# Fermionic WDM reproduces galaxy observations because of quantum mechanics

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Summary Warm Dark Matter, WDM:  $m \sim \text{keV}$ 

- Large Scales, structures beyond ~ 100 kpc: WDM and CDM yield identical results which agree with observations
- Intermediate Scales: WDM give the correct abundance of substructures.
- Inside galaxy cores, below ~ 100 pc: N-body classical physics simulations are incorrect for WDM because of important quantum effects.
- Quantum calculations (Thomas-Fermi) give galaxy cores, galaxy masses, velocity dispersions and densities in agreement with the observations.
- Direct Detection of the main WDM candidate: the sterile neutrino. Beta decay and electron capture. <sup>3</sup>H, Re, Ho. So far, not a single valid objection arose against WDM.
  Baryons (=16%DM) expected to give a correction to WDM

#### **Quantum physics in Galaxies**

de Broglie wavelength of DM particles  $\lambda_{dB} = \frac{\hbar}{m \sigma}$ 

 $d= {\rm mean\ distance\ between\ particles,\ }\sigma={\rm mean\ velocity}$   $d=\left(\frac{m}{\rho}\right)^{\frac{1}{3}}\quad,\quad Q=\rho/\sigma^3\quad,\quad Q={\rm phase\ space\ density.}$ 

ratio:  $\mathcal{R} = \frac{\lambda_{dB}}{d} = \hbar \left(\frac{Q}{m^4}\right)^{\frac{1}{3}}$ 

Observed values:  $2 \times 10^{-3} \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}} < \mathcal{R} < 1.4 \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}}$ 

The larger  $\mathcal{R}$  is for ultracompact dwarfs. The smaller  $\mathcal{R}$  is for big spirals.

 ${\cal R}$  near unity (or above) means a QUANTUM OBJECT.

Observations alone show that compact dwarf galaxies are quantum objects (for WDM).

No quantum effects in CDM:  $m \gtrsim \text{GeV} \Rightarrow \mathcal{R} \lesssim 10^{-8}$ 

Quantum pressure vs. gravitational pressure quantum pressure:  $P_q =$ flux of momentum = n v p, v = mean velocity, momentum  $= p \sim \hbar/\Delta x \sim \hbar n^{\frac{1}{3}}$ , particle number density =  $n = \frac{M_q}{\frac{4}{2}\pi R_a^3 m}$ galaxy mass  $= M_q$ , galaxy halo radius  $= R_q$ gravitational pressure:  $P_G = \frac{G M_q^2}{R_q^2} \times \frac{1}{4 \pi R_q^2}$ Equilibrium:  $P_q = P_G \Longrightarrow$  $R_q = \frac{3^{\frac{5}{3}}}{(4\pi)^{\frac{2}{3}}} \frac{\hbar^2}{Gm^{\frac{8}{3}}M^{\frac{1}{3}}} = 10.6\dots \operatorname{pc}\left(\frac{10^6 M_{\odot}}{M_q}\right)^{\frac{1}{3}} \left(\frac{\operatorname{keV}}{m}\right)^{\frac{8}{3}}$  $v = \left(\frac{4\pi}{81}\right)^{\frac{1}{3}} \frac{G}{\hbar} m^{\frac{4}{3}} M_q^{\frac{2}{3}} = 11.6 \frac{\mathrm{km}}{\mathrm{s}} \left(\frac{\mathrm{keV}}{m}\right)^{\frac{4}{3}} \left(\frac{M_q}{10^6 M_{\odot}}\right)^{\frac{2}{3}}$ for WDM the values of  $M_q$ ,  $R_q$  and v are consistent with the dwarf galaxy observations !! .

Dwarf spheroidal galaxies can be supported by the fermionic quantum pressure of WDM.

#### **Self-gravitating Fermions in the Thomas-Fermi approach**

WDM is non-relativistic in the MD era. A single DM halo in late stages of formation relaxes to a time-independent form especially in the interior.

Chemical potential:  $\mu(r) = \mu_0 - m \phi(r)$ ,  $\phi(r) = \text{grav. pot.}$ 

Poisson's equation:  $\frac{d^2\mu}{dr^2} + \frac{2}{r} \frac{d\mu}{dr} = -4\pi G m \rho(r)$ 

