Comparison of Nb, Nb$_3$Sn, MgB$_2$, cuprate, and pnictide for future SRF cavities

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Issues: beyond Nb for sustainability and higher performance

- Niobium material is getting more and more expensive
- Cryogenic costs: infrastructure, helium, and electricity

Another point: HTS market is growing
Three different families of superconductors

- Copper oxide cuprate d-wave
- Conventional BCS s-wave
- Iron-based s±-wave
How to estimate surface resistance at low B-field and lower RF frequency?
Optical conductivity in the Meissner state

\[ \sigma_1 = \frac{2\sigma_n}{\hbar\omega} \int_0^\infty [f(\epsilon) - f(\epsilon + \hbar\omega)][ReG^R(\epsilon)ReG^R(\epsilon + \omega) + ReF^R(\epsilon)ReF^R(\epsilon + \omega)]d\epsilon \]

S. N. Nam, Phys Rev 156 470 (1967)

\[ \sim \frac{2\sigma_n}{\hbar\omega} (1 - e^{-\omega/T}) \int_0^\infty e^{-\epsilon/kT}N(\epsilon)N(\epsilon + \hbar\omega)d\epsilon \]


Conventional s-wave (Nb, Nb\textsubscript{3}Sn)

\[ \frac{N(\epsilon)}{N_0} = \text{Re}\left( \frac{\epsilon + i\delta}{\sqrt{(\epsilon + i\delta)^2 - \Delta_0^2}} \right) \]

P. Coleman "Introduction to Many-Body Physics"

\[ \Delta(\theta) = \Delta_0 \cos 2\theta \]

Clean MgB\textsubscript{2}

\[ \frac{N(\epsilon)}{N_0} = \sum_{\alpha=\sigma,\pi} N_\alpha \text{Re}\left( \frac{\epsilon + i\delta}{\sqrt{(\epsilon + i\delta)^2 - \Delta_\alpha^2}} \right) \]

K. Watanabe and T. Kita, JPSJ 73 2239 (2004)

Cuprate d-wave

\[ \frac{N(\epsilon)}{N_0} = \text{Re}\left( \frac{\epsilon + i\delta}{\sqrt{(\epsilon + i\delta)^2 - \Delta^2(\theta)}} \right) \]

\[ \Delta(\theta) = \Delta_0 \cos 2\theta \]

Pnictide s\textsubscript{\pm}-wave

\[ \frac{N(\epsilon)}{N_0} = \text{Re}\left( \frac{\epsilon + i\delta}{\sqrt{(\epsilon + i\delta)^2 - \Delta^2_{\alpha_1,\beta_1,\phi_1,\phi_2}(\phi_1,\phi_2)}} \right) \]

\[ \Delta_{\alpha_1,\beta_1,\phi_1,\phi_2}(\phi_1,\phi_2) = \Delta_0 \Phi_{\alpha_1,\beta_1,\phi_1,\phi_2} \]

\[ \Phi_{\alpha_1,\beta_1} = -\Phi_a \]

\[ \Phi_{\beta_1,\phi_2} = \frac{1 + \Phi_{\beta_{\text{min}}}}{2} \pm \frac{(1 - \Phi_{\beta_{\text{min}}})}{2} \cos(2\phi_1,\phi_2) \]

Fermi surface and gap structure

- **BCS**
  - Gap-full

- **Cuprate**
  - Gapless nodes

- **Pnictide**
  - Double gap but gap-full

A. Carrington, Physique 12 (2011) 502–514
Summary of density of states

The energy is normalized to $T_c(Nb) = 9.25$ K.

Substitute $N(\epsilon)$ to

$$\frac{\sigma_1}{\sigma_n} \sim \frac{2\sigma_n}{\hbar\omega} (1 - e^{-\omega/T}) \int_0^\infty e^{-\epsilon/kT} N(\epsilon)N(\epsilon + \hbar\omega) d\epsilon$$

And perform numerical integral
$\sigma_1 \text{ vs } T$: an example ($\omega = 0.02 \sim 600 \text{ MHz}$)

Best fitting functions

gap-full: $\frac{\sigma_1(T)}{\sigma_n} = \frac{A}{T} \exp\left(-\frac{\Delta}{T}\right) + B$

Gapless: $\frac{\sigma_1(T)}{\sigma_n} = CT^\alpha + B$

$\frac{\sigma_1(T)}{\sigma_n} = \frac{2}{3} \left(\frac{T}{\Delta_0}\right)^2 \ln^2 \frac{4\Delta_0}{T}$

PRL 71 3705 (1993)

