Tests of Quantum Mechanics with Atom Interferometers

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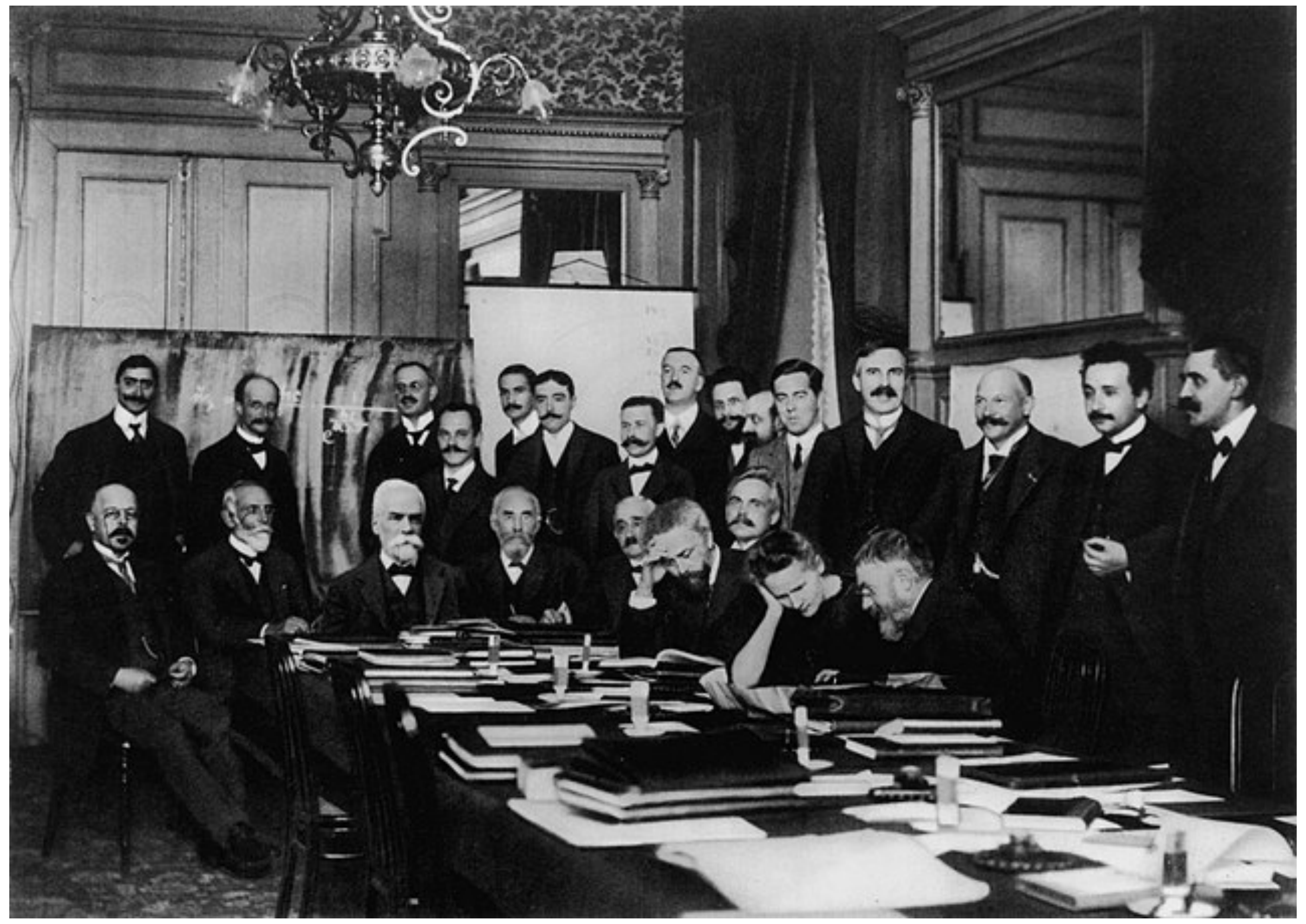
Why?



Quantum Mechanics

Theory built on observations in the 1900s
Why should it be "the absolute truth"?

What?

Two Postulates of Quantum Mechanics

Probability → **Fact**

Linearity → **Why not?**

Causality and Entanglement

Trial Non-Linear Term

$$i\frac{\partial \Psi}{\partial t} = H_L \Psi + \epsilon \left(\Psi^2 + \Psi^{*2}\right) \Psi$$

Entanglement is fundamental to quantum mechanics

$$\Psi(x, y; t) = \sum_{i,j} c_{ij}(t)\, \alpha_i(x)\, \beta_j(y)$$

Apply some local operation on x: $a_i(x)$ -> U $a_i(x)$

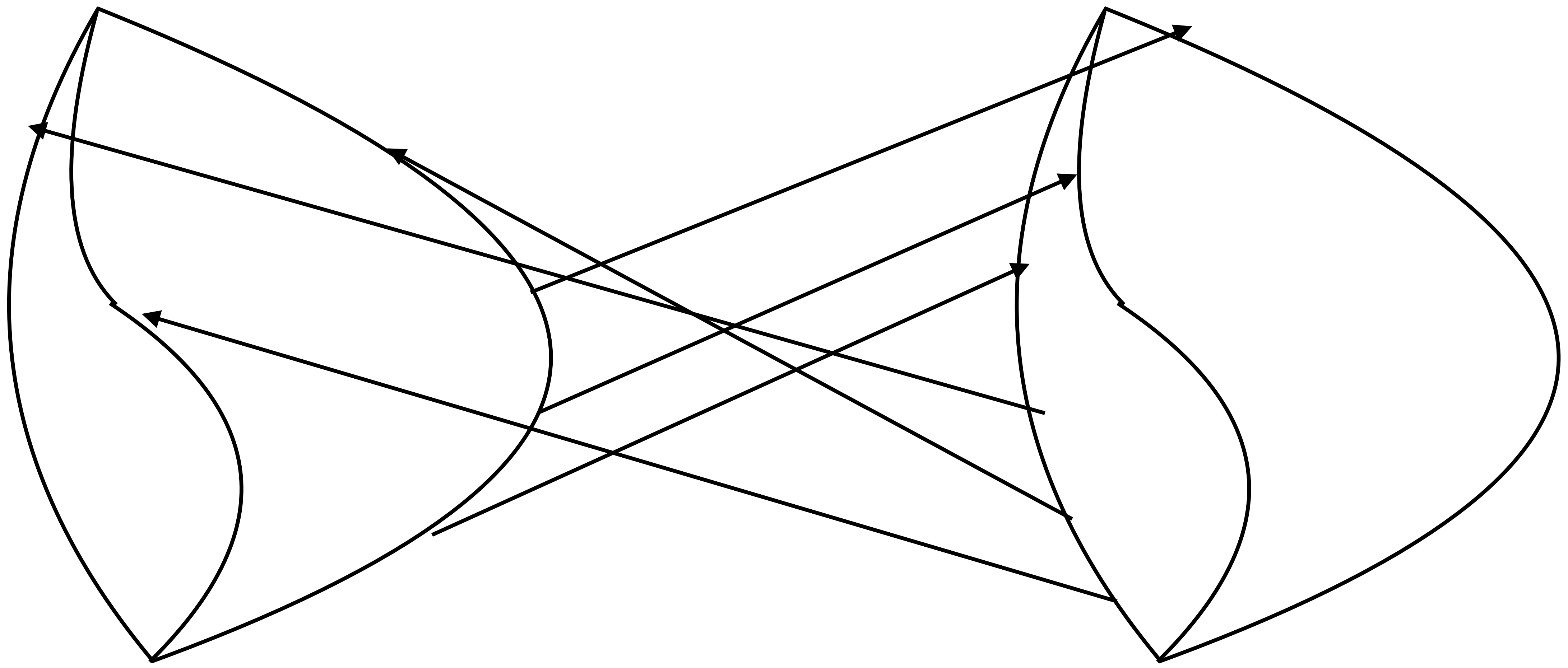
Does it instantly change the time evolution of y?

YES
Not causal

Causality and Non-Linearity

Linear Quantum Mechanics

Electron Coupled to Electromagnetism



Electron paths do not interact via electromagnetism

Paths of two electrons interact causally (QFT)

Why can't path talk to itself?
Formulate directly into QFT

The Framework

The Schrodinger Picture of Quantum Field Theory

$|\chi(t)\rangle$ **Quantum State of Fields (e.g. in Fock states)**

$\phi(x)$ **Time Independent Operators**

$$H = \int d^3x\, \mathcal{H}(\phi(x), \pi(x))$$

Time Evolution

$$i\frac{\partial|\chi(t)\rangle}{\partial t} = H|\chi(t)\rangle$$

Action

$$S = \int dt\, \left(i\langle\chi|\dot{\chi}\rangle - \langle\chi|H|\chi\rangle\right)$$

The Framework

Yukawa $H \supset \int d^3x\, y\, \phi\,(x)\,\bar{\Psi}\,(x)\,\Psi\,(x)$

Action

$$S = \int dt\ \left(i\langle\chi|\dot{\chi}\rangle - \langle\chi|H|\chi\rangle\right) \supset \langle\chi\,(t)\,| \left(\int d^3x\, y\, \phi\,(x)\,\bar{\Psi}\,(x)\,\Psi\,(x)\right) |\chi\,(t)\rangle$$

$$\supset \left(\int d^3x\, y\, \langle\chi\,(t)\,|\phi\,(x)\,\bar{\Psi}\,(x)\,\Psi\,(x)\,|\chi\,(t)\rangle\right)$$

Quantum Field Theory $\supset \left(\int d^3x\, y\, \langle\chi\,(t)\,|\phi\,(x)\,\bar{\Psi}\,(x)\,\Psi\,(x) + \frac{\phi^2}{\Lambda}\bar{\Psi}\Psi + \ldots\,|\chi\,(t)\rangle\right)$

