KALMAN FILTER PERFORMANCE STUDY

ND-GAR MEETING 19TH SEPTEMBER 2023

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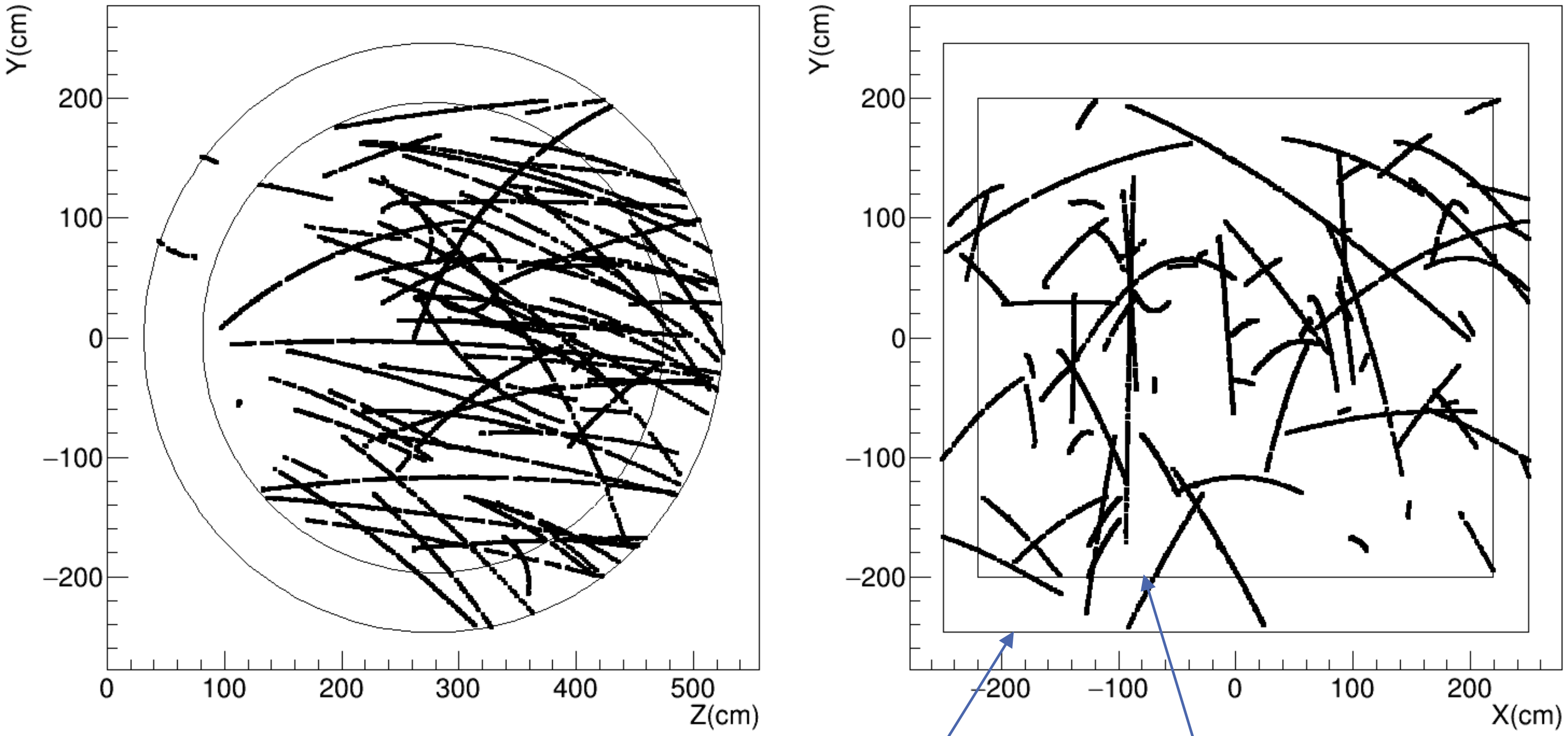
ALICE BASED KALMAN FILTER FOR ND-GAR: PERFORMANCE STUDY

- In today's presentation:
  - Study on sample of primary particles (μ^-, p, π^+) produced in $\nu_\mu CC$ interactions inside the TPC fiducial volume:
    - Discussion of bug fixes for new ALICE-based Kalman Filter
    - Comparison of momentum reconstruction performance of new ALICE-based Kalman Filter with current GArSoft reconstruction

- Previous presentations include:

1. Dune Collaboration meeting 26th January 2022: https://indico.fnal.gov/event/50215/contributions/232480/
2. ND-GAr weekly meeting 15th March 2022: https://indico.fnal.gov/event/53600/contributions/236685/
3. DUNE Collaboration meeting 18th May 2022: https://indico.fnal.gov/event/50217/contributions/241519/
4. ND-GAr weekly meeting 9th August 2022: https://indico.fnal.gov/event/55842/
5. ND-GAr weekly meeting 25th October 2022: https://indico.fnal.gov/event/56687/
6. ND-GAr weekly meeting 28th February 2023: https://indico.fnal.gov/event/58350/
7. Dune Collaboration meeting 25th May 2023: https://indico.fnal.gov/event/57487/contributions/267579/

SAMPLE 2: MUONS FROM $\nu_\mu CC$ IN TPC FIDUCIAL



Total TPC Volume
$R_{TPC} \leq 246.6cm;$
$|z_{TPC}| \leq 249.6cm;$

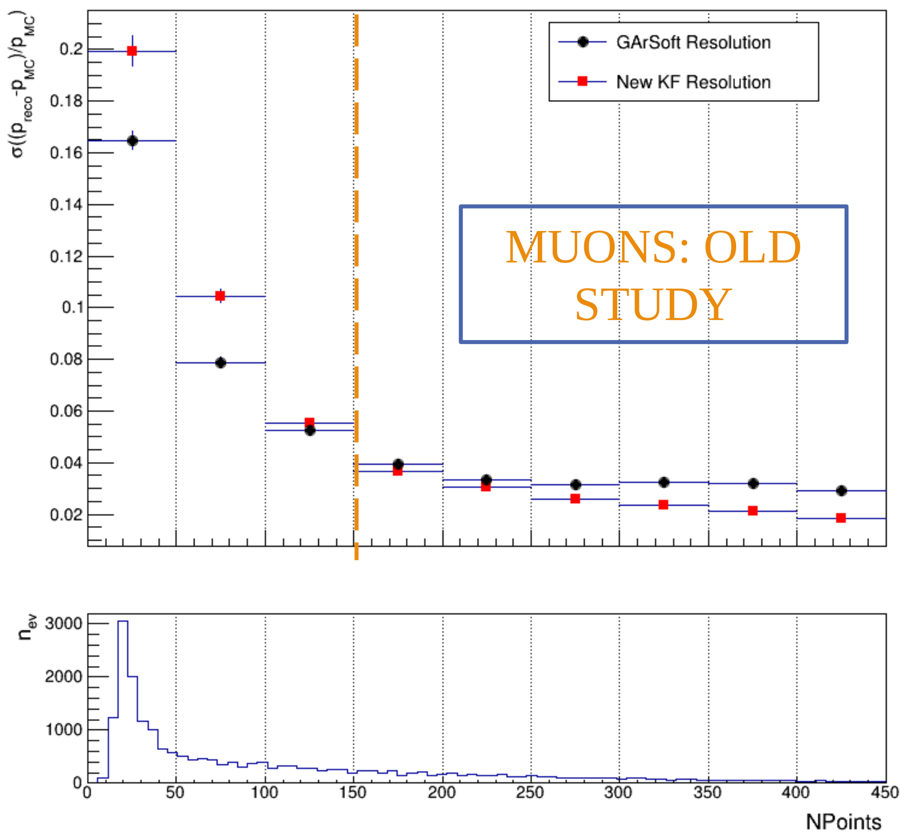
Fiducial Volume
$R_{fid} \leq (R_{TPC} - 50cm);$
$|z_{fid}| \leq (|z_{TPC}| - 30cm);$

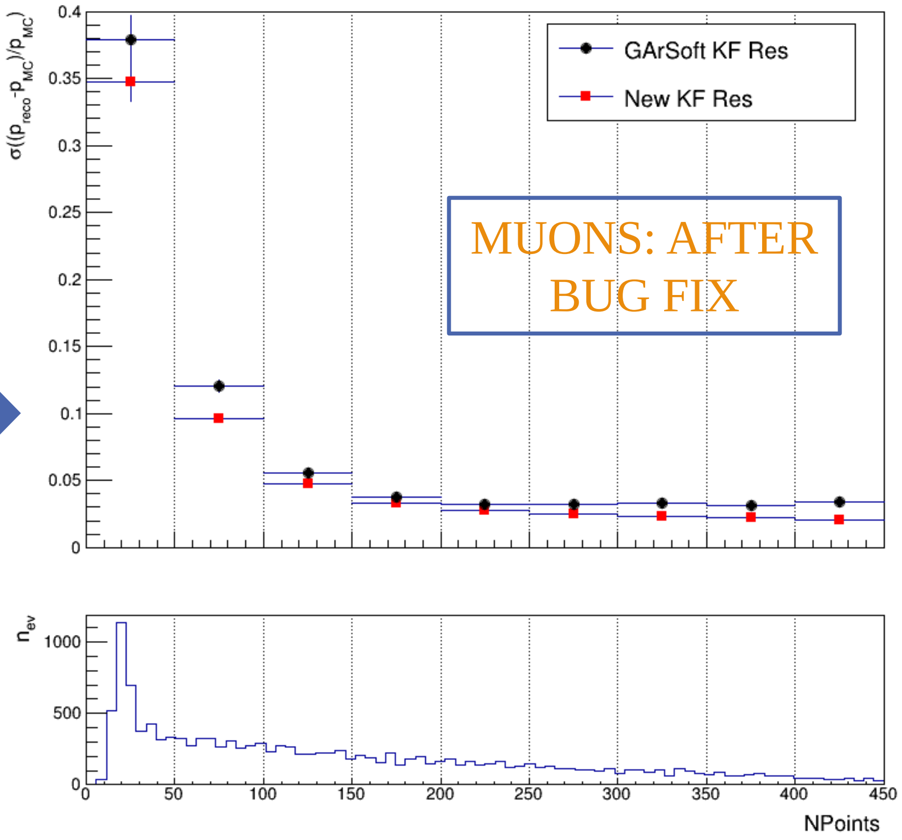
- SAMPLE : 4.35×10^4 neutrino interactions in active TPC volume produced using GENIE module in GArSoft v2_18_00 with standard flux
- Selected only $\nu_\mu CC$ interactions with reconstructed vertex in TPC fiducial volume as defined in ND-CDR :

$$R_{fid} \leq (R_{TPC} - 50cm);$$
$$|z_{fid}| \leq (|z_{TPC}| - 30cm);$$

- Considered primary particles from interactions: μ^-, p, π^+ (previous study only included muons)

NEW RESULTS AFTER BUG FIXES



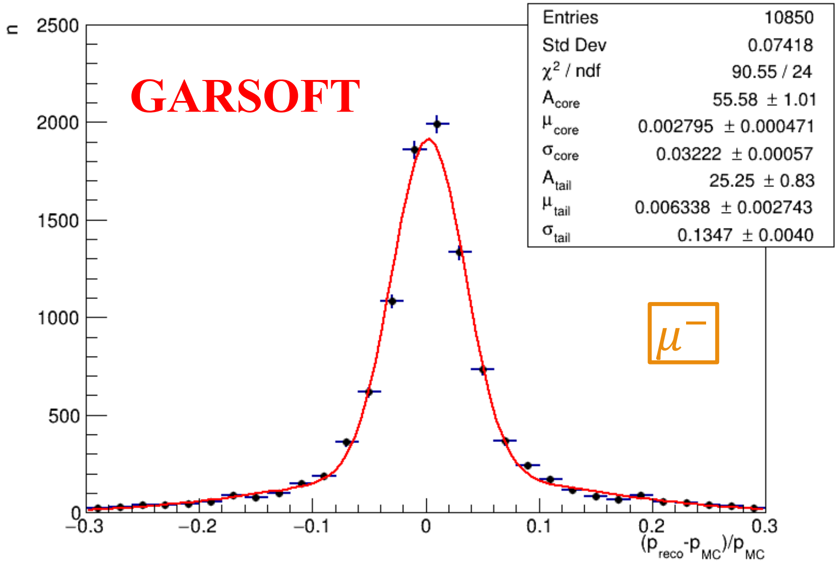


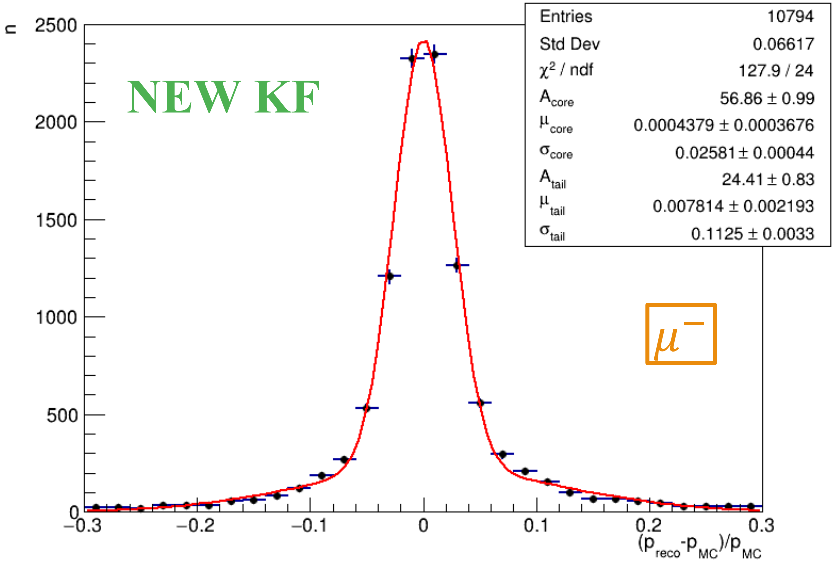
Profile plots for resolution : (σ) from momentum residual Gauss fit in each NPoints slice

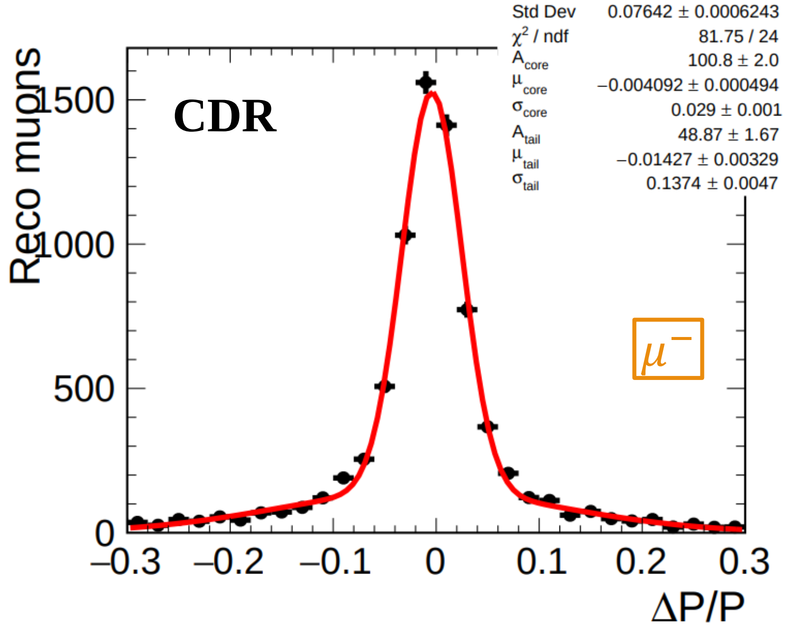
NPoints1D distribution (NB: in old study garsoft tracks associated with wrong number of points)

- Momentum resolution should go as $\propto 1/\sqrt{NPoints}$ (https://indico.fnal.gov/event/58350/)
- Old study seemed to indicate that the new KF out-performed the current garsoft one only for long tracks: new study shows new KF outperforms old one over the whole spectrum.
- Two major bugs found:
  - In track point ordering wrong cutoff parameter was used, reducing length of tracks (fSortDistCut = 10cm instead of 20 cm)
  - Cross Length between points was calculated incorrectly for energy loss corrections

MUONS MOMENTUM RESOLUTION : GARSOFT VS NEW KF VS CDR





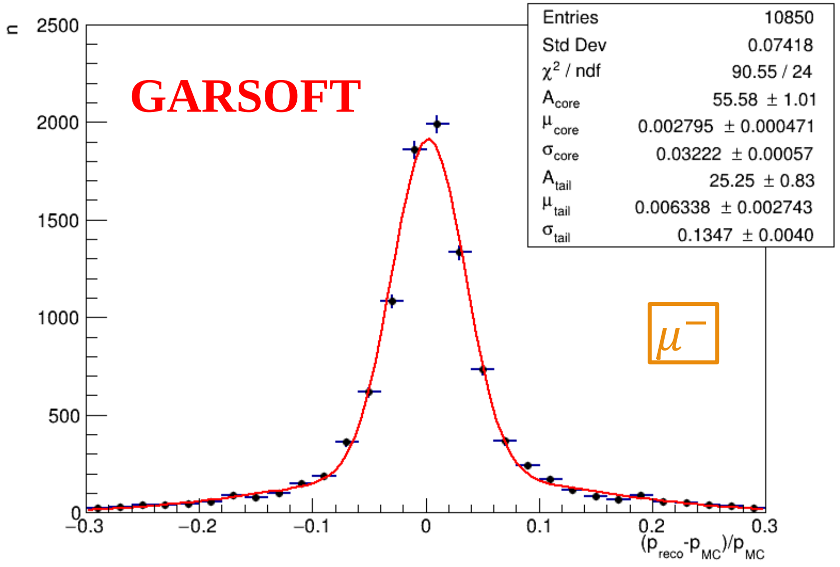


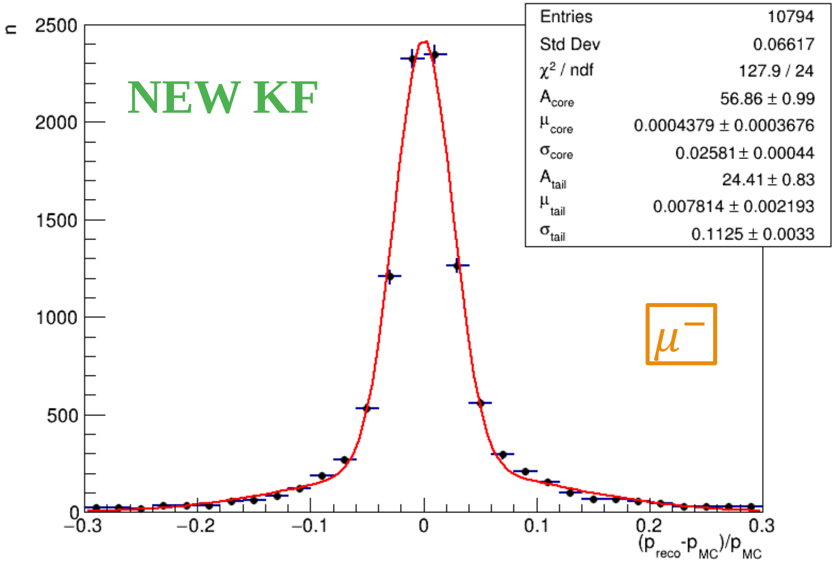
- Momentum fractional residuals for muon sample define reconstruction and resolution biases

