Nested Head Tail Vlasov Solver:

Impedance, Damper, Radial Modes, Coupled Bunches, Beam-Beam and more…

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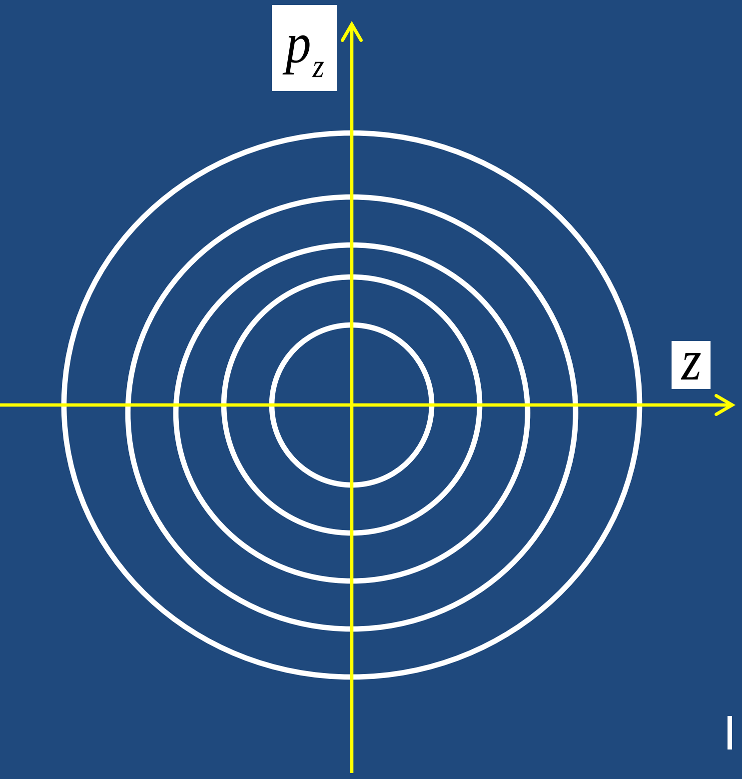
Fermilab-LARP

special thanks - to V.Danilov, E.Metral, N.Mounet, S.White, X.Buffat, and T.Pieloni

LARP CM20/HiLumi, April 2013, Napa Valley, CA

Nested Head-Tail Basis



$$\psi_{l\alpha} \propto \exp(il\phi + i\chi_\alpha \cos\phi - i\omega_b t)\,;$$

$$\chi_\alpha = \frac{Q'\omega_0 r_\alpha}{c\eta}\;;$$

I am using n_r equally populated rings which radii r_α are chosen to reflect the phase space density.

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Starting Equation, single bunch

- In the air-bag single bunch approximation, beam equations of motion can be presented as in Ref [A. Chao, Eq. 6.183]:

$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X$$

where X is a vector of the HT mode amplitudes,

$$(\hat{S} + \hat{Z})_{lm\alpha\beta} = -il\delta_{lm}\delta_{\alpha\beta} - i^{l-m} \frac{\kappa}{n_r} \int_{-\infty}^{\infty} d\omega Z(\omega) J_l(\omega\tau_\alpha - \chi_\alpha) J_m(\omega\tau_\beta - \chi_\beta)$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta)$$

d is the damper gain in units of the damping rate, $\kappa = \dfrac{N_b r_0 R_0}{8\pi^2 \gamma Q_b Q_s}$

time is in units of the angular synchrotron frequency.

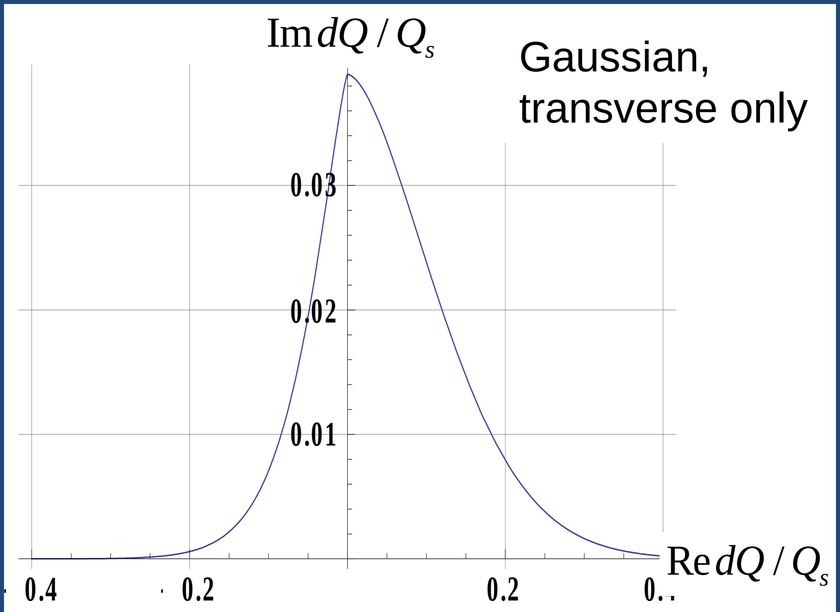
Analysis of solutions

1. For every given gain and chromaticity, the eigensystem is found for the provided impedance tables or functions.

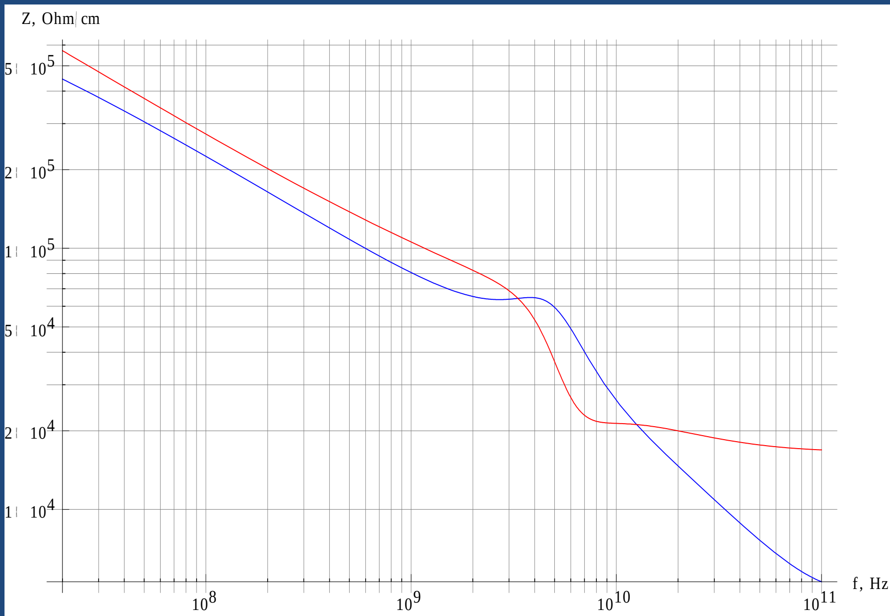
2. The complex tune shifts are found from the eigenvalues $\Delta\Omega_{l\alpha} = \Omega_{l\alpha} - l$

3. The stabilizing octupole current is found from the stability diagram for every mode, then max is taken.

Stability diagram at +200 A of octupoles



LHC Horizontal Impedances (N. Mounet)



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Coupled Equidistant Bunches

Main idea:

For LHC, wake field of preceding bunches can be taken as flat within the bunch length.

The only difference between the bunches is CB mode phase advance, otherwise they are all identical.

Thus, the CB kick felt by any bunch is proportional to its own offset, so the CB matrix $\hat{C}$ has the same structure as the damper matrix $\hat{D}$:

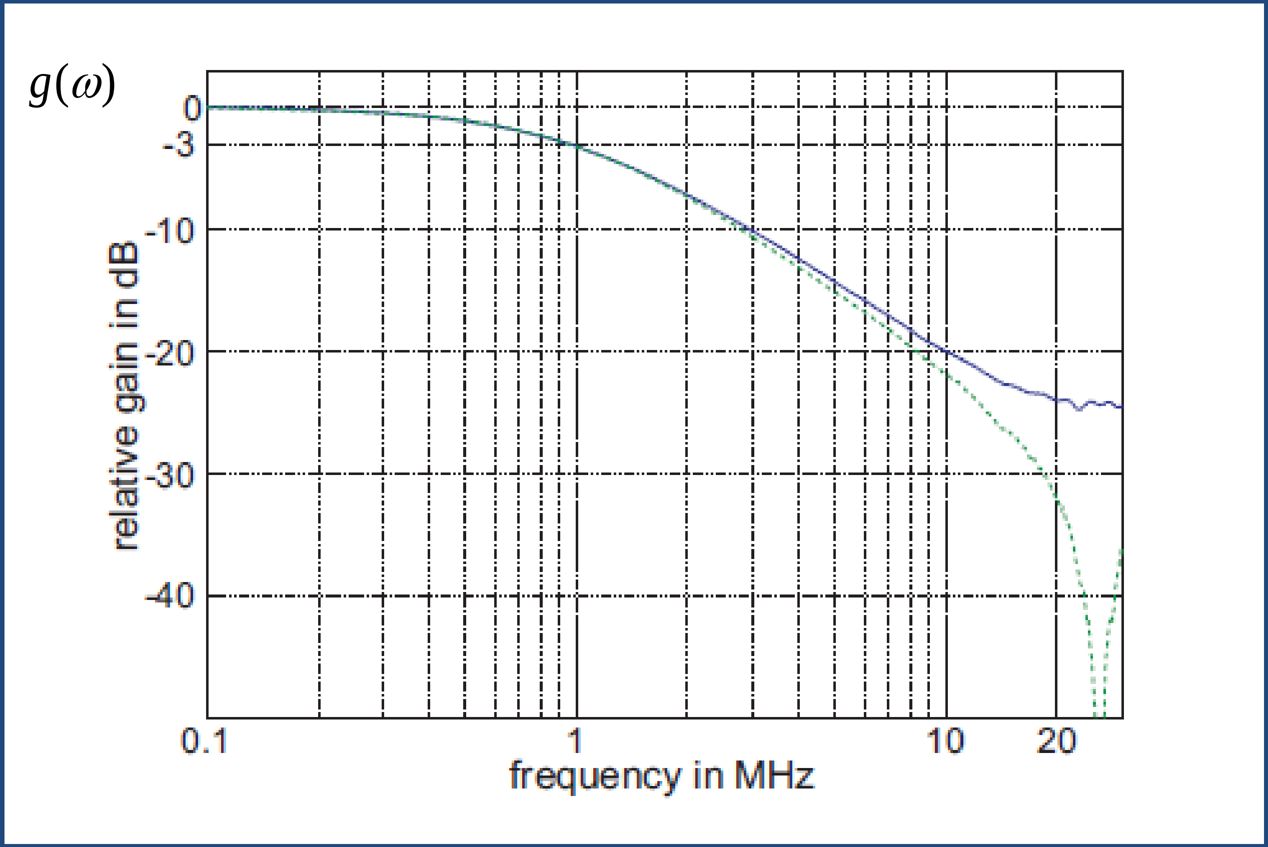
$$\dot{X} = \hat{S} \cdot X + \hat{Z} \cdot X + \hat{D} \cdot X + \hat{C} \cdot X ;$$

$$\hat{D}_{lm\alpha\beta} = -i^{m-l} \frac{d_\mu}{n_r} J_l(\chi_\alpha) J_m(\chi_\beta); \qquad \hat{C} = 2\pi i \kappa W(\varphi_\mu) \hat{D} / d_\mu;$$

$$W(\varphi_\mu) = \sum_{k=1}^{\infty} W(-ks_0) \exp(-ik\varphi_\mu); \quad \varphi_\mu = \frac{2\pi(\mu + \{Q_x\})}{M_b}; \quad 0 \le \mu \le M_b - 1 .$$

Wake and impedance are determined according to A. Chao book.

Old damper gain



Old narrow-band ADT gain profile (W. Hofle, D. Valuch) .
At 10 MHz it drops 10 times. The new damper is bbb for 50ns beam.

Below gain is measured in omega_s units, max gain=1.4 is equivalent to 50 turns of the damping time.

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CB Mode Damping Rate

With $g(\omega)$ as the frequency response function of the previous plot, the time-domain damper's "wake" is

$$G(\tau) = \int_0^\infty g(\omega)\cos\left(\omega\tau\right)d\omega / \pi ,$$

assuming this response to be even function of time (no causality for the damper!).

From here (equidistant bunches!):

$$d_\mu = d\,\frac{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)\cos(k\varphi_\mu)}{G(0) + 2\sum_{k=1}^{\infty} G(k\tau_0)};$$

where d is the rate provided for low-frequency CB zero-head-tail modes at zero chromaticity.