Non-linearities in the operators but not in the state

The Framework

Yukawa $H \supset \int d^3x \, y \, \phi(x) \, \bar{\Psi}(x) \, \Psi(x)$

Linear QFT: $S \supset \left(\int d^3x \, y \, \langle \chi(t) | \phi(x) \, \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

Non-Linear QFT: $S_{NL} \supset \epsilon \left(\int d^3x \, \langle \chi(t) | \phi(x) | \chi(t) \rangle \langle \chi(t) | \bar{\Psi}(x) \, \Psi(x) | \chi(t) \rangle \right)$

$$i \frac{\partial |\chi\rangle}{\partial t} = H |\chi\rangle$$

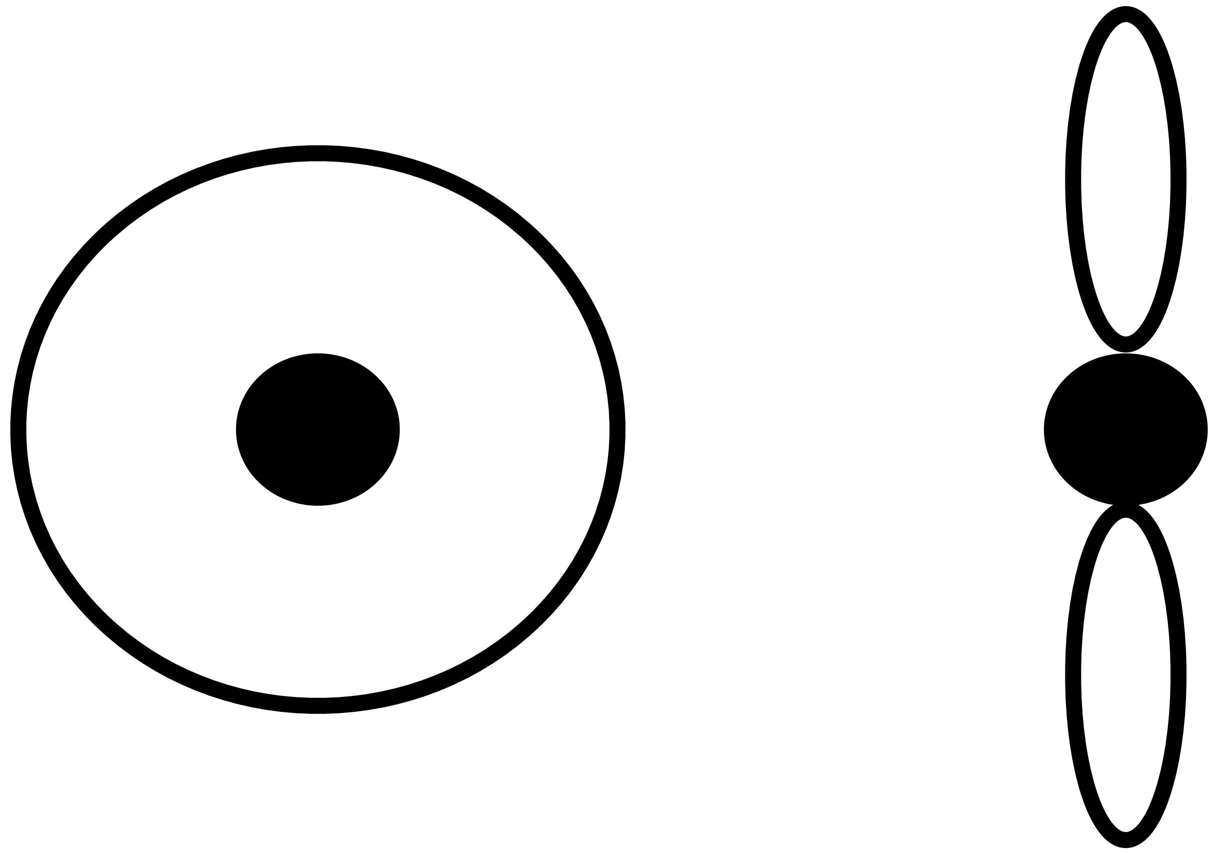
$$\mathcal{H} \supset y \Phi \bar{\Psi} \Psi = \left(y\phi + \epsilon \langle \chi | \phi | \chi \rangle \right) \bar{\Psi} \Psi$$

Causality Preserved: Nonlinearity enters via lower order expectation value. Causal observable of QFT

Constraints and Tests

Constraints

What does this do to the Lamb Shift?



Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

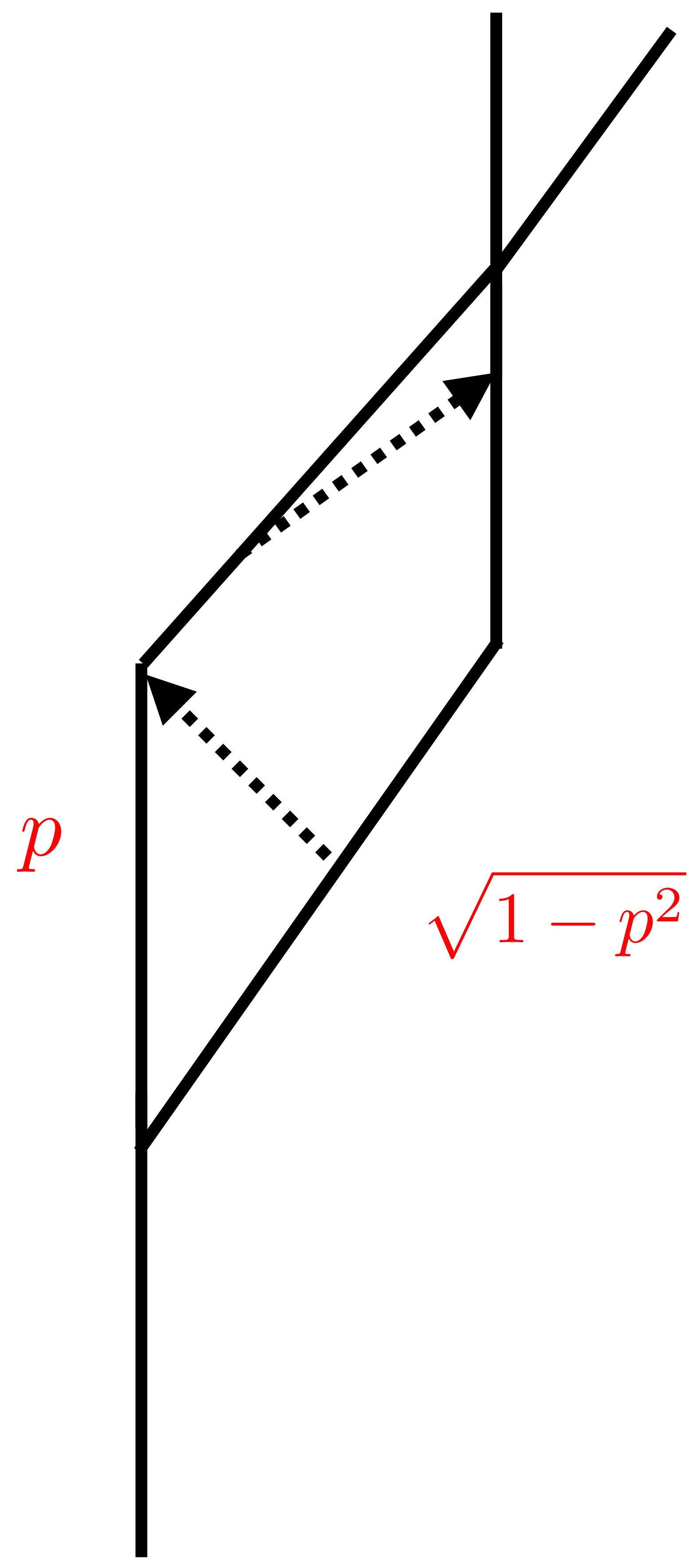
$\varepsilon < 10^{-2}$

Similarly, kills possible bounds on QCD and gravity

Experimental Tests

Interferometry - interaction between paths

Take an ion - split its wave-function



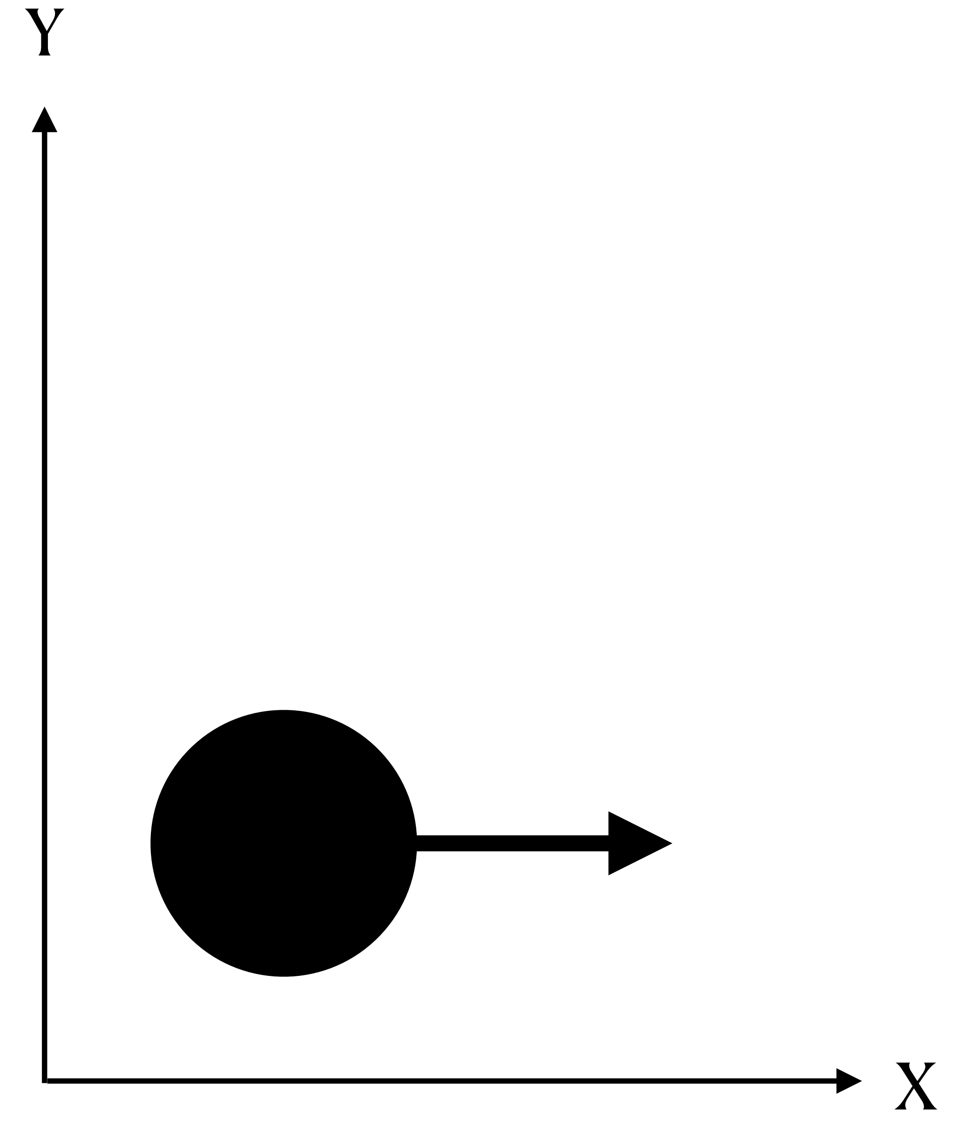
Coulomb Field of one path interacts with the other path

Gives rise to phase shift that depends on the intensity p of the split

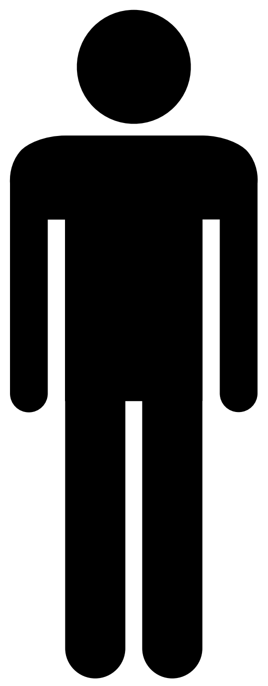
Use intensity dependence to combat systematics

Macroscopic Effects

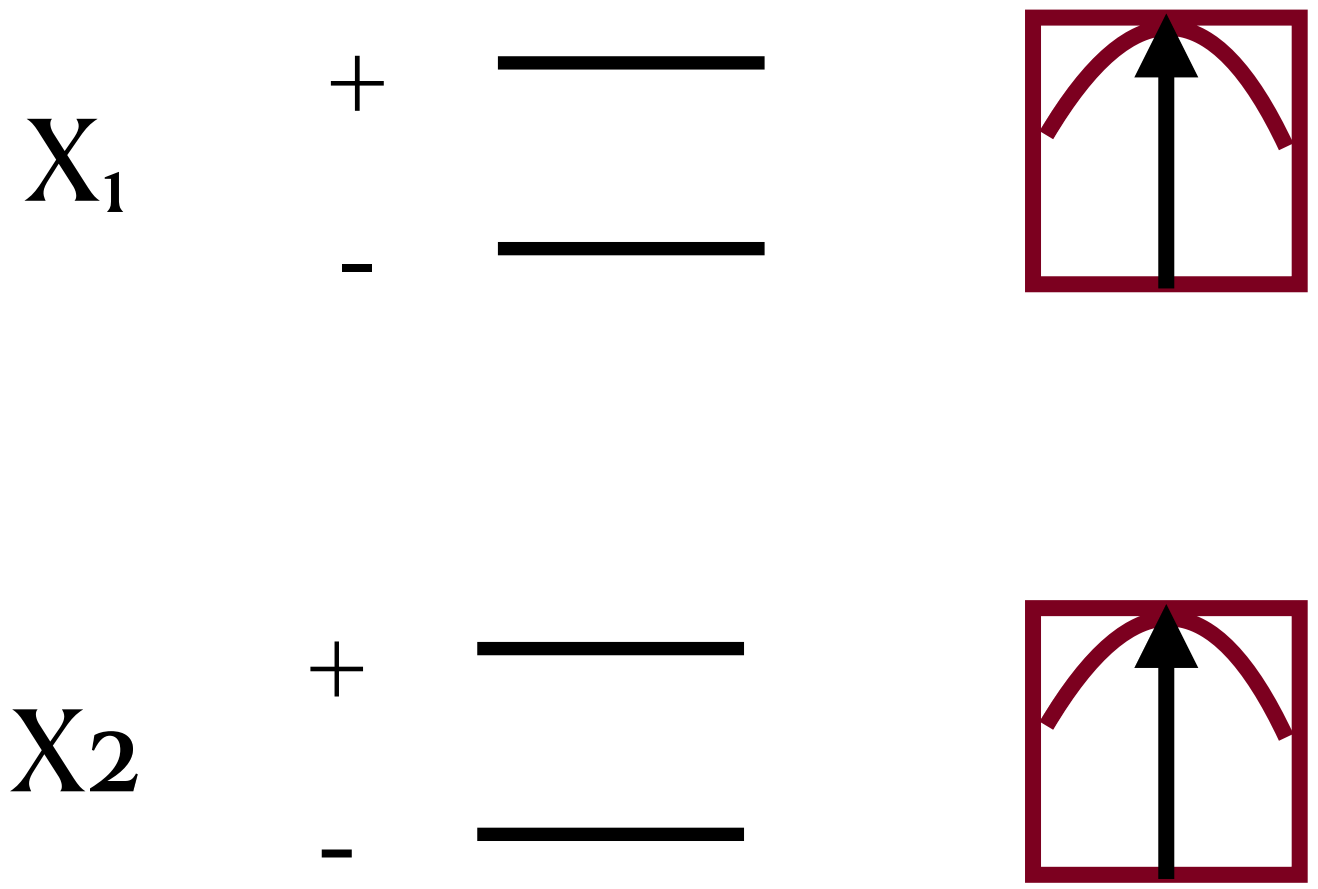
Linear Quantum Mechanics



Spin
Along x



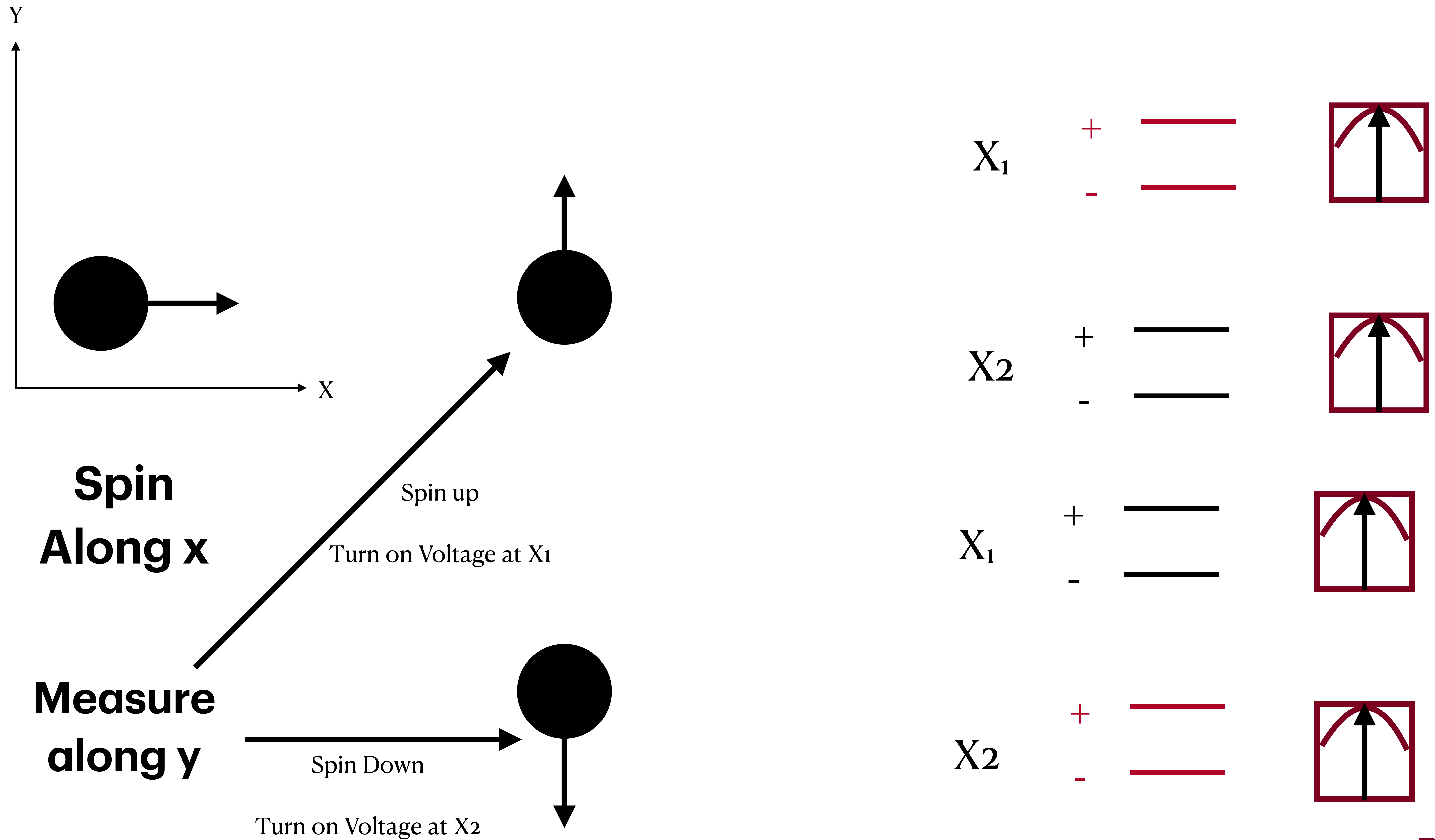
Experimentalist



Initial State : $|\chi(0)>$
Represents Full Quantum State (spin, experimentalist...)