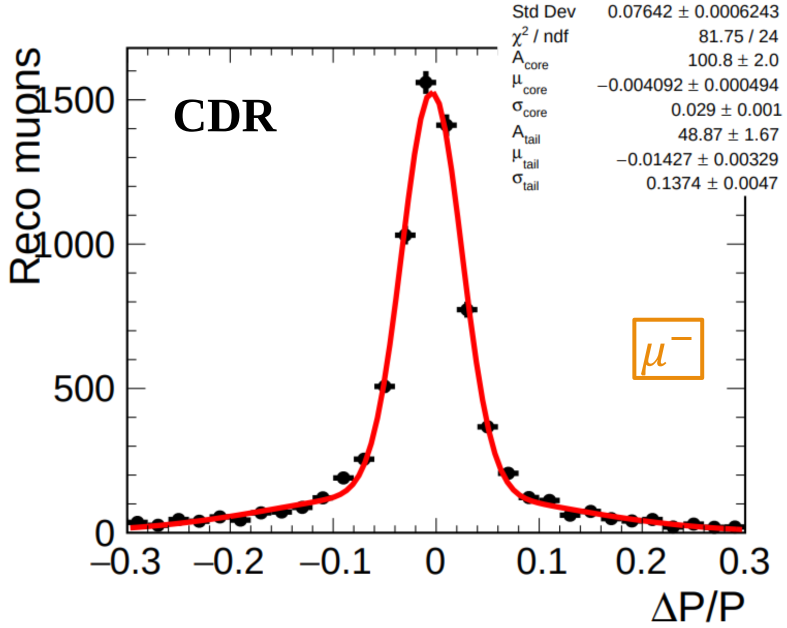
- $(p_{reco} - p_{MC})/p_{MC}$ distributions are fitted with a double Gauss fit, like in CDR, defining a core and tails sample:
  - GArSoft: $(A_{core}, \mu_{core}, \sigma_{core}) = (56, 0.3\%, 3.2\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (25, 0.6\%, 13\%)$
  - New KF: $(A_{core}, \mu_{core}, \sigma_{core}) = (57, 0.04\%, 2.6\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (24, 0.8\%, 11\%)$
  - CDR: $(A_{core}, \mu_{core}, \sigma_{core}) = (100, -0.4\%, 3\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (49, -1.4\%, 14\%)$

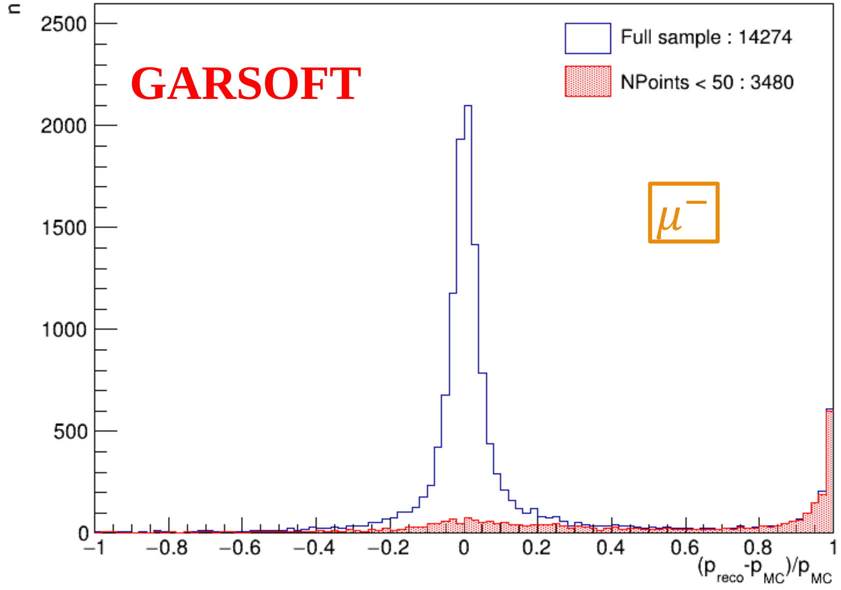
- New KF improves biases and resolutions overall for the muon sample

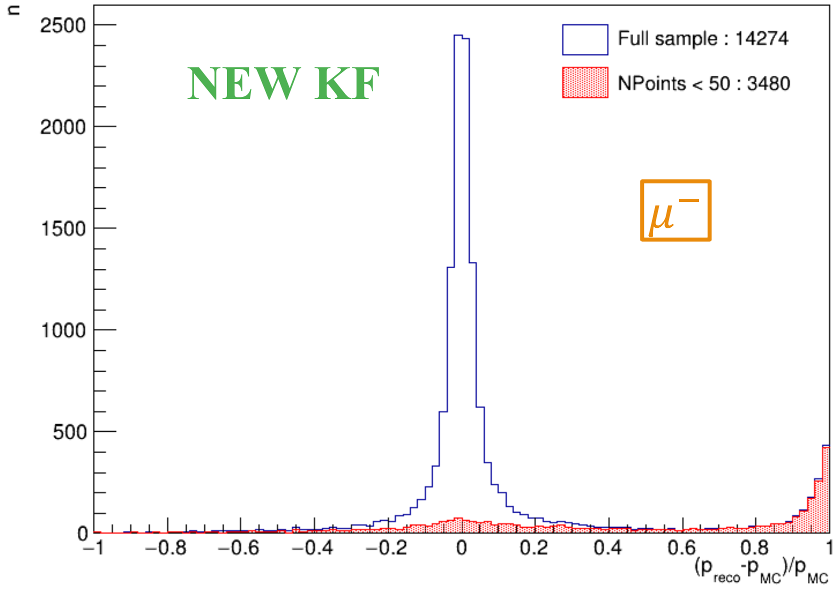
MUONS MOMENTUM RESOLUTION : GARSOFT VS NEW KF VS CDR





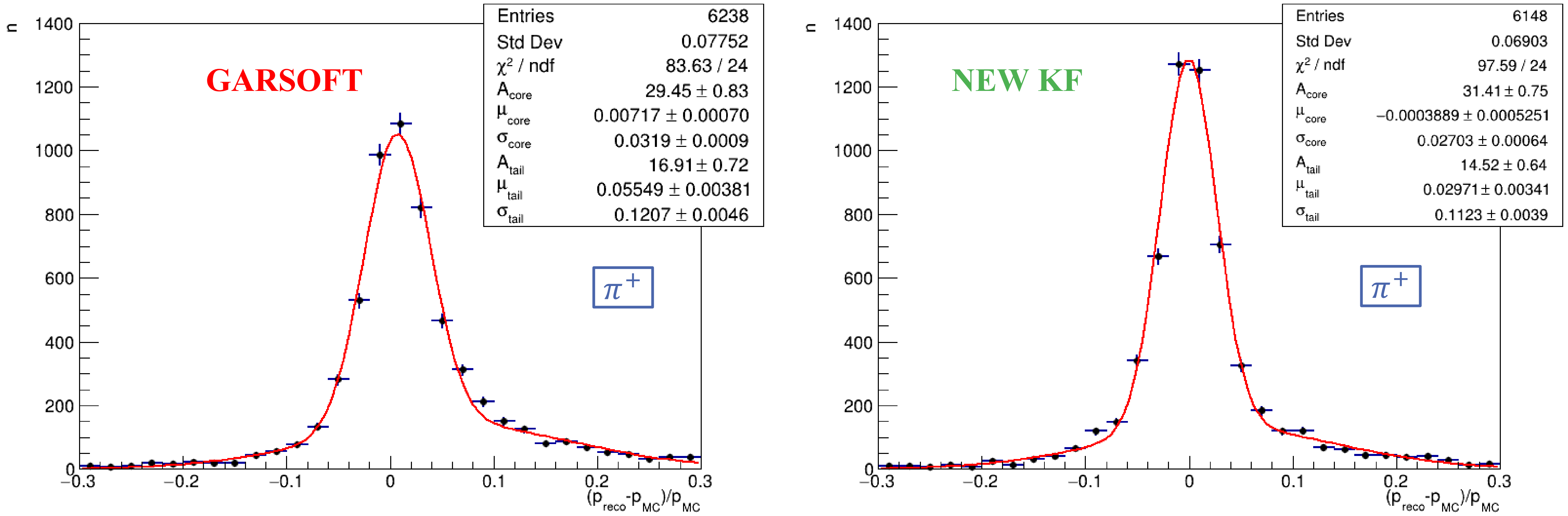






NOTE: tracks with $N_{points} < 50$ excluded from the sample because highly problematic; Results much more in agreement with CDR compared to previous study for which the portion of core sample was closer to $1/2$ than $2/3$

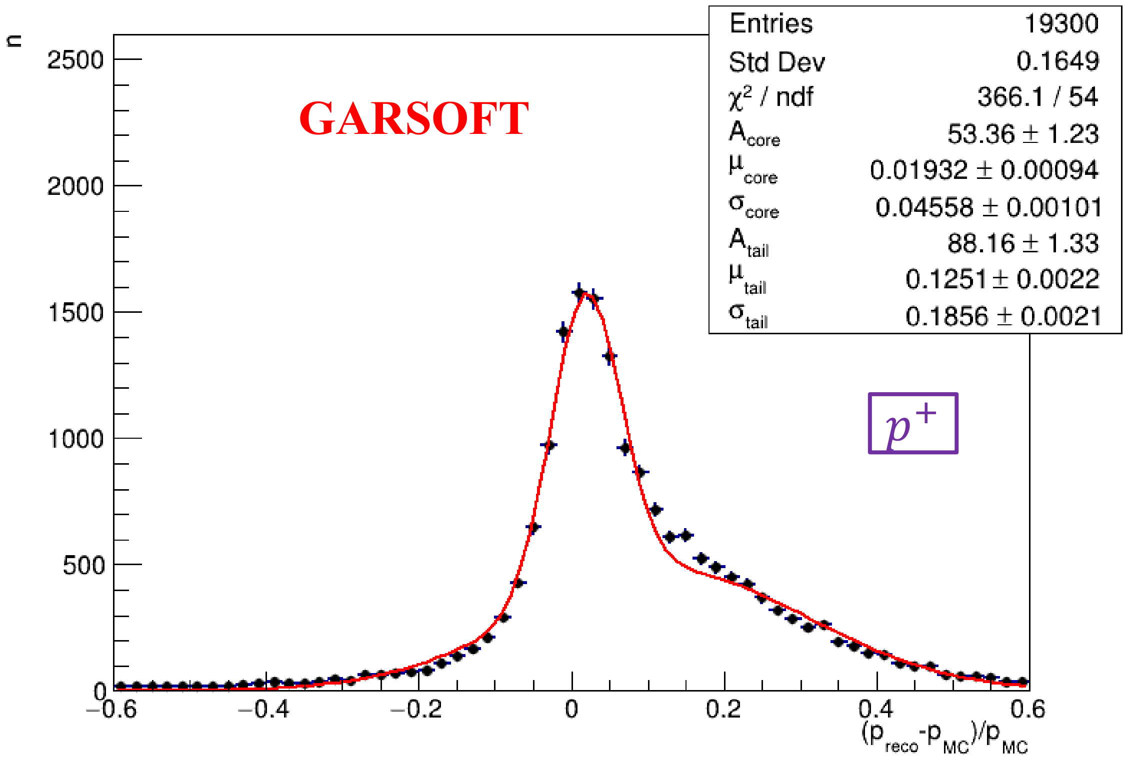
PIONS MOMENTUM RESOLUTION : GARSOFT VS NEW KF VS CDR

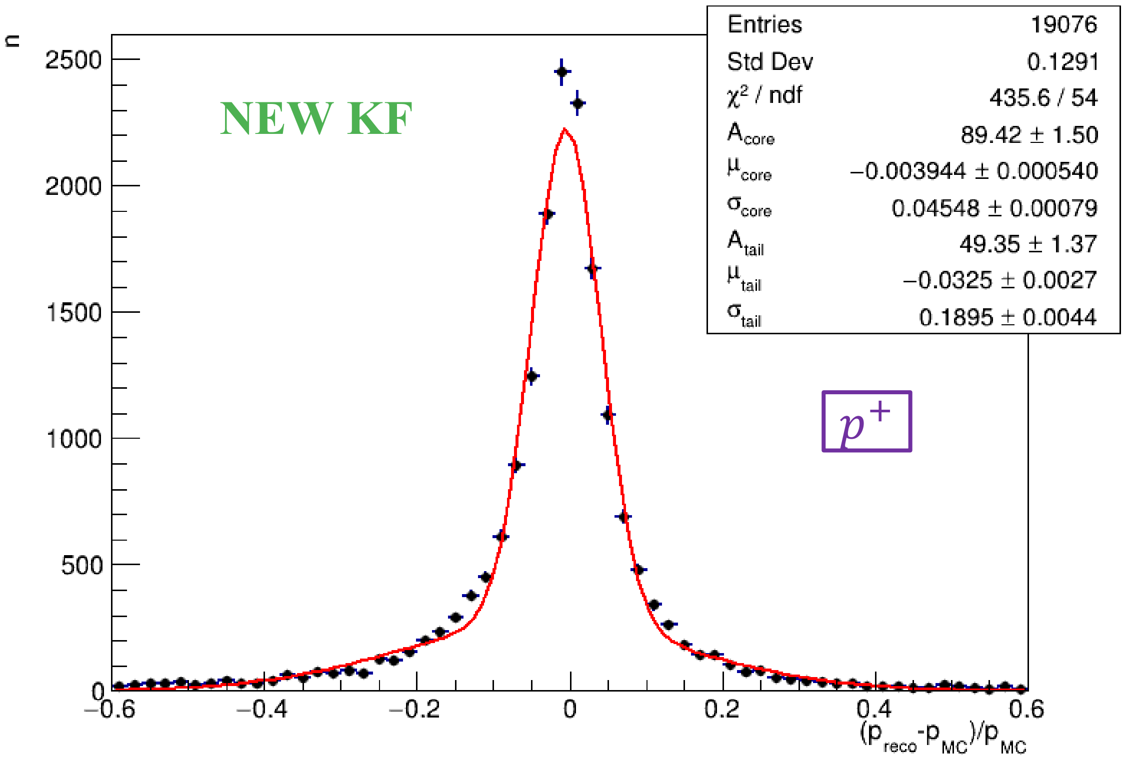


- $(p_{reco} - p_{MC})/p_{MC}$ distributions are fitted with a double Gauss fit, like in CDR, defining a core and tails sample:
  - GArSoft: $(A_{core}, \mu_{core}, \sigma_{core}) = (30\,, 0.7\%\,, 3.2\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (17\,, 6\%\,, 12\%)$
  - New KF: $(A_{core}, \mu_{core}, \sigma_{core}) = (31\,, -0.04\%\,, 2.7\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (15\,, 3\%\,, 11\%)$

- New KF improves biases and resolutions overall for the pion sample very similarly to muon sample (similar dEdx and mass)
- NOTE1: No CDR Results available: no direct comparison possible
- NOTE2: Tracks with less than 50 points are removed as for the muons

PROTONS MOMENTUM RESOLUTION : GARSOFT VS NEW KF





- $(p_{reco}-p_{MC})/p_{MC}$ distributions are fitted with a double Gauss fit, like in CDR, defining a core and tails sample:
  - GArSoft: $(A_{core}, \mu_{core}, \sigma_{core}) = (53\,, 2\%\,, 4.6\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (88\,, 12\%\,, 19\%)$
  - New KF: $(A_{core}, \mu_{core}, \sigma_{core}) = (89\,, -0.4\%\,, 4.5\%)$ $(A_{tails}, \mu_{tails}, \sigma_{tails}) = (49\,, -3\%\,, 19\%)$

- New KF majorly improves biases and resolutions for the proton sample (similar dEdx and mass)
- NOTE1: No CDR Results available: no direct comparison possible
- NOTE2: Tracks with less than 50 points are removed as for the muons

RESOLUTION DEPENDENCIES

- Analytical formulas derived directly from PDG chapter 34 on detectors give realistic expectations for the q/p_T resolution dependency: https://pdg.lbl.gov/2019/reviews/rpp2019-rev-particle-detectors-accel.pdf

POINT RESOLUTION

$$\sigma_N(q/p_T) = \frac{q\sigma_r}{0.3BL_{arm}}\sqrt{\frac{720}{N}}$$

MULTIPLE SCATTERING

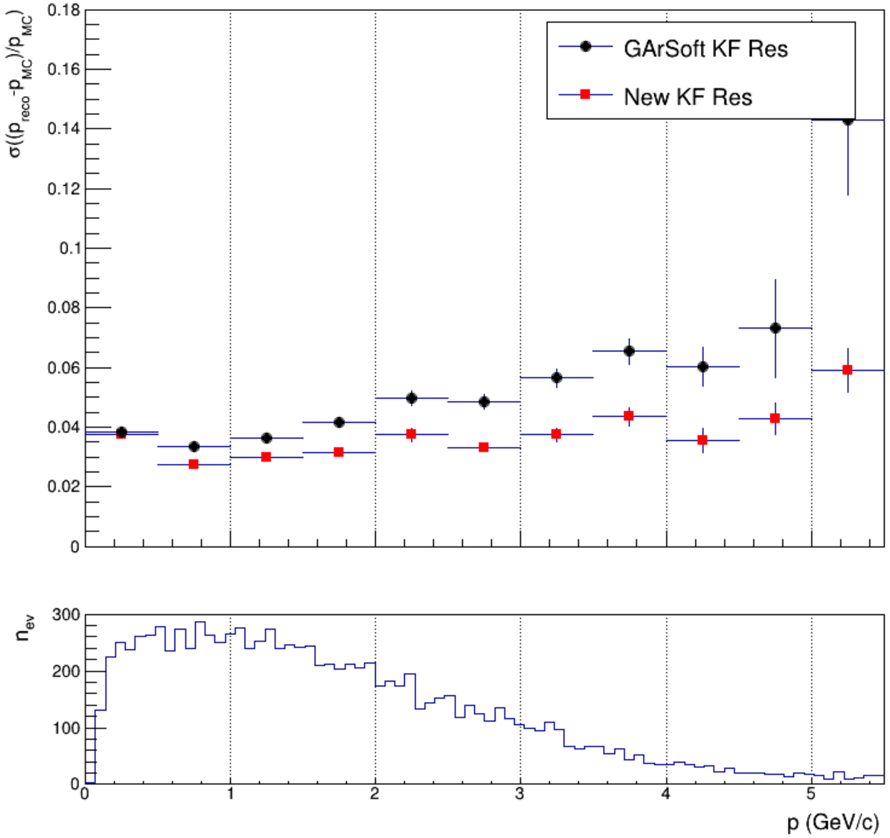
$$\sigma_{MS}(q/p_T) = \left\langle \frac{1}{\beta p_T} \right\rangle \frac{q \times 0.016 \times B \times 0.3}{L}\sqrt{\frac{L}{X_0}}$$

