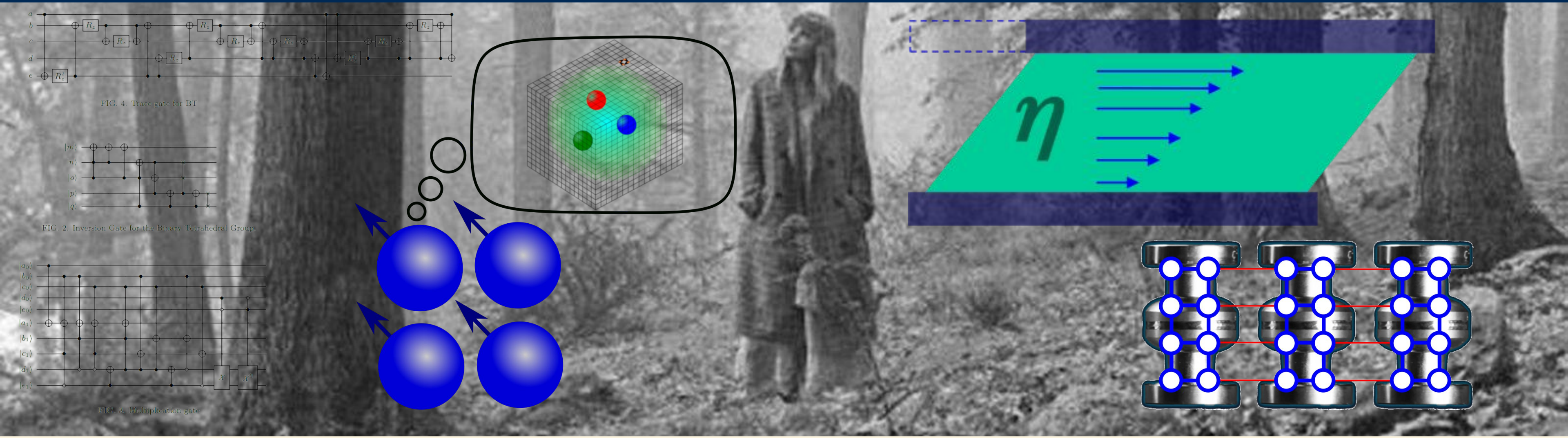


I’m the problem, it’s me:
Applications of Quantum Computing to High Energy Physics

Hank Lamm

August 12, 2023

Why am I here?

If this is a nuclear physics lab, where are the **bombs? :(**

With all this security, you must be doing something **classified? :(**

So when are you all gonna **destroy** the world? **:(**

Why am I here?

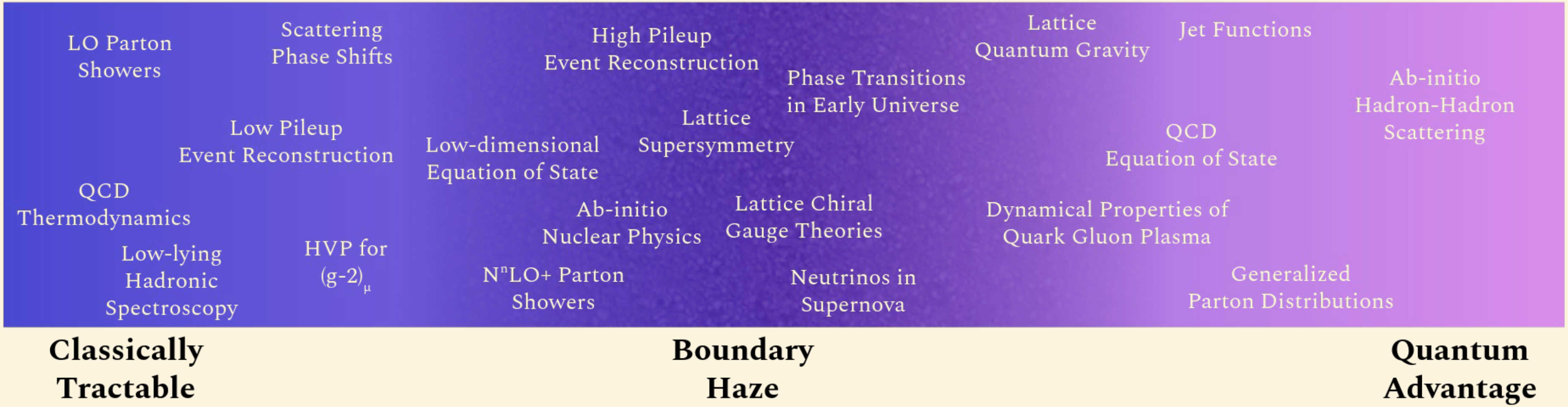

Fermilab
Home | About | Science | Jobs | Contact | Phone Book
Newsroom | DUNE at LBNF | Come visit us | Resources for
Fermilab is America's particle physics and accelerator laboratory
We bring the world together to solve the mysteries of matter, energy, space and time.

Long history here of computational physics

In 1989, 5 Gflop ACPMAPS was about **2000x less powerful** than your phone

Quantum Computing for Particle Physics, it's a need

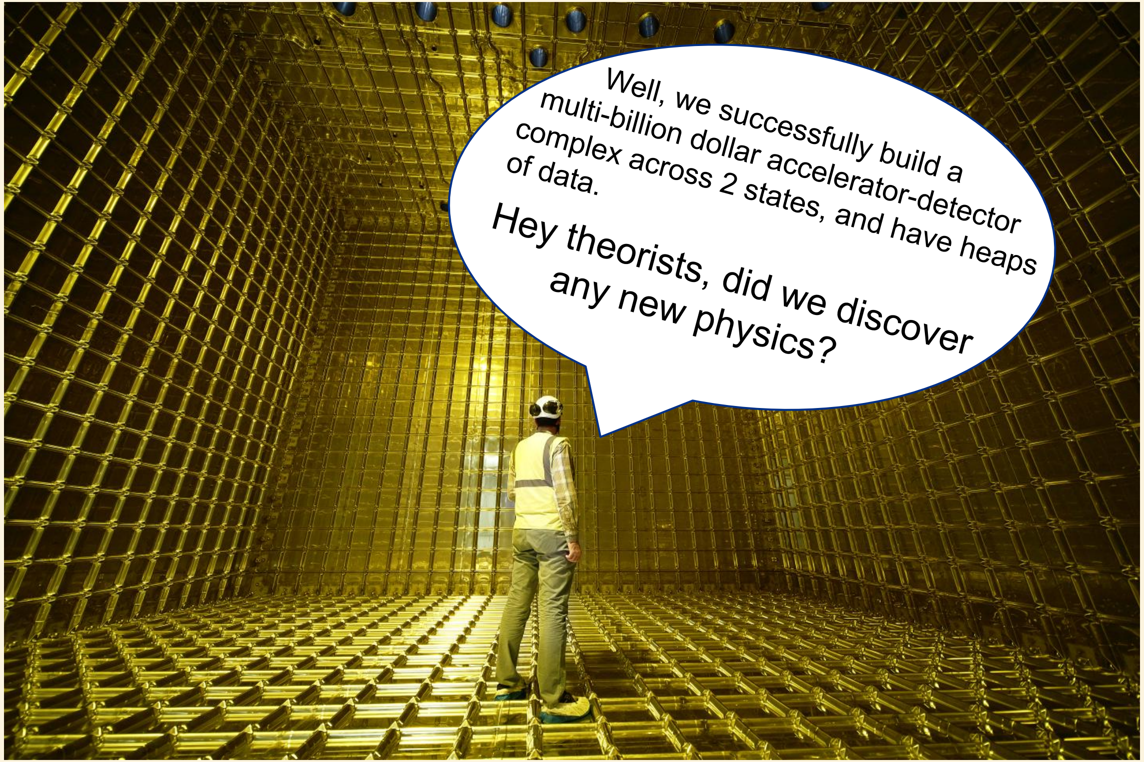
- **The world is quantum, and we are lucky anything is amenable to classical computers**
  - Large-scale quantum computers can tackle computations in HEP otherwise **inaccessible**
  - This opens up new frontiers & extends the reach of LHC, LIGO, EIC & DUNE



While broad, these topics often are formulated as **lattice field theories**

Quantum Simulation for High-Energy Physics
Bauer, Davoudi *et al.* - *PRX Quantum 4 (2023) 2, 027001*
Wonderful survey of physics questions, methods, and outstanding problems in field

Stated succinctly….



Gut Check!

Suppose we wanted to run a circuit on **100q** with each qubit acted on by a **2q** entangling gate.

Could we achieve **70% overall success** is the gate fidelity is **95%? 99%?**

Gut Check!

Suppose we wanted to run a circuit on **100q** with each qubit acted on by a **3q** entangling gate.

Could we achieve **70% overall success** is the gate fidelity is **95%? 99%?**

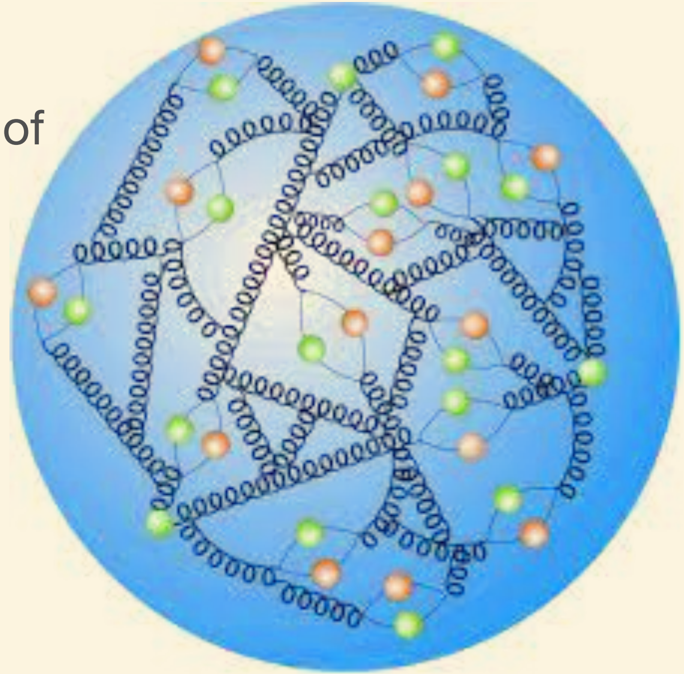
Gut Check!

Suppose we wanted to run a circuit on **50 ququarts** with each ququart acted on by a **2q** entangling gate.

Could we achieve **70% overall success** is the gate fidelity is **95%? 99%?**

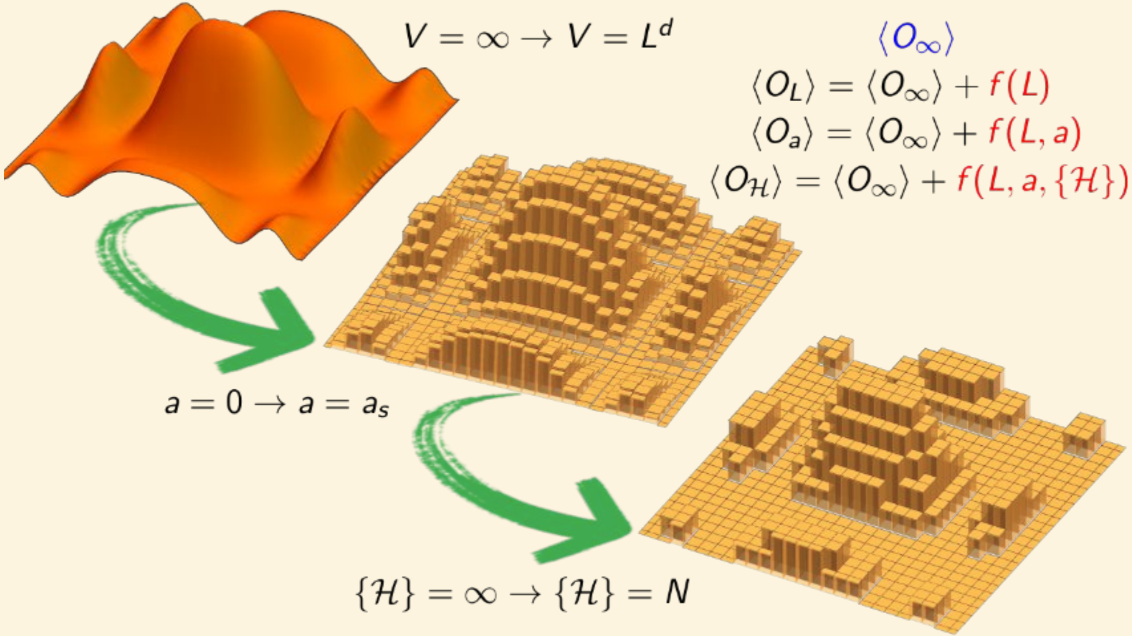
Quantum Chromodynamics – the theory of strong interactions

- Fundamental theory of the interactions of quarks (fermions) & gluons (boson)
- In analogy to charge in QED, has color
  - Instead of + or - we have **R**,**B**,**G** and **aR**, **aB**, **aG**
- Unlike QED, coupling is very large, precluding lots of perturbative calculations
- This leads us to needing nonperturbative methods



