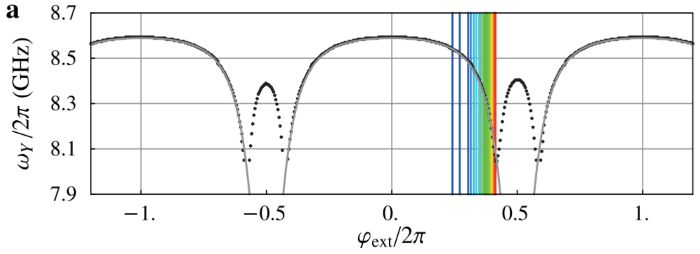
- **Four-wave mixing**: $2\omega_p = \omega_s + \omega_i$ (for example in non dc biased JJs or SQUID)
- **Three-wave mixing**: $\omega_p = \omega_s + \omega_i$ (for example in dc biased JJs or SQUID, JPC, SNAILs)
- **Non-degenerate amplifier**: signal and idler live in separate resonators
- **degenerate amplifier**: signal and idler live in the same resonator
- **Singly-degenerate vs doubly- degenerate amplifier**: degenerate amplifier using 3WM or 4WM
- **Phase-preserving amplifier**: correlation between signal and idler are not used, amplifiy both quadratures, adds noise
- **Phase-sensitive amplifier**: correlation between signal and idler are used, single quadrature amplifiers, noiseless.

Outline

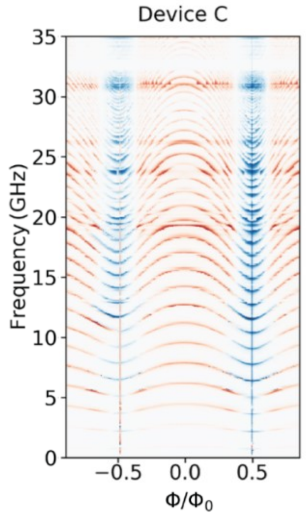
- Dispersive Readout of superconducting qubits
- Intro to parametric amplifiers:
  - Resonant parametric amplifiers
  - Traveling-waves parametric amplifiers
- Future directions

Parametric amplifier requirements

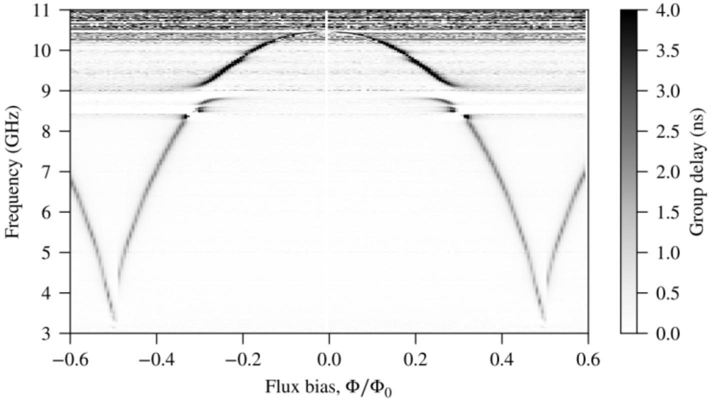
- Tunability and bandwidth
- Power handling
- High enough gain
- Low system added noise
- Directionality



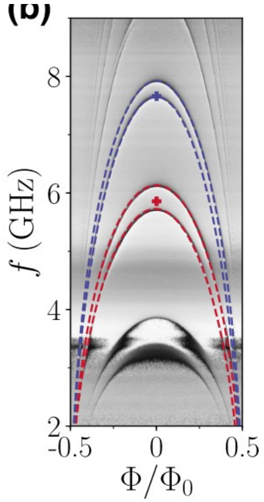
*N. Roch, **PRL** (2012)*



*V. V. Sivak, **PRapplied** (2020)*



*G. Peterson, **Thesis** (2020)*



*P. Winkel, **PRapplied** (2020)*

Parametric amplifier requirements

- Tunability and bandwidth
- Power handling
- High enough gain
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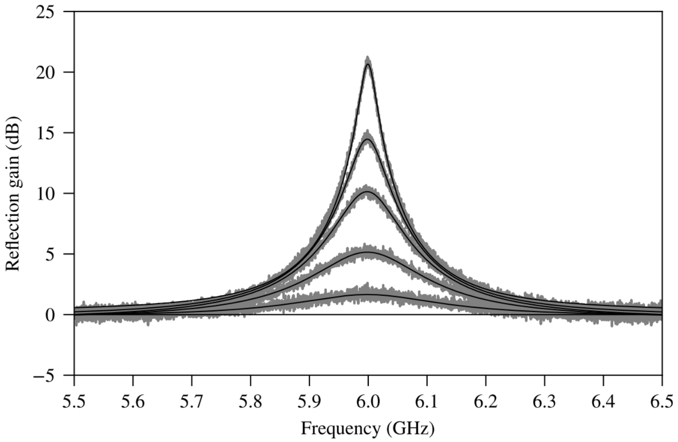
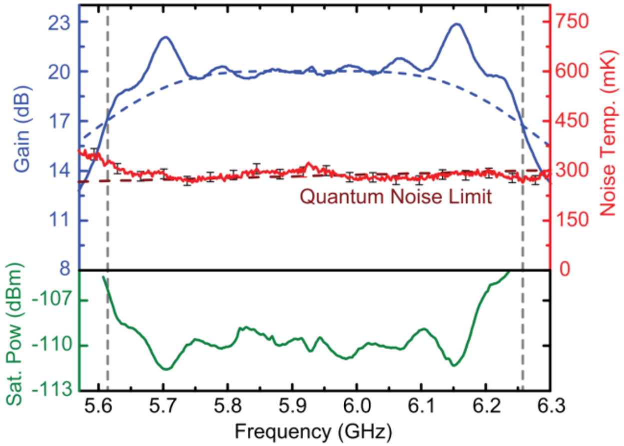
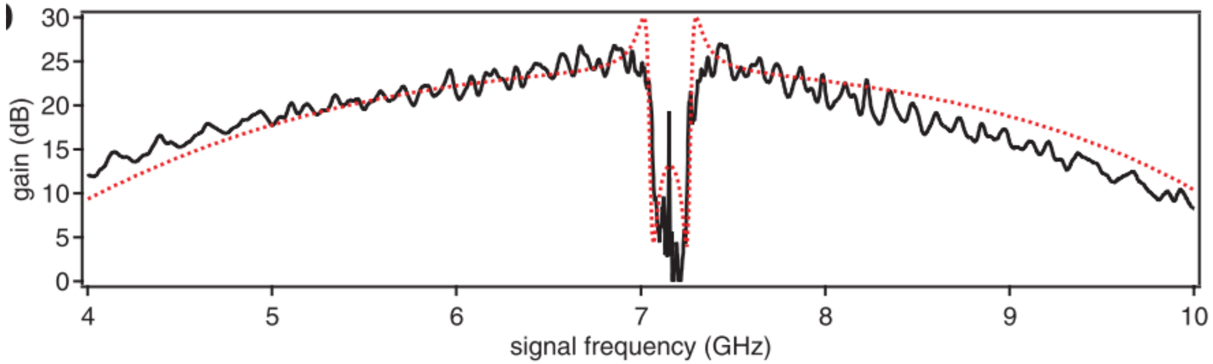
- Directionality



*G. Peterson, **Thesis** (2020)*



*T. Roy, **APL** (2015)*

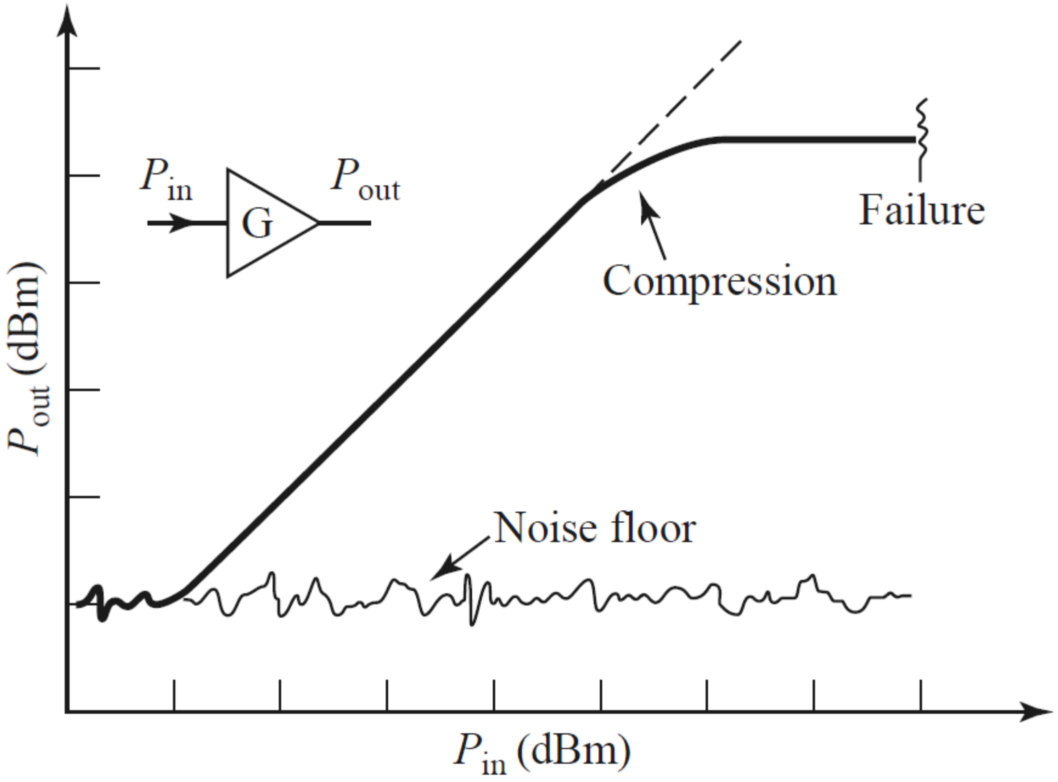


*C. Macklin, **Science** (2015)*

*Also: Naaman & Aumentado, **PRXQ** 3 (2022), R. Kaufman **arXiv** 2305.17816 (2023)*

Parametric amplifier requirements

- Tunability and bandwidth
- Power handling
- High enough gain
- Low system added noise
- Directionality



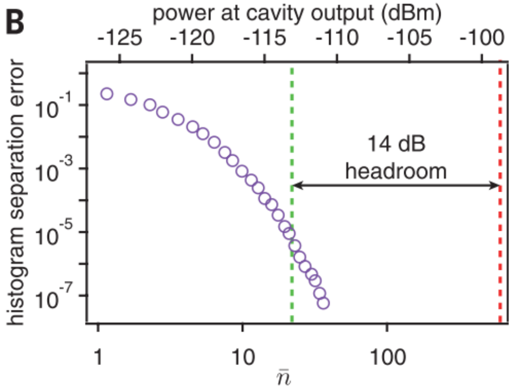
1dB Compression point: Power when G is reduced by 1dB (20%)

$$P_{readout} \approx -120dBm \approx P_{1dB}^{JPA,20dB\ gain}$$

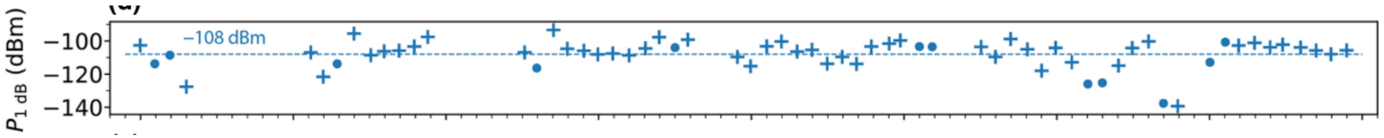
Parametric amplifier requirements

- Tunability and bandwidth
- **Power handling**
- High enough gain
- Low system added noise
- Directionality

tailoring nonlinearity to increase power handling



*C. Macklin, **Science** (2015)*



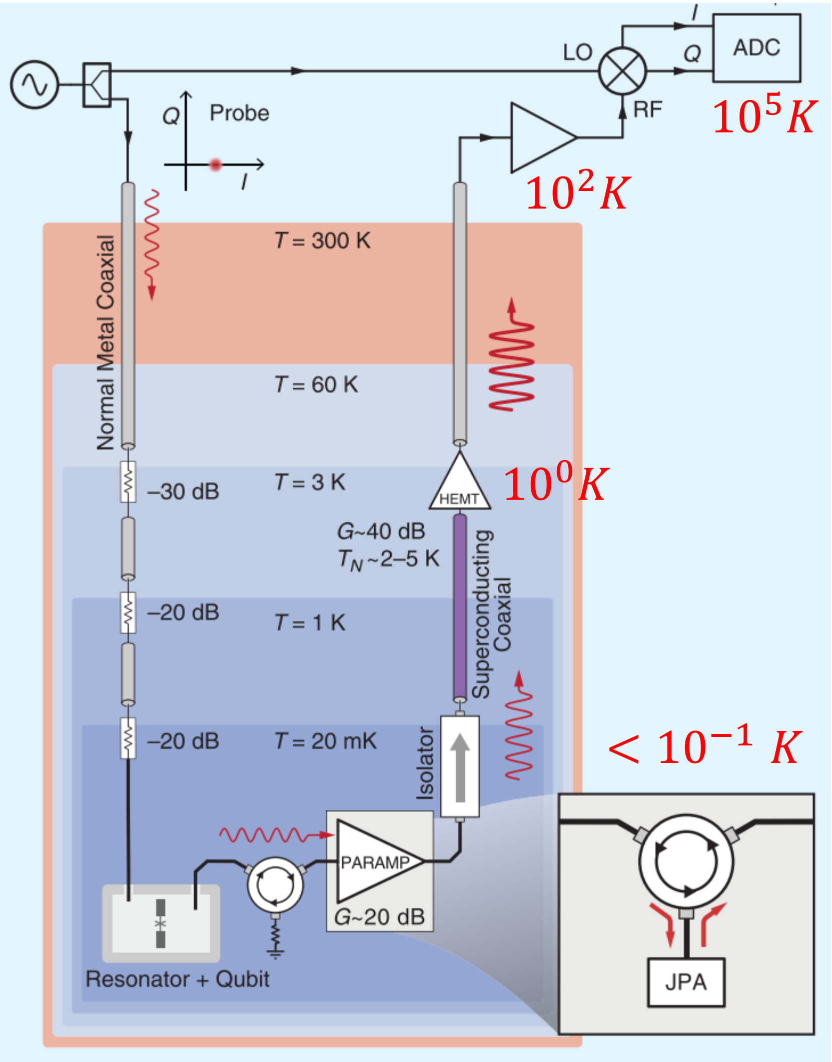
*V. V. Sivak, **PRapplied** (2020)*

Goal: $P_{1dB}\sim-100dBm\ to\ -90dBm$ for readout 10 to 100 qubits

*R Kaufman, … , M Hatridge, **In Preparation** (2023)*

Parametric amplifier requirements

- Tunability and bandwidth
- Power handling
- High enough gain
- Low system added noise
- Directionality

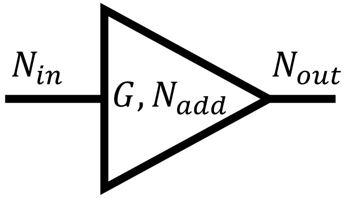


*J. Aumentado, **IEEE MW magazine** 21 (2020)*

Definitions and formulas

linear measurements, classical power spectral densities (therefore account for vacuum noise)

Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

$$N = \frac{1}{2}\langle a^\dagger a + a a^\dagger \rangle = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{1}{2}$$

$$\boldsymbol{\eta = \frac{1}{1 + 2N_{add}}}$$

Units and conversions:

N in $quanta/s/Hz \sim quanta$

$PSD = \hbar\omega N$ in W/Hz

And $10 * \log_{10}(PSD \times 10^3)$ in dBm/Hz

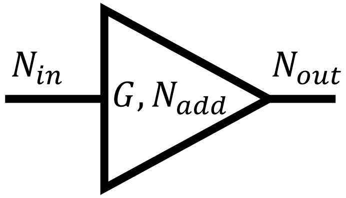
Typical values @ 6GHz:

- Vacuum noise PSD $\sim -207\ dBm/Hz$
- Room temp noise PSD $\sim -174\ dBm/Hz$
- Typical Signal Analyzer / Digitizer PSD $\sim -147\ dBm/Hz$

Noise of a parametric amplifier is set by the idler

linear measurements, classical power spectral densities (therefore account for vacuum noise)

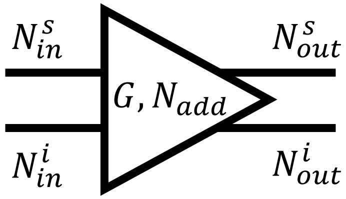
Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

$$N = \frac{1}{2}\langle a^\dagger a + a a^\dagger \rangle = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{1}{2}$$

$$\begin{pmatrix} a_{out} \\ b_{out}^\dagger \end{pmatrix} = \begin{pmatrix} \sqrt{G} & \sqrt{G-1} \\ \sqrt{G-1} & \sqrt{G} \end{pmatrix} \begin{pmatrix} a_{in} \\ b_{in}^\dagger \end{pmatrix}$$



$$\boldsymbol{\eta = \frac{1}{1 + 2N_{add}}}$$

$$N_{out}^s = G N_{in}^s + (G-1) N_{in}^i$$

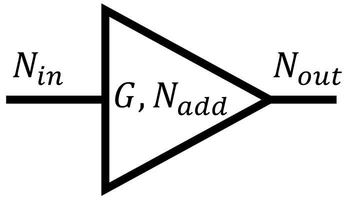
$$N_{out}^s = G\left(N_{in}^s + \frac{G-1}{G} N_{in}^i\right) \approx G\left(N_{in}^s + N_{in}^i\right) \quad \boldsymbol{\Rightarrow N_{add} \geq \frac{1}{2}}$$

