Fermi National Accelerator Laboratory

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Low Energy Muon Capture and Transport from a Heavy Target

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Abstract

The MeV Test Area (MTA) secondary beamline is currently designed to generate a low-energy pion beam using the 400 MeV primary proton Linac beam on a tungsten target, leading to a tertiary muon beam from decay along its secondary path. However, a key challenge lies in the significant muon decay angles emitted relative to the transported pion beam which is a result of the very low momentum and small beam phase space of the captured pion beam. This, coupled with a flat pion production beam profile in angle, restricts the beamline elements' ability to effectively capture muons outside the limited acceptance of the secondary beamline. To address this limitation, a low-energy MTA muon beam collaboration group is actively engaged in research and development for incorporating newly acquired, stronger-gradient quadrupoles to enlarge the capture solid angle of the secondary beamline. Their primary goal is to upgrade the lattice structure of the MTA's secondary beamline, aiming to enhance its muon capture efficiency and enhance the available muon flux well beyond the current configuration.

Introduction

The MeV Test Area (MTA), located at the end of the current Linear Accelerator (LINAC), is the primary location for smaller-scale, sub-GeV experiments. In its present state, the MTA has been set up to receive a primary beam of Hydrogen ions (H-) originating from the LINAC, which is then incident on a Tungsten or Graphite target. A secondary pion beam is thus generated that undergoes decay while in flight, producing a tertiary beam of μ^- for π- production and capture. The current Fermilab LINAC beam has a kinetic energy of 400 MeV, and the secondary pion beam produced from the target has a mean kinetic energy of approximately 32.128 MeV. The calculated initial elliptical emittance originating from the target measures around 5π mm-mrad (Geometric, flat, non-Gaussian distribution). This emittance result corresponds to a spot size with an approximate radius of 5mm about the reference trajectory at the target and an elliptical capture angle of only $\pm$1mr in the current low-strength quadrupole configuration (a larger non-elliptical phase space is captured in simulations). With the stronger quadrupoles, this emittance can be increased to as much as 60π mm-mr and a capture angle of $\pm$12mr. The LINAC generates 5.3x10^{12}ac per pulse with a pulse duration of 32 μs. To adhere to existing allowed radiation limits, the maximum pulses allowed per minute is 8. Based on an initial study conducted earlier in the year, the production rate per pulse for π^- is estimated at approximately 5300 events per pulse, while for π+, with a ratio of 2 π+ per π^-, the production reaches 10600 events per pulse [1]. This latter ratio increases significantly with lower atomic mass for the target (graphite vs. tungsten) as π^- is produced by a proton charge-changing with a neutron in the nucleus.

The primary challenge encountered by the current low focusing strength configuration of the MTA pertains to the decay angle of the $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ process. Given the pion's relatively low kinetic energy and momentum, the angle between the two resulting daughter products is substantial and not forward-directed as is the case for a more relativistic pion beam. This leads to significant muon capture losses and dependence on the lattice design, strength, and spacing of the quadrupole components. The existing elements cannot effectively capture and transport muons from pions that are outside ±10 mrad at the target. This suboptimal level of acceptance originates from their considerable length of 12.75 inches per element coupled with low magnetic gradient which is a characteristic of air core quadrupoles. To address this issue, the MTA collaboration has initiated a study to explore the utilization of recently acquired steel core quadrupoles which are both stronger and significantly shorter (3”). The existing lattice configuration for the MTA's Secondary Beamline comprises a capture solenoid, a matching quadrupole, and a half FODO cell which extends ~5m in length. The individual quadrupole elements consist of multiple air core quadrupoles in series to form a single "effective quadrupole," as depicted in Figures 1 and 2. The air core quadrupoles cannot generate a sufficient magnetic field individually. Hence, to achieve the quadrupole strength needed for efficient transport, they must be arranged in a sequential stack to create a single "effective quadrupole."

A consequence of this extended distance is a low-angular acceptance resulting in high loss of muons from in-flight decay. Particularly for the lower production rate of π-/ μ^- particles, the loss level is substantial raising concerns about supporting experiments such as muon catalyzed fusion. With the proposed quadrupole upgrade for the MTA, the stronger, shorter quadrupoles are projected to enable a reduced spacing between elements, owing to their decreased length. In addition, their enhanced magnetic gradient strength should facilitate a broader angular acceptance, resulting in the capture of muons with significantly larger decay angles.