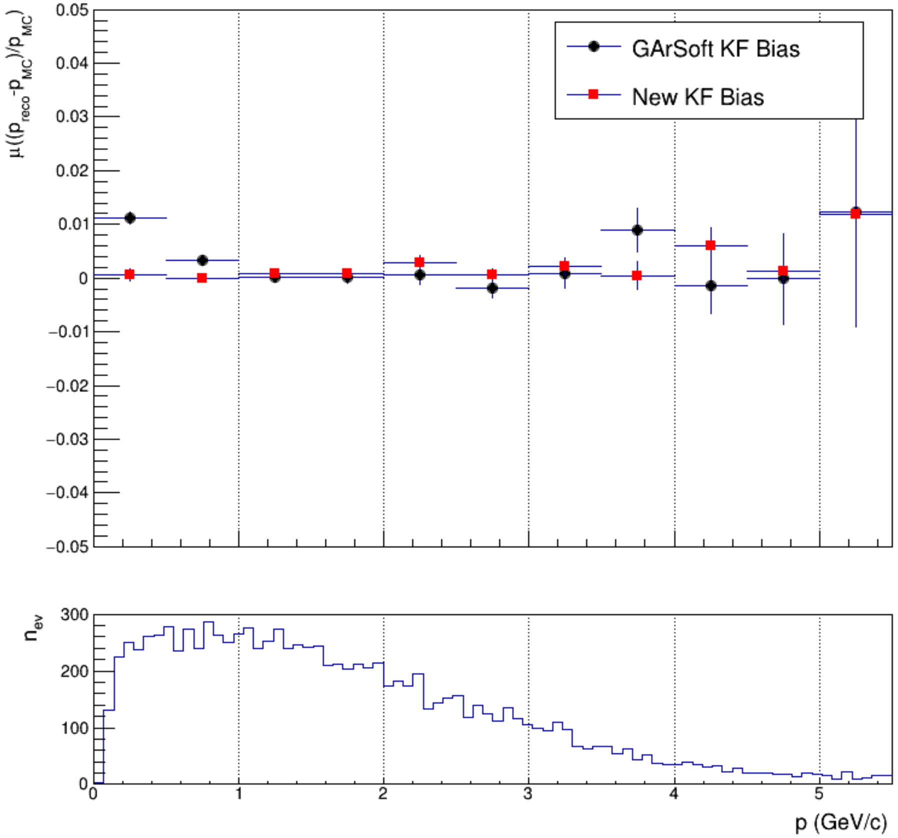
$$\sigma_{ana}(q/p_T) = \sqrt{\sigma_N^2 + \sigma_{MS}^2}$$

- σ_r = radial resolution
- B = magnetic field
- N = number of points measured
- L_{arm} = Lever arm on XY plane
- L = Length of the track on XY plane
- X_0 =Radiation length in cm
- β = velocity
- **NOTE: <$1/\beta p_T$>** = value of $1/(\beta p_T)$ averaged along the trajectory to take into account energy loss

NB: q/p_T scaling for high density materials, such as ND-GAr's gas mixture, should be dominates by the σ_{MS} component

MOMENTUM RESOLUTION AND BIAS VS P: MUONS μ^-



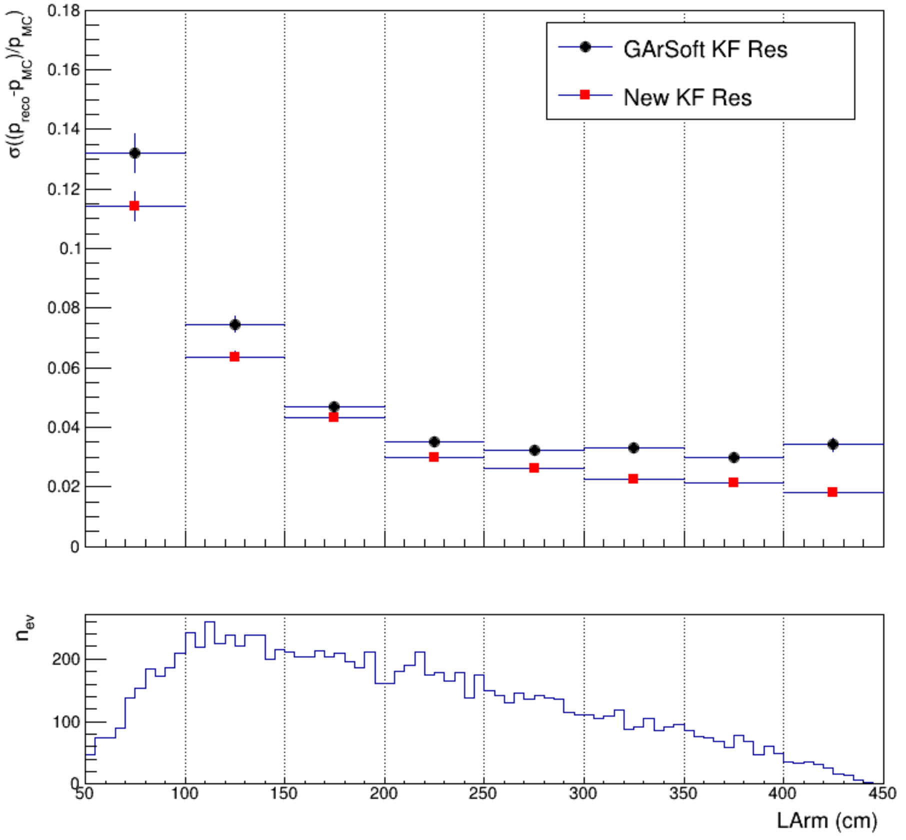


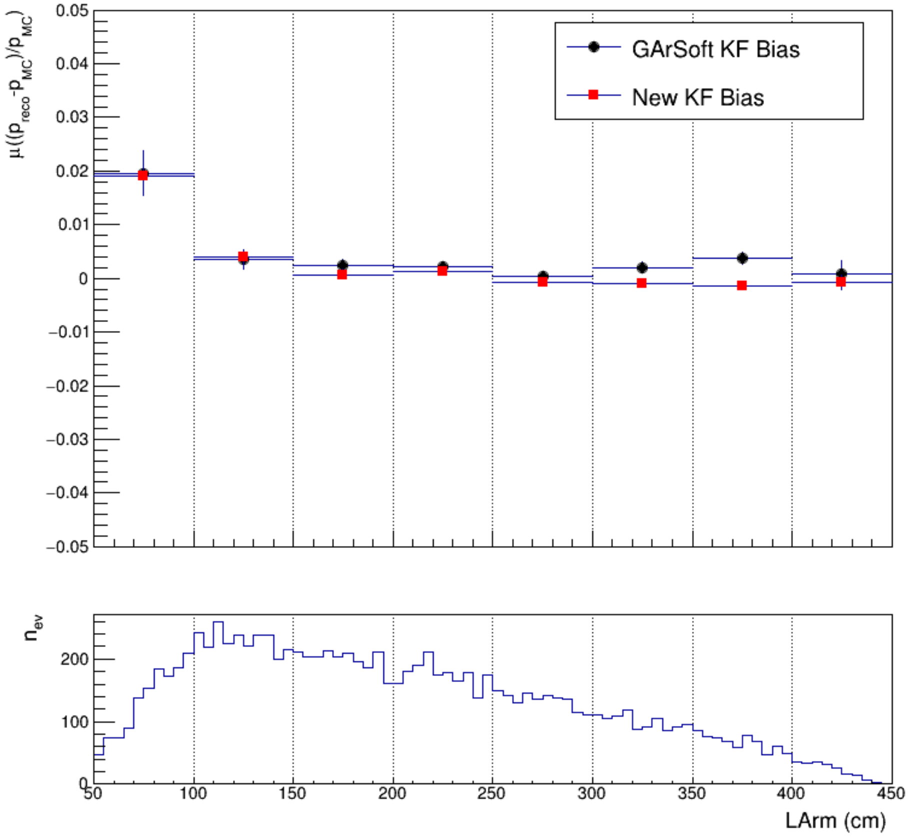
Profile plots for resolution and bias : (σ, μ) from momentum residual Gauss fit in each p slice

Correspondent p (GeV/c) distribution (NB: Tracks with $N_{points} < 50$ are cut)

- Momentum resolution should be mostly momentum independent in this range and at these densities. This is largely true for the new KF but not in garsoft
- Note that the pT should be averaged through the whole track, which wasn't done here.
- New KF improves resolution over the whole spectrum and keeps bias mostly the same

MOMENTUM RESOLUTION AND BIAS VS LARM: MUONS μ^-



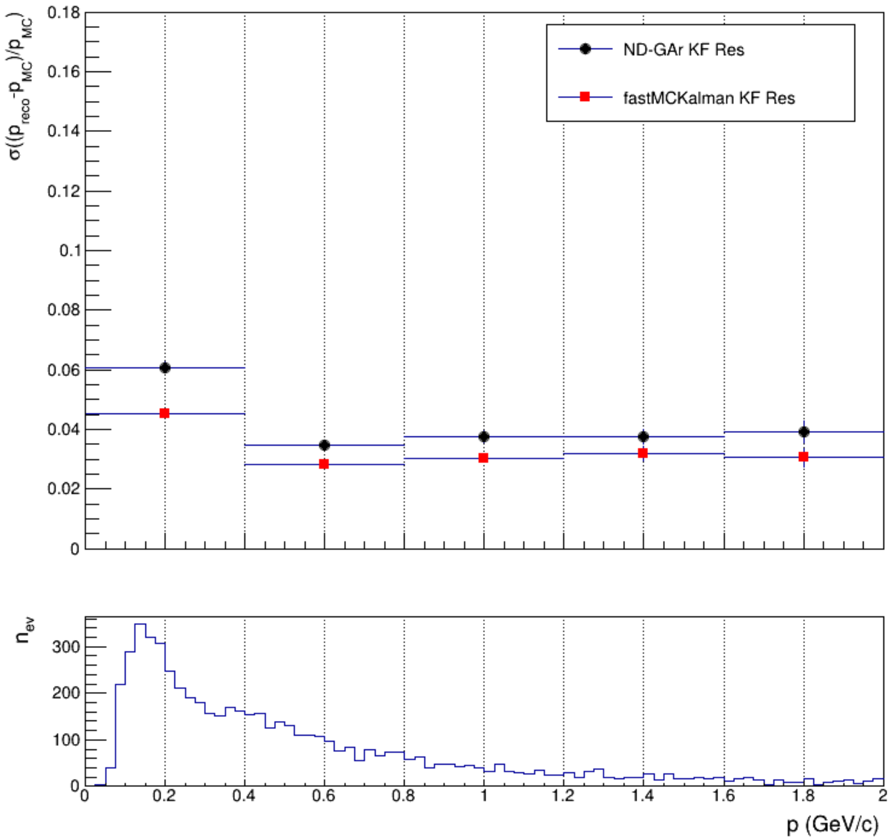


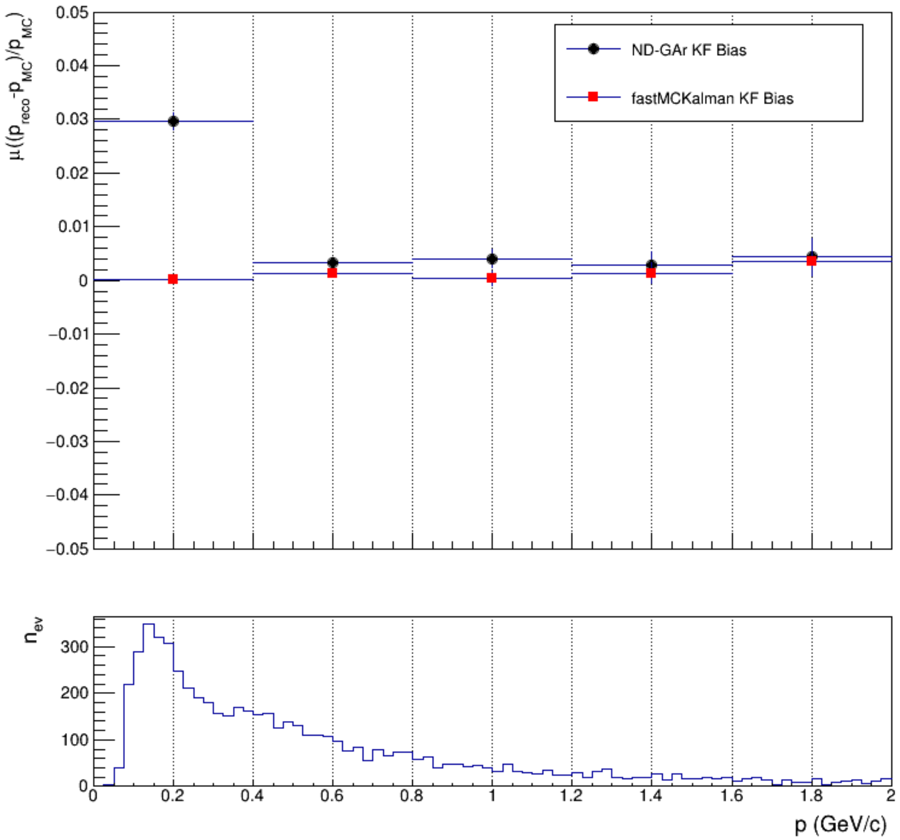
Profile plots for resolution and bias : (σ, μ) from momentum residual Gauss fit in each p slice

Correspondent p (GeV/c) distribution (NB: Tracks with $N_{points} < 50$ are cut)

- Lever Arm: distance in transverse (yz) plane between first and last point in the track
- Momentum resolution in the range $p \in [0,6]$ GeV/c should be multiple scattering dominated and go as $\propto 1/\sqrt{LArm}$ (dependencies on Npoints and Length are similar; see back-up)
- New KF improves resolution over the whole spectrum and keeps bias mostly the same

MOMENTUM RESOLUTION AND BIAS VS P: PIONS π^+





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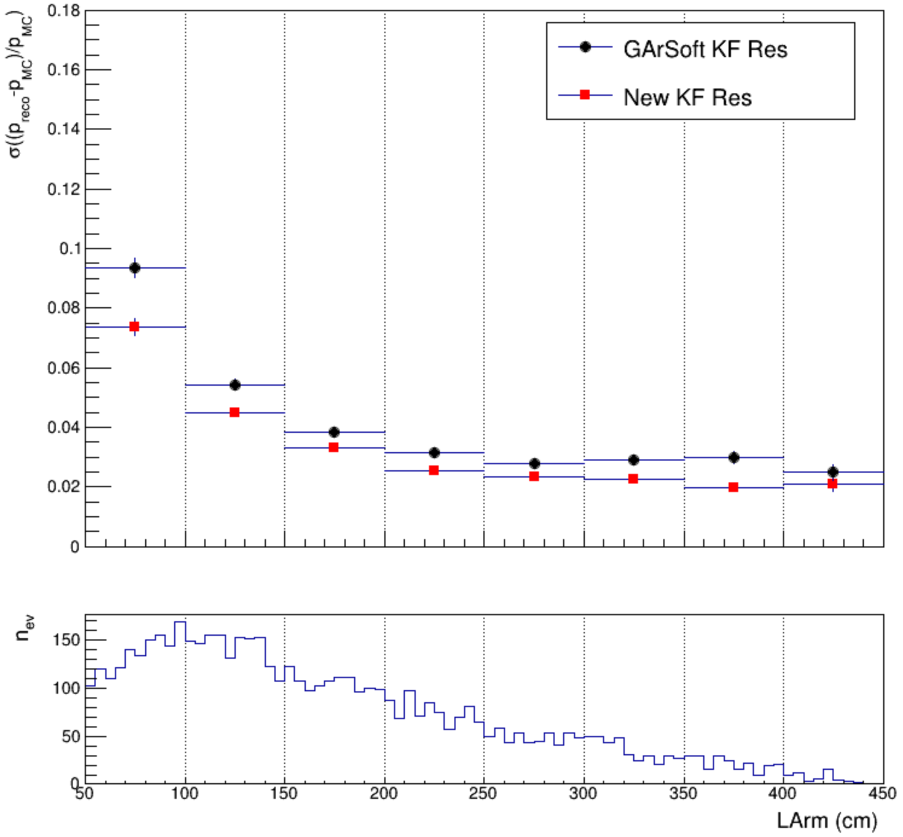
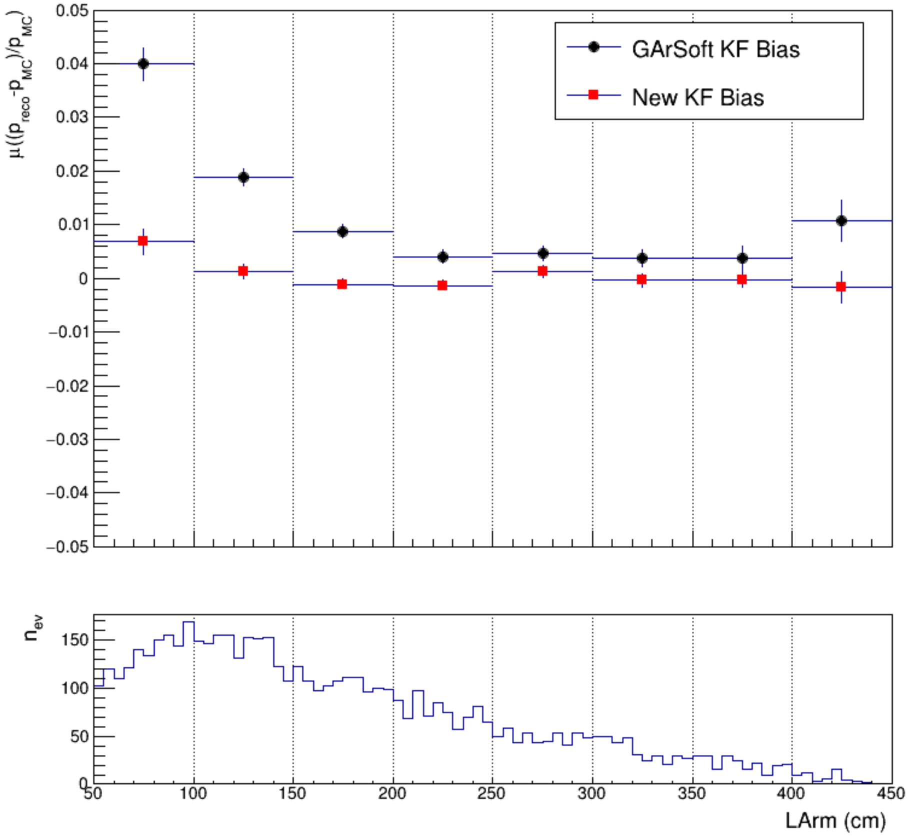
Correspondent p (GeV/c) distribution (NB: Tracks with $N_{points} < 50$ are cut)

