

State of the art developments in computations of direct scattering

RICHARD HILL



THE UNIVERSITY OF
CHICAGO

Cosmic Frontier meeting

6 March 2013

outline

- remarks on SM extensions
- heavy particle expansions and DM interactions
- worked example: “wino” - like DM
- quarks in nucleons, nucleons in nuclei

effective field theory = “QM + relativity + calculus”

based largely on work with M.P. Solon PLB 707 539 (2012), and to appear

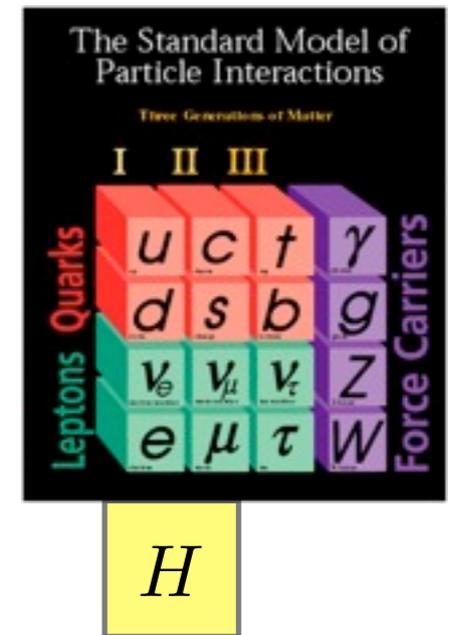
interesting work I will not cover

- contact interaction dark matter
- derivative interaction dark matter
- factoring out astrophysics
- SUSY and model building

too many contributions to list here (apologies)

*in defense of simple models**

- sometimes simple models work very well (e.g. Standard Model higgs)



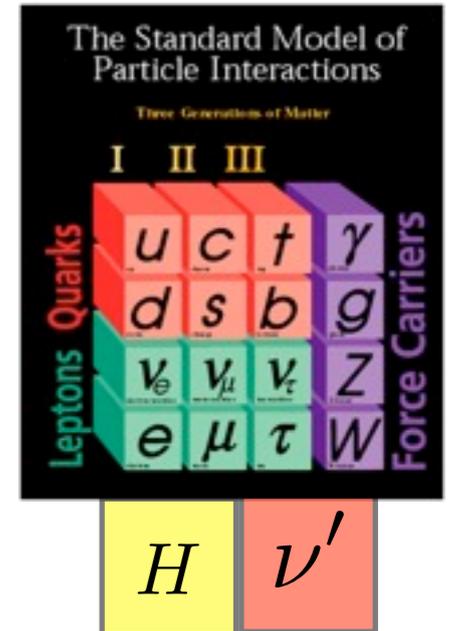
	$SU(3)$	$SU(2)$	$U(1)$
Q	3	2	1/6
\bar{u}	$\bar{3}$	1	-2/3
\bar{d}	$\bar{3}$	1	1/3
L	1	2	-1/2
\bar{e}	1	1	1
H	1	2	1/2

* for present purposes, simple model
 ~ UV completion whose form is RG invariant

- guidance into the unknown

neutrino mass problem

SM gauge symmetries allow dimension five operator



$$\mathcal{L} \sim \frac{1}{\Lambda} HHLL$$

$$\rightarrow \mathcal{L} \sim m_\nu \nu\nu, \quad m_\nu \sim \frac{v_{\text{weak}}^2}{\Lambda}$$

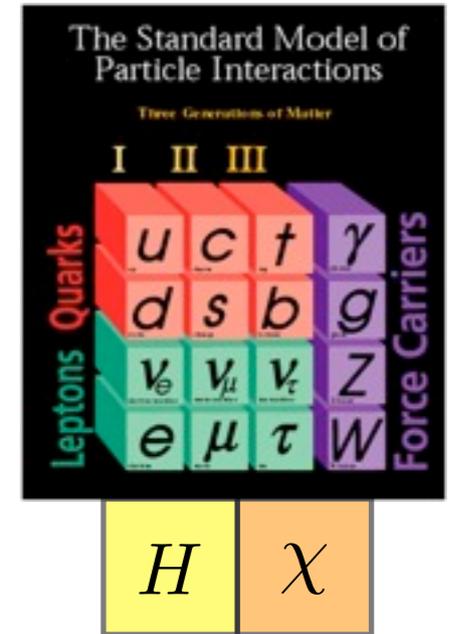
	$SU(3)$	$SU(2)$	$U(1)$
Q	3	2	1/6
\bar{u}	$\bar{3}$	1	-2/3
\bar{d}	$\bar{3}$	1	1/3
L	1	2	-1/2
\bar{e}	1	1	1
H	1	2	1/2
ν'	1	1	0

- seesaw UV completion a simple guide to possible size of neutrino mass

- guidance into the unknown

dark matter problem

at very low energies, interactions with SM given by contact interactions



$$\mathcal{L}_{\chi,SM} = \chi^* \chi \left\{ \sum_q c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_\mu v_\nu O_{1q}^{(2)\mu\nu} + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_\mu v_\nu O_2^{(2)\mu\nu} + \dots \right\}$$

$$O_{1q}^{(0)} = m_q \bar{q} q,$$

$$O_2^{(0)} = (G_{\mu\nu}^A)^2,$$

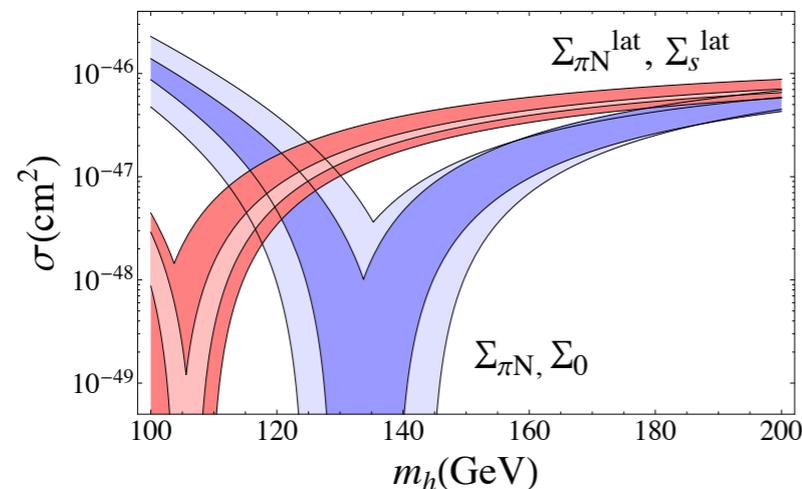
$$O_{1q}^{(2)\mu\nu} = \bar{q} \left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i \not{D} \right) q,$$

$$O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G_{\lambda\nu}^A + \frac{1}{d} g^{\mu\nu} (G_{\alpha\beta}^A)^2.$$

to understand strength of coupling and to relate different processes, need guidance from underlying interactions

in defense of calculating

Naive dimensional estimates can be very wrong for some basic numbers



VS.

