Electromagnetic Response of Disordered Superconducting Cavities

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Summary

- Use microscopic theory of superconductivity $\rightarrow$ calculate the current response $J = KA$ [Rainer and Sauls (1995)]
- Apply boundary value conditions on Electromagnetic field at the specular vacuum-metal interface,
- Predict the observed $\delta f = f_s - f_n$ with a precision of order several 10Hz over the temperature range $0 \lesssim T \leq T_c$, including the negative frequency shift anomalies observed very near T_c [Bafia et al. (2021)]
- The non-monotonic dependence of the quality factor on the quasiparticle scattering rate is studied.

Current Response of an SRF Cavity

$$\vec{J}(\vec{r},t) = -\frac{c}{4\pi}\int_{-\infty}^{t} K(t-t')\,\vec{A}(\vec{r},t')dt'$$

$$K(t-t',\vec{r}-\vec{r}\,') \equiv \Theta(t-t')\,K(t-t')\delta(\vec{r}-\vec{r}\,') \quad (1)$$

the local limit i.e. $K(\omega,q) \approx K(\omega)$

Sufficient disorder : reduces the coherence length, ξ, increases the London penetration depth: $q \leq 1/\lambda_{\mathrm{L}} \ll 1/\xi_0$

$$\left(\nabla^2 + \frac{\omega^2}{c^2} - K(\omega)\right)\vec{A}(\vec{r},\omega) = 0\,, \quad (2)$$

Specular Boundary conditions: the continuity of $\vec{A}$ *and* $\partial_n\vec{A}$

Response function, conductivity, penetration depth

- The imaginary part of K is proportional to the dissipative conductivity σ_1

$$\mathrm{Im}(K(\omega)) = -\frac{4\pi\omega}{c^2}\,\sigma_1(\omega)\,, \tag{3}$$

- The real part of K is proportional the penetration depth

$$\mathrm{Re}(K(\omega)) = \frac{4\pi\omega}{c^2}\,\sigma_2(\omega) \quad \xrightarrow[T\to T_c^-]{\hbar\omega\ll 2\Delta(T)} \quad \frac{1}{\lambda_{\mathrm{L}}(T,\tau)^2}\,, \tag{4}$$

The pair formation time $\tau_0 \equiv \hbar/2\pi k_B T_c$

- Clean limit $\tau > \tau_0$ and near T_c: $\lambda_{\mathrm{L}}(T,\tau) \simeq \lambda_{\mathrm{L}_0}/\sqrt{1 - T/T_c}$

  $\lambda_{L_0} = c/\omega_p$ is the pure London length

- Dirty limit $\tau < \tau_0$:

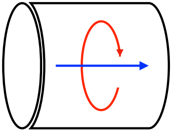
$\lambda_{\mathrm{L}}(T,\tau) \simeq \lambda_{\mathrm{L}(T)}\sqrt{1+\tau_0/\tau} \simeq \lambda_{\mathrm{L}(T)}\sqrt{1+\xi_0/\ell}$ where $\ell = v_F\tau$ and $\xi_0 \approx v_F\tau_0$

- SRF cavities: *Tesla geometry*
- For analytic calculations we consider *cylindrical cavities* of radius R and length L and *adjust for the geometric factor* G.

TM_{010} is the lowest energy mode

$\vec{E}(\vec{r},\omega) = (i\omega/c)\, A_0\, J_0(\omega\rho/c)\,\hat{z},$

$\vec{B}(\vec{r},\omega) = A_0\,(\omega/c)\, J_1(\omega\rho/c)\,\hat{\varphi}.$



Eigenvalue equation:

$$\Lambda(\varpi) \equiv \sqrt{K(\varpi) - (\varpi/c)^2} = \left(\frac{\varpi}{c}\right) \frac{J_1(\varpi R/c)}{J_0(\varpi R/c)}$$

$\varpi \equiv \omega + \delta\omega - i\omega/2Q$ Q is the quality factor

$\delta\omega = 2\pi(f_s - f_0)$ $f_s - f_0$ frequency shift with respect to ideal metal

(5)

Impedance, Resistance and Reactance

$$Z/Z_0 \equiv E_{||}(R)/H_{||}(R) = \frac{J_0(\varpi R/c)}{J_1(\varpi R/c)} = \frac{\varpi}{c}\Lambda(\varpi)^{-1}$$
$$Z = R + iX \qquad (6)$$