- Momentum resolution should be mostly momentum independent in this range and at these densities.
- Primary pions are on average much lower in momentum than muons
- NB: at lower momenta, for higher mass particles the tracks will tend to be shorter and the resolution will degrade
- NB: pT should be averaged through the whole track, which wasn't done here.
- New KF improves resolution over the whole spectrum as well as the bias

MOMENTUM RESOLUTION AND BIAS VS LARM: PIONS π^+





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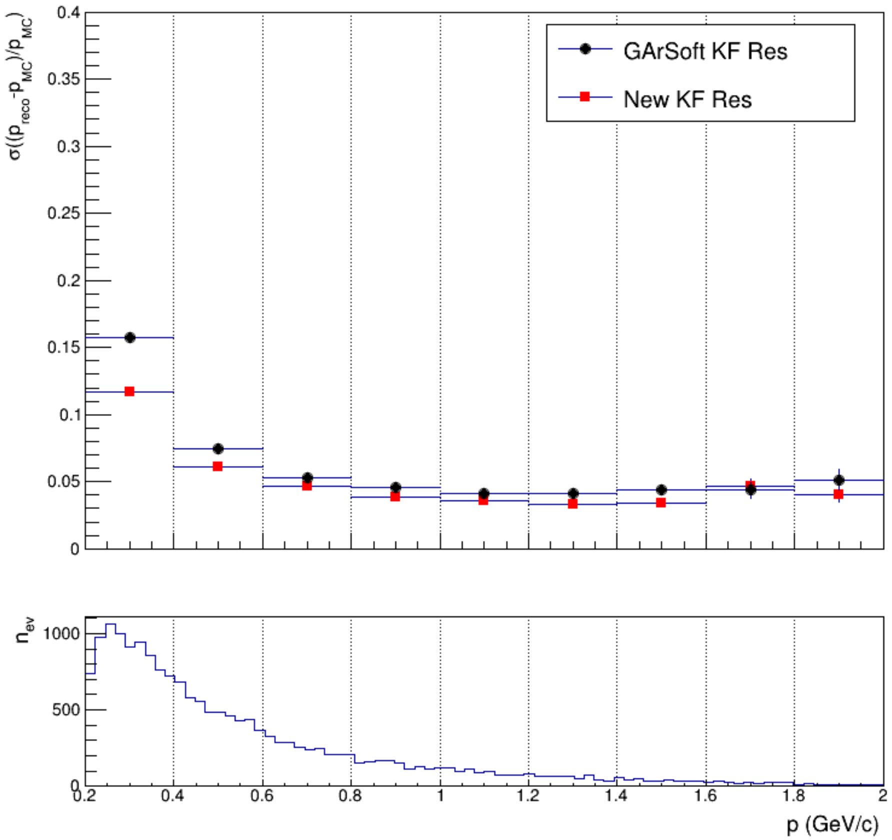
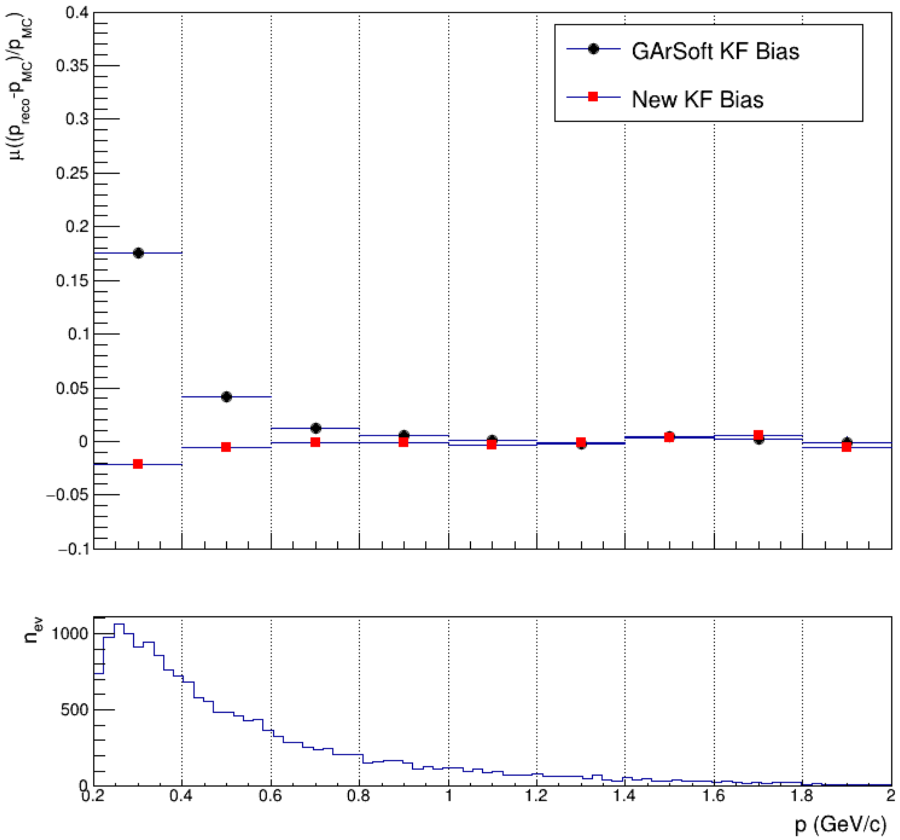
- New KF improves resolution over the whole spectrum as well as the bias: as the mass of the pions is higher than the one of the muons, the dEdx and MS components of the new KF start having a bigger effect

MOMENTUM RESOLUTION AND BIAS VS P: PROTONS p^+





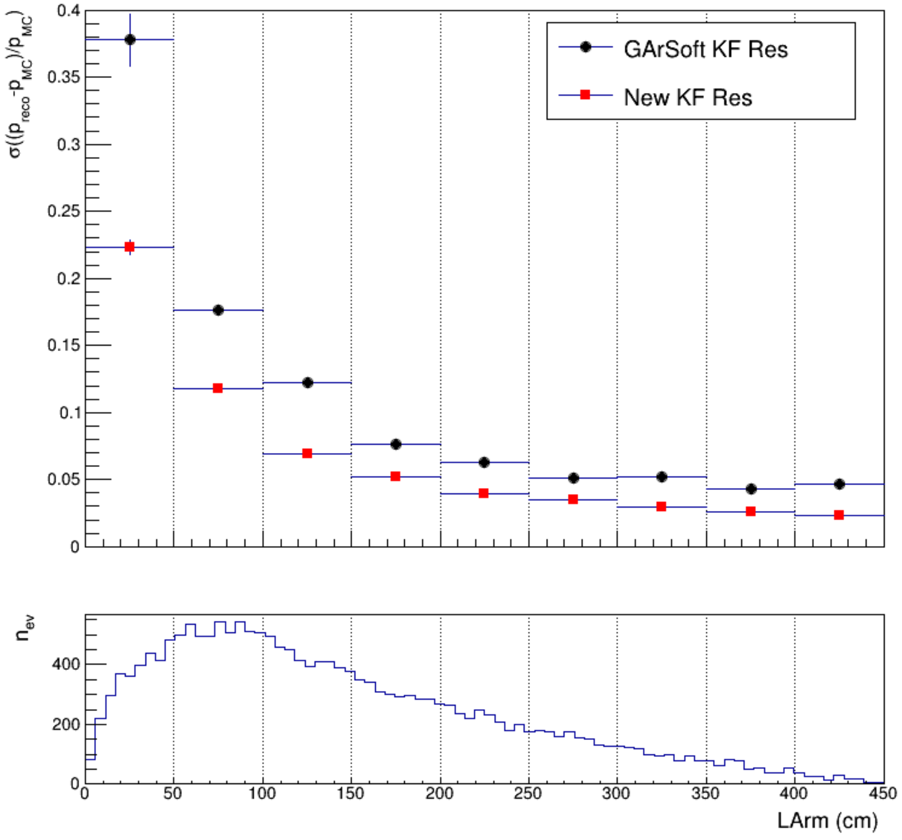
Profile plots for resolution and bias : (σ, μ) from momentum residual Gauss fit in each p slice

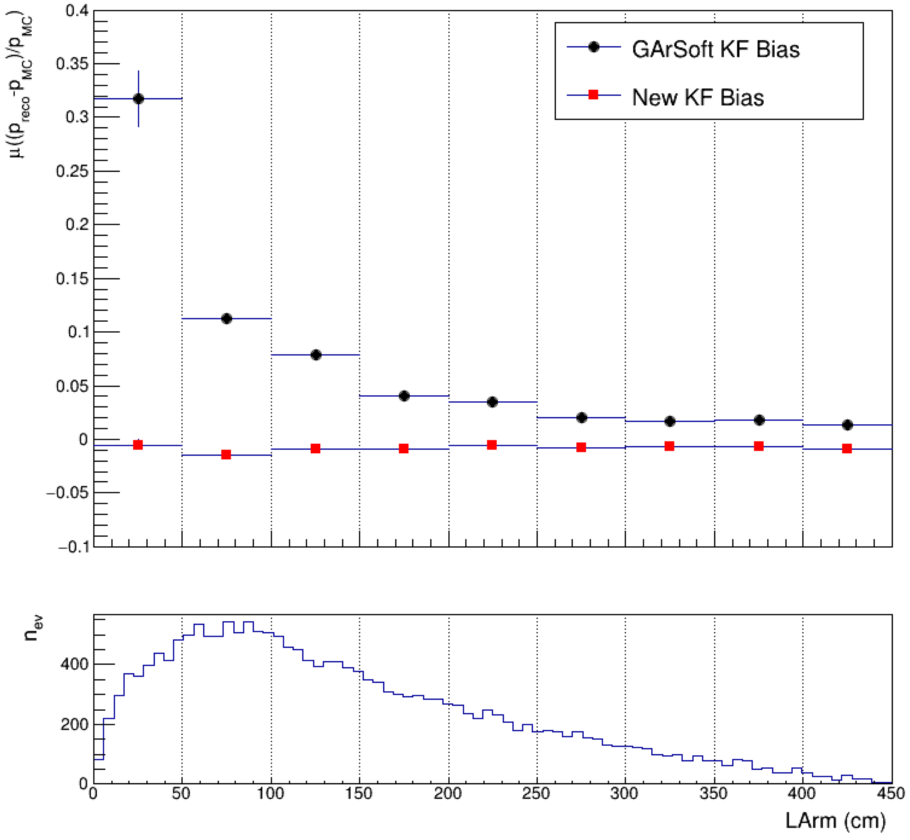
NB: Y axis range is wider as the resolution is worse for protons

Correspondent p (GeV/c) distribution (NB: Tracks with $N_{points} < 50$ are cut)

- Momentum resolution should be mostly momentum independent in this range and at these densities.
- Primary protons are on average much lower in momentum than muons and much more similar to pions
- NB: at lower momenta, for higher mass particles the tracks will tend to be shorter and the resolution will degrade
- NB: pT should be averaged through the whole track, which wasn't done here.
- New KF improves resolution over the whole spectrum and especially the bias

MOMENTUM RESOLUTION AND BIAS VS LARM: PROTONS p^+





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SUMMARY AND CONCLUSIONS

- New ALICE-BASED Kalman Filter was tested and compared to the current GArSoft Reconstruction over a sample of primary particles from $\nu_\mu CC$ interactions with MC vertex in TPC fiducial volume:
  - Selected from a sample of 4.35×10^4 neutrino interactions in active TPC volume;
  - Produced using GENIE module in GArSoft v2_18_00 with standard flux;
  - Primary protons, pions and muons were considered

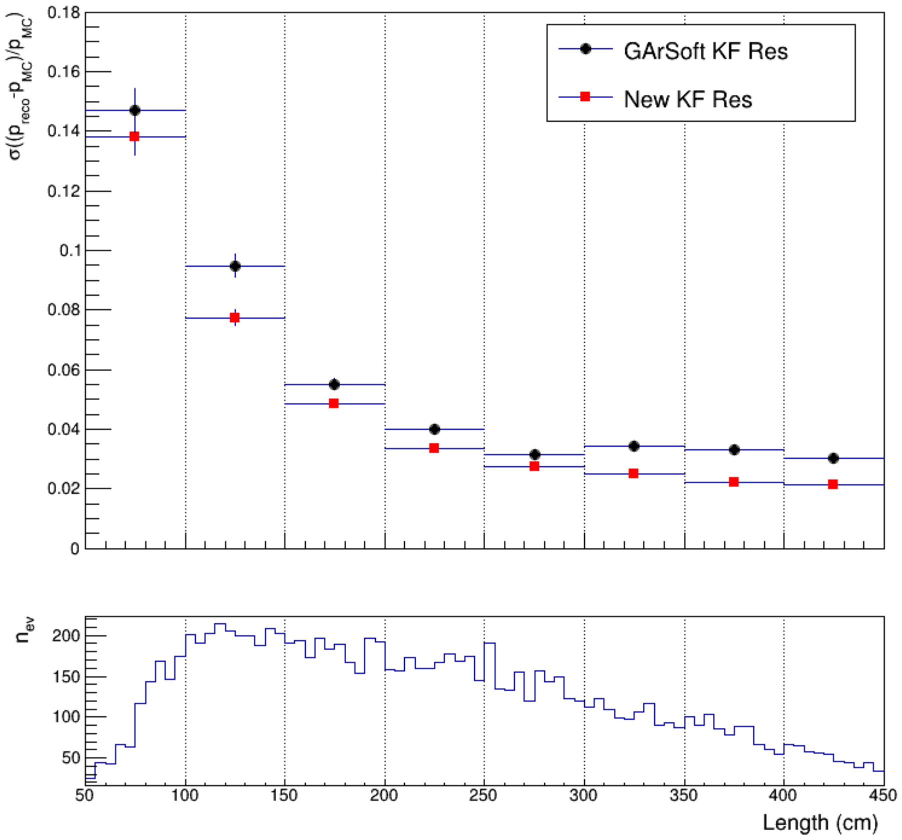
- Main Takeaways:
  1. After bug fixes, new KF shown to provide significant performance benefits for all analyzed particle types from the core sample of $\nu_\mu CC$ interactions
  2. Proton reconstruction is especially biased at the current state and the new KF can improve this

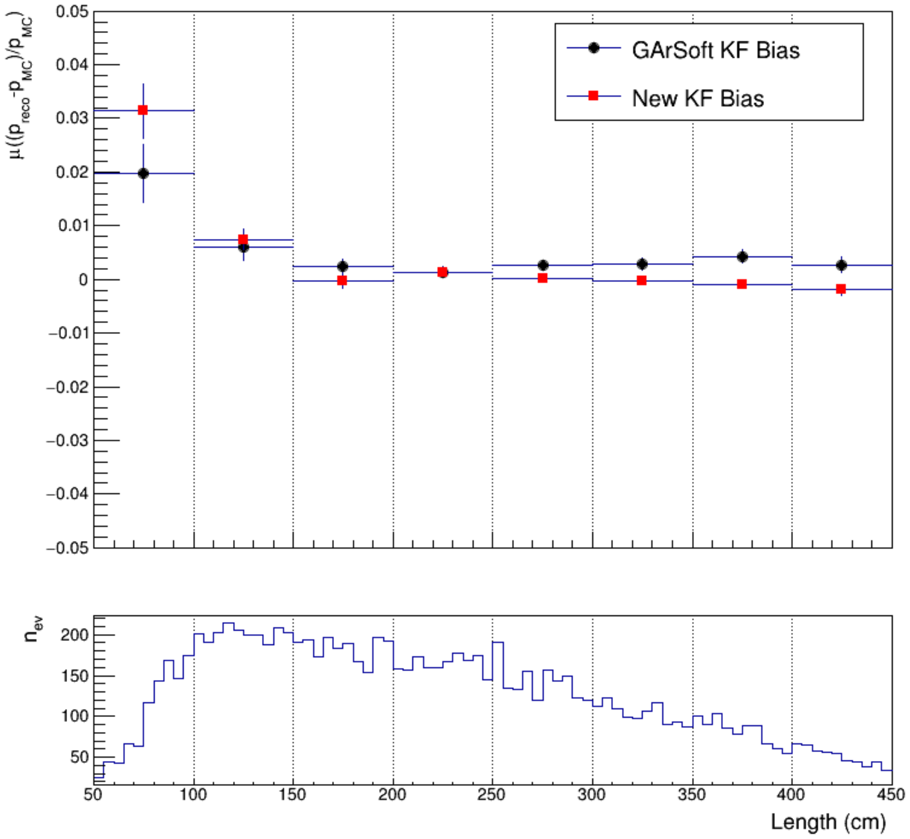
- Next steps:
  1. Finish improving pull tests so that they are as expected for all particle types (not discussed in this presentation)
  2. Explore benefits of the improved performance (e.g. TKI hydrogen study https://indico.fnal.gov/event/59667/)
  3. Implement in GArSoft

