Modeling a Thermionic Electron Source Using a Physics-Informed Neural Network

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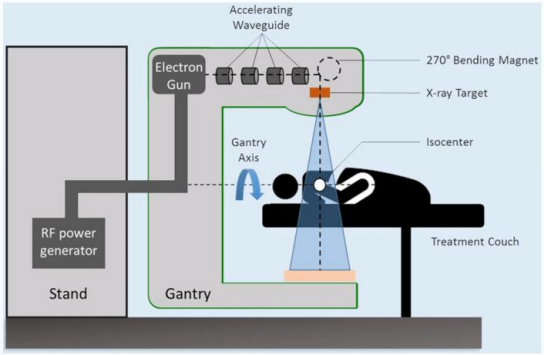
Accelerator Physics: Brief Overview

Applications

- Research
- Medicine
- Industry

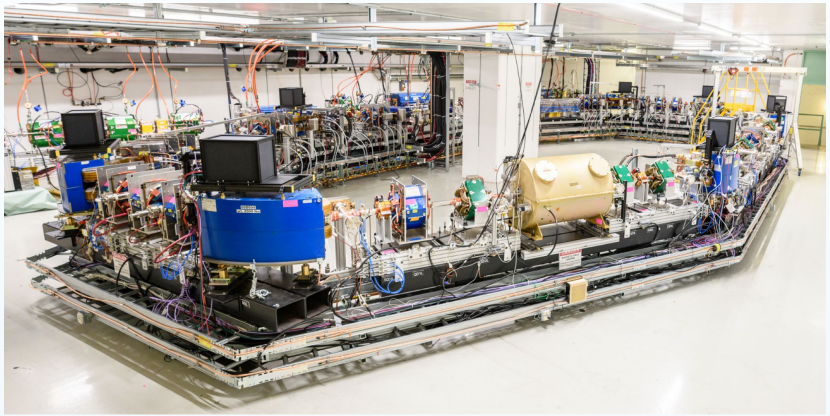
} 97% Accelerators



Challenges

1. *Beam Intensity*
2. *Beam Quality*
3. *Beam Control*
4. *Beam Prediction*

IOTA + The Electron Lens



The Integrable Optics Test Accelerator (IOTA)

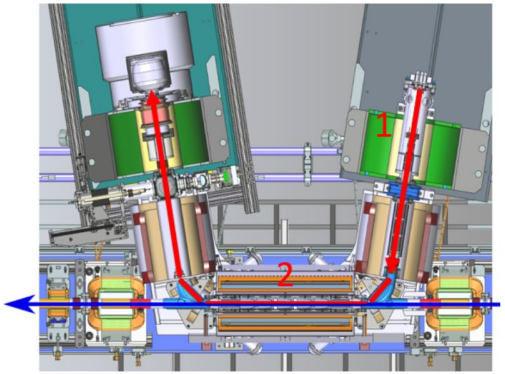
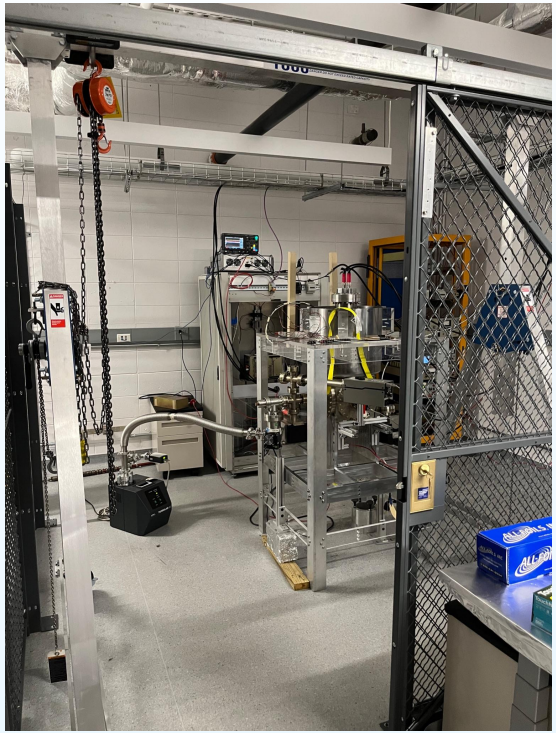


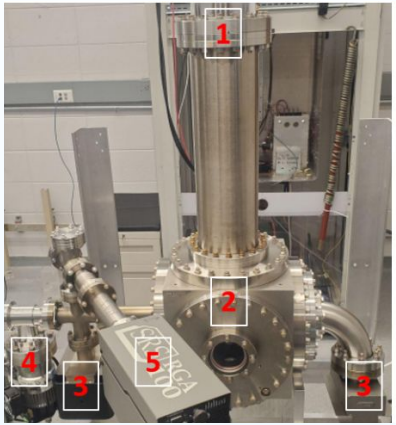
FIG. 2. Electron lens scheme. Electrons, produced in a thermionic source (1), travel with the circulating proton beam (2).