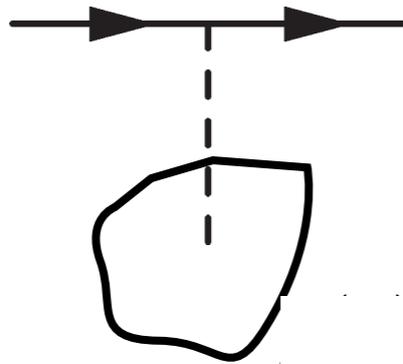
$$\sigma \sim \frac{\pi \alpha_2^4 m_N^4}{m_W^2} \left(\frac{1}{m_W^2} + \frac{1}{m_h^2} \right)^2 \sim 10^{-44} \text{ cm}^2$$

Large logarithms of QCD perturbation theory can cause large effects

Given our present knowledge of SM, can now make robust predictions for how BSM particles interact with, e.g., nuclei

heavy particles

Universal interactions with **heavy particles**



$$\mathcal{L} = \psi^\dagger (i\partial_t + gA^0 + \dots) \psi$$

- hydrogen spectroscopy

$$E_n(H) = -\frac{1}{2}m_e\alpha^2 + \dots$$

- heavy meson transitions

$$F^{B \rightarrow D}(v' = v) = 1 + \dots$$

- DM interactions

$$\sigma(\chi N \rightarrow \chi N) = ?$$

LHC: New physics may be heavy (compared to m_W)

- in this regime, m_W/M expansion becomes meaningful, universal behavior emerges
- in SUSY language, pure bino/wino/higgsino scattering suppressed (no tree level higgs exchange). This case becomes “generic” when $M \gg m_W$ ($M_1 - M_2 \sim m_W$ not generic)
- heavy particle methods efficient in particular models (e.g. relic abundance $\rightarrow m_\chi \gtrsim \text{TeV}$ for wino-like, higgsino-like DM)
- but applicable to general case where UV completion unspecified

Standard Model anatomy of direct detection

Generic dark matter candidate described by extending SM by finite number of particles in representations of SM gauge groups

As prototype, consider Lorentz-scalar, $SU(2)$ electroweak multiplet

- $M \gtrsim \text{TeV}$ from thermal relic abundance. $M \gg m_W$: model-independent analysis, predictive scattering cross section

- large gluon matrix element:
2 loop required for leading analysis

$$\mathcal{L} = c_1 \text{ (diagram)} + c_2 \text{ (diagram)} + \dots$$

$$\text{diagram} + \text{diagram} = c_1 \text{ (diagram)} + \dots$$

- Scattering on nucleon is completely determined, up to controlled corrections

$$m_W/M, \quad \Lambda_{\text{QCD}}^2/m_c^2, \quad m_b/m_W \dots$$

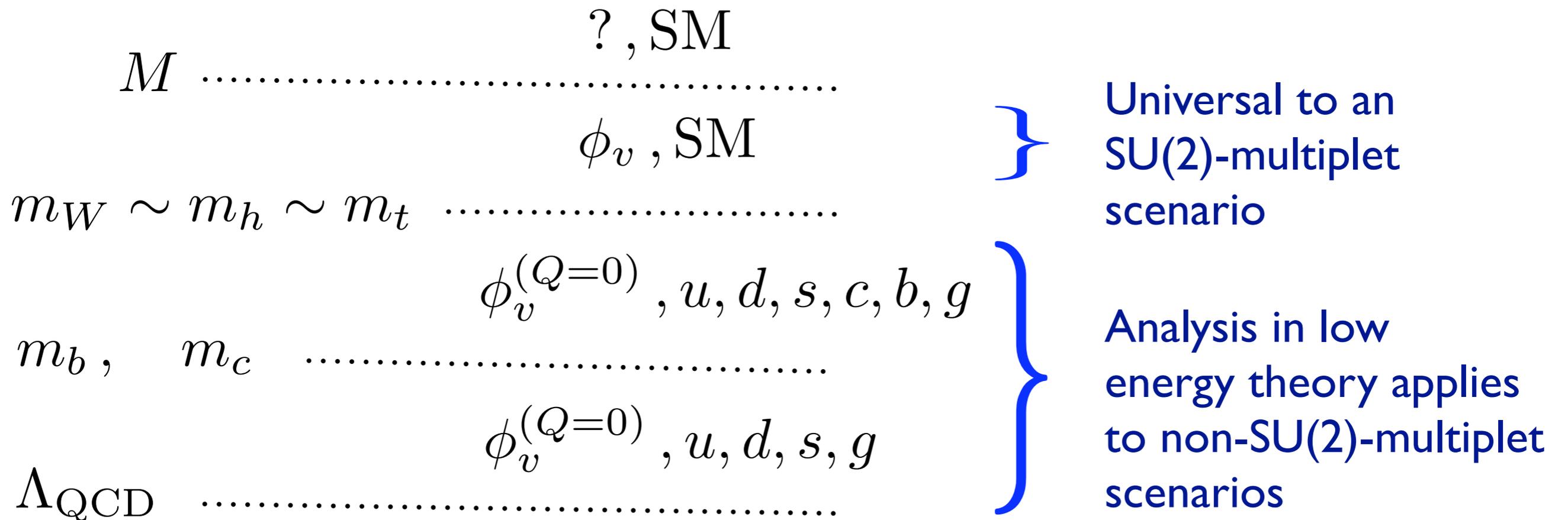
$$= c_2 \text{ (diagram)} + c_1 \left[\text{diagram} + \text{diagram} \right] + \dots$$

Multiple scales:

Renormalization analysis required to sum large logarithms

$$\alpha_s(\mu) \log \frac{m_t}{\mu} \sim \alpha_s(1 \text{ GeV}) \log \frac{170 \text{ GeV}}{1 \text{ GeV}}$$

Consider effective theory at each scale:



(EW symmetric) heavy DM effective theory:

Operator basis

Building blocks: $\phi_v(x)$, v^μ , $D_{\perp\mu} = D_\mu - v^\mu v \cdot D$

Everything not forbidden is allowed:

$$\begin{aligned} \mathcal{L}_\phi = \phi_v^* \left\{ & i v \cdot D - c_1 \frac{D_\perp^2}{2M} + c_2 \frac{D_\perp^4}{8M^3} + g_2 c_D \frac{v^\alpha [D_\perp^\beta, W_{\alpha\beta}]}{8M^2} + i g_2 c_M \frac{\{D_\perp^\alpha, [D_\perp^\beta, W_{\alpha\beta}]\}}{16M^3} \right. \\ & + g_2^2 c_{A1} \frac{W^{\alpha\beta} W_{\alpha\beta}}{16M^3} + g_2^2 c_{A2} \frac{v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta}}{16M^3} + g_2^2 c_{A3} \frac{\text{Tr}(W^{\alpha\beta} W_{\alpha\beta})}{16M^3} + g_2^2 c_{A4} \frac{\text{Tr}(v_\alpha v^\beta W^{\mu\alpha} W_{\mu\beta})}{16M^3} \\ & + g_2^2 c'_{A1} \frac{\epsilon^{\mu\nu\rho\sigma} W_{\mu\nu} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A2} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu W_{\nu\alpha} W_{\rho\sigma}}{16M^3} + g_2^2 c'_{A3} \frac{\epsilon^{\mu\nu\rho\sigma} \text{Tr}(W_{\mu\nu} W_{\rho\sigma})}{16M^3} \\ & \left. + g_2^2 c'_{A4} \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu \text{Tr}(W_{\nu\alpha} W_{\rho\sigma})}{16M^3} + \dots \right\} \phi_v, \end{aligned}$$

Lorentz invariance: $c_1 = c_2 = 1$, $c_M = c_D$

⇒ Through $\mathcal{O}(1/M^3)$, heavy gauged scalar determined by 2 numbers (mass and “charge radius”), plus polarizabilities

Standard model interactions

$$\begin{aligned}
 \mathcal{L}_{\phi, \text{SM}} = \phi_v^* \left\{ & c_H \frac{H^\dagger H}{M} + \dots + c_Q \frac{t_J^a \bar{Q}_L \tau^a \psi Q_L}{M^2} + c_X \frac{i \bar{Q}_L \tau^a \gamma^\mu Q_L \{t_J^a, D_\mu\}}{2M^3} + c_{DQ} \frac{\bar{Q}_L \psi i v \cdot D Q_L}{M^3} \right. \\
 & + c_{Du} \frac{\bar{u}_R \psi i v \cdot D u_R}{M^3} + c_{Dd} \frac{\bar{d}_R \psi i v \cdot D d_R}{M^3} + c_{Hd} \frac{\bar{Q}_L H d_R + h.c.}{M^3} + c_{Hu} \frac{\bar{Q}_L \tilde{H} u_R + h.c.}{M^3} \\
 & + g_3^2 c_{A1}^{(G)} \frac{G^{A\alpha\beta} G_{\alpha\beta}^A}{16M^3} + g_3^2 c_{A2}^{(G)} \frac{v_\alpha v^\beta G^{A\mu\alpha} G_{\mu\beta}^A}{16M^3} + g_3^2 c_{A1}' \frac{\epsilon^{\mu\nu\rho\sigma} G_{\mu\nu}^A G_{\rho\sigma}^A}{16M^3} + g_3^2 c_{A2}' \frac{\epsilon^{\mu\nu\rho\sigma} v^\alpha v_\mu G_{\nu\alpha}^A G_{\rho\sigma}^A}{16M^3} \\
 & \left. + \dots \right\} \phi_v.
 \end{aligned}$$

Lorentz invariance:

$$c_Q = c_X$$

All of these are suppressed by 1/M

Low energy theory

Operator basis

$$\mathcal{L} = \mathcal{L}_{\phi_0} + \mathcal{L}_{\text{SM}} + \mathcal{L}_{\phi_0, \text{SM}} + \dots ,$$

Heavy neutral scalar:

$$\mathcal{L}_{\phi_0} = \phi_{v, Q=0}^* \left\{ i v \cdot \partial - \frac{\partial_{\perp}^2}{2M_{(Q=0)}} + \mathcal{O}(1/m_W^3) \right\} \phi_{v, Q=0}$$

SM interactions:

$$\mathcal{L}_{\phi_0, \text{SM}} = \frac{1}{m_W^3} \phi_v^* \phi_v \left\{ \sum_q \left[c_{1q}^{(0)} O_{1q}^{(0)} + c_{1q}^{(2)} v_{\mu} v_{\nu} O_{1q}^{(2)\mu\nu} \right] + c_2^{(0)} O_2^{(0)} + c_2^{(2)} v_{\mu} v_{\nu} O_2^{(2)\mu\nu} \right\} + \dots$$

Convenient to choose basis of definite spin

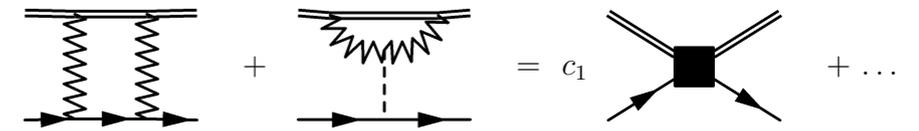
$$\begin{aligned} O_{1q}^{(0)} &= m_q \bar{q} q , & O_2^{(0)} &= (G_{\mu\nu}^A)^2 , \\ O_{1q}^{(2)\mu\nu} &= \bar{q} \left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i \not{D} \right) q , & O_2^{(2)\mu\nu} &= -G^{A\mu\lambda} G^{A\nu}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G_{\alpha\beta}^A)^2 . \end{aligned}$$

Matching ($\mu \approx m_W$)

quark operators

$$c_{1U}^{(0)}(\mu_t) = \mathcal{C} \left[-\frac{1}{x_h^2} \right], \quad c_{1D}^{(0)}(\mu_t) = \mathcal{C} \left[-\frac{1}{x_h^2} - |V_{tD}|^2 \frac{x_t}{4(1+x_t)^3} \right],$$