Measure spin along y.
Based on outcome, turn on voltage source at X_1 or X_2.
What is the quantum state after measurement?

Macroscopic Superposition



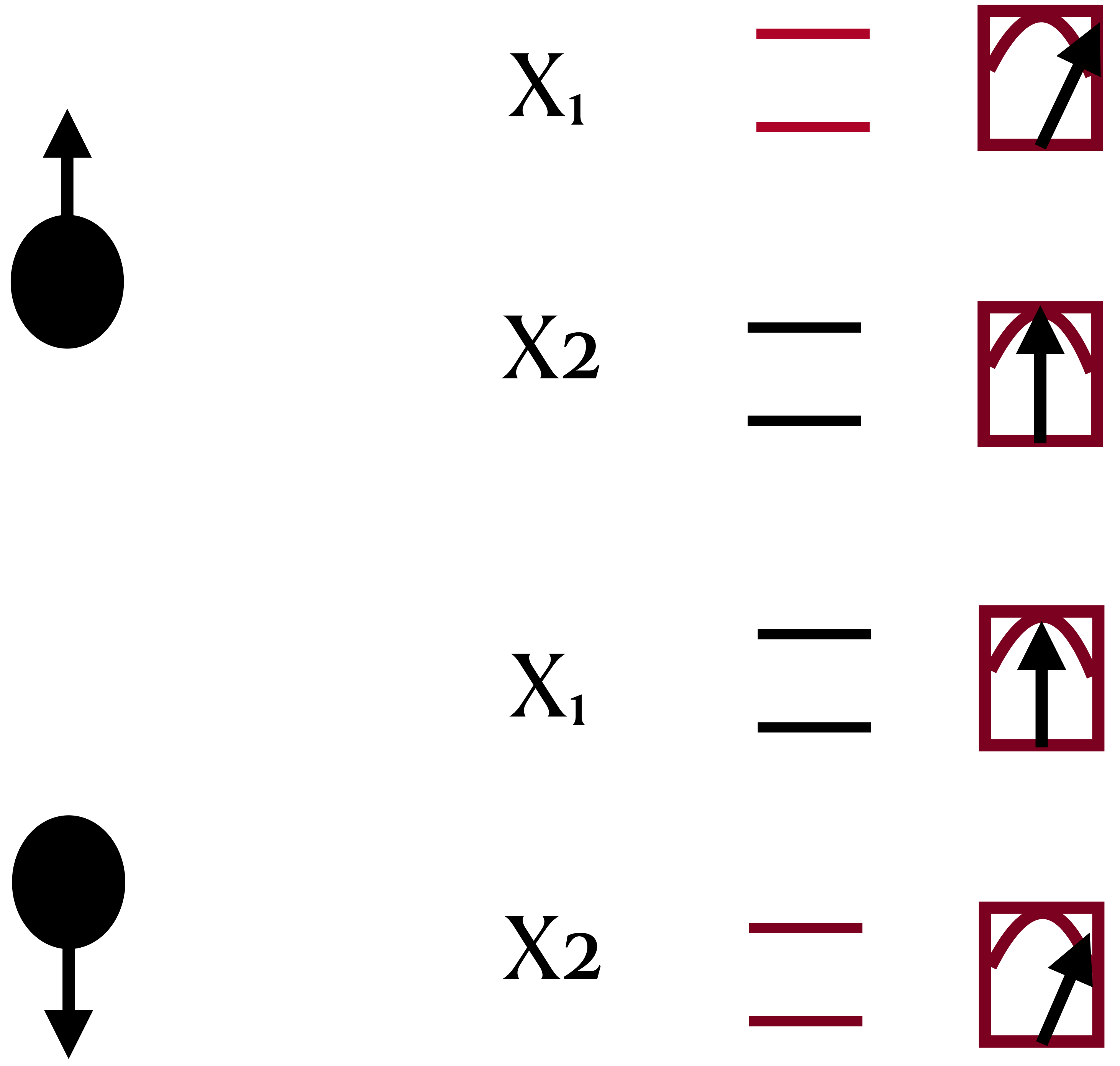
Final State: $|\chi\rangle = |U\rangle|V_1\rangle|E_1\rangle + |D\rangle|V_2\rangle|E_2\rangle$

Prediction of QM (Many Worlds)

Linear Quantum Mechanics

Which Voltage sensors light up?

$|\chi> = |U>|V_1>|E_1> + |D>|V_2>|E_2>$



$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi$$

Compute Transition Matrix Elements

$$\langle U|\langle V_1|\langle E_1|eA_\mu\left(x_1\right)\bar{\Psi}\left(x_1\right)\gamma^\mu\Psi\left(x_1\right)|\chi\rangle \neq 0$$

$$\langle U|\langle V_1|\langle E_1|eA_\mu\left(x_2\right)\bar{\Psi}\left(x_2\right)\gamma^\mu\Psi\left(x_2\right)|\chi\rangle = 0$$

$$\downarrow$$

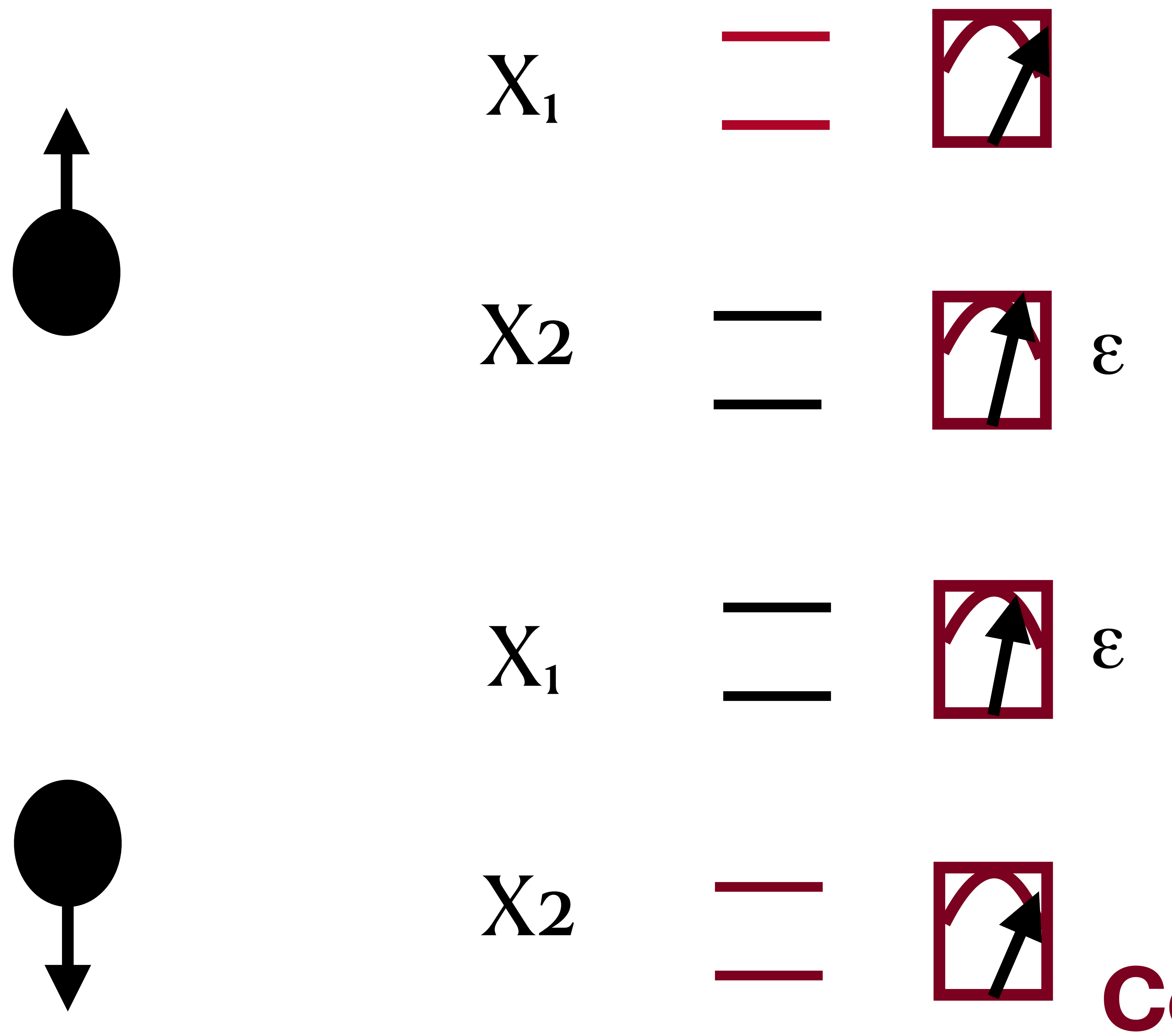
$$\langle V_1|A_\mu\left(x_2\right)|V_1\rangle = 0$$

But in both $|V_1>$, $|V_2>$:

$$\langle\chi|A_\mu\left(x_1\right)|\chi\rangle \neq 0, \langle\chi|A_\mu\left(x_2\right)|\chi\rangle \neq 0$$