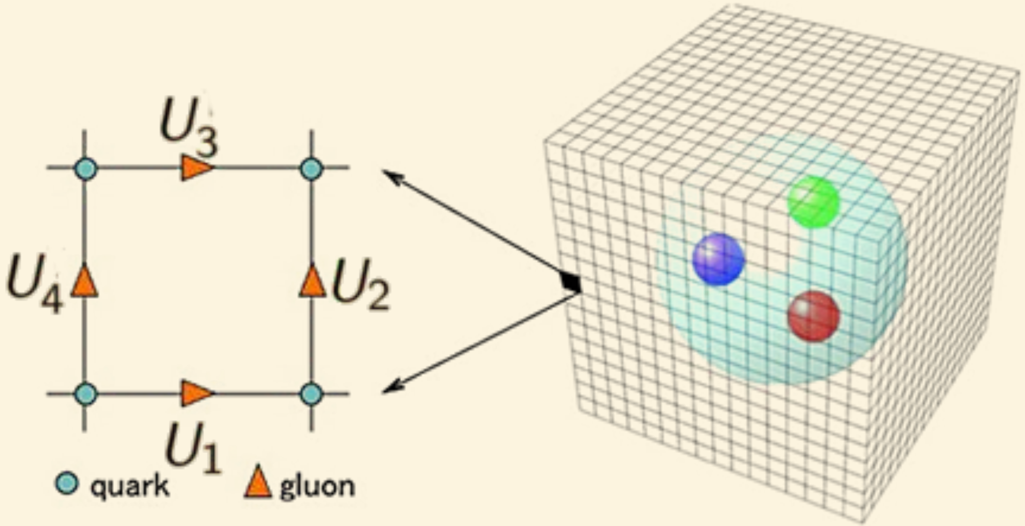
Lattice Field Theory

Take it to the limit



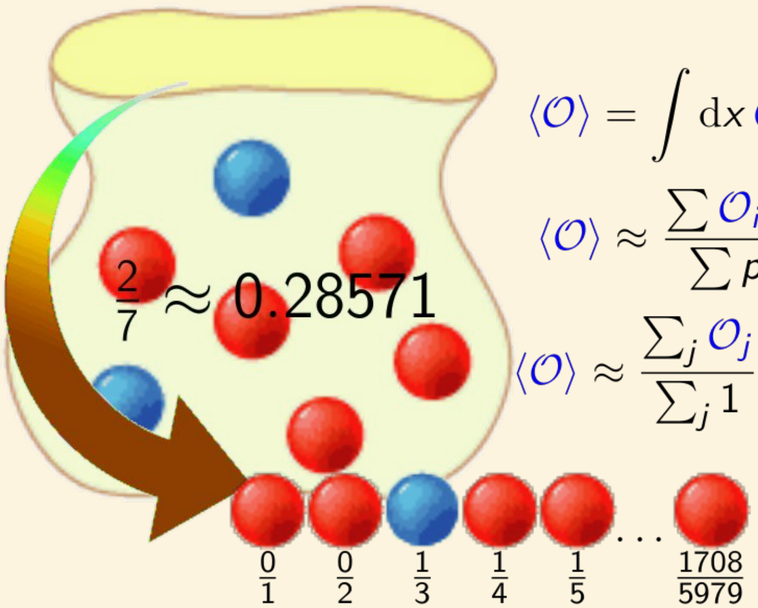
- O(L,a,$\mathcal{H}$) is an **approximation** for HEP
- Truncations leads to **systematic errors**
- **Extrapolating** is done on results, reducing computational resources…
- …but **obscures** precise resource estimates

3+1d Lattice QCD



- Put the **4-component, 3 color spinor** for quark on each site in lattice
- Put the **3x3 matrix of complex numbers** for each gluon on each link of lattice
- Perform a Monte Carlo by sampling field configurations
- Modern simulations performed on **100^4 lattices** w/ **yrs** of supercomputing time

Monte Carlo presents a practical solution…



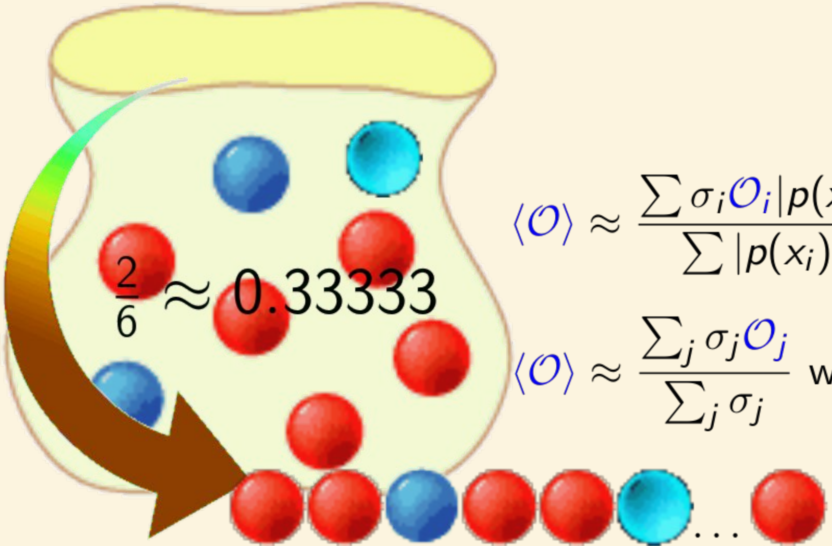
$$\langle \mathcal{O} \rangle = \int \mathrm{d}x \, \mathcal{O} p(x)$$

$$\langle \mathcal{O} \rangle \approx \frac{\sum \mathcal{O}_i p(x_i)}{\sum p(x_i)}$$

$$\langle \mathcal{O} \rangle \approx \frac{\sum_j \mathcal{O}_j}{\sum_j 1} \text{ where } j \text{ sampled with } p(x_j)$$

- As $N \to \infty$, $\langle \mathcal{O} \rangle \to \mathcal{O}_{\mathrm{exact}}$.
- Computable uncertainty which decreases as N grows!
- …but what if $p(x_i) \neq [0, 1]$ (e.g. e^{-S} is not real)

Monte Carlo presents a practical solution…



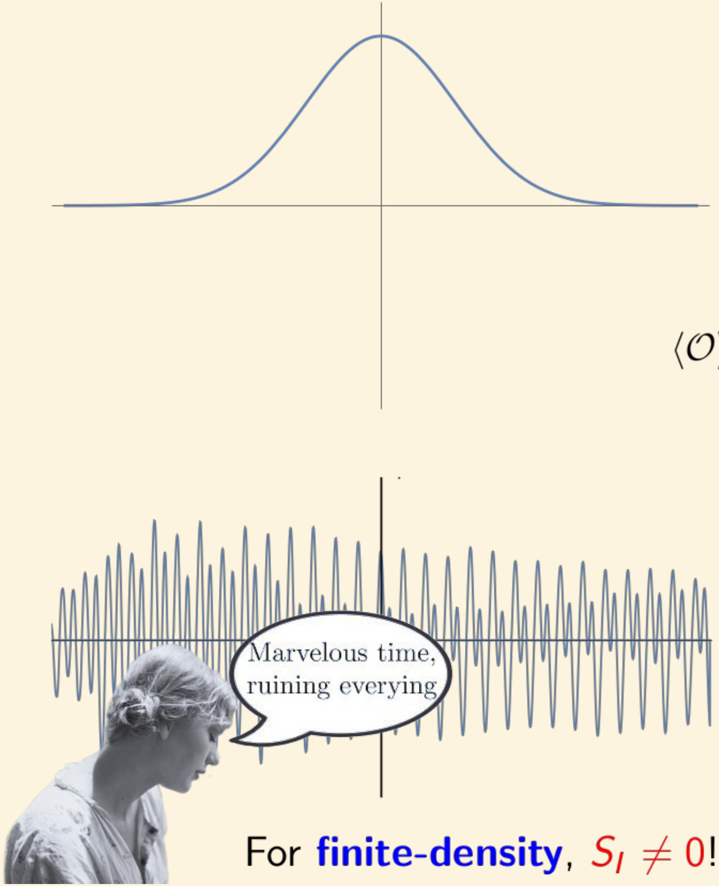
$$\langle \mathcal{O} \rangle \approx \frac{\sum \sigma_i \mathcal{O}_i |p(x_i)|}{\sum |p(x_i)|} \frac{\sum |p(x_i)|}{\sum \sigma_i |p(x_i)|}$$

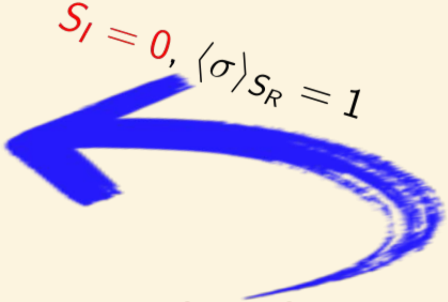
$$\langle \mathcal{O} \rangle \approx \frac{\sum_j \sigma_j \mathcal{O}_j}{\sum_j \sigma_j} \text{ where } j \text{ sampled with } |p(x_j)|$$

$$\frac{0}{1} \quad \frac{0}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{4} \quad \ldots \quad \frac{1708}{5125} \approx 0.33327$$

- Reweighting: assign probabilities $|p(x_i)|$ and make the relative sign, σ_i part of the observable
- …but what happens when the cancellations are strong?

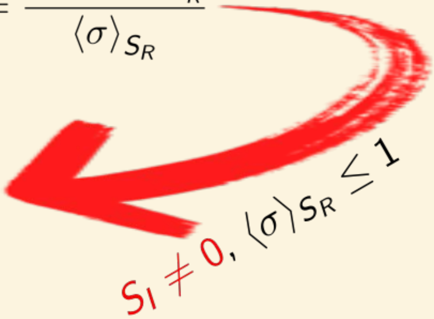
Sign problems stymie HEP





$$\langle \mathcal{O} \rangle = \frac{\int \mathcal{D}\phi\ e^{-iS_I} \mathcal{O} e^{-S_R}}{\int \mathcal{D}\phi\ e^{-S_R}} \frac{\int \mathcal{D}\phi\ e^{-S_R}}{\int \mathcal{D}\phi\ e^{-S_R} e^{-iS_I}}$$

$$= \frac{\left\langle \mathcal{O} e^{-iS_I} \right\rangle_{S_R}}{\langle \sigma \rangle_{S_R}}$$



For **finite-density**, $S_I \neq 0$! For **dynamics**, $S_R = 0$!

Put pithily….

$|\psi\rangle$ is a **complex-valued** probability amplitude

All I need is…(the industrial workforce of a small country)

$$\langle \psi_0 | e^{-iHt} \mathcal{O} e^{iHt} | \psi_0 \rangle$$

- **Prepare a state**
- **Time evolve the state**
- **Perform a measurement**

Where did all the matter come from (aka baryogenesis)?

As a closer target, consider the viscosity of QCD

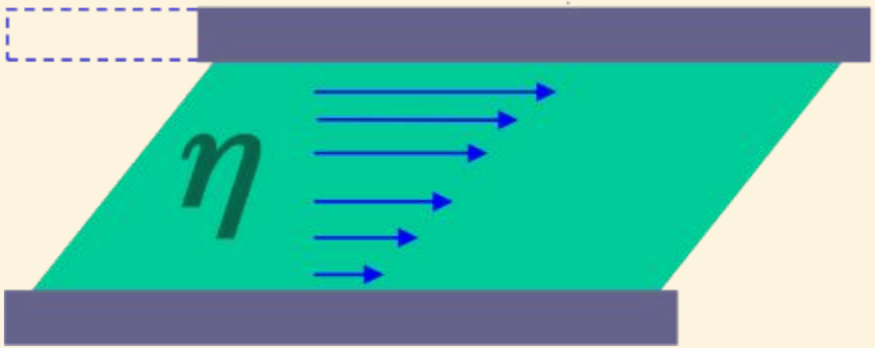
- $\eta = \frac{V}{T} \int_0^\infty \langle T_{12}(t) T_{12}(0) \rangle$

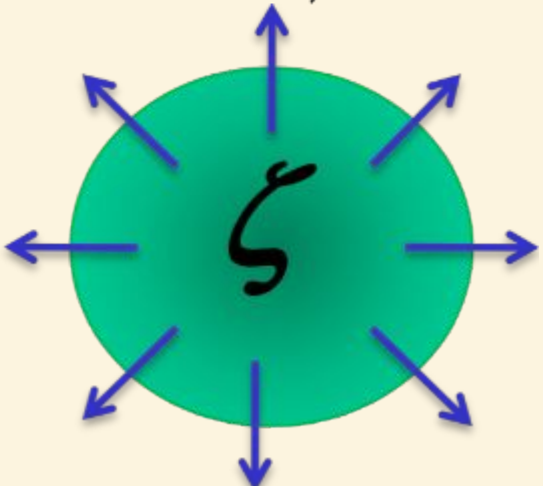
Quantum algorithms for transport coefficients in gauge theories
NuQS Collaboration - *Phys.Rev.D 104 (2021) 9, 094514*
Formulates lattice operators and propose correlators

- I believe its a **“near-term”** goal and allows for focus…
- …while introducing **all** the necessary pieces

Viscosity of pure-glue QCD from the lattice
Altenkort *et al.* - *2211.08230 [hep-lat]*
State of the art lattice results, but massive uncertainties persist