THANK YOU

BACK-UP

MOMENTUM RESOLUTION AND BIAS VS LENGTH: MUONS μ^-



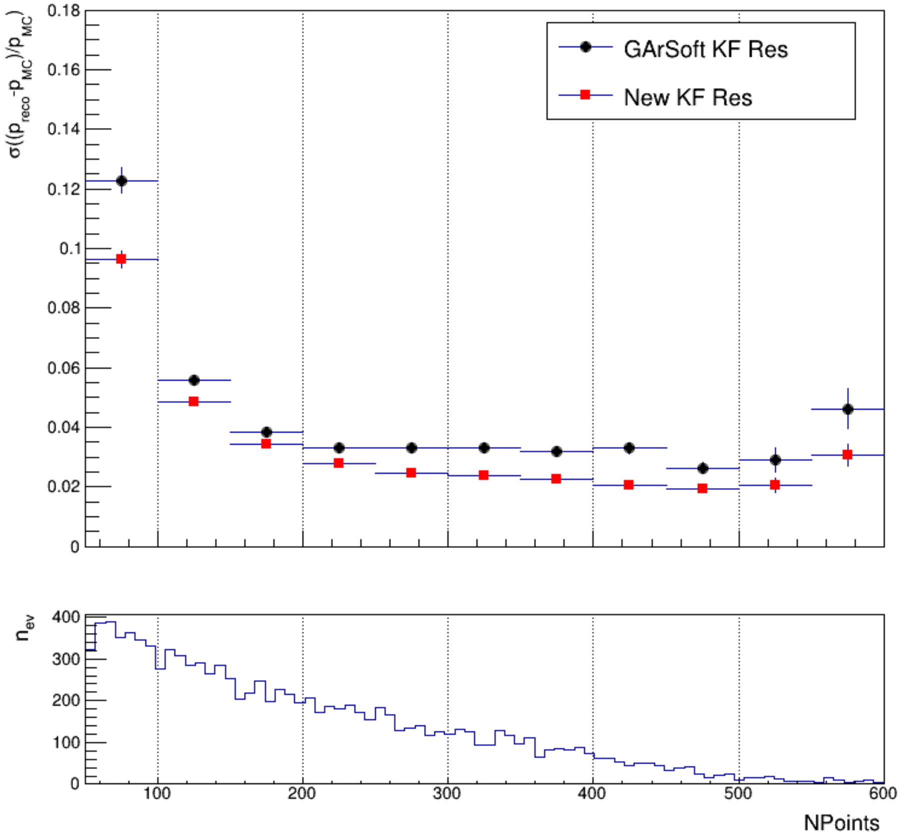


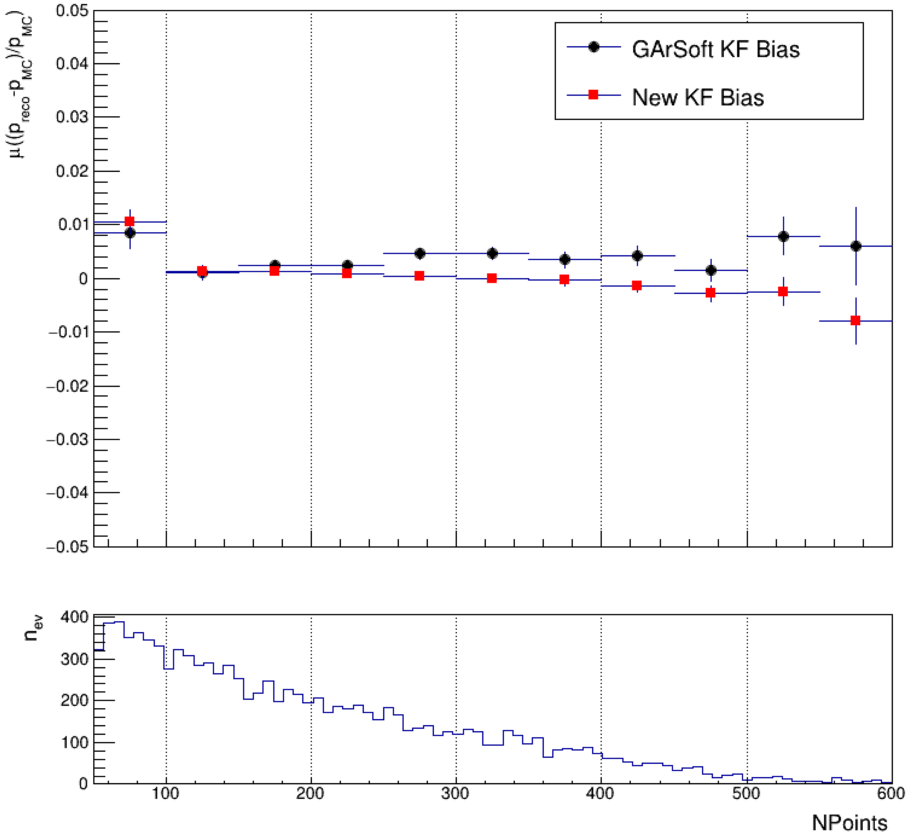
Profile plots for resolution and bias : (σ, μ) from momentum residual Gauss fit in each p slice

Correspondent p (GeV/c) distribution (NB: Tracks with $N_{points} < 50$ are cut)

- Momentum resolution in the range $p \in [0,6]$ GeV/c should be multiple scattering dominated and go as $\propto 1/\sqrt{Length}$
- New KF improves resolution over the whole spectrum and keeps bias mostly the same

MOMENTUM RESOLUTION AND BIAS VS NPOINTS: MUONS μ^-





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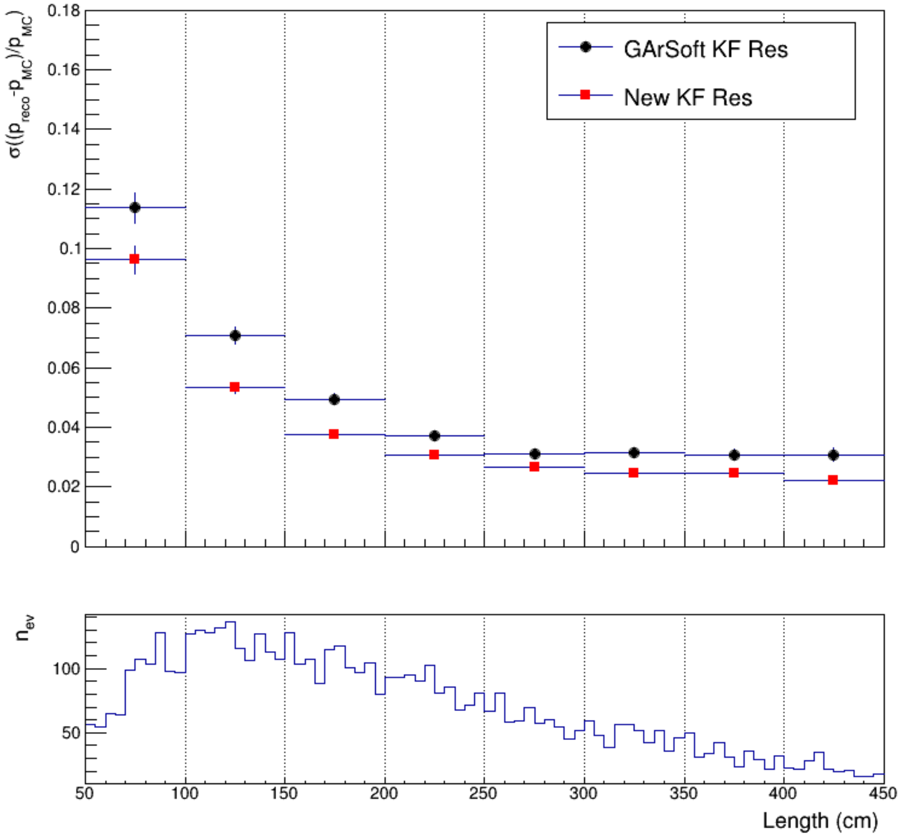
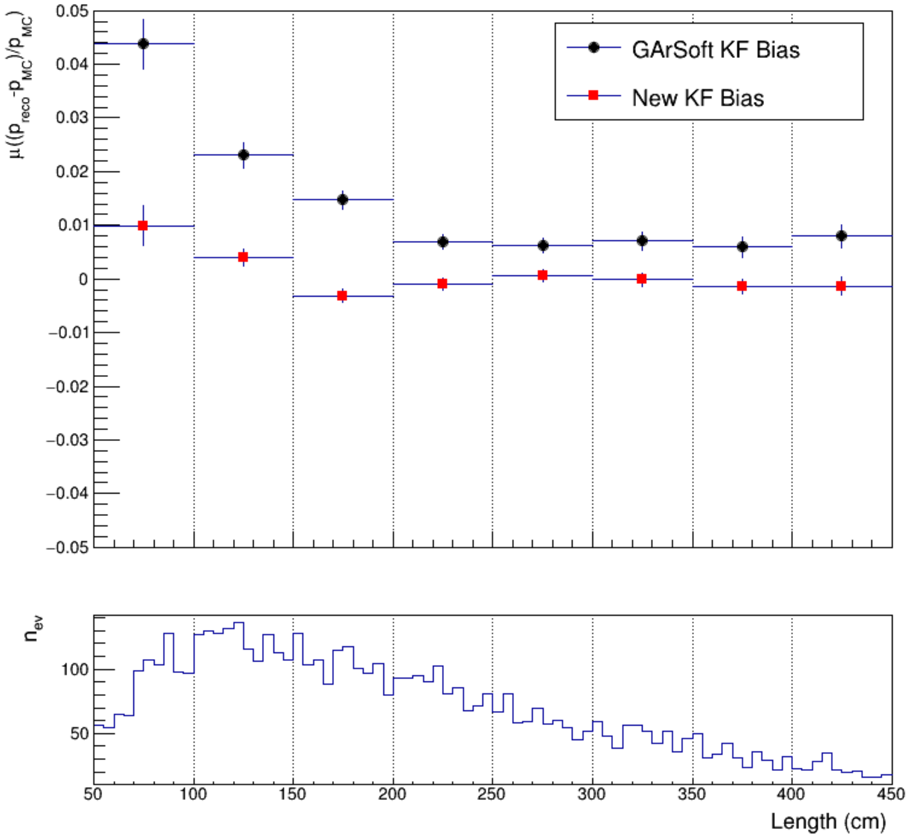
MOMENTUM RESOLUTION AND BIAS VS LENGTH: PIONS π^+





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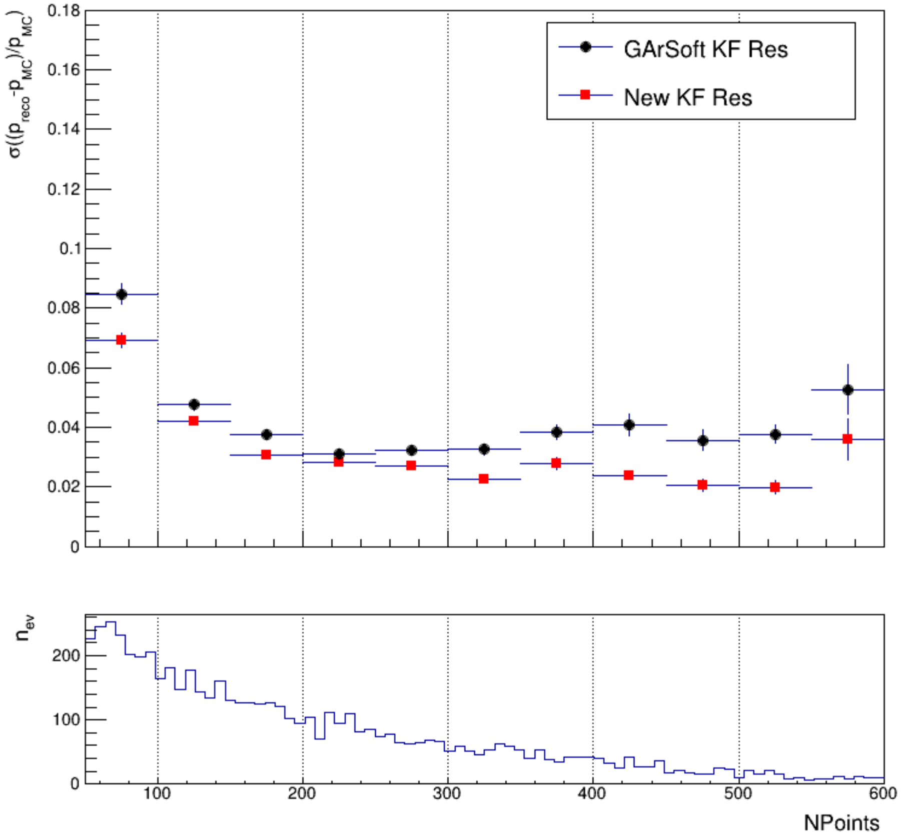
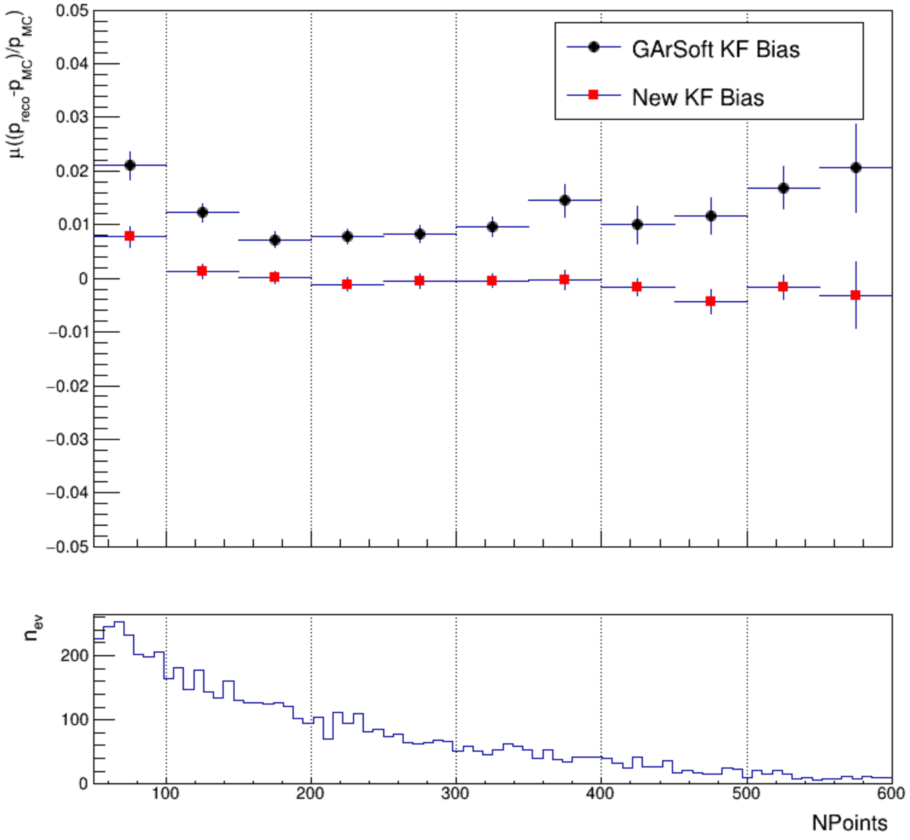
MOMENTUM RESOLUTION AND BIAS VS NPOINTS: PIONS π^+





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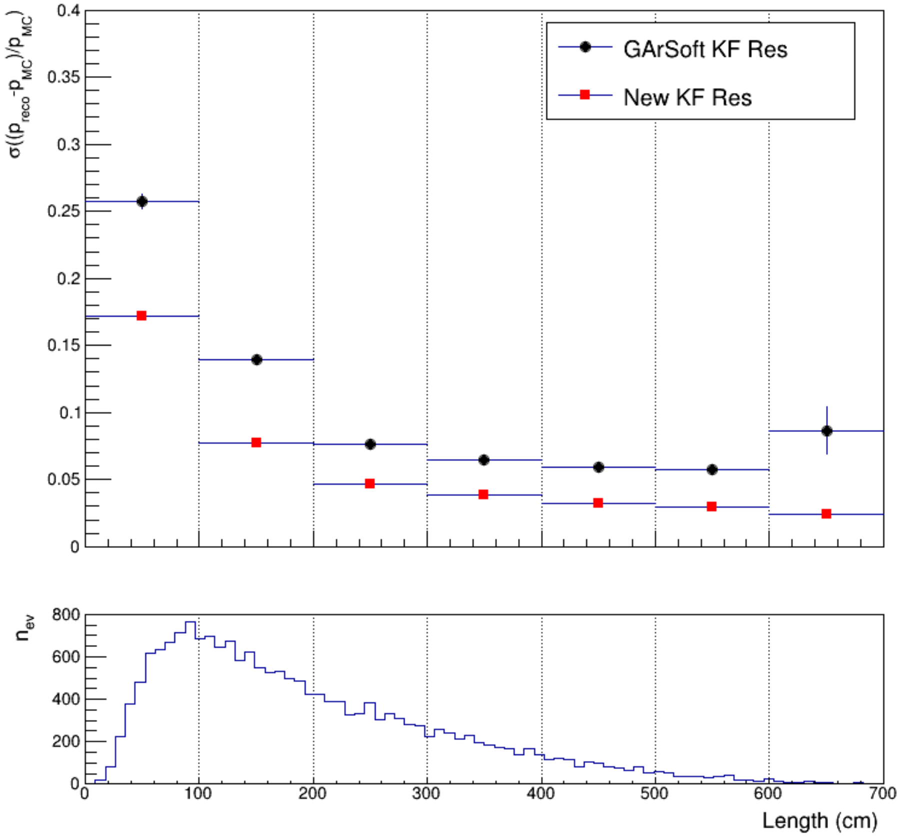