Non-Linear Quantum Mechanics

$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi + \epsilon_\gamma e \langle \chi | A_\mu | \chi \rangle \bar{\Psi}\gamma^\mu \Psi$$



State Dependent Non-linear Term

But in both $|V_1\rangle$, $|V_2\rangle$:

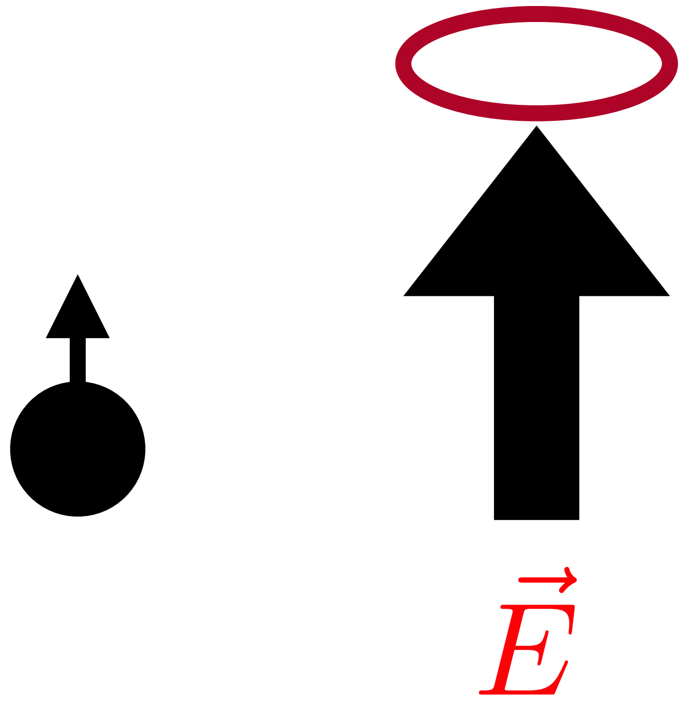
$$\langle \chi | A_\mu (x_1) | \chi \rangle \neq 0, \langle \chi | A_\mu (x_2) | \chi \rangle \neq 0$$

Communication between "worlds"
Consequence of Causality - trace over entangled particles

Non-linearity visible despite Environmental De-coherence!
Polchinski: "Everett Phone"

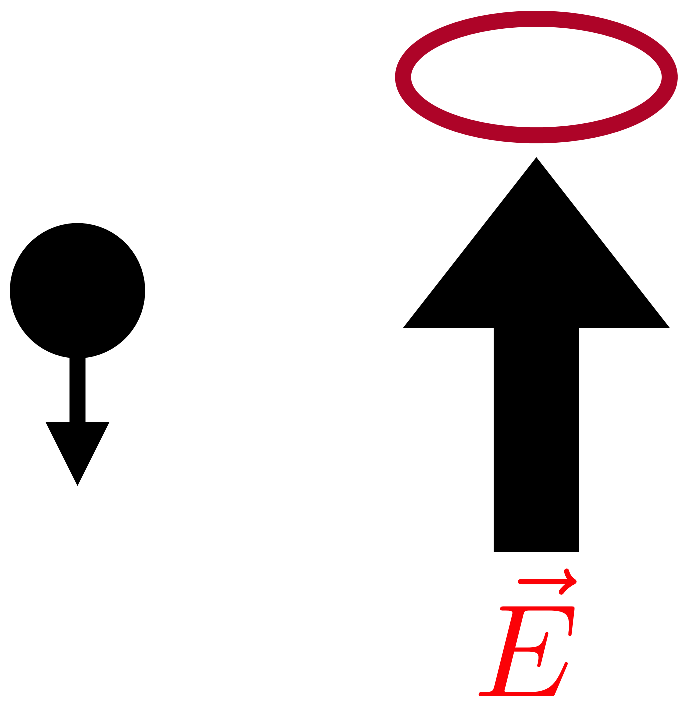
Experimental Tests

Key Point: Create macroscopic superposition
Create Expectation value of EM/Gravity
Search for Expectation value



On: {T, T + dT}
Off: {T + dT, T + 2 dT}

Voltmeter
ε E in {T + dT, T+ 2 dT}?



Off: {T, T + dT}
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Voltmeter
ε E in {T, T+ dT}?

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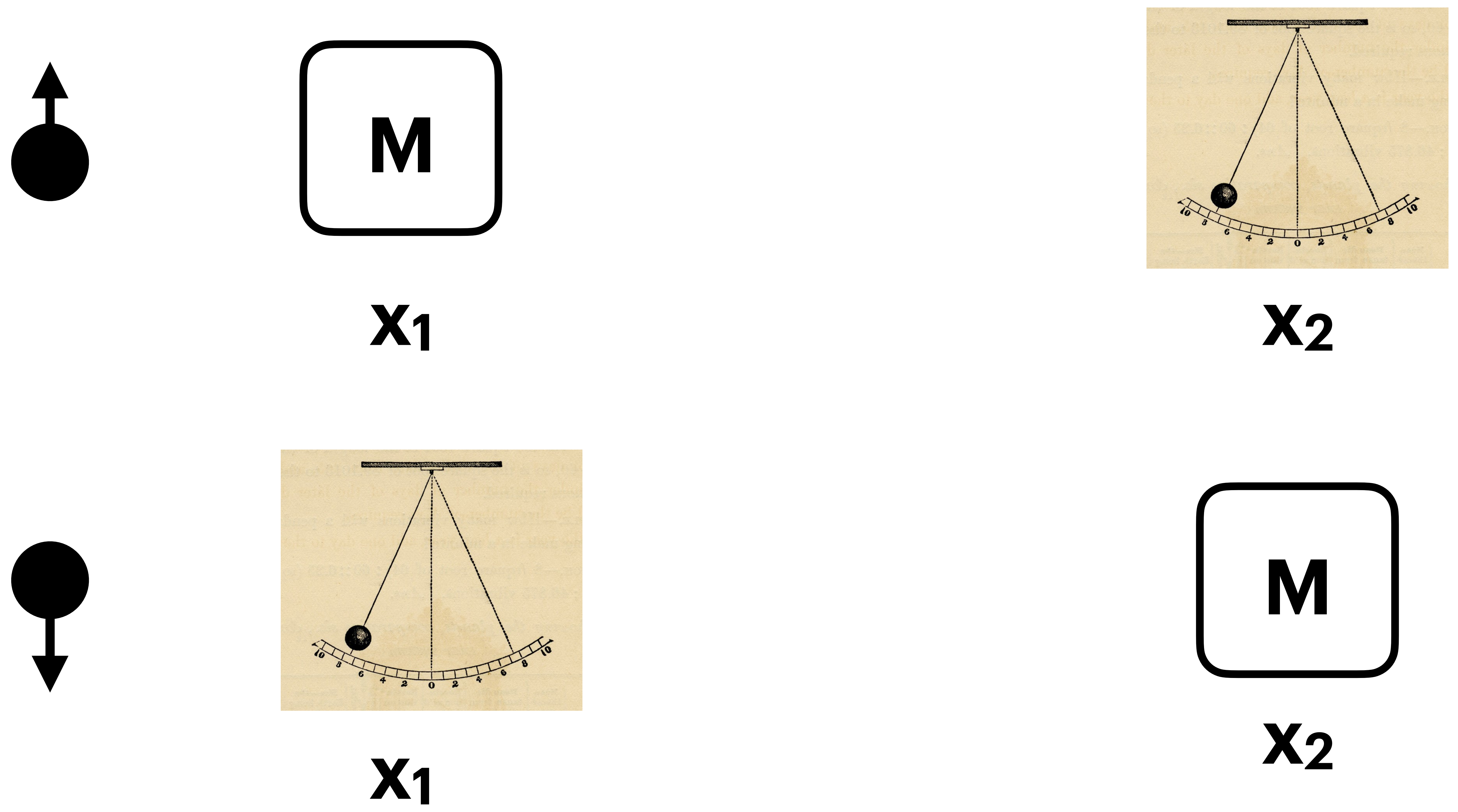


Even Null Result is Interesting: $G_{\mu\nu} = \langle T_{\mu\nu} \rangle$

Conclusions

1. Quantum Field Theory can be generalized to include non-linear, state dependent time evolution

2. Conventional tests of quantum mechanics in atomic and nuclear systems do NOT probe causal non-linear quantum mechanics

3. Straightforward set of experimental tests possible to probe non-linear quantum mechanics

4. Motivation to test other extensions as well - e.g. Lindblad Decoherence

Backup

Perturbation Theory

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

$$i\frac{\partial|\chi\rangle}{\partial t} = H|\chi\rangle$$

At zeroth order, this is just standard QFT

At first order, use zeroth order solution - expectation value is simply a background field

Perform standard QFT on this background field to compute first order correction

Applies to all orders : To compute term of given order, only need lower order terms
Lower order terms enter as background fields

Single Particle states? Causality for Multi-particle states?