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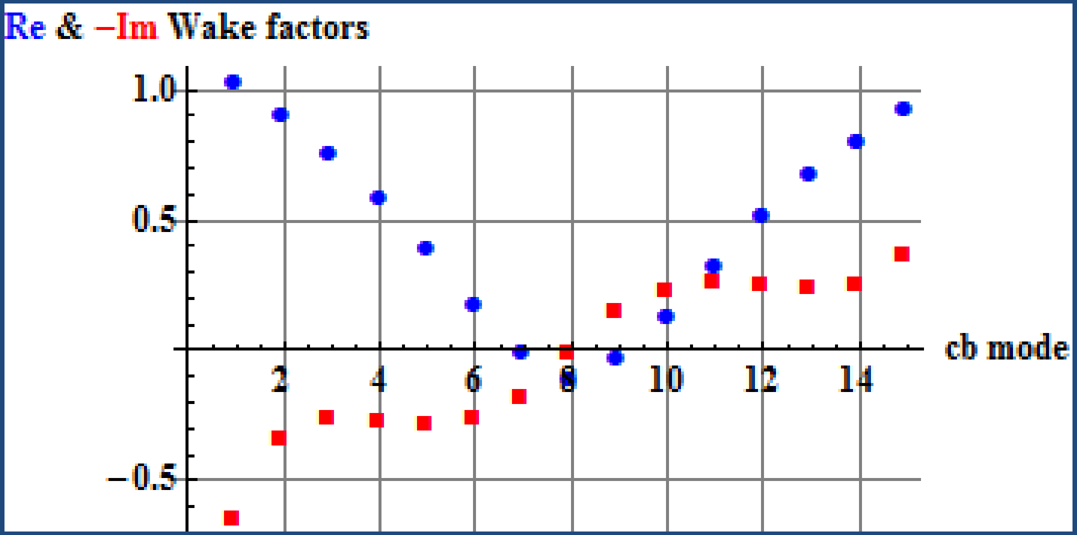
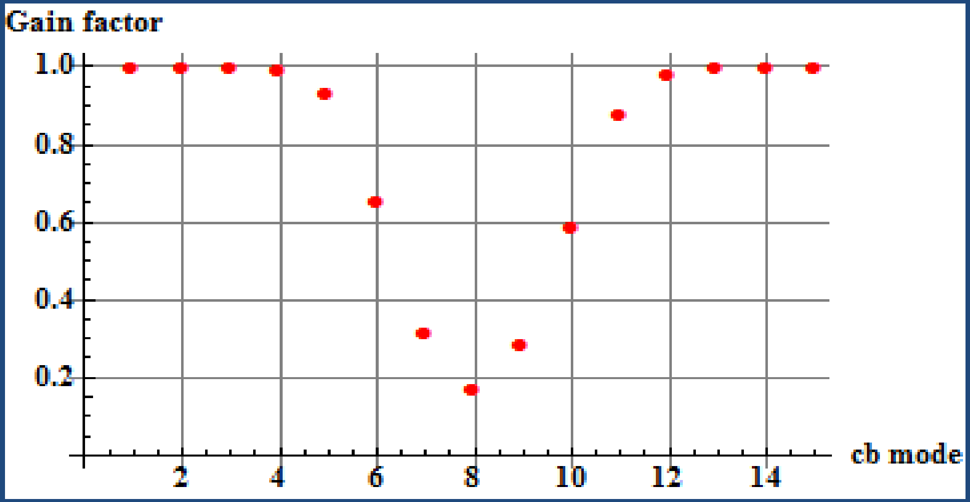
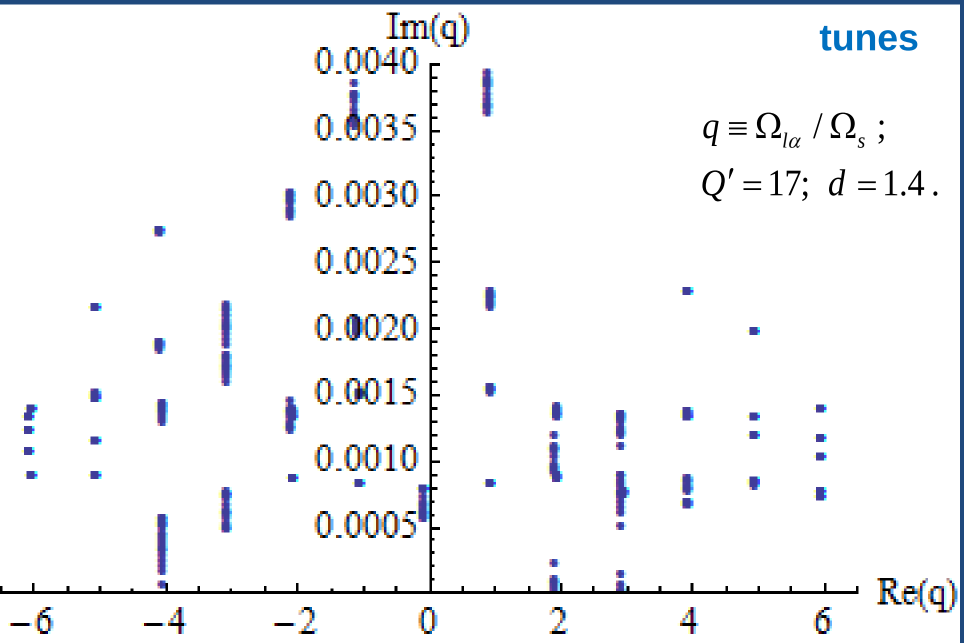
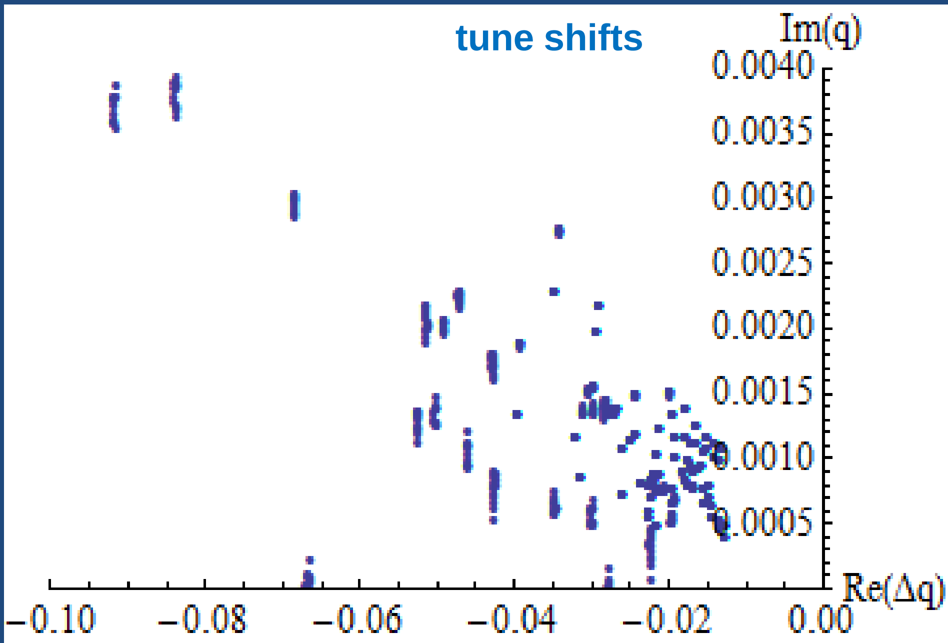
CB Wake and Gain Factors for the Old ADT

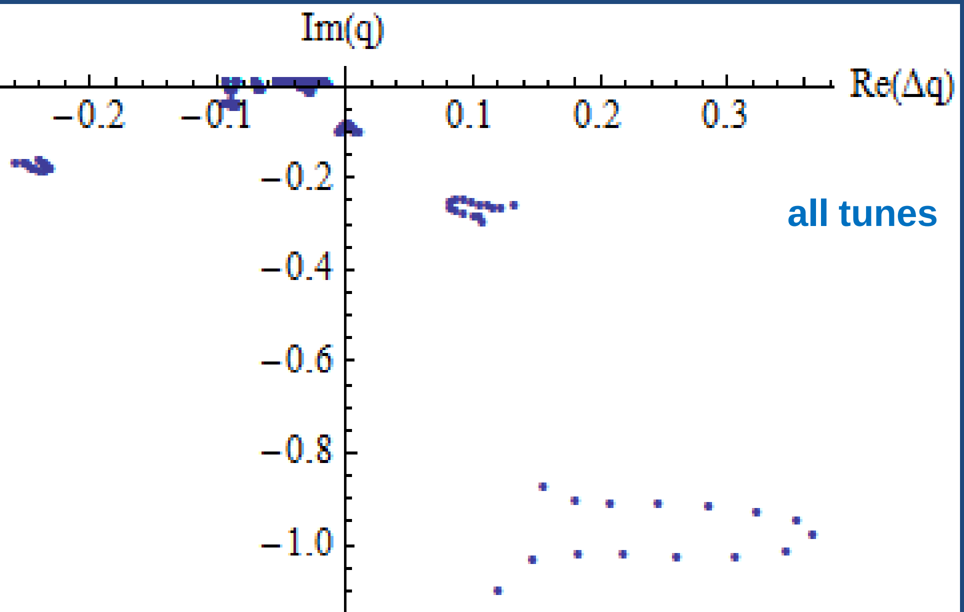




2⊗(SB and CB), flat ADT, Tunes at the Plateau







- For unstables $-0.1 < \mathrm{Re}(\Delta q) < 0$.

$$\frac{\mathrm{Im}(q)}{|\,\mathrm{Re}(\Delta q)\,|} \simeq 20 - 50$$

- Weak head-tail is justified at the plateau.
- Most unstable mode (MUM) has ~max tune shift as well.

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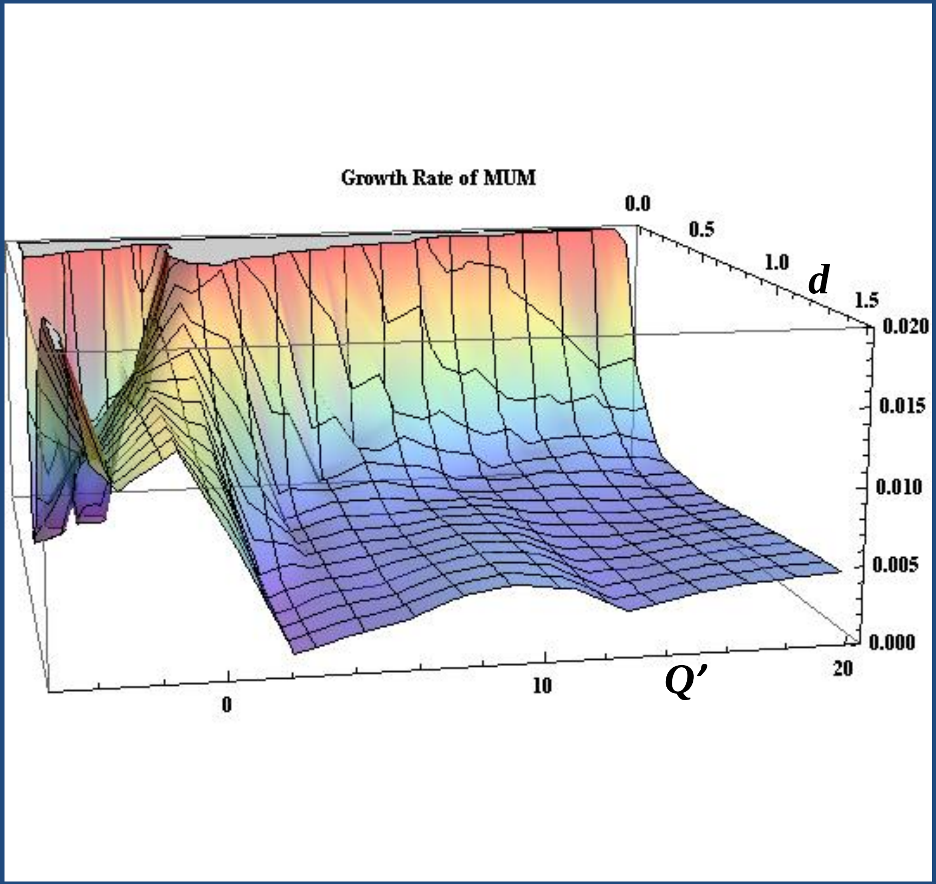
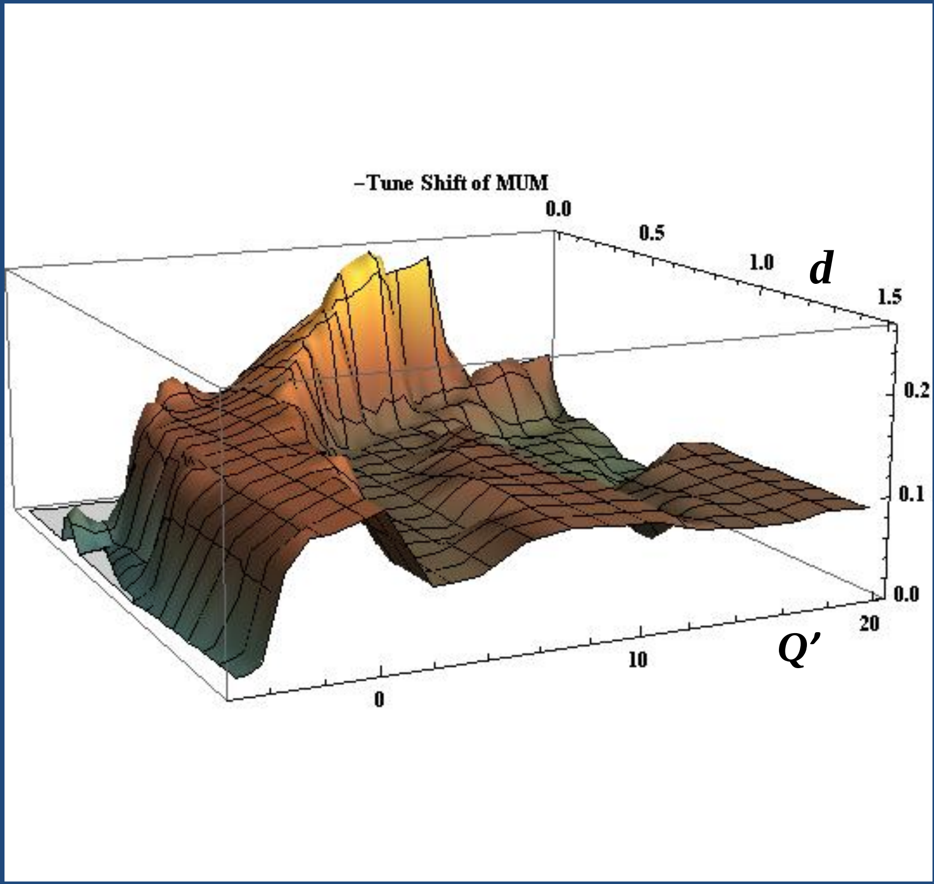
2⊗(SB and CB), flat ADT, MUM