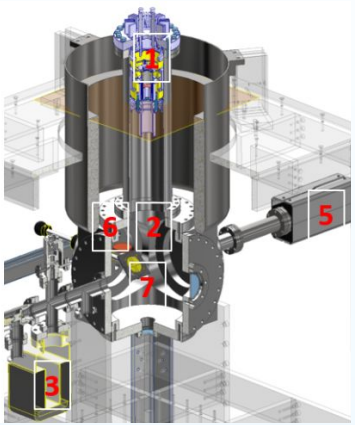
Stancari, G., *et al.* Beam physics research with the IOTA electron lens. *Journal of Instrumentation*, *16*(05), P05002.

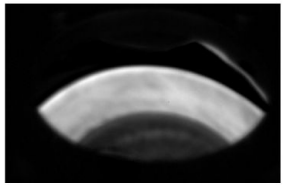
Test Stand being researched at UChicago

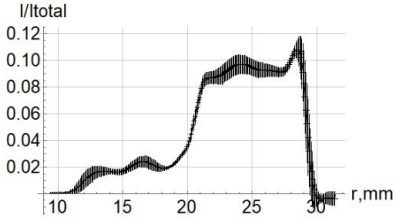
UChicago Setup





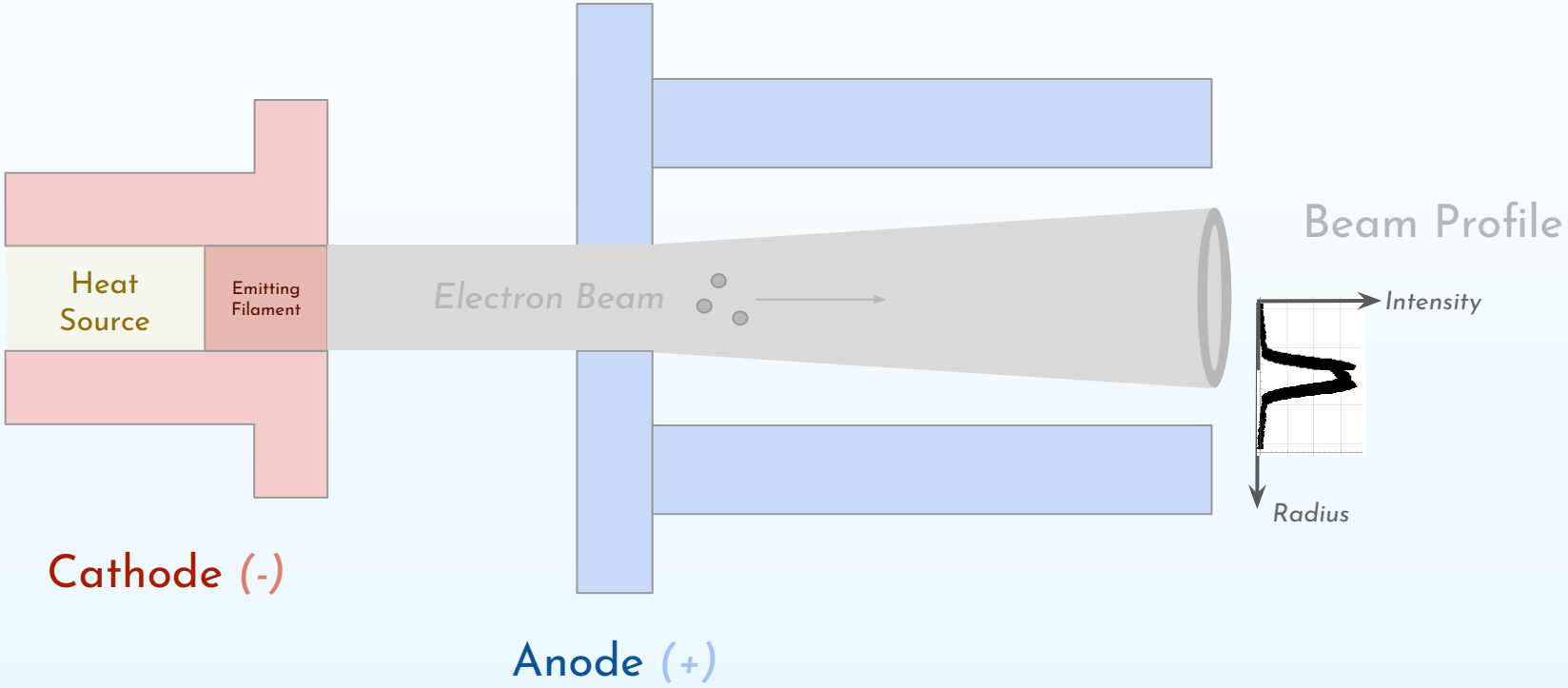




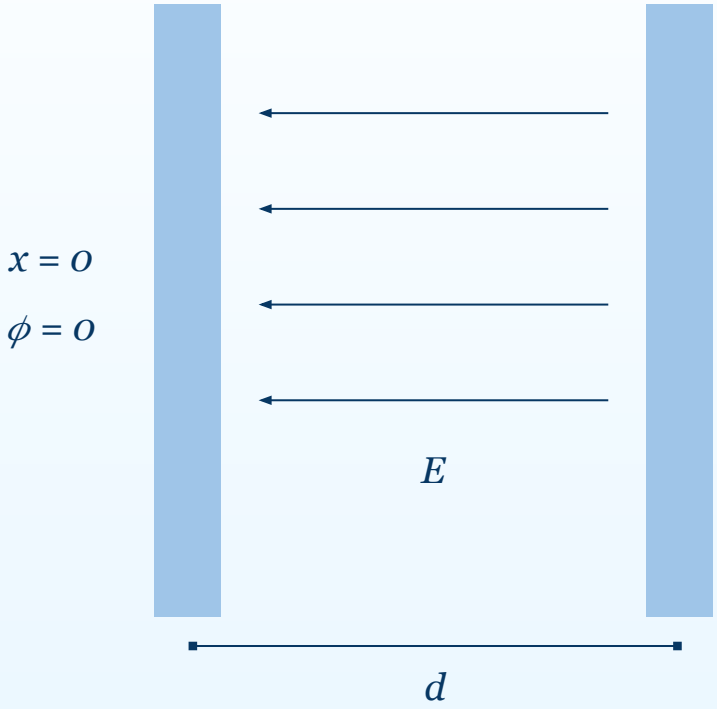


Kladov, S., *et al.* (2023, July). Commissioning of the Low Energy Electron Gun Test Stand at the University of Chicago. *14th International Particle Accelerator Conference*.