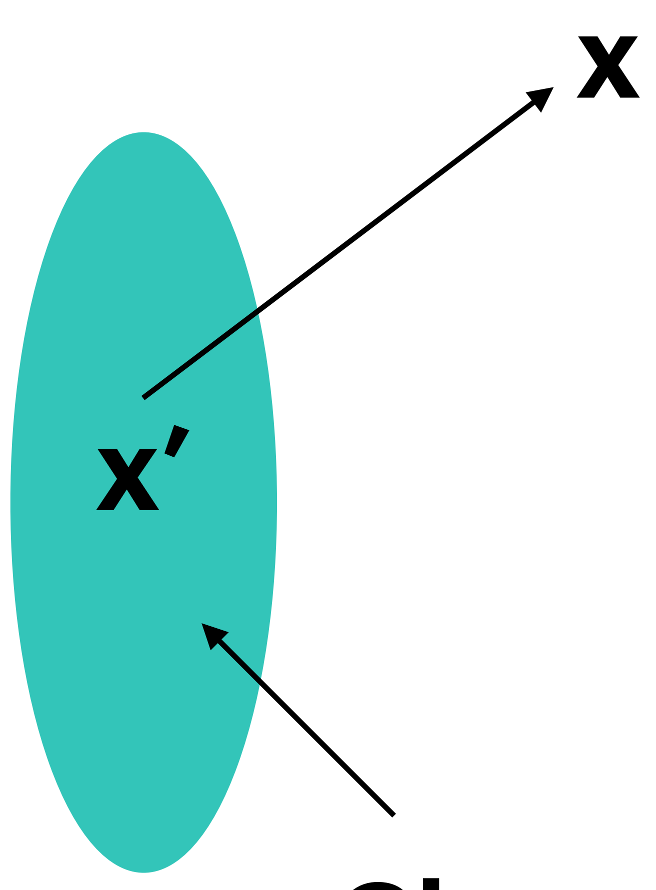
Single Particle

$$H \supset y\Phi\bar{\Psi}\Psi = y\left(\phi + \tilde{\epsilon}\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Suppose we have a ψ particle - how does its wave-function evolve?

To zeroth order, ψ just sources the Φ field

Straightforward Computation of Expectation Value



$$\langle\chi|\phi\left(x\right)|\chi\rangle = \int d^4x'\psi^*\left(x'\right)\psi\left(x'\right)G_R\left(x - x'\right)$$

Charge Density of ψ

Causal Green's Function

Schrodinger Equation

$$\mathcal{H} \supset y\Phi\bar{\Psi}\Psi = \left(y\phi + \epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

Single particle equation derived from field theory
Equation depends upon theory (Yukawa, Φ^4 etc)

$$i\,\frac{\partial\Psi(t,\mathbf{x})}{\partial t} = \left(H + \tilde{\epsilon}y\int d^4x'\,\Psi^*\left(x\right)\Psi\left(x'\right)G_R\left(x;x'\right)\right)\Psi\left(t,\mathbf{x}\right)$$

Hermitean Form of Hamiltonian implies conserved norm

Maintain Probabilistic Interpretation

Entangled Systems

$$\Psi\left(x,y;t\right)=\sum_{i,j}c_{ij}\left(t\right)\alpha_i\left(x\right)\beta_j\left(y\right)$$

How do multi-particle systems evolve?

$$\mathcal{H}\supset y\Phi\bar{\Psi}\Psi=\left(y\phi+\epsilon\langle\chi|\phi|\chi\rangle\right)\bar{\Psi}\Psi$$

$$\langle\chi|\phi|\chi\rangle=\int d^3x_1d^3y_1d\tau|\Psi\left(x_1,y_1;\tau\right)|^2\left(G_R\left(t,x;\tau,x_1\right)+G_R\left(t,y;\tau,x_1\right)+G_R\left(t,x;\tau,y_1\right)+G_R\left(t,y;\tau,y_1\right)\right)$$

To change evolution, need to change ϕ

Φ changes via causal Green’s function - naturally comes from field theory!

Gauge Theories and Gravitation

Linear QFT Lagrangian, Shift bosonic field by expectation value

To Path Integral, add:

$$e^{iS_0+i\int d^4x\left(e^{\frac{(A_\mu+\epsilon_\gamma\langle\chi|A_\mu|\chi\rangle)}{1+\epsilon_\gamma}}J^\mu+\epsilon_{\tilde{\gamma}}\langle\chi|F_{\mu\nu}|\chi\rangle F^{\mu\nu}\right)}$$

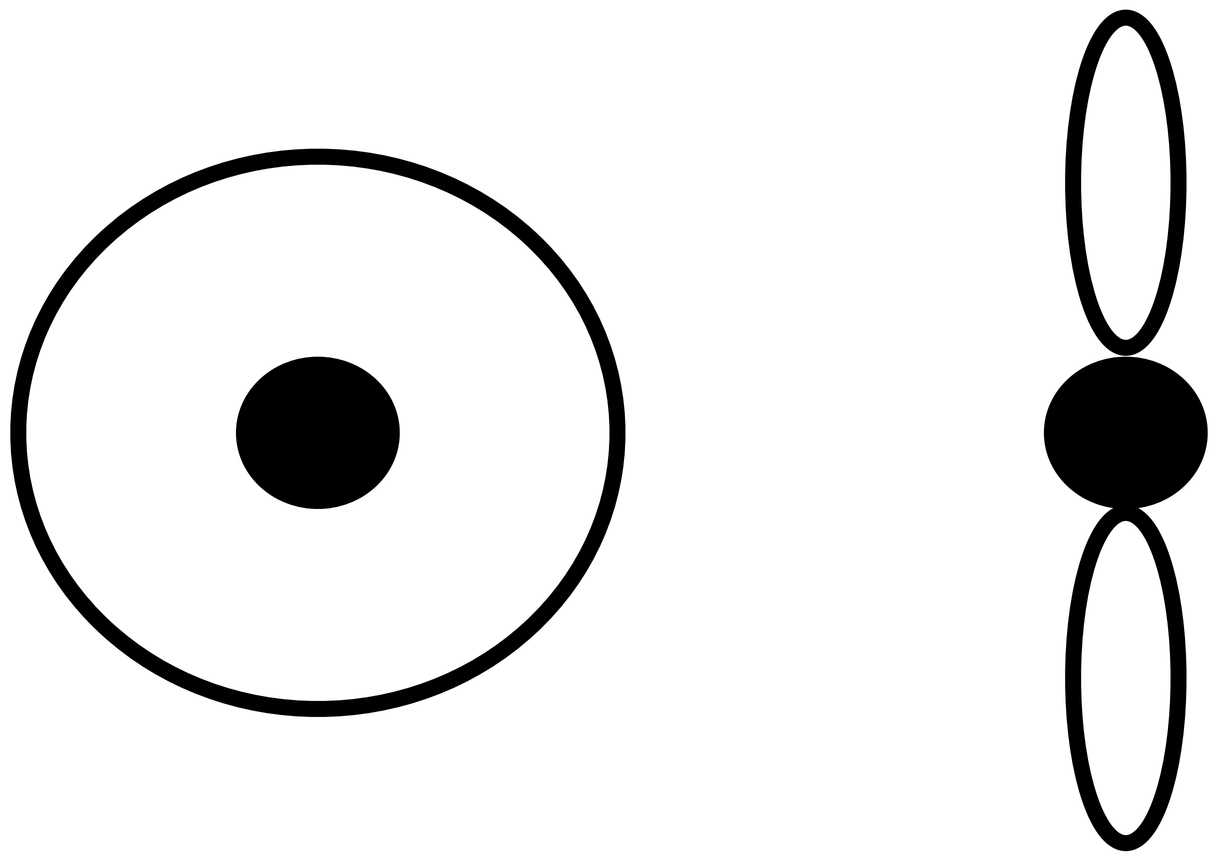
Background Field

Gravitation

$$e^{iS_0+i\int d^4x(\epsilon_G\langle\chi|g_{\mu\nu}|\chi\rangle\partial^\mu\phi\partial^\nu\phi)}$$

Constraints

What does this do to the Lamb Shift?



$$\langle \chi | A_\mu | \chi \rangle J^\mu$$

Proton at Fixed Location

2S and 2P electron have different charge distribution

Different expectation value of electromagnetic field

Level Splitting!

BUT: Cannot decouple center of mass and relative co-ordinates

Proton wave-function spread over some region (e.g. trap size ~ 100 nm)
Expectation value of electromagnetic field diluted
In neutral atom - heavily suppressed, except at edges!

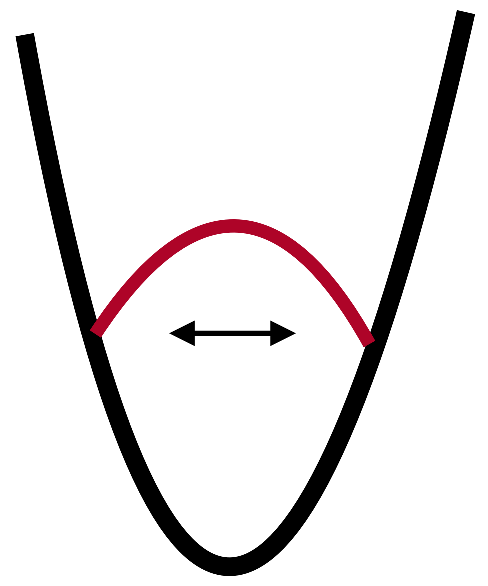
$\varepsilon < 10^{-2}$

Similarly, kills possible bounds on QCD and gravity

Constraints

Leading Constraint?

For ε > 0 (repulsive interaction)



Too large a repulsion, Cant trap ion in trap

$\varepsilon < 10^{-5}$

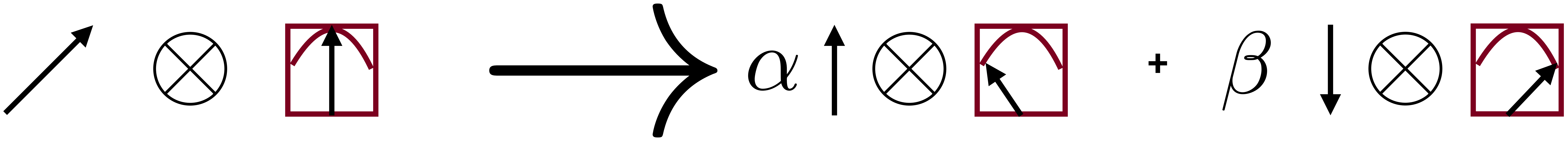
No direct limit on ε < 0 (attractive interaction)

Perhaps from mapping of ion in trap?