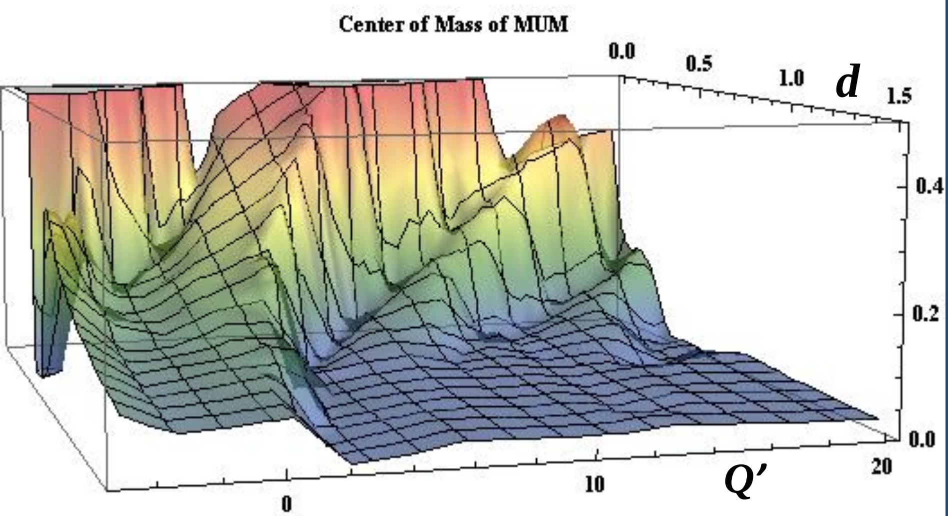




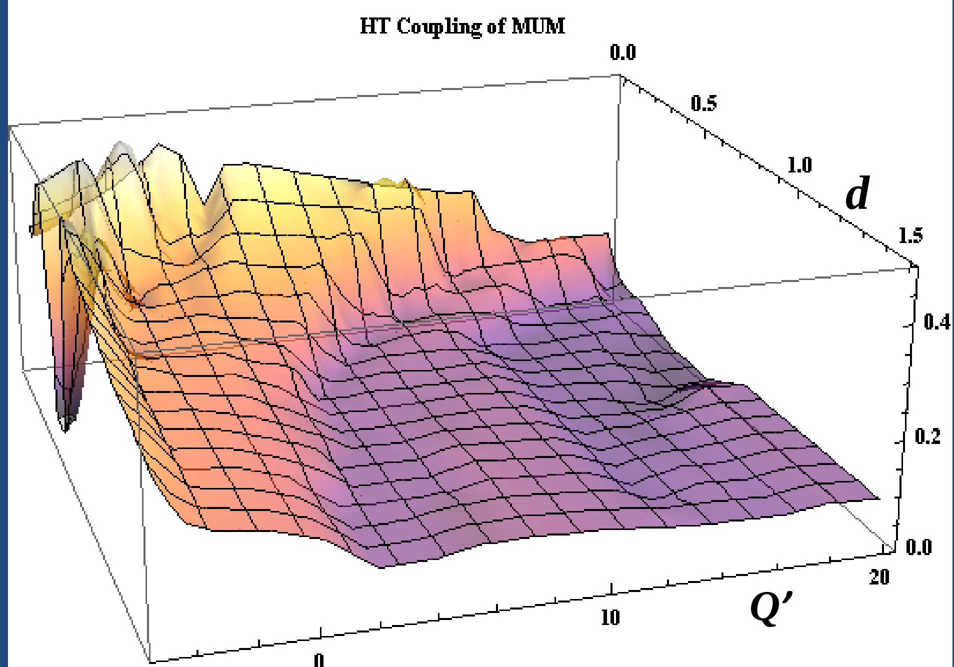
Growth rate and -tune shift of the most unstable mode (MUM) vs chroma and gain.
Both are in units of Qs.

Note that at the plateau the rate (Im[dQ_c]) is ~20-30 times smaller than the shift (Re[dQ_c]).

2⊗(SB and CB), flat ADT, MUM CM and Coupling



$$A_{l\alpha} = i^l J_l(\chi_\alpha)/\sqrt{n_r}; \quad \bar{x} = X \cdot A.$$



$$|X_l|^2 \equiv \sum_{\alpha=1}^{n_r} |X_{l\alpha}|^2; \quad \sum_l |X_l|^2 = 1;$$

$$l_m : |X_{l_m}|^2 = \max_l |X_l|^2; \quad \mathrm{HTC} = \sqrt{1 - |X_{lm}|^2}$$

Center of mass (CM) and head-tail coupling parameters for MUM.

Note strong suppression of CM at the plateau by the damper.
Note that at plateau the weak head-tail approximation is well-justified.

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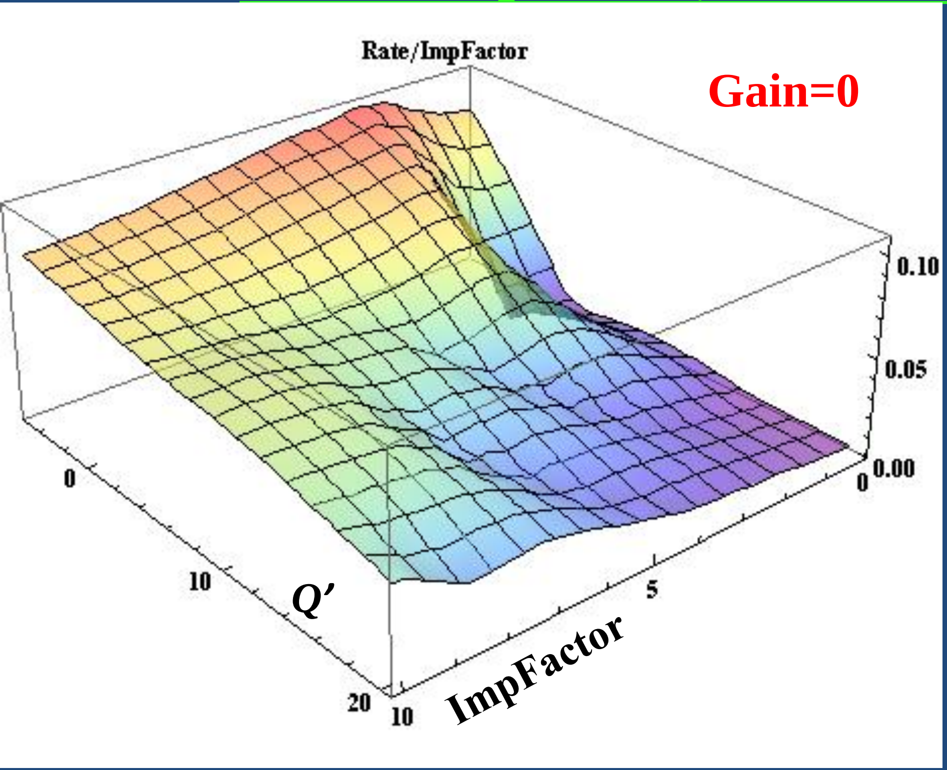
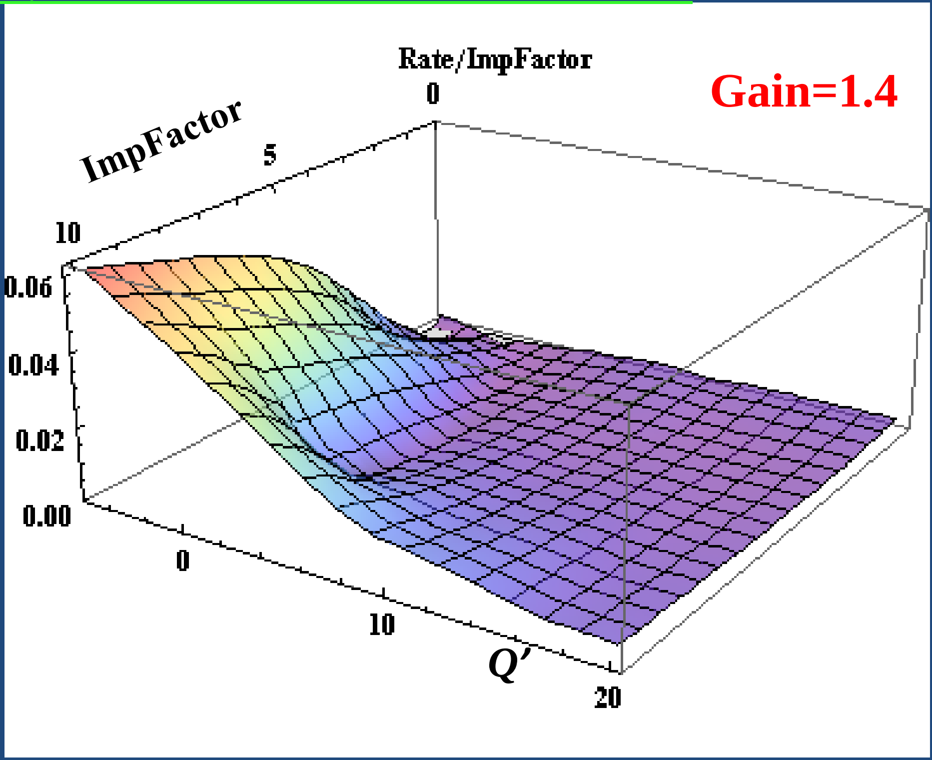
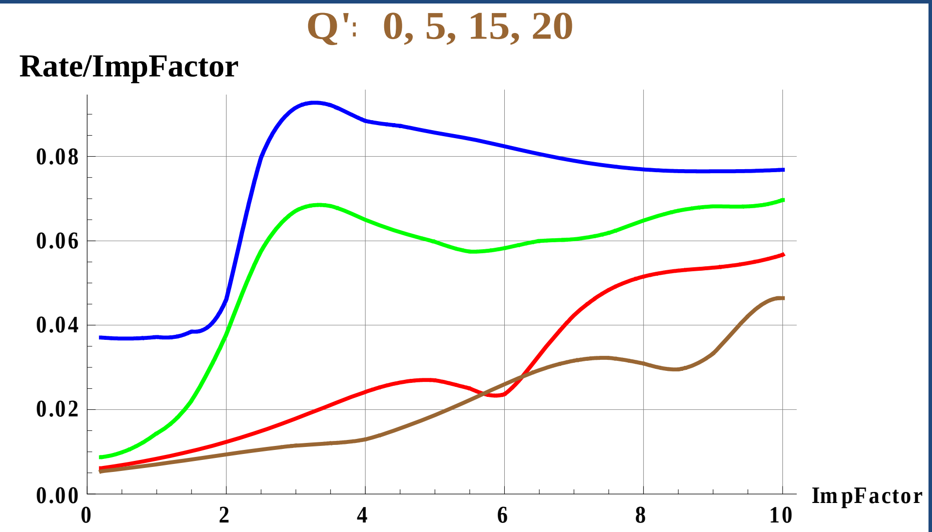
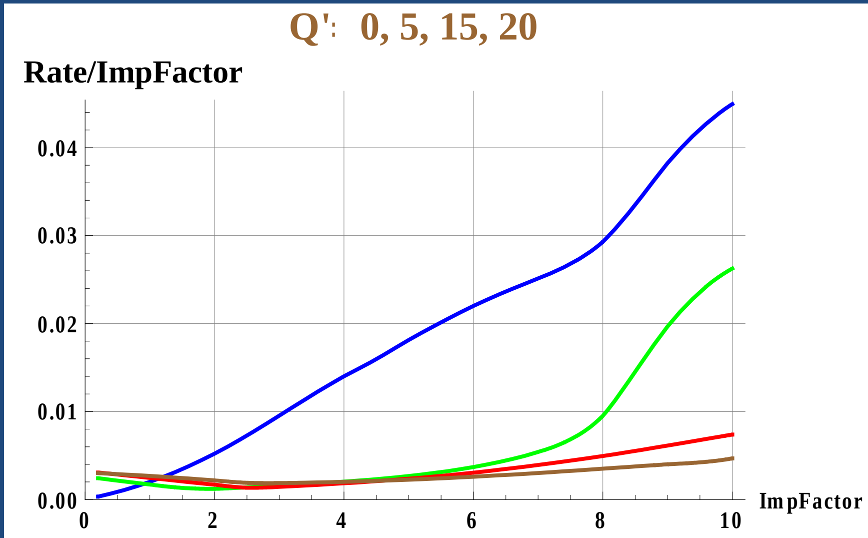
Intensity scan, flat ADT, MUM: where is TMCI?









