



Preliminary Design of the $\mu^+\mu^-$ Higgs Factory Ring Lattice

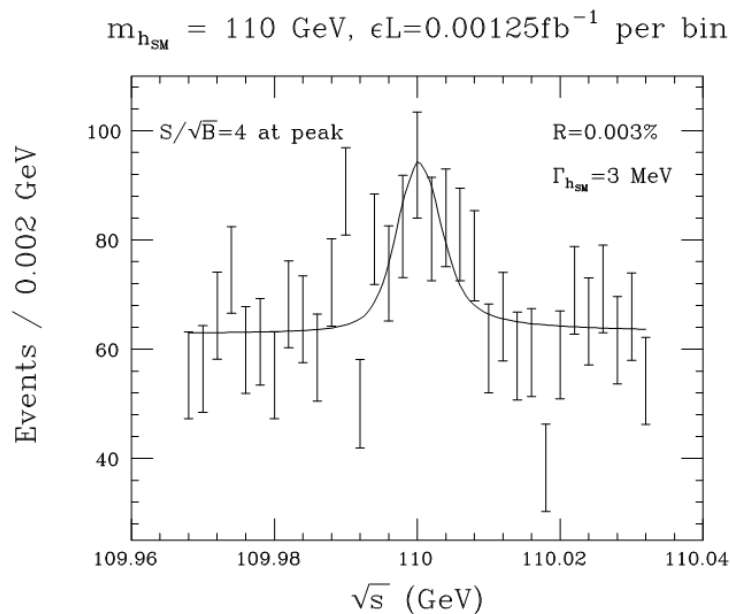
Y. Alexahin (FNAL APC)



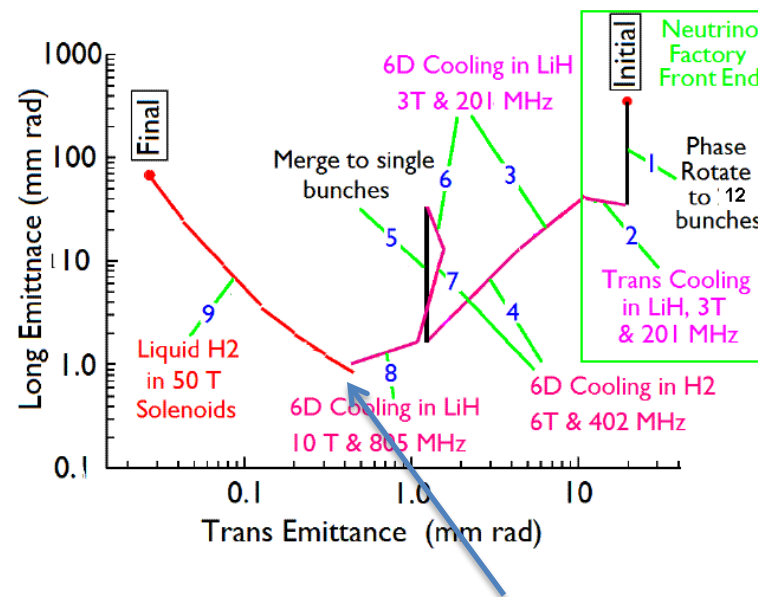
125 GeV Higgs Factory (D.Neuffer)

The major advantage of a $\mu^+\mu^-$ Higgs Factory – the possibility of direct measurement of the Higgs boson width ($\Gamma \sim 3\text{MeV}$ FWHM expected):

a very small beam energy spread is required, $R \sim 0.003\%$



Number of events and statistical errors in the final state $b\bar{b}$ as a function of \sqrt{s} (M.Berger, 2008)



Dave's proposal: stop cooling here:

$$\epsilon_{\perp N} = 0.3\pi \cdot \text{mm} \cdot \text{rad}, \epsilon_{\parallel N} = 1\pi \cdot \text{mm} \cdot \text{rad}$$

The machine must be able to digest even higher emittances (at the price of larger β^*)