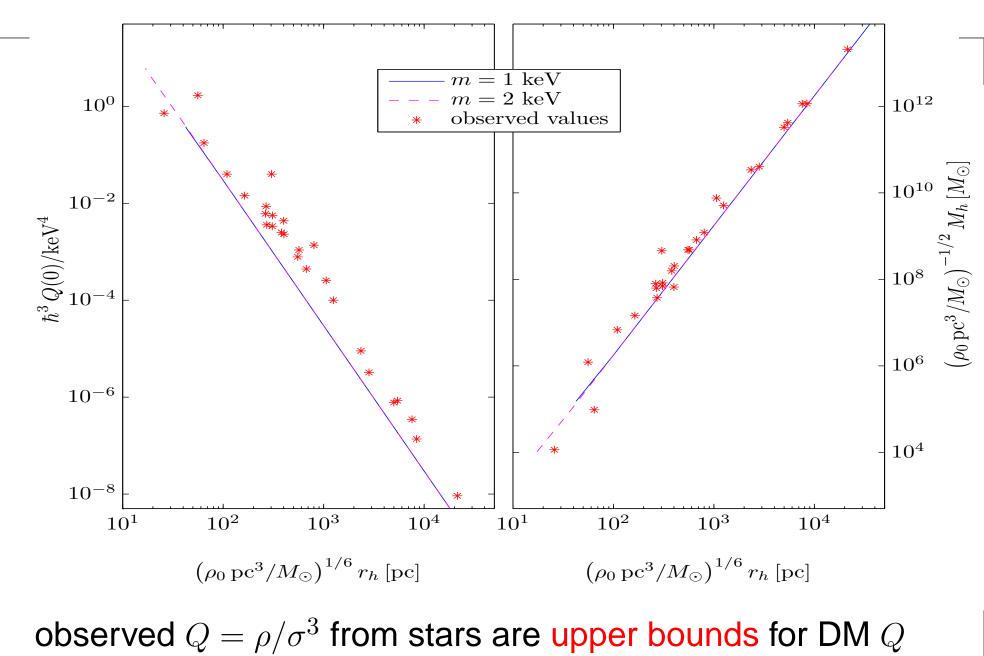
 $\rho(0) = \text{finite for fermions} \Longrightarrow \frac{d\mu}{dr}(0) = 0.$ 

Density  $\rho(r)$  and pressure P(r) in terms of the distribution function f(E):

$$\rho(r) = \frac{m}{\pi^2 \hbar^3} \int_0^\infty p^2 \, dp \, f[\frac{p^2}{2m} - \mu(r)]$$
$$P(r) = \frac{m}{3\pi^2 \hbar^3} \int_0^\infty p^4 \, dp \, f[\frac{p^2}{2m} - \mu(r)]$$

Boundary condition at  $r = R = R_{200} \sim R_{vir}$ ,  $\rho(R_{200}) \simeq 200 \ \bar{\rho}_{DM}$ 

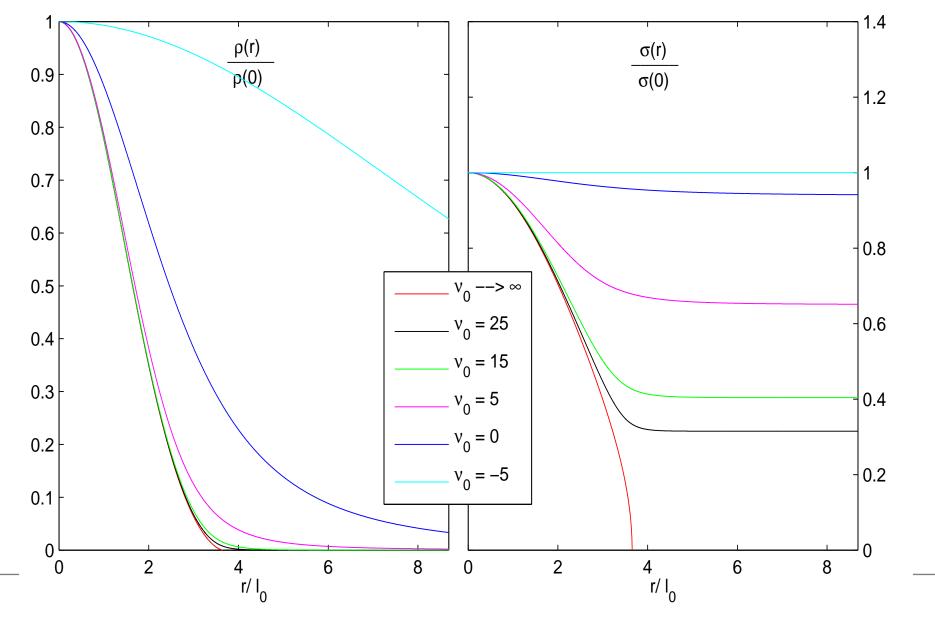
#### Q vs. halo radius. Galaxy observations vs. Thomas-Fermi



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#### **Density and velocity profiles from Thomas-Fermi**

**Cored** density profile and velocity profile obtained from Thomas-Fermi.



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#### Galaxy data vs. Thomas-Fermi

Mass, halo radius, velocity dispersion and central density from a broad variety of galaxies: ultracompact galaxies to giant spirals, Willman 1, Segue 1, Canis Venatici II, Coma-Berenices, Leo II, Leo T, Hercules, Carina, Ursa Major I, Draco, Leo I, Sculptor, Boötes, Canis Venatici I, Sextans, Ursa Minor, Fornax, NGC 185, NGC 855, NGC 4478, NGC 731, NGC 3853, NGC 499 and a large number of spiral galaxies.