Standard quantum limit

Noise of a parametric amplifier is set by the idler

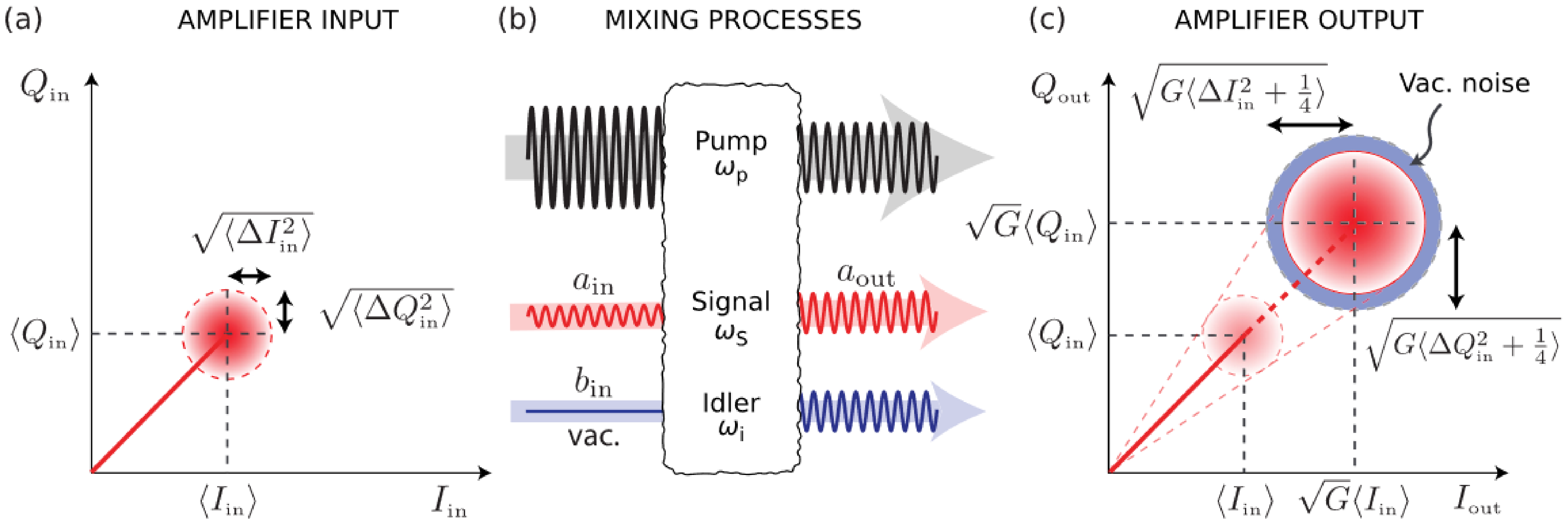
linear measurements, classical power spectral densities (therefore account for vacuum noise)

Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

$$N = \frac{1}{2}\langle a^\dagger a + a a^\dagger \rangle = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{1}{2}$$

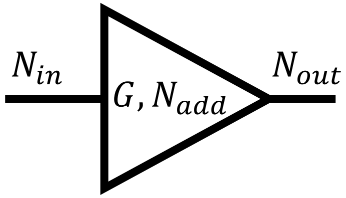


P. Krantz, … , W. D. Oliver, ***App. Phys. Rev.*** *6 (2019)*

Phase sensitive amplifier can be noiseless

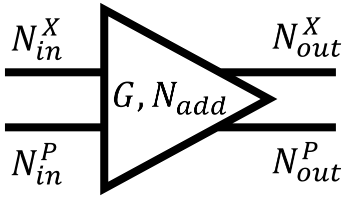
linear measurements, classical power spectral densities (therefore account for vacuum noise)

Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

$$\begin{pmatrix} X_{out} \\ P_{out} \end{pmatrix} = \begin{pmatrix} \sqrt{G} & 0 \\ 0 & \frac{1}{\sqrt{G}} \end{pmatrix} \begin{pmatrix} X_{in} \\ P_{in} \end{pmatrix}$$



$$N_{out}^X = G N_{in}^X \quad \Longrightarrow N_{add} \geq 0$$

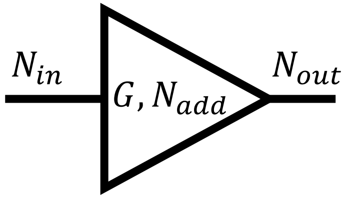
$$N_{out}^P = N_{in}^P / G$$

Phase sensitive amplification can be noiseless (unitary, reversible)

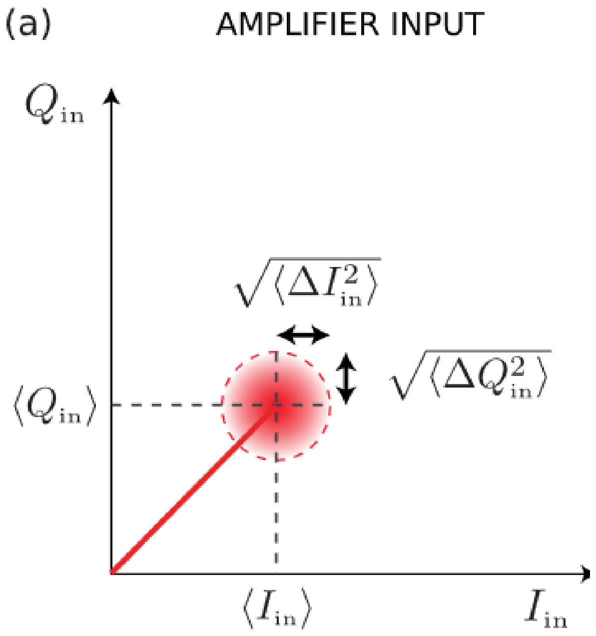
Phase sensitive amplifier can be noiseless

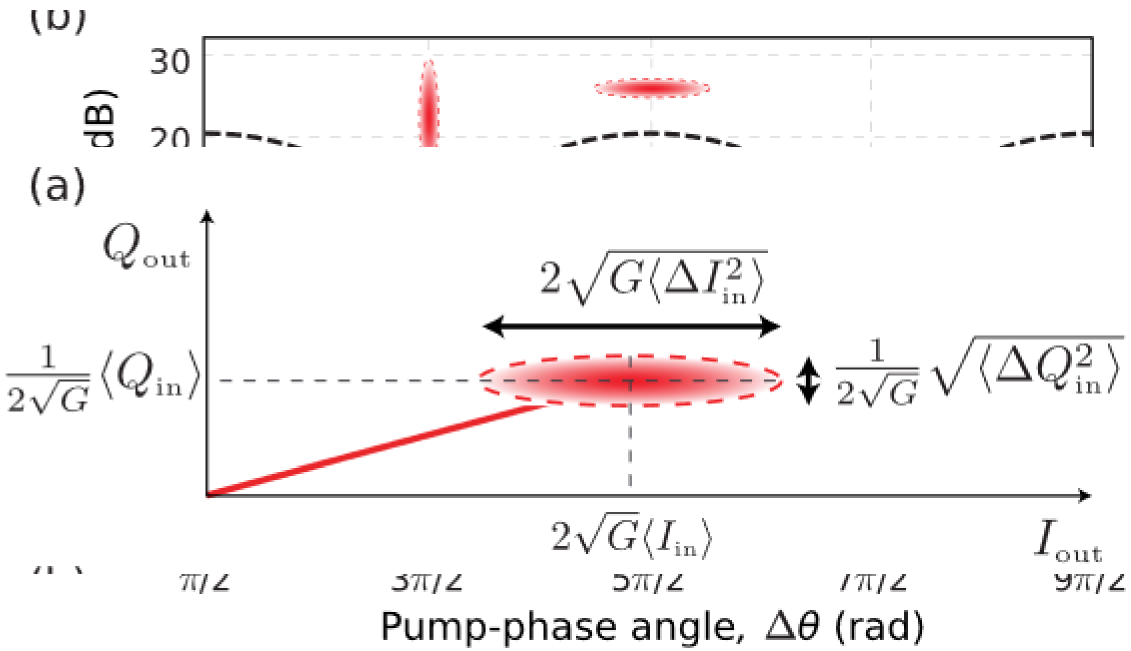
I will consider that we are performing linear measurements, therefore account for vacuum noise

<u>Noise of an amplifier:</u>



$$N_{out} = G(N_{in} + N_{add})$$



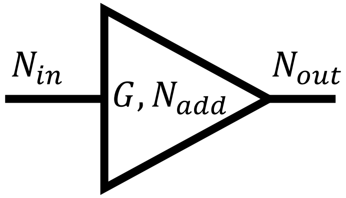


P. Krantz, … , W. D. Oliver, ***App. Phys. Rev.*** *6 (2019)*

Definitions and formulas

linear measurements, classical power spectral densities (therefore account for vacuum noise)

Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

Noise of an attenuator:

$$N_{out} = \eta N_{in} + (1-\eta)N_{attn} = \eta(N_{in} + N_{add}) \text{ with } \begin{cases} N_{add} = \frac{1-\eta}{\eta} N_{attn} \\ N_{attn} = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \frac{1}{2} \end{cases}$$

Units and conversions:

N in $quanta/s/Hz \sim quanta$

$PSD = \hbar\omega N$ in W/Hz

And $10 * \log_{10}(PSD \times 10^3)$ in dBm/Hz

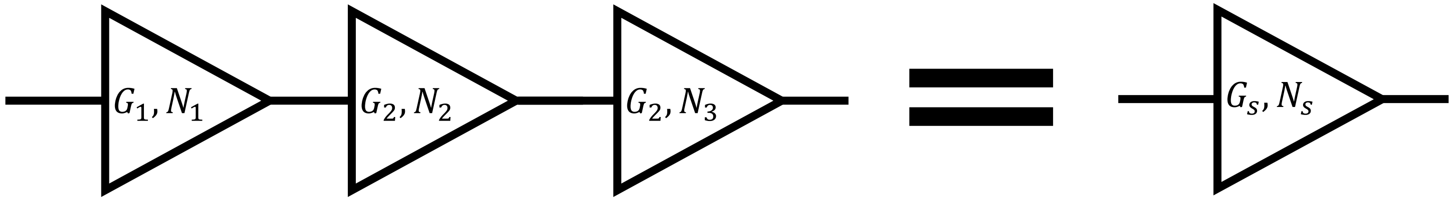
Typical values @ 6GHz:

$$\frac{\hbar\omega}{k_B T} \underset{1\,K}{\xRightarrow[20\,GHz]{}} 1$$

- Vacuum noise PSD $\sim -207\ dBm/Hz$
- Room temp noise PSD $\sim -174\ dBm/Hz$
- Typical Signal Analyzer / Digitizer PSD $\sim -147\ dBm/Hz$

Friis formula

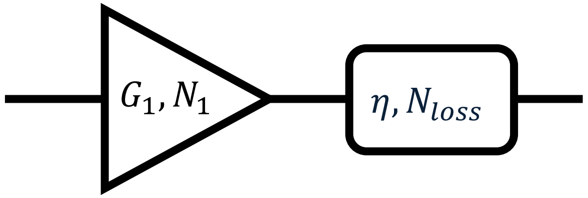
Friis formula:



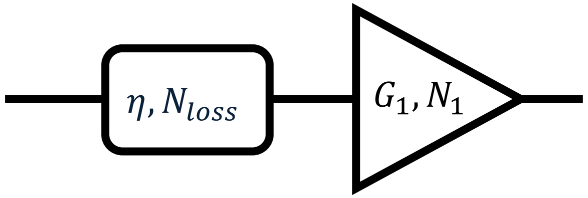
$$G_s = G_1 G_2 G_3 \ldots$$

$$N_s = N_1 + \frac{N_2}{G_1} + \frac{N_3}{G_1 G_2} + \cdots$$

Consequence 1: loss before amplification is bad, but loss after amplification can be ok

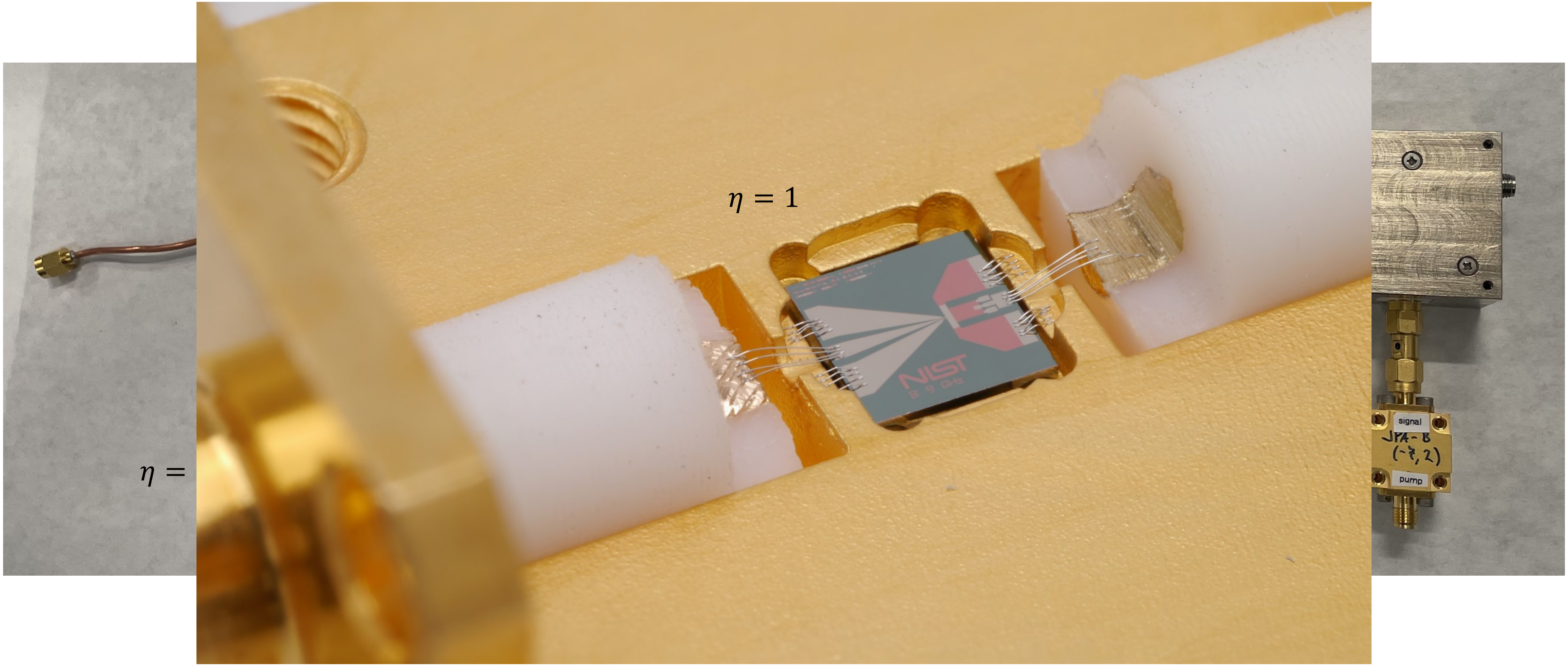


$$\begin{cases} G_1 = 10^4 = 40dB \\ N_1 = 10 \\ \eta = 0.5, N_{loss} = 0.5 \end{cases} \Longrightarrow N_s = N_1 + \frac{1-\eta}{\eta G_1} N_{loss} = 10.00005$$



$$\begin{cases} G_1 = 10^4 = 40dB \\ N_1 = 10 \\ \eta = 0.5, N_{loss} = 0.5 \end{cases} \Longrightarrow N_s = \frac{1-\eta}{\eta} N_{loss} + \frac{N_1}{\eta} = 20.5$$

What is the amplifier?



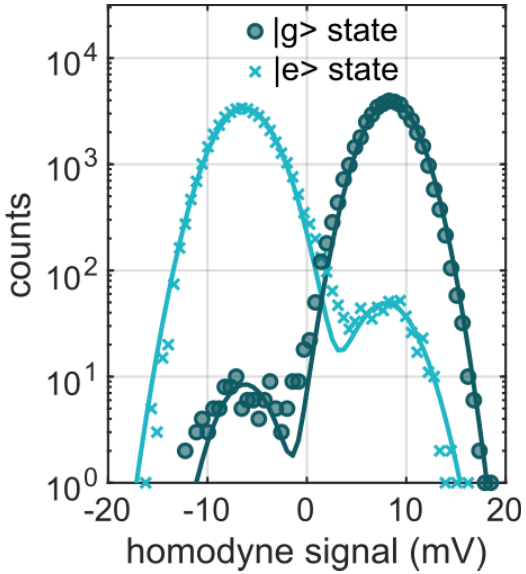
$\eta = 1$

$\eta =$

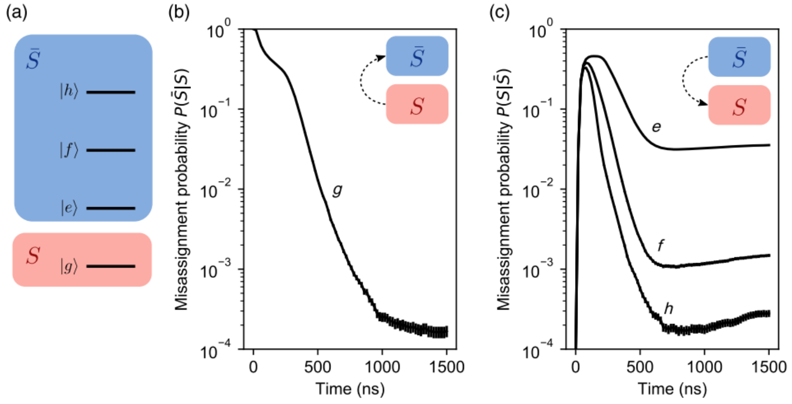
The truth about amplifiers

- And that’s ok for typical qubit projective readout:

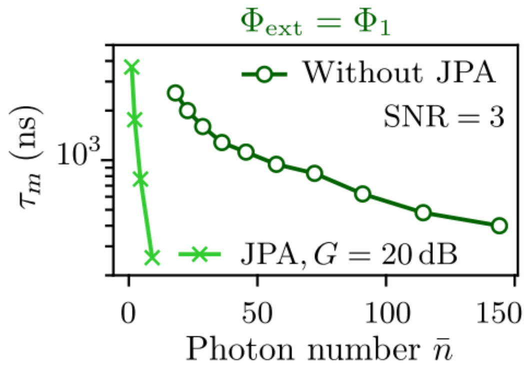
$$F = erf(\sqrt{\eta\tau\Gamma_m/2})$$



High fidelity can be achieved without perfect efficiency



*S. S. Elder, **PRX** (2020)*



*D. Gusenkova, **PRapplied** (2021)*

J. Aumentado

*Ancilla based readout: Z. K. Minev, **Nature** (2019) and R. Dassonneville, **PRX** (2020)*