Coherent Beam-Beam

Main assumption: bunch length << beta-function. For transversely dipolar modes, CBB is a cross-talk of bunch CM – thus, intra-bunch matrix structure is similar to the ADT and CB:

$$\dot{X}_1 = \hat{S}\cdot X_1 + \hat{Z}\cdot X_1 + \hat{D}\cdot X_1 + \hat{C}\cdot X_1 + b_{12}\hat{B}\cdot X_2;$$

$$\dot{X}_2 = \hat{S}\cdot X_2 + \hat{Z}\cdot X_2 + \hat{D}\cdot X_2 + \hat{C}\cdot X_2 + b_{21}\hat{B}\cdot X_1;$$

$$\hat{B} = -i\Delta\omega_{\mathrm{bb}}(\hat{D}/d_\mu)\sum_{k=-K}^{K}\frac{\beta_k}{\rho_k^2}\cos(k\phi_\mu)\Big/\sum_{k=-K}^{K}\frac{\beta_k}{\rho_k^2};$$

$$b_{12} = b_{21}^* = 1-\exp(-i\psi).$$

Here 2 identical opposite IRs are assumed (IR1 and IR5 for LHC) with $2K+1$ LR collisions for each, every one with its beta-function and separation β_k, ρ_k .

Alternating x/y collision for IR1/IR5 is assumed with ψ as a difference between the two phase advances, while $\Delta\omega_{\mathrm{bb}}$ is the incoherent beam-beam tune shift per IR.

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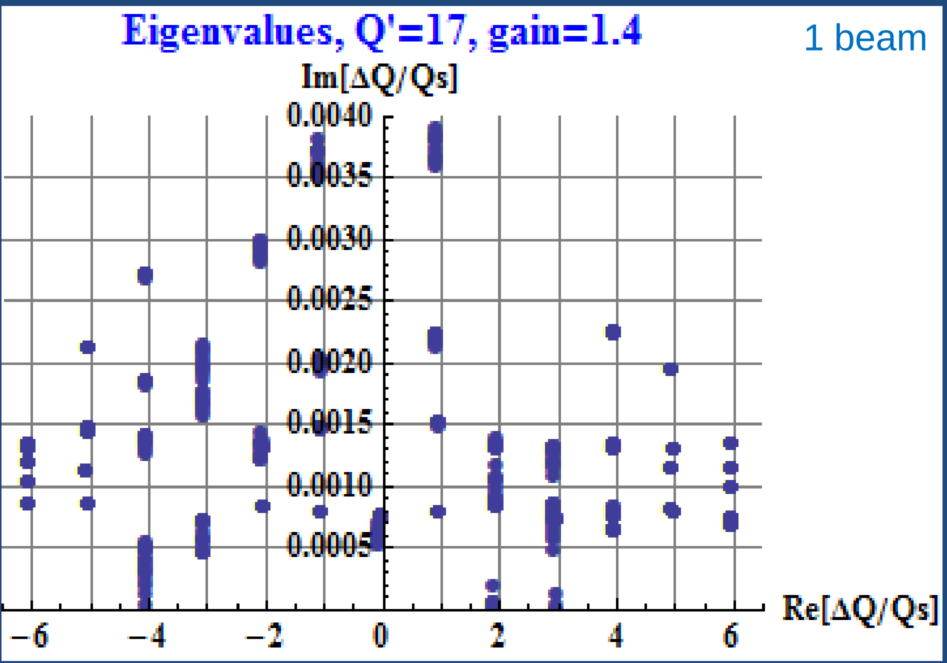
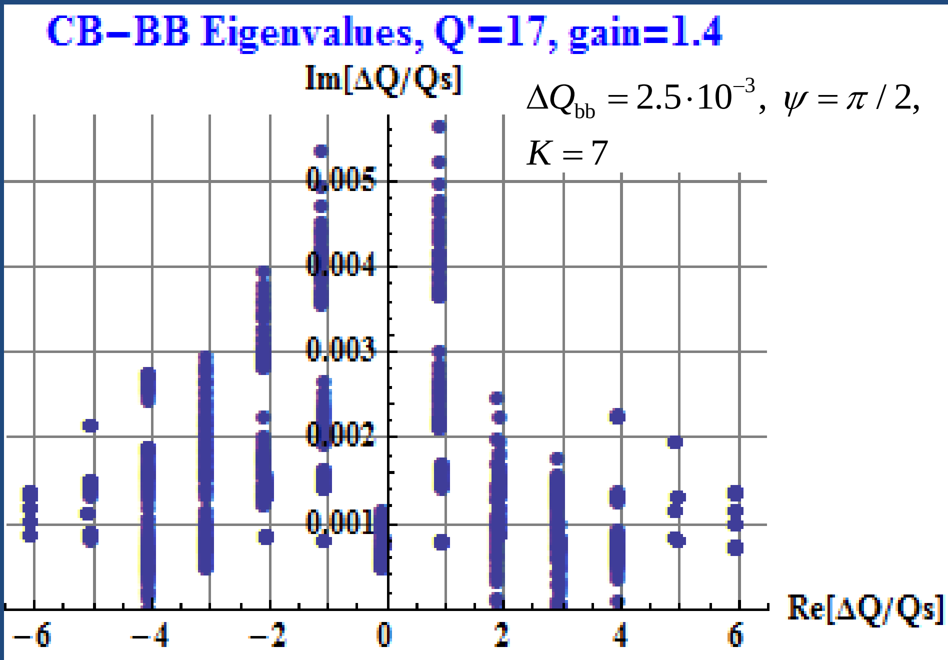
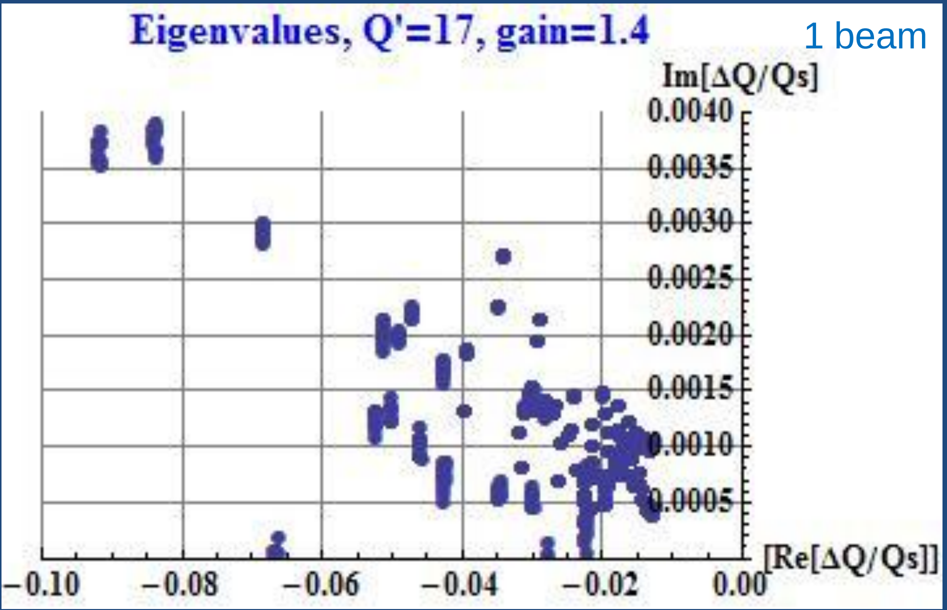
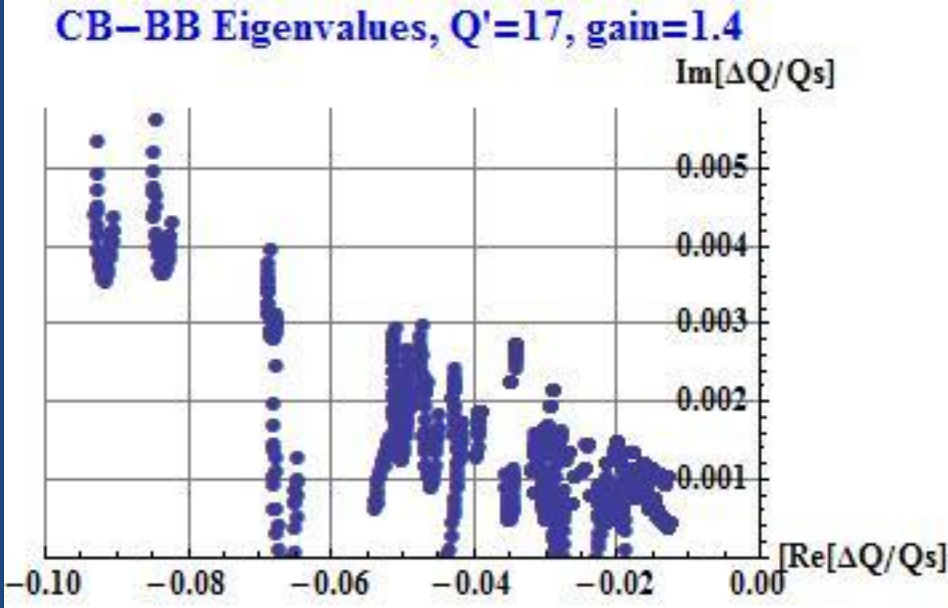
Coherent BB at Plateau: effect ~30%









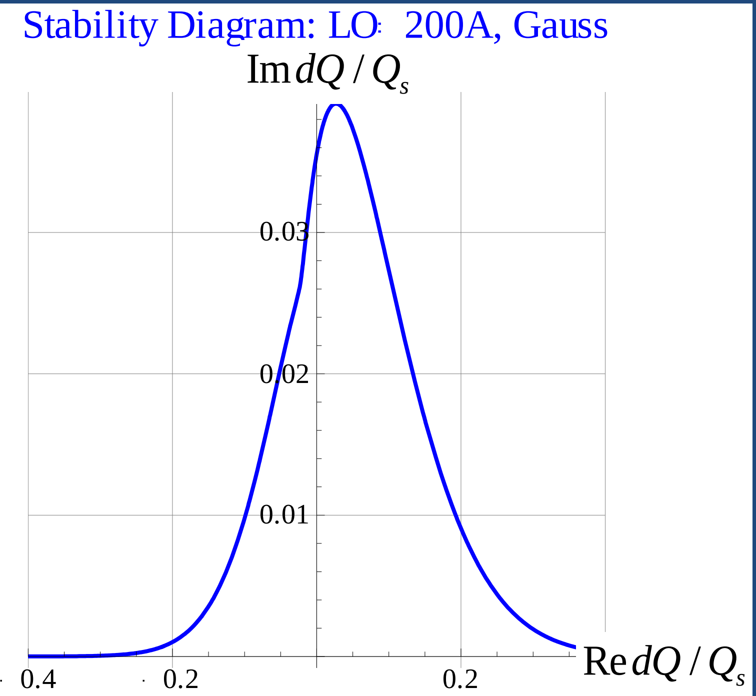
Stability Diagram

Stability diagram (SD) is defined as a map of real axes Ω on the complex plane:

$$\mathrm{D} = \left(-\int \frac{J_x \partial F / \partial J_x}{\Omega - l\omega_s - \delta\omega_x + io} d\Gamma \right)^{-1}$$

$$\mathrm{D} = \Omega_c - l\bar{\omega}_s$$

To be stable, the coherent tune shift has to be inside the SD.