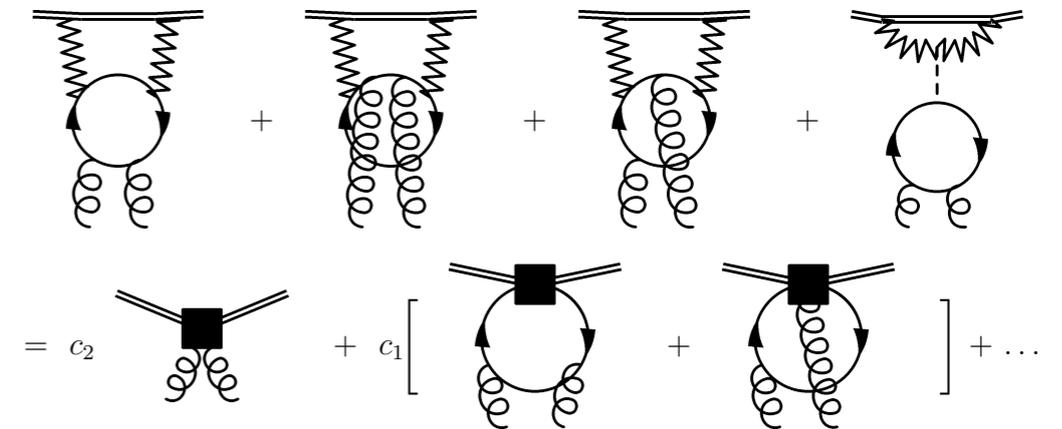
$$c_{1U}^{(2)}(\mu_t) = \mathcal{C} \left[\frac{2}{3} \right], \quad c_{1D}^{(2)}(\mu_t) = \mathcal{C} \left[\frac{2}{3} - |V_{tD}|^2 \frac{x_t(3+6x_t+2x_t^2)}{3(1+x_t)^3} \right],$$



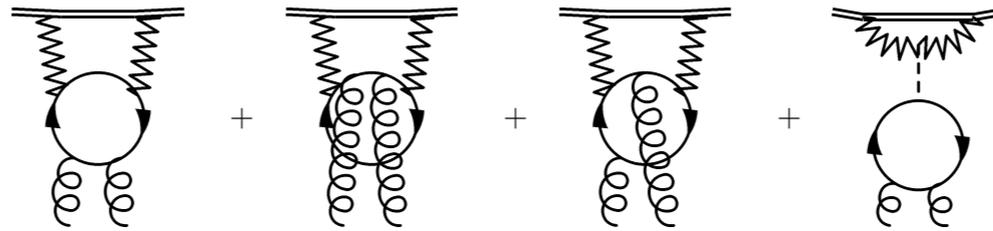
gluon operators

$$c_2^{(0)}(\mu_t) = \mathcal{C} \frac{\alpha_s(\mu_t)}{4\pi} \left[\frac{1}{3x_h^2} + \frac{3+4x_t+2x_t^2}{6(1+x_t)^2} \right],$$

$$c_2^{(2)}(\mu_t) = \mathcal{C} \frac{\alpha_s(\mu_t)}{4\pi} \left[-\frac{32}{9} \log \frac{\mu_t}{m_W} - 4 - \frac{4(2+3x_t)}{9(1+x_t)^3} \log \frac{\mu_t}{m_W(1+x_t)} \right. \\ \left. - \frac{4(12x_t^5 - 36x_t^4 + 36x_t^3 - 12x_t^2 + 3x_t - 2)}{9(x_t - 1)^3} \log \frac{x_t}{1+x_t} - \frac{8x_t(-3+7x_t^2)}{9(x_t^2 - 1)^3} \log 2 \right. \\ \left. - \frac{48x_t^6 + 24x_t^5 - 104x_t^4 - 35x_t^3 + 20x_t^2 + 13x_t + 18}{9(x_t^2 - 1)^2(1+x_t)} \right].$$



Heavy particle Feynman rules simplify matching calculations



$$i\mathcal{M} = -g_2^2 \int (dL) \left[\frac{1}{-v \cdot L + i0} + \frac{1}{v \cdot L + i0} \right] \frac{1}{(L^2 - m_W^2 + i0)^2} v_\mu v_\nu \Pi^{\mu\nu}(L)$$

electroweak polarizability tensor
in background gluon field

Electroweak gauge invariance is immediate:

$$v^\mu \left[g_{\mu\mu'} - (1 - \xi) \frac{L_\mu L_{\mu'}}{L^2 - \xi m_W^2} \right] = v_{\mu'} + \mathcal{O}(v \cdot L)$$

crossed and uncrossed diagrams cancel

gluon Fock-Schwinger gauge ($x \cdot A = 0$) in dim.reg.:

$$iS(p) = \frac{i}{\not{p} - m} + g \int (dq) \frac{i}{\not{p} - m} i\mathcal{A}(q) \frac{i}{\not{p} - \not{q} - m} \\ + g^2 \int (dq_1)(dq_2) \frac{i}{\not{p} - m} i\mathcal{A}(q_1) \frac{i}{\not{p} - \not{q}_1 - m} i\mathcal{A}(q_2) \frac{i}{\not{p} - \not{q}_1 - \not{q}_2 - m} + \dots$$

Solution to RG equations

$$O_{1q}^{(0)} = m_q \bar{q}q,$$

$$O_2^{(0)} = (G_{\mu\nu}^A)^2,$$

$$O_{1q}^{(2)\mu\nu} = \bar{q} \left(\gamma^{\{\mu} i D^{\nu\}} - \frac{1}{d} g^{\mu\nu} i \not{D} \right) q,$$

$$O_2^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}_{\lambda} + \frac{1}{d} g^{\mu\nu} (G_{\alpha\beta}^A)^2.$$

$$\frac{d}{d \log \mu} O_i^{(S)} = - \sum_j \gamma_{ij}^{(S)} O_j$$

$$\frac{d}{d \log \mu} c_i^{(S)} = \sum_j \gamma_{ji}^{(S)} c_j^{(S)}$$

Spin 0:

$$c_2^{(0)}(\mu) = c_2^{(0)}(\mu_t) \frac{\frac{\beta}{g}[\alpha_s(\mu)]}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

$$\hat{\gamma}^{(0)} = \left(\begin{array}{ccc|c} 0 & & & 0 \\ & \ddots & & \vdots \\ & & 0 & 0 \\ \hline -2\gamma'_m & \cdots & -2\gamma'_m & (\beta/g)' \end{array} \right)$$

$$c_1^{(0)}(\mu) = c_1^{(0)}(\mu_t) - 2[\gamma_m(\mu) - \gamma_m(\mu_t)] \frac{c_2^{(0)}(\mu_t)}{\frac{\beta}{g}[\alpha_s(\mu_t)]}$$