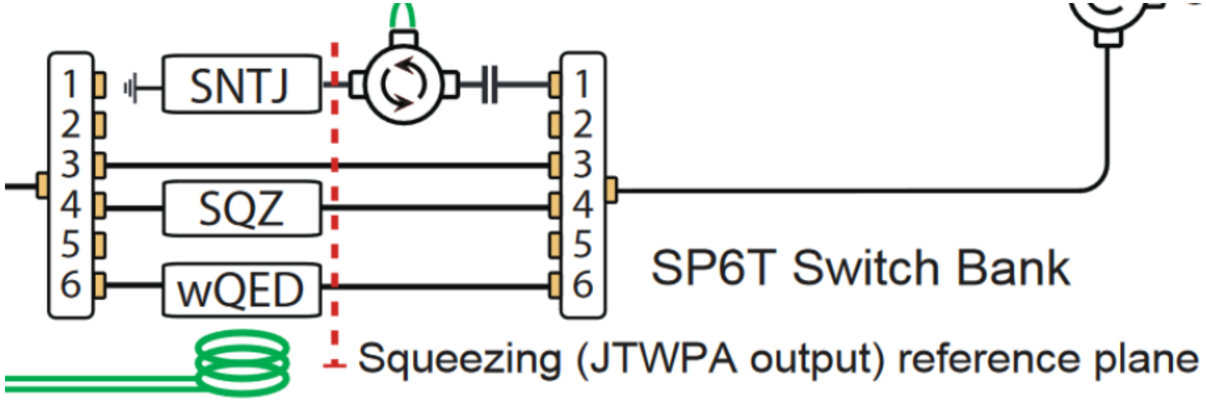
The truth about amplifiers

- But critical for specific experiments
  - New “quantum-limited” amplifier
  - Vacuum squeezing
  - Analog quantum feedback
  - Quantum sensing

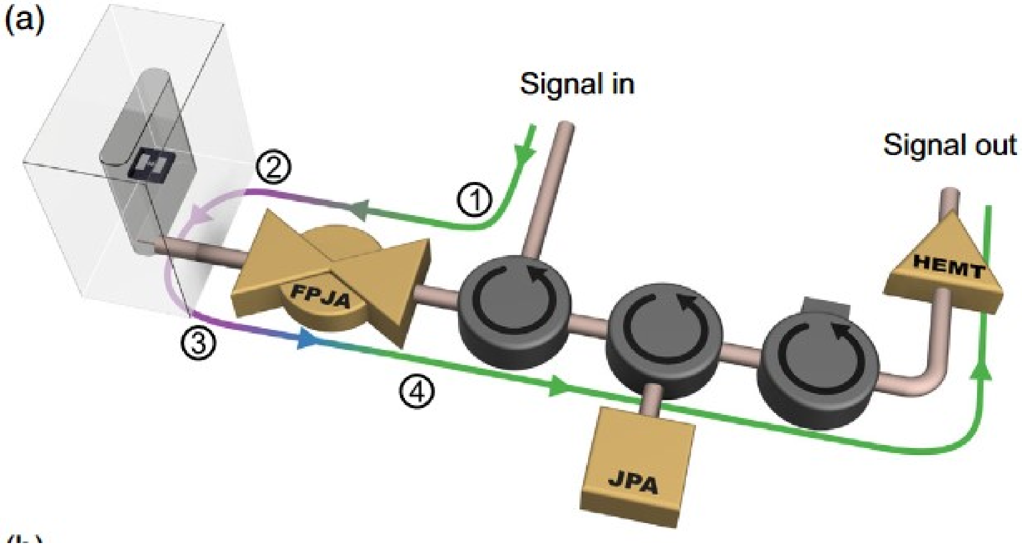
J. Aumentado

How to characterize system noise

- The right way to do it: use a calibrated noise or signal source



J. Qiu, ***Nature Physics*** *19 (2023)*



F. Lecocq, ***PRL*** *126 (2021)*

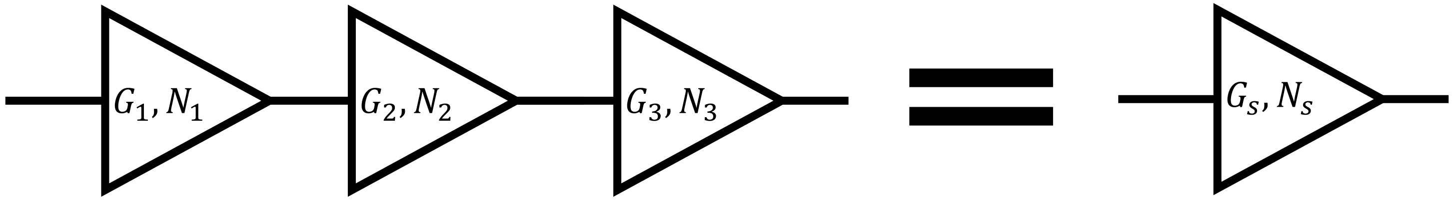
Many pitfalls, look out for a review on that coming up

or reach out to the noise police (J. Aumentado, M. Malnou, or myself)

How to characterize system noise

- The sanity check: check roughly your “noise rise”

Friis formula:



$$G_s = G_1 G_2 G_3 \ldots$$

$$N_s = N_1 + \frac{N_2}{G_1} + \frac{N_3}{G_1 G_2} + \cdots$$

$$N_{tot} = G_3(G_2(G_1(N_{in} + N_1) + N_2) + N_3)$$

- Monitor power spectral density on a spectrum analyzer (or measurement histogram standard deviation)
- Sequentially turn on amplifiers, starting from the closest to your spectrum analyzer or digitizer, ending with “coldest” amplifier (HEMT or parametric amplifier). Measure how much the noise rised.
- Compare measurement with spec sheets and best estimate of loss

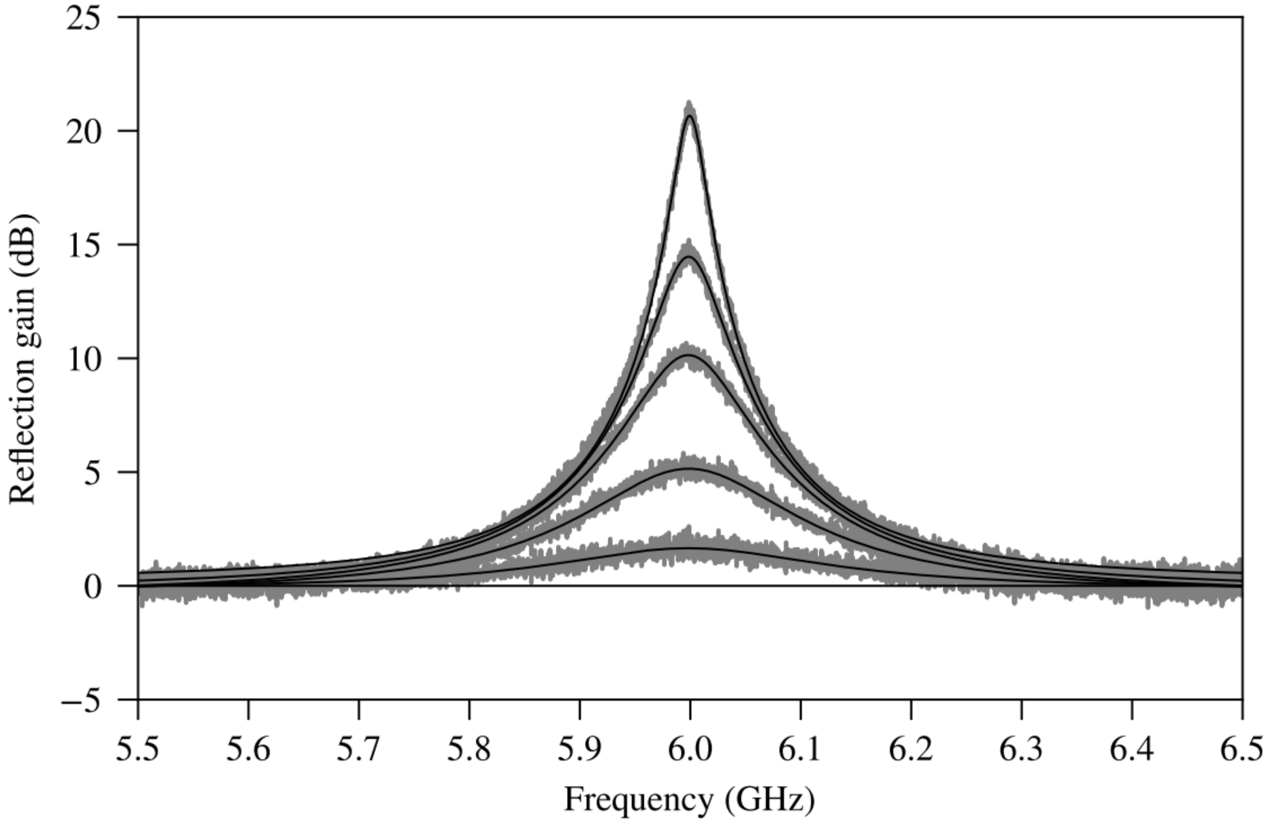
Red flag: I turned on my parametric amplifier, see gain, but do not see a noise rise

See questions at the end

Parametric amplifier requirements

- Tunability and bandwidth
- Power handling
- High enough gain
- Low system added noise
- Directionality

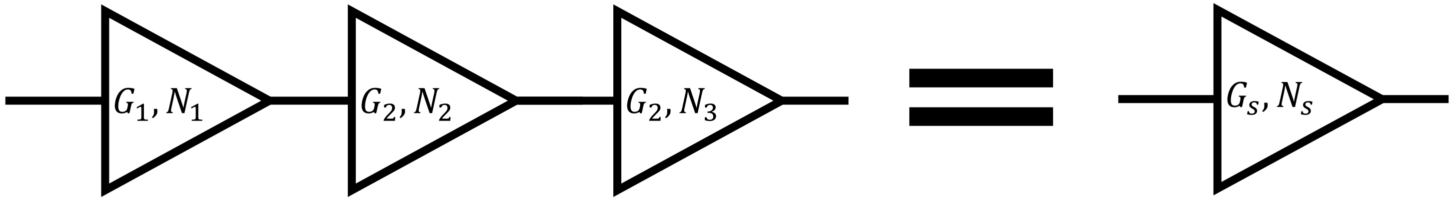
Increasing gain typically reduce bandwidth and power handling



*G. Peterson, **Thesis** (2020)*

How much gain is enough gain?

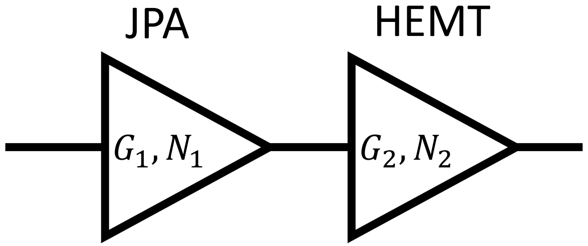
Friis formula:



$$G_s = G_1 G_2 G_3 \ldots$$

$$N_s = N_1 + \frac{N_2}{G_1} + \frac{N_3}{G_1 G_2} + \cdots$$

Consequence 2: diminishing returns after overwhelming the noise of the following amplifier



$N_1 = \frac{1}{2}$ for $G_1 \gg 1$

$N_2 = 20$

$$N_s = N_1 + \frac{N_2}{G_1}$$

$$G_1 = 0dB \Longrightarrow N_s = 0 + \frac{20}{1} = 20$$

$$G_1 = 10dB \Longrightarrow N_s = \frac{1}{2} + \frac{20}{10} = 2.5$$

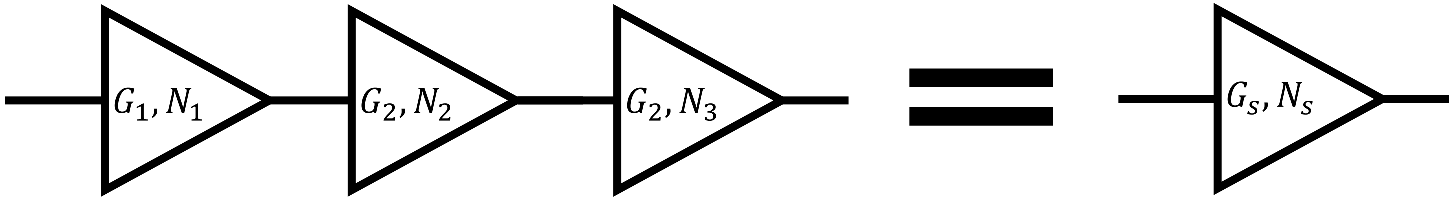
$$G_1 = 20dB \Longrightarrow N_s = \frac{1}{2} + \frac{20}{100} = 0.7$$

$$G_1 = 30dB \Longrightarrow N_s = \frac{1}{2} + \frac{20}{1000} = 0.52$$

$$G_2 = 40dB \Longrightarrow N_s = \frac{1}{2} + \frac{20}{10000} = 0.502$$

How much gain is enough gain?

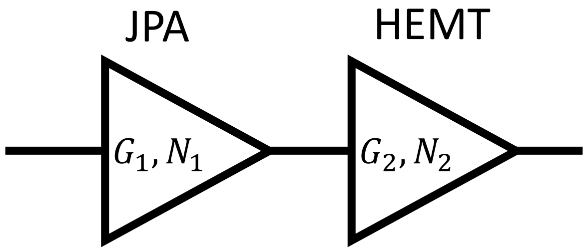
Friis formula:



$$G_s = G_1 G_2 G_3 \ldots$$

$$N_s = N_1 + \frac{N_2}{G_1} + \frac{N_3}{G_1 G_2} + \cdots$$

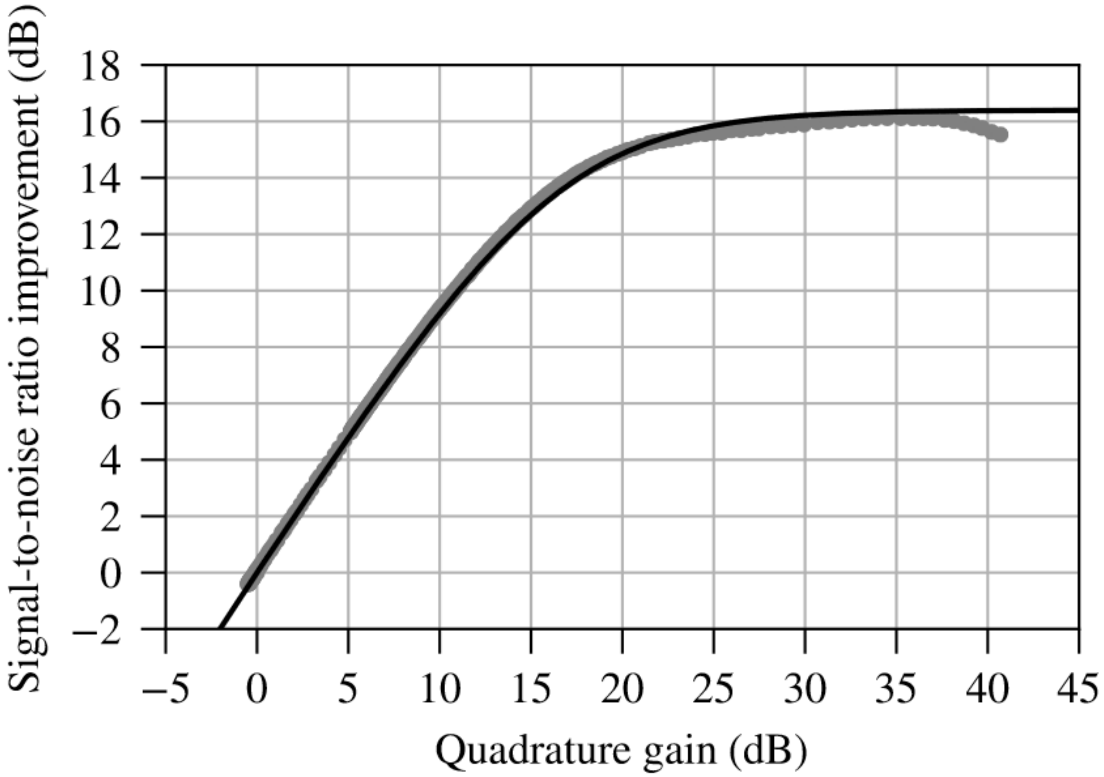
Consequence 2: diminishing returns after overwhelming the noise of the following amplifier