Phase-Space distribution function  $f(E/E_0)$ : Fermi-Dirac  $(F(x) = \frac{1}{e^x+1})$  and out of equilibrium sterile neutrinos give similar results.

 $E_0 = effective galaxy temperature (energy scale).$ 

 $E_0$  turns to be  $10^{-3} \ ^{o}\text{K} < E_0 < 10 \ ^{o}\text{K}$ 

colder = ultracompact, warmer = large spirals.

 $E_0 \sim m < v^2 >_{\text{observed}}$  for  $m \sim 2 \text{ keV}$ .

## **Self-gravitating Fermions in the Thomas-Fermi approach**

The Thomas-Fermi approach gives physical galaxy magnitudes: mass, halo radius, phase-space density and velocity dispersion fully compatible with observations from the largest spiral galaxies till the ultracompact dwarf galaxies for a WDM particle mass around 2 keV independent of the particle physics model.

Compact dwarf galaxies are close to a degenerate WDM Fermi gas while large galaxies are classical WDM Boltzmann gases.

Thomas-Fermi approach works in the classical (Boltzmann) regime too: we always obtain cores with observed sizes.

Fermionic WDM treated quantum mechanically is able to reproduce the observed galaxies.

C. Destri, H. J. de Vega, N. G. Sanchez, arXiv:1204.3090, New Astronomy 22, 39 (2013) and arXiv:1301.1864.

### **Minimal galaxy mass from degenerate WDM**

The halo radius, the velocity dispersion and the galaxy mass take their minimum values for degenerate WDM:

$$r_{h \min} = 24.51 \dots \text{ pc } \left(\frac{m}{\text{keV}}\right)^{\frac{4}{3}} \left[\rho(0) \frac{\text{pc}^{3}}{M_{\odot}}\right]^{\frac{1}{6}}$$
$$M_{min} = 2.939 \dots 10^{5} M_{\odot} \left(\frac{\text{keV}}{m}\right)^{4} \sqrt{\rho(0) \frac{\text{pc}^{3}}{M_{\odot}}}$$
$$\sigma_{min}(0) = 2.751 \dots \frac{\text{km}}{\text{s}} \left(\frac{\text{keV}}{m}\right)^{\frac{4}{3}} \left[\rho(0) \frac{\text{pc}^{3}}{M_{\odot}}\right]^{\frac{1}{3}}.$$

These minimum values correspond to the observations of compact dwarf galaxies.

Lightest known compact dwarf galaxy is Willman I:  $M_{Willman~I} = 2.9~10^4~M_{\odot}$ 

Imposing  $M_{Willman I} > M_{min}$  yields the lower bound for the WDM particle mass: m > 1.91 keV.

#### **Dark Matter Particles**

DM particles decouple due to the universe expansion, their distribution function freezes out at decoupling.

The characteristic length scale is the free streaming scale (or Jeans' scale). For DM particles decoupling UR:

 $r_{Jeans} = 57.2 \,\mathrm{kpc} \,\frac{\mathrm{keV}}{m} \,\left(\frac{100}{g_d}\right)^{\frac{1}{3}}$ , solving the linear Boltz-V eqs.

 $g_d$  = number of UR degrees of freedom at decoupling.

DM particles can freely propagate over distances of the order of the free streaming scale.

Therefore, structures at scales smaller or of the order of  $r_{Jeans}$  are erased.

The size of the DM galaxy cores is in the  $\sim 50 \text{ kpc scale} \Rightarrow m$  should be in the keV scale (Warm Dark Matter particles, WDM).

**Structure Formation in the Universe** 

Structures in the Universe as galaxies and cluster of galaxies form out of the small primordial quantum fluctuations originated by inflation with power P(k).

These small primordial fluctuations grow due to gravitational unstabilities (Jeans) and then classicalize.

Structures form through non-linear gravitational evolution. Hierarchical formation starts from small scales first.

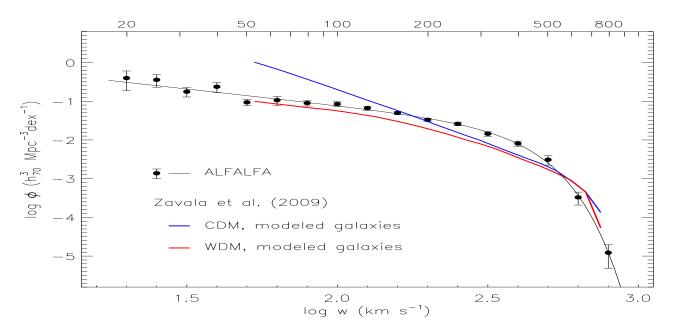
In WDM hierarchical formation dominates.