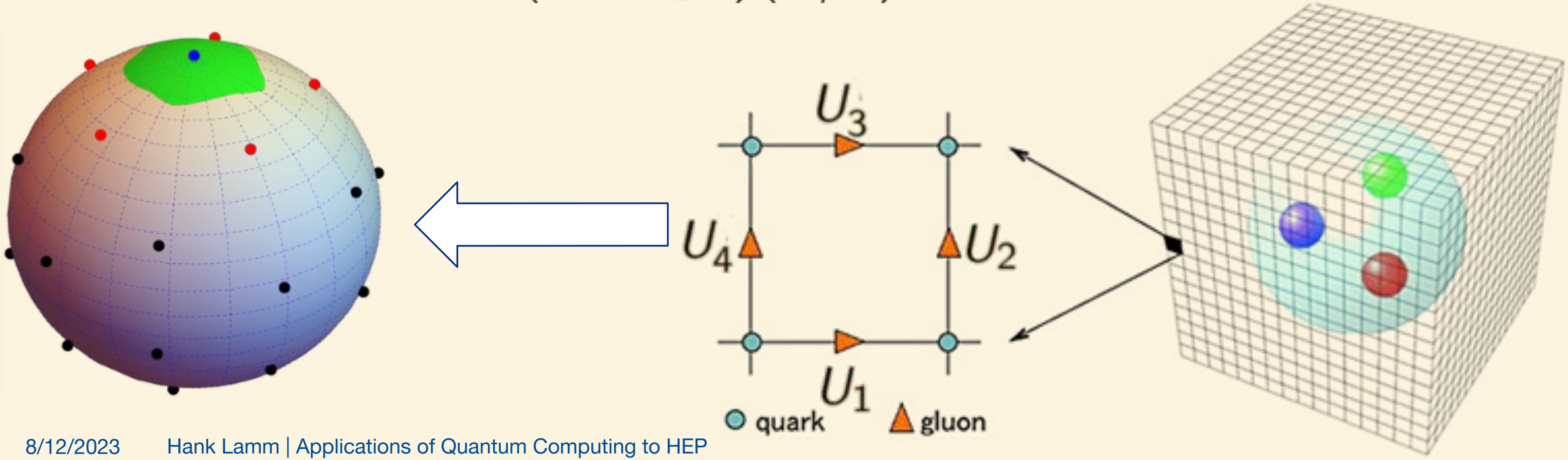
$$\eta/s = 0.15 - 0.48, T = 1.5T_c$$
$$\zeta/s = 0.017 - 0.059, T = 1.5T_c$$





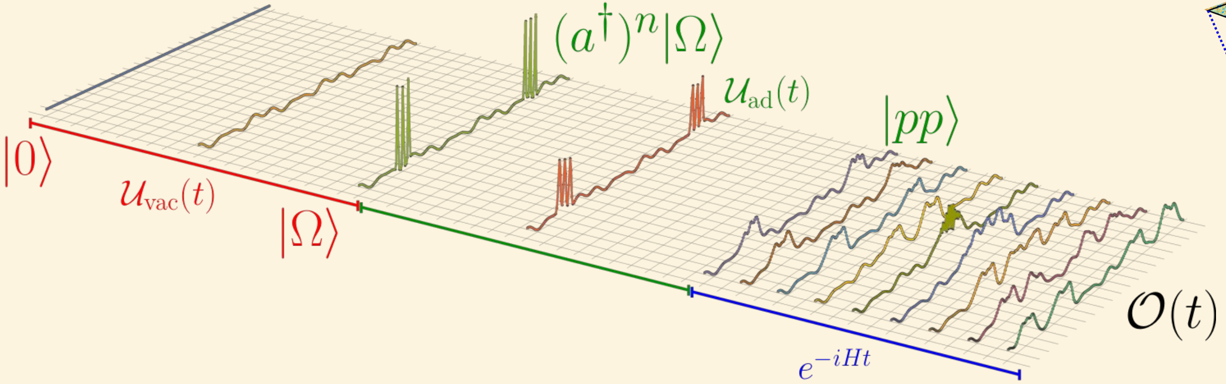
Qubit Costs for Lattice Field Theory

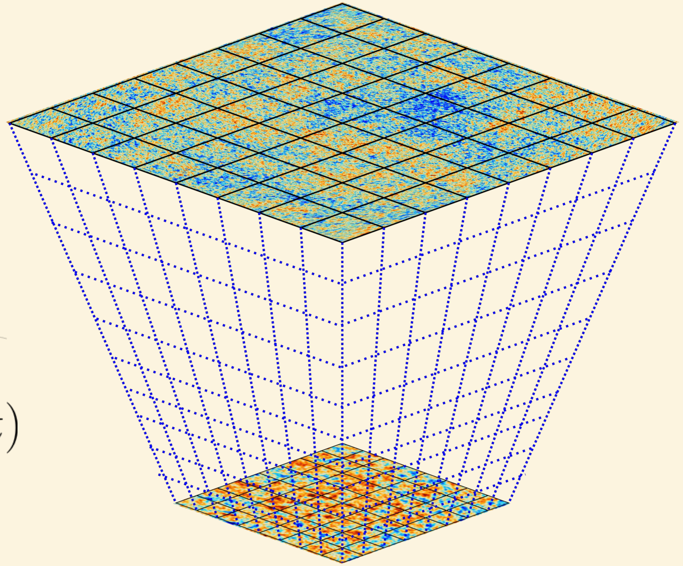
- Lattice field theory discretizes spacetime into a lattice of $(L/a)^d$ sites
  - $L\rightarrow\infty$ and $a\rightarrow 0$ must be taken
- Matter fields are placed on sites, gauge fields on links
  - Fermionic matter need **Ƒ=Spin x Color x Flavor qubits per site** e.g. 12 for staggered QCD
  - Gauge links are bosonic and need efficient truncation **Λ qubits per link** e.g. SU(3) ~ **???**q
- So **logical** qubit cost is: $(d\Lambda + F)(L/a)^d$



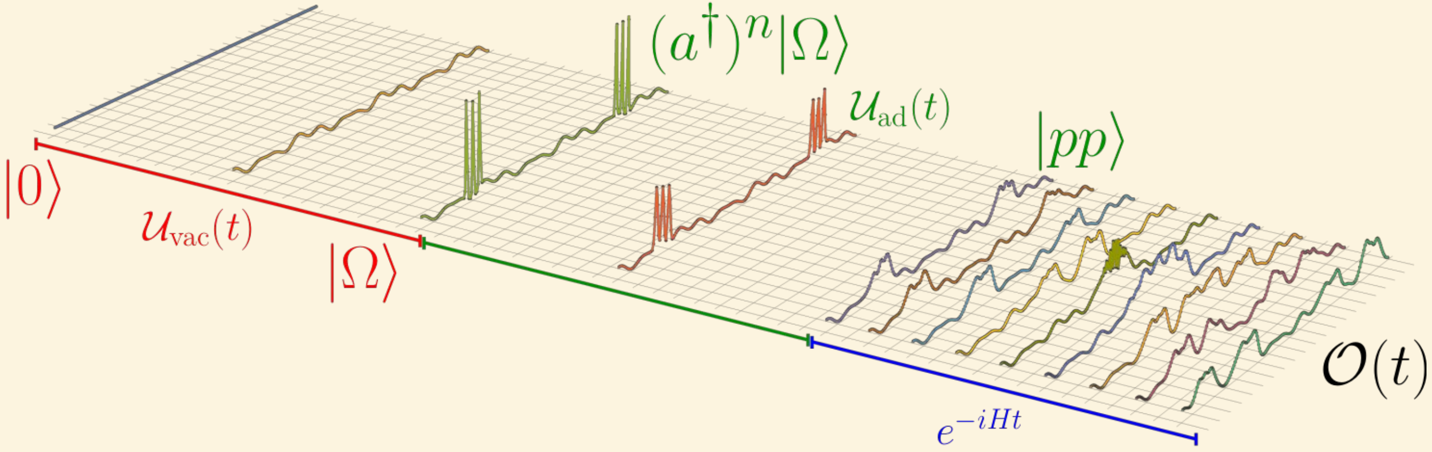
Gate Costs for Lattice Field Theory

- Approximating $U(T) = e^{-iHT}$ *can* corresponds to a lattice of size Ta_t
  - $a_t \to 0$ or equivalent limit must be taken
  - Trottertization has this property, others less clear i.e. potentially variable temporal spacing
- **Logical** gate cost is heuristically: $\frac{T}{a_t} \times [\mathcal{O}(1)(d\Lambda + F)(L/a)^d]^{\mathcal{O}(1)}$





It’s one calculation, Hank. What could it cost?



O(10^{11}) q and **O(10^{55})** T-gates

99.998% cost is **QFOPs** for **< 3 yrs** on an **exascale** QC

Lattice Quantum Chromodynamics and Electrodynamics on a Universal Quantum Computer
Kan and Nam - 2107.12769 [quant-ph]
Rough, conservative, model- and algorithm-dependent estimates for viscosity and heavy-ion collisions

Compare to **O(10^7) q** and **O(10^{20})** T-gates for RSA Cracking and Chemistry

Exercise 1: What will QCD viscosity take?

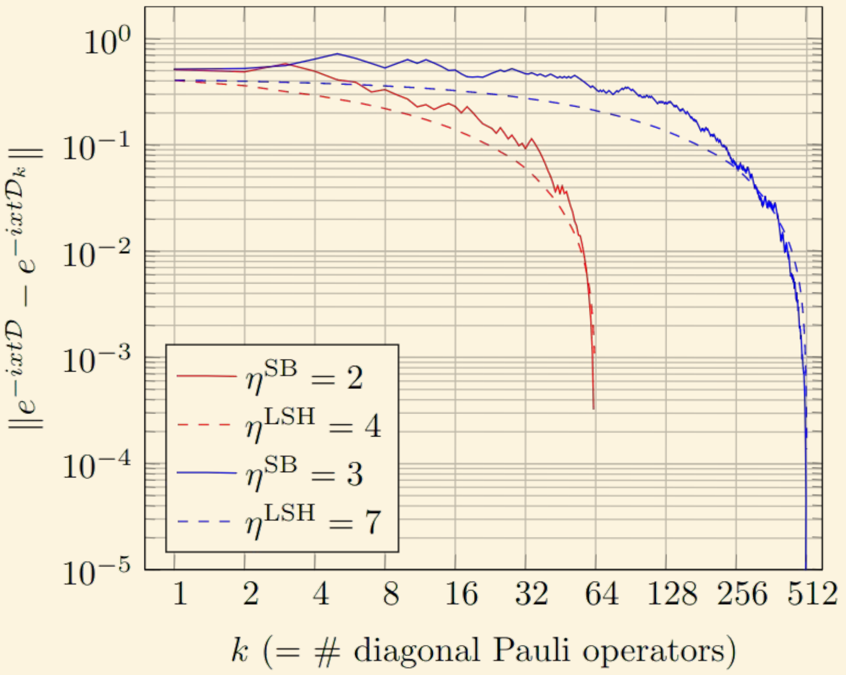
Qubits: $\mathcal{E}(d\Lambda + \mathcal{F})(L/a)^d$ Gates: $\frac{T}{a_t} \times [G_q \mathcal{E}(d\Lambda + \mathcal{F})(L/a)^d]^{\mathcal{G}}$

- What dimension, d? (3)
  - Note: **universality errors**
- How will you truncate Λ? (9 64-bit $\mathbb{C}$ ~ 10^3)
  - Note: **truncation errors**
- How large will you take L? (r_{proton} ~1fm)
  - Note: **finite volume errors**
- What QEC is needed, $\mathcal{E}$? (1-10^8)
  - Note: **quantum noise errors**
- How small will you take a_t?
  - Note: **Trotter errors**
- What is Ƒ? (Staggered=12; Wilson=24)
  - Note: **a_t scaling of errors**
- How small will you take a? (1fm^{-1} ~ 200 MeV)
  - Note: **discretization errors**
- How efficient is your algorithm (N_g/N_q) G_q?
  - Note: ***shrug* errors**
- How well approximated are your gates $\mathcal{G}$?
  - Note: **gate synthesis errors**
- How long do you need to run for (T)?
  - Note: **Signal resolution errors**

Knowledge of optimal choices are probably **years away**...**if you want a research project**

What didja get?

- **Qubit costs: 10^3-10^9**
  - **10q** for SU(3) might be reasonable
  - a~**0.5 fm**, L~**3 fm**
  - Perhaps we **drop** fermions
  - Perhaps **lower** dimensions
- **Gate costs: 10^7-10^{60}**
  - a_t~**0.1 fm**, T~**1 fm**
  - **Quantum arithmetic** hurts
  - Perhaps **sloppy** synthesis
  - Perhaps **improved** algorithms



General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

But we don't today have a good sense of **theoretical** errors…

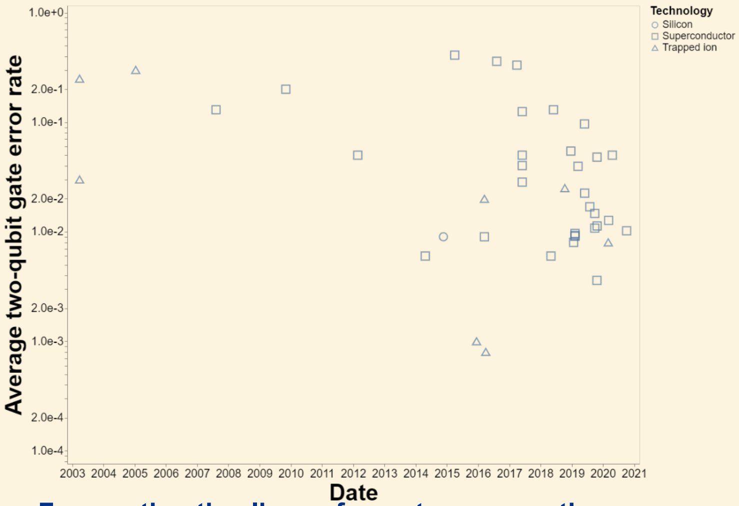
Exercise 2: What gate fidelities do you need?