$N_1 = \frac{1}{2}$ for $G_1 \gg 1$

$N_2 = 20$

$$N_s = N_1 + \frac{N_2}{G_1}$$

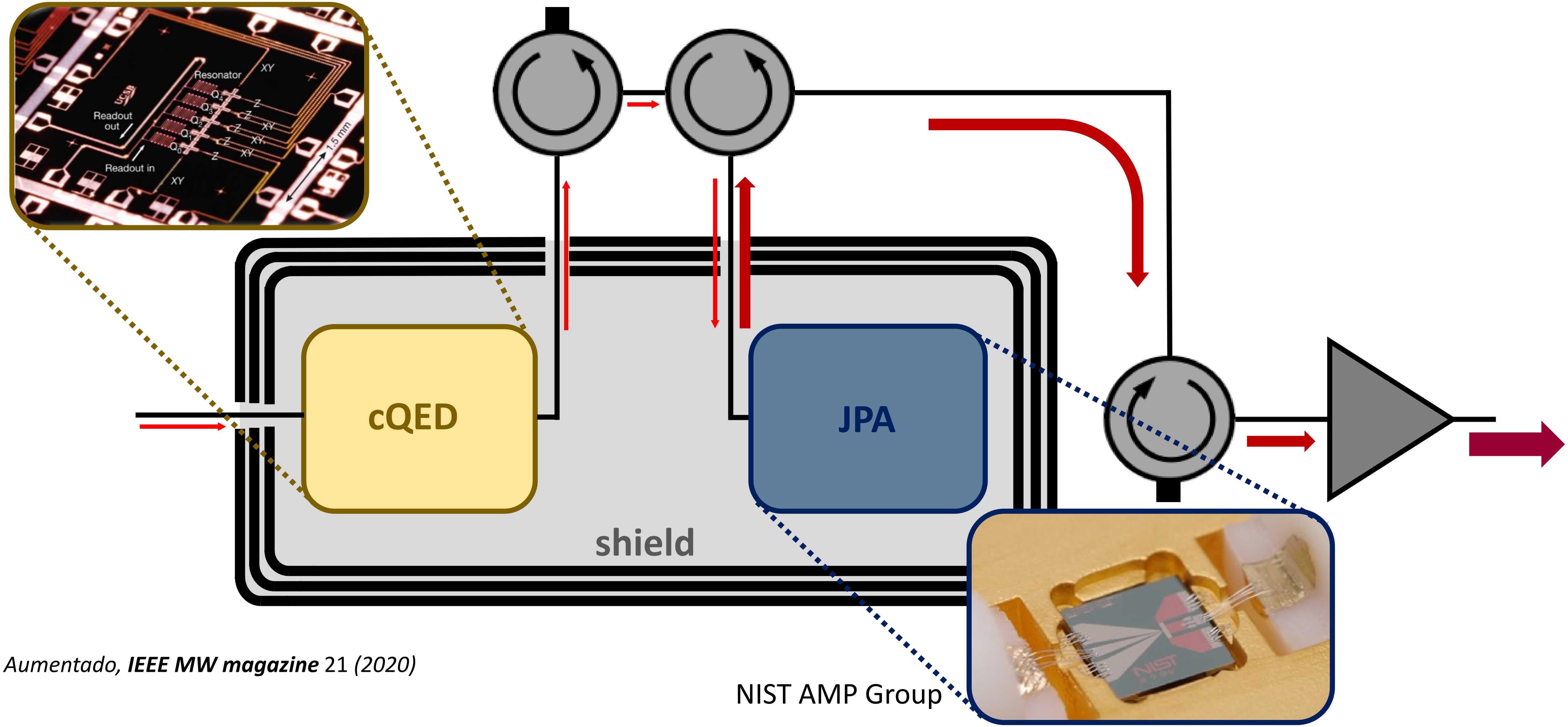


Parametric amplifier requirements

- Tunability and bandwidth
- Power handling
- High enough gain
- Low system added noise
- Directionality

Typical dispersive qubit readout

Paik et al. *PRL*, **107** (2011)

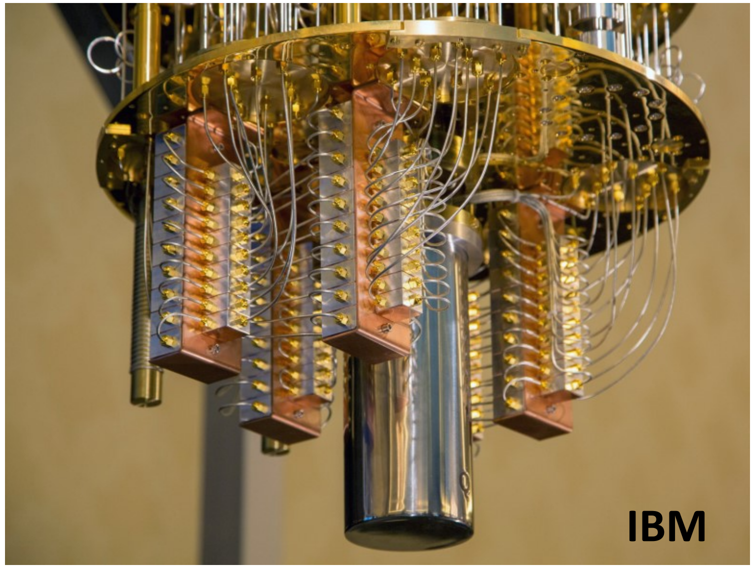


NIST AMP Group

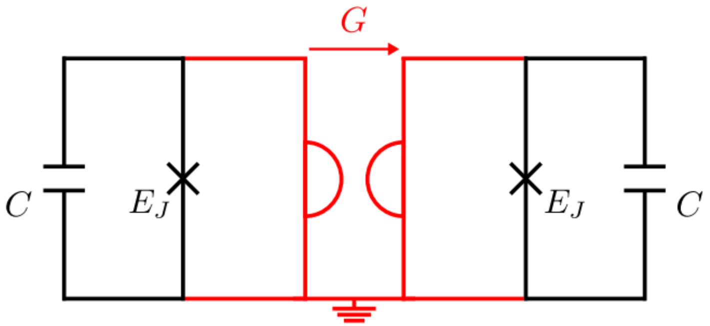
*J. Aumentado, **IEEE MW magazine** 21 (2020)*

The circulator problem?

Large size take up real-estate



Magnetic fields prevent integration

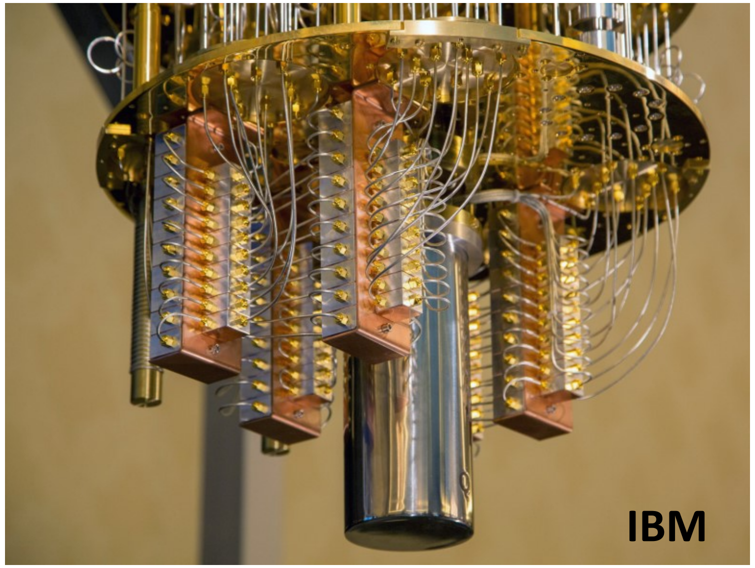


Rymarz,..., DiVincenzo, **PRX 11** (2021)

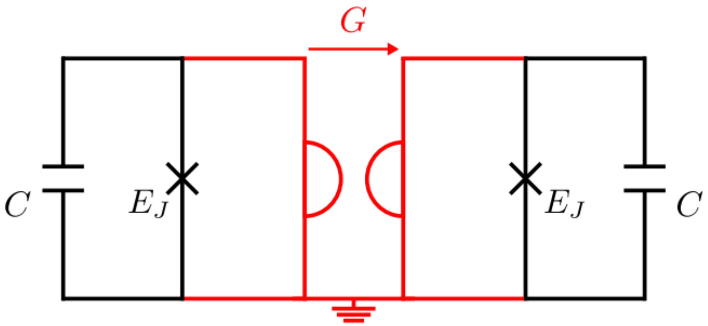
See also: Roushan,..., Martinis, **Nature Phys. 13** (2017)

The circulator problem?

Large size take up real-estate



Magnetic fields prevent integration

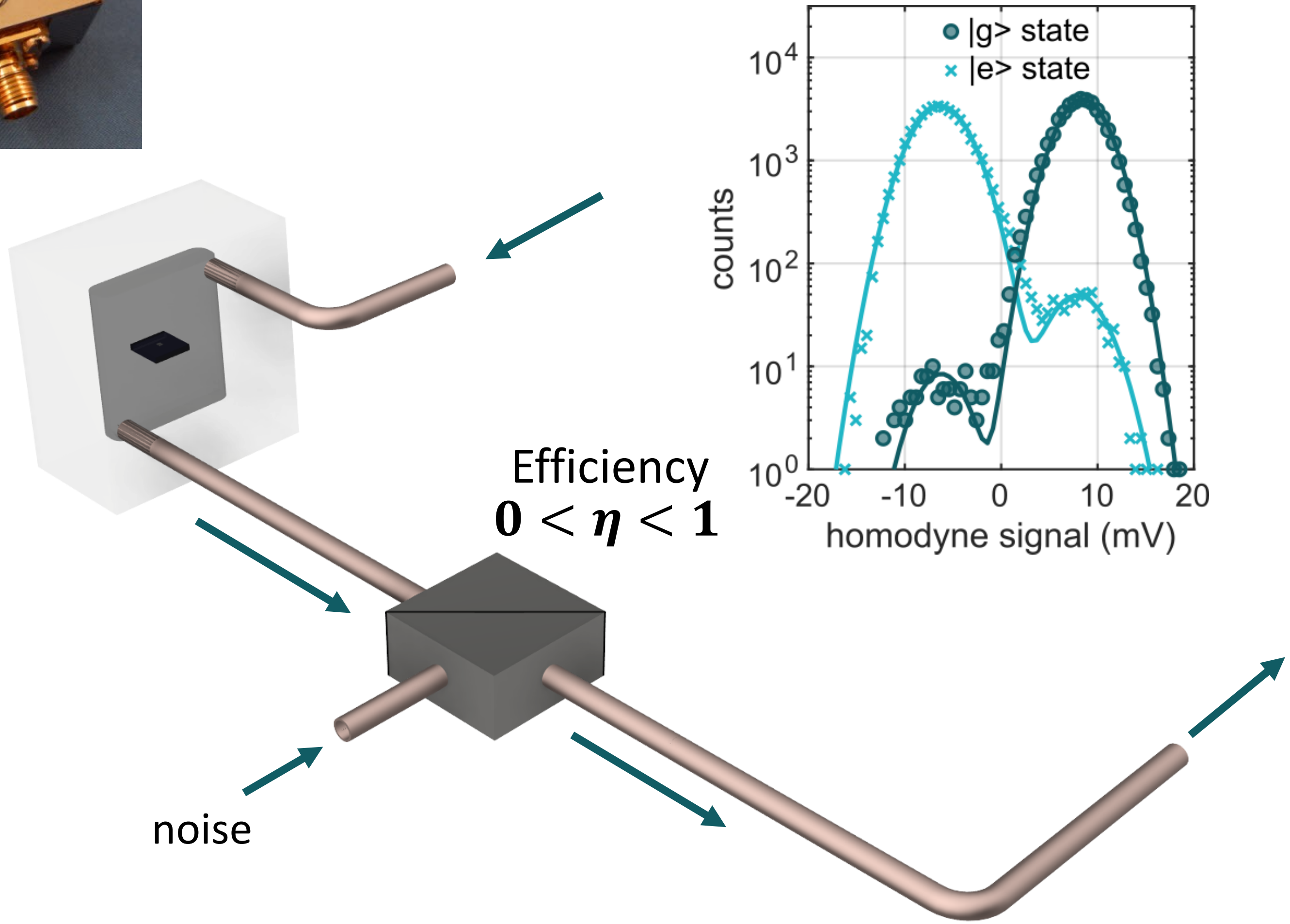


Rymarz,..., DiVincenzo, **PRX 11** (2021)

See also: Roushan,..., Martinis, **Nature Phys. 13** (2017)

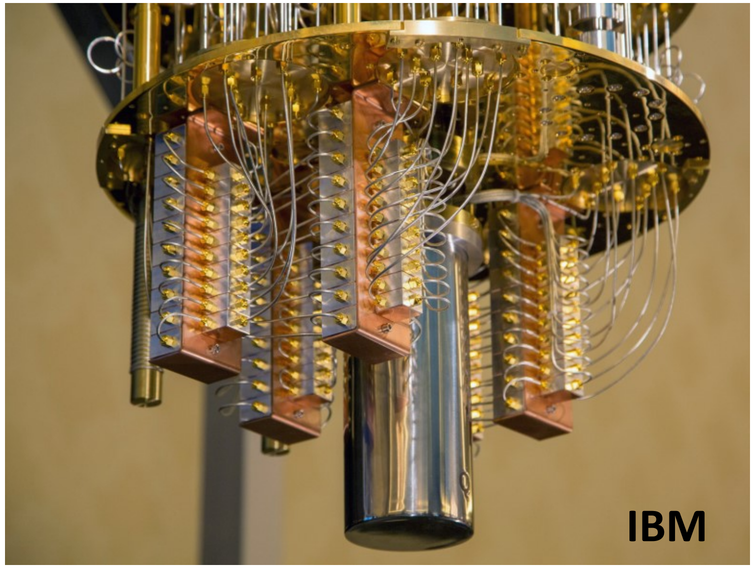
Loss limit efficiency

$$F = erf(\sqrt{\eta\tau\Gamma_m/2})$$

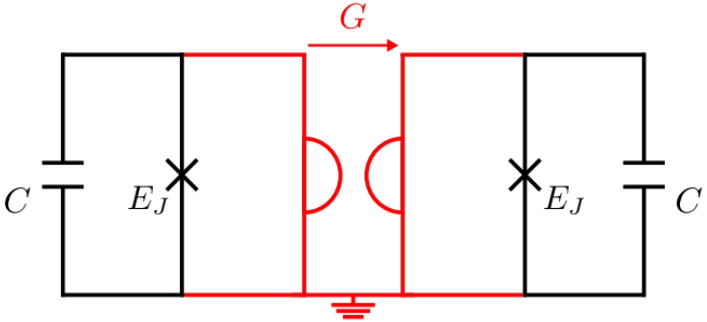


The circulator problem?

Large size take up real-estate



Magnetic fields prevent integration



Rymarz,..., DiVincenzo, **PRX 11** (2021)

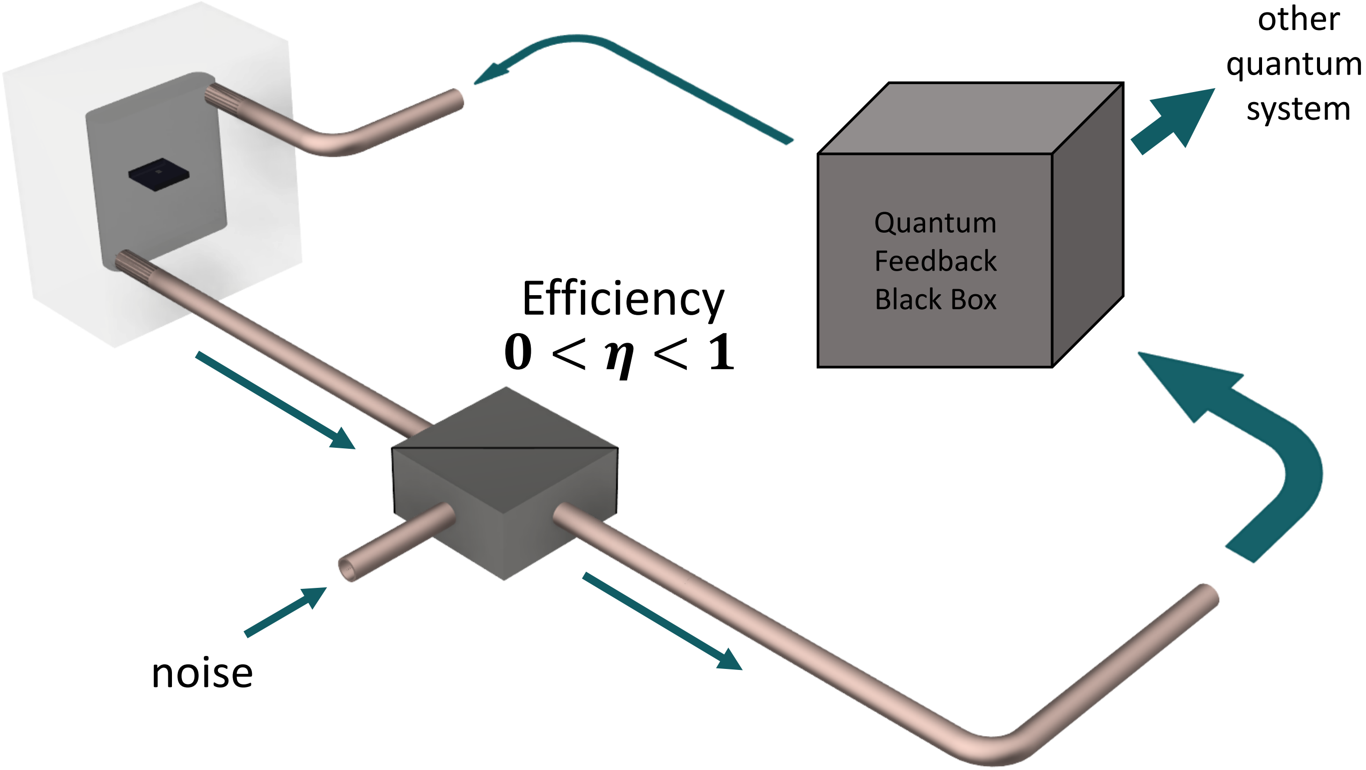
See also: Roushan,..., Martinis, **Nature Phys. 13** (2017)