Both *N*-body WDM and CDM simulations yield identical and correct structures for scales larger than some kpc.

WDM predicts correct structures for small scales (below kpc) when its quantum nature is taken into account.

WDM cuts P(k) on small scales  $r \leq 100 \; (\text{keV}/m)^{4/3}$  kpc. CDM and WDM identical for CMB.

### **Velocity widths in galaxies: test substructure formation**

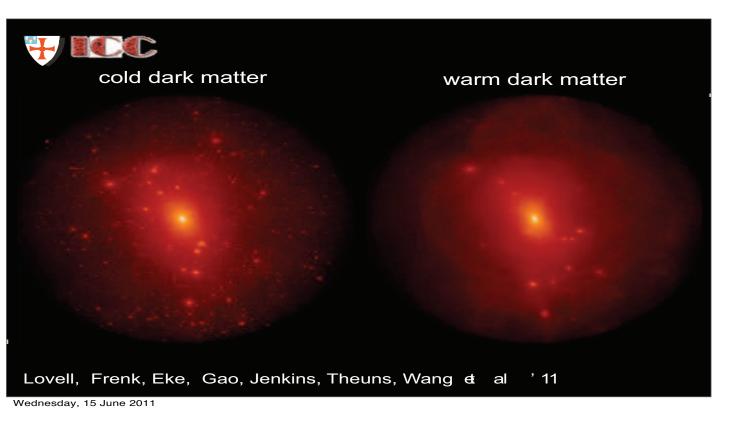


Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. (Papastergis et al. ApJ, 2011, Zavala et al. ApJ, 2009).

Notice that the WDM red curve is for m = 1 keV WDM particle decoupling at thermal equilibrium.

The 1 keV WDM curve falls somehow below the data suggesting a slightly larger WDM particle mass.

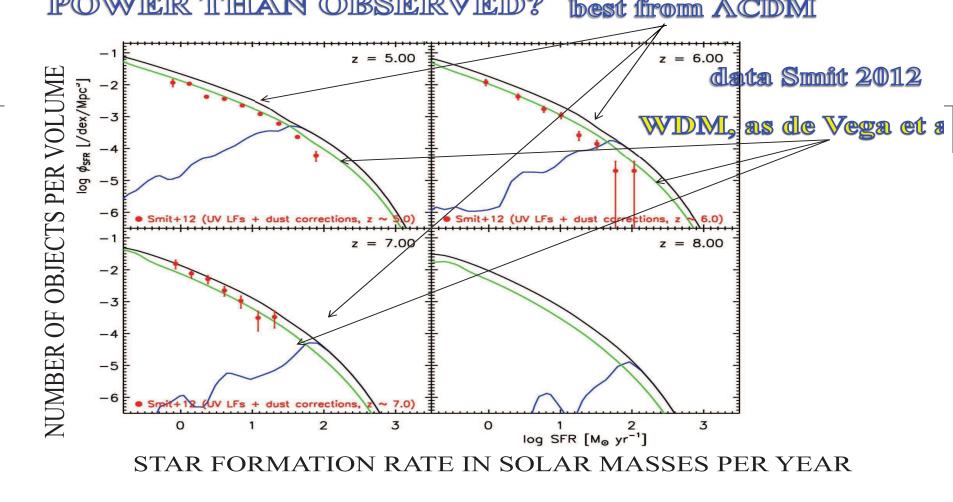
## **N-body WDM Simulations: substructure formation**



WDM subhalos are less concentrated than CDM subhalos.

WDM subhalos have the right concentration to host the bright Milky Way satellites. Lovell et al. MNRAS (2012).

Summary: WDM produces correct substructure abundance.



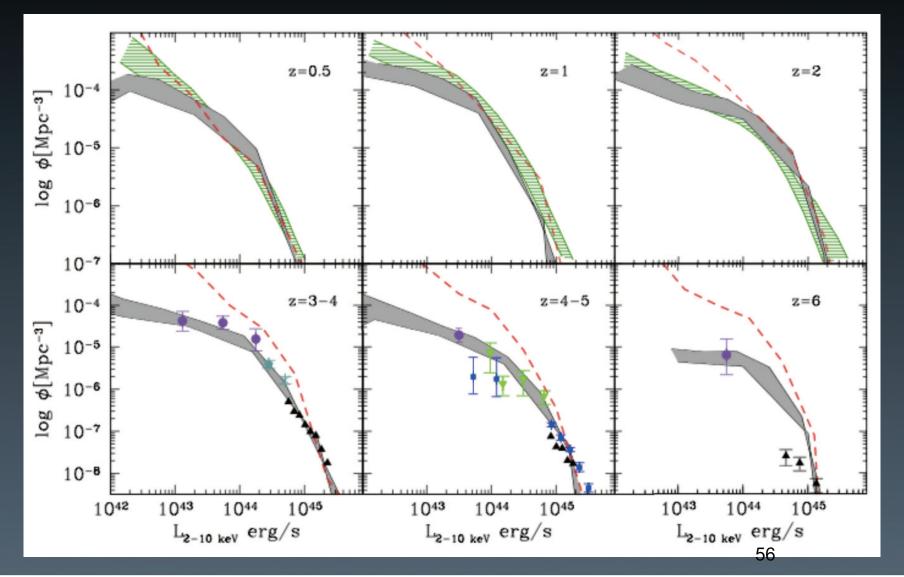
Small scale structures at high redshift.