Measurement in Quantum Mechanics

Not some mysterious process

Interaction between quantum state and measuring device



$|\Psi\rangle$ $|A_0\rangle$ $|i\rangle \otimes |A_i\rangle$

$$|\Psi\rangle \otimes |A_0\rangle \to \sum_i c_i |i\rangle \otimes |A_i\rangle$$

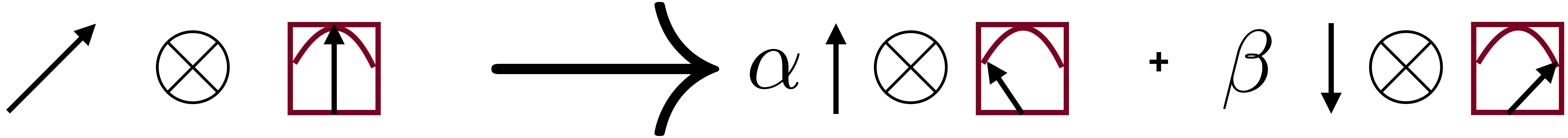
Prediction of Quantum Mechanics ("Many Worlds"), Not an interpretation

Pick: $\langle A_j | A_i \rangle = \delta_{ij} \implies \rho_{|\Psi\rangle} = \sum_i c_i c_i^* |i\rangle\langle i|$

"Interpret" as direct sum of "worlds"

Measurement in Non-Linear Quantum Mechanics

Interaction between quantum state and measuring device



$|\Psi\rangle$ $\quad$ $|A_0\rangle$ $\quad$ $|i\rangle \otimes |A_i\rangle$

In linear QM, just need to know the basis vectors

Interaction Hamiltonian independent of unknown quantum state

$$\text{Pick: } \langle A_j | A_i \rangle = \delta_{ij}$$

Key Point: Non-linear Hamiltonian depends upon unknown quantum state

$$\text{No Guarantee:} \langle A_j | A_i \rangle = 0$$

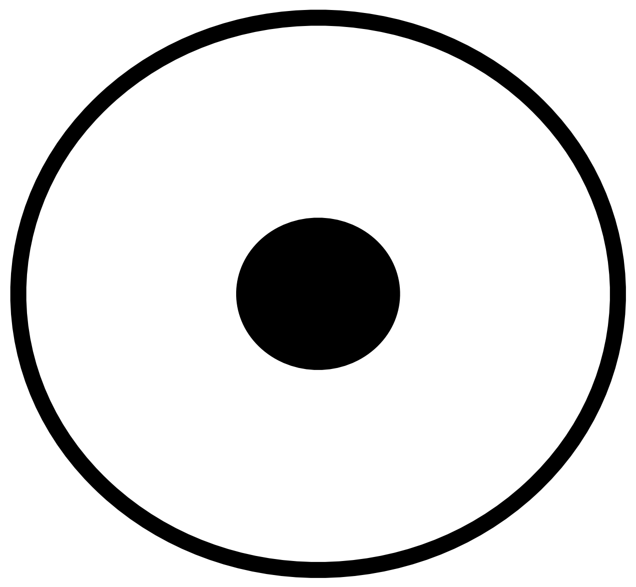
$$|\Psi\rangle \otimes |A_0\rangle \rightarrow \sum_i c_i |i\rangle \otimes |A_i\rangle + \epsilon \sum_{i,j} d_{i,j} |i\rangle \otimes |A_j\rangle$$

Measurement Noise

Atom Aging

Interferometry - interaction between paths

Decaying Radioactive nucleus



$$\langle\chi|A_\mu|\chi\rangle J^\mu$$

$$|\chi(0)\rangle = |X\rangle$$

$$|\chi(t)\rangle = \alpha(t)\, e^{-\frac{\Gamma t}{2}}|X\rangle + \beta(t)\,|Y\rangle$$

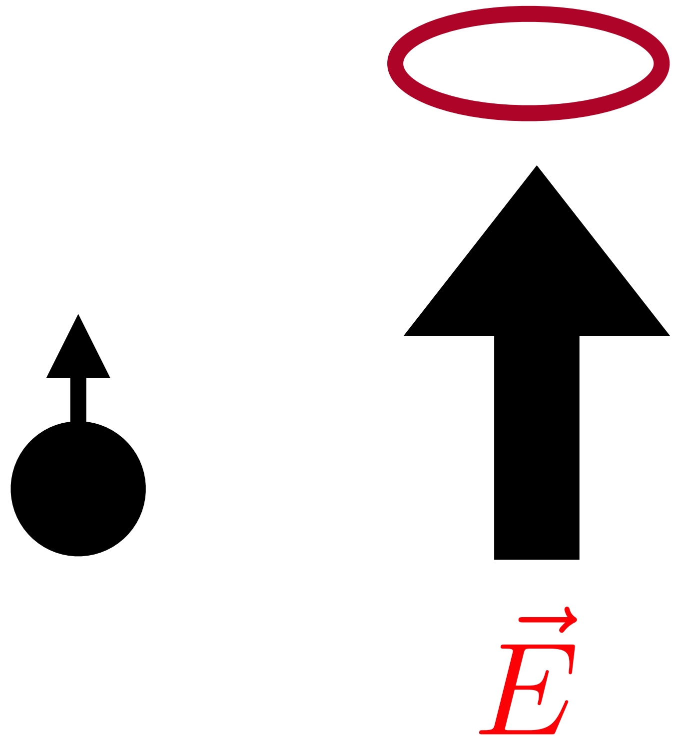
$$\langle\chi|A_\mu|\chi\rangle = \langle X|A_\mu|X\rangle \propto e^{-\Gamma t}$$

Time dependent self-interaction - time dependent shift to the energy of atomic states!

Delicate Non-Linearity

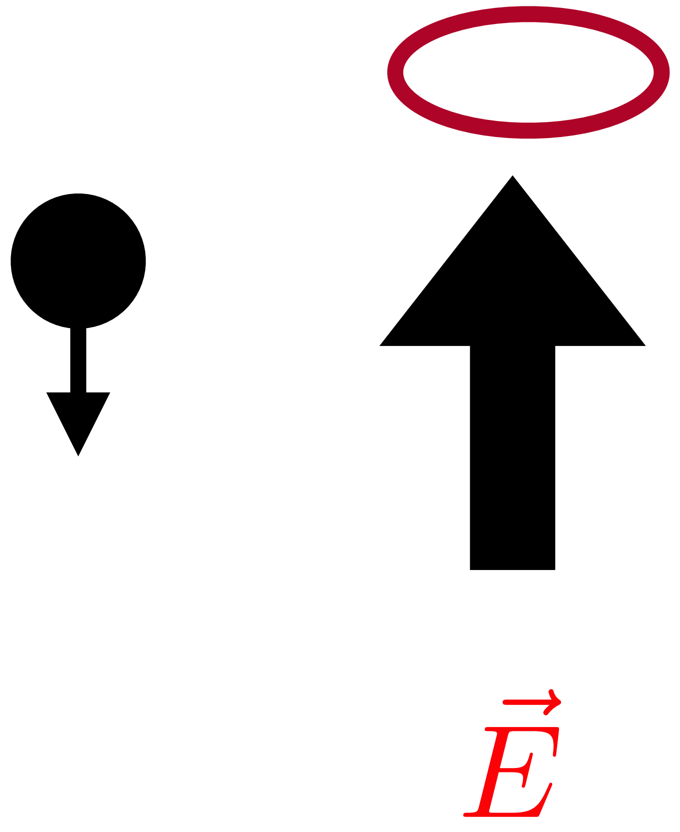
Suppose |X> = |U>

Alex performs experiment on July 6 - discovers non-linear quantum mechanics!



$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle + |D\rangle|O_D\rangle\right)$$

O wants to repeat experiment



Suppose $|O_U>$ decides to run experiment at 9 AM on July 10
But $|O_D>$ runs experiment on 9 AM on July 20

State on 9 AM on July 10

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle\frac{(|U\rangle|T\rangle + |D\rangle|R\rangle)}{\sqrt{2}} + |D\rangle|O_D\rangle\right)$$

Delicate Non-Linearity

State on 9 AM on July 10

$$|\chi\rangle = \frac{1}{\sqrt{2}}\left(|U\rangle|O_U\rangle\frac{(|U\rangle|T\rangle + |D\rangle|R\rangle)}{\sqrt{2}} + |D\rangle|O_D\rangle\right)$$

Compare with State on July 6

$$|\chi\rangle = \frac{1}{\sqrt{2}}(|U\rangle|T\rangle + |D\rangle|R\rangle)$$

$$\langle U|\langle O_U|\langle U\langle T|\langle E_T|eA_\mu(x_R)\,\bar{\Psi}(x_R)\,\gamma^\mu\Psi(x_R)\,|\chi\rangle = \frac{1}{2}\langle U|\langle T|\langle E_T|eA_\mu(x_R)\,\bar{\Psi}(x_R)\,\gamma^\mu\Psi(x_R)\,|\chi\rangle$$

Effect is 1/2 of prior effect!

But, full effect if O_U and O_D perform experiment at same time!

Quantum Pollution: Without adequate care, superpositions may diverge wildly, preventing exploitability. Not automatic - but need careful protocols!

Particles have been scattering for 13 billion years. Cosmological dilution?

Cosmological Relaxation of Non-Linear QM?

