Harmonic Slip-Stacking

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Abstract

This report involves enhancing the stability of slip-stacking using the new approach "Harmonic Slip Stacking"[3]. We achieved these improvements by adding a new term to the harmonic equation of motion. This term accounts for the interactions between the particles, which helps stabilize the stacking process. We Performed a number of simulations and measurements to test the proposal By optimizing the values of voltage and the new harmonic term, we managed to show a significant expansion in the stable area and improvement by 50%. The results accounts for errors and shows that harmonic slip-Stacking still outperforms conventional slip-stacking, even in the presence of various degrees of error.

Acknowledgments

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Contents

1 introduction

Slip-stacking is a technique that involves combining and storing multiple particle bunches in an accelerator ring to increase the overall beam intensity[2]. In slip-stacking, two particle beams with slightly different energies are injected into the accelerator at different times. Overlapping is typically achieved by fine-tuning the accelerator's magnetic fields and controlling the acceleration rate. Particles from the higher-energy batch transfer energy to particles in the lower-energy batch. Once the slip-stacking process is complete, the combined beam is extracted from the Main Injector and directed to the desired experimental area for research purposes[2]. "slip stacking" is a crucial strategy for achieving high-intensity operation at Fermilab. The Fermilab Proton Improvement Plan(PIP-II)[1] proposes a 70% increase in beam power, requiring a commensurate reduction in slip-stacking losses. Recent work has shown a modification on slip-Stacking that is called "harmonic slip- stacking"[3].Harmonic slip stacking reduces particle loss using 106MHz 20KV cavity. the harmonic cavity operates at twice the average of the upper and lower frequency main rf cavities[3].
In this paper, a python tracking code is written to simulate bunches of particles using the harmonic equation of motion and compare with conventional slip-stacking equation of motion. Our result demonstrates that harmonic slip stacking could increase the Bucket Area by 50%. These results exceed the loss requirements of the Fermilab PIP-II upgrade.

2 Results

2.1 Background And Methods Used

The Equation of motion associated with a the trajectory of a single particle inside the rf cavity for Slip-Stacking are given by[**1**]:

$$\phi' = 2\pi f_{\text{rev}} h\eta\delta, \tag{1}$$
$$\delta' = f_{\text{rev}} \tfrac{eV_M}{\beta^2 E} [\sin(\phi) + \sin(\phi - \omega_\phi t)]$$

The Phase ϕ is the phase of the particle relative to the electromagnetic field in the rf cavity, and δ is the fractional deviation from the selected reference momentum[**1**].

Like slip-Stacking,The Equation of motion associated with a the trajectory of a single particle inside the harmonic rf cavity for Harmonic Slip-Stacking are given by[**2**]:

$$\phi' = 2\pi f_{\text{rev}} h\eta\delta, \tag{2}$$
$$\delta' = f_{\text{rev}} \tfrac{eV_M}{\beta^2 E} [\sin(\phi) + \sin(\phi - \omega_\phi t) + \lambda \sin(2\phi - \omega_\phi t)]$$

The new added term correspond to the new harmonic rf cavity[**2**], Where λ is the ratio between the harmonic rf voltage and the main rf voltage. Adjusting the ratio between the harmonic rf voltage and the main rf voltage (λ) can have effects on the behavior of the harmonic rf cavity. A negative (λ) leads to a specific defocusing effect at an average phase, which helps counteract disturbances caused by other components and reduces unwanted resonance effects. This adjustment contributes to better system performance and stability as we will show in this paper.

We can use both set of equation to trace the behaviour of as many particles as we want, One in conventional slip-Stacking and the other in Harmonic slip-Stacking. Using Python libraries, Numpy and Matplotlib, we created an array of initial values of particles then proceeded to trace them. using the following code:

```

import numpy as np
import matplotlib.pyplot as plt

def Lamda_Function(z, K):
  delta_height = 0.002 * np.sqrt(z)
  x = np.linspace(-np.pi, np.pi, 1000)
  y = np.linspace(-0.002, 0.002, 1000)
  phi0, delta0 = np.meshgrid(x, y)
  phi0 = np.ravel(phi0)
  delta0 = np.ravel(delta0)
```