WDM (green continuous line) reproduces the observed small scale structures for redshifts up to eight where observations are available.

Danese, de Vega, Lapi, Salucci, Sanchez (in preparation).

#### The evolution of the AGN luminosity function in WDM vs CDM WDM: shaded grey area CDM red dashed line

NM, Fiore, Lamastra 2012b



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#### **Sterile Neutrinos** $\nu_s \simeq \nu_R + \theta \ \nu_L$

Sterile neutrinos  $\nu_s$ : named by Bruno Pontecorvo (1968). WDM  $\nu_s$  are produced from active neutrinos by mixing. Mixing angles:  $\theta \sim 10^{-3} - 10^{-4}$  (depending on the model) are appropriate to produce enough  $\nu_s$  accounting for the observed total DM.

Smallness of  $\theta$  makes sterile neutrinos difficult to detect.

Sterile neutrinos can be detected in beta decay and in electron capture (EC) when a  $\nu_s$  with mass in the keV scale is produced instead of an active  $\nu_a$ .

Beta decay: the electron spectrum is slightly modified at energies around the mass ( $\sim$  keV) of the  $\nu_s$ .

 $^{3}H_{1} \Longrightarrow ^{3}He_{2} + e^{-} + \bar{\nu}_{e} \quad , \quad ^{187}Re \Longrightarrow ^{187}Os + e^{-} + \bar{\nu}_{e}.$ 

The electron energy spectrum is observed.

#### **Electron Capture and Sterile Neutrinos**

Electron capture:  ${}^{163}Ho \Longrightarrow {}^{163}Dy^* + \nu_e$ 

The nonradiative de-excitation of the  $Dy^*$  is observed and is different for  $\nu_s$  in the keV range than for active  $\nu_a$ .

Available energies:

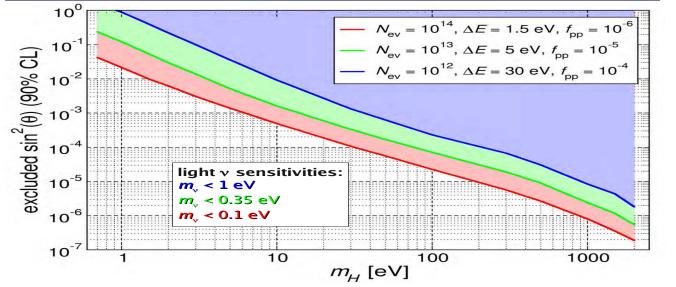
 $Q(^{187}Re) = 2.47 \text{ keV}, Q(^{3}H_{1}) = 18.6 \text{ keV}, Q(^{163}Ho) \simeq 2.5 \text{ keV}.$ 

Theoretical analysis of  $\nu_s$  detection in Rhenium and Tritium beta decay: H J de V, O. Moreno, E. Moya, M. Ramón Medrano, N. Sánchez, Nucl. Phys. B866, 177 (2013).

Present experiments searching the small active neutrino mass also look for sterile neutrinos in the keV scale:

MARE (Milan, Italy), Rhenium beta decay and Holmiun EC. KATRIN (Karlsruhe, Germany), Tritium beta decay. ECHo (Heidelberg, Germany), Holmiun EC. Project 8, (MIT, USA) Tritium beta decay (still in project).

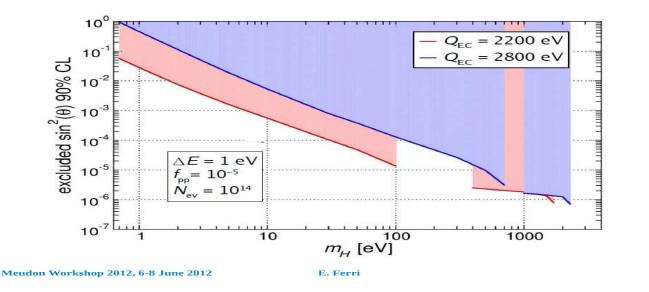
#### /IARE searchs in Re187 eta decay and Ho163 electron capture



MARE sensitivity to heavy neutrinos: 187 Re option

A. Nucciotti, Meudon Workshop 2011, 8-10 JUNE 2011 36

MARE sensitivity to heavy neutrinos: Ho option 2



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#### **Sterile neutrino models**

- DW: Dodelson-Widrow model (1994) sterile neutrinos produced by non-resonant mixing from active neutrinos.
- Shi-Fuller model (1998) sterile neutrinos produced by resonant mixing from active neutrinos.
- Models based on: Froggatt-Nielsen mechanism, flavor symmetries, Q<sub>6</sub>, split see-saw, extended see-saw, inverse see-saw, loop mass. Furthermore: scotogenic, LR symmetric, etc. Review by A Merle (2013).