Input Parameters

Parameter	Unit	Value
Beam energy	GeV	62.5
Number of IPs	-	1
Number of bunches / beam	-	1
Number of muons / bunch	10^{12}	1.5 → 2
Normalized emittance, $\varepsilon_{\perp N}$	$\pi \cdot \text{mm} \cdot \text{rad}$	0.3
Long. emittance, $\varepsilon_{\parallel N}$	$\pi \cdot \text{mm} \cdot \text{rad}$	1.0
Beam energy spread	%	0.003*
Bunch length, σ_s	cm	5.64
Repetition rate	Hz	10 → 30
p-driver power	MW	1 → 4

*) For initial scan a factor of 3-5 higher energy spread is needed, the machine must handle it.

The goal for the ring circumference is $C=300\text{m}$

HF Lattice Design Issues

- Large $\varepsilon_{\perp N}$ → small β^* to achieve the required luminosity → very large IR magnet apertures (ID=50cm set as a limit)
- Detector protection from backgrounds → it's desirable that longest & strongest IR quad was defocusing → quadruplet FF (also good for smaller β_{\max})
- β^* variation in wide range (2cm – 10cm)
- Preservation of small σ_E in the presence of strong self-fields ($I_{\text{peak}} \sim 500\text{A}$!) → requirements on RF and momentum compaction
- Do we need chromaticity correction for σ_E/E as low as $3 \cdot 10^{-5}$? – There are effects which may require it
 - Let us start with the last two items

Momentum Compaction Factor

To obtain small σ_E with high $\varepsilon_{||} \rightarrow$ high $\beta_{||}$ is required:

$$\sigma_E / E = \sqrt{\varepsilon_{||} / \beta_{||}} = 3 \cdot 10^{-5} \rightarrow \beta_{||} \approx 1880m, \beta_{||} = \frac{\alpha_c C}{2\pi Q_s} = C \sqrt{\frac{\alpha_c E}{2\pi h V_{RF} \cos \varphi_s}}$$

\rightarrow low V_{RF} and/or high momentum compaction α_c is required.

But V_{RF} should be high enough to minimize the effect of strong self-fields ($\sim 50kV$ for $Z_{||}/n \sim 0.1\Omega$ in the GHz range*). For $V_{RF} = 100kV$ and $f_{RF} = 200MHz$

$$\alpha_c = \frac{2\pi h e V_{RF} \cos \varphi_s}{E} \left(\frac{\beta_{||}}{C} \right)^2 \approx 0.08 \cos \varphi_s$$

Another way to look at this is to use the Keil-Schnell criterion (+ Boussard conjecture)

$$\left| \frac{Z_{||}}{n} \right| \leq \frac{2\pi E |\alpha_c|}{e I_{peak}} \left(\frac{\sigma_E}{E} \right)^2 \rightarrow |\alpha_c| \geq 0.13$$

Obviously we have to limit $Z_{||}/n$ to well below 0.1Ω and make α_c as large as possible (I set the goal at $\alpha_c > 0.05$)