- Consider your **gate cost** N_g
- Assume that every gate has a **infidelity** of p
- **"Simulation fidelity"** is $(1-p)^{N_g}$ i.e the probability your result is without error.

What must p be such that the simulation fidelity is 50%

10^{-8}-10^{-60}

Today, we talk about p ~10^{-3}

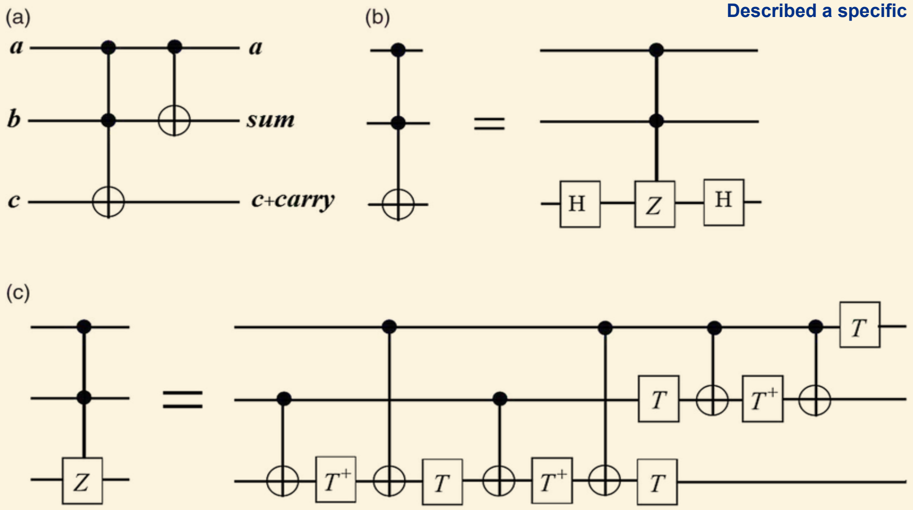


Forecasting timelines of quantum computing
Sevilla and Riedel - 2009.05045 [quant-ph]
Current hardware properties and projections for future

Regardless of your choice, you will need to do some math

- Arithmetic is **expensive** in qubits and gates
- Consider the half-adder

A transmon-based quantum half-adder scheme
Chatterjee and Roy- PTEP 2015 9, September 2015, 093A02
Described a specific hardware implementation of the general half-adder algorithm



"Never use a **quad**. Never use a **double** when **single** will do.
Never use a **float** when an **int** will do."

- Guy that learned to program in 1970

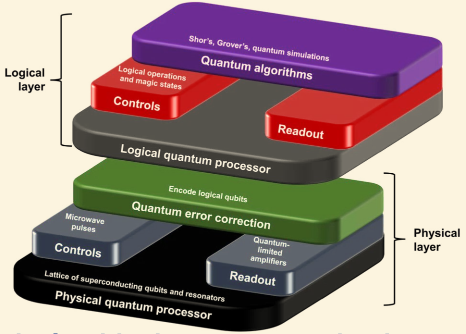
Noisy Intermediate-Scale Quantum vs Fault-Tolerance

NISQ

- Exists today!
- Limited number of qubits
  - Probably $<10^4$
- Basic gate set is native one
  - Often included arbitrary rotations
- Speed limited by 2q gate
- Errors tolerated or mitigated
  - Probably $>10^{-7}$
  - Measurement slow
  - Count CNOTs

FT

- Scalable, networked qubits
  - No limits on number of logical qubits
- Requires error correction
  - Potentially huge overhead
  - Threshold error rates
  - Measurement + Classical compute
- Gate set limited
  - Must synthesize
  - Count nontransverse T-gates



Building logical qubits in a superconducting quantum computing system
Gambetta, Chow, Steffen - npj Quantum Information 3, 2 (2017)
Discusses possible architectures for FT devices

Now, you may be depressed. Why choose such a hard problem?

Why does Rice play Texas?

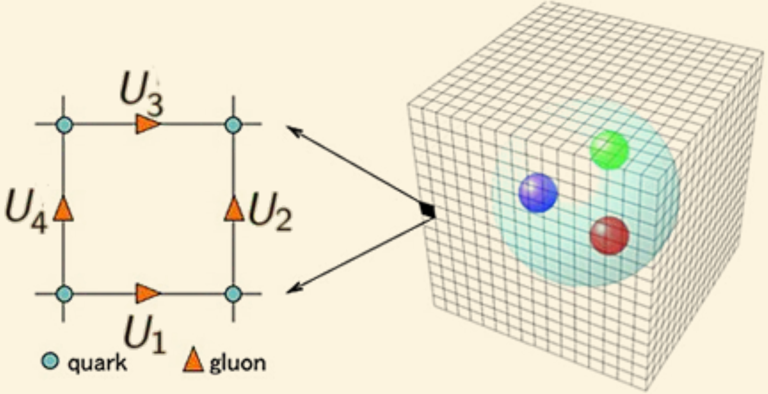
Now, you may be depressed. Why choose such a hard problem?

Why do we choose to go to the moon?

not because they are **easy**, but because they are **hard**, because that goal will serve to organize and measure the **best of our energies and skills**

- Discovering the Higgs Boson - 50 years
- Direct Detection of Gravitational Waves - 40 years
- *High-Precision Lattice QCD Confronts Experiment* - 30 years

Kogut-Susskind Hamiltonian



$$H_{KS} = \sum_n m_n \psi_n^\dagger \psi + \sum_{n,k} [\psi_n^\dagger U_{nk} \psi_{n+k} + h.c.]$$

Fermionic mass term   Fermionic kinetic or *hopping* term

$$+ \sum_n E_n^2 + \sum_{n,k} \mathrm{Re}\mathrm{Tr}\, U_p$$

Gauge **E** field   Gauge **B** or *plaquette* term

Hamiltonian Formulation of Wilson's Lattice Gauge Theories
Kogut & Susskind *Phys.Rev.D 11 (1975) 395-408*
Formulated $O(a^2)$ lattice Hamiltonian for LGT with staggered matter

So many choices of fermions…

Nielsen-Ninomiya theorem

Assuming **locality, hermiticity, and translational symmetry**, any lattice **chiral** fermions have **doublers**

- *Staggered (KS) Fermions*: Spin-taste components on different lattice sites in hypercube
- *Wilson Fermions:* add a new term to give additional mass to doublers
- *Domain wall Fermions*: Increase dimensionality
- *Overlap Fermions*: Use nonlocal operator to remove doublers
- ...others

These are **categories**, which can be **improved** to remove **lattice artifacts**

Not all are formulated in Hamiltonian (aka for QC) **...if you want a research project**

Fermions, someone else's problem?

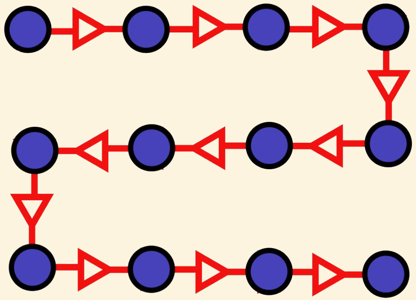
Most quantum computers are built from **bosonic** degrees of freedom

This is a **problem**…. since fermions **anticommute**!

Fermionic state are **fully** antisymmetric $\Longrightarrow$ **nontrivial** map to qudits

Most common…but there are others

- *Jordan-Wigner:* $a_j = -(\otimes_{k=1}^{j-1} Z_k) \otimes \sigma_j \Longrightarrow$ Good in **1+1d** but…



What about fermionic QC or QEC+fermionic encodings? **…if you want a research project**

Fermion-qudit quantum processors for simulating lattice gauge theories with matter
Zache, González-Cuadra, Zoller 2110.10280 [quant-ph]
Coupling fermionic atoms to qudit architecture

Logical fermions for fault-tolerant quantum simulation
Landahl & Morrison 2110.10280 [quant-ph]
How can fermionic logical states be efficiently constructed from physical bosonic states

Hamiltonians for Gluons

$$H = \int d^d x \, \mathrm{Tr}(\mathbf{E}^2 + \mathbf{B}^2)$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu] \qquad E_i = \frac{1}{2} F_{ii} \qquad B_i = \frac{1}{2}\epsilon_{ijk} F_{jk}$$

- Approximating A_μ is **fraught** with danger
- Instead ***average*** along a particular direction $\mathcal{A}_\mu = \frac{1}{a}\int_\mu d\mathbf{x} \cdot \mathbf{A}$
- Then we have a ***Wilson line*** or **gauge link**:

$$U_l(x) = e^{ieaA_l} \approx 1 + ieaA_l(x) - \frac{e^2 a^2}{2!} A_l(x) A_l(x)$$

- From this, one can derive the **lattice electric field**

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Lattice Kinetic Energy

- With this definition and imposing gauge invariance, we find:

$$\mathrm{Tr}[\mathbf{E}^2(\mathbf{x})] \approx \frac{g^2}{2a}\mathrm{Tr}[X\mathcal{E}_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}) + Y\mathcal{E}_i(\mathbf{x})U_i(\mathbf{x})\mathcal{E}_i(\mathbf{x}+a\hat{i})U_i^\dagger(\mathbf{x})]$$

- Expanding E and U in terms of their continuum fields, we find

$$K = \frac{X+Y}{2}E_i^2 + \frac{5Y-X}{12}E_i\partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

- For X=1, Y=0 we obtain the E_{KS} with **errors $O(a_s^2)$**

Exercise 3a: Improved Lattice Kinetic Energy

- What values of X,Y would cancel of all classical a^2 errors?

$$K = \frac{X+Y}{2} E_i^2 + \frac{5Y-X}{12} E_i \partial_i^2 E + \mathcal{O}(ea^2, a^4)$$

Lattice Potential Energy

- Only **closed** loops of Wilson lines
- The simplest *Wilson loop* is the **plaquette**:

$$P_{xy} = 1 - \frac{1}{N}\mathrm{ReTr}[U_x(\mathbf{x})U_y(\mathbf{x}+a\hat{\mathbf{x}})U_x^\dagger(\mathbf{x}+a\hat{\mathbf{y}})U_y^\dagger(\mathbf{x})]$$

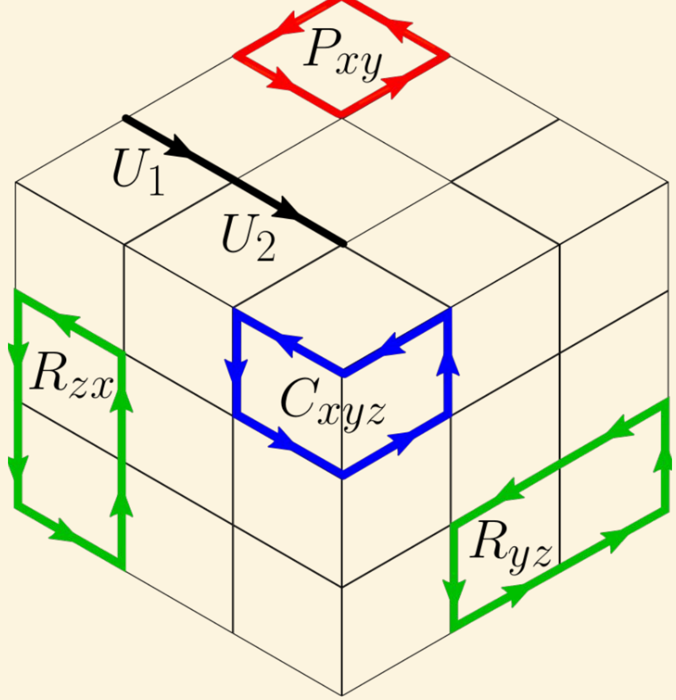
- In the continuum limit, this becomes $Tr(\mathbf{B}^2)$
- Including **rectangles** yields:

$$V = \frac{2N}{ag^2}[XP_{ij}(\mathbf{x}) + \frac{Y}{2}(R_{ij}(\mathbf{x}) + R_{ji}(\mathbf{x})]$$

- Which is related to the continuum:

$$V \approx a^d[(X+4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2+D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)]$$

- If satisfied with **$O(a_s^2)$ errors**, X=1,Y=0 yields the B_{KS}



Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

Exercise 3b:

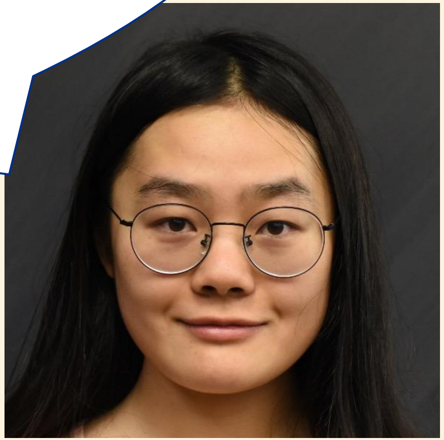
- **What values of X and Y will yield an a² improved Hamiltonian?**

$$V \approx a^d[(X+4Y)\mathrm{Tr}(F_{ij}^2) + \frac{a^2}{12}(X+10Y)\mathrm{Tr}(F_{ij}\{D_i^2+D_j^2\}F_{ij}) + \mathcal{O}(e^2a^2, a^4)]$$

Improvement and analytic techniques in Hamiltonian lattice gauge theory
Carlsson - PhD thesis, 0309138 [hep-lat]
Derivation of KS and Improved Hamiltonians and variational techniques

So what did we gain by doing math?