For LHC, with Landau octupoles (LO):

(E.Metral, N.Mounet, B.Salvant, 2010)

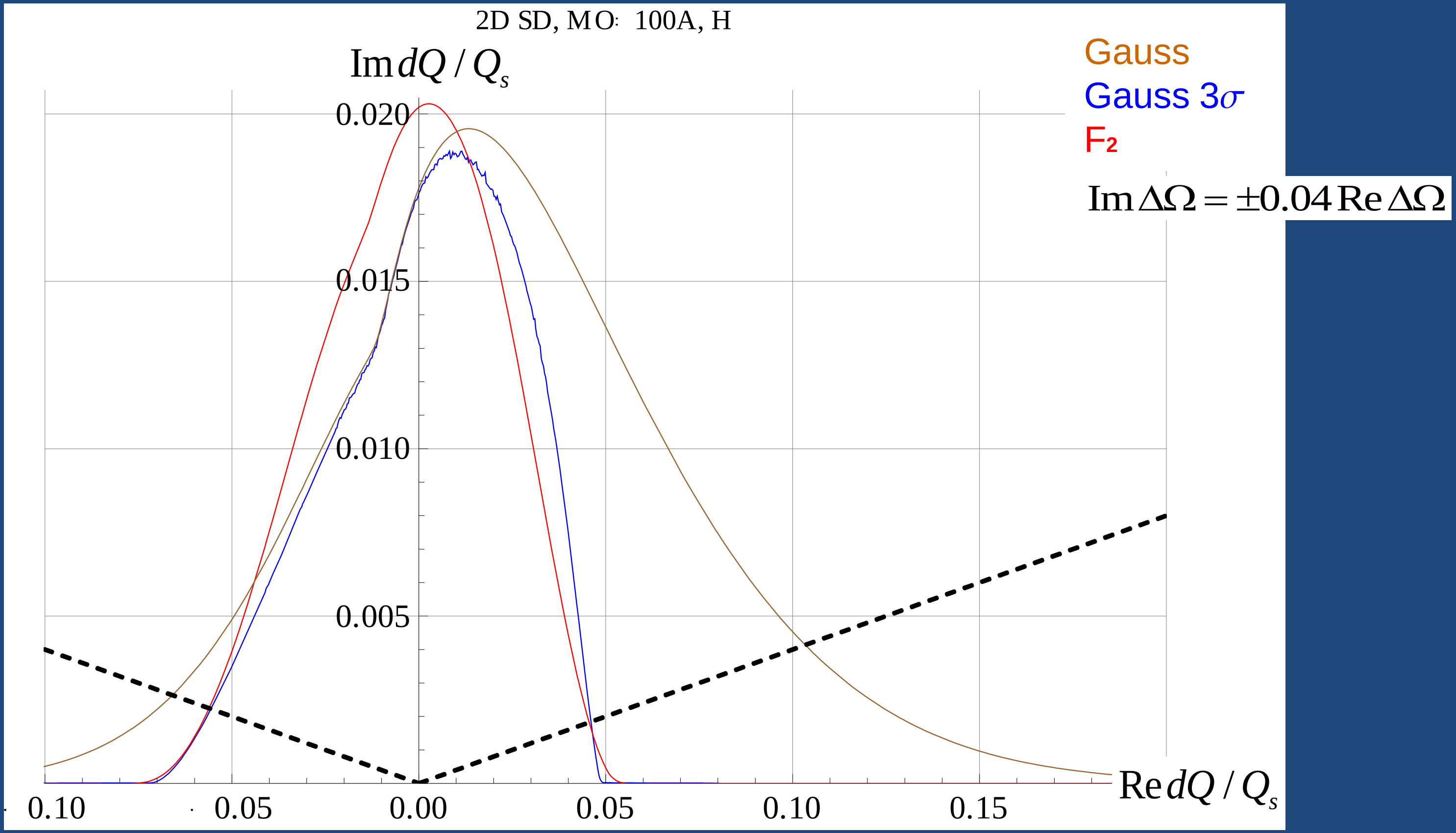
$$\Delta\mathbf{Q} \equiv \left(\Delta Q_x, \Delta Q_y\right)^{\mathrm{T}}; \qquad \mathbf{J} \equiv (J_x, J_y)^{\mathrm{T}};$$

$$\Delta\mathbf{Q} = \hat{\mathbf{A}} \cdot \mathbf{J} / \varepsilon; \qquad \hat{\mathbf{A}} = Q_s \frac{I_{\mathrm{LO}}}{100\mathrm{A}} \begin{pmatrix} a_{xx} & a_{xy} \\ a_{yx} & a_{yy} \end{pmatrix};$$

$$a_{xx} = a_{yy} = 1.8 \cdot 10^{-2} \varepsilon / (2\mu\mathrm{m});$$

$$a_{xy} = a_{yx} = -1.3 \cdot 10^{-2} \varepsilon / (2\mu\mathrm{m});$$

Stability Diagrams



LHC stability diagrams for both emittances $2\mu\mathrm{m}$ and 100A of the octupole current.

$$F_n(J_x, J_y) = a_n \left(1 - \frac{J_x + J_y}{b_n}\right)^n; \ \int F_n(J_x, J_y)\, dJ_x dJ_y = 1; \ \int J_x F_n(J_x, J_y)\, dJ_x dJ_y = 1.$$

see more on SD with F_n at E. Metral & A. Verdier, 2004

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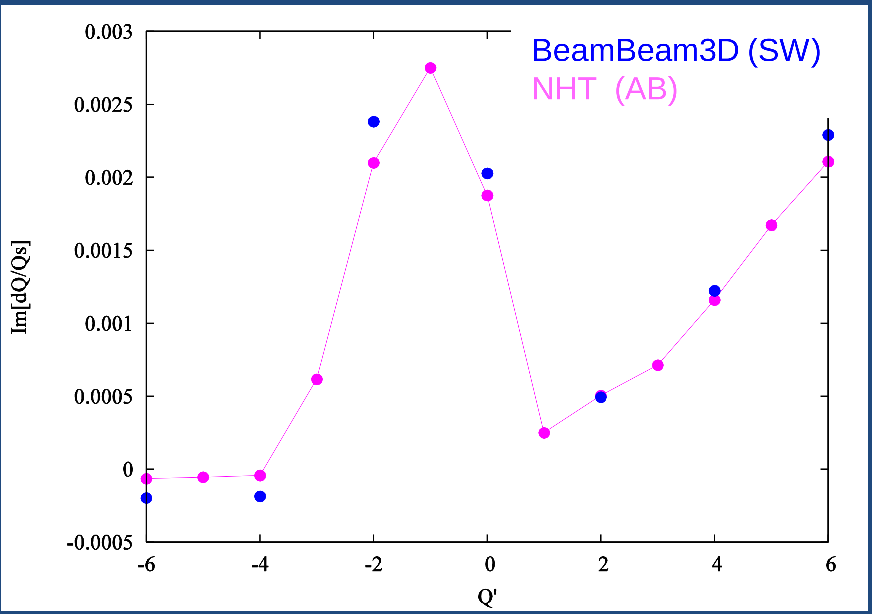
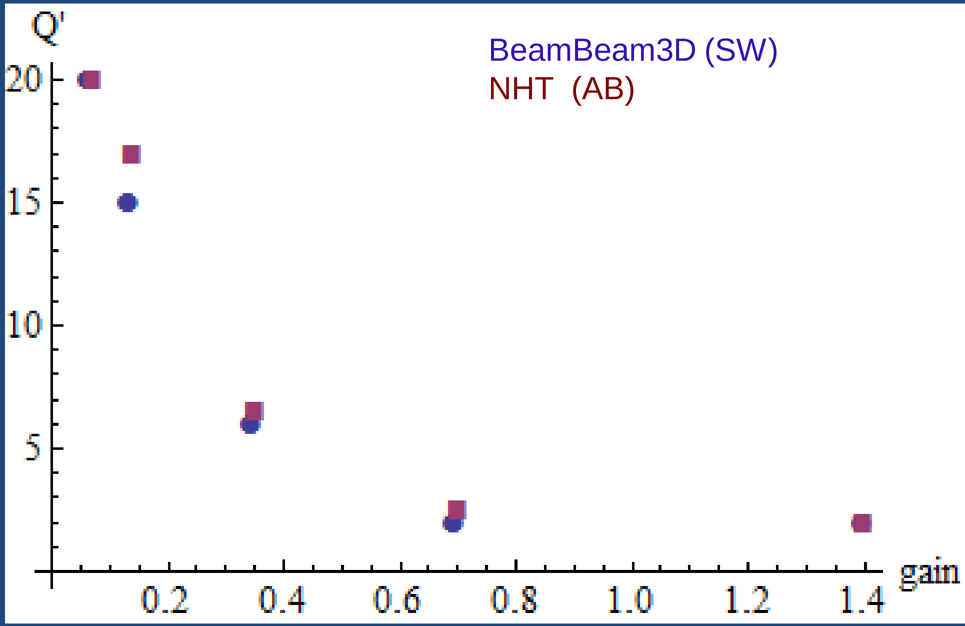
Benchmarking: NHT vs BeamBeam3D (S.White)





Highest growth rates for

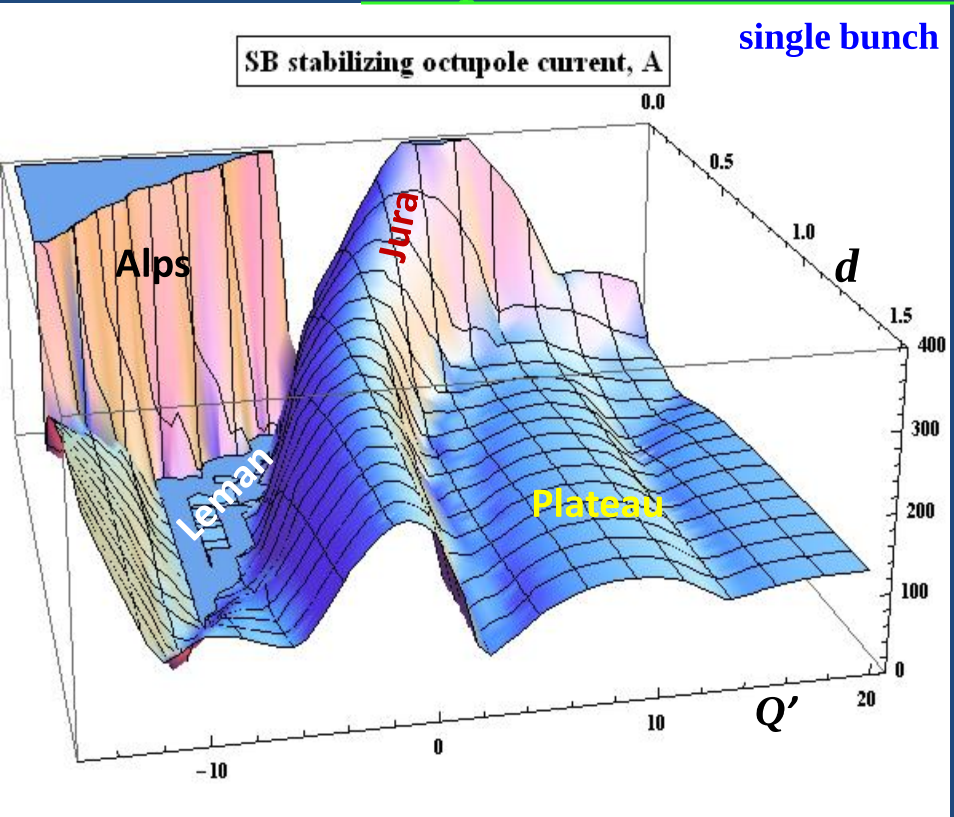
single beam, single bunch,
maximal gain and nominal impedance