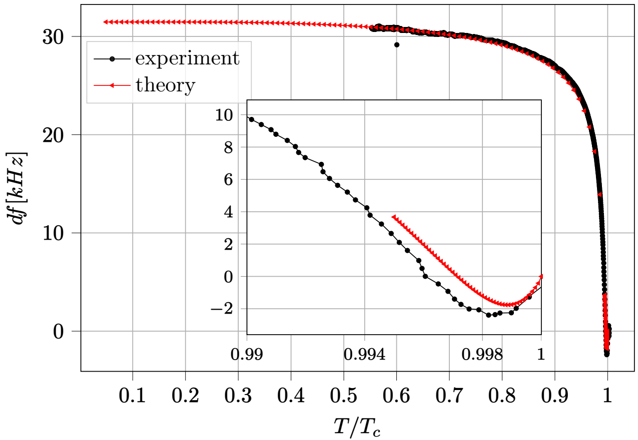
The change in the reactance upon cooling through the superconducting transition generates a shift in the resonance frequency,

$$\delta f \equiv f_s - f_n = \frac{f}{2G}(X_n - X_s)\,, \qquad (7)$$

while the surface resistance below T_c determines the quality factor,

$$Q_s = \frac{G}{R_s}\,, \qquad (8)$$

Frequency shift, theory and experiment



experiment (black) : [Bafia et al. 2021] reports $R_n \simeq 7.1 \times 10^{-3}\Omega$, corresponding to $\tau = 0.28\,ps$, from $\tau = \pi f \left(\frac{\lambda_{L_0}}{c}\frac{Z_0}{R_n}\right)^2$

theorey (red) : $\tau = 0.26ps$

$f = 2.6$ GHz

Nitrogen dopped cavities

See ref. Ueki *et al*, arXiv:2207.14236

$\tau_0 \equiv \hbar/2\pi k_B T_c$,

T_c [K]	v_f [10^8 cm/s]	τ_0 [ps]	ξ_0 [nm]	λ_{L_0} [nm]	Δ_0 [meV]
9.33	0.257	0.131	38.0	33.0	1.55

$f[GHz]$	$\tau[ps]$	$\ell[nm]$	τ/τ_0	$R_n[m\Omega]^*$	$\tau_{R_n}[ps]$
0.65	0.173	45.092	1.327	4.37	0.184
1.30	0.224	58.235	1.713	5.45	0.236
2.60	0.257	66.700	1.963	7.10	0.279
3.90	0.249	64.730	1.905	8.96	0.262

* R_n from D. Bafia [private communication].

The origin of the anomalous frequency shift

dissipative conductivity and the normal metal skin-depth

$$\mathrm{Im}(K(\omega)) = -\frac{4\pi\omega}{c^2}\,\sigma_1(\omega) \qquad \xrightarrow[T\to T_c^-]{\hbar\omega\ll 2\Delta(T)} \qquad -\frac{1}{2\lambda_{\mathrm{n}}(\omega,\tau)^2}\,, \tag{9}$$

$\lambda_n(\omega,\tau) \equiv \delta_n(\omega,\tau)/2$

$$\delta(\omega) \equiv c/\sqrt{2\pi\sigma_{\mathrm{D}}\omega} = \frac{\lambda_{\mathrm{L}_0}}{\sqrt{\omega\tau/2}}\,. \tag{10}$$

the penetration depth

$$\mathrm{Re}(K(\omega)) = \frac{4\pi\omega}{c^2}\,\sigma_2(\omega) \qquad \xrightarrow[T\to T_c^-]{\hbar\omega\ll 2\Delta(T)} \qquad \frac{1}{\lambda_{\mathrm{L}}(T,\tau)^2}\,, \tag{11}$$

LSU

SQMS

Explaining anomalous frequency shift

in the limit $\delta\omega/\omega \ll 1$ and $1/Q \ll 1$ for $|T - T_c| \ll T_c$

$$f_s - f_0 = -\frac{f}{R_{\mathrm{eff}}}\,\mathrm{Re}\left\{\left(\frac{1}{\lambda_{\mathrm{L}}(T,\tau)^2} - \frac{i}{2\lambda_n(\omega,\tau)^2}\right)^{-\frac{1}{2}}\right\}$$

$$f_n - f_0 = -(f/R_{\mathrm{eff}})\,\lambda_n = -2(f/R_{\mathrm{eff}})\,\delta_n \tag{12}$$

where $R_{\mathrm{eff}} = R\,G_{\mathrm{Tesla}}/G_{\mathrm{cyl}}$ and R is the radius of a cylindrical SRF cavity with a TM_{010} mode (for $f = 2.6\,\mathrm{GHz}$).

Competition between the normal-state penetration depth λ_n and the London penetration depth λ_L.