Comparing to the commonly used H_{KS}, H_I should allow quantum simulations with $>2^d$ fewer qubits…[with a gate count] comparable or less than that of H_{KS} for theories with $d \geq 2$…

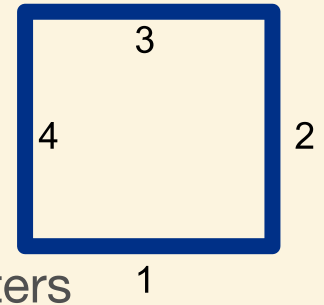


Wanqiang Liu
Graduate Student
U Chicago

Improved Hamiltonians for Quantum Simulation of Gauge Theories
Carena, Lamm, Li, Liu *PRL 129 (2022) 5*
Developed quantum circuits for $O(a^4)$ pure-gauge Hamiltonian

Primitives as subroutines

$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \text{ReTr} U_1 U_2 U_3^\dagger U_4^\dagger$$



- Lattice ***gauge*** theories require gauge group operations on registers
  - Think **native gates** for gauge theories

- U_i and E_i are related* by **group Fourier transform** (gFT)
  - *Depending on your digitization, relations are broken an “approximate” gFT exists

- Further we need to
  - **Inversion**: g -> g^{-1}
  - **Multiplication**: g,h -> gh
  - **Trace**: Tr(g)

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Primitives as gates

Imagine instead of qubits, you have collection of qubits – *registers*

Alternatively think of a datatype with operations like `int`, `bool`, `single`, `array`… **but not `double`**

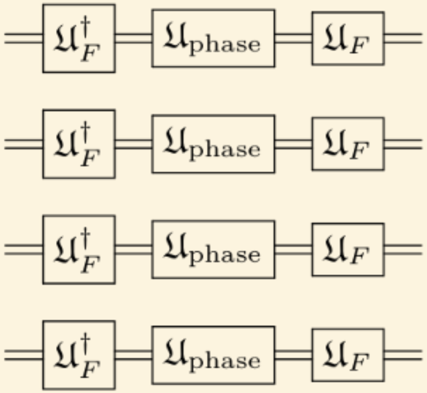
- Inversion gate: $\mathfrak{U}_{-1}\left|g\right\rangle = \left|g^{-1}\right\rangle$
- Multiplication gate: $\mathfrak{U}_{\times}\left|g\right\rangle\left|h\right\rangle = \left|g\right\rangle\left|gh\right\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)\left|g\right\rangle = e^{i\theta\,\mathrm{Re}\,\mathrm{Tr}\,g}\left|g\right\rangle$
- Fourier Transform gate: $\mathfrak{U}_{F}\sum_{g\in G} f(g)\left|g\right\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij}\left|\rho, i, j\right\rangle$

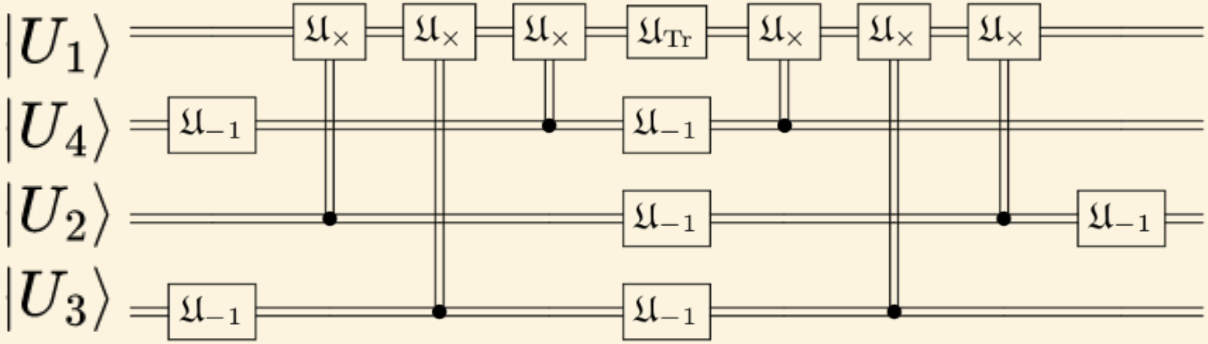
General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Circuits for Kogut-Susskind without regard for connectivity

- With these gates, the evolution operators are given for Kogut-Susskind by:

$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \mathrm{ReTr} U_1 U_2 U_3^\dagger U_4^\dagger$$





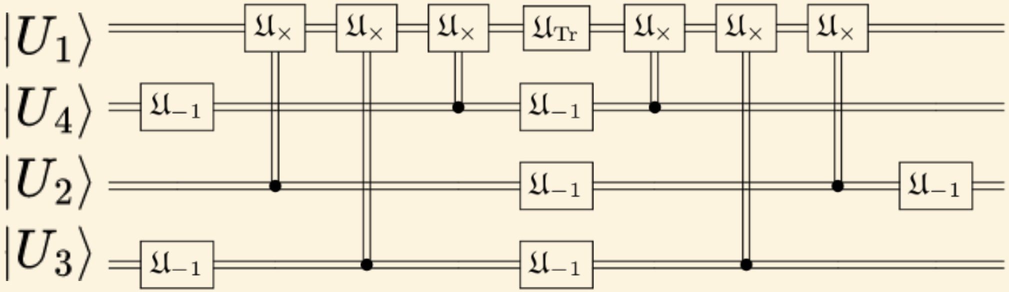
- Need **Λ A2A in-register** and **1:(2d) register** connectivity

General Methods for Digital Quantum Simulations of Gauge Theories
Lamm, Lawrence, Yamauchi - *Phys.Rev.D* 100 (2019) 3, 034518
Constructed this general formalism for group independent implementation

Exercise 6: $U_{V,KS}$ with only linear register connectivity

- Real hardware has limited connectivity.
- The $U_{V,KS}$ assumed 1:3 register connectivity

$$\mathrm{Re}\,\mathrm{Tr}\,U_1 U_2 U_3^\dagger U_4^\dagger$$



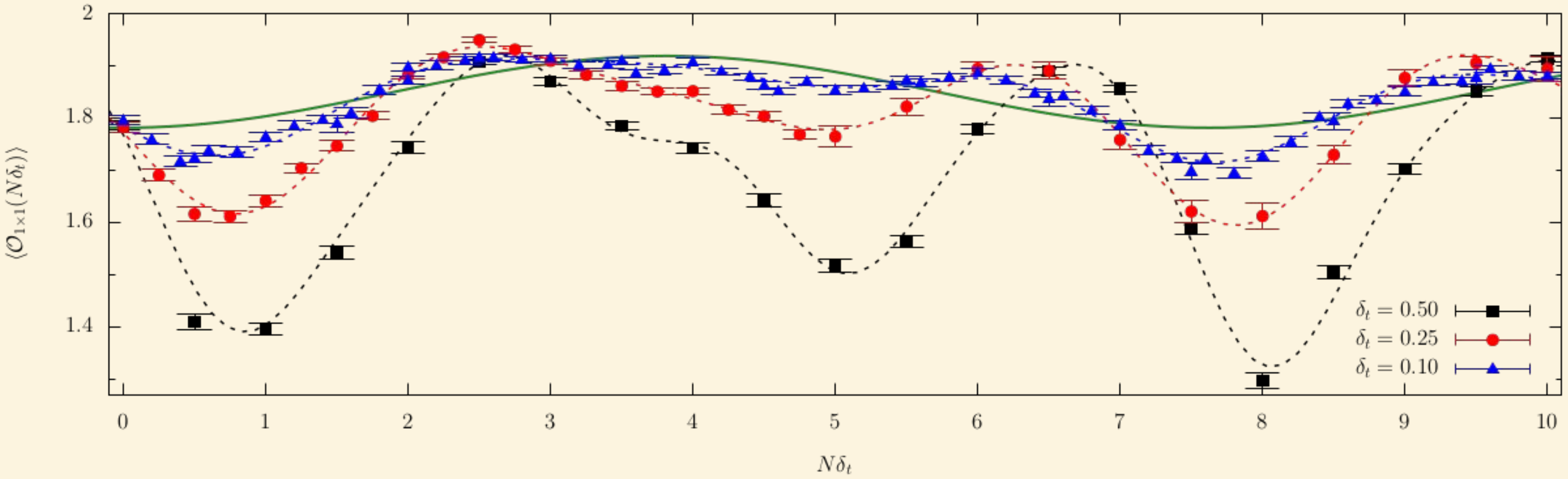
- Inversion gate: $\mathfrak{U}_{-1}|g\rangle = |g^{-1}\rangle$
- Multiplication gate: $\mathfrak{U}_{\times}|g\rangle|h\rangle = |g\rangle|gh\rangle$
- Trace gate $\mathfrak{U}_{\mathrm{Tr}}(\theta)|g\rangle = e^{i\theta\,\mathrm{Re}\,\mathrm{Tr}\,g}|g\rangle$

- **Can you construct a $U_{V,KS}$ where only linear (nearest-neighbor) register interactions?**
  - **It might prove useful to consider** $\mathcal{U}_{\times}^{R}|g\rangle|h\rangle = |gh\rangle|h\rangle$

What is trotterization?

$$\mathcal{U}(t) = e^{-iHt} \approx \left(e^{-i\delta t \frac{H_V}{2}} e^{-i\delta t H_K} e^{-i\delta t \frac{H_V}{2}} \right)^{\frac{t}{\delta t}}$$

$$\approx \exp\left\{ -it \left(H_K + H_V + \frac{\delta t^2}{24} (2[H_K, [H_K, H_V]] - [H_V, [H_V, H_K]]) \right) \right\}$$



How to estimate Trotter errors

- Loose error bounds obtained from

$$||U(t) - U_{trott}(t))|| \leq (\delta t)^n \sum_{i,j,\cdots}[[H_i, H_j], \cdots]$$

General quantum algorithms for Hamiltonian simulation with applications to a non-Abelian lattice gauge theory
Davoudi, Shaw, Stryker - 2212.14030 [hep-lat]
Understanding the synthesis and Trotter errors, along with algorithmic choices in 1+1 SU(2)

$$\begin{aligned}
\|\mathcal{O}_3\| &= \left\|\left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r)]\right]\right\| && \leq 4x^3 \quad (k > j),\\
\|\mathcal{O}_5\| &= \left\|\left[H_I^{(j)}(r), [H_I^{(j)}(r), H_I^{(k)}(r+1)]\right]\right\| && \leq 4x^3,\\
\|\mathcal{O}_{13}\| &= \left\|\left[H_I^{(l)}(r), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| && \leq 4x^3 \quad (k > j, l > j),\\
\|\mathcal{O}_{14}\| &= \left\|\left[H_M(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| && \leq 4x^2\mu \quad (k > j),\\
\|\mathcal{O}_{15}\| &= \left\|\left[H_I^{(l)}(r+1), [H_I^{(k)}(r), H_I^{(j)}(r)]\right]\right\| && \leq 4x^3 \quad (k > j),\\
\|\mathcal{O}_{19}\| &= \left\|\left[H_I^{(l)}(r), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| && \leq 4x^3 \quad (l > j),\\
\|\mathcal{O}_{20}\| &= \left\|\left[H_M(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| && \leq 4x^2\mu,\\
\|\mathcal{O}_{22}\| &= \left\|\left[H_I^{(l)}(r+1), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| && \leq 4x^3,\\
\|\mathcal{O}_{24}\| &= \left\|\left[H_I^{(l)}(r+2), [H_I^{(k)}(r+1), H_I^{(j)}(r)]\right]\right\| && \leq 4x^3.
\end{aligned}$$

- **Overly** conservative: cutoff states are largest EV
  - Empirically, we find MUCH smaller

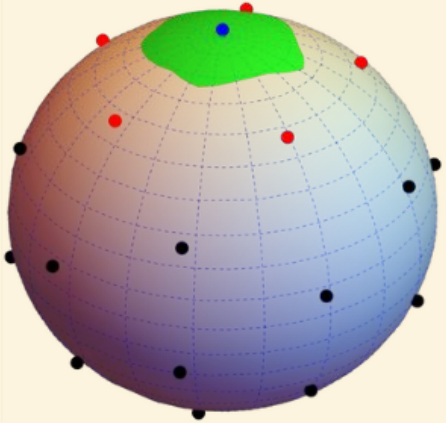
State-dependent error bound for digital quantum simulation of driven systems
Hatomura - PRA 105, L050601 (2022)
Compares trotter errors for given initial state to norm-based estimates

- Can we use Euclidean calculations to compute?