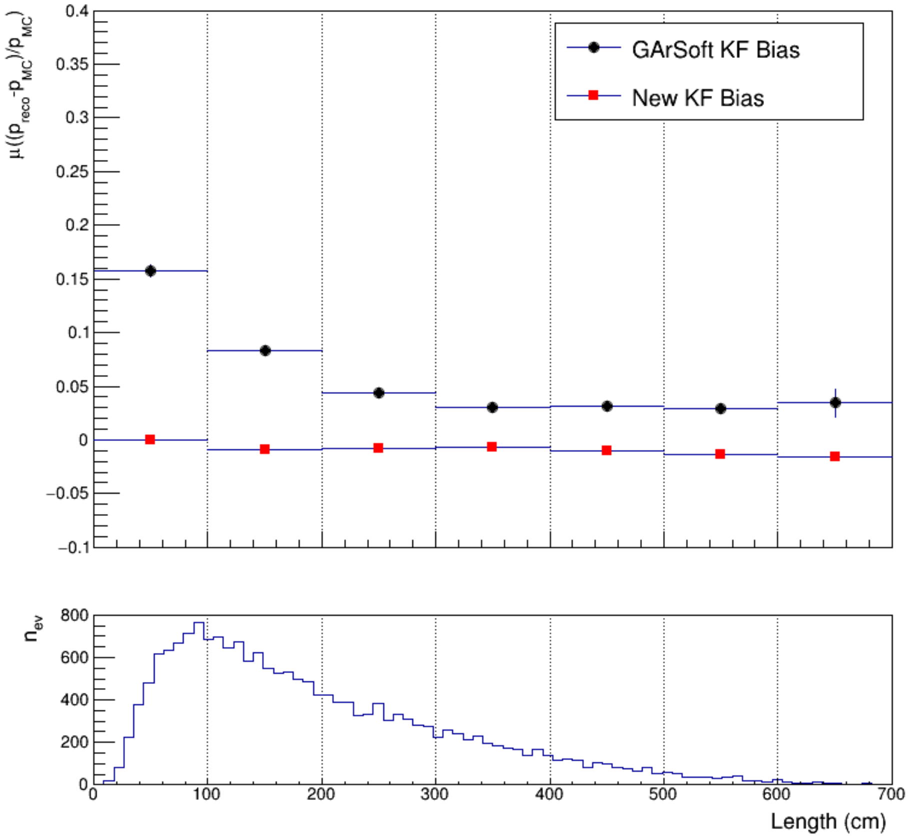

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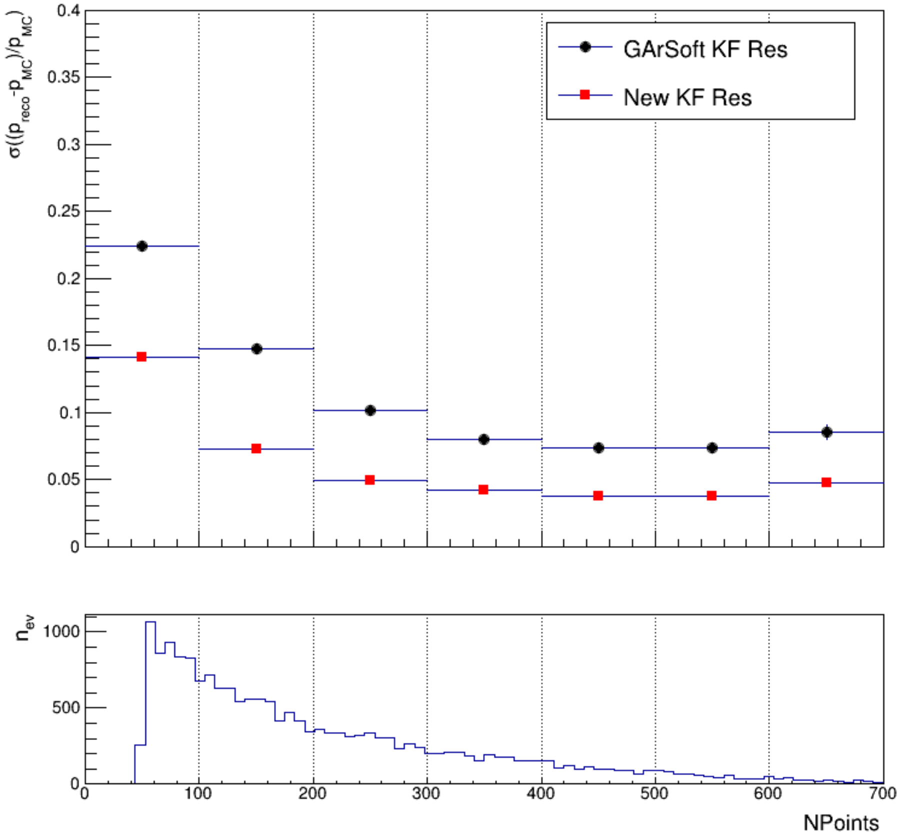

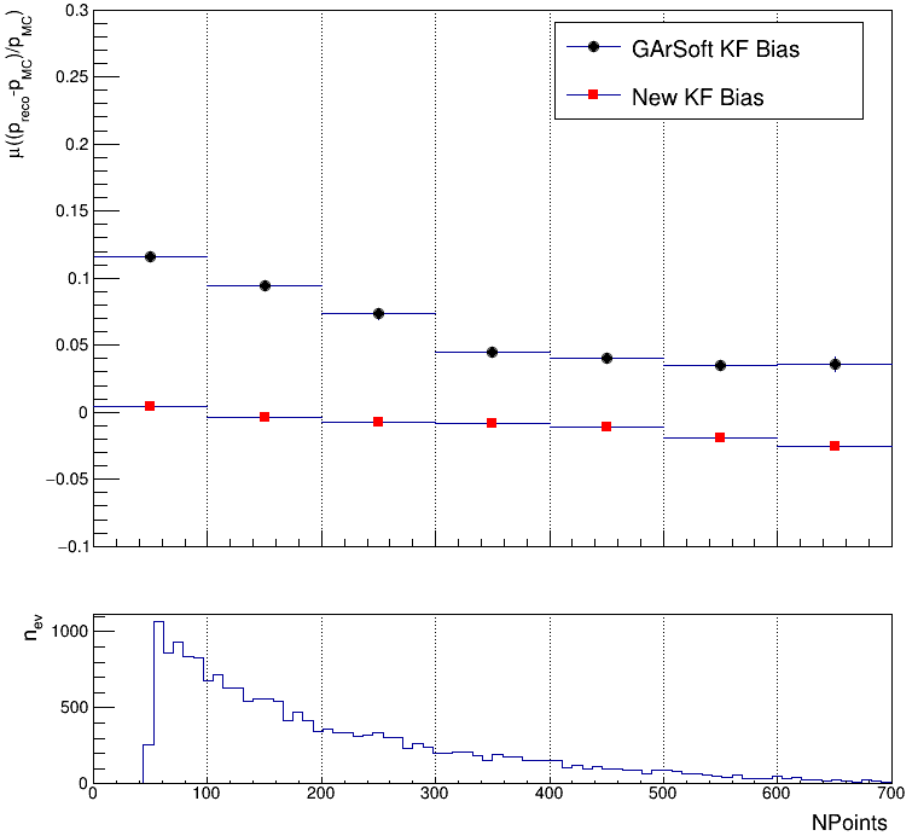

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- New KF improves resolution over the whole spectrum as well as the bias

RECONSTRUCTION

KALMAN FILTER BASICS

$$\hat{s}_k^+ = \hat{s}_k^- + K_k \tilde{r}$$

Kalman Filter Update Equation

A Posteriori

A Priori

Kalman Gain: small if confidence in model (determined by covariance matrix P) high, large if confidence low

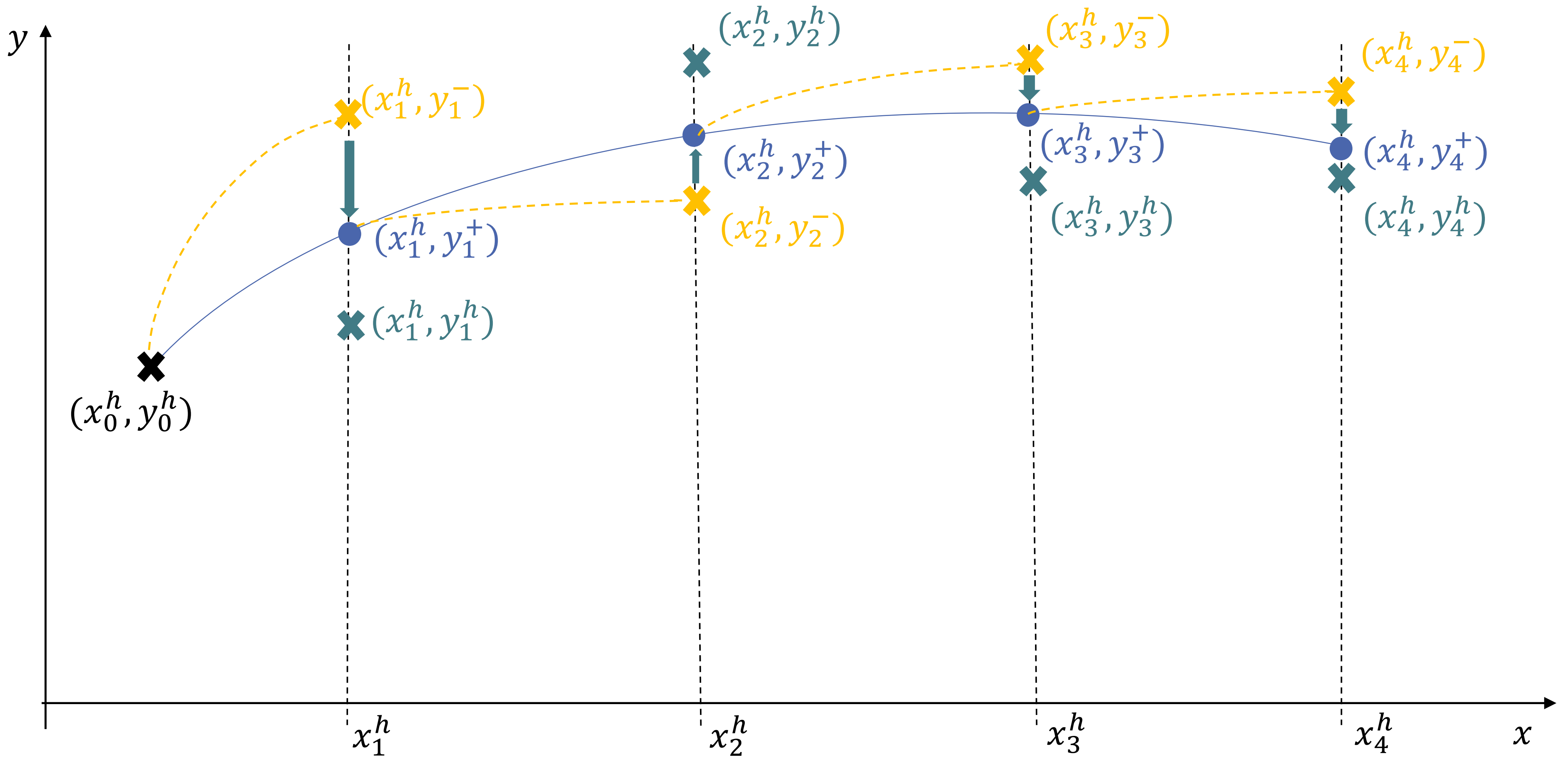
Residual: distance between measurement and a priori

- **Kalman filter:** iterative Bayesian algorithm which mediates between system knowledge and measurement. Each iteration divided in three steps:

  1. Make **A Priori prediction** of the state of the system using evolution model for the particle's trajectory
  2. Calculate **Residual:** distance between measurement and prediction
  3. Mediate between the a priori prediction and the measurement calculating **Kalman Gain** and produce **A Posteriori estimate**

Note: See back-up for further reading

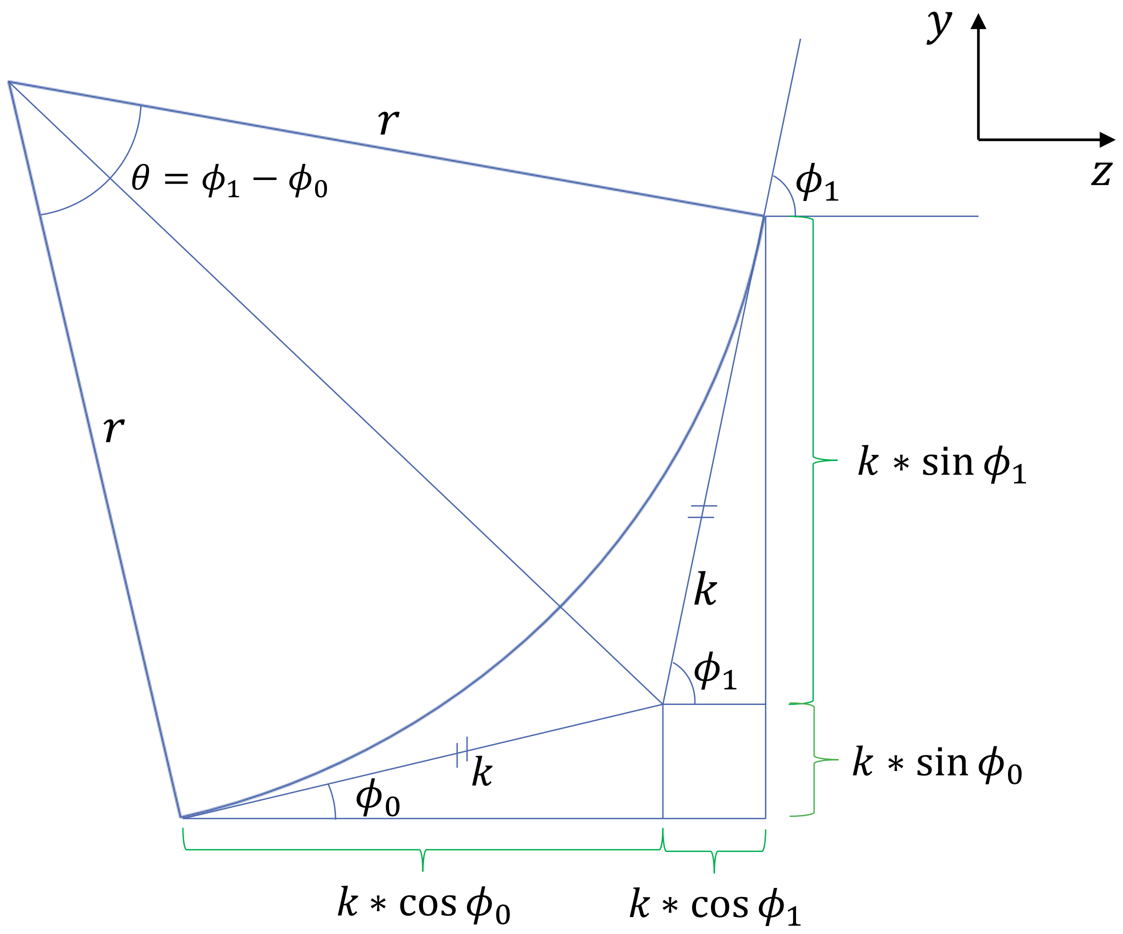
KALMAN FILTER BASICS



KALMAN FILTER MODEL AND APPLICATION

- Use parametrization used in ALICE: state vector updated by the Kalman filter is $s = (y, x, sin\phi, \tan\lambda, \frac{q}{p_T})$

- ALICE uses no approximations in the propagation, unlike current ND-GAr model which uses small angle approximation (for full description check back-up and first ND-GAr-Lite presentation https://indico.fnal.gov/event/50215/contributions/232480/)



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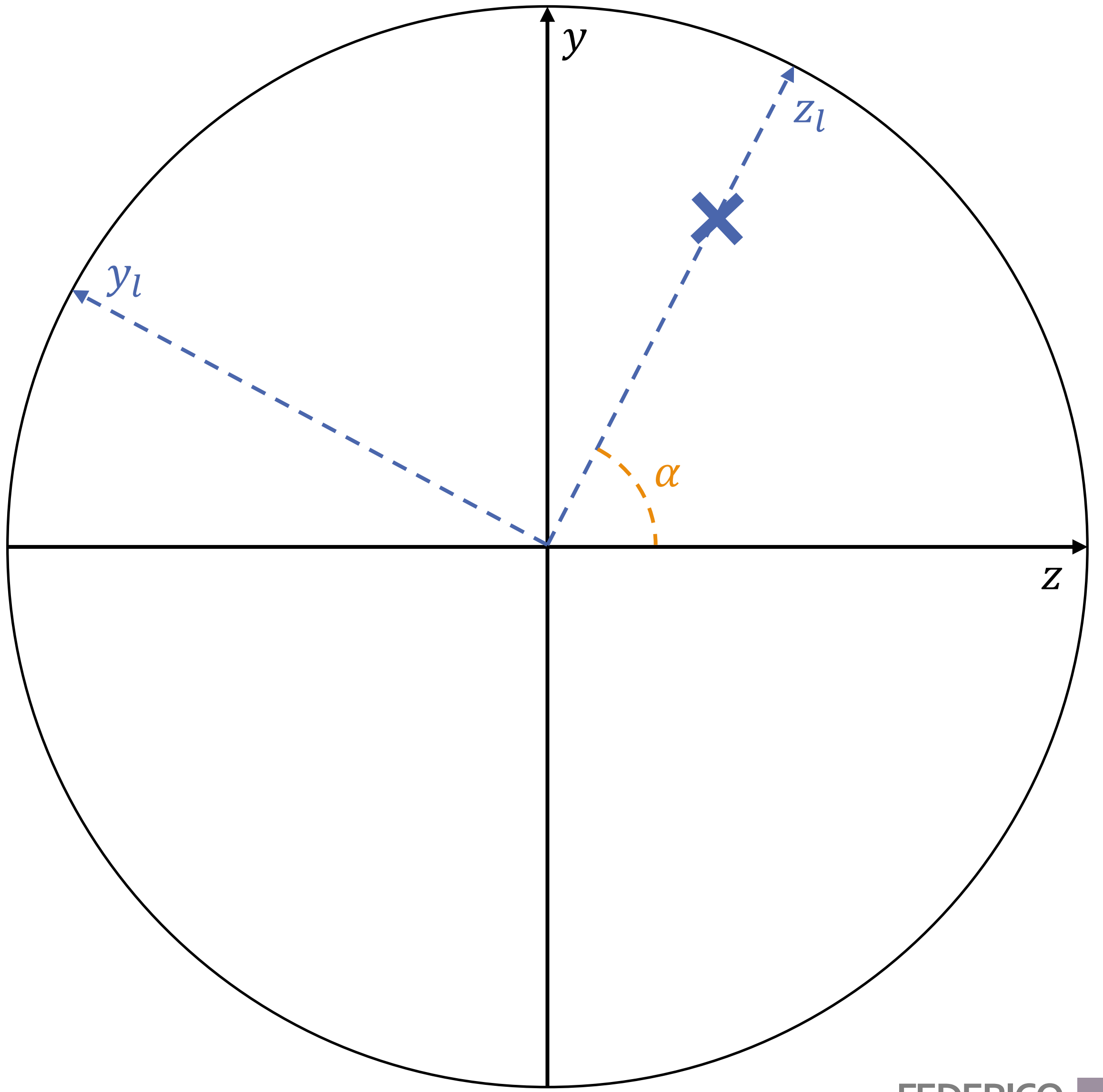
- Kalman filter propagated radially: before each propagation, the coordinate system is rotated by an angle $\alpha = \tan(y/z)$, so that the track point “sits” on the local z axis (i.e. z coordinate becomes the radius from center of the detector)



KALMAN FILTER MODEL AND APPLICATION

- Local $sin\phi$ defines two yz semi-planes with "mirrored representations": the line separating the two is the one connecting the center of the detector and the center of curvature of the track

- As the track approaches one of the two semi-planes, $sin\phi$ reaches a point where it cannot be propagated further: $\sin\phi \in [-1,1]$