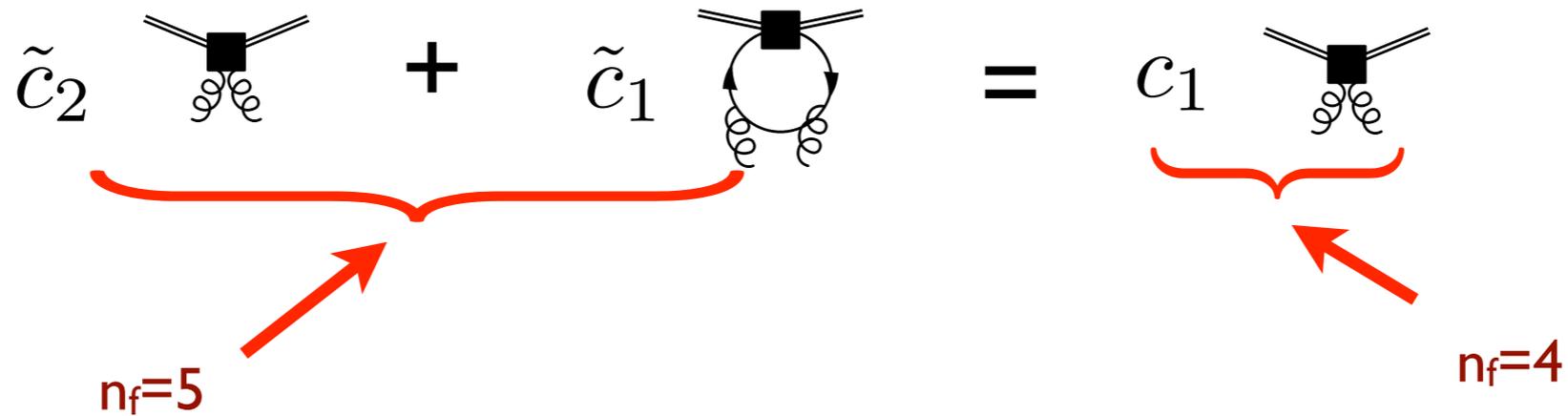
Spin 2:

Diagonalize anomalous dimension matrix
(familiar from PDF analysis)

As check, can evaluate spin-2 matrix elements at high scale (spin-0 and spin-2 decoupled)

$$\hat{\gamma}^{(2)} = \frac{\alpha_s}{4\pi} \left(\begin{array}{ccc|c} \frac{64}{9} & & & -\frac{4}{3} \\ & \ddots & & \vdots \\ & & \frac{64}{9} & -\frac{4}{3} \\ \hline -\frac{64}{9} & \cdots & -\frac{64}{9} & \frac{4n_f}{3} \end{array} \right) + \dots$$

Integrate out heavy quarks ($\mu \approx m_b$)



$$c_2^{(0)}(\mu_b) = \tilde{c}_2^{(0)}(\mu_b) \left(1 + \frac{4\tilde{a}}{3} \log \frac{m_b}{\mu_b} \right) - \frac{\tilde{a}}{3} \tilde{c}_{1b}^{(0)}(\mu_b) \left[1 + \tilde{a} \left(11 + \frac{4}{3} \log \frac{m_b}{\mu_b} \right) \right] + \mathcal{O}(\tilde{a}^3)$$

$$c_{1q}^{(0)}(\mu_b) = \tilde{c}_{1q}^{(0)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

$$c_2^{(2)}(\mu_b) = \tilde{c}_2^{(2)}(\mu_b) - \frac{4\tilde{a}}{3} \log \frac{m_b}{\mu_b} \tilde{c}_{1b}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}^2),$$

$$c_{1q}^{(2)}(\mu_b) = \tilde{c}_{1q}^{(2)}(\mu_b) + \mathcal{O}(\tilde{a}),$$

Contribution to gluon operators familiar from $h \rightarrow gg$

Heavy quark mass scheme enters at higher order

Charm quark treated similarly (after running to m_c)

Spin - 0

$$\langle N(k) | T^{\mu\nu} | N(k) \rangle = \frac{k^\mu k^\nu}{m_N} = \frac{1}{m_N} \left(k^\mu k^\nu - \frac{1}{4} g^{\mu\nu} m_N^2 \right) + m_N \frac{1}{4} g^{\mu\nu}$$

Spin-0 operators determine contributions to nucleon mass

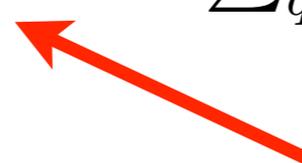
$$m_N = (1 - \gamma_m) \sum_q \langle N | m_q \bar{q}q | N \rangle + \frac{\beta}{2g} \langle N | (G_{\mu\nu}^a)^2 | N \rangle$$

$$\langle N | O_{1q}^{(0)} | N \rangle \equiv m_N f_{q,N}^{(0)}, \quad \frac{-9\alpha_s(\mu)}{8\pi} \langle N | O_2^{(0)}(\mu) | N \rangle \equiv m_N f_{G,N}^{(0)}(\mu)$$

significant uncertainty in this quantity



$$m_N (f_{u,N}^{(0)} + f_{d,N}^{(0)}) \approx \Sigma_{\pi N}, \quad m_N f_{s,N}^{(0)} = \frac{m_s}{m_u + m_d} (\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$

$$f_{G,N}^{(0)}(\mu) \approx 1 - \sum_{q=u,d,s} f_{q,N}^{(0)}$$


but NLO, NNLO corrections significant

Spin - 2

$$\langle N(k) | T^{\mu\nu} | N(k) \rangle = \frac{k^\mu k^\nu}{m_N} = \frac{1}{m_N} \left(k^\mu k^\nu - \frac{1}{4} g^{\mu\nu} m_N^2 \right) + m_N \frac{1}{4} g^{\mu\nu}$$

Spin-2 operators determine momentum fraction carried by partons

$$\langle N | O_{1q}^{(2)\mu\nu}(\mu) | N \rangle \equiv \frac{1}{m_N} \left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4} m_N^2 \right) f_{q,N}^{(2)}(\mu)$$

$$\langle N | O_2^{(2)\mu\nu}(\mu) | N \rangle \equiv \frac{1}{m_N} \left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4} m_N^2 \right) f_{G,N}^{(2)}(\mu)$$