The acquisition of the new quadrupoles, depicted in Figure 3, are surplus inventory from a completed upgrade for the current LINAC circa 1993. These components built for the 200 to 400 MeV upgrade of the Fermilab Linac serve as spares in the event of a Linac quadrupole unrepairable failure. Recently, the MTA secondary beamline project acquired seven spare inventory quadruples, as the new PIP-II Linac project is approaching operational readiness. Each element has a physical aperture radius of 0.787 inches and a steel length of 2.75 inches. The recently obtained quadrupoles are approximately 14 times shorter in length and 2.5 times narrower in aperture compared to the dimensions of the existing elements. One of the most significant advantages of the new quadrupoles is their pole construction, comprised of laminated low-carbon steel plates. The specific steel alloy selected features a carbon composition within its matrix ranging from 0.02% to 0.10%. Each lamination forms a quarter of the entire element, as depicted in Figure 4. A complete element comprises four pole assemblies, each wound in enameled copper wire. The primary advantage of the chosen alloy lies in its minimal trace element content, resulting in an iron percentage reaching up to a minimum concentration of 98.814% [2]. Due to its high purity, the material demonstrates magnetic properties resembling iron. Iron's high relative magnetic permeability allows significant density of magnetic field flux lines from a current source. A ferromagnetic material aligns its magnetic domains to the direction of the field, thereby increasing the number of flux lines achievable through an area and thus increasing the net achievable magnetic field strength. Moreover, the gradient field integrated along the length of the quad greatly exceeds the integrated strength of the gradient observed in an air core quadrupole.

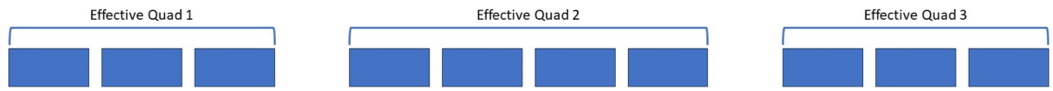

Figure 1 *Current FODO cell configuration consisting of a matching quadrupole as the first cell, a focusing quadrupole for the second cell, and a defocusing quadrupole for the third cell.*

Figure 2 Picture of current air core quadrupole located in the MTA Hall.

Figure 3 Future upgraded low carbon steel core quadrupoles.

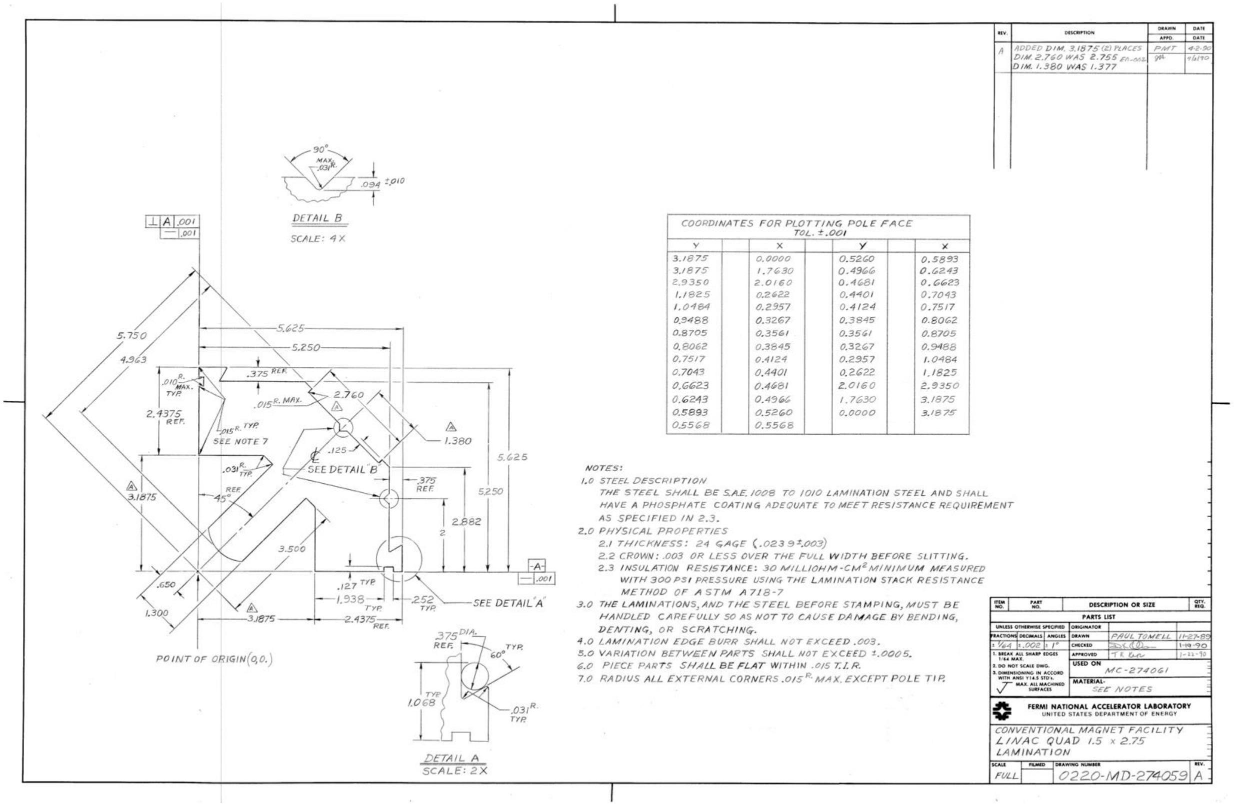

Figure 4 Technical drawing of a single pole, with materials and construction specifications.