We translate Eq.1 and Eq.2 using the library Numpy. We use for loop on each equation to trace each particle independently, see the following code:

```
tnum = 1000
phi = phi0
delta = delta0
#conventional Slip-Stacking equation of motion translated in Python
↪  Numpy
for lp in range(tnum-1):
      n = lp
      t = n*t_i
      delta = delta + V*(np.sin(phi) + np.sin(phi - 2*np.pi * dF *
      ↪  t) + lamda*np.sin(2*phi - 2*np.pi * dF * t ))
      phi = phi + L*delta
#Slip-Stacking equation of motion translated in Python Numpy
for lp in range(tnum-1):
      delta = delta + V*(np.sin(phi) + np.sin(phi - 2*np.pi * dF * t)
      phi = phi + L*delta
```

2.2 Harmonic Slip-Stacking Vs Slip-Stacking

We trace each particle independently to find out its final position which indicates whether the particle is stable or unstable. Eventually, we create a stability map of all particles that are only stable which visually shows the regions of stability and instability in a parameter space (See Fig 2 and Fig 3). This involves simulating the movement of particles based on the initial conditions and the forces acting on them by their equation of motions mentioned in the previous section.

A particle is considered "lost" if its phase (position in the oscillatory cycle) in relation to the upper rf cavity, lower rf cavity, and the average of the two rf cavities becomes unbounded. In other words if the particle's behavior becomes longitudinally unstable, it's considered lost. The white region in the stability map in Fig 2 and Fig 3 shows the "Bucket Area" that corresponds to the stable particles. Darker region correspond to instability or that particle is considered "lost".

For conventional slip-stacking, We can vary the size of the Bucket by varying the Voltage . However, For harmonic we have to account for the new added term λ. Fig 1 shows how the bucket area, on the Y-axis, varies as λ varies between -0.5 to 0.2, on the X-axis, running on 4 different Voltages (Blue runs on 100KV, Orange runs on 150KV, green runs on 200KV, Red runs on 250KV).

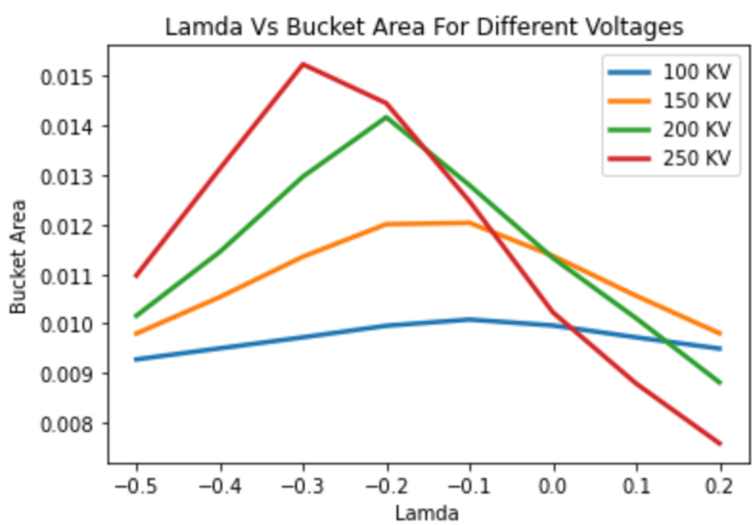

Figure 1: Bucket Area using Harmonic Cavities for different Voltages by varying λ from -0.5 to 0.2.

The Purpose of Fig 1 is to compare conventional slip-Stacking with Harmonic Slip-Stacking. The peak on the line When a 250KV is being used for Harmonic Slip-Stacking (indicated by the red line) shows the largest Bucket Area out of all the different Voltages. On the other hand, if λ is zero the new set of motion equations are simply just conventional slip-Stacking. Tracing the red line, when Voltage is 250KV, when λ (conventional Slip-Stacking) is zero, the Bucket Area is 0.010. By comparison to Harmonic Slip-stacking, the Bucket Area is 0.015. Our results have shown a 50% improvement of the bucket the Area using Harmonic Slip-Stacking .

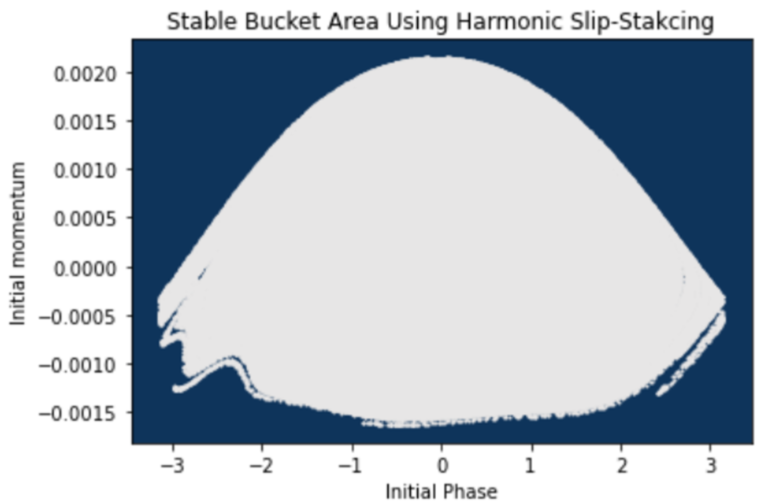

Figure 2: Stability Map of initial particles using the Harmonic Cavities (Where $\lambda \approx -0.3$, $V \approx 0.015$)