Thermionic Electron Source

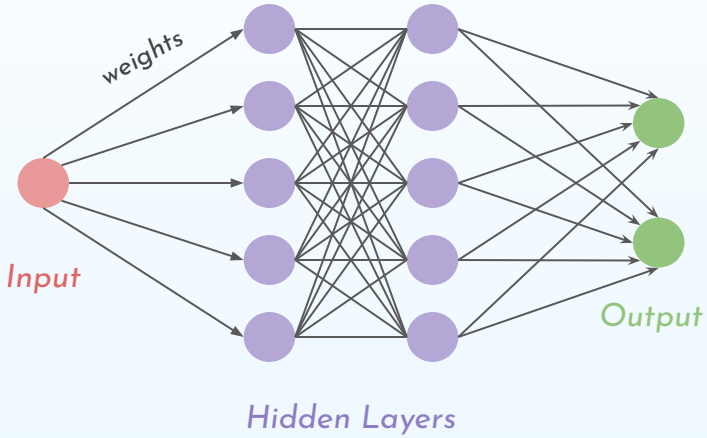


Planar Diode Model



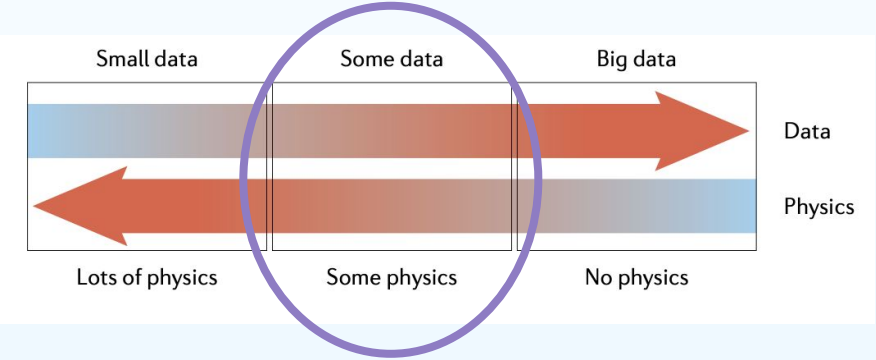
$\Rightarrow$ Useful to Simulate

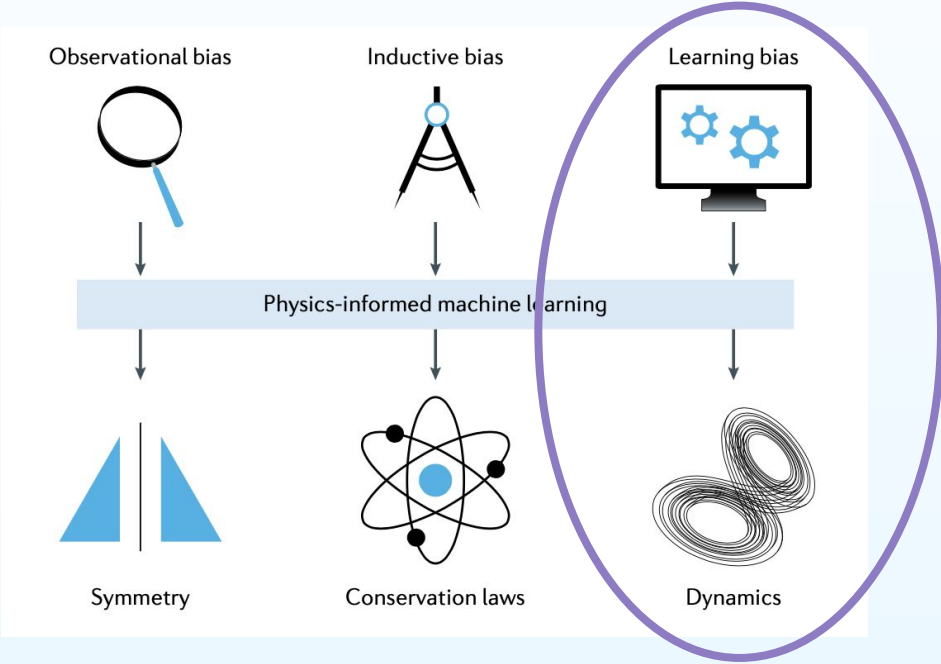
Neural Networks



- *Goal: approximating some unknown function*
- *Loss function: Defining what the function needs to minimize*