WDM particles in the first 3 models behave primordially just as if their masses were different (FD = thermal fermions):  $\frac{m_{DW}}{\text{keV}} \simeq 2.85 \left(\frac{m_{FD}}{\text{keV}}\right)^{\frac{4}{3}}, m_{SF} \simeq 2.55 m_{FD}, m_{\nu\text{MSM}} \simeq 1.9 m_{FD}.$ H J de Vega, N Sanchez, Warm Dark Matter cosmological fluctuations, Phys. Rev. D85, 043516 and 043517 (2012).

#### **X-ray detection of DM sterile neutrinos**

Sterile neutrinos  $\nu_s$  decay into active neutrinos  $\nu_a$  plus X-rays with a lifetime  $\sim 10^{11} \times$  age of the universe.

These X-rays may be seen in the sky looking to galaxies !

recent review: C. R. Watson et al. JCAP, (2012).

Talk by Loewenstein in CF3, 7 March 11.30. Future observations:

- DM bridge between M81 and M82  $\sim 50$  kpc. Overlap of DM halos. Satellite projects: Xenia (NASA).
- CMB: WDM decay distorts the blackbody CMB spectrum. The projected PIXIE satellite mission (A. Kogut et al.) can measure WDM sterile neutrino mass.

Results from Supernovae:  $\theta$  unconstrained, 1 < m < 10 keV, (G. Raffelt & S. Zhou, PRD 2011).

#### **Summary: keV scale DM particles**

- Reproduce the phase-space density observed in dwarf spheroidal and spiral galaxies (de Vega, Sanchez, MNRAS 2010).
- Fermionic WDM treated quantum mechanically reproduces the main physical galaxy magnitudes: mass, core radius, phase-space density, velocity dispersion, fully consistent with observations and points to a DM particle mass 2 keV (Destri, de Vega, Sanchez, New Astronomy 2012, and 2013).
- The galaxy surface density  $\mu_0 \equiv \rho_0 r_0$  is universal up to  $\pm 10\%$  according to the observations. Its value  $\mu_0 \simeq (18 \text{ MeV})^3$  is reproduced by WDM (de Vega, Salucci, Sanchez, New Astronomy, 2012). CDM simulations give 1000 times the observed value of  $\mu_0$  (Hoffman et al. ApJ 2007).

#### **Summary: keV scale DM particles**

- Alleviate the CDM satellite problem (Avila-Reese et al. 2000, Götz & Sommer-Larsen 2002, Markovic et al. JCAP 2011) and the CDM voids problem (Tikhonov et al. MNRAS 2009).
- Velocity widths in galaxies from 21cm HI surveys. ALFALFA survey clearly favours WDM over CDM. Papastergis et al. ApJ 2011, Zavala et al. ApJ 2009
- ▲ All direct searches of DM particles look for m ≥ 1 GeV. DM mass in the keV scale explains why nothing has been found ... e<sup>+</sup> and p̄ excess in cosmic rays may be explained by astrophysics: P. L. Biermann et al. PRL (2009), P. Blasi, P. D. Serpico PRL (2009).
- Highlights and conclusions of the Chalonge Meudon Workshop 2011: Warm dark matter in the galaxies, arXiv:1109.3187 and the 16th Paris Cosmology Colloquium 2011 arXiv:1203.3562, H. J. de V., N. G. S.

#### **Future Perspectives**

WDM particle models must explain the baryon asymmetry of the universe. An appealing mass neutrino hierarchy appears:

- Active neutrino:  $\sim$  mili eV
- **•** Light sterile neutrino:  $\sim eV$
- **Dark Matter:**  $\sim$  keV
- Unstable sterile neutrino:  $\sim$  MeV....

Need WDM simulations showing substructures, galaxy formation and evolution including quantum dynamical evolution. Quantum pressure must be included ! WDM simulations should be performed matching semiclassical Hartree-Fock (Thomas-Fermi) dynamics in regions where  $Q/m^4 > 0.1$  with classical evolution in regions where  $Q/m^4 \ll 1$ . Not easy but unavoidable!