$\mu(\text{GeV})$	$f_{u,p}^{(2)}(\mu)$	$f_{d,p}^{(2)}(\mu)$	$f_{s,p}^{(2)}(\mu)$	$f_{G,p}^{(2)}(\mu)$
1.0	0.404(6)	0.217(4)	0.024(3)	0.36(1)
1.2	0.383(6)	0.208(4)	0.027(2)	0.38(1)
1.4	0.370(5)	0.202(4)	0.030(2)	0.40(1)

[MSTW 0901.0002]

$$f_{q,p}^{(2)}(\mu) = \int_0^1 dx x [q(x, \mu) + \bar{q}(x, \mu)]$$

Approximate isospin symmetry:

$$f_{u,n}^{(2)} = f_{d,p}^{(2)}, \quad f_{d,n}^{(2)} = f_{u,p}^{(2)}, \quad f_{s,n}^{(2)} = f_{s,p}^{(2)}$$

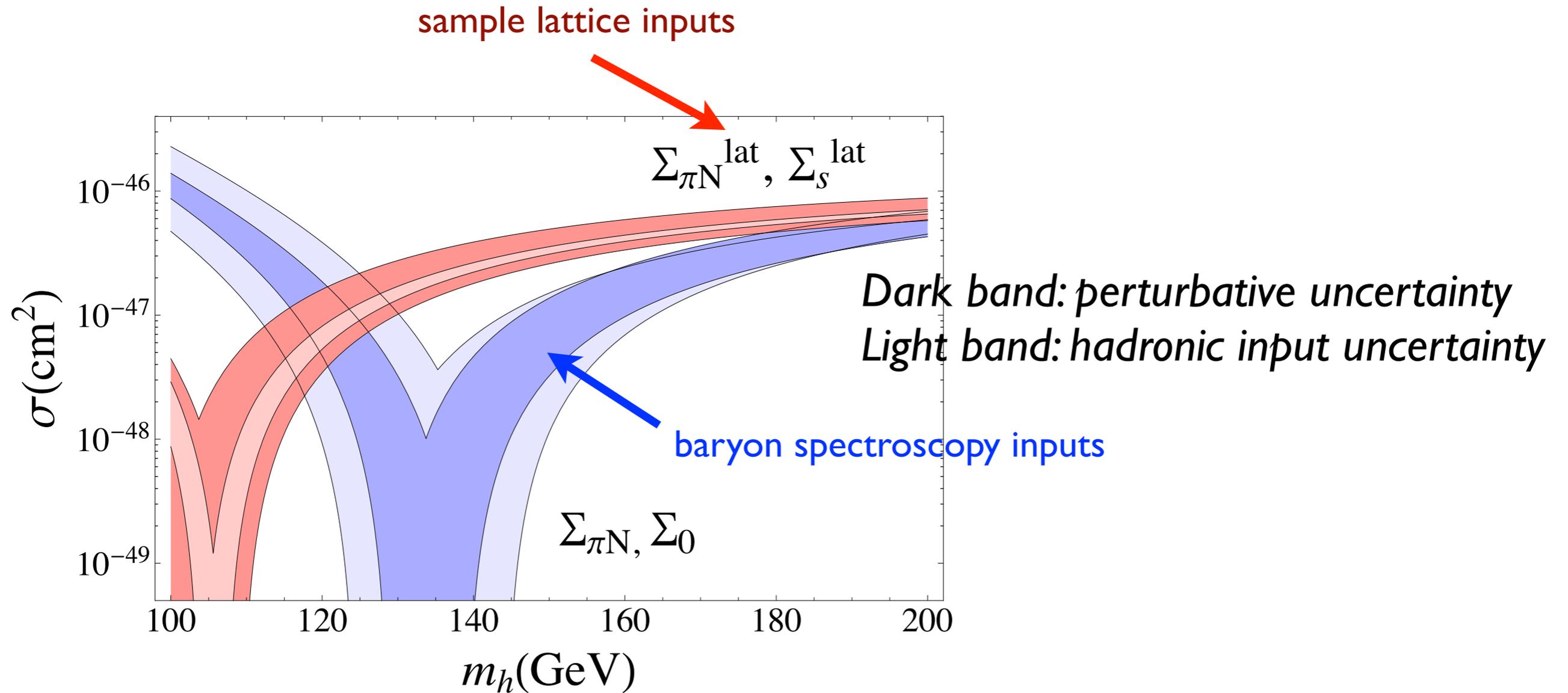
Numerical benchmark: low velocity, spin independent cross section on nucleon

Parameter	Value
$ V_{td} $	~ 0
$ V_{ts} $	~ 0
$ V_{tb} $	~ 1
m_u/m_d	0.49(13)
m_s/m_d	19.5(2.5)
$\Sigma_{\pi N}^{\text{lat}}$	0.047(9) GeV
Σ_s^{lat}	0.050(8) GeV
$\Sigma_{\pi N}$	0.064(7) GeV
Σ_0	0.036(7) GeV
m_W	80.4 GeV
m_t	172 GeV
m_b	4.75 GeV
m_c	1.4 GeV
m_N	0.94 GeV
$\alpha_s(m_Z)$	0.118
$\alpha_2(m_Z)$	0.0338
m_h	?