...if you want a research project

Digitization of lattice gauge theories

- Need to map **infinite-dimensional** Hilbert space of gauge field to **finite** quantum register built from qudits



- This is **not** a trivial decision, it breaks some symmetries and are simulating

$$H + \hat{O}_{trunc}$$

How to choose a side?

$$H_{KS,1} = \sum_{i=\text{color}} E^2_{1,i}(n) + \sum_{k=direction} \text{ReTr} U_1 U_2 U_3^\dagger U_4^\dagger$$

E (irreducible representation) basis

Trailhead for quantum simulation of SU(3) Yang-Mills lattice gauge theory in the local multiplet basis
Ciavarella, Klco, Savage *Phys.Rev.D 103 (2021) 9, 094501*
Qubit implementation of SU(3) with irrep truncations

Mixed basis

A new basis for Hamiltonian SU(2) simulations
Bauer, D'Andrea, Freytsis, Grabowska
Formulated an alternative basis that contains parts of E & B basis

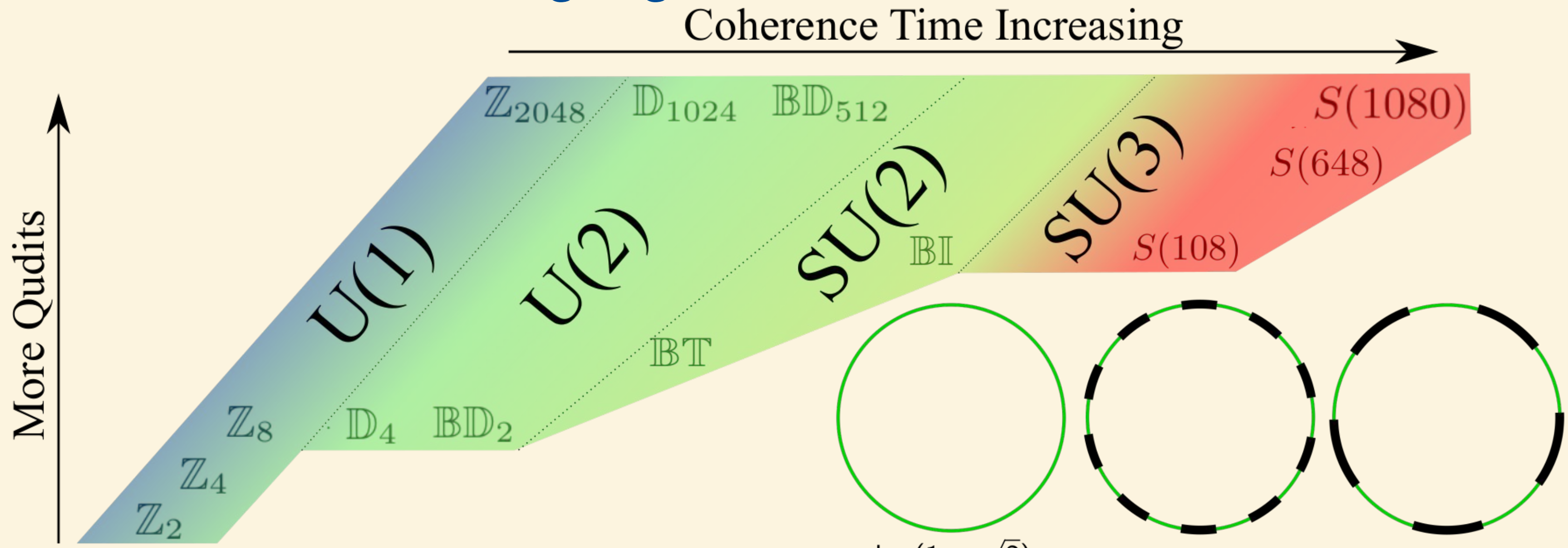
B (group element) basis

Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT
Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501
Qubit and Qudit gates for approximating SU(2) with subgroups

Well, what keeps **you** up at night?

arbitrary precision, gauge fixing, quantum noise, error correction, gate costs, classical simulatability

The ladder of discrete gauge theories in HEP calculations



Gluon Field Digitization for Quantum Computers
NuQS collaboration - *Phys.Rev.D 100 (2019) 11, 114501*
Demonstrated that S(1080) approximates certain 3+1d SU(3) observables

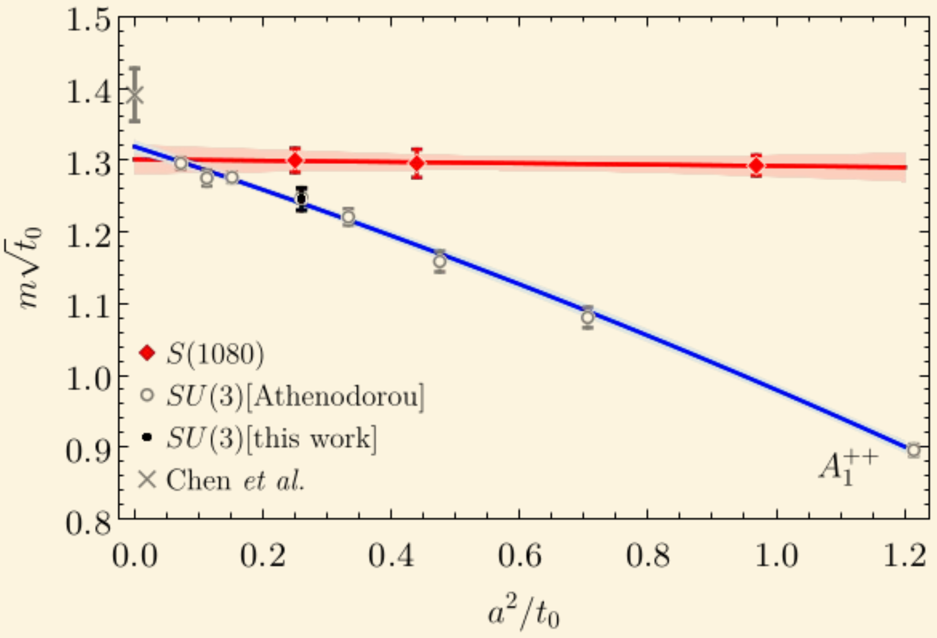
$$\beta_{f,U(1)} = \frac{\log(1+\sqrt{2})}{1-\cos\left(\frac{2\pi}{N}\right)} \approx \kappa_2 N^2, \text{ which extends to } \beta_{f,SU(N_c)} \approx \kappa N^{\frac{N_c^2-1}{2}}$$

But whereas $\mathbb{Z}_N$ can be **taken to ∞**, **limited** number for $SU(N_c)$

$$\beta \propto \frac{1}{\log(a)} \implies a_f \propto e^{-\beta_f}$$

Digitising SU(2) gauge fields and the freezing transition
Hartung *et al.* - *Eur.Phys.J.C 82 (2022) 3, 237*
Understanding the scaling of freezing transitions with approximations

…but why use Kogut-Susskind Hamiltonian?



Glueballs at $a = 0.08$ fm $\rightarrow$ 10^3 **lattices** $\sim 10^5$ **lq**

Spectrum of digitized QCD: Glueballs in a S(1080) gauge theory
Alexandru *et al.* - *Phys.Rev.D 105 (2022) 11, 114508*
Can the low-lying spectrum of an S(1080) approximate SU(3)

How do we represent discrete groups?

- Ordered product of generators

$$h_{\{o_k\}} = \prod_k \lambda_k^{o_k} = h_d$$

$$\mathbb{D}_4 : \qquad h_d = \boldsymbol{s}^a \boldsymbol{r}^b$$

$$\mathbb{Q}_8 : \qquad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c$$

$$\mathbb{BT} : \qquad h_d = (-1)^a \mathbf{i}^b \mathbf{j}^c \mathbf{l}^d$$

$$\Sigma(36 \times 3) : \qquad h_d = \omega_3^a \mathbf{C}^b \mathbf{E}^c \mathbf{V}^d$$

$$\rightarrow |abcd \cdots\rangle$$

These integers are not all binary, so naturally more robust and easier on **qudits!**

Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Discusses quantum registers with qubits, qudits for U(1), SU(2), SU(3)

Exercise 7: Inverse operation for D_4

Consider the group $\mathbb{D}_4$: $\quad h_d = s^a r^b$

which have the relations: $srs = r^{-1} = r^3, \; sr = r^3 s, \; s = s^{-1}$

What is $h_d^{-1} = (s^a r^b)^{-1}$ in the standard presentation?

Exercise 8 : Inverse primitive gate for D_4

$$(s^a r^b)^{-1} = s^a r^{(3-b)(1-a)+ab}$$

- Can you construct a

$$\mathcal{U}_{-1}|ab_0b_1\rangle \rightarrow |a'b_0'b_1'\rangle$$

On three qubits ? one quoctit? one qubit & one ququart?

Group Primitives for BT

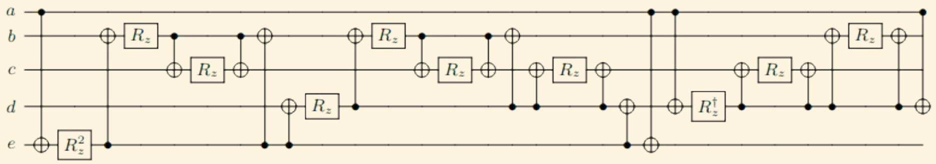


FIG. 4. Trace gate for BT

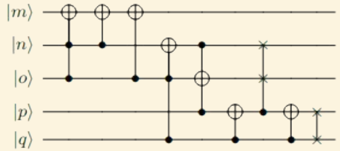


FIG. 2. Inversion Gate for the Binary Tetrahedral Group.

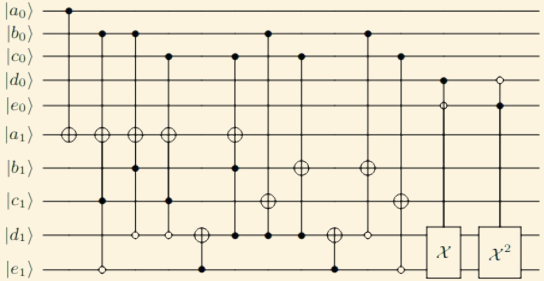
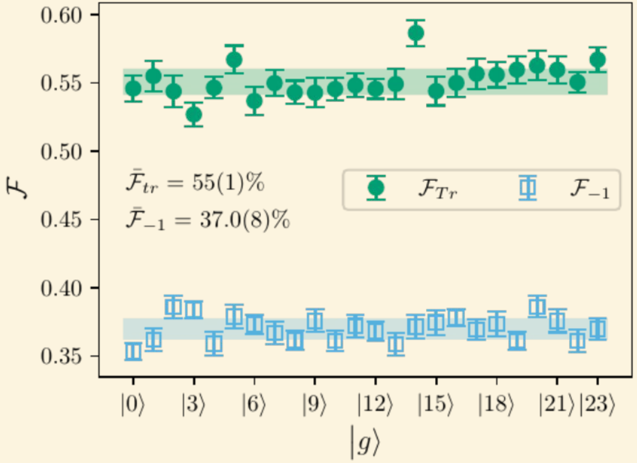
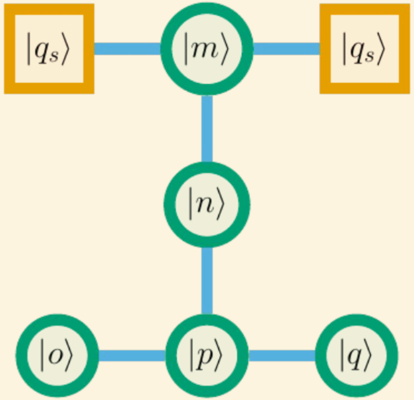


FIG. 3. Multiplication gate

Felicity Lovelace
University of Illinois, Chicago





Primitive Quantum Gates for an SU(2) Discrete Subgroup: BT

Gustafson, Lamm, Lovelace, Musk - *Phys.Rev.D* 106 (2022) 11, 114501

Derived and implemented using custom QEM necessary primitives for HEP simulations

Resource Estimation for Lattice Simulations of Z_2, BT, S(1080)

TABLE I. C^nNOT gates required for $\mathbb{BT}$ (top) primitive gates (bottom) H_I simulations per link per δt.