Loss limit efficiency

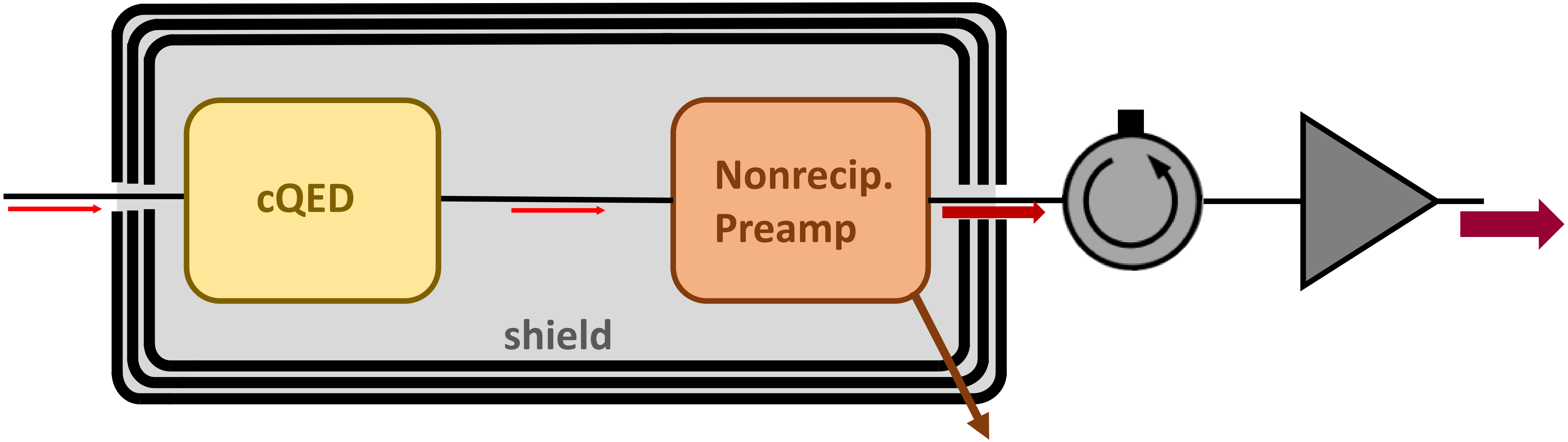
Liu,..., Devoret, **PRX** 6 (2016)

Rossi, .. , Schliesser, **Nature** 563 (2018)

Roch,..., Siddiqi, **PRL** 112 (2014)



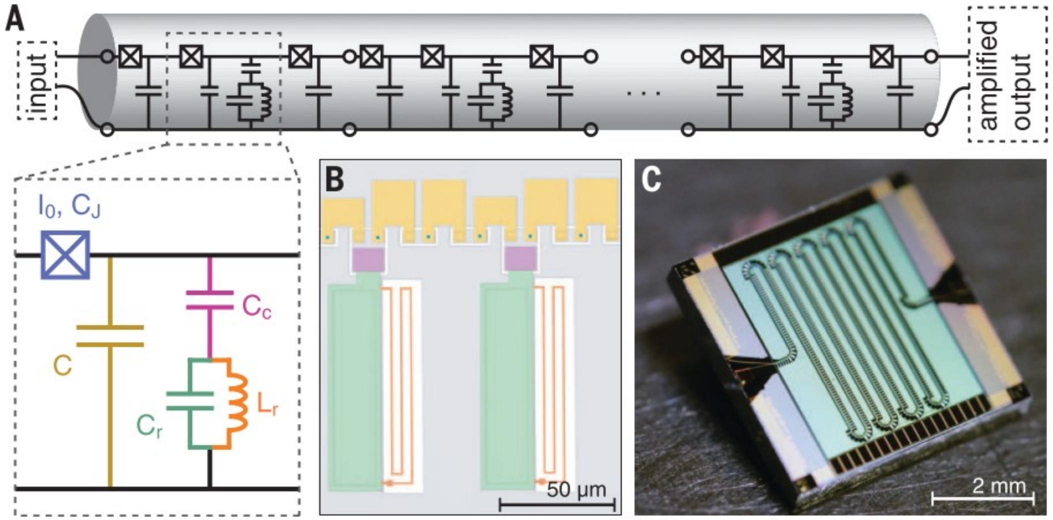
The need for directional amplifiers



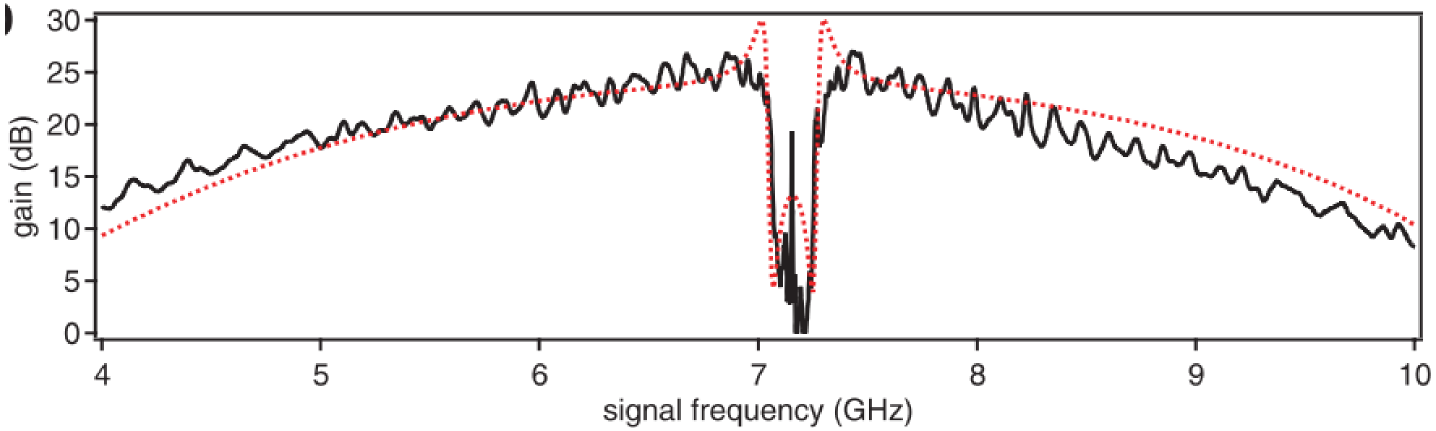
- **Traveling wave devices**
- **Multi-pump parametric devices**

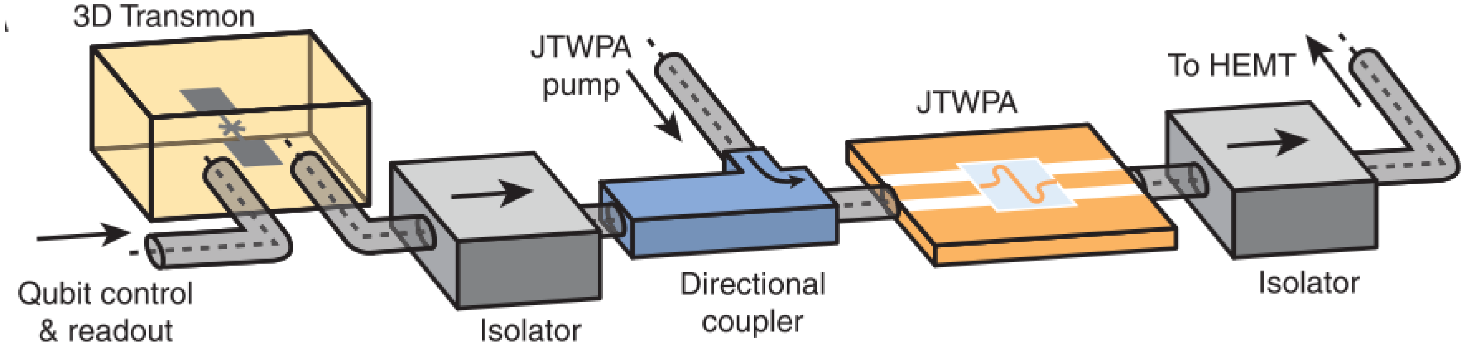
*L. Ranzani, J. Aumentado, **IEEE MW magazine** 20 (2019)*
*J. Aumentado, **IEEE MW magazine** 21 (2020)*

Traveling wave amplifiers



*Macklin, … , Siddiqi, **Science** 350 (2015)*





Pros:

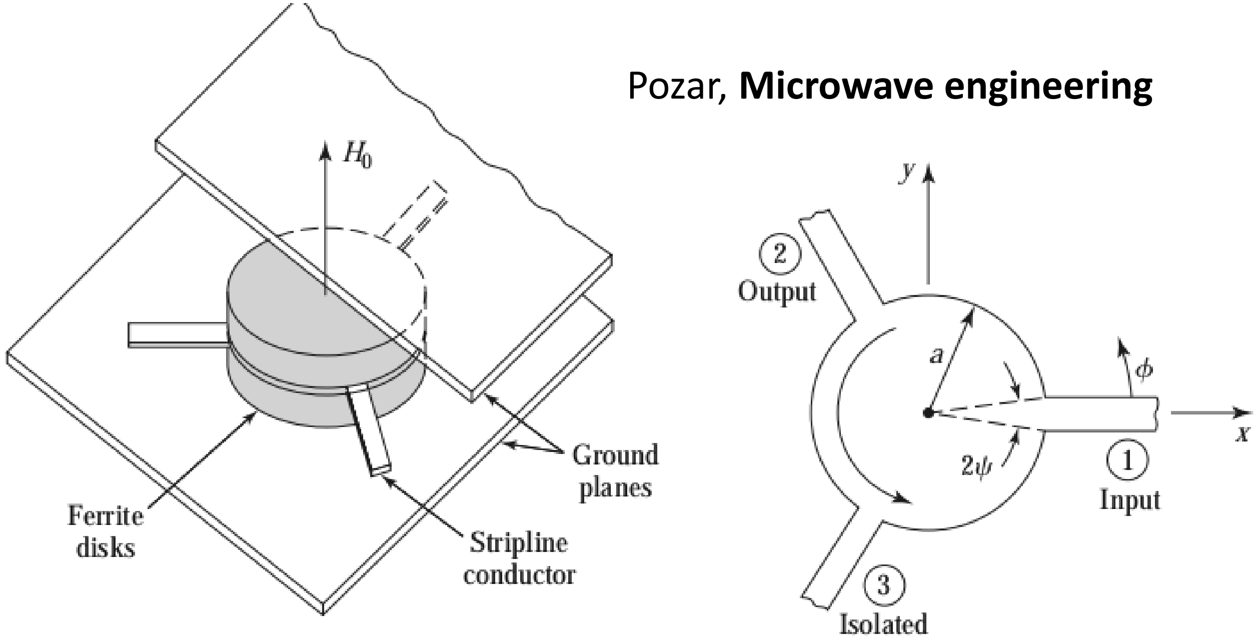
- **Many GHz of bandwidth**
- **High dynamic range**

Cons:

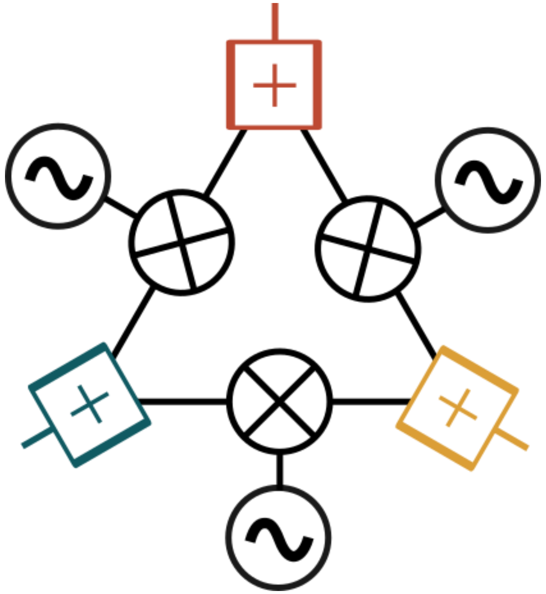
- **High pump power**
- **Residual reverse gain**

Also: *Planat, **PRX** 10 (2020), Ranadive, **Nat. Com.** 13 (2021), Esposito, **Applied. Phys Lett.** 119 (2021)* and *Ranzani, **PRapplied** (2021)*

Parametric nonreciprocity



Pozar, **Microwave engineering**



Necessary ingredients:

- **Interferometer**
- **Nonreciprocal phase shift**

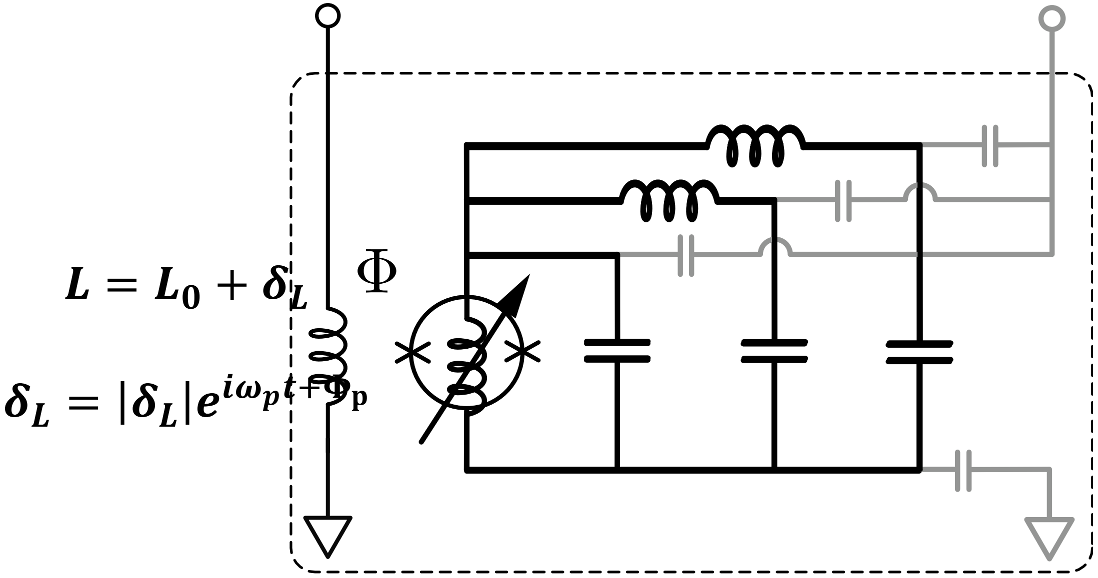
Parametric implementation:

- **Superconducting resonators**
- **Parametric frequency conversion**

Field Programmable Josephson Amplifier

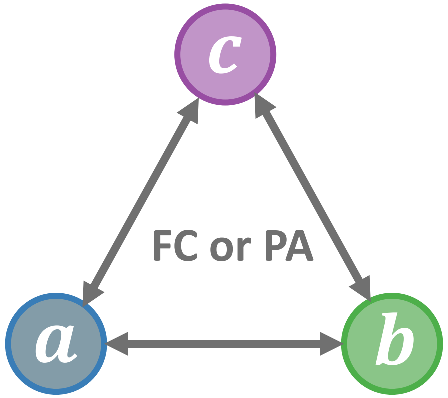
FC: Frequency Conversion
PA: Parametric Amplification

Theory:
Ranzani and Aumentado, **NJP** 17 (2015)
Metelmann and Clerk, **PRX** 5 (2015)



$$L = L_0 + \delta_L$$

$$\delta_L = |\delta_L| e^{i\omega_p t + \phi_p}$$



If $\omega_p = \omega_b - \omega_a$

$$H_I \propto \delta_L a b^\dagger + \boxed{\delta_L^*} a^\dagger b$$

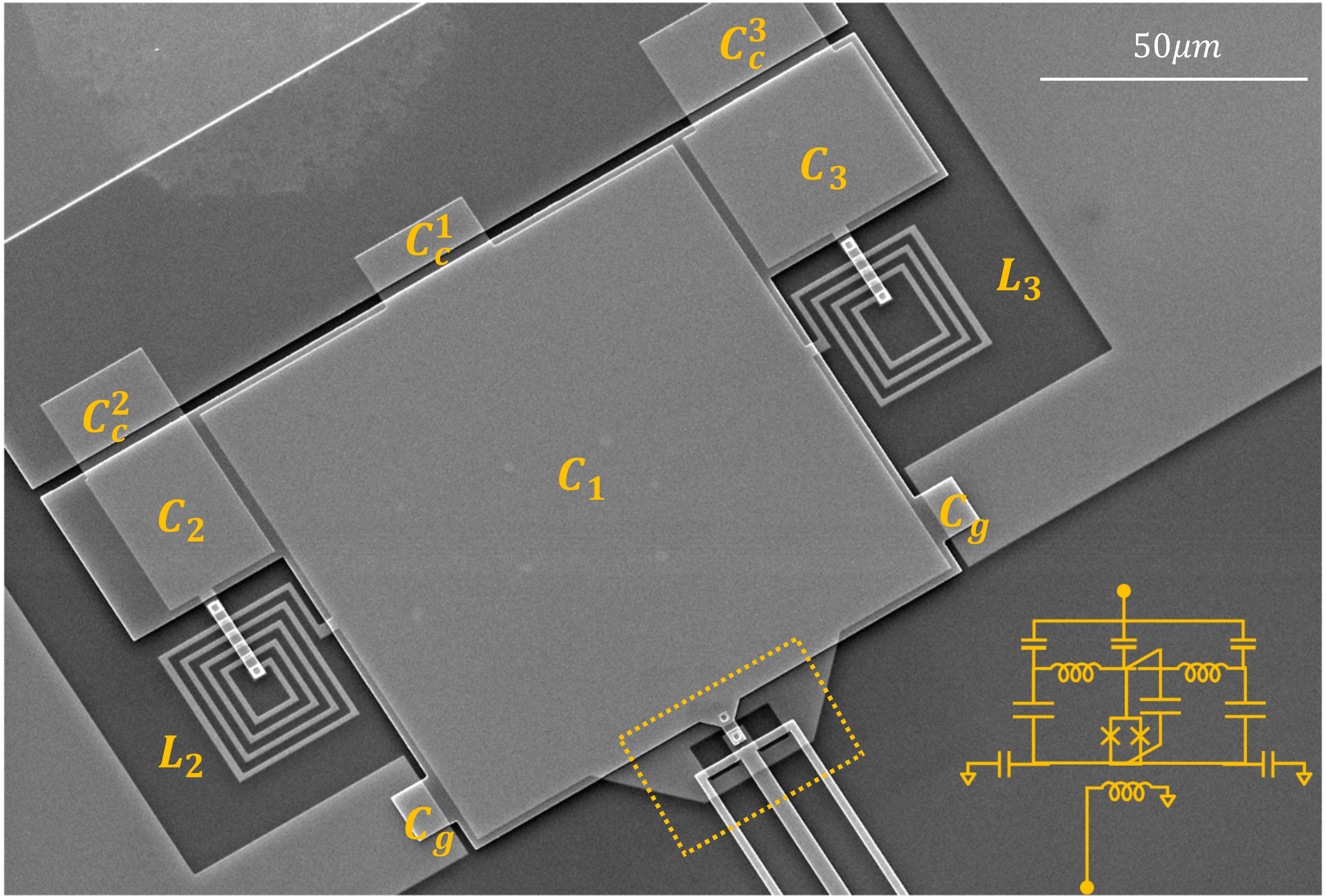
Frequency Conversion (FC)