Cross section is completely determined, given standard model inputs

Consider result as a function of higgs boson mass

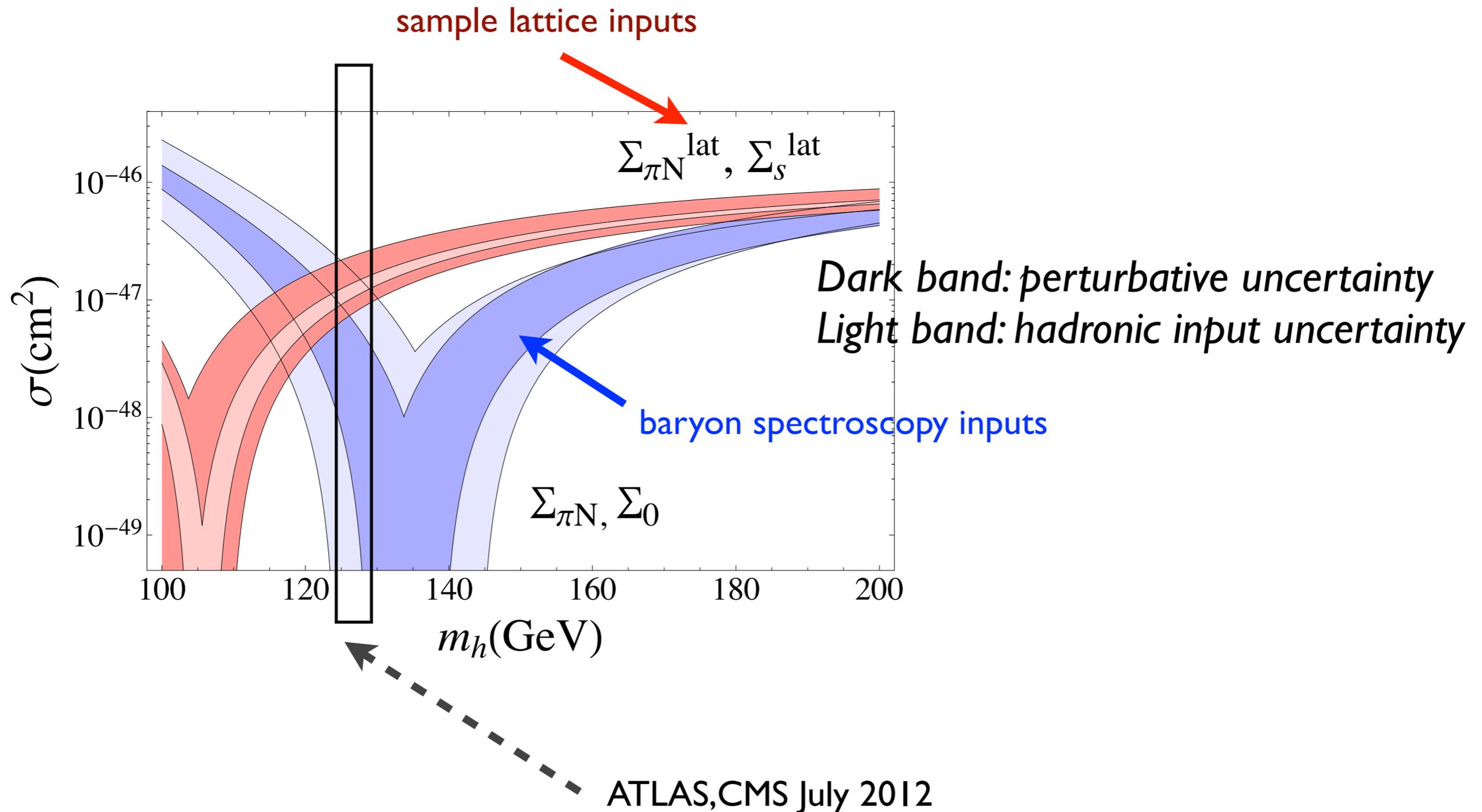
Numerical benchmark: low velocity, spin independent cross section on nucleon



$$\sigma \sim \frac{\pi \alpha_2^4 m_N^4}{m_W^2} \left(\frac{1}{m_W^2} + \frac{1}{m_h^2} \right)^2 \sim 10^{-44} \text{ cm}^2$$

Previous estimates range over several orders of magnitude, errors not specified

Numerical benchmark: low velocity, spin independent cross section on nucleon

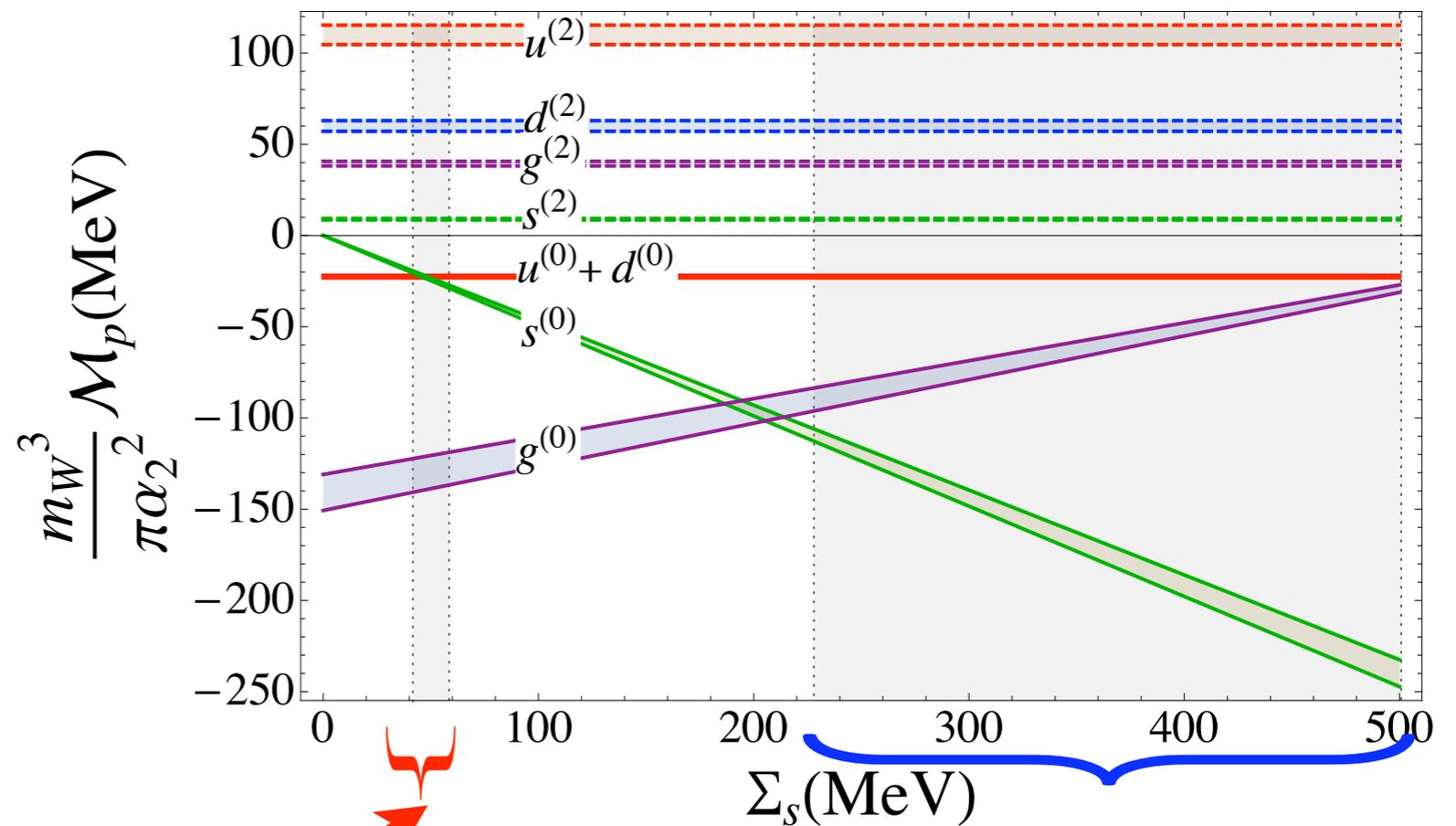


$$\sigma \sim \frac{\pi \alpha_2^4 m_N^4}{m_W^2} \left(\frac{1}{m_W^2} + \frac{1}{m_h^2} \right)^2 \sim 10^{-44} \text{ cm}^2$$