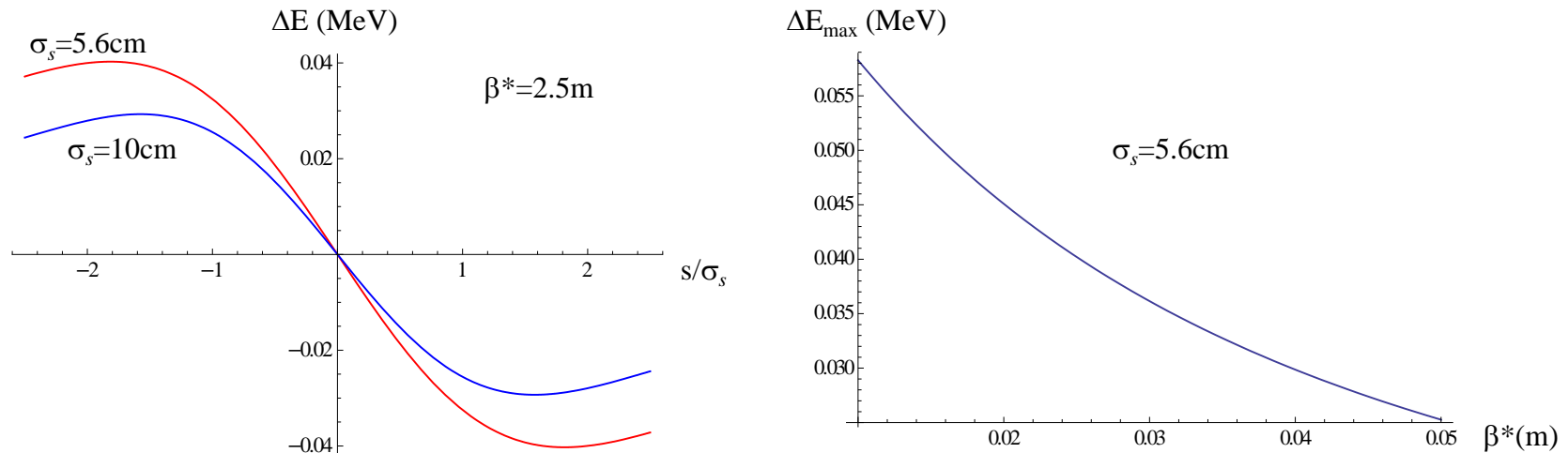
*) Characteristic bunch frequency is $c/4\sigma_s \sim 1GHz$ whereas the rotational frequency is $\sim 1MHz \rightarrow n \sim 10^3$

Longitudinal Beam-Beam Effect (Derbenev & Skrinsky, 1972)

Collision with a thin slice of N_s particles leads to energy change

$$\Delta E = \frac{e^2 N_s}{2\beta_\perp} \frac{d\beta_\perp}{ds} \Big|_{\text{collision point}}, \quad \Delta E_{\text{max}} = \frac{e^2 N_s}{2\beta^*} \sim 58\text{kV} \text{ for } N_s = 2 \cdot 10^{12} \text{ and } \beta^* = 2.5\text{cm}$$

For $\alpha_c > 0$ the effect is defocusing (good), but it is strongly nonlinear (not so good).
The finite bunch length reduces it somewhat:



Thanks to $\alpha_c > 0$, $\beta^* = 1.5\text{cm}$ is probably still admissible

Chromaticity Correction?

Thanks to small $\sigma_E/E \sim 3 \cdot 10^{-5}$ the chromaticity by itself is not a problem (though it may be such for larger σ_E/E needed for initial scan).

But there are other effects which require chromaticity correction, most notably the path length dependence on betatron amplitude (L. Emery, 1992) which translates into additional energy spread:

$$\frac{\Delta E}{E} \approx -\frac{1}{2\alpha_c R} (Q'_x I_x + Q'_y I_y) \rightarrow \left\langle \frac{\Delta E}{E} \right\rangle_{r.m.s.} = \frac{|Q'_\perp| \varepsilon_\perp}{\sqrt{2\alpha_c R}}, \quad \varepsilon_x = \langle I_x \rangle$$