Purpose

The current beamline setup is customized to fulfill the requirements of NK Labs' Muon Catalyzed Fusion experiment. The experiment's central focus is to reevaluate the sticking factor associated with a muon during DD and DT fusion processes. Their fabricated target consists of a liquid deuterium sample 2 mm in diameter, enclosed and pressurized between two diamond anvil cells [5]. The MTA's current objective is to deliver a stable beam of low-energy muons to the NK lab target; however, to produce an experimentally viable beam, the beamline must be capable of capturing more muons and delivering higher flux per unit area/sec. The upgraded quadrupoles are anticipated to significantly enhance both the transport and capture of decay muons. The implementation involves conducting a design and study of a new lattice design. This foundational groundwork is essential to smoothly integrating the upgraded components within the MTA. This new lattice and integration of stronger quadrupoles can support future experiments involving antimuons and muonium.

Methods

During the initial phase of the design process, the primary emphasis centers around increasing the acceptance of the secondary beam of pions by focusing on efficiently transporting a larger phase space of pions from the target to the final drift without consideration of decay. By designing for a larger emittance pion beam using conventional envelope lattice techniques, the foundation is established for generating and capturing more muons along the entire length of the line. The first step to accomplish this lattice design is calculating the independent parameter "k," which represents the magnetic focusing strength [m^{-2}] in "normalized" or energy-independent units (the strength or current required in beamline elements however is dependent on the central momentum of the beam). Establishing a rudimentary thin-lens approximation for the magnetic focusing strength (desired focal length and phase advance for a FODO-based lattice) allows CERN's Methodical Accelerator Design (MAD) to have a starting point to effectively apply an optimization/fitting algorithm. MAD will execute fits to lattice constraints to find the best "fit" to user-defined Twiss Parameters at strategic locations – such as reflective symmetry and small waist optics at the experiment. Utilizing the given dimensions of the quadrupoles and initial conditions for the pion beam, the thick lens equations will compute B' (integrated field strength) for the quadrupole which matches the user-defined constraints. The focal length is based on a 90 deg FODO cell and quadrupoles were initially positioned from 1-2' center to center with strengths that produce a focal length of 1.414 x center to center distance. Note that a half quad length must be used for the focal length (parallel to point focusing).

$$kl = \frac{1}{f} \quad 1.1$$

$$k = \frac{eB\prime}{cp_0} \quad 1.2$$

$$f = 1.414(d) \quad 1.3$$

To compute the integrated magnetic strength B', the combination of equations 1.1 and 1.2 is used to produce an equation solving for B' equation 1.1 establishes the thin lens approximation link between the quadrupole length l, focusing strength k, and the reciprocal of the focal length f. Subsequently, equation 1.2 equates k with the elementary charge e and B' divided by the speed of light constant c and the initial momentum p_0 in units of [GeV/c].

$$B' = \frac{p_0}{0.4242(0.5l)(d)} \quad 1.4$$

Using the derived equation 1.4, the integrated field strength is calculated for distances of 1 ft, 1.5 ft, and 2 ft. Based on the resulting B' values, the magnetic focusing strength is determined and presented in Table 1.

Quad Sep Dist. [Ft]	Quad Sep Dist. [m]	Integrated Field Strength [T/m]	k [m^-2]
1.00	0.304785	22.0977	66.2931
1.50	0.457178	14.7318	44.1954
2.00	0.609570	11.0488	33.1464

Table 1 Calculations of Integrated field strength for a quadrupole half-length and the resulting magnetic focusing strength, dependent on quadrupole separation distance.

After obtaining the preliminary values, they are input into MAD. The idea behind initiating the design process in MAD is to use its optimizers to establish a lattice configuration that produces optimal Twiss Parameter values specified by the user. In this study, the beta function (which is proportional to the square of the beam envelope) must be as small as possible to maximize the phase space acceptance of the pion beam that fits within the beam pipe. Alpha must begin at zero and end at zero to create what's known as a waist, and one starts with a phase advance of 90 degrees for both the x and y envelopes to preserve a base FODO cell. Collectively, these parameter and lattice cell values contribute to the achievement of a stable, minimal beam envelope function.

After establishing a stable beta function envelope for the initial FODO cell, as shown in Figure 5, this can be used to fit the quadrupole parameters to a center "symmetry" or alpha=0 point at the center of the optics in the beamline. Then a reverse propagation can be applied to reverse the optics to a small beta-function waist at the location of the experiment – this represents the smallest beam size and largest momentum acceptance. To achieve this, the beta value at the first focusing quadrupole serves as the initial condition. A particle is simulated as it traverses through a drift and then to a defocusing quadrupole, shown in Figure 6, with the requirement that it converges to a waist point by bringing alpha to zero at the midpoint in the lattice. The script employed to simulate this process is outlined in Appendix B1. To modify separation distance as an adjustable parameter, the design process necessitates the utilization of MAD8, which permits the parameterization and fits to drift lengths. Upon identifying the initial values for k in both the matching quadrupoles and the FODO elements, MAD8 is utilized to design the complete lattice. The beta function at the target is found by back-propagating this FODO cell to the target position. The reflection optics guarantee that the minimum beta function is also established at the experimental location.