Gate	CNOT	C^2NOT	C^3NOT
$\mathcal{U}_{-1}$	6	4	0
$\mathcal{U}_{\times}$	5	8	4
$\mathcal{U}_{Tr}$	20 QFT→O(100)	0	0
$\mathcal{U}_{FT}$	1025	0	0
$e^{-iH_I\delta t}$	$226d+3906$	$252d-212$	$104d-88$

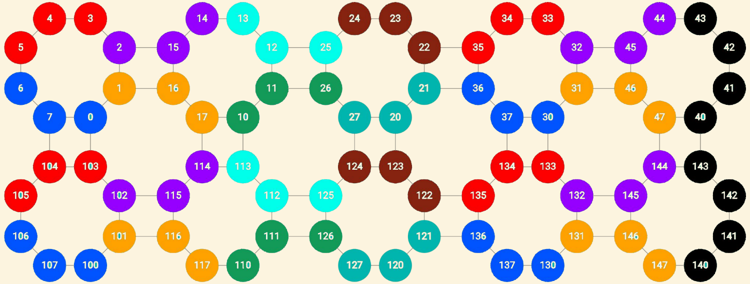
Gate depth rather than memory limits options

Proof of concept — **2x1 Lattice**
- 7 links → {7, 35, 77}q
- N_t=1→ {28, 70k, ~200m} CNOTs*

Nontrivial Physics — **4x4 Lattice**
- 24 links → {24, 120, 264}q
- N_t=10→{96, 220k, ~700m} CNOTs*

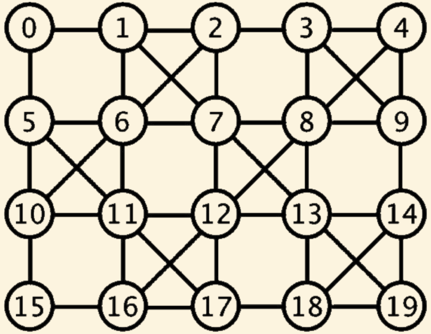
HEP Applications — **10^3 Lattice**
- 3k links → {3k, 15k, 33k}q
- N_t=10→{4m, 450m, ~10t} CNOTs*

Qudits, Qudits, my kingdom for some qudits…

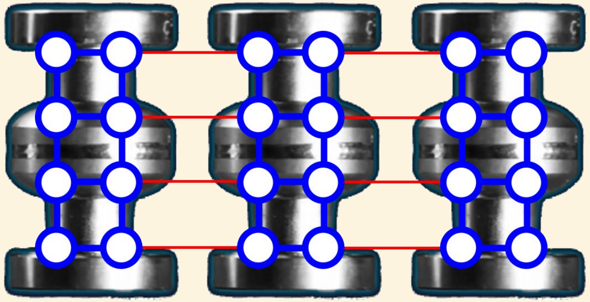


Multi-qubit |g>

Single qudit |g>

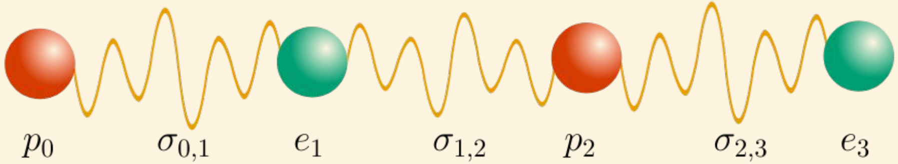


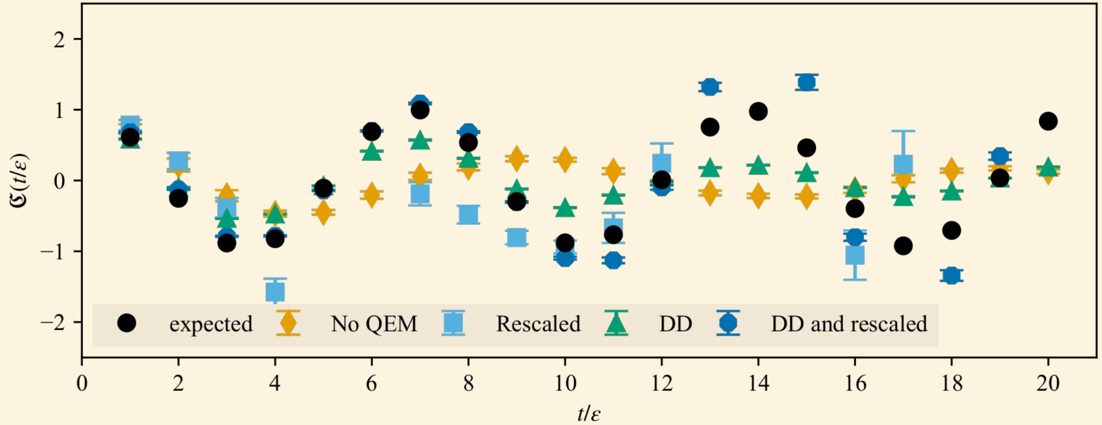
$$\mathcal{F}_2^{N_q(N_q-1)} = \mathcal{F}_{2^N_q}$$

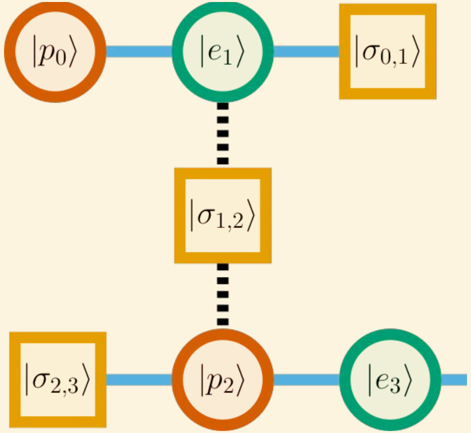


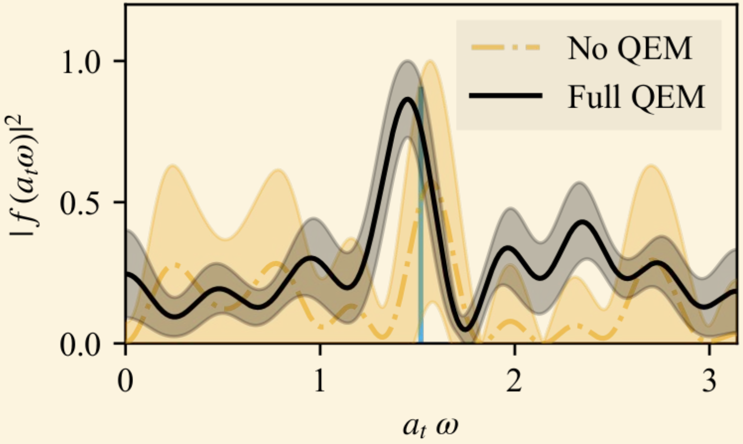
Multi-|g> qudit

Let see a real example! 1+1 Z_2 gauge theory + matter on 7q





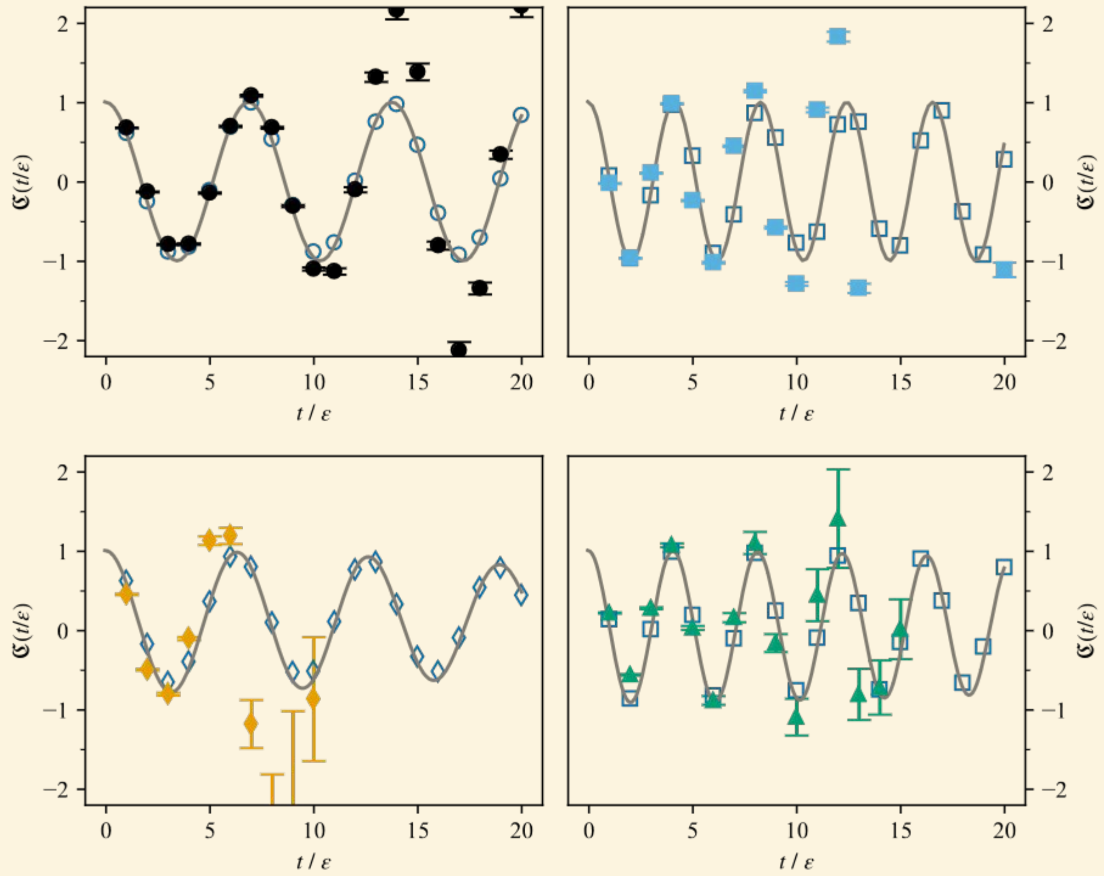


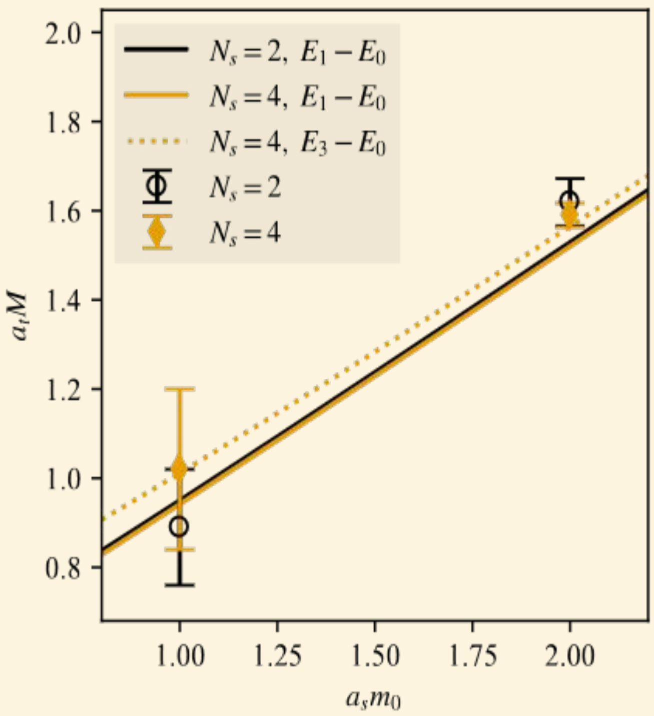


Error mitigation allows up to **6x** longer evolution

Simulating Z2 lattice gauge theory on a quantum computer
Charles et al. - 2305.02361 [hep-lat]
Performed scale setting and multiple QEM on LGT

Multiple volumes, multiple masses



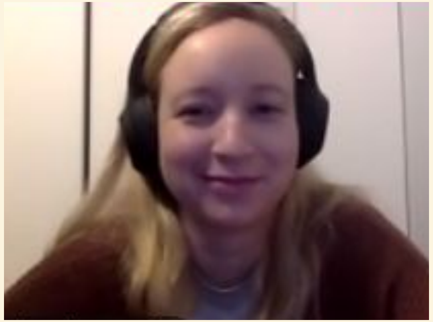


N_s	m_0	ε	$a_t M$	$(a_t M)_{\text{exact}}$
2	1.0	0.3	0.89(13)	0.9473
4	1.0	0.3	1.02(18)	0.9386
2	2.0	0.3	1.619(53)	1.5204
4	2.0	0.3	1.591(27)	1.5168

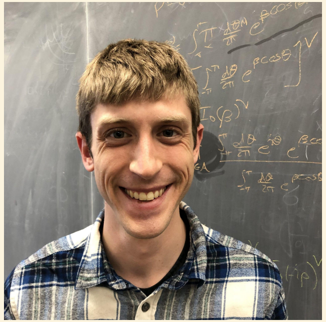
That work is an outcome of the QCIPU program @ Fermilab

Ruth Van de Water
Fermilab

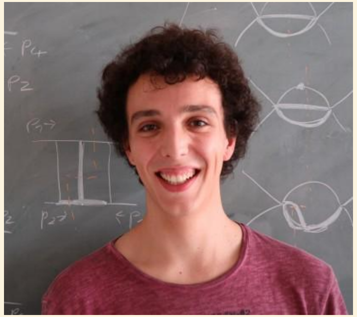
Clement Charles
Now Grad Student @ Maryland