<table>
<thead>
<tr>
<th></th>
<th>Nb</th>
<th>Nb$_3$Sn</th>
<th>MgB$_2$</th>
<th>pnictide</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>81.8 ± 2.1</td>
<td>204.8 ± 2.9</td>
<td>34.0 ± 0.6</td>
<td>106.3 ± 2.3</td>
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<tr>
<td>$\Delta$</td>
<td>20.8 ± 0.11</td>
<td>41.8 ± 0.11</td>
<td>26.8 ± 0.1</td>
<td>67.0 ± 0.11</td>
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<td>B</td>
<td>0.0053 ± 0.0002</td>
<td>0.0015 ± 0.0001</td>
<td>0.0018 ± 0.0001</td>
<td>0.0006 ± 0.00006</td>
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<table>
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<tr>
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<th>cuprate</th>
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<tr>
<td>C</td>
<td>0.0001 ± 0.00005</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>2.341 ± 0.013</td>
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<tr>
<td>B</td>
<td>0.0038 ± 0.00036</td>
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</table>
Surface resistance \( Z_s = \sqrt{\frac{i\omega \mu_0}{\sigma_1 - i\sigma_2}} \), \( T \ll T_c, \sigma_1 \ll \sigma_2 \) 

\[
Z_s = \frac{\mu_0}{\omega \sigma_2} \left( \frac{1}{2} \sigma_1 + i\sigma_2 \right) \rightarrow R_s = \text{Re}(Z_s) = \frac{\mu_0 \omega^2 \lambda^3}{2\sigma_1(T)}
\]

The penetration depth is factor 10 longer in HTS than Nb \( \rightarrow \) RF field looks more materials.

The diagram shows the penetration depth \( \lambda \) and the behavior of various materials at different temperatures. The graph plots the resistivity ratio \( R_s/\sigma_n \) against temperature \( T \) for niobium, \( \text{Nb}_3\text{Sn} \), \( \text{MgB}_2 \), pnictide, and cuprate.

Thin film, multilayer materials are shown, with the magnetic field \( B_0 \) penetrating the surface and the bulk, with \( B_x(z) = B_0 \exp(-z/\lambda_T) \).
HTS: between SC and NC → pulse operation (?)

Nb $Q_0 \approx 10^{10}$, ms-CW

HTS $Q_0 \approx 10^{5-8}$, 10-100 µs

Cu $Q_0 \approx 10^4$, 1 µs

Lack of experimental data at high RF field
Microstrip resonator (200 µm, t350 nm, 8 GHz)~$10^{11}$ A/m²
→ t1 µm $10^5$ A/m ~ 0.1T ~ 25 MV/m (?)

→ High gradient test at SLAC (I.FAST IIIF) with the mushroom cavity
Summary

• Current SRF technology is at the risk of sustainability for the future
• Materials beyond Nb or even beyond Nb$_3$Sn could be useful to
  • Operate cavities at higher temperature
  • Ultimately high performance at low temperature
• A phenomenological model calculation was developed to compare s-wave, d-wave, and s$_{\pm}$-wave superconductors’ RF performance in the Meissner state
  • High resistance due to long penetration depth \(\rightarrow\) multilayer
  • Pulse operation
• More measurements are necessary!

Discussions

• More realistic modeling of the material
  • residual resistance, influence of grain boundaries, band-structure, e-ph counting, etc
• FeSe is also studied for higher gradient \(\rightarrow\) $H_{sh}$ calculation
  • Z. Lin et al SUST 34 015001 (2021)
• Deposition process: CVD, PVD (?)
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backup
<table>
<thead>
<tr>
<th>Material</th>
<th>$T_C$ (K)</th>
<th>$\rho_n$ ((\mu\Omega\text{cm}))</th>
<th>$\mu_0H_{C1}$ (mT)*</th>
<th>$\mu_0H_{C2}$ (mT)*</th>
<th>$\mu_0H_C$ (mT)*</th>
<th>$\mu_0H_{SH}$ (mT)*</th>
<th>$\lambda$ (nm)*</th>
<th>$\xi$ (nm)*</th>
<th>$\Delta$ (meV)</th>
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<td>n.a.</td>
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<td>48</td>
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<td>400</td>
<td>200</td>
<td>219</td>
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<td>756</td>
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<td>s/d wave**</td>
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</table>
\[ N(\epsilon)N(\epsilon + \hbar \omega) \text{ and } e^{-\epsilon/kT} N(\epsilon)N(\epsilon + \hbar \omega) \quad (T = 0.2, \omega = 0.02) \]

\[ \frac{\sigma_1}{\sigma_n} \sim \frac{2\sigma_n}{\hbar \omega} \left( 1 - e^{-\omega/T} \right) \int_{0}^{\infty} e^{-\epsilon/kT} N(\epsilon)N(\epsilon + \hbar \omega) \, d\epsilon \]

- **s-wave** (Nb)
- **s_\pm-wave** (pnictide)
- **d-wave** (cuprate)

Due to Dynes parameter $\delta > 0$ → residual resistance

Due to d-wave gapless