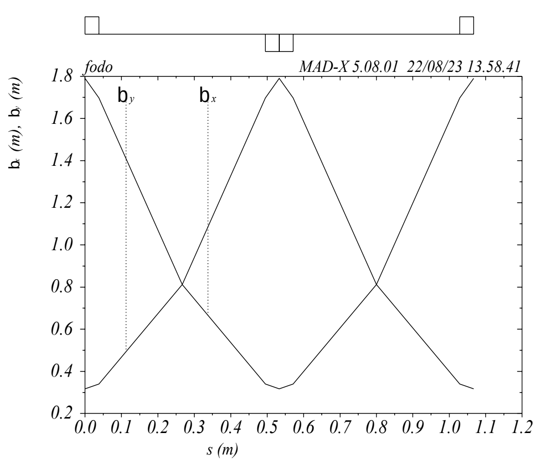

Figure 5 Beta function for initial FODO cell found using MADX and a separation distance of 1.5 feet.

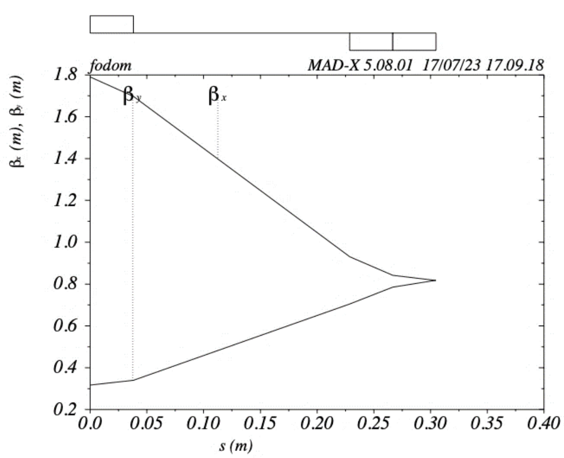

Figure 6 5 Beta function for reverse propagation of QF to MQ element found using MADX and a separation distance of 1.5 feet.

Using MAD8, Dr. Johnstone was able to produce a 2m stable line with five elements; however, this produced a reduction of muons produced in the decay in flight process. A beam of pions with a momentum of 100 MeV/c will have a total batch decay of 59.163% at 5 meters. At 2 meters that value drops to only 30.096%, refer to Appendix A1 for calculations. Currently, design efforts are being made to lengthen the line, but current efforts use the ratio of decay to scale produced models to predict how a 5-meter line will behave at 100 MeV/c. Future efforts, addressed later, will discuss how a 5-meter line may be achieved.

Once the initial 2m line has been designed, its parameters are integrated into G4Beamline for simulating the decay characteristics of muons. The line's representation can be observed in Figure 7. Through G4Beamline, it becomes possible to evaluate the capturing potential of the new elements and to determine the line's capability for transporting particles to the intended target, this code can be found in Appendix B2. During the simulation process, a previously developed package known as "jjsm-fermi" was employed to facilitate multicore processing and data extraction from the output generated by G4Beamline [7].