With uncorrected $Q'_\perp \sim -100$ and $\alpha_c = 0.05$ we would have

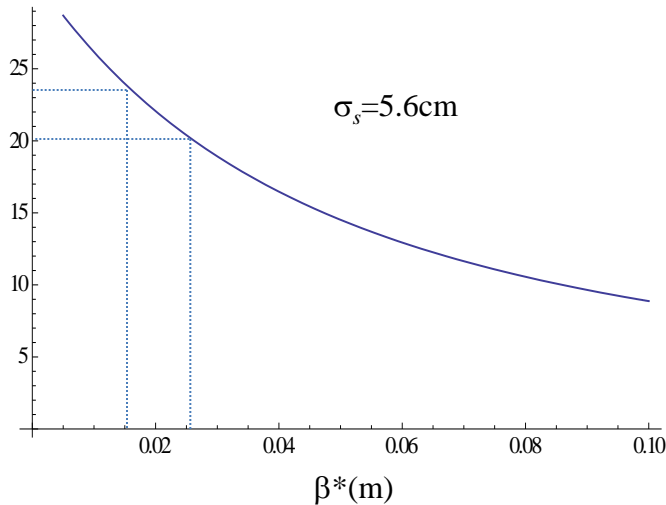
$$\left\langle \frac{\Delta E}{E} \right\rangle \sim 2 \cdot 10^{-5}$$

→ we need chromaticity correction! - But in a way that will not compromise the dynamic aperture.

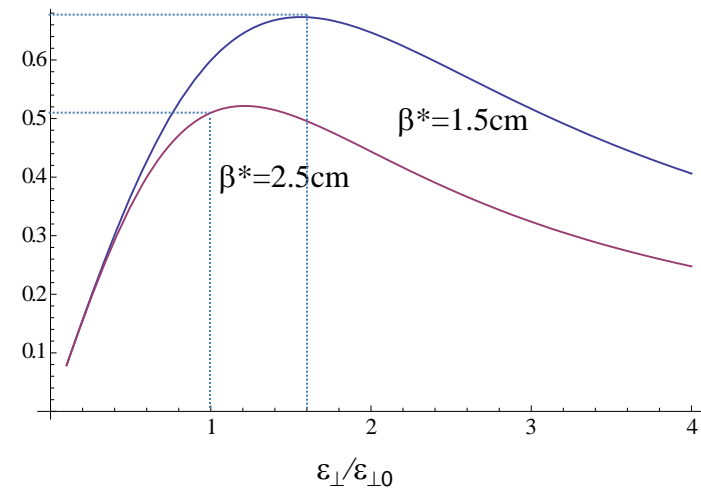
Chromaticity correction is also needed from operational considerations.

Optimum β^* and Emittance Ratio

Hourglass Factor / $\beta^*(m)$



Hourglass Factor $\times \epsilon_{\perp 0} / \epsilon_{\perp}$



$$\epsilon_{6D} = \epsilon_{\perp}^2 \epsilon_{\parallel} = \text{const},$$

$$\sigma_E = \text{const}$$

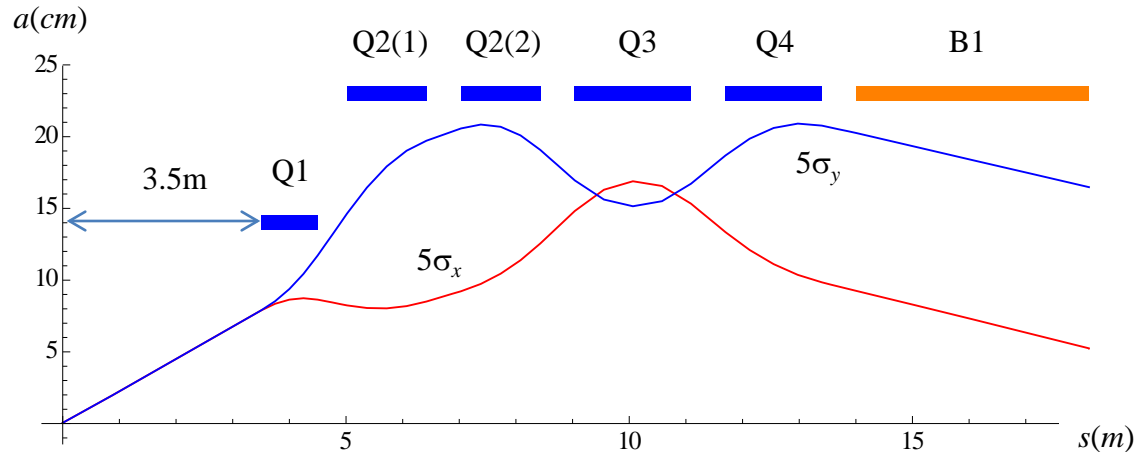
The hourglass factor takes away half of the luminosity gain from β^* reduction: $\beta^*=1.5\text{cm}$ gives just 17% higher luminosity compared to $\beta^*=2.5\text{cm}$ (still can be worthwhile).