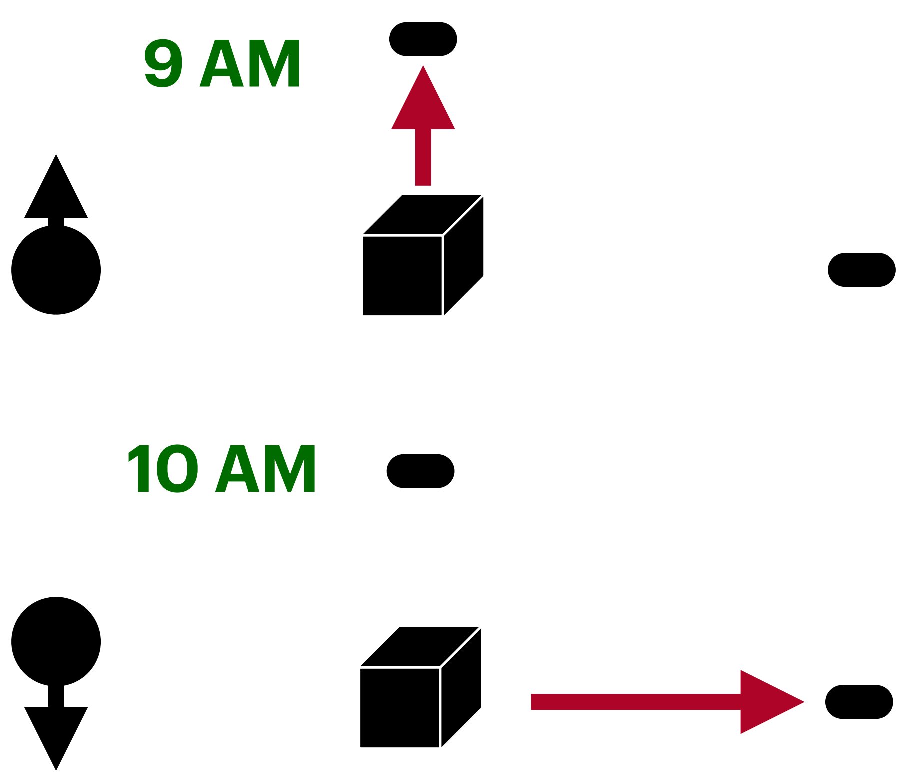
$$\mathcal{L} \supset eA_\mu \bar{\Psi}\gamma^\mu \Psi + \epsilon_\gamma e\langle\chi|A_\mu|\chi\rangle \bar{\Psi}\gamma^\mu\Psi$$

All we need is the expectation value. Non-Linear effects are resistant to decoherence.

For e.g. when we repeat the experiment, it is ok for O_U and O_D to be two different individuals - all we care is that the fields are turned on at the same space-time points

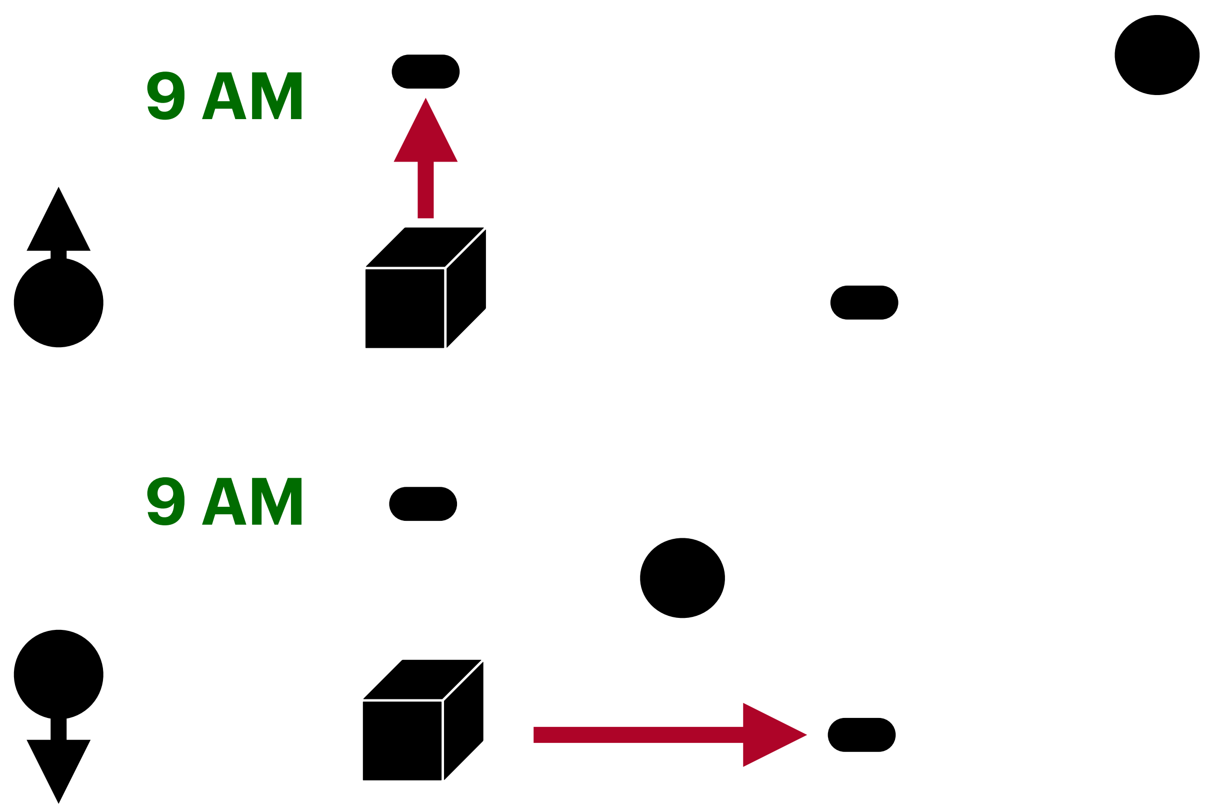
Relevant

Superpositions where expectation values of fields are very different



Irrelevant

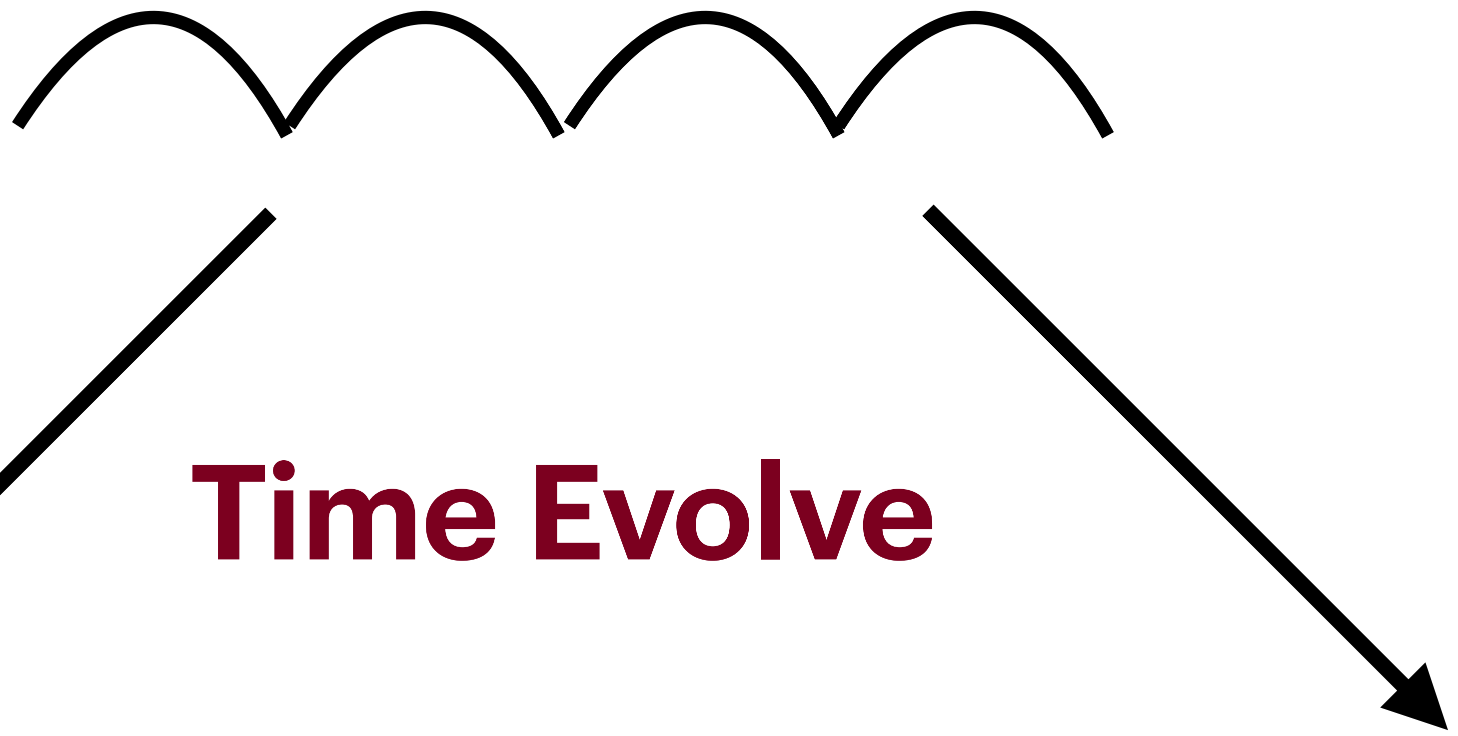
Scattering where expectation values are not significantly changed



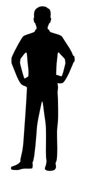
Classical Universe?

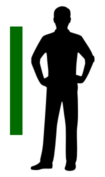
Suppose |X> = |U>

|U(t = 0)> =



Time Evolve

|U(t)> = |> + δ |☠ >

|U(t)> = δ|> + |☠>

Can quantum events (scattering, decay etc.) lead to wildly different classical outcomes?

Clearly Possible - e.g. Human choosing to act differently based on quantum event

But, fundamentally - this is because humans can be quantum amplifiers

Are there natural quantum amplifiers, for e.g. in chaotic systems?