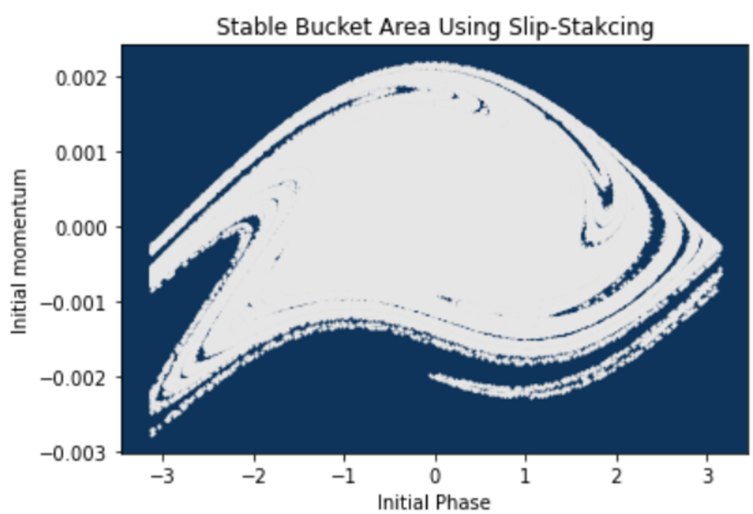

Figure 3: Stability Map of initial particles using conventional slip-Stacking (Where $\lambda = 0$, $V \approx 0.010$)

Fig 2 Shows the number of Synchrotron periods a test particle survives before it is lost. The white color indicates the mapping of the Bucket Area, Or stable Area, while the blue area indicates unstable Area. Fig 2 is the Optimal λ value while using 250KV (see Fig 1, Peak of red line). However, Fig 3 is the bucket area when $\lambda = 0$ (Conventional Slip-Stacking) while also using 250KV. Both Figures show that Harmonic Bucket Area (Fig 2) is more stable than the Conventional Slip-Stacking (Fig 3) running on 250KV.

2.3 Optimal Voltage and λ For A Bigger Bucket Area

Fig 4 displays phase space area (right bar) as a function of λ and V. Each data point is calculated from its own stability map. This figure illustrates how the phase space area varies based on different parameters. It's observed from Fig 1 and Fig 4 that, for any given value of V (Voltage), there exists an optimal value of λ (harmonic-main voltage ratio) that maximizes the phase-space area. This optimal condition is referred to as the "balanced" condition for λ[2].

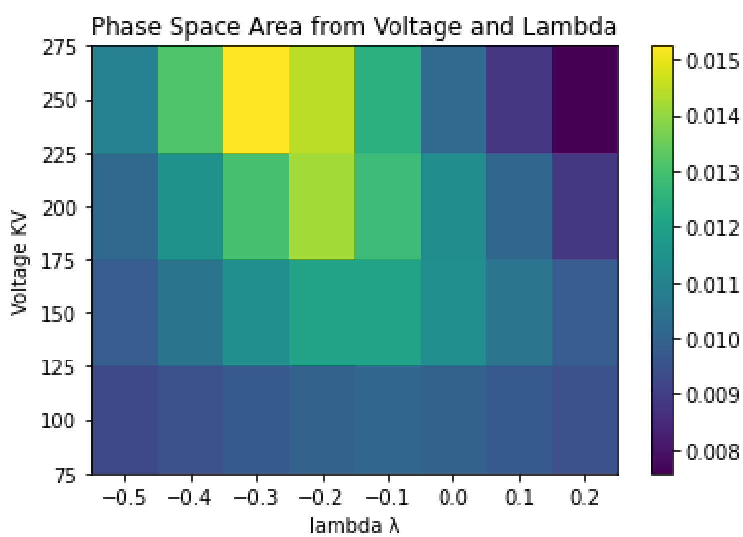

Figure 4: Harmonic Slip-Stacking Phase Space Area as a Function of Voltage and . The left bar indicates an increase in Phase Space Area. The Graph shows that the best combination to obtain an Optimal phase space Area

Fig 4 suggests that when , at least 90% of the stable phase-space area can be achieved using the balanced condition for . This indicates that the balanced condition is effective in preserving stability. Fig 4 shows that the best combination to obtain an Optimal phase space Area is by having a high voltage with negative λ. The closer to yellow on the graph indicates the highest Phase space Area; Having around -0.3 with High Voltage, and the darker the color the worst the case is; Having high λ with High Voltage.