The gain could be higher if emittances are redistributed so as to reduce the longitudinal one, but this would require even larger aperture of the FF quads.

$\beta^*=2.5\text{cm}$ was chosen as the nominal value.

Quadruplet Final Focus

$$\beta^* = 2.5\text{cm}, \quad \varepsilon_{\perp N} = 0.3\text{mm}$$

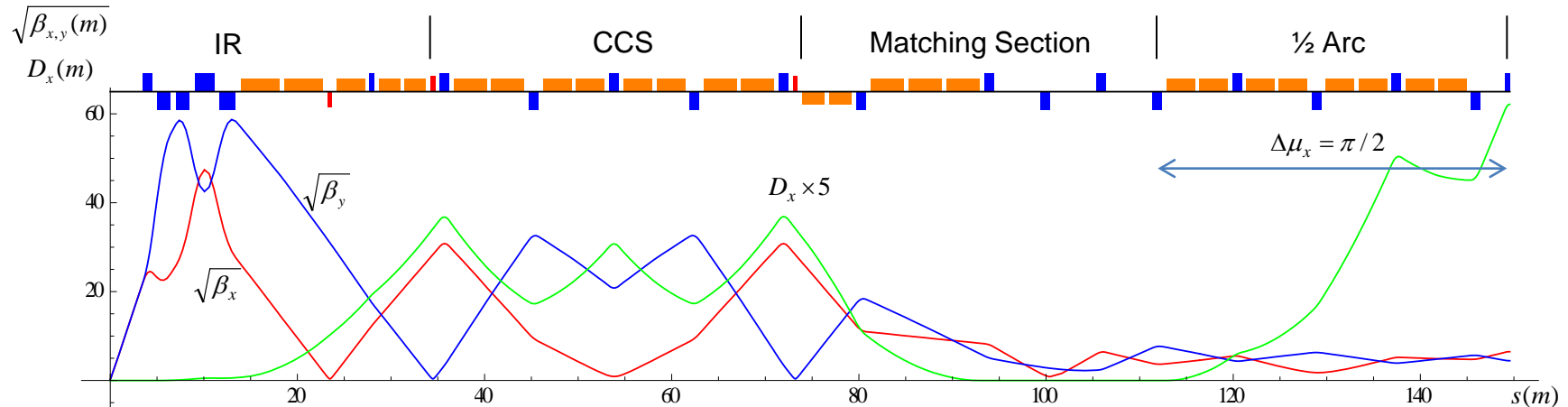


	Q1	Q2	Q3	Q4
aperture (cm)	27	45	45	45
gradient (T/m)	74	-36	44	-25
dipole field (T)	0	2	0	2
length (m)	1.0	1.4	2.05	1.7