- *Training: Adjusting weights to lower loss*

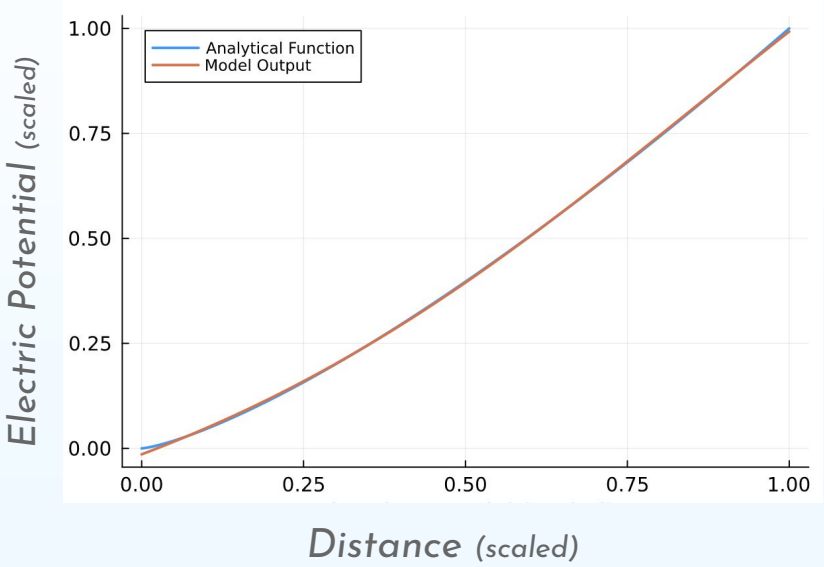
Physics-Informed Neural Networks





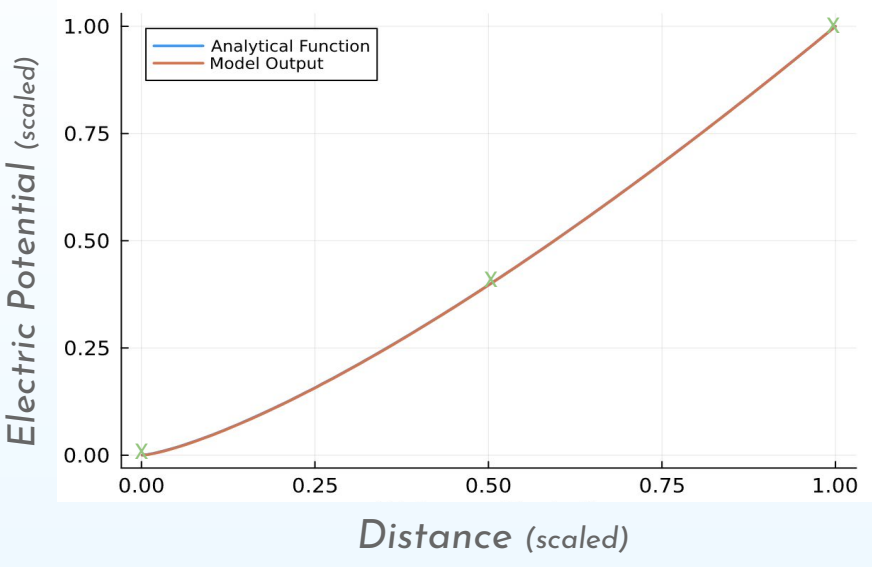
Karniadakis, G. E., Kevrekidis, I. G., Lu, L., Perdikaris, P., Wang, S., & Yang, L. (2021). Physics-informed machine learning. *Nature Reviews Physics*, 3(6), Article 6. https://doi.org/10.1038/s42254-021-00314-5