Previous estimates range over several orders of magnitude, errors not specified

Strange quark scalar matrix element dependence

strange matrix element
(and correlated gluon
matrix element) a
prominent uncertainty

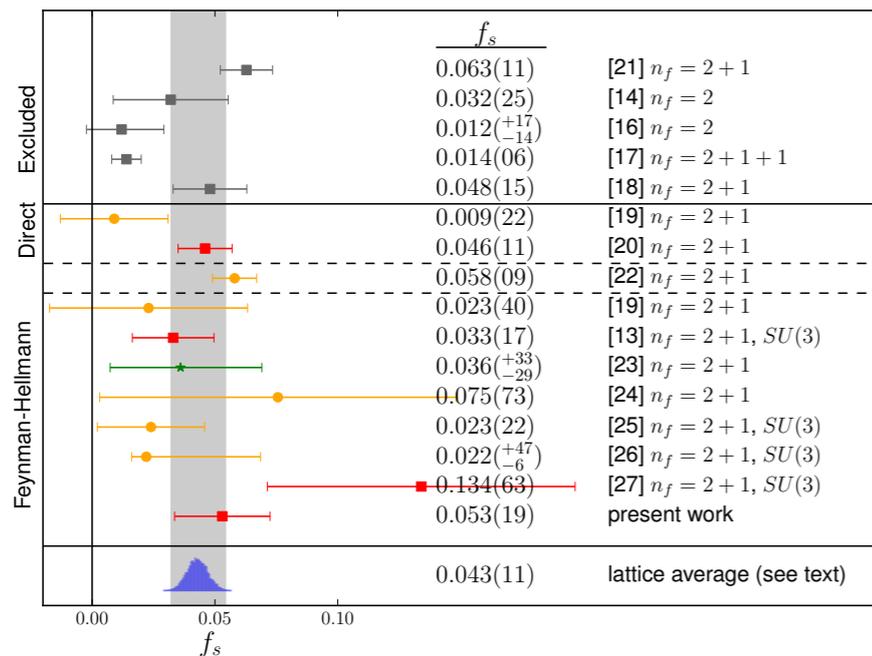


sample lattice inputs

baryon spectroscopy inputs

Nucleon matrix elements

- strange quark scalar matrix element the subject of controversy

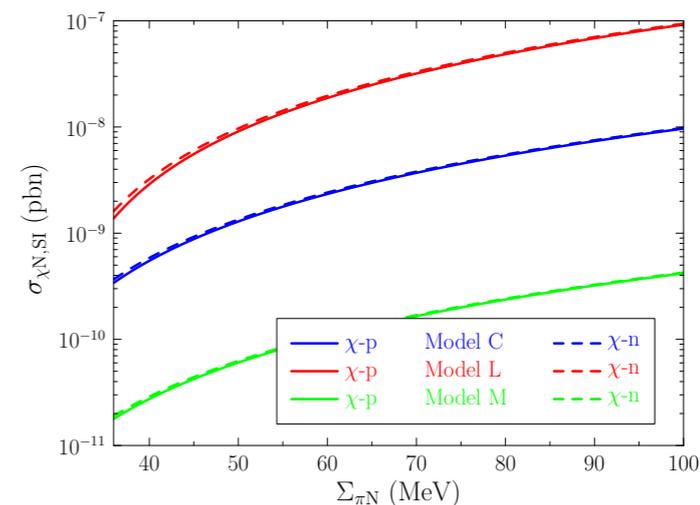


summary plot: Junnarkar and Walker-Loud, I301.1114

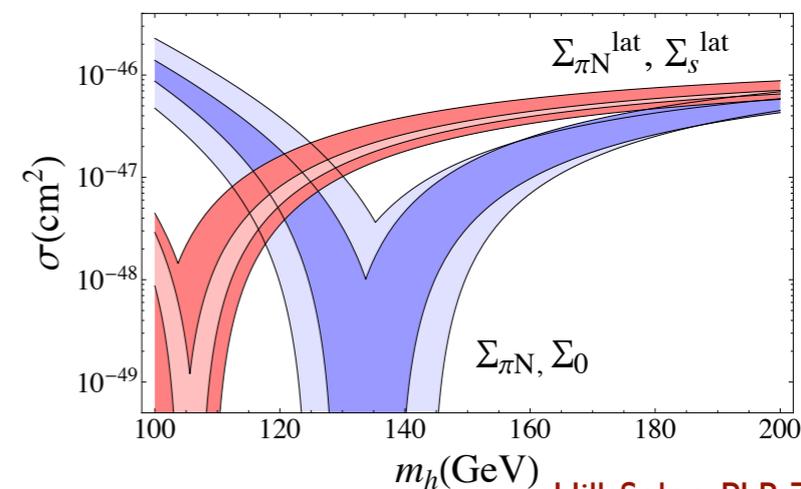
$$\Sigma_{\pi N} = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d) | p \rangle \quad \Sigma_0 = \frac{m_u + m_d}{2} \langle p | (\bar{u}u + \bar{d}d - 2\bar{s}s) | p \rangle$$

$$m_N (f_{u,N}^{(0)} + f_{d,N}^{(0)}) \approx \Sigma_{\pi N}$$

$$m_N f_{s,N}^{(0)} = \frac{m_s}{m_u + m_d} (\Sigma_{\pi N} - \Sigma_0) = \Sigma_s$$



Ellis, Olive, Savage, PRD 77 065026 (2008)



Hill, Solon, PLB 707 539 (2012)

- lattice results still noisy but converging on small value compared traditional SU(3) Ch.P.T. (cf. Alarcon et al., I209.2870)