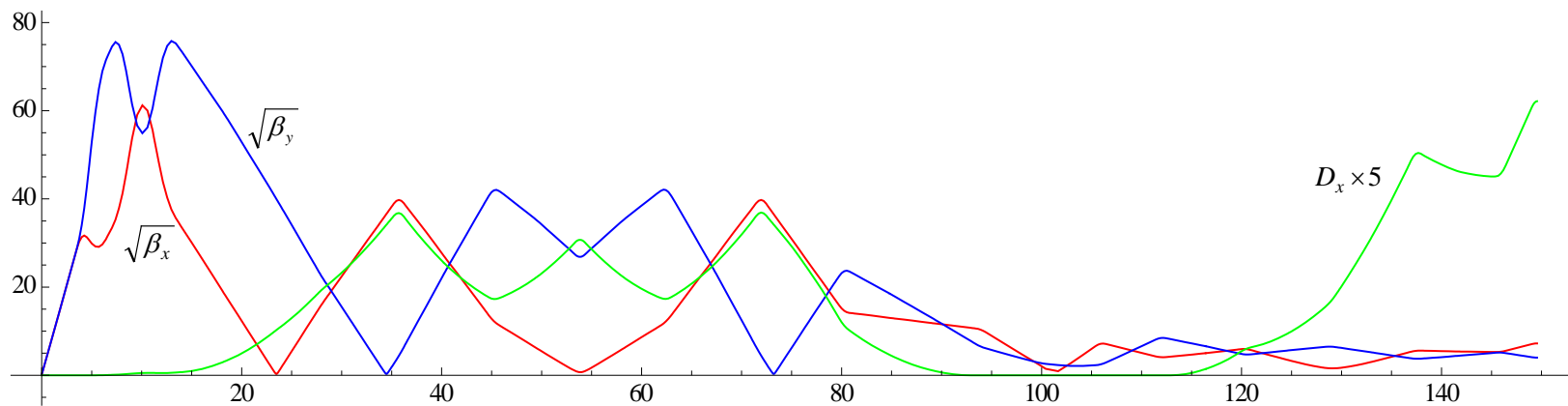
Obviously the design can be improved: β_{y_max} can be reduced to match β_{x_max}

Optics Functions (from IP to SP)

$\beta^*=2.5\text{cm}$

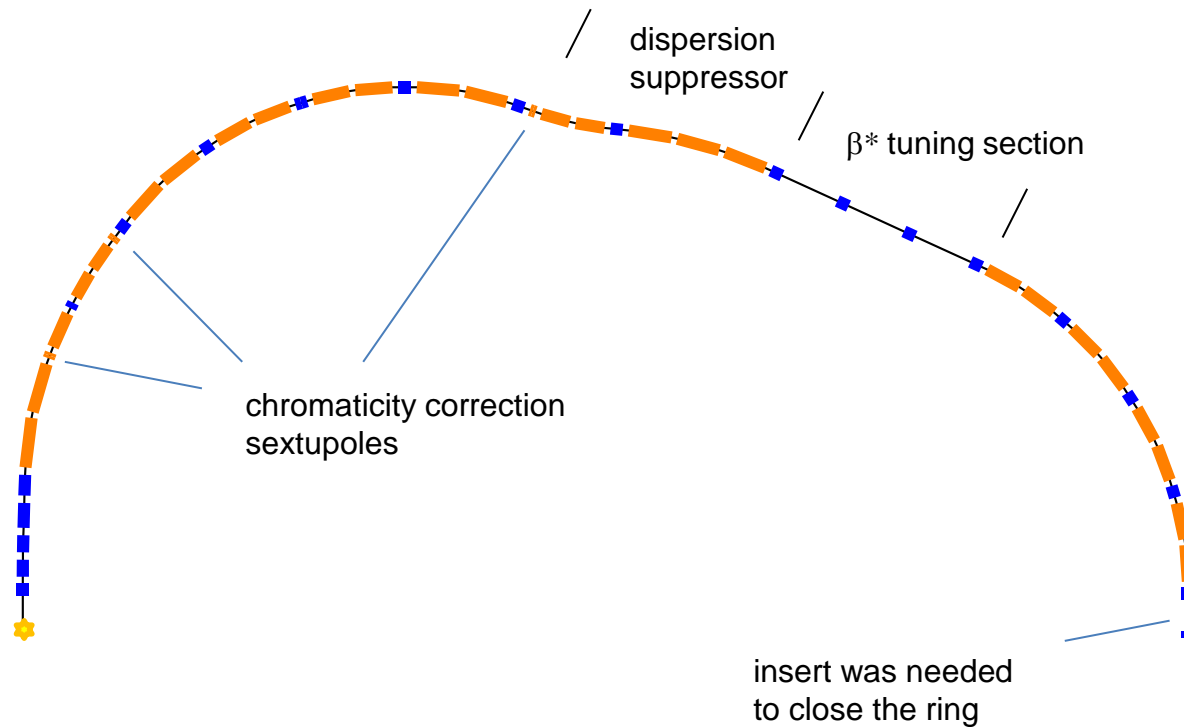


$\beta^*=1.5\text{cm}$



β^* can be increased up to 10 cm by changing quad gradients in the matching section