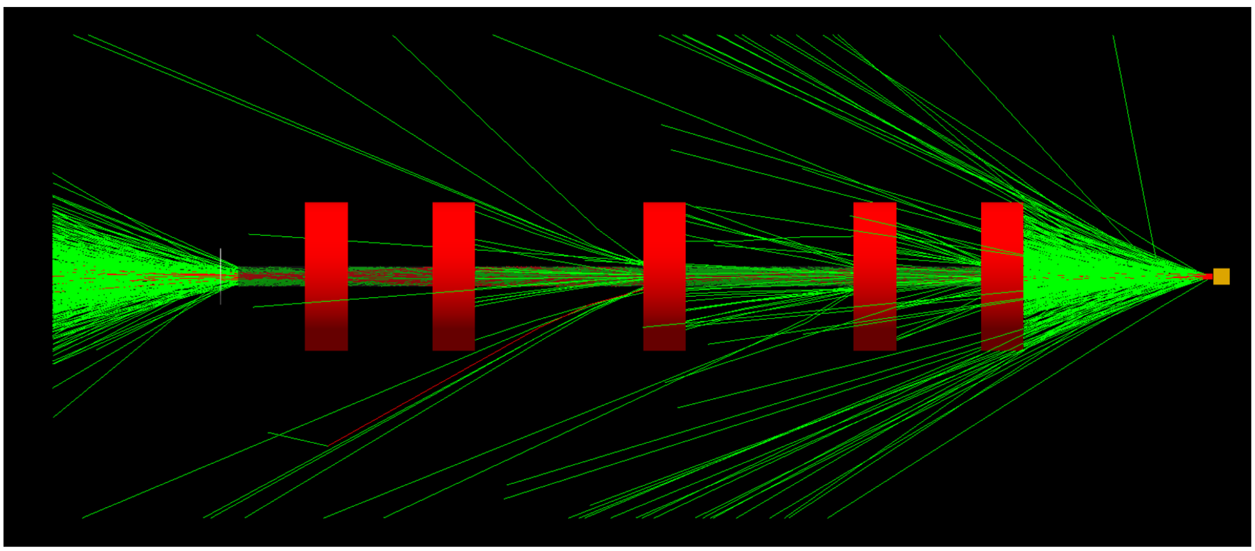

Figure 7 Simulation of the 2m line was carried out using the G4Beamline GUI, and an image displaying the trajectories of muons within the beampipe during the simulation is available.

Results and Conclusion

Through G4Beamline simulations, it became evident that the new elements effectively expanded the capture radius to about ± 50 mRads. This expansion notably increased the count of captured muons at the end of the line. The measurements taken were with virtual detectors staring at the beampipe's center and radiated outward by 5 mm, 15 mm, and 50 mm. A graphical representation of the detected muons is presented in Figure 8. It's important to note that the results from the shortened line were adjusted based on the decay ratio between a 2 m line and a 5 m line. Despite this adjustment, the preliminary outcomes indicate an acceptance growth of approximately a factor of 5 and display a large increase in muon counts at the end of the line.

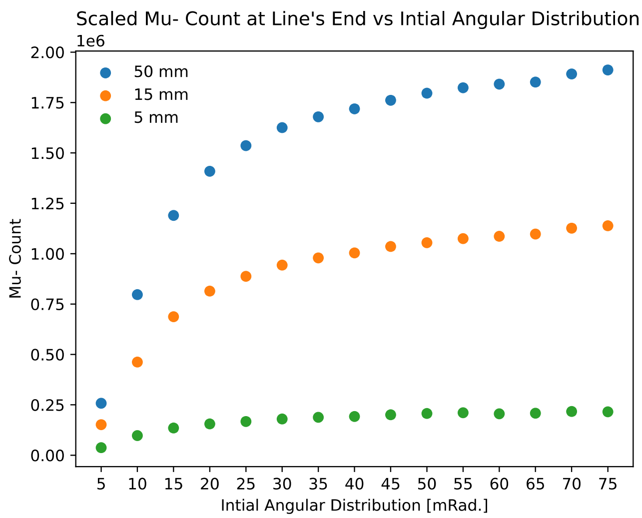

Figure 8 Graph exhibiting the relationship between Muon Count and Angular Distribution, derived from data extracted by jjsm-fermi and information sourced from the NewBeamline-V2 script.

Future Work

In the context of the MTA secondary beamline, the primary future objective is the extension of the line to optimize both capture and number of decays. Another approach is to reduce the momentum of a pion by implementing a beam degrader to reduce the momentum of the pions at the exit of the target. The decay in flight then is increased and permits an extension of the line to around 3 meters without compromising the decay fraction. This extension plus the reduction to 50MeV/c ensures that there is one mean lifetime's worth of decay and also aligns with the momentum requirements for NK Lab's test assembly. Subsequent MAD and G4Beamline simulations are planned to validate a production model for the beam degrader and to assess the lattice for the extended line – two more quadrupoles are available for this extension without compromising angular acceptance or quad spacing. (The lower momentum reduces the magnetic rigidity of the beam and a slower beam allows for a shorter decay time. After the performance of the new line is confirmed for μ^- studies, the performance will be studied for surface μ^+ production for future experiments involving Muonium (this is ~ a 4 MeV muon energy). Prospective Muonium experiments, such as antimatter gravity interactions using surface spin muons are proposed. Furthermore, this configuration also allows the MTA to be adapted as a facility for muon spin spectroscopy (Muon Spin Resonance or MuSR), similar to the user muon facilities at the Paul Scherrer Institute (PSI).

In the context of future experiments centered on Muon Catalyzed Fusion, there is a proposition for a reactor design using a recirculating accelerator with an internal target for production once studies are complete. This design incorporates non-scaling Fixed Field Gradient Accelerators (FFGAs), resembling those proposed for muon colliders. The central concept is to recover the primary protons after the target and reuse them to produce pions with an RF cavity replacing the energy loss. The approach for delivery involves utilizing an internal target. To facilitate this, a 100 kHz RF cavity would be employed within the FFGA, aiding in both cooling the emittance blowup primary proton beam after the target and replacing the energy lost in the target. This cooling process enhances the potential for increasing the number of muons per incident proton and reducing the energy consumption required for fusion. The prospect of a Superconducting design appears promising as well, as it has the potential to further improve the efficiency of a muon-catalyzed fusion power plant design by incorporating a recirculating FFGA with an internal target.

