Hardware-Efficient Decomposition of Qudit Gates

Sean Dougherty, Northwestern University

Tanay Roy, Fermilab

FERMILAB-POSTER-23-232-SQMS-STUDENT

Introduction: Computing with Qubits

Most contemporary quantum computing efforts have focused on encoding quantum information in two-level systems, called qubits. Processing information on d-dimensional systems or qudits, provides a larger Hilbert space to store and process information which can reduce circuit complexity as well as improve the information density of the processor.

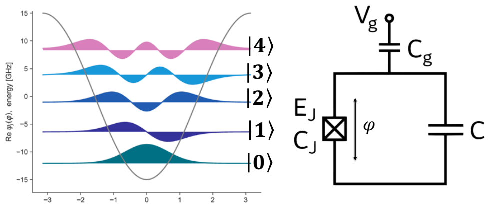


Fig. 1 Transmon energy levels (left) and circuit diagram (right)

Gate Decomposition

All qudit operations (or gates) can be represented by a unitary matrix. Before an algorithm can be implemented on a quantum computer, the gates must be decomposed into elementary operations which are executable on the quantum hardware.

Gate decomposition for qubits is well known and is equivalent to performing a Euler angle decomposition on the transformation.

For qudits, the process is more convoluted since not all transitions are possible. In transmon qudits, for example, only transitions between adjacent energy levels are allowed. In general, many decompositions exist for a given unitary gate.

The goal of this work is to find a hardware efficient gate decomposition scheme for an arbitrary unitary.

Example: Qutrit Hadamard Gate

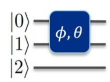
The Hadamard gate (H) is an essential component of most quantum algorithms. H can be conveniently constructed in terms of three SU(2) rotations [1]:

$$\omega = e^{2\pi i/3} \qquad H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$$

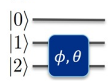
$$H = R_{12}\left(0, \frac{\pi}{2}\right) \cdot R_{01}(0, \beta) \cdot \Theta\left(\pi, \frac{\pi}{2}\right) \cdot R_{12}\left(0, \frac{\pi}{2}\right) \cdot \Theta(0, \pi)$$

$$\text{with } \beta = 2\tan^{-1}(\sqrt{2}).$$

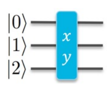
$$R_{01}(\phi,\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -e^{-i\phi}\sin(\theta/2) & 0 \\ e^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \equiv$$



$$R_{12}(\phi,\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\theta}{2} & -e^{-i\phi}\sin(\theta/2) \\ 0 & e^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix} \equiv$$



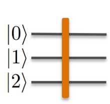
$$\Theta(x,y) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{ix} & 0 \\ 0 & 0 & e^{i(x+y)} \end{bmatrix} \equiv$$



The decomposition consists of two rotations in the $\{|1\rangle, |2\rangle\}$ subspace and one rotation in the $\{|0\rangle, |1\rangle\}$ subspace. However, higher levels typically have shorter coherence times and so a decomposition involving more low-energy levels is preferred.

A preferred decomposition can be obtained by performing a change of coordinates $|0\rangle \leftrightarrow |2\rangle$ with transition matrix, T.

$$T = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \equiv$$



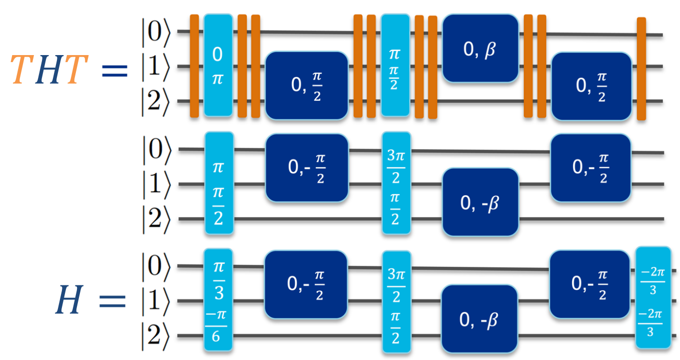


Fig.2 Decomposition visualization for Hadamard gate into SU(2) rotations (dark blue), generalized phase gates (cyan) and T matrices (orange). The process takes the Hadamard into the transformed coordinates (top), applies the transformation to each constituent operation (middle) and then returns to the standard basis (bottom).

This T matrix scheme produces a decomposition with two $|0\rangle \leftrightarrow |1\rangle$ transitions and only one $|1\rangle \leftrightarrow |2\rangle$ transition.

For transmons, this decomposition is more hardware efficient since low energy states are less susceptible to decoherence.

The phase gates can be applied virtually in software and so the additional phase gate will not contribute to the duration of the pulse sequence. [2]

This scheme can be easily extended to ququart and beyond. The generalized T matrix takes $|m\rangle \rightarrow |n-m\rangle$ for $n \geq m$.

References and Acknowledgments

1. Roy, Tanay, et al. "Two-Qutrit Quantum Algorithms on a Programmable Superconducting Processor." Physical Review Applied, vol. 19, no. 6, June 2023.
2. David C. McKay et al. "Efficient Z gates for quantum computing". Phys. Rev. A 96 (2 2017)

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.
This work was supported in part by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists (WDTS) under the Science Undergraduate Laboratory Internships Program (SULI).

Future Work

1. Construct similar symmetry arguments for other common qutrit gates or sections of other qutrit gates
2. Find analytical form for Hadamard decomposition for a qudit

Title of Poster – Helvetica / Arial Bold 110pt

Author(s) of poster, Affiliation – Helvetica / Arial 60pt Regular

Subtitles – Helvetica / Arial Bold 50pt RGB 0,76,151

Body text – Helvetica / Arial Regular 40pt / Black

This template is to be used as a guide when designing your poster. The layout can be rearranged and elements can be resized within the columns. Overall design should follow this grid with 1 inch gutter and margin.

Body text – Helvetica / Arial Regular 40pt / Black

This template is to be used as a guide when designing your poster. The layout can be rearranged and elements can be resized within the columns. Overall design should follow this grid with 1 inch gutter and margin.

Subtitles – Helvetica / Arial Bold 50pt RGB 0,76,151

Body text – Helvetica / Arial Regular 40pt / Black

This template is to be used as a guide when designing your poster. The layout can be rearranged and elements can be resized within the columns. Overall design should follow this grid with 1 inch gutter and margin.

Subtitles – Helvetica / Arial Bold 50pt RGB 0,76,151

Body text – Helvetica / Arial Regular 40pt / Black

This template is to be used as a guide when designing your poster. The layout can be rearranged and elements can be resized within the columns. Overall design should follow this grid with 1 inch gutter and margin.