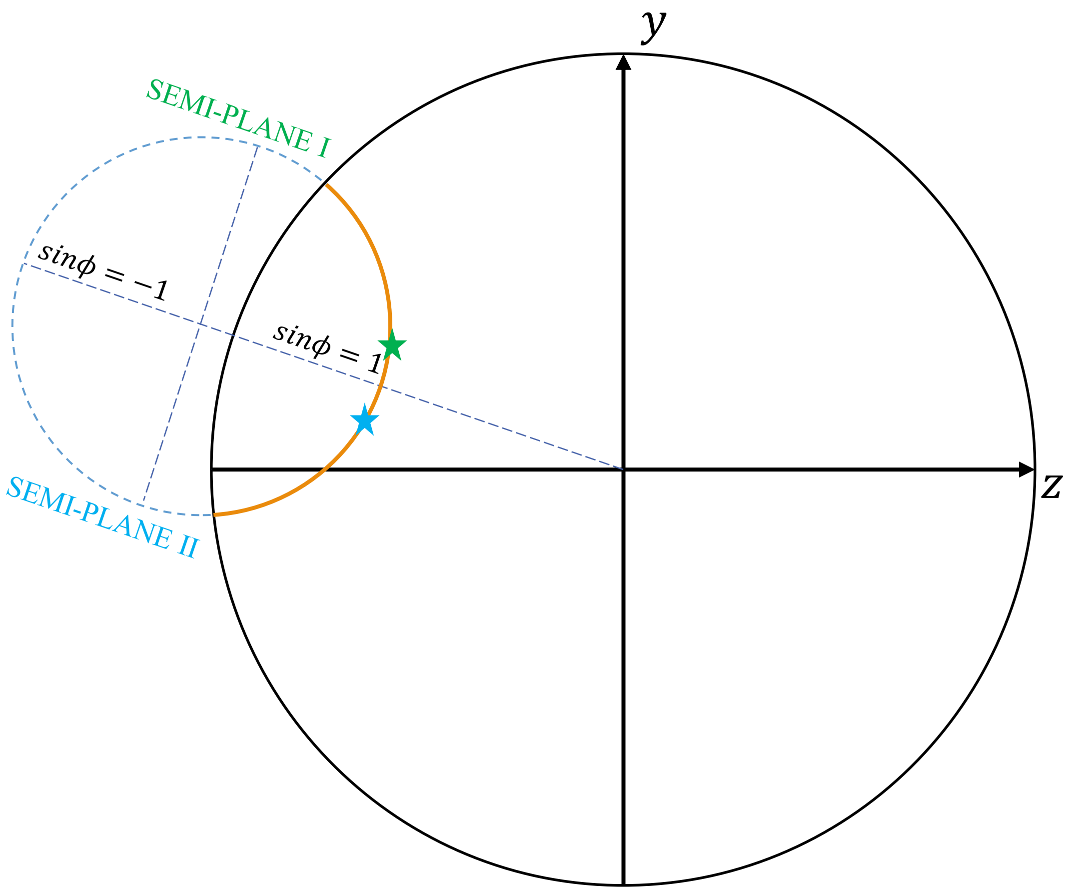
- Once the limit is reached, the state-vector and Covariance associated with the last reconstructed track point are "mirrored":

$$s_{k+1}^{-} = R s_k^{+} \qquad P_{k+1}^{-} = R P_k^{+} R^T$$

$$with\ R = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix}$$

- Finally, the local x coordinate is propagated by calculating the arch between the two mirrored points:

$$x_{k+1}^{-} = x_k^{+} + arch * tan\lambda$$



ENERGY LOSS CORRECTION

Bethe-Bloch (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\frac{1}{\rho}\frac{dE}{dx} = K \times \frac{Z}{A} \times \frac{z^2}{\beta^2}\left[\frac{1}{2} ln\left(\frac{2m_e\gamma^2\beta^2T_{max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right] \quad [\text{GeV/(g/cm^2)}]$$

- Energy loss correction applied to helix fit:

1. Get dE/dx with Bethe-Bloch and evaluate momentum loss over trajectory in small "momentum-loss" steps
2. Calculate multiplicative factor to update q/p_T:

$$\frac{q}{p_T} *= cP4 = \left(1 + \frac{\Delta E}{p_{mean}^2}(\Delta E + 2 \times E_{in})\right)$$

2. Add factor to diagonal element of 5x5 Covariance Matrix P correspondent to q/p_T (found through error propagation):

$$P[4][4] += \left(\frac{\sigma_E}{p_{mean}^2} \times \frac{q}{p_T}\right)^2$$

- Note 1: These formulas are the same as the ones used by Geant4
- Note 2: Applied to both Kalman Filter "step-by-step" and Seeding "globally"

MS CORRECTION

Molière Formula (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\theta_0 = \frac{13.6MeV}{\beta p} z\sqrt{x/X_0}[1 + 0.038\ln(x/X_0)]$$

- Multiple Scattering correction applied to Helix fit:

  1. Calculate width of the angular gaussian distribution produced by MS: θ_0 from Molière formula
  2. Propagate the error to the relevant Helix parameters, obtaining their respective σ's ($\sigma_{sin\phi}, \sigma_{tan\lambda}, \sigma_{q/p_T}$)
  3. Update covariance matrix diagonal elements:

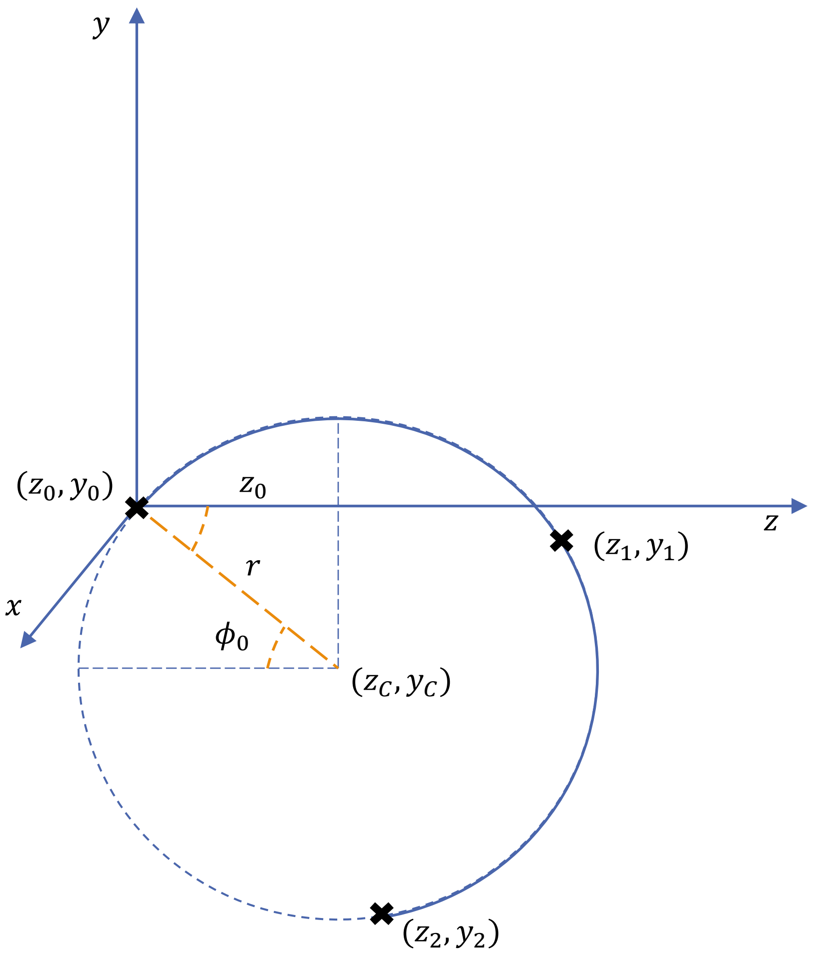
$$\begin{cases} P[2][2] += \sigma^2_{sin\phi} \\ P[3][3] += \sigma^2_{\tan\lambda} \\ P[4][4] += \sigma^2_{q/p_T} \end{cases}$$

- Note 1: These formulas are the same as the ones used by Geant4
- Note 2: Applied to both Kalman Filter "step-by-step" and Seeding "globally"

GLOBAL HELIX FIT AND INITIAL COVARIANCE ESTIMATION

- Seeding for Kalman done with simple 3-point helix fit:
  - $c = 1/r$ and $\sin\phi_0$ estimated by finding (z_c, y_C) and r of the yz plane circumference:

$$c = 1/r \qquad \sin\phi_0 = \frac{z_0}{r}$$



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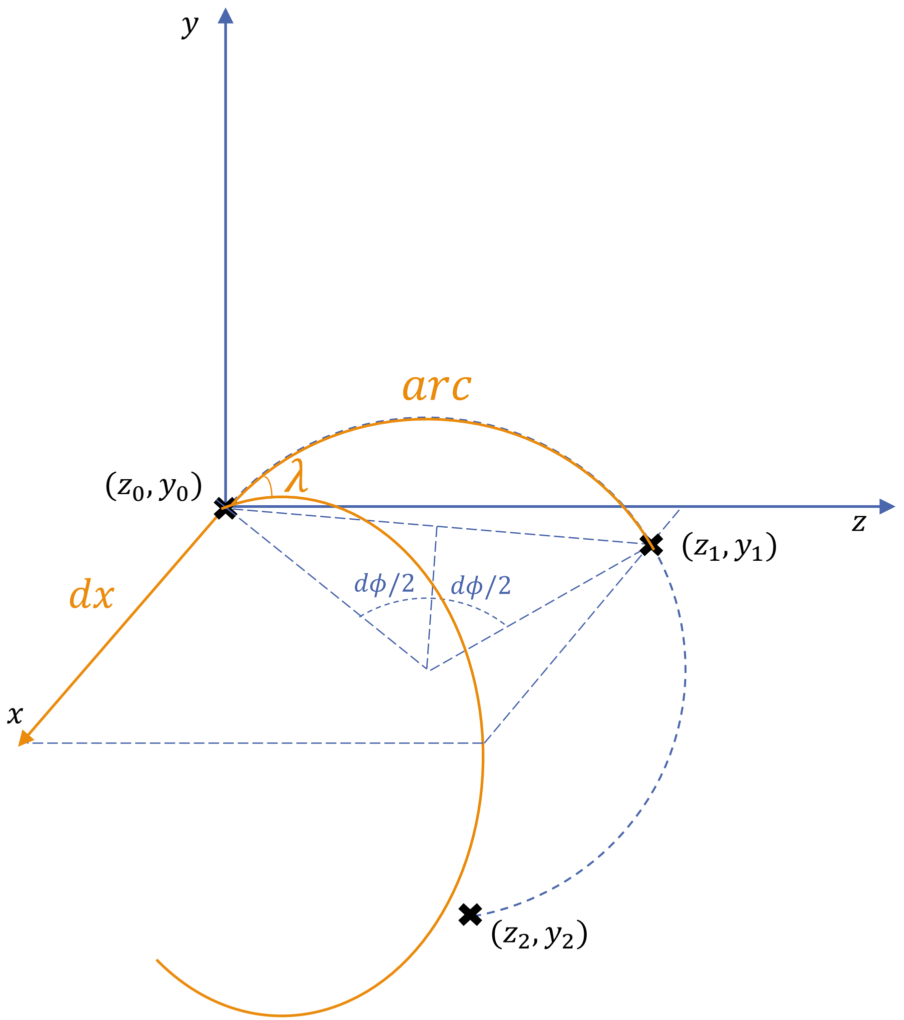
  - $\tan\lambda$ from the yz plane arc between the first two points and the correspondent movement in the x direction:

$$\tan\lambda = \frac{dx}{arc} = \frac{dx}{d\phi * r}$$

- Note: Energy loss and MS corrections applied similarly to Kalman Filter



ENERGY LOSS AND MS

ENERGY LOSS: BETHE-BLOCH FORMULA

Bethe-Bloch (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\frac{1}{\rho}\frac{dE}{dx} = K \times \frac{Z}{A} \times \frac{z^2}{\beta^2}\left[\frac{1}{2} ln\left(\frac{2m_e c^2\gamma^2\beta^2 W_{max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right] \quad [\text{GeV}/(\text{g}/cm^2)]$$

- $\rho = 1.032\ g/cm^3$ — Plastic scintillator density
- $K = 4\pi N_A r_e^2 m_e c^2 = 0.307\ 075\ MeV\ mol^{-1} cm^2$ — Bethe Bloch constant coefficient
- $Z/A = 0.54141\ mol/g$ — Mean atomic number/mass of plastic scintillator
- z — Atomic number of incident particle
- $m_e c^2 = 0.511$ MeV — Mass of electron
- $W_{max} = 2m_e c^2\ \beta^2\gamma^2$ — Low energy approximation of maximum energy transfer
- $I = 64.7 \times 10^{-9}\ GeV$ — Mean excitation energy

$$\frac{\delta}{2} = \begin{cases} 0 & ln\beta\gamma < 2.303x_0 \\ \ln\beta\gamma - 1/2\,C & ln\beta\gamma > 2.303x_1 \\ \ln\beta\gamma - 1/2\,C + (1/2\,C - 2.303X_0) \times \left(\frac{2.303X_1 - \ln\beta\gamma}{2.303(X_1 - X_0)}\right)^3 & ln\beta\gamma \in [2.303x_0, 2.303x_1] \end{cases}$$

DENSITY CORRECTION

$$with\ C = 2 - \ln\left(\frac{28.816 \times 10^{-9}\sqrt{\rho(Z/A)}}{I}\right)$$

$x_0 = 0.1469 \quad x_1 = 2.49$

1st and 2nd junction points for plastic scintillator

ENERGY LOSS CORRECTION

Bethe-Bloch (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\frac{1}{\rho}\frac{dE}{dx} = K \times \frac{Z}{A} \times \frac{z^2}{\beta^2}\left[\frac{1}{2} ln\left(\frac{2m_e c^2\gamma^2\beta^2 W_{max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right] \quad [\mathrm{GeV}/(\mathrm{g}/cm^2)]$$

- Step by step procedure:

1. Convert into: $dp/dx = dE/dx \times \beta^{-1}$
2. Calculate number of steps: $n_{steps} = 1 + (dp/dx \times \Delta x)/step$ with $step = 0.005$
3. Calculate step-wise total momentum loss: $\Delta p_{tot} = \sum_{i=0}^{n_{steps}} \Delta p_i = \sum_{i=0}^{n_{steps}} \frac{\mathrm{dp}}{\mathrm{dx_i}} \Delta x_i$
4. Calculate total energy loss $\Delta E = E_{in} - \sqrt{p_{out}^2 + m^2}$ with $p_{out} = p_{in} - \Delta p_{tot}$
5. Apply multiplicative factor:
$$\frac{q}{p_T} *= cP4 = \left(1 + \frac{\Delta E}{p_{mean}^2}(\Delta E + 2 \times E_{in})\right)$$
6. Apply correction to covariance matrix:
$$P[4][4] += \left(\frac{\sigma_E}{p_{mean}^2} \times \frac{q}{p_T}\right)^2$$

KALMAN FILTER: MS CORRECTION

Molière Formula (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\theta_0 = \frac{13.6MeV}{\beta p} z\sqrt{x/X_0}[1 + 0.038\ln(x/X_0)]$$

- $X_0 = 42.54cm$ Radiation length of plastic scintillator in cm
- x is the step length
- z is the charge of incident particle
- Formulas for propagated σ's:

$$\begin{cases} \sigma_{\sin\phi} = \theta_0 \cos\phi \sqrt{1 + \tan^2\lambda} \\ \sigma_{\tan\lambda} = \theta_0 (1 + \tan^2\lambda) \\ \sigma_{q/p_T} = \theta_0 \tan\lambda \frac{q}{p_T} \end{cases}$$

KALMAN FILTER: ENERGY LOSS CORRECTION

Bethe-Bloch (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\frac{1}{\rho}\frac{dE}{dx} = K \times \frac{Z}{A} \times \frac{z^2}{\beta^2}\left[\frac{1}{2} ln\left(\frac{2m_e\gamma^2\beta^2T_{max}}{I^2}\right) - \beta^2 - \frac{\delta}{2}\right] \quad [\mathrm{GeV/(g/cm^2)}]$$

- Energy loss correction:

  1. Use multiplicative factor $cP4$ (see slide 7) to update q/p_T
  2. Add factor to diagonal element of 5x5 Covariance Matrix P correspondent to q/p_T (found through error propagation):

$$P[4][4] \mathrel{+}= \left(\frac{\sigma_E}{p_{mean}^2} \times \frac{q}{p_T}\right)^2$$

- **NOTE:** $\sigma_E = k \times \sqrt{|\Delta E|}$ where k is a tunable parameter set at 0.07

KALMAN FILTER: MS CORRECTION

Molière Formula (PDG)
https://pdg.lbl.gov/2005/reviews/passagerpp.pdf

$$\theta_0 = \frac{13.6MeV}{\beta p} z\sqrt{x/X_0}[1 + 0.038\ln(x/X_0)]$$

- Multiple Scattering smearing simulated in three steps:

  1. Obtain parameter σ's ($\sigma_{sin\phi}, \sigma_{tan\lambda}, \sigma_{q/p_T}$) through error propagation as described in slide 6

  2. Update covariance matrix diagonal elements:

$$\begin{cases} P[2][2] += \sigma^2_{sin\phi} \\ P[3][3] += \sigma^2_{\tan\lambda} \\ P[4][4] += \sigma^2_{q/p_T} \end{cases}$$

KALMAN FILTER

KALMAN FILTER IN GENERAL

1. Make a priori predictions for the current step's state and covariance matrix using the a posteriori best estimate of the previous step (i.e. updated using measurement)

STATE VECTOR $s_k^- = f(s_{k-1}^+, X_{k-1})$

COVARIANCE MATRIX $P_k^- = F_{k-1} P_{k-1}^+ F_{k-1}^T + Q$

$$F_{k-1} = \left.\frac{\partial f}{\partial s}\right|_{s_{k-1}^+, X_{k-1}}$$

JACOBIAN

$$Q$$

PROCESS NOISE COVARIANCE

Note: In the first iteration step we use step 0 estimates for the state vector and the covariance matrix (s_0, P_0), which can be made very roughly

KALMAN FILTER IN GENERAL

2. Calculate the measurement residual and the Kalman Gain

RESIDUAL	$\tilde{y}_k = m_k^h - H(s_k^-)$
KALMAN GAIN	$K_k = P_k^- H^T (R + H P_k^- H^T)^{-1}$