Results: The PINN



Traditional Network

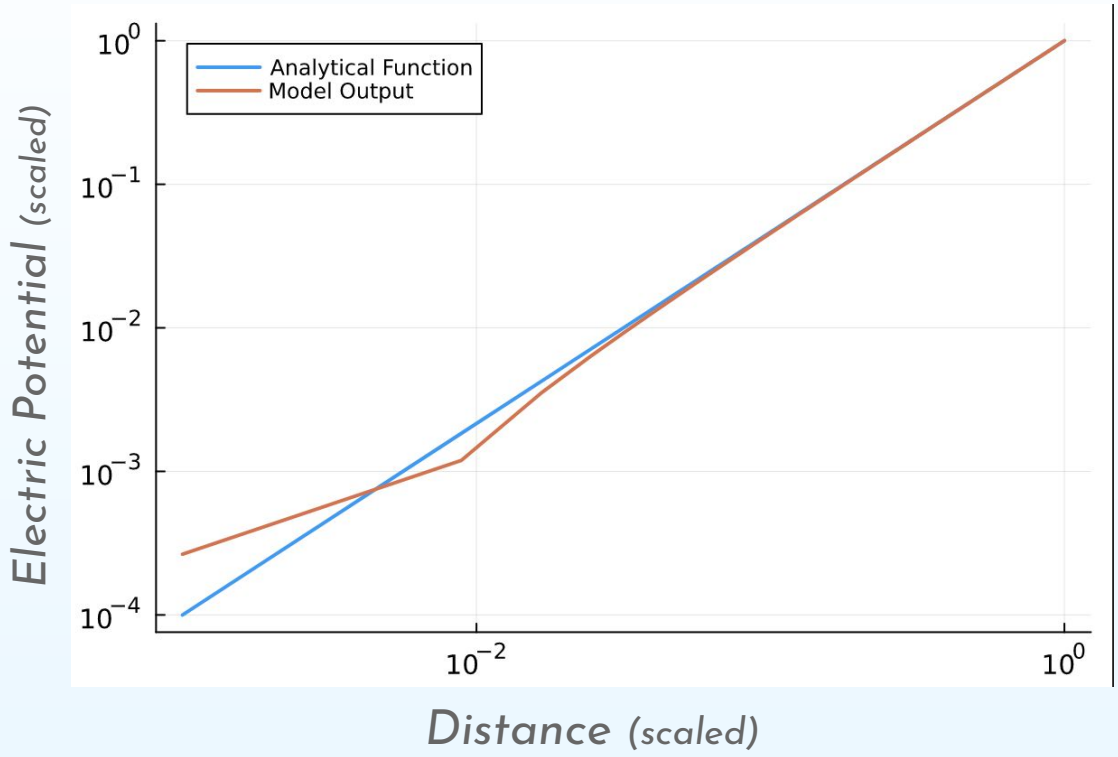
512 Training Points



PINN Model

3 Training Points

Results: The PINN



Conclusion + Next Steps

Takeaways

- PINN model proof of concept demonstrated; will greatly speed up simulation time
- Model behaviour around x = 0 a notable issue

Future Focus

- Generating data based on the thermionic source beam
- Developing a PINN for the thermionic source
  - Parameterize based on current density
- Possible generalization to other accelerator components

Acknowledgements

*Major thanks **Nilanjan Banerjee** and **Christopher Pierce** for their leadership and guidance of me through this project. I'd also like to thank Professor **Young-Kee Kim** and the whole accelerator group for hosting me this summer.*

Thank you for listening!