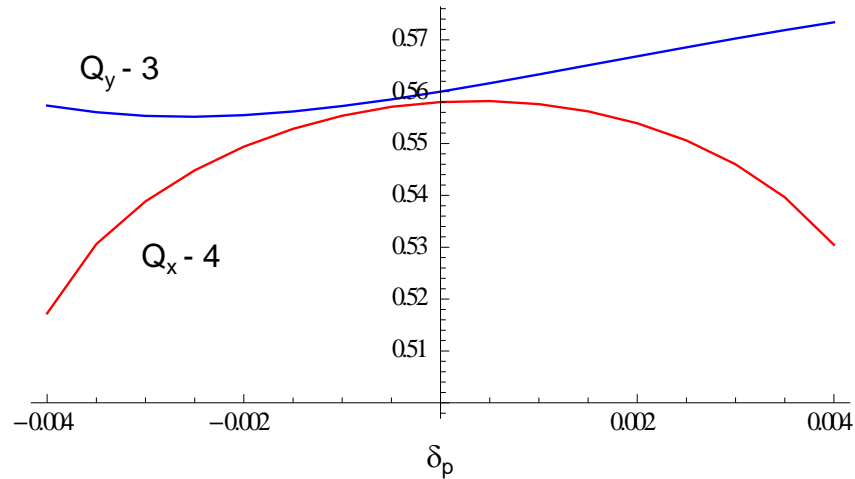
(Half) Ring Layout



Dispersion suppressor and β^* tuning section drastically increase the ring circumference, but they are probably indispensable