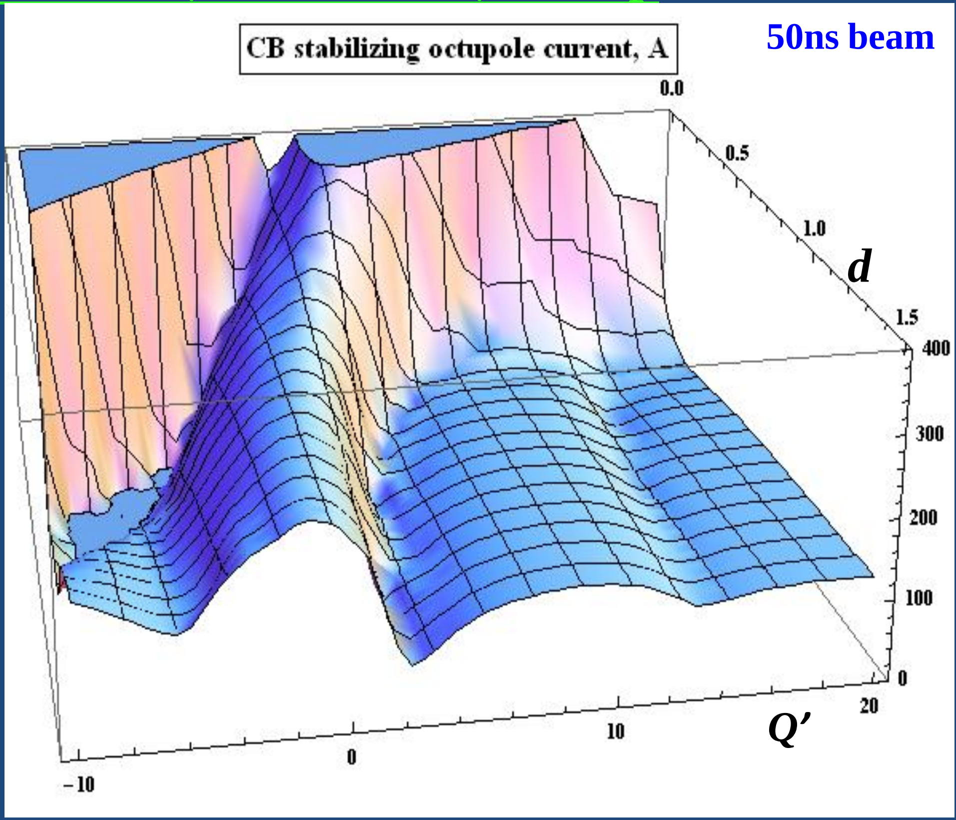
Threshold chromaticity vs gain for

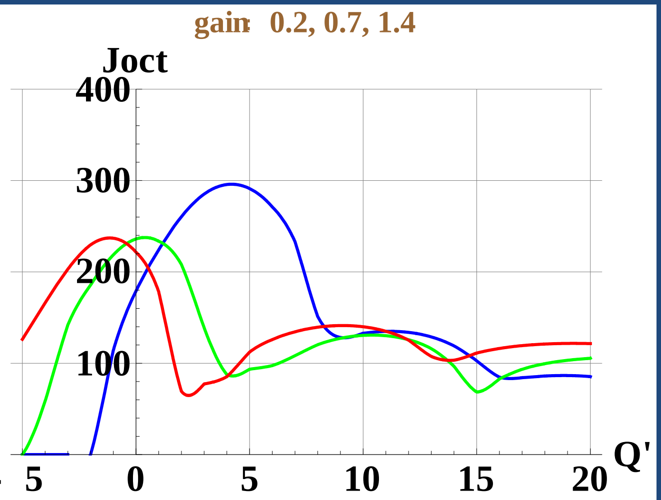
two single-bunch LR-colliding beams,
end of the squeeze parameters,
no octupoles.

BeamBeam3D data – ICE mtg, 07/11/2012.

Couple Bunch Factor: LO+, bbb ADT, 2Imp

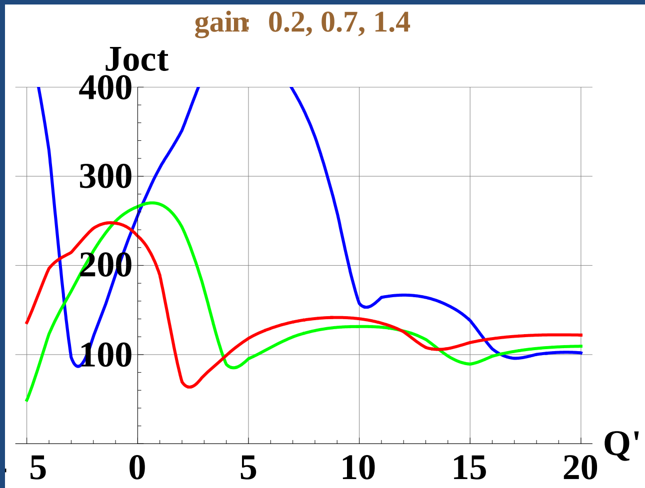




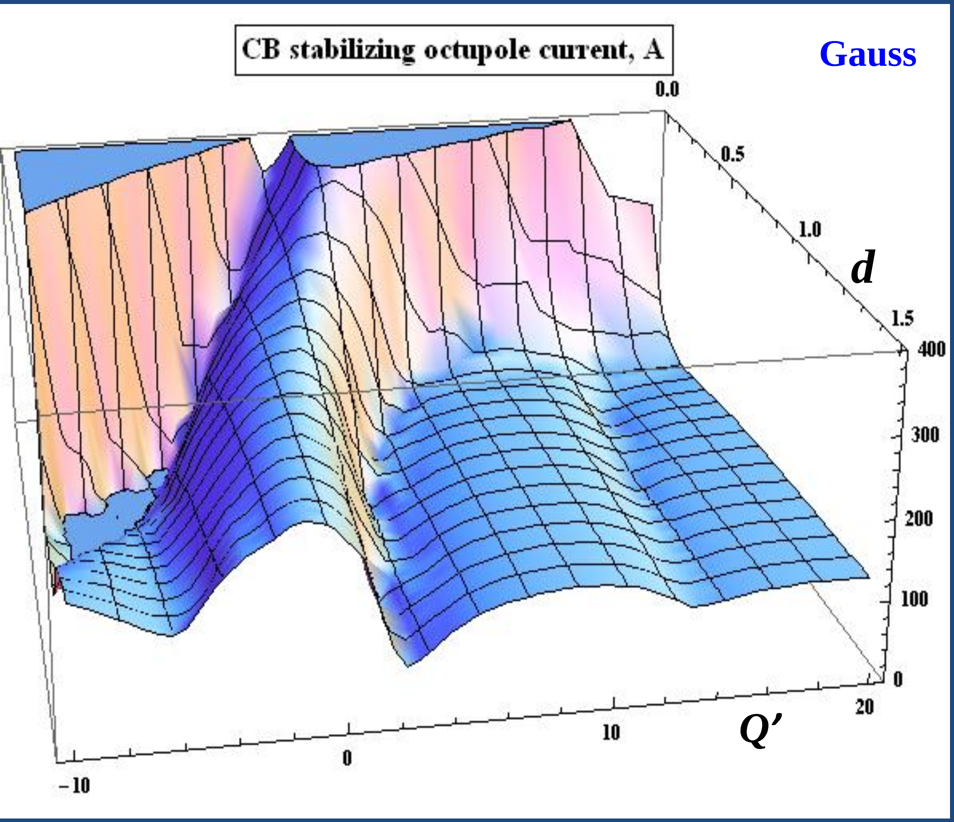


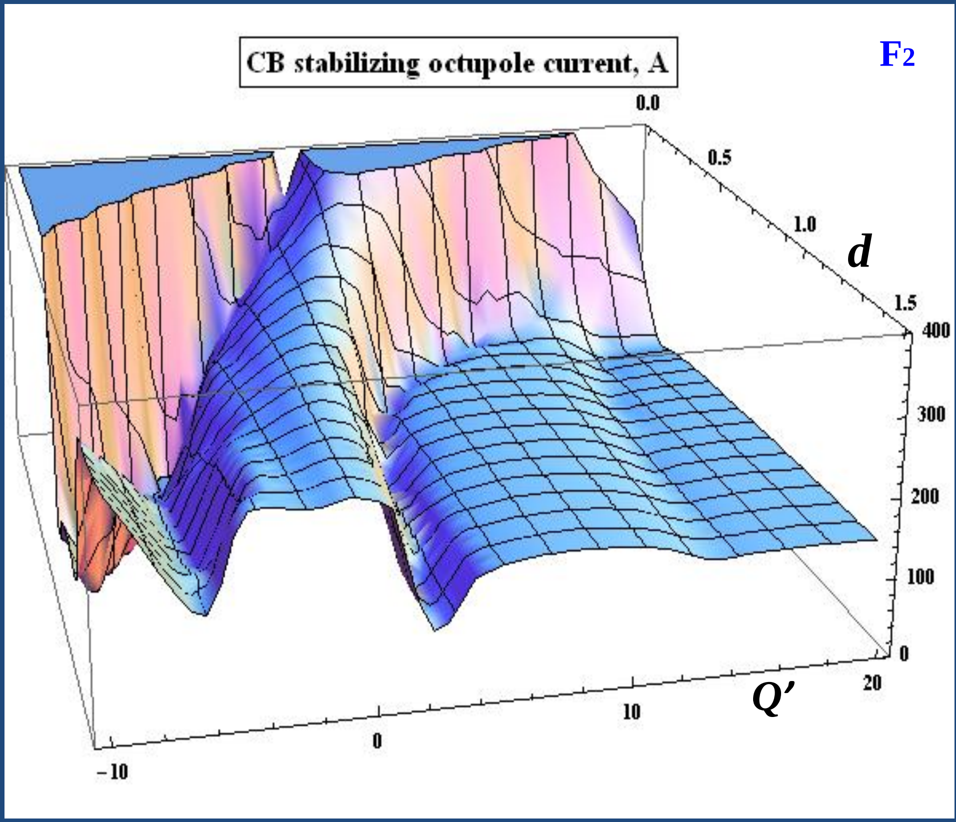
Almost no difference at Plateau.

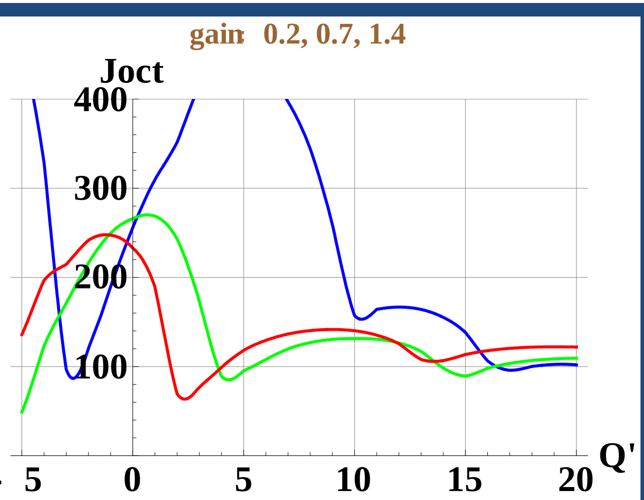
All the plots – for 1.5E11 p/b, emittances of 2μm, 50ns beam.