Appendix: Planar Diode

$$\nabla^2\phi = \frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon_0} \qquad \textit{(Poisson's Equation)}$$

$$J = \rho\frac{dx}{dt} \implies \rho = \frac{J}{\frac{dx}{dt}} \qquad \textit{(Current Density Definition)}$$

$$\frac{m}{2}\left(\frac{dx}{dt}\right)^2 = e\phi(x) \implies \frac{dx}{dt} = \sqrt{\frac{2e\phi(x)}{m}} \qquad \textit{(Motion Equation)}$$

↓

$$\frac{d^2\phi}{dx^2} = \frac{J}{\epsilon_0\sqrt{\frac{2e\phi}{m}}} \rightarrow J = \frac{4\epsilon_0\sqrt{2em}}{9}\frac{V_0^{\frac{3}{2}}}{d^2} = (2.33\times10^{-6})\frac{V_0^{\frac{3}{2}}}{d^2}$$

(For Electrons)

↓

$$\phi(x) = V_0\left(\frac{x}{d}\right)^{\frac{4}{3}}$$

↓ *Using* $E = -\nabla\phi$

$$E = -\frac{4V_0}{3d^{\frac{4}{3}}}x^{\frac{1}{3}}$$

Appendix: Loss Function

$$\phi = V_0\tilde{\phi}$$
$$x = \tilde{x}d$$
$$J = J_0\tilde{J}$$

(Initial Condition #1)

$$\phi(0) = 0$$
$$V_0\tilde{\phi}(0) = 0$$
$$\tilde{\phi}(0) = 0$$

(Initial Condition #2)

$$\left.\frac{d\phi}{dx}\right|_{x=0} = 0$$
$$\left.\frac{dV_0\tilde{\phi}}{dx}\right|_{x=0} = 0$$
$$\left.\frac{d\tilde{\phi}}{d\tilde{x}}\right|_{\tilde{x}=0} = 0$$

(ODE)

$$\frac{d^2V_0\tilde{\phi}}{dx^2} = \frac{J}{\epsilon_0\sqrt{\frac{2eV_0\tilde{\phi}}{m}}}$$
$$\frac{V_0}{d^2}\frac{d^2\tilde{\phi}}{d\tilde{x}^2} = \frac{J}{\epsilon_0\sqrt{\frac{2eV_0\tilde{\phi}}{m}}}$$
$$\frac{d^2\tilde{\phi}}{d\tilde{x}^2} = \frac{d^2}{\epsilon_0\sqrt{\frac{2eV_0^3\tilde{\phi}}{m}}}J$$

use $J_0 = \frac{\epsilon_0\sqrt{\frac{2eV_0^3}{m}}}{d^2}$

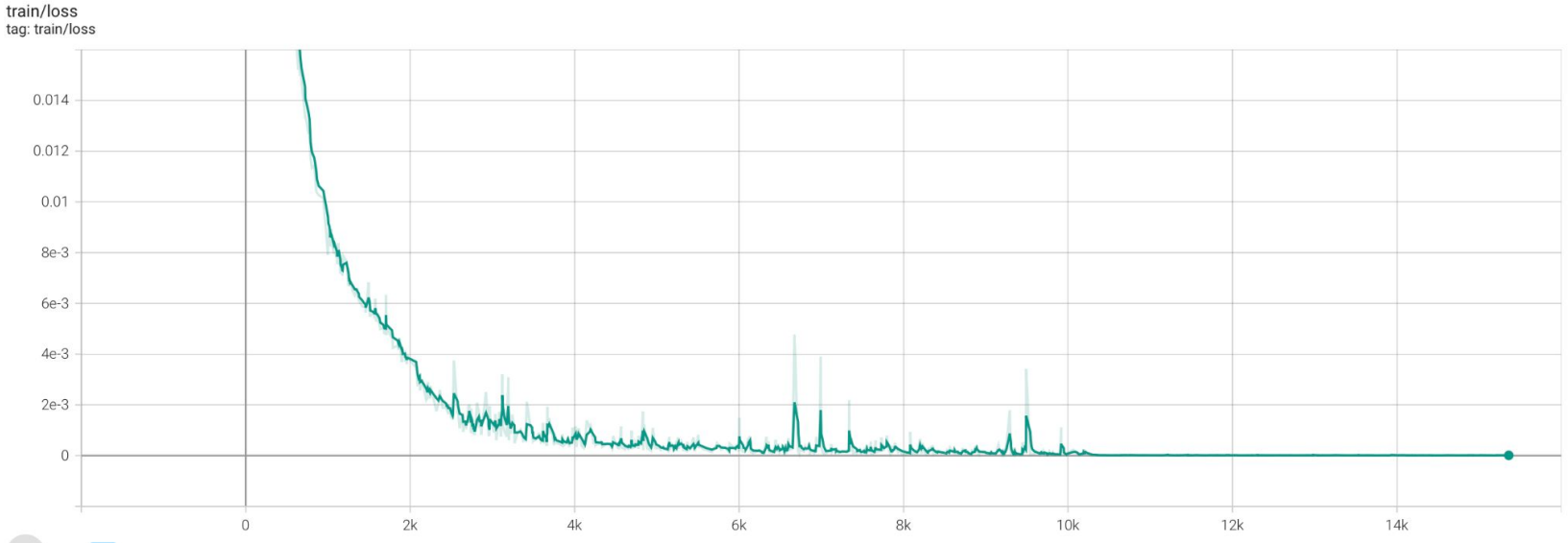
$$\frac{d^2\tilde{\phi}}{d\tilde{x}^2} = \frac{\tilde{J}}{\sqrt{\tilde{\phi}}}$$
$$\frac{d^2\tilde{\phi}}{d\tilde{x}^2} - \frac{\tilde{J}}{\sqrt{\tilde{\phi}}} = 0$$