Momentum Acceptance

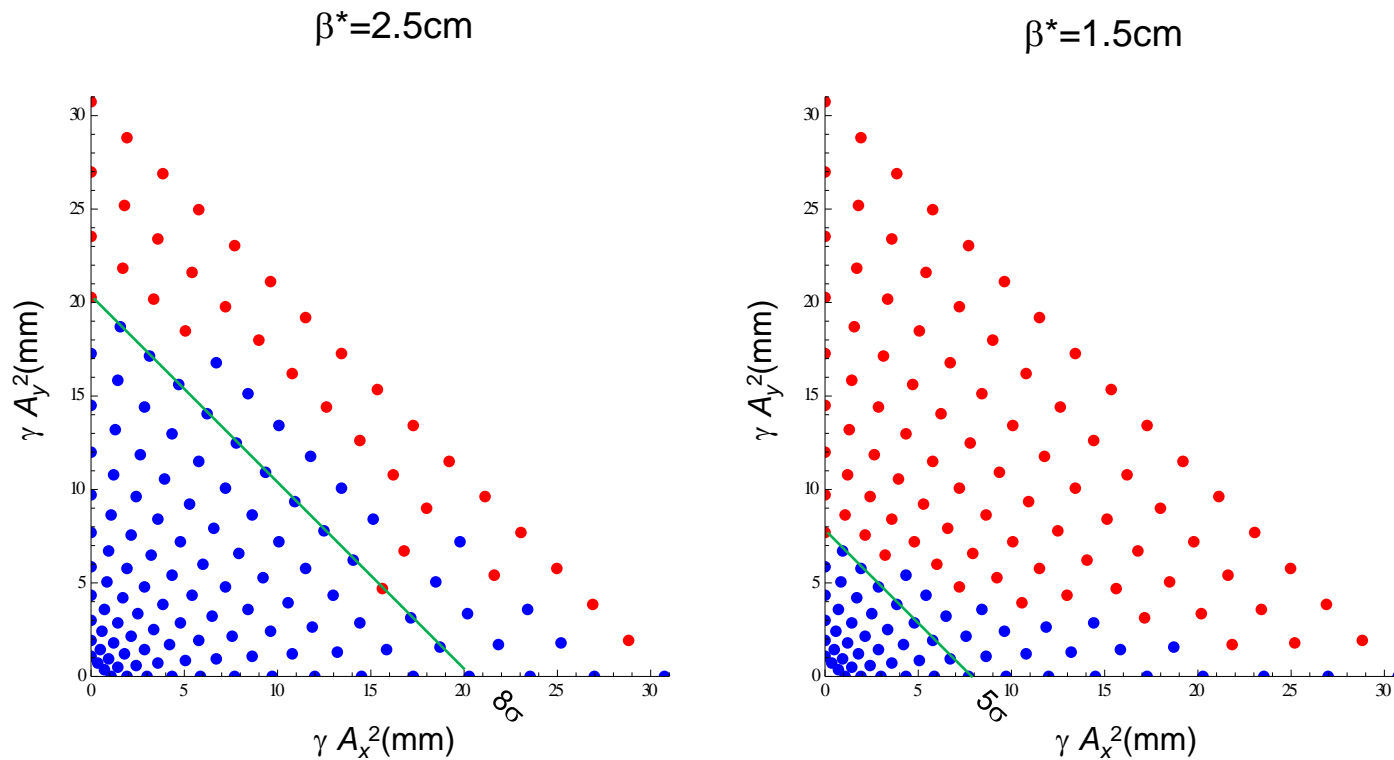
$$\beta^* = 1.5 \text{ cm}$$



No attempt to correct nonlinear chromaticity has been made (should not be a problem).
Acceptance at $\beta^* = 2.5 \text{ cm}$ exceeds $\pm 0.5\%$

N.B. Chromaticity computed with MAD8 TWISS command is completely wrong!

Dynamic Aperture



2048 turns Dynamic Aperture (DA) computed with MAD8 LIE4 method. Octupole correctors were used to correct vertical detuning with amplitude. In both cases the DA significantly exceeds the physical aperture (5σ and 4σ respectively)

Parameters of the Present Design

Parameter	Unit	Value
Circumference, C	m	299
β^*	cm	2.5 (1.5-10)
Momentum compaction, α_p	-	0.0793
Betatron tunes	-	4.56 / 3.56
Bare lattice chromaticity	-	-124 / -197
Synchrotron tune (100kV, 200MHz)	-	0.002
Number of muons / bunch	10^{12}	1.5 \rightarrow 2
Normalized emittance, $\varepsilon_{\perp N}$	$\pi \cdot \text{mm} \cdot \text{rad}$	0.3
Long. emittance, $\varepsilon_{\parallel N}$	$\pi \cdot \text{mm}$	1.0
Beam energy spread	%	0.003
Bunch length, σ_s	cm	5.64
Beam-beam parameter	-	0.0054 \rightarrow 0.0072
Repetition rate	Hz	10 \rightarrow 30
Average luminosity	$10^{31}/\text{cm}^2/\text{s}$	0.46 \rightarrow 2.5

Summary & Outlook

- Unlike high-energy MC, the Higgs Factory needs highly non-isochronous ring ($\alpha_c \sim 0.1$) to preserve small energy spread
- Larger $\varepsilon_{\perp N}$ in D.Neuffer's proposal \rightarrow smaller β^* to achieve the required luminosity \rightarrow very large IR magnet apertures (up to 50cm) \rightarrow gradients \sim twice lower than in the early designs
- A quadruplet Final Focus is proposed for better detector protection from backgrounds
- The proposed design is versatile enough to adapt to changes in the input parameters, β^* can be varied in a wide range (1.5cm – 10cm)
- MARS simulations of the detector backgrounds is the critical next step
- Other important issues to address: fringe field effects, longitudinal dynamics with wake-fields and beam-beam