#### **Future Perspectives: Detection!**

Sterile neutrino detection depends upon the particle physics model. There are sterile neutrino models where the keV sterile is stable. Very hard to detect.

Astronomical observation of steriles: X-ray data from galaxy halos.

**Direct** detection of steriles in Lab:

Bounds on mixing angles from Mare, Katrin, ECHo and Project 8 are expected.

For a particle detection a dedicated beta decay or electron capture experiment looks necessary to search sterile neutrinos with mass around 2 keV.

Calorimetric techniques seem well suited.

Best nuclei for study:

Electron capture in  $^{163}$ Ho, beta decay in  $^{187}$ Re and Tritium.

Richard P. Feynman foresaw the necessity to include quantum physics in simulations in 1981

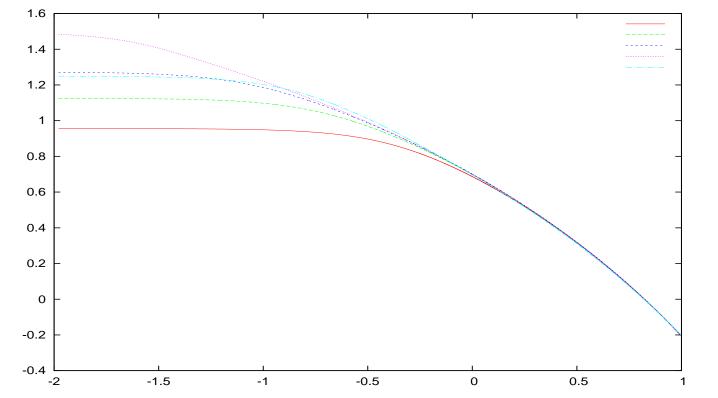
"I'm not happy with all the analyses that go with just the classical theory, because nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy."

Feynman again:

"It doesn't matter how beautiful your theory is, it doesn't matter how smart you are. If it doesn't agree with experiment, it's wrong. R. P. Feynman"

# THANK YOU VERY MUCH FOR YOUR ATTENTION!!

#### The expected overdensity The expected overdensity within a radius R in the linear regime $\sigma^2(R) = \int_0^\infty \frac{dk}{k} \Delta^2(k) W^2(kR)$ , W(kR): window function.



 $\log \sigma(R)$  vs.  $\log(R/h \text{ Mpc})$  for CDM, 1 keV, 2 keV, 4 keV DM particles decoupling in equil, and 1 keV (light-blue) sterile neutrinos. WDM flattens and reduces  $\sigma(R)$  for small scales.

**Galaxy Density Profiles: Cores vs. Cusps** Astronomical observations always find cored profiles. Selected references: J. van Eymeren et al. A&A (2009), M. G. Walker, J.Peñarrubia, ApJ(2012). N. Amorisco, N. Evans, MNRAS(2012). Galaxy profiles in the linear regime: core size  $\sim$  free streaming length (de Vega, Salucci, Sanchez, 2010) halo radius  $r_0 = \begin{cases} \sim 0.05 \text{ pc cusps for CDM (m > GeV)}. \\ \sim 50 \text{ kpc cores for WDM (m ~ keV)}. \end{cases}$ 

N-body simulations for CDM give cusps (NFW profile).

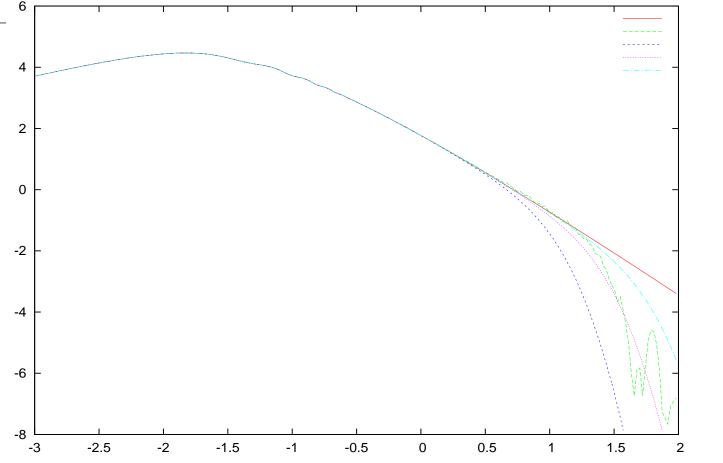
N-body simulations for WDM : quantum physics needed for fermionic DM !!! (Destri, de Vega, Sanchez, 2012)

CDM simulations give a precise value for the concentration  $\equiv R_{virial}/r_0$ .

CDM concentrations disagree with observed values.