$$l(\tilde{x}, d, V_0, \theta) = \underbrace{\left(\frac{d^2 f_{\tilde{\phi}}(\tilde{x}, d, V_0, \theta)}{d\tilde{x}^2} - \frac{f_{\tilde{J}}(\tilde{x}, d, V_0, \theta)}{\sqrt{f_{\tilde{\phi}}(\tilde{x}, d, V_0, \theta)}}\right)^2}_{\text{ODE Loss}} + \underbrace{k_1(f_{\tilde{\phi}}(0, d, V_0, \theta))^2}_{\text{IC \#1 Loss}} + \underbrace{k_2\left(\frac{df_{\tilde{\phi}}(\tilde{x}, d, V_0, \theta)}{d\tilde{x}}\right)^2}_{\text{IC \#2 Loss}}$$

k_1 , k_2 = constant coefficients (to weight the losses)

f = model output (J or ϕ) based on weights and biases (θ)

Appendix: Training Curves



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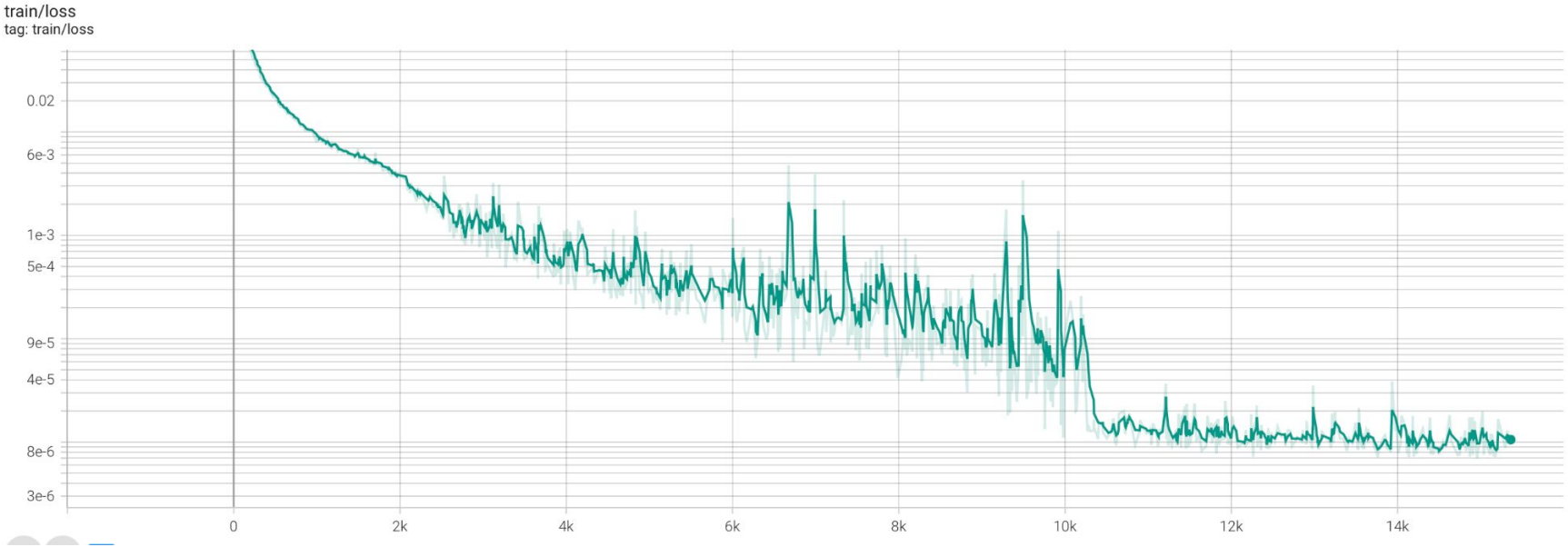
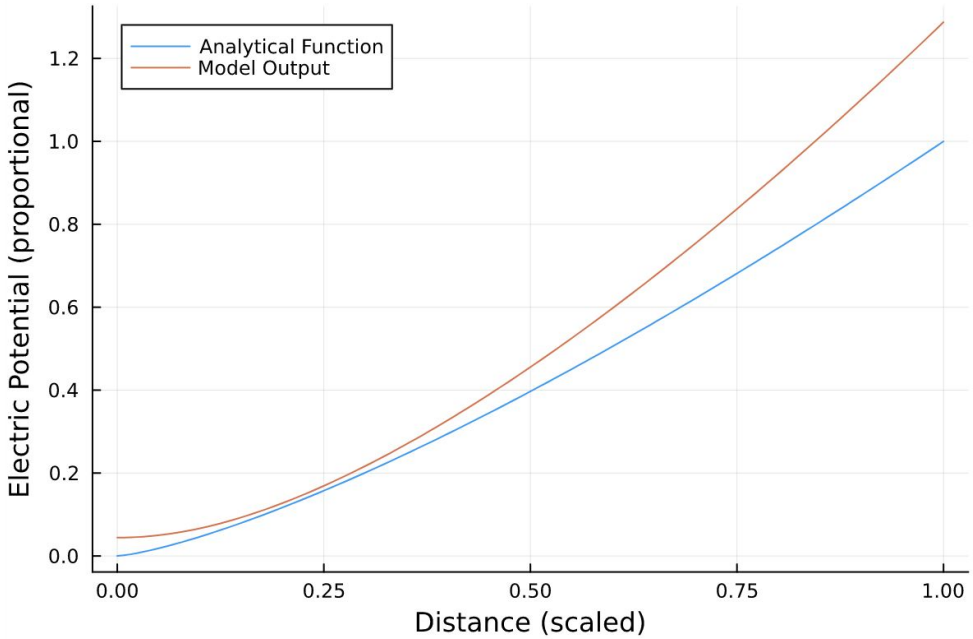
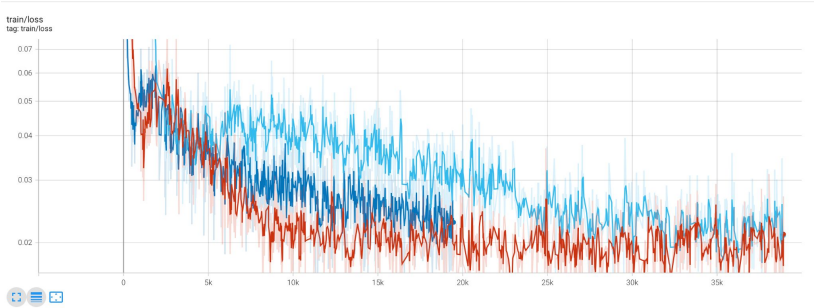
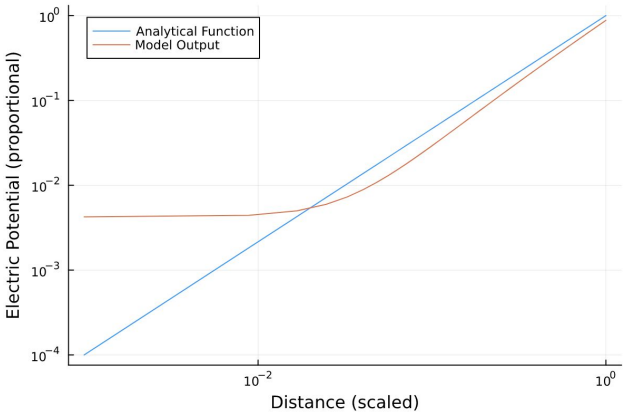


Appendix: Early Trials







Appendix: IOTA Layout

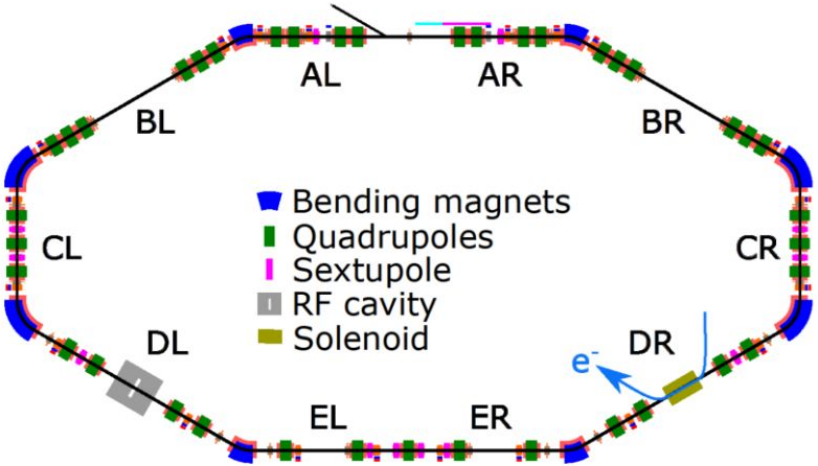


FIG. 1. IOTA ring layout. The lens will be installed in the DR line with zero dispersion.