Appendix A (Equations)

A1 Percent Decay for π^- Within the MTA

$P_0 = 100\ [MeV/c]$ (Initial longitudinal momentum of Pion)

$RE = 139.57\ [MeV/c^2]$ (Rest mass for a charged Pion)

$\tau = 26.0\ [ns]$ (Mean lifetime of a charged Pion)

$$E_{tot}^2 = (P_0 c)^2 + RE^2 \quad \text{(A1.1)}$$

$$E_{tot} = KE + RE \quad \text{(A1.2)}$$

$$KE = \sqrt{(P_0 c)^2 + RE^2} - RE \quad \text{(A1.3)}$$

Equations (E1.1) through (E1.3) introduce the variable E_{tot}, signifying the total energy of a particle, encompassing both kinetic energy (KE) and rest energy (RE). Within this set of equations, KE remains undetermined, prompting the utilization of P_0 to deduce the total energy through equation (E1.1). By reordering (E1.2) to solve for KE and subsequently substituting (E1.1), equation (E1.3) emerges.

$$KE = (\gamma - 1)RE \quad \text{(A1.4)}$$

$$\gamma = \frac{KE}{RE} + 1 \quad \text{(A1.5)}$$

$$\gamma = \frac{\sqrt{(P_0 c)^2 + RE^2} - RE}{RE} + 1 \quad \text{(A1.6)}$$

$$\beta = \sqrt{1 - \left(\frac{1}{\gamma}\right)^2} \quad \text{(A1.7)}$$

$$\gamma = 1.23\ \&\ \beta = 0.582$$

Upon determining the value of KE, the equation linking RE to KE through γ (E1.4) is manipulated to solve for γ, the Lorentz factor. Substituting (E1.3) into (E1.5) results in (E1.6), where γ is computed as 1.23, rounded to three decimal points.

Subsequently, this γ value is employed to derive β in equation (E1.7), yielding a value of 0.582 for β.

$$\tau_\gamma = \gamma\tau\ [ns] \tag{A1.8}$$

$$\lambda = \frac{1}{\tau_\gamma}\ [ns^{-1}] \tag{A1.9}$$

$$\tau_\gamma = 31.98\ [ns]\ \&\ \lambda = 0.03127\ [ns^{-1}]$$

Once γ and β are established, γ is employed to adjust the mean lifetime due to relativistic effects stemming from the velocity of the Pions. Utilizing the γ value of 1.23 in equation (E1.8), along with a mean lifetime τ of 26.00 [ns], yields the dilated mean lifetime τ_γ, which amounts to 31.98 [ns] at β = 0.582 or 58.2% the speed of light. To determine the decay factor λ, the reciprocal of the dilated mean lifetime is taken, resulting in a value of 0.03127 [ns^{-1}].

$$\Delta\ell = [2\ [m], 5[m]]$$

$$\bar{v} = \beta c\ [m/s] \tag{A1.10}$$

$$t = \frac{\Delta\ell}{\beta c} \tag{A1.11}$$

$$t = \left[11.45\ [ns], 28.69\ [ns]\right]$$

$$Percent\ Decay = \left(1 - e^{-\lambda t}\right) \cdot 100 \tag{A1.12}$$

$$Percent\ Decay = \{30.096\ \%, 59.163\ \%\}$$

Having defined λ, the time of flight and subsequent percentage decay is calculated using β from equation (E1.7) and λ. Line lengths Δl of 2 [m] and 5 [m] are employed, and the $\bar{v}$ represents β multiplied by the speed of light in units of [m/s] (equation E1.10). With these values, flight time is determined using equation (E1.11). At 2 [m], the Pion travels for 11.45 [ns], and at 5 [m], the Pion travels for 28.69 [ns]. The percentage decay is computed by plugging these flight times into equation (E1.12) along with λ. At 2 [m], 30.096% of Pions decay, and at 5 [m], 59.163% of Pions decay.

Appendix B (Code)