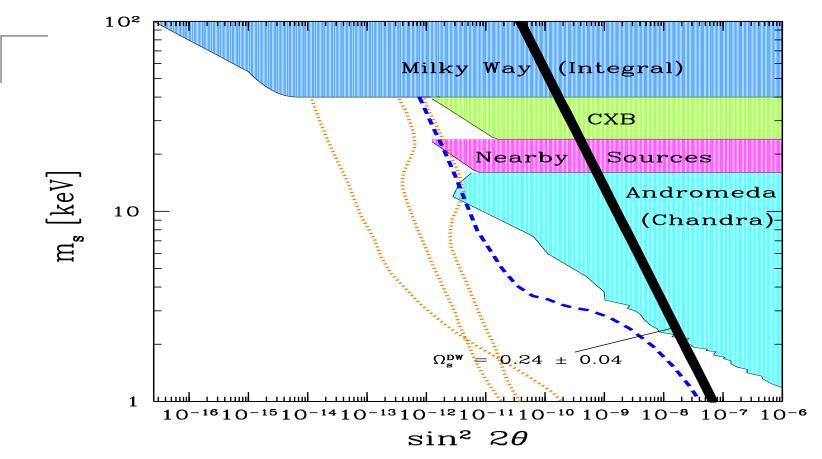
- **CDM free streaming scale** For CDM particles with  $m \sim 100 \text{ GeV} \Rightarrow r_{Jeans} \sim 0.1 \text{ pc.}$ Hence CDM structures keep forming till scales as small as the solar system.
- This is a robust result of N-body CDM simulations but never observed in the sky. Including baryons do not cure this serious problem. There is over abundance of small structures in CDM (also called the satellite problem).
- CDM has many serious conflicts with observations:
- Galaxies naturally grow through merging in CDM models. Observations show that galaxy mergers are rare (< 10%). Pure-disk galaxies (bulgeless) are observed whose formation through CDM is unexplained.
- CDM predicts cusped density profiles:  $\rho(r) \sim 1/r$  for small r. Observations show cored profiles:  $\rho(r)$  bounded for small r. Adding by hand strong enough feedback from baryons can eliminate cusps but spoils the star formation rate.

#### **Linear primordial power today** P(k) vs. k Mpc h



 $\log_{10} P(k)$  vs.  $\log_{10}[k \text{ Mpc } h]$  for CDM, 1 keV, 2 keV, light-blue 4 keV DM particles decoupling in equil, and 1 keV sterile neutrinos. WDM cuts P(k) on small scales  $r \leq 100 \ (\text{keV}/m)^{4/3}$  kpc. CDM and WDM identical for CMB.

### **Constraints on the sterile neutrino mass and mixing angle**



Dashed = Shi-Fuller model. Dotted = Dodelson-Widrow for fermion asymmetry L = 0.1, 0.01 and 0.003. Allowed sterile neutrino region in the right lower corner. Main difficulty: to distinguish the sterile neutrino decay -X-ray from narrow X-lines emitted by hot ions.

#### In the presence of angular momentum $L^2$

Adds the centrifugal pressure:  $P_L = \frac{L^2}{M R^3} \frac{1}{4 \pi R^2}$ Equilibrium:  $P_q + P_L = P_G$ :

$$\left(\frac{3}{4\pi}\right)^{5/3} \frac{\hbar^2 M^{5/3}}{m^{8/3} R^5} + \frac{L^2}{4\pi M R^5} = \frac{G M^2}{4\pi R^4} \Rightarrow R = \frac{L^2}{G M^3} + \frac{3^{5/3}}{(4\pi)^{2/3}} \frac{\hbar^2}{G m^{8/3} M^{1/3}}$$

We estimate  $L^2$  as  $L^2 \sim \frac{1}{2} M^2 R^2 3 \sigma^2$ .

The  $\frac{1}{2}$  factor comes from averaging the  $\sin^2$  of the angle between the momentum  $\vec{p}$  and the particle position  $\vec{r}$ .

$$R = 10.6 \operatorname{pc} \left(\frac{10^6 \ M_{\odot}}{M}\right)^{\frac{1}{3}} \left(\frac{\operatorname{keV}}{m}\right)^{\frac{8}{3}} + 3.48 \operatorname{pc} \frac{10^6 \ M_{\odot}}{M} \left(\frac{\sigma}{10 \ \frac{\operatorname{km}}{\operatorname{s}}} \ \frac{R}{10 \ \operatorname{pc}}\right)^2$$

The angular momentum increases the size R. For dwarf galaxies, R and  $\sigma$  have the same order of magnitude for L > 0 and for L = 0.

#### **Summary: keV scale DM particles**

- Reproduce the phase-space density observed in dwarf spheroidal and spiral galaxies (de Vega, Sanchez, MNRAS 2010).
- Fermionic WDM treated quantum mechanically reproduces the main physical galaxy magnitudes: mass, core radius, phase-space density, velocity dispersion, fully consistent with observations and points to a DM particle mass 2 keV (Destri, de Vega, Sanchez, New Astronomy 2012, and 2013).
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