Elizabeth Hardt
Now Argonne



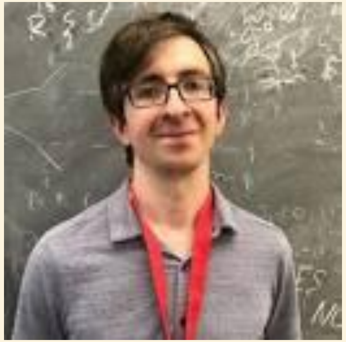
Michael Wagman
Fermilab



Florian Herren
Fermilab

Sara Starecheski
Undergrad @ UIUC

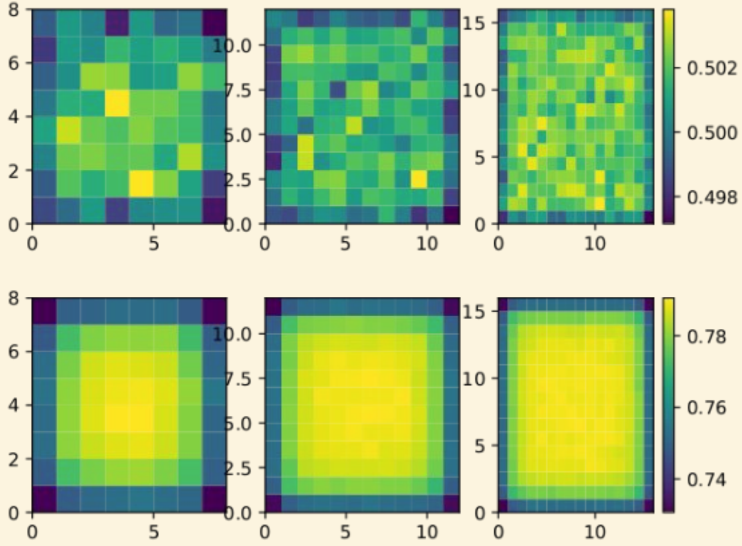
Norman Hogan
Now Grad Student @ NCSU



Erik Gustafson
NASA

Periodic Boundary Conditions are *HIGHLY* desirable

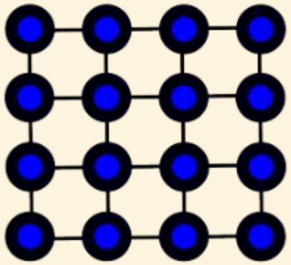
$$\langle O(t)\rangle_{OBC} \approx \langle O(t)\rangle_{PBC} + Ae^{-mT/2}\cosh m\left(\frac{T}{2}-t\right)$$



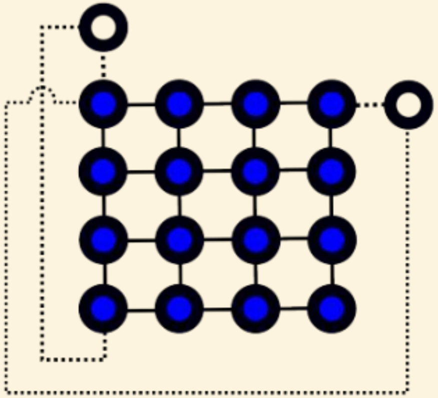
To obtain same results as L^d_{PBC} requires $[x(a)L]^d_{OBC}$

where $x(a) > 1$ grows with a

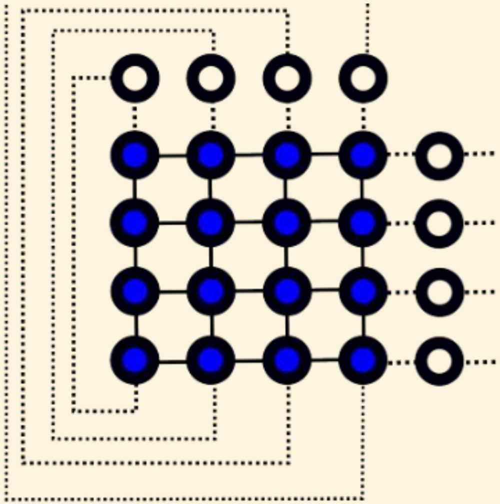
SWAPs, Routes, and *Circuit Cutting*



SWAP all boundaries



SWAP thru routing



Boundaries connected

Going to right you are **infuriating** experimentalists more

For gauge registers, should determine fidelity thresholds

Circuit Cutting

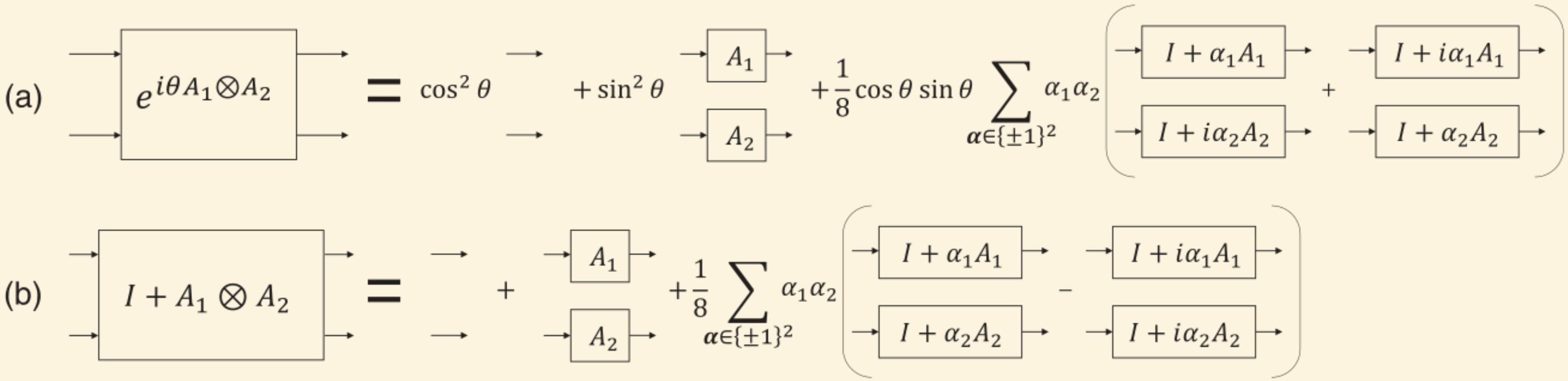
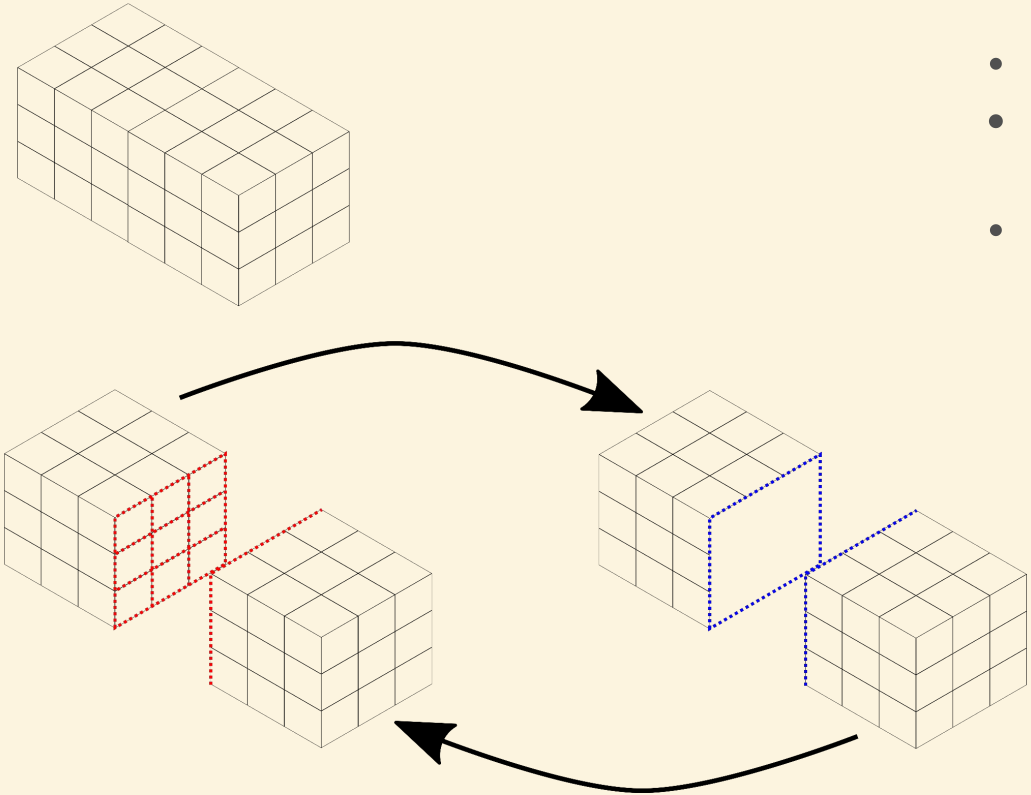


Figure 1. Decomposition of (a) a non-local gate and (b) a non-local non-destructive measurement into a sequence of local operations. A_1 and A_2 are operators such that $A_1^2 = I$ and $A_2^2 = I$.

Constructing a virtual two-qubit gate by sampling single-qubit operations
Mitarai, Fujii - New J. Phys. 23 023021 2021
A particularly good explanation and lit review of topic

Multigrid and Circuit Knitting



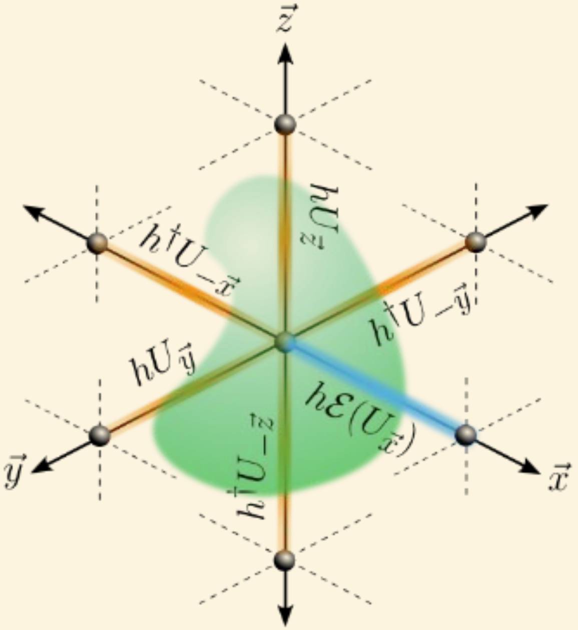
- Circuit Knitting has **<O(9^N) scaling**
- **Quasiprobabilities** will increase costs
  - Sign problem!
- Reduce this for LFT through **multigrid** techniques?
  - Split larger **lattice** → **sublattices**, 1 per QPU
  - Spatially average ***a*** → **larger *a'*** for fixed L
  - **Circuit Knitting** U(t) on *a'* lattice
  - Rediscretize ***a'*** → ***a*** with pseudorandom sampling

…if you want a research project

Partial Error Correction, Probabilistic Error Mitigation for LFT

- Given a register, **prioritize error channels** for mitigation and correction
- Reduction of **large theoretical error** at **lower cost**

...if you want a research project



Robustness of Gauge Digitization to Quantum Noise
Gustafson, Lamm - 2301.10207 [hep-lat]
Classification of Gauge Violating noise for qubits, qudits for U(1), SU(2), SU(3)

TABLE I. $\mathcal{N}_i$ vs. $\mathbb{G}$ for $U(1)$ subgroups: $\mathbb{Z}_N$ where $N = 2^n$

Binary	Gray	Qudits	$\mathbb{G}$
$\hat{Y}_0$	$\hat{Y}_0$	$\hat{B}^{(i,j)}$, $\hat{Z}^{(i)}$	—
—	$\hat{B}_{a\neg 0}$, $\hat{Z}_a$	$\hat{\mathcal{V}}^m$	$\mathbb{Z}_2$
$\hat{X}_{a\neg 0}$	—	—	$\mathbb{Z}_{2^{n-a}}$
$\hat{Z}_a$, $\hat{Y}_{a\neg 0}$	$\hat{X}_0$	—	$\mathbb{Z}_{2^{n-1}}$
$\hat{X}_0$	—	$\hat{\chi}^m$	$\mathbb{Z}_N$

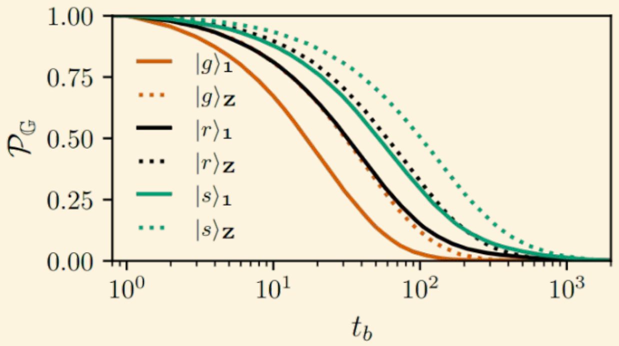
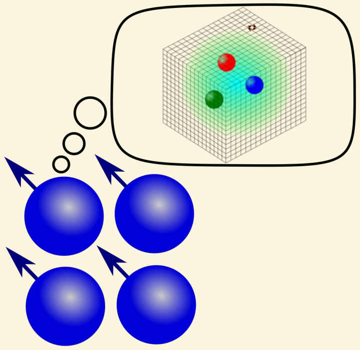


FIG. 2. $\mathcal{P}_{\mathbb{G}}(t_b)$ for $\mathbb{Z}_8$ versus t_b using $|g\rangle$, $|r\rangle$, and $|s\rangle$ for depolarizing and dephasing channels.

Endgame

- The road to **quantum practicality in HEP** will be **long** and **winding**
- We **do not have** anything close to realistic game plan
- Material fabrication, cryogenics, hardware design, quantum software stack, and classical communication **all profoundly affect** the questions we can ask in HEP

But also can be affected by HEP



There is so much for you to do.