B1 MADX MQ_MTA_FODO

```
!----------------------SECONDARY BEAMLINE PARAM----------------------
beam, MASS=0.13957039, CHARGE=-1., PC=0.1;
!SETPLOT, LWIDTH=5.0, ASCALE=1.0, LSCALE=2.0, RSCALE=2.0, SSCALE=5;
!----------------------FODO Cell Optic Parameters--------------------
! Quad Steel length
lq = 0.076200152;

! k and ld for 1.5 ft (18 in.)
qk := 36.5894;
ld := 0.457178;
!----------------------FODO Cell Optic Parameters--------------------
D:  DRIFT,  L :=0.5*ld; !Half drift
QF: QUADRUPOLE, L=0.5*lq, K1:=qk; ! Half cell w/ postive k
QD: QUADRUPOLE, L=0.5*lq, K1:=-qk; ! Half Cell w/ negative k

M1: MARKER;
M2: MARKER;
M3: MARKER;

FODO: line = (QF, D, M1, D, QD, M3, QD, D, M2, D, QF);
!-----------------------------Simulation----------------------------
use, period=FODO;

! For 1.5 scaled ft
match, sequence=FODO;
VARY, NAME=qk, STEP=0.000001, LOWER = 0.0, UPPER= 88.67151355;
VARY, NAME=ld, STEP=0.000001, LOWER= 0.304785, UPPER= 0.609570;
!constraint, sequence=FODO, range=M1, BETX = 0.7825303793, BETY = 0.7825303793;
!constraint, sequence=FODO, range=M3, MUX = 0.15, MUY = 0.15;
!constraint, sequence=FODO, range=M2, BETX = 0.7825303793, BETY = 0.7825303793;
constraint, sequence=FODO, range=#e, mux = 0.25, muy = 0.25;
LMDIF, CALLS=100000000, tolerance=0.00000001;
!SIMPLEX, CALLS=100000000, tolerance=0.00000001;
endmatch;

!twiss, save, BETX = 2.179582125, BETY = 0.1787507416, MUX = 0, MUY = 0,
!file=MatchTwiss_Scaled.out;
```

```
twiss, save, file=MatchTwiss_Scaled.out;

plot, haxis=s, vaxis=betx, bety, file=mbeta_plot_Scaled;
plot, haxis=s, vaxis= mux, muy, file=mmu_plot_Scaled;
plot, haxis=s, vaxis= alfx, alfy, file=maplf_plot_Scaled;

USE ,SEQUENCE=FODO;
SAVEBETA, LABEL=END, PLACE=#e, SEQUENCE=FODO;
TWISS;
!-----------------------FODOM Cell Optic Parameters---------------------
qkm := 36.5894;
ldm := 0.190488924; !Intial is 0.457178 at 0.266689076 BETX = BETY (0.8121493798)

DM: DRIFT, L :=ldm;
QM: QUADRUPOLE, L=0.5*lq, K1:=-qkm;

M1: MARKER;

FODOM: Line = (QF, DM, QM, QM);
!-------------------------------FODOM Simulation-----------------------------
USE, period = FODOM;

match, sequence=FODOM, BETA0=END;
VARY, NAME=qkm, STEP=0.000001, LOWER = 0.0, UPPER= 88.67151355;
VARY, NAME=ldm, STEP=0.000001, LOWER = 0.0, UPPER= 0.4125;
constraint, sequence=FODOM, range=#e, ALFX =0.0, ALFY =0.0;
!LMDIF, CALLS=1000000, tolerance=1.0e-18;
SIMPLEX, CALLS=100000, tolerance=1.0e-18;
!MIGRAD, CALLS=1000000, TOLERANCE=1.0e-18, STRATEGY=1;
endmatch;

TWISS, BETA0 = END, RMATRIX=True, save, file=SolMatchTwiss_Scaled.out;
!----------------------------------Plot-------------------------------
plot, haxis=s, vaxis=betx, bety, file=Solmbeta_plot_Scaled;
plot, haxis=s, vaxis= mux, muy, file=Solmmu_plot_Scaled;
plot, haxis=s, vaxis= alfx, alfy, file=Solmaplf_plot_Scaled;
```