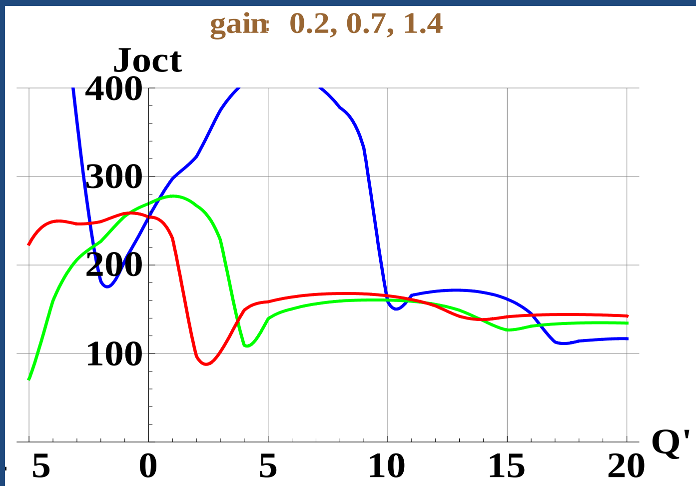
Tails Factor: LO+, CB, bbb ADT, 2Imp



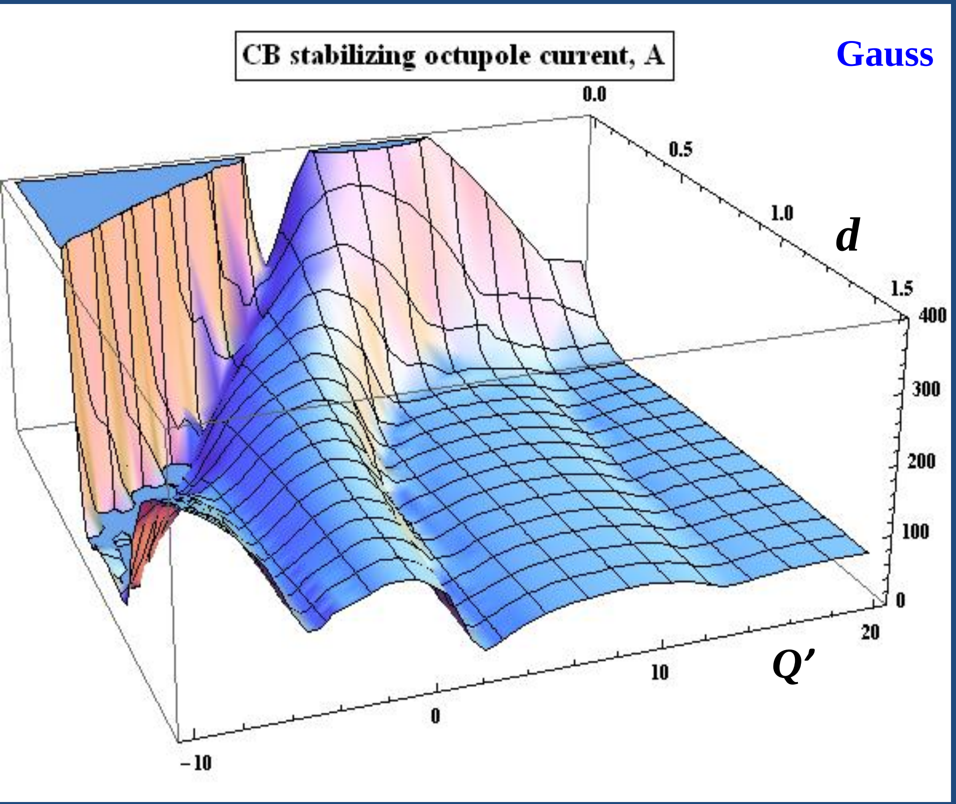


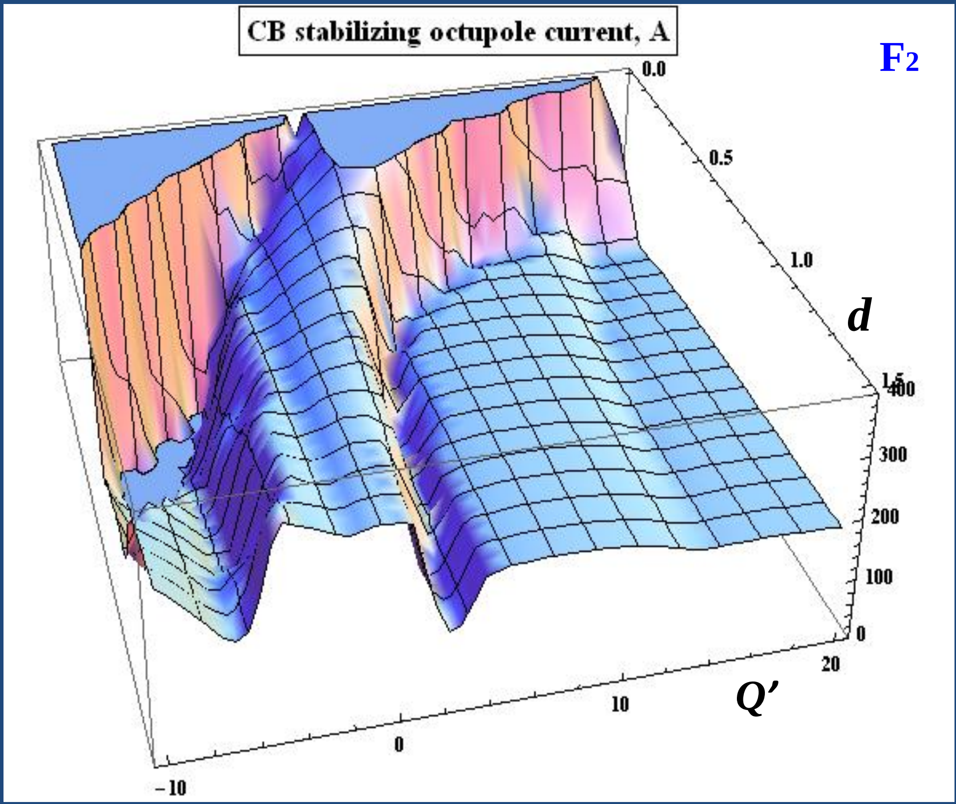


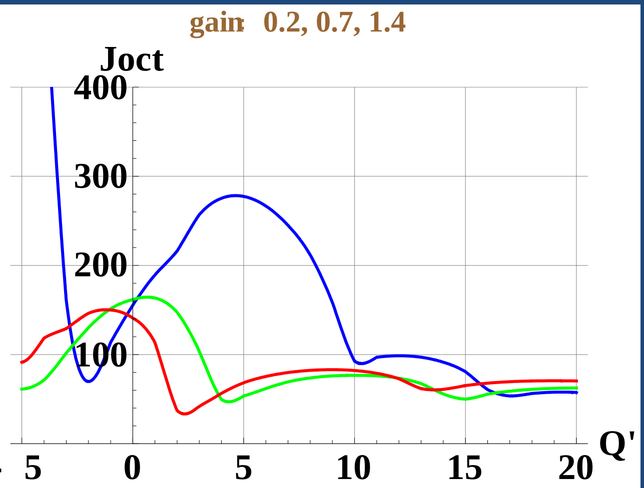
Almost no difference at this polarity.



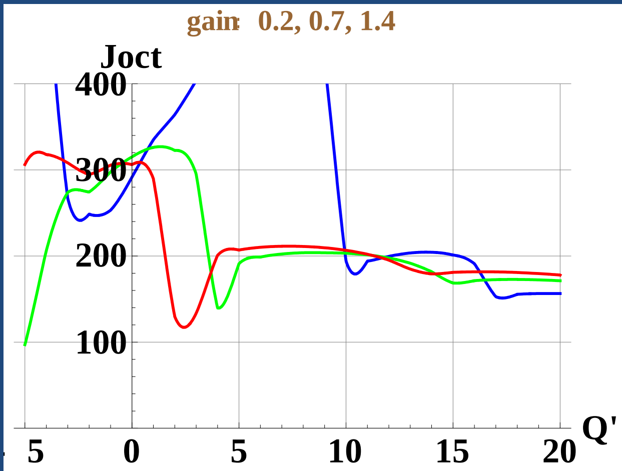
Tails Factor: LO-, bbb ADT, 2Imp



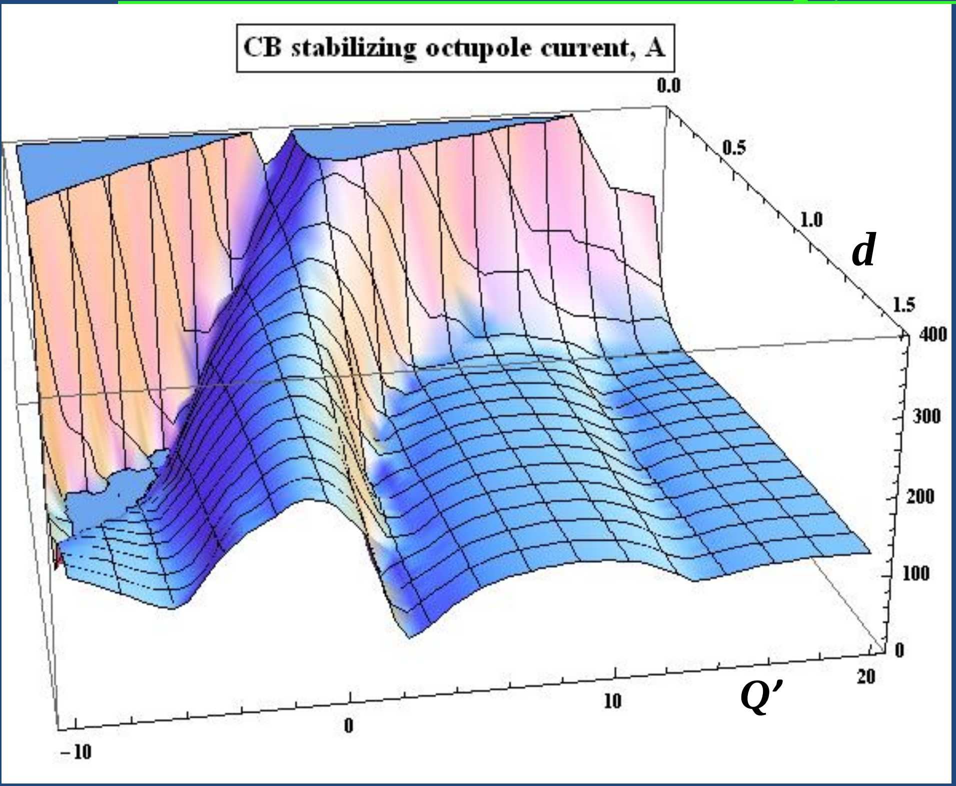


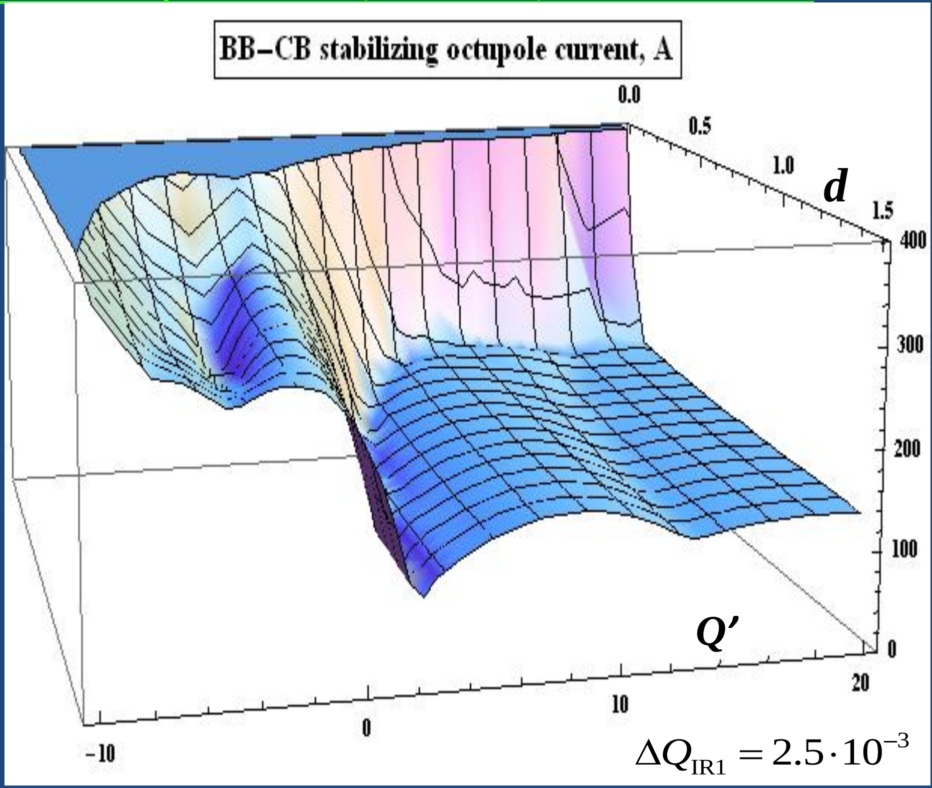


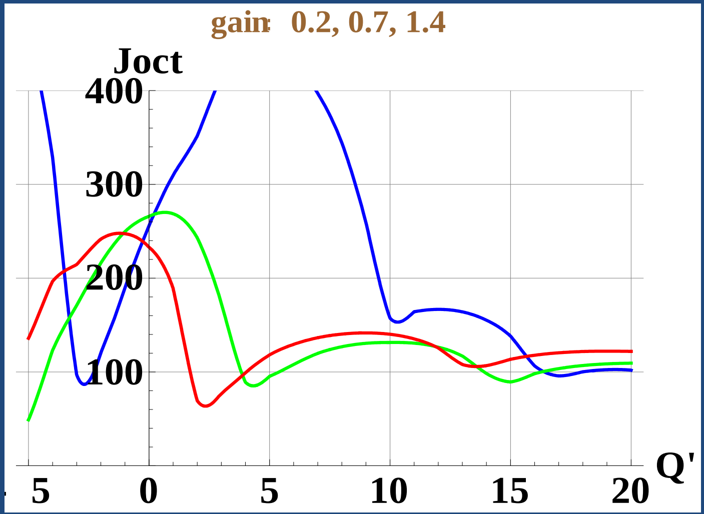
About a factor of 3 difference at the plateau!