If $\omega_p = \omega_b + \omega_a$

$$H_I \propto \delta_L a b + \boxed{\delta_L^*} a^\dagger b^\dagger$$

Parametric Amplification (PA)

Field Programmable Josephson Amplifier

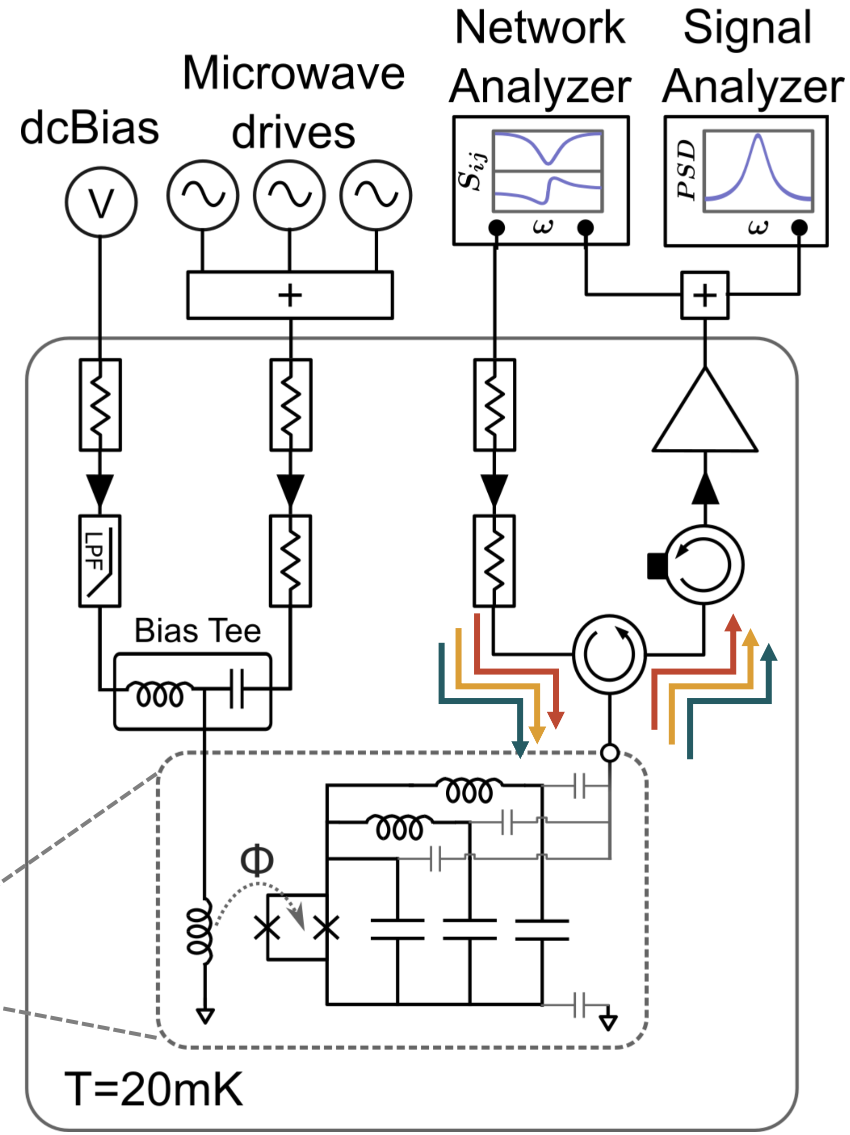


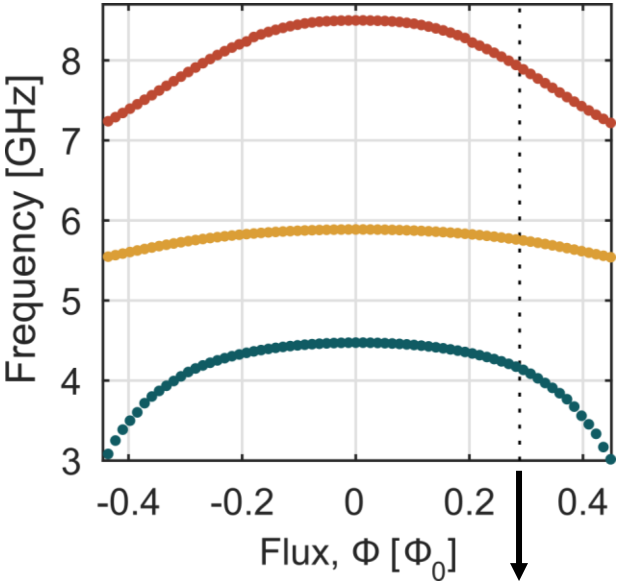
Key info:

- Nb/Al/Nb trilayer
- aSi dielectric
- Gradiometric SQUID
- On-chip bias

FPJA: three tunable resonances

Lecocq, at al **Phys. Rev. Applied** 7 (2017)





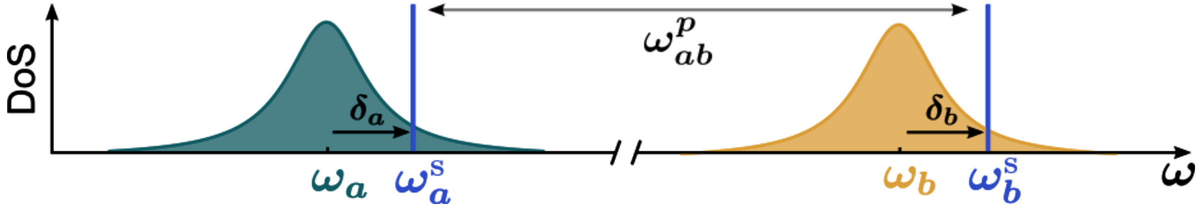
Mode i	$\frac{\omega_i}{2\pi}$ [GHz]	$\frac{\kappa_i}{2\pi}$ [MHz]	$\frac{\kappa_i^{int}}{2\pi}$ [MHz]
a	4.155	29.8	1.4
b	5.756	29.5	1.2
c	7.915	59.3	2.4

$$g_{jk}(t) = \frac{\delta\Phi_{jk}(t)}{4}\sqrt{\frac{\partial\omega_j}{\partial\Phi}\frac{\partial\omega_k}{\partial\Phi}}$$

FPJA, first building block: frequency conversion

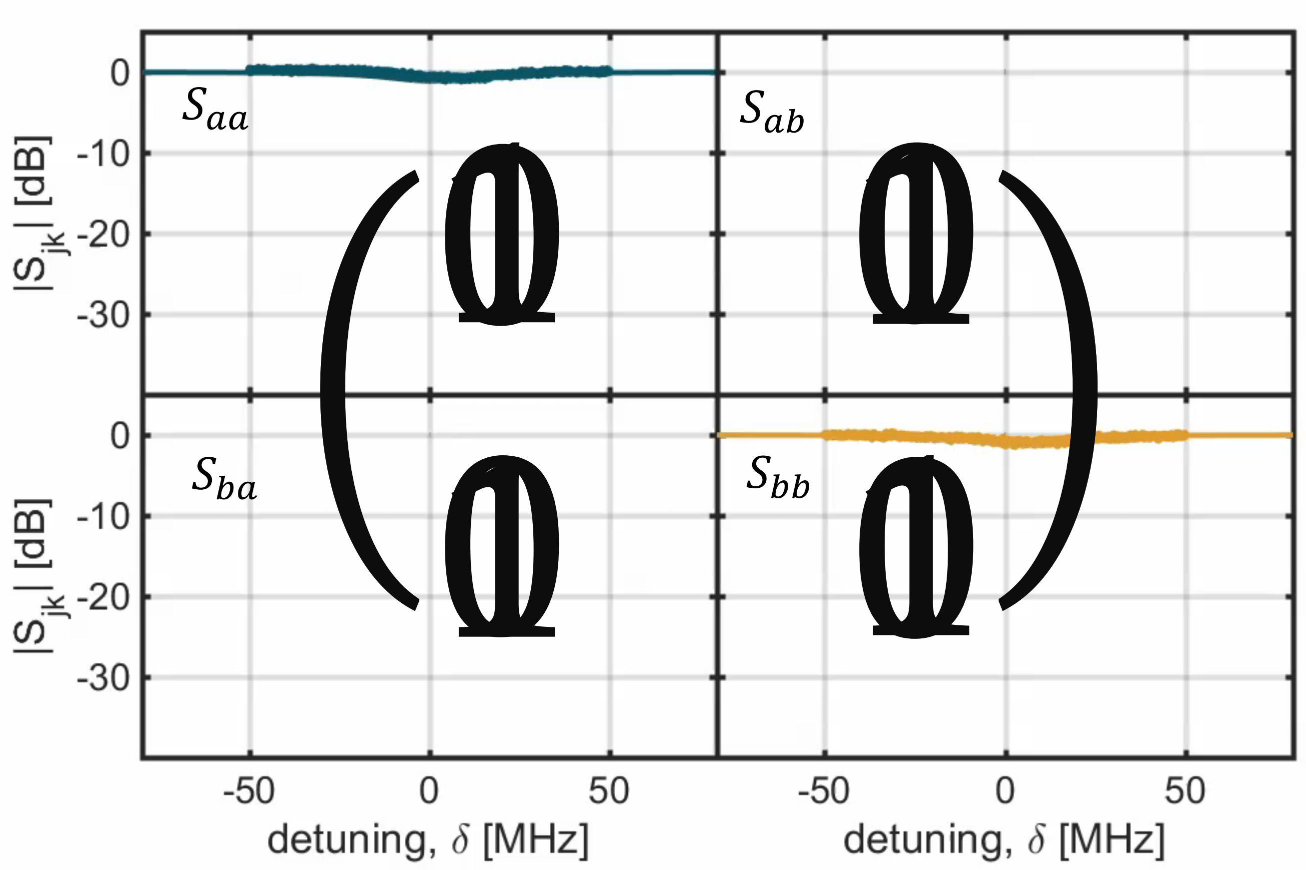
Lecocq, at al **Phys. Rev. Applied** 7 (2017)

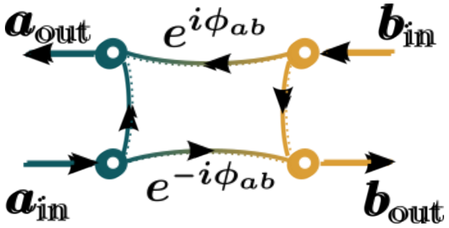
See also: Abdo, .., Devoret, **PRL** 110 (2013)
Lecocq, .., Teufel, **PRL** 116 (2016)



$$\omega_{ab} = \omega_b - \omega_a$$

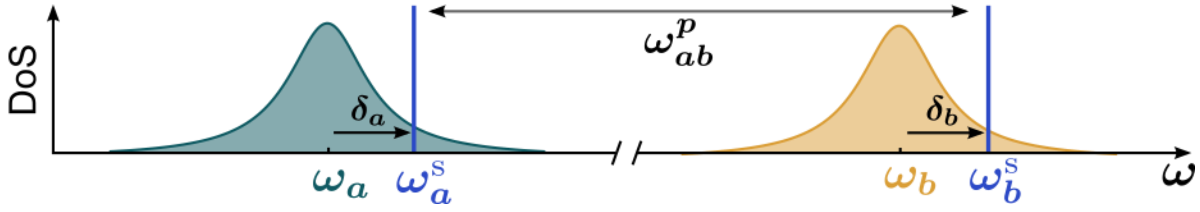
$$g_{ab} \sim \sqrt{\kappa_a \kappa_b}$$





FPJA, first building block: frequency conversion

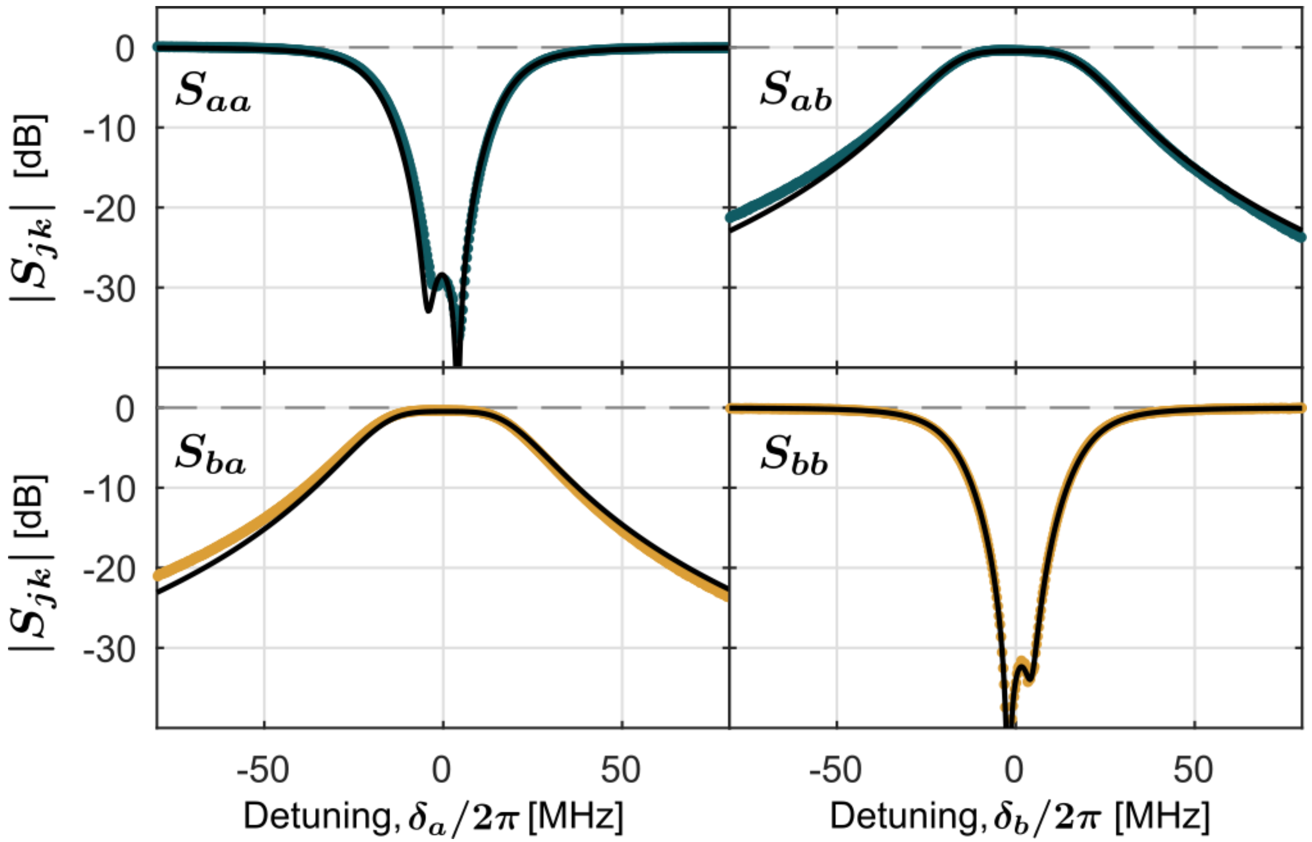
Lecocq, at al **Phys. Rev. Applied** 7 (2017)

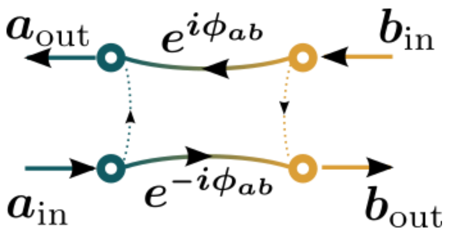


$$\omega_{ab} = \omega_b - \omega_a$$

$$g_{ab} \sim \sqrt{\kappa_a \kappa_b}$$

See also: Abdo, .., Devoret, **PRL** 110 (2013)
Lecocq, .., Teufel, **PRL** 116 (2016)





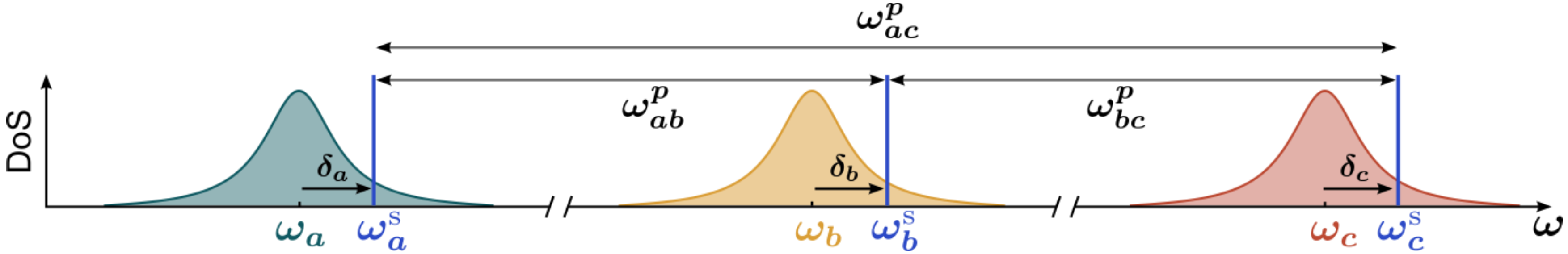
Near ideal conversion (loss<0.5dB)

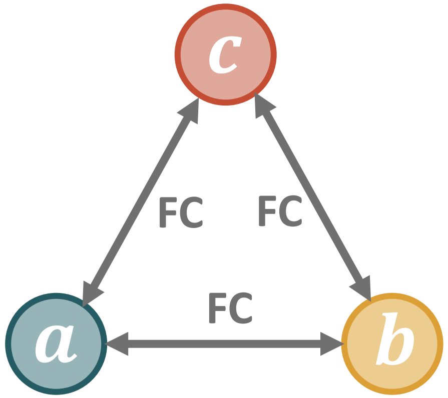
Pump phase imprinted in the conversion

FPJA: circulation

Lecocq, at al **Phys. Rev. Applied** 7 (2017)

See also: Sliwa, **PRX** 5 (2015)