R

MEASUREMENT NOISE COVARIANCE

H

CONVERSION MATRIX

3. Update the estimate

STATE VECTOR	$s_k^+ = s_k^- + K_k \tilde{y}$
COVARIANCE MATRIX	$P_k^+ = (1 - K_k H) P_k^-$

Note: in the case where R is a null matrix $s_k^+ = s_k^h$ and $P_k^+ = 0$

Note: the conversion matrix is needed to make the dimensions of vectors and matrixes turn out right. For exemple if s_k^h is a 2-D vector and s_k^- is 5-D, then H would be a 2×5 matrix:

$$H = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

KALMAN FILTER MODEL

- Use parametrization used in ALICE: free parameter z, state vector s = $(y, x, sin\phi, \tan\lambda, \frac{q}{p_T})$ (ϕ azimuthal angle, λ dip-angle, p_T transverse momentum in yz plane), evolution function:

0

$$\frac{dy}{dz} = \frac{k * (sin\phi_0 + sin\phi_1)}{k * (\cos\phi_0 + \cos\phi_1)}$$

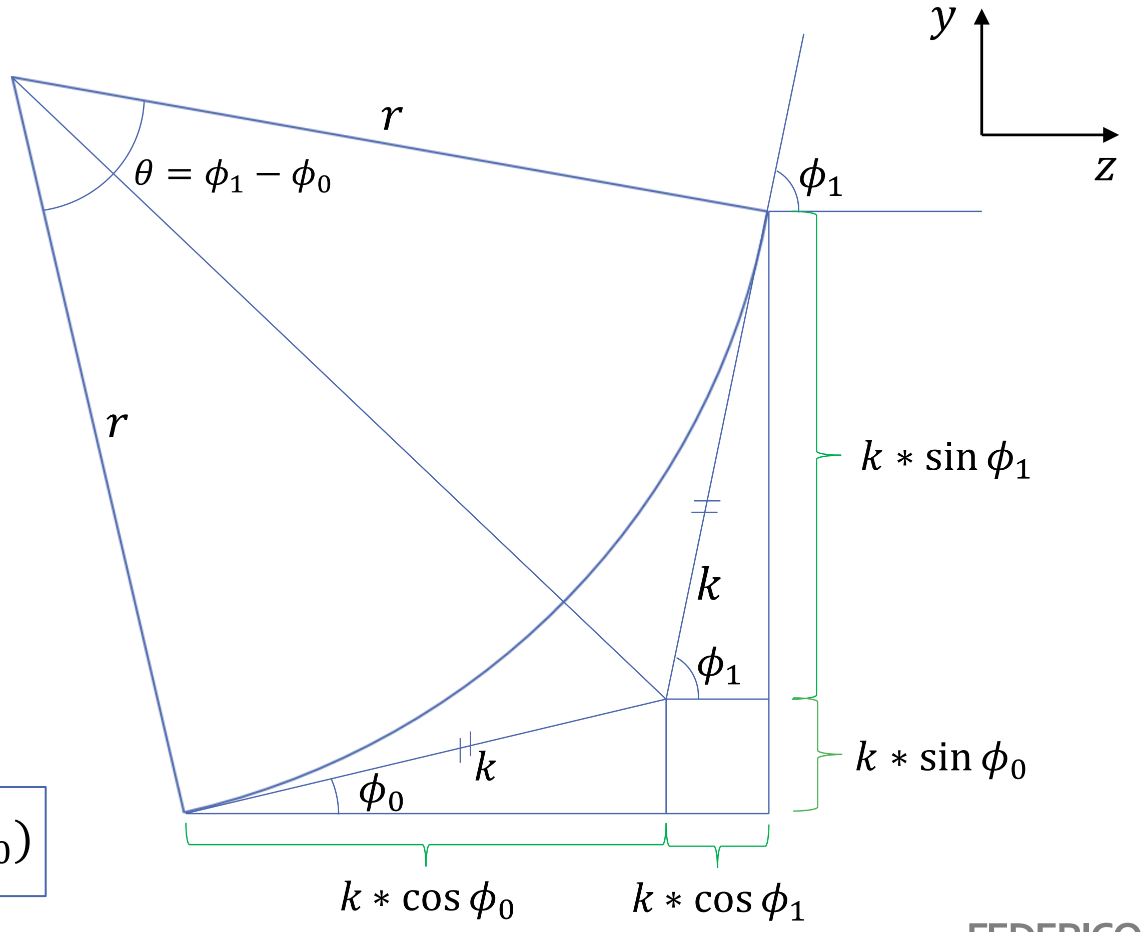
$$y_1 = y_0 + \frac{(sin\phi_0 + sin\phi_1)}{(\cos\phi_0 + \cos\phi_1)} * dz$$

1

$$dx = arch * tan\lambda = \theta * r * tan\lambda$$

$$\theta = \phi_1 - \phi_0 = \arcsin(\sin(\phi_1 - \phi_0)) =$$
$$= \arcsin(\cos\phi_0 \sin\phi_1 - \cos\phi_1 \sin\phi_0)$$

$$x_1 = x_0 + \tan\lambda * \frac{r}{q} * \arcsin(\cos\phi_0 \sin\phi_1 - \cos\phi_1 \sin\phi_0)$$



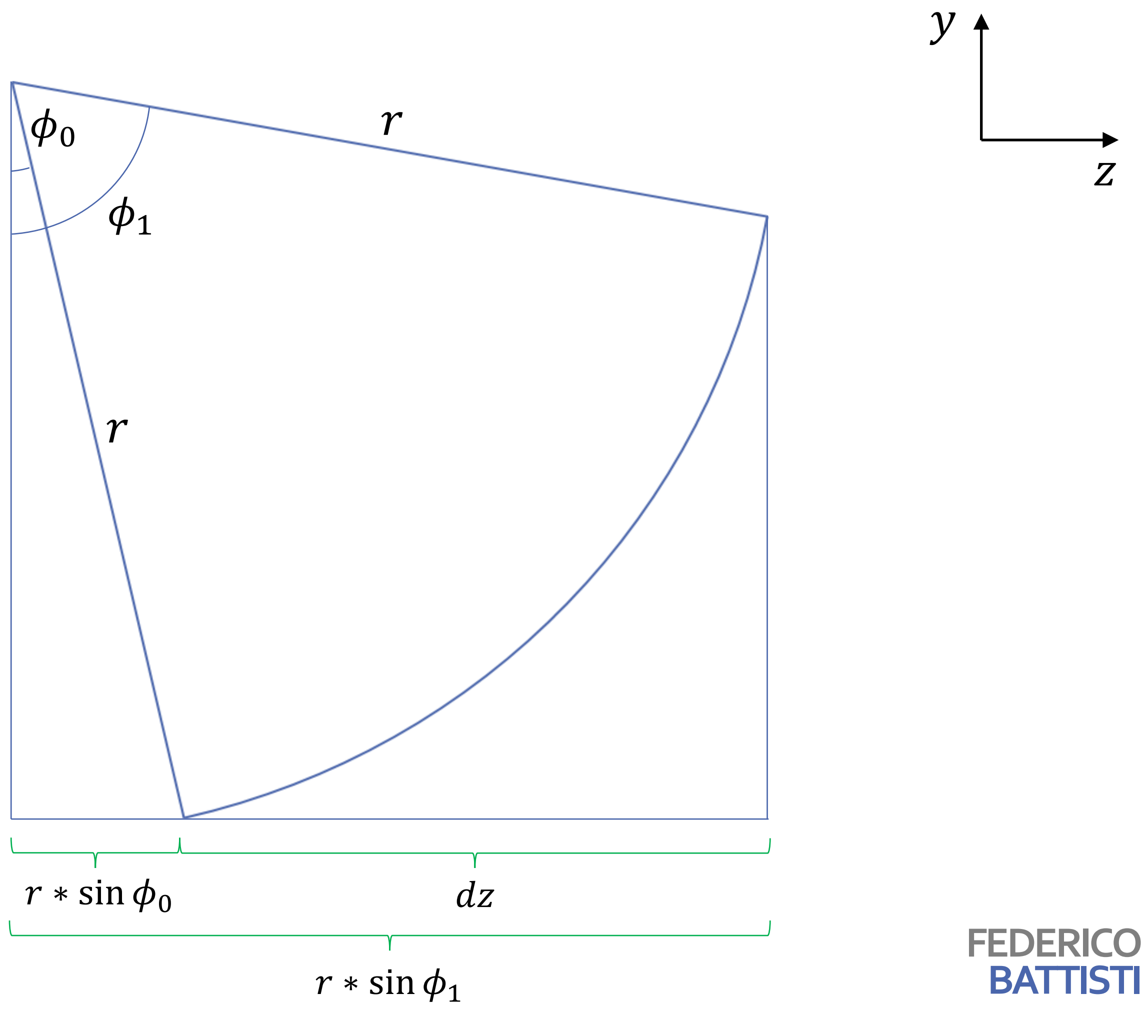
KALMAN FILTER MODEL

- Use parametrization used in ALICE: free parameter z, state vector $(y, x, sin\phi, \tan\lambda, \frac{q}{p_T})$ (ϕ azimuthal angle, λ dip-angle, p_T transverse momentum in yz plane), evolution function:

2 $$dz = r * \sin\phi_1 - r * \sin\phi_0$$

$$\sin\phi_1 = \sin\phi_0 + \frac{dz}{r}$$

3 & 4 are static



HELIX FIT

K-F UPDATE: GLOBAL HELIX FIT AND INITIAL COVARIANCE ESTIMATION

- $c = 1/r$ and $\sin\phi_0$ estimated by finding (z_C, y_C) and r of the yz plane circomference passing through the first, last and middle hit point of the particle trajectory

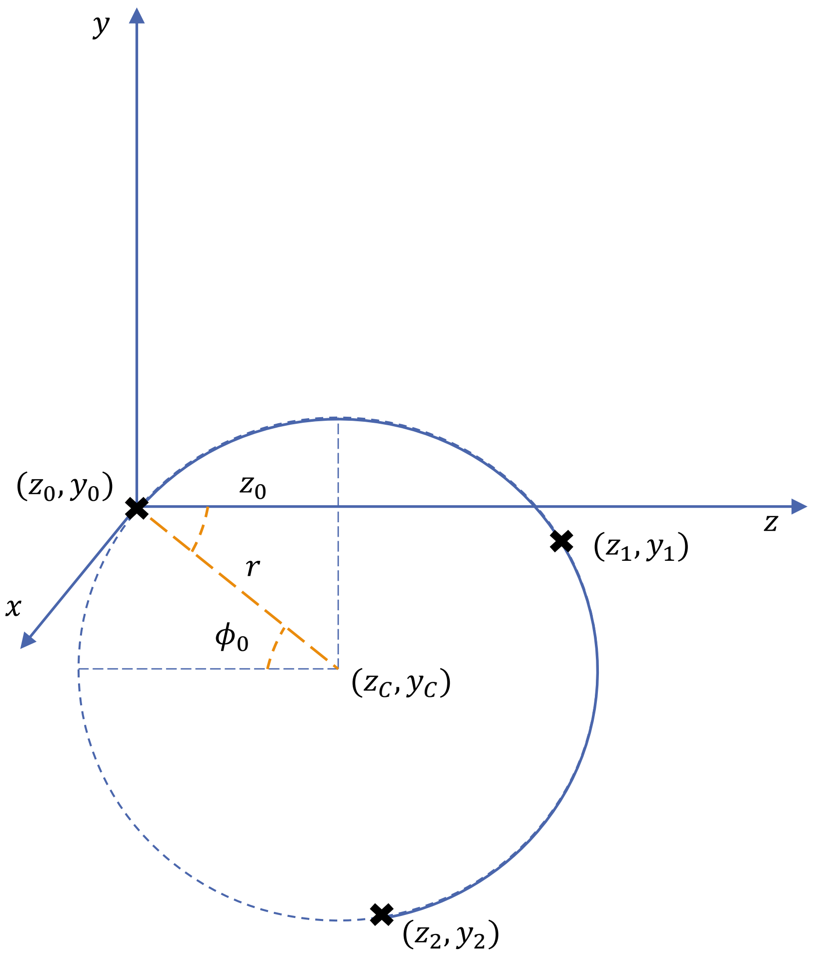
- After traslating the coordinate system to have the origin on the first point $(z_0, y_0) \rightarrow (0{,}0)$ we have the circumference equations:

$$\begin{cases} z_C^2 + y_C^2 = r^2 \\ (z_1 - z_C)^2 + (y_1 - y_C)^2 = r^2 \\ (z_2 - z_C)^2 + (y_2 - y_C)^2 = r^2 \end{cases}$$

$$\begin{cases} z_C = \frac{1}{2}\left(z_2 - y_2 \frac{z_1(z_1 - z_2) + y_1(y_1 - y_2)}{z_2 y_1 - z_1 y_2}\right) \\ y_C = \frac{1}{2}\left(z_2 - y_2 \frac{z_1(z_1 - z_2) + y_1(y_1 - y_2)}{z_2 y_1 - z_1 y_2}\right) \\ r = \sqrt{z_C^2 + y_C^2} \end{cases}$$

$$c = 1/r$$

$$\sin\phi_0 = \frac{z_0}{r}$$



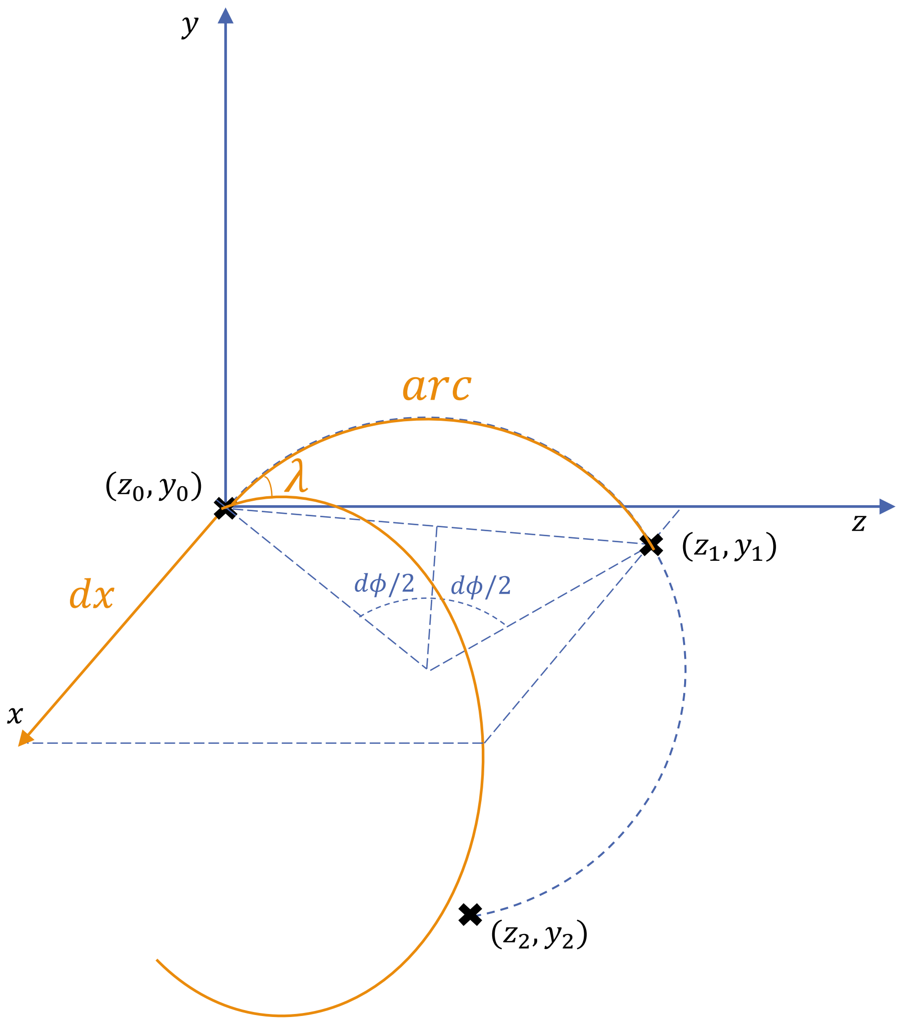
K-F UPDATE: GLOBAL HELIX FIT AND INITIAL COVARIANCE ESTIMATION

- We evaluate $\tan\lambda$ from the yz plane arc between the first two points and the correspondent movement in the x direction (magnetic field direction) using r estimate from previous step:

$$d\phi = 2\arcsin\left(\frac{chord}{2r}\right)$$

$$= 2\arcsin\left(\frac{\sqrt{(y_1 - y_0)^2 + (z_1 - z_0)^2}}{2r}\right)$$

$$\tan\lambda = \frac{dx}{arc} = \frac{dx}{d\phi * r}$$



K-F UPDATE: GLOBAL HELIX FIT AND INITIAL COVARIANCE ESTIMATION

- Given parameter estimation from global helix fit, estimate uncertainties through error propagation
- Uncertainties associated with x and y: σ_{xy} ; z free parameter with no uncertainty $\sigma_z = 0$ (as in the Kalman filter)
- Formula for $\sin\phi_0$ estimation is function of $f(z_0, y_0, z_1, y_1, z_2, y_2)$ but since $\sigma_z = 0$, consider only $f(y_0, y_1, y_2) \rightarrow$ From error propagation we get:

$$\sigma_{\sin\phi_0} = \sqrt{\left(\frac{\partial f(y_0, y_1, y_2)}{\partial y_0}\right)^2 \sigma_{xy}^2 + \left(\frac{\partial f(y_0, y_1, y_2)}{\partial y_2}\right)^2 \sigma_{xy}^2 + \left(\frac{\partial f(y_0, y_1, y_2)}{\partial y_3}\right)^2 \sigma_{xy}^2}$$

- This can be approximated as:

$$\sigma_{\sin\phi_0} = \sqrt{\left(\frac{f(y_0 + \sigma_{xy}, y_1, y_2)}{\sigma_{xy}}\right)^2 \sigma_{xy}^2 + \left(\frac{f(y_0, y_1 + \sigma_{xy}, y_2)}{\sigma_{xy}}\right)^2 \sigma_{xy}^2 + \left(\frac{f(y_0, y_1, y_2 + \sigma_{xy})}{\sigma_{xy}}\right)^2 \sigma_{xy}^2}$$

K-F UPDATE: GLOBAL HELIX FIT AND INITIAL COVARIANCE ESTIMATION

- Repeat the process with other parameters to get respective uncertainties
- Estimate for covariance matrix P_0 is diagonal matrix with:

$$P_0 = \begin{pmatrix} \sigma_{xy}^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{xy}^2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{sin\phi}^2 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{tan\lambda}^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{q/p_T}^2 \end{pmatrix}$$

- Note: off-diagonal elements could also be calculated, but are not at the moment