2.4 Harmonic Slip-Stacking Including Errors

In this section we introduce a modified set of equation that correspond to Harmonic Slip-Stacking. By adding the new term "Error" on Eq.2 the following set of equation becomes:

$$\phi' = 2\pi f_{\text{rev}} h\eta\delta, \tag{3}$$

$$\delta' = f_{\text{rev}} \tfrac{eV_M}{\beta^2 E}[\sin(\phi) + \sin(\phi - \omega_\phi t) + \lambda \sin(2\phi - \omega_\phi t) + Error]$$

Where Error is the degree shift in the opposite direction of the phase-space motion and is calculated by:

$$Error = Z^\circ, Z = \text{is the number of degree shift} \tag{4}$$

We can use the new equation to anticipate and account for any error that could occur in the process. Fig 5 display values of the Bucket Area as λ varies between -0.5 and 0.1 running on 250KV.

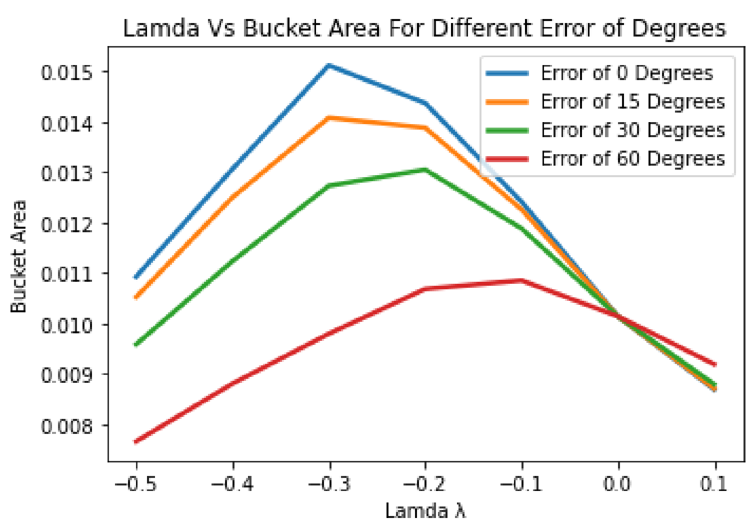

Figure 5: Bucket Area using Harmonic Cavities for different Voltages by varying λ from -0.5 to 0.1. including different Error Degrees

Choosing z in Eq.4 to be [0,15,30,60], where each number indicate the degree of Error, Each line in Fig 5 correspond to the chosen value z (Error Degree). The blue line indicates the optimal situation where $Error = 0$. The purpose of Fig 5 is to show that even if errors where to be accounted for, harmonic slip-Stacking still shows an a larger Bucket area than conventional Slip-Stacking. Each peak in Fig 5 indicates the Optimal λ that runs on 250KV. Each peak accounting for each error degree shows a larger bucket Area compared to conventional slip-Stacking ($\lambda = 0$) on each line. At 15 degrees, most of the benefit is retained. But that at 60 degrees, the harmonic cavity helps only marginally (but doesn't make anything worse) Moreover, beam-loading effects are expected to account for about 15 degrees of phase-error in the Recycler, which means that harmonic cavity is applicable to slip-stacking in a real machine.

3 Conclusion

our results have demonstrated improved stability Area compared to conventional Slip-Stacking. We began by showing stability maps of traced particle. We varied V (Voltage) and the new Harmonic term (λ). The Bucket Area has shown 50% improvement using optimal λ value compared to conventional Slip-Stacking (when $\lambda = 0$). We have demonstrated that when and given value of V (Voltage), there exists an optimal value of λ, and the best combination to achieve the largest bucket area value is by choosing a high V value and a lower λ value. In the end, we accounted errors in our studies and introduced a modified version of the Harmonic Equation of motion by adding the new "Error" term. Our results still have shown an larger Bucket Area value compared to conventional Slip-Stacking on various Degrees of Error. Our studies indicate that the harmonic RF cavity provides a clear benefit for slip-stacking in the Recycler, and the appropriate funding and timeline for installing that RF cavity is under active consideration

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