The negative frequency shift near T_c

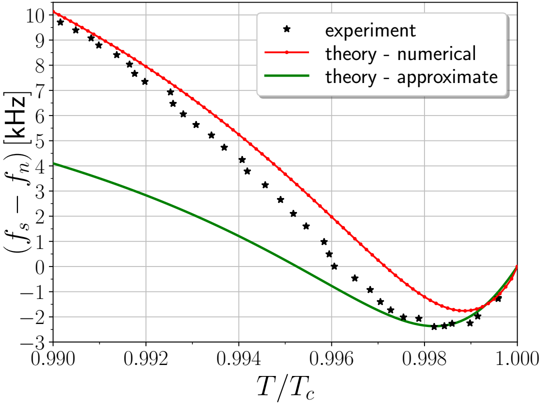


Figure: $f = 2.6\,\text{GHz}$ cavity

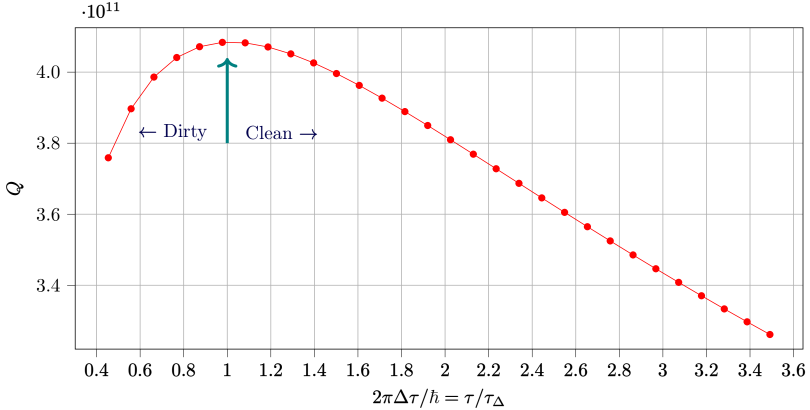
Minumum frequency shift: $-2.39\,\text{kHz}$ at $T = 0.9982\,T_c$

dip extends over $|\delta T| \approx 4 \times 10^{-3}\,T_c$.

$\lambda_L \approx \lambda_{L_0}/\sqrt{1 - T/T_c}$

When λ_L exceeds λ_n the fequency shift is negative.

Quality factor



$f = 2.6GHz$ cavity $T/T_c = 0.15$

$\tau_\Delta \equiv \hbar/2\pi\Delta(T)$,

$\tau/\tau_\Delta \ll 1$: dirty normal state, *pair-breaking* suppression of $n_s \propto 1/\lambda^2$

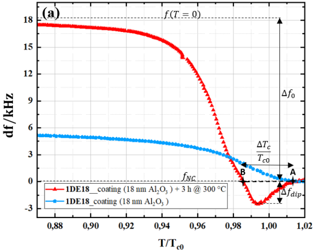
$\tau/\tau_\Delta \gg 1$: clean limit, beyond local approximation

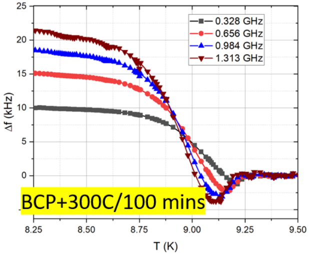
Conclusion

- The origin of anomalies in frequency shift is shown to be *the competition between the normal metal skin depth and the London penetration depth* which diverges as $T \to T_c^-$.
- An analytic approximation to the full current response, valid for $|T - T_c| \ll T_c$, accounts for the negative frequency shift near T_c.
- δf *anomaly is good way to characterize disorder in Nb resonators*
- The pair-breaking effect of disorder leads to the non-monotonic dependence of the quality factor on the quasiparticle scattering rate.

Future work

- Recent results: wider anomalous frequency shift up to $\delta T / T_c \approx 0.1$.
  - inhomogeneous impurites? [Ngampruetikorn and Sauls (2019)]
  - nonlocal effects?
  - TLS contribution?
  - extended impurites?





Ghanbari *et al*
doi:10.18429/JACoW-SRF2023-MOPMB021

N. Raut *et al*
unpulished, Jefferson Lab

Thank you!

EM linear response function $K(\vec{q}, \omega)$

Current response K [Rainer and Sauls 1995] in local limit

$$K(\omega; \tau, T) = \frac{4\pi\omega}{ic^2}(\sigma_1 + i\sigma_2) = \frac{\pi\sigma_\mathrm{D}}{ic^2\tau}\int_{-\infty}^{+\infty} d\epsilon I(\epsilon, \omega; \tau, T)$$

$$I(\epsilon, \omega; \tau, T) \equiv \Bigg\{ \tanh\left(\frac{\epsilon - \omega/2}{2T}\right) \frac{1}{D_+^R + D_-^R + 1/\tau}\left(\frac{\epsilon^2 + \Delta^2 - \omega^2/4}{D_+^R D_-^R} + 1\right)$$

$$- \tanh\left(\frac{\epsilon + \omega/2}{2T}\right) \frac{1}{D_+^A + D_-^A + 1/\tau}\left(\frac{\epsilon^2 + \Delta^2 - \omega^2/4}{D_+^A D_-^A} + 1\right) +$$

$$\left[\tanh\left(\frac{\epsilon + \omega/2}{2T}\right) - \tanh\left(\frac{\epsilon - \omega/2}{2T}\right)\right] \frac{1}{D_+^R + D_-^A + 1/\tau}\left(\frac{\epsilon^2 + \Delta^2 - \omega^2/4}{D_+^R D_-^A} +$$

$$D_\pm^{R/A} \equiv \sqrt{\Delta^2 - (\epsilon \pm \omega/2 \pm i\delta)^2} \qquad (13)$$

where $\sigma_\mathrm{D} = ne^2\tau/m^*$ is the Drude result for the d.c. conductivity

the normal state conductivity $\sigma_n(\omega) = \sigma_\mathrm{D}/(1 - i\omega\tau)$