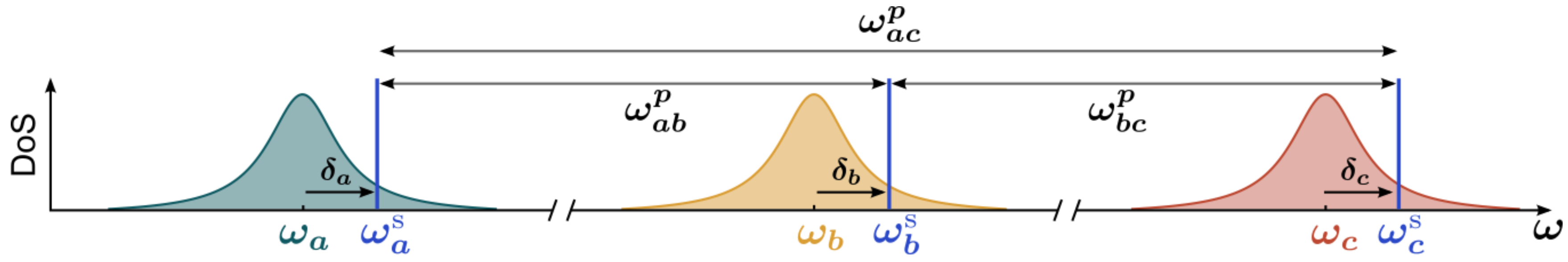
Loop phase: $\phi_{loop} = \phi_{ab} + \phi_{bc} - \phi_{ac}$

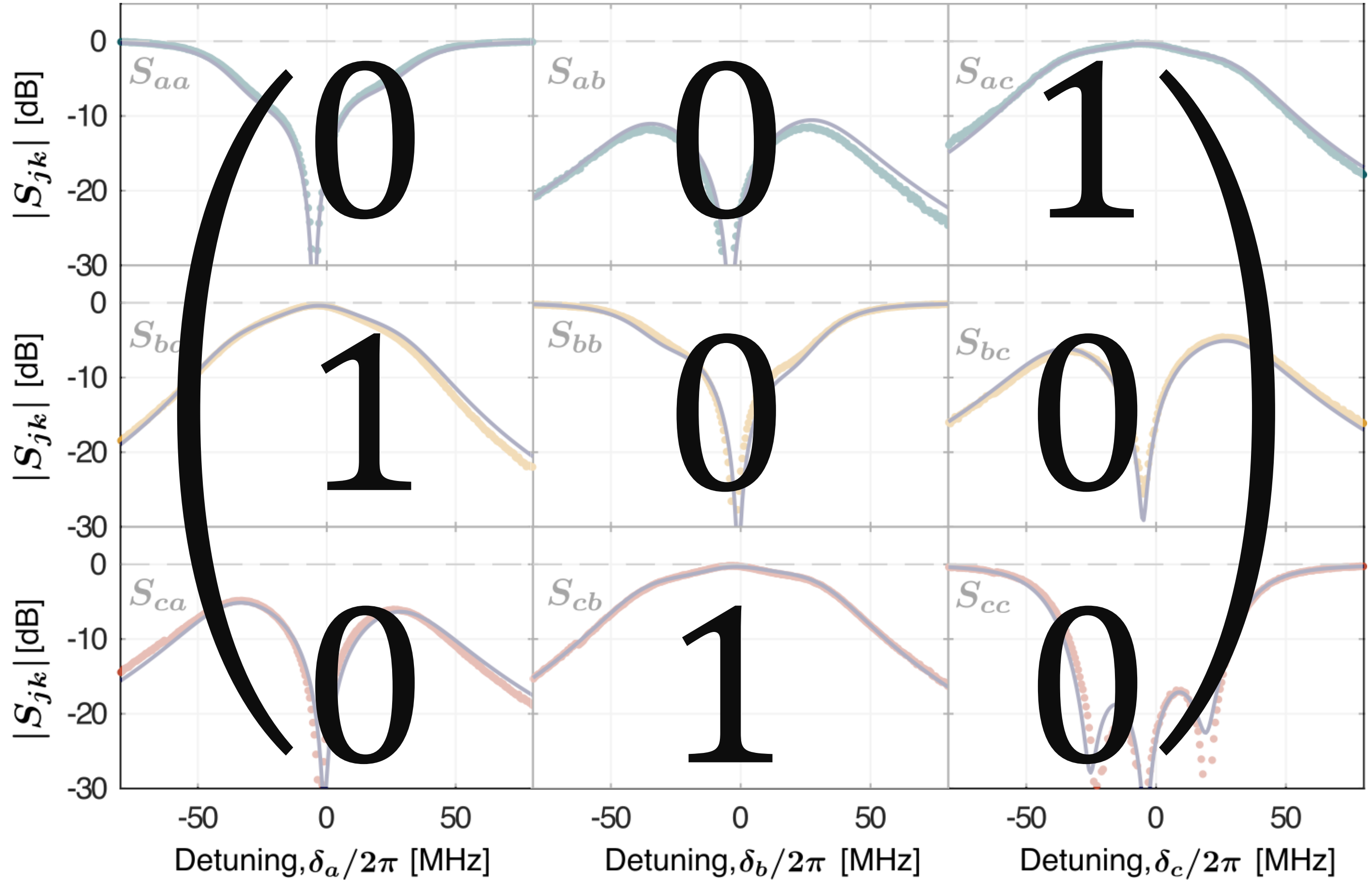
Interference happens for $\phi_{loop} = \pm 90^{\circ}$

FPJA: **circulation**

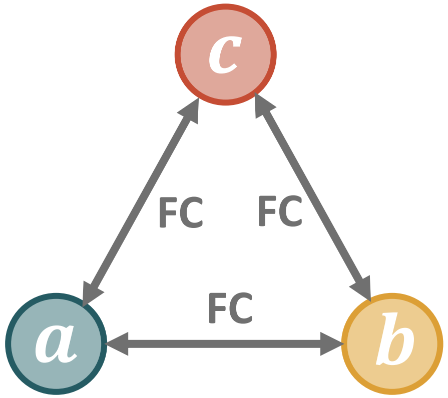
Lecocq, at al **Phys. Rev. Applied** 7 (2017)

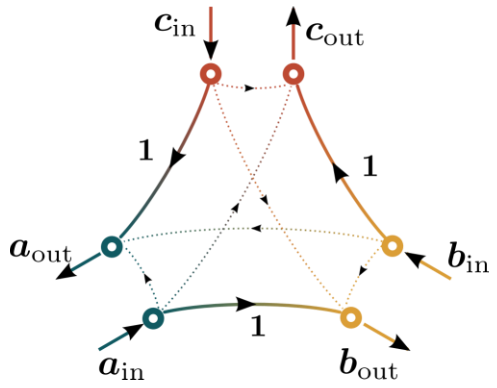
See also: Sliwa, **PRX** 5 (2015)





$$\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

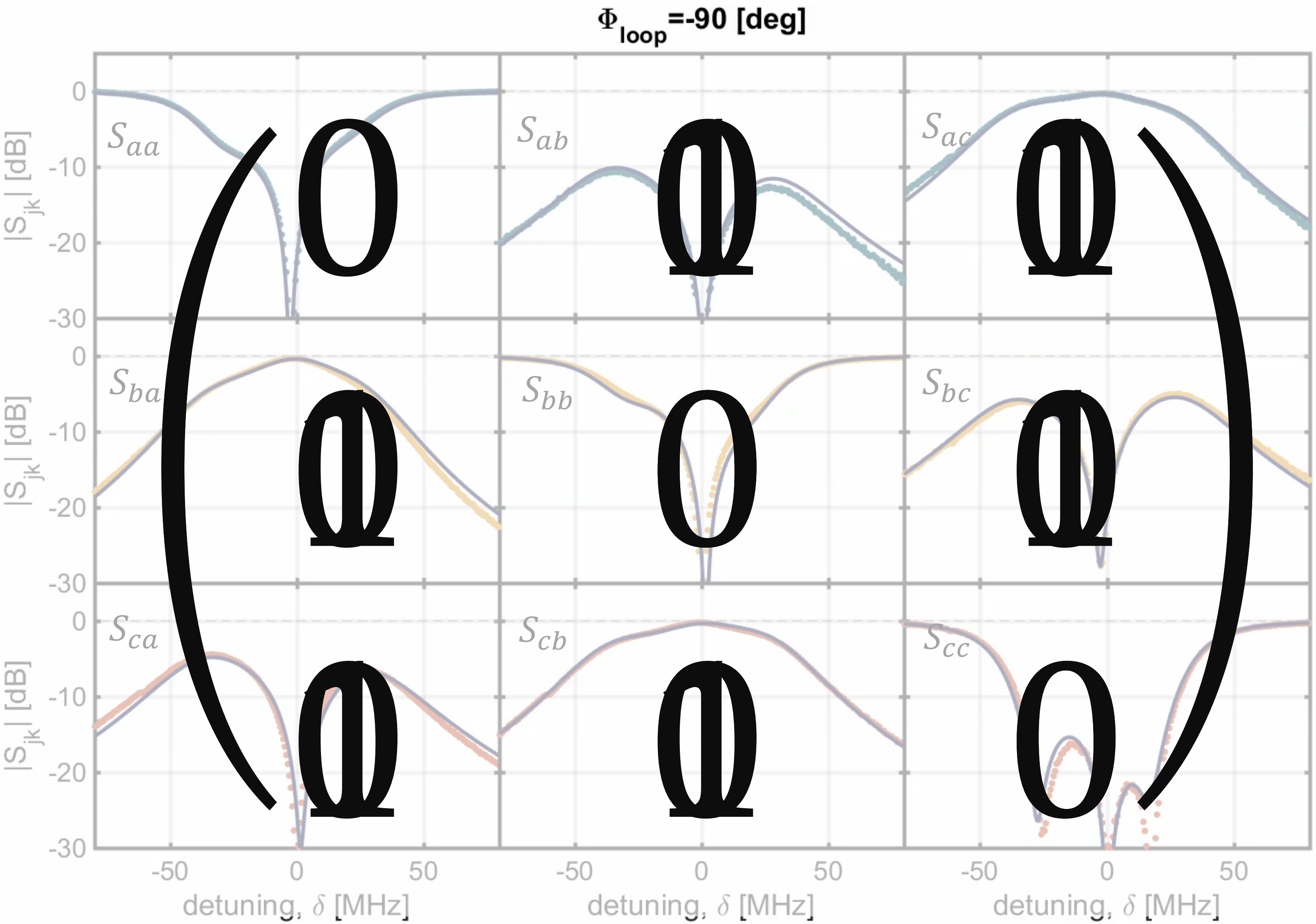




FPJA: **circulation**

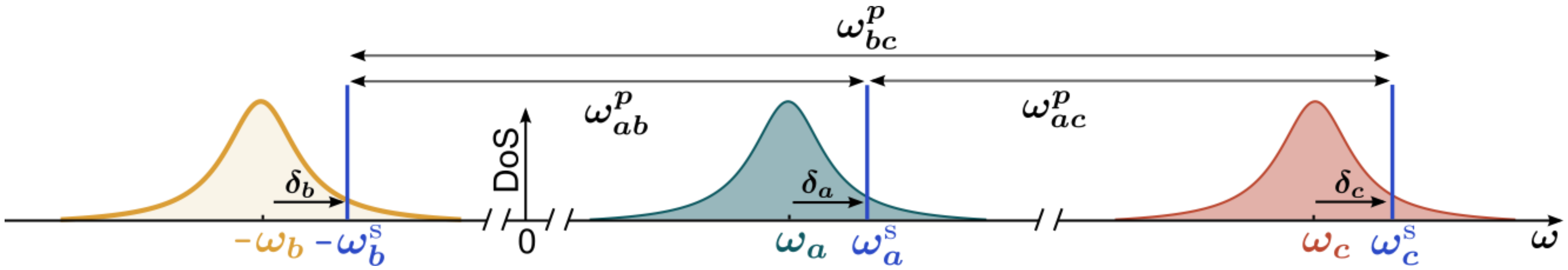
Lecocq, at al **Phys. Rev. Applied** 7 (2017)

<u>See also</u>: Sliwa, **PRX** 5 (2015)

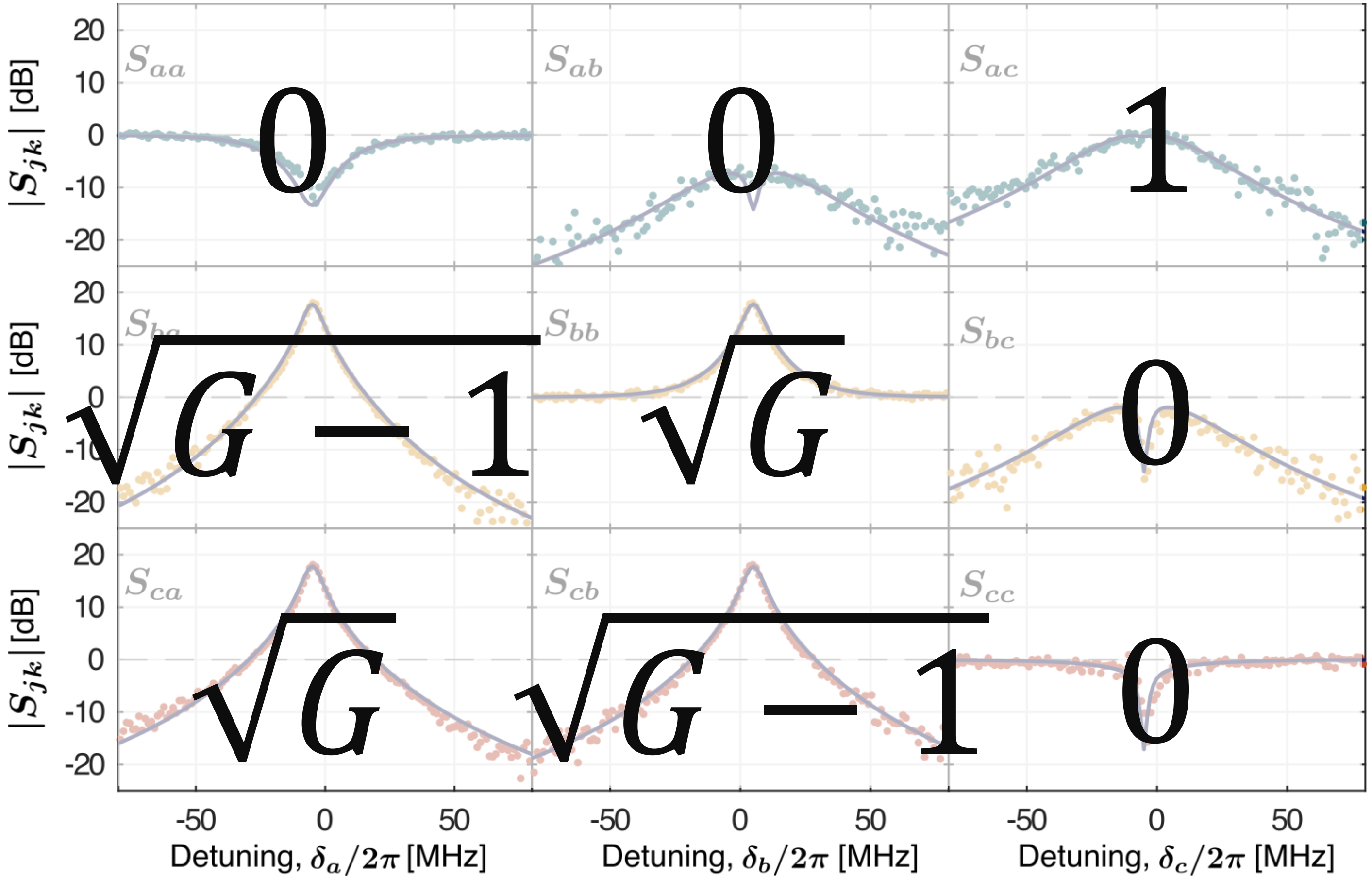


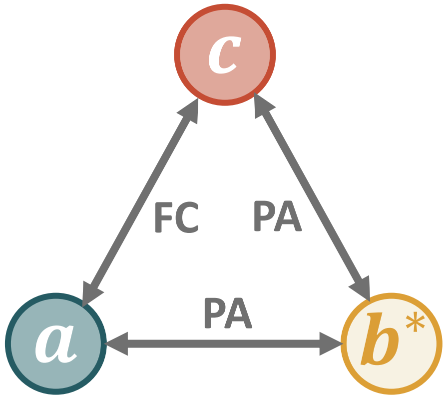
FPJA: directional amplification

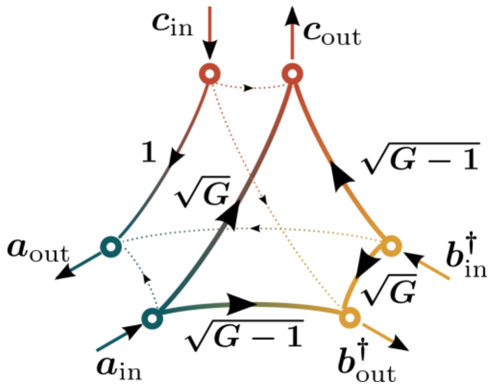
Lecocq, at al **Phys. Rev. Applied** 7 (2017)



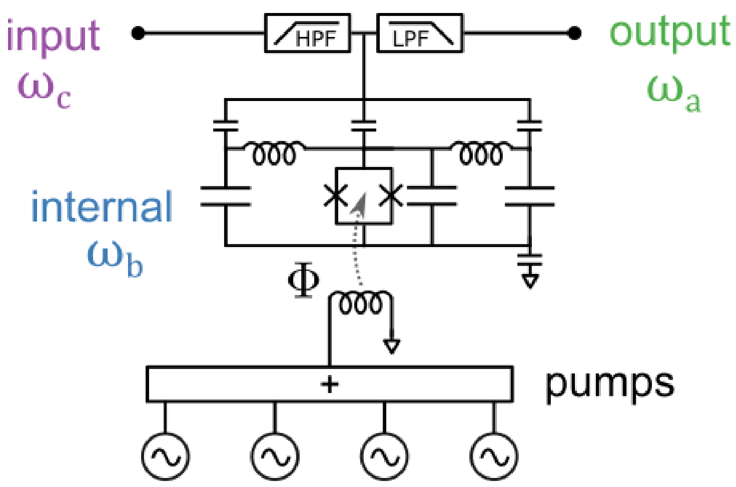
See also: Sliwa, **PRX** 5 (2015)