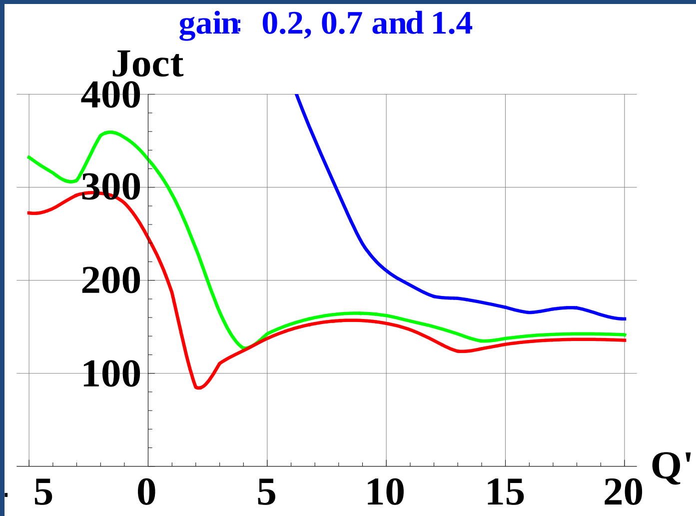
Beam-Beam Factor: 2Imp, CB, CBB $\psi=\pi/2$, LO+, bbb ADT



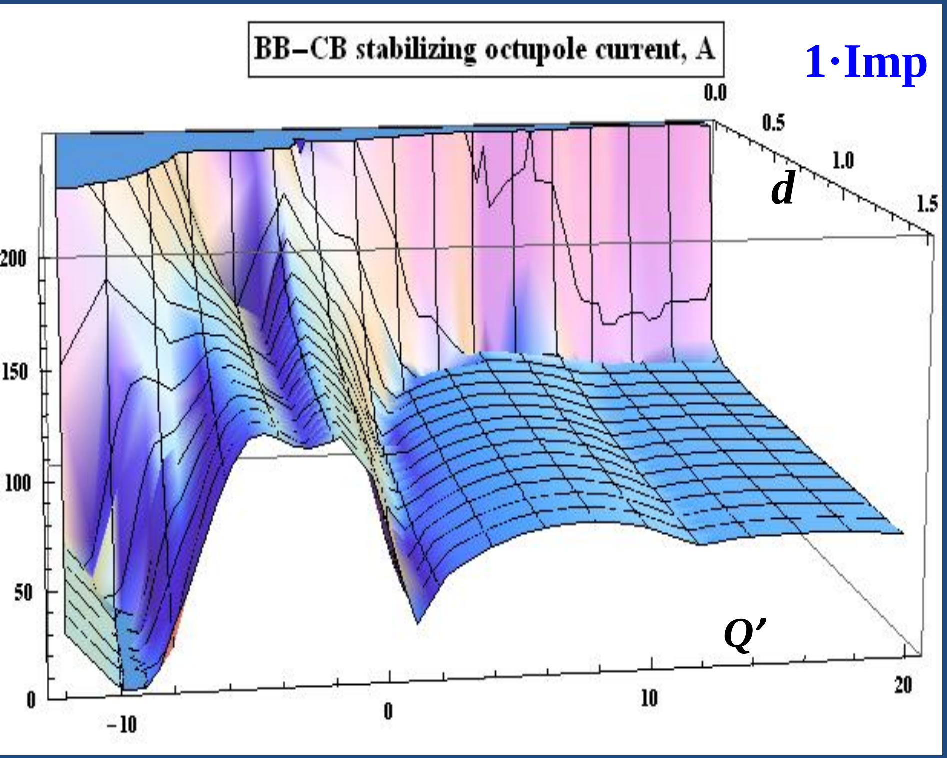


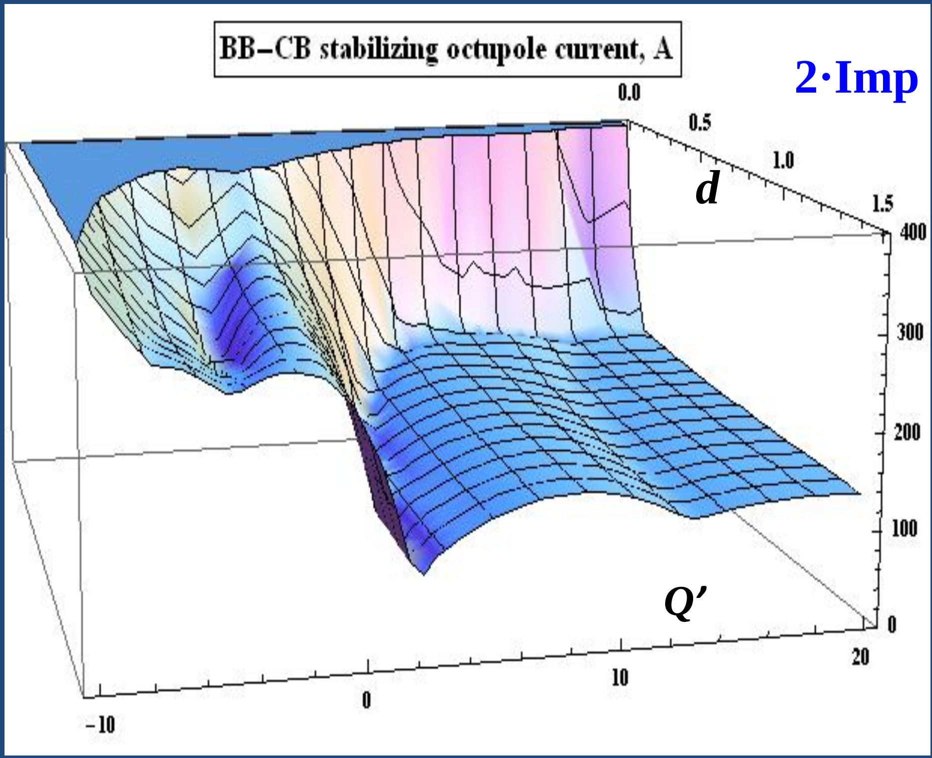


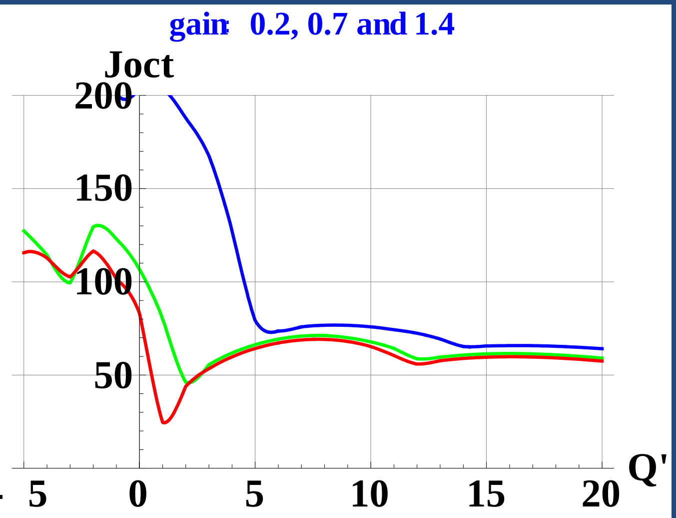
CBB effect ~ 30% at the Plateau.



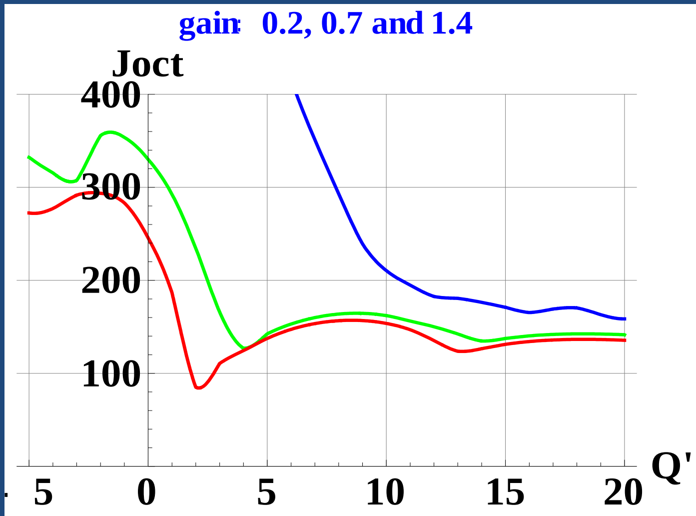
Impedance Factor: CB, CBB $\psi=\pi/2$, LO+, bbb ADT







At the Plateau it scales ~ linearly



Long-Range Beam-Beam Tune Spread

- For the alternating x/y IR1/IR5 collision scheme, the octupolar LR tune spread is
$$\Delta Q_{4x} = \frac{3\Delta Q_{\text{bb}}}{r^2}\frac{J_x - 2J_y}{\varepsilon}.$$
Here ΔQ_{bb} is the linear LR bb tune shift per IR, $r \gg 1$ is beam separation in units of their rms size at that point. Round betas are assumed.

- For LHC at the end of the squeeze $\Delta Q_{\text{bb}} = 2.5 \cdot 10^{-3}$, $r = 9.5$.

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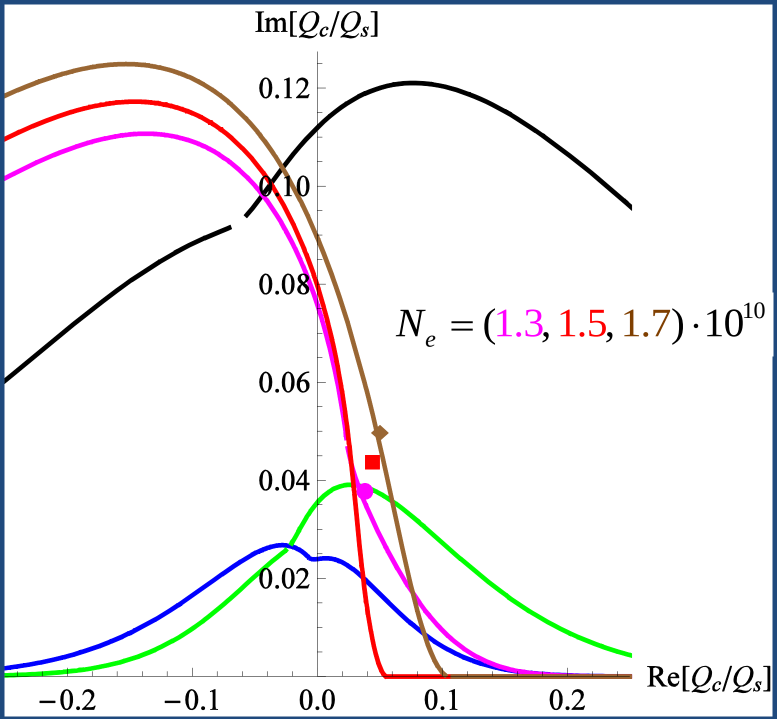
Beam-Beam-Beam Effect in LHC



LO=200A – computed threshold

(Pacman) BB only, LO=0

BB and LO=500A

BB, LO=500A, dQe0=6.0E-4

BB, LO=500A, dQe0=8.0E-4

BB, LO=500A, dQe0=1.0E-3

Markers – MUMs, colors correspond

Instability is driven by e-cloud attracted by 2 beams in the high-beta area of IR1&5.

It happens due to a right-collapse of the SD + low-frequency e-wake with positive coherent tune shifts.

Electron wake:

$$W(\tau) \simeq W_0 \sin(\omega_e \tau) \exp\left(\omega_e \tau / 2Q\right);$$

$$W_0 = \frac{N_e r_e c}{4\sigma_\perp^4 \omega_e}, \quad Q \sim 3-5, \;\; \tau < 0$$

$$\psi_e \equiv \omega_e \sigma_z / c = \begin{cases} 6.5 \text{ rad} & \text{for } \beta\text{=300m} \\ 1.4 \text{ rad} & \text{for } \beta\text{=4km} \end{cases}$$

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Summary: power of the model

- Method of nested head-tail modes (NHT) is implemented on a base of Mathematica. It allows to find coherent tunes for all the modes, solving the eigenproblem at its 4D set:

  azimuthal $\otimes$ radial $\otimes$ coupled-bunch $\otimes$ beam-beam.

- The external data: impedance/wake, damper frequency profile, distribution functions and nonlinearities, beam-beam scheme.

- Based on that, all the coherent modes with all the details are computed.

- For given machine and beam parameters, computation takes ~1s at my 3 year-old laptop.

- NHT is successfully benchmarked with BeamBeam3D tracking.

Next steps

- To include longitudinal nonlinearities.
- To include detuning wakes/impedances.
- To include train structure.
- To make all that user friendly and public.

Many thanks!