A2 G4Beamline NewBeamline_V2

```
#lengths are in mm, fields is Tesla !  Momentum in MeV

#________________________________________________________________________


#------------------------------- Setup -------------------------------#

#Must add kill=$kill_aperture to every element to use this feature.

param kill_aperture=1


#physics QGSP_BERT disable=Decay
physics QGSP_BERT
#---------------------------- Define World and Beam -------------------------#
# Set default environment material
param worldMaterial=Vacuum


#BEAM
# Ellipse beam only takes X, Xp, Y, Yp, T, E
# Xp is x prime, which is decay angle
param -unset _meanMomentum=100
param -unset _sigmaP=20
param -unset _nEvents=10E6
param -unset _sigmaX=-5
param -unset _sigmaY=-5
param -unset _meanXp=0
param -unset _meanYp=0
param -unset _sigmaXp=-0.02
param -unset _sigmaYp=-0.02
param -unset _beamZ=16

#Launch Parameters
beam gaussian meanMomentum=$_meanMomentum sigmaP=$_sigmaP nEvents=$_nEvents
particle=pi- sigmaX=$_sigmaX sigmaY=$_sigmaY \\
meanXp=$_meanXp meanYp=$_meanYp sigmaXp=$_sigmaXp sigmaYp=$_sigmaYp beamZ=$_beamZ
```

```
# Only pay attention to the following particles (saves on calculation time), specifically with trackcuts
particlecolor pi-=1,0,0 mu-=0,1,0
trackcuts keep=mu-,pi-

#--------------------------- Bckground Color --------------------------#
#Scaling GUI view from pg 19 of the manual

g4ui when=4 "/vis/viewer/scaleTo 10 10 1"
#--------------------------- Define Beamline Elements --------------------------#
#Parameters
param Tw=15 Th=30 Tl=30
param Tm=W

param QfieldLength=152.4
param QironLength=76.200152
param QironRadius=133.25
param QpoleTipRadius=19.05
param QcoilRadius=80.9625
param QcoilHalfWidth=50.8


param MDQgradient=-10.7574665
param FQgradient=15.5351385
param DQgradient=-14.8896381

box Target width=$Tw height=$Th length=$Tl material=W color=1.0,0.75,0.0 kill=$kill_aperture \

genericquad MDQuad fieldLength=$QfieldLength ironLength=$QironLength ironRadius=$QironRadius\
poleTipRadius=$QpoleTipRadius coilRadius=$QcoilRadius coilHalfWidth=$QcoilHalfWidth
ironColor=1,0,0\
maxStep=1.0\
gradient=$MDQgradient fringeFactor=10.0 kill=$kill_aperture

genericquad FQuad fieldLength=$QfieldLength ironLength=$QironLength ironRadius=$QironRadius\
poleTipRadius=$QpoleTipRadius coilRadius=$QcoilRadius coilHalfWidth=$QcoilHalfWidth
ironColor=1,0,0 \
maxStep=1.0\
gradient=$FQgradient fringeFactor=10.0 kill=$kill_aperture
```

```
genericquad DQuad fieldLength=$QfieldLength ironLength=$QironLength ironRadius=$QironRadius\
poleTipRadius=$QpoleTipRadius coilRadius=$QcoilRadius coilHalfWidth=$QcoilHalfWidth
ironColor=1,0,0\
maxStep=1.0\
gradient=$DQgradient fringeFactor=10.0 kill=$kill_aperture



tubs ToEnd material=STAINLESS-STEEL outerRadius=19.05 innerRadius=17.4625 length=5000
color=.2,.2,.2,.3 kill=$kill_aperture
#---------------------- Define Virtual Detectors for scoring -----------------------#
#ascii is normal readable characters when foramt is in ascii is saying will output the data in that (Not the
most comp efficent)
#saying they are a vaccum means they won't interact with the particle because kill is on
#virtualdetector Det1 radius=50 length=1 color=1,1,1 material=Vacuum format=ascii

#virtualdetector Track1 radius=17.4625 length=1 color=1,1,1 material=Vacuum format=ascii
#virtualdetector Track2 radius=17.4625 length=1 color=1,1,1 material=Vacuum format=ascii
#virtualdetector Track3 radius=17.4625 length=1 color=1,1,1 material=Vacuum format=ascii
#virtualdetector Track4 radius=17.4625 length=1 color=1,1,1 material=Vacuum format=ascii
#virtualdetector Track5 radius=17.4625 length=1 color=1,1,1 material=Vacuum format=ascii


virtualdetector Det2 radius=50 length=1 color=1,1,1 material=Vacuum format=ascii
virtualdetector Det3 radius=15 length=1 color=1,1,1 material=Vacuum format=ascii
virtualdetector Det4 radius=5 length=1 color=1,1,1 material=Vacuum format=ascii
#---------------------- Place Beamline Elements -----------------------#
#intial target
place Target rename=target z=0.0
#place Det1 z=30.1 rename="DetectorAtTarget"

#First Quad 14" away

place ToEnd rename=EndPipe z=2530
place MDQuad rename=QD1 z=393.700076
#place Track1 z=393.700076 rename="DetectorAtQD1|$_sigmaXp"
place FQuad rename=QF1 z=622.300328
#place Track2 z=508.000202 rename="DetectorAtQF1|$_sigmaXp"
place DQuad rename=MQD1 z=1000.44878
#place Track3 z=1000.44878 rename="DetectorAtMQD1|$_sigmaXp"
```

```
place FQuad rename=QF2 z=1378.597232
#place Track4 z=1378.597232 rename="DetectorAtQF2|$_sigmaXp"
place MDQuad rename=QD2 z=1607.197484
#place Track5 z=1378.597232 rename="DetectorAtQD2|$_sigmaXp"
#---------------------- Place End Virtual Detectors -----------------------#
#Want detector at 6" away from old quad
place Det2 z=1797.69756 rename="DetectorAtEnd50|$_sigmaXp|$_nEvents"
place Det3 z=1796.69756 rename="DetectorAtEnd15|$_sigmaXp|$_nEvents"
place Det4 z=1795.69756 rename="DetectorAtEnd5|$_sigmaXp|$_nEvents"
#profile zloop=0:6000:50 particle=pi- file=BeamEllipseTransport.txt


#2.5 inches solenoid, +-1milirad, +-0.5cm add require=PDGID!=-211 when you build your virtual detector to make it not count pi-
#20 milirad and 60 and turn off pions, put detector at target
```

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