Field Programmable Josephson Amplifier



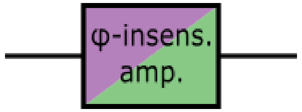
- 3 resonators
- 1 SQUID
- All-to-all parametric coupling

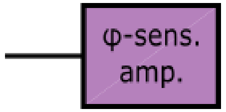
Phys. Rev. Applied, ***7*** *024028 (2017)*
Phys. Rev. Applied, ***13*** *044005 (2020)*

No pump: open circuit

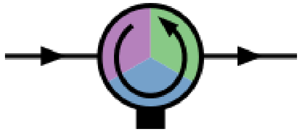
1 pump: Frequency converter

Phase sensitive or insensitive amplifier

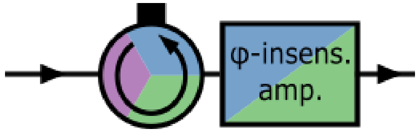




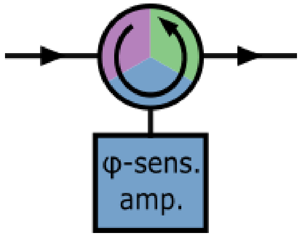
3 pumps: Circulator



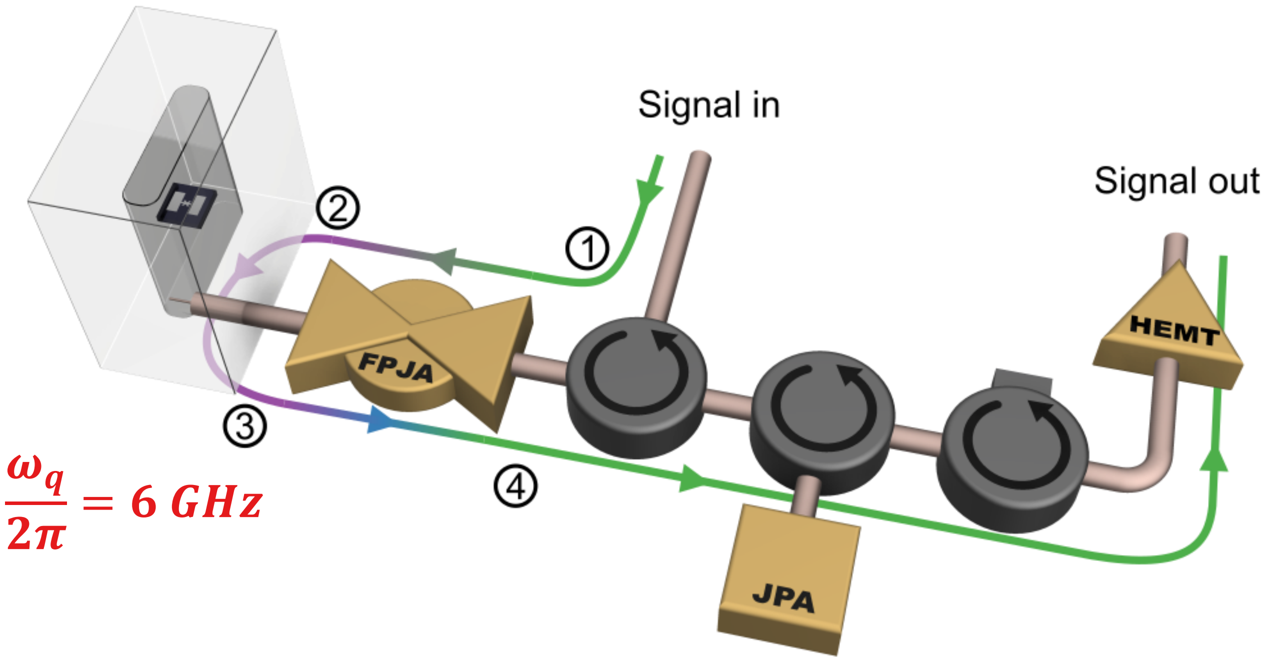
Directional phase insensitive amplifier



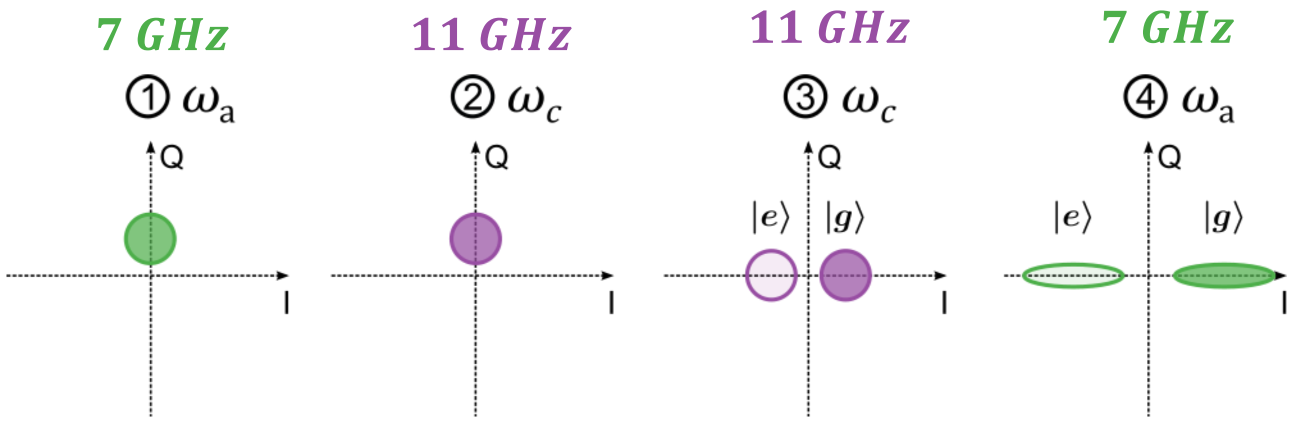
4 pumps: Directional phase sensitive amplifier



Field Programmable Josephson Amplifier



$\frac{\omega_q}{2\pi} = 6\ GHz$



Lecocq, at al **Phys. Rev. Lett.** 126 (2021)

Pros:

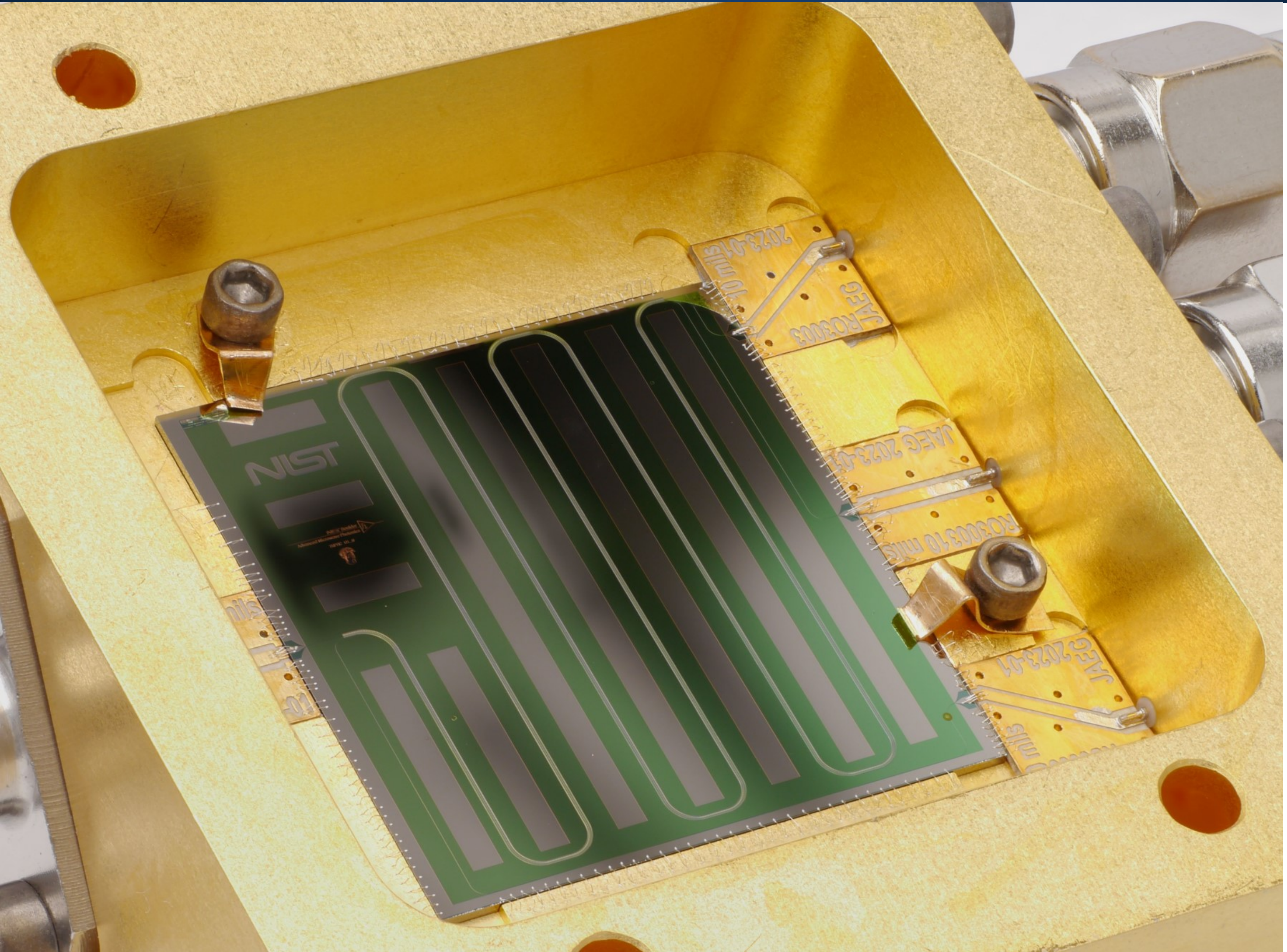
- **Ultra-low noise**
- **Fully integrable on-chip**

Cons:

- **Limited bandwidth**
- **Limited dynamic range**

Efficiency $\boldsymbol{\eta_m} = \boldsymbol{\Gamma_m/\Gamma_\phi^m} = 72\%$

Conclusion



Amplifier as enabling technologies:

- **High fidelity readout**
- **Quantum sensing**
- **Quantum feedback**
- **Scaling**

Amplifier research:

- **Parametric interactions**
- **nonreciprocity**

The Advanced Microwave Photonics Group (The Whole Team)

PIs:
Joe Aumentado
Florent Lecocq
Tony McFadden
Ray Simmonds
John Teufel

Fab Team:
Kat Cicak
Kristen Genter

Postdocs:
Akash Dixit
Jose Estrada
Stephen Gill
Bradly Hauer
Trevyn Larson
Maxime Malnou
Sudhir Sahu

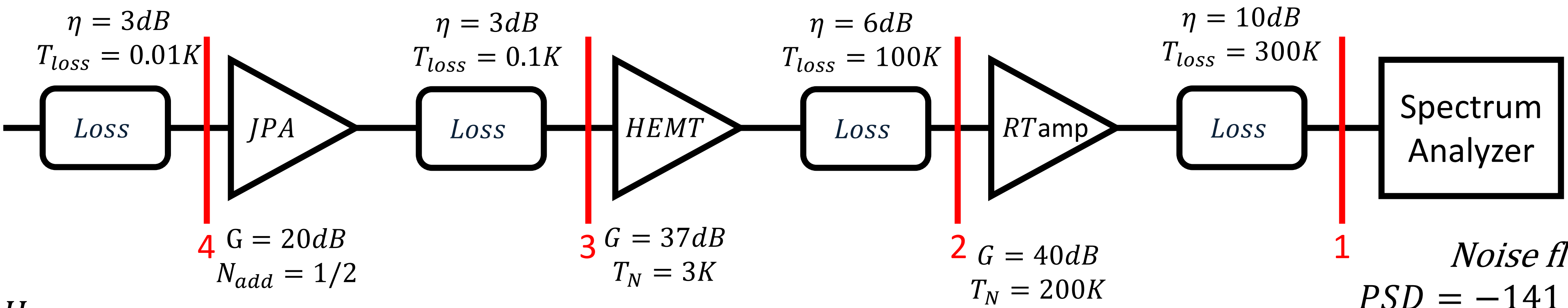
PhD Students:
Kaixuan Ji
Benton Miller
Zachary Parrott
Tongyu Zhao

NIST NATIONAL INSTITUTE OF STANDARDS AND TECHNOLOGY
U.S. DEPARTMENT OF COMMERCE

University of Colorado Boulder

NIST Boulder
Advanced **M**icrowave **P**hotonics Group

Problem: noise rise



$$\frac{\omega}{2\pi} = 6\ GHz$$

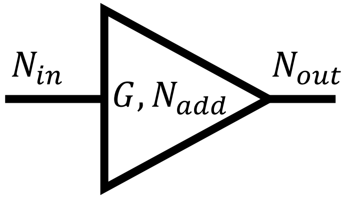
Noise floor

$PSD = -141\ dBm/Hz$

1. What the system added noise of this amplifier chain?
2. What is the noise rise on the spectrum analyzer when turning on the RT amp only, then adding the HEMT, then adding the JPA
3. What happens to the system noise and HEMT noise rise if I do not use the RT amp?
4. What happens to system noise and noise if the first attenuator goes form 3dB to 10dB?

Cheat sheet

Noise of an amplifier:



$$N_{out} = G(N_{in} + N_{add})$$

Noise of an attenuator:

$$N_{out} = \eta N_{in} + (1-\eta)N_{attn} = \eta(N_{in} + N_{add}) \text{ with } \begin{cases} N_{add} = \dfrac{1-\eta}{\eta} N_{attn} \\ N_{attn} = \dfrac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} + \dfrac{1}{2} \end{cases}$$

Units and conversions:

N in $quanta/s/Hz \sim quanta$

$PSD = \hbar\omega N$ in W/Hz

And $10 * \log_{10}(PSD \times 10^3)$ in dBm/Hz

Typical values @ 6GHz:

- Vacuum noise PSD $\sim -207\ dBm/Hz$
- Room temp noise PSD $\sim -174\ dBm/Hz$

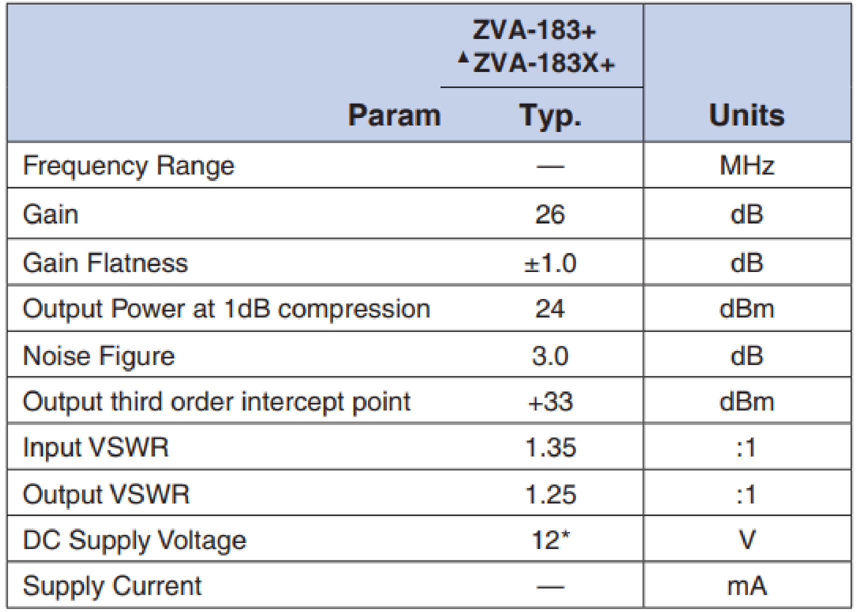
$$\frac{\hbar\omega}{k_B T} \underset{\substack{20\ GHz \\ 1\ K}}{\Longrightarrow} 1$$

Noise temperature : $\mathrm{k_B T_N} = \hbar\omega N_{add}$ NOT A GOOD METRIC WHEN $k_B T_N \leq \hbar\omega$

How to read an amplifier spec sheet

ZVA-183-S+
ZVA-183X-S+

Generic photo used f



	ZVA-183+ ▲ZVA-183X+	
Param	**Typ.**	**Units**
Frequency Range	—	MHz
Gain	26	dB
Gain Flatness	±1.0	dB
Output Power at 1dB compression	24	dBm
Noise Figure	3.0	dB
Output third order intercept point	+33	dBm
Input VSWR	1.35	:1
Output VSWR	1.25	:1
DC Supply Voltage	12*	V
Supply Current	—	mA

Parameter	Test Condition	Value	Unit
Gain	4-8GHz	42	dB
Noise	4-8 GHz	1.5	K
IRL	4-8 GHz	13	dB
ORL	4-8 GHz	20	dB
P_{1dB}	5 GHz	-12	dBm
OIP3	5 GHz	-2	dBm

Noise temperature : $k_B T_N = \hbar\omega N_{add}$ NOT A GOOD METRIC WHEN $k_B T_N \leq \hbar\omega$

Noise factor (referenced to RT noise): $F = \frac{SNR_{in}}{SNR_{out}} = 1 + \frac{T_N}{290} > 1$

Noise figure: $NF = 10 * \log_{10}(F)$

$$NF = 3\text{dB} \rightarrow T_N = 290\ K$$

Passive device with X dB of loss has a $NF = X$

Voltage Standing Wave Ratio: a measure of input/output impedance match

$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$ with $\Gamma = \frac{Z_l - Z_0}{Z_l + Z_0}$ the reflection coefficient at input/output (aka return loss)

$VSWR = 1.35 \rightarrow \Gamma = -16.5dB$

Directivity: D = $S_{21} \times S_{12}$ ($S_{21} =$ G and S_{12} is rarely spec'd)

Compression, Third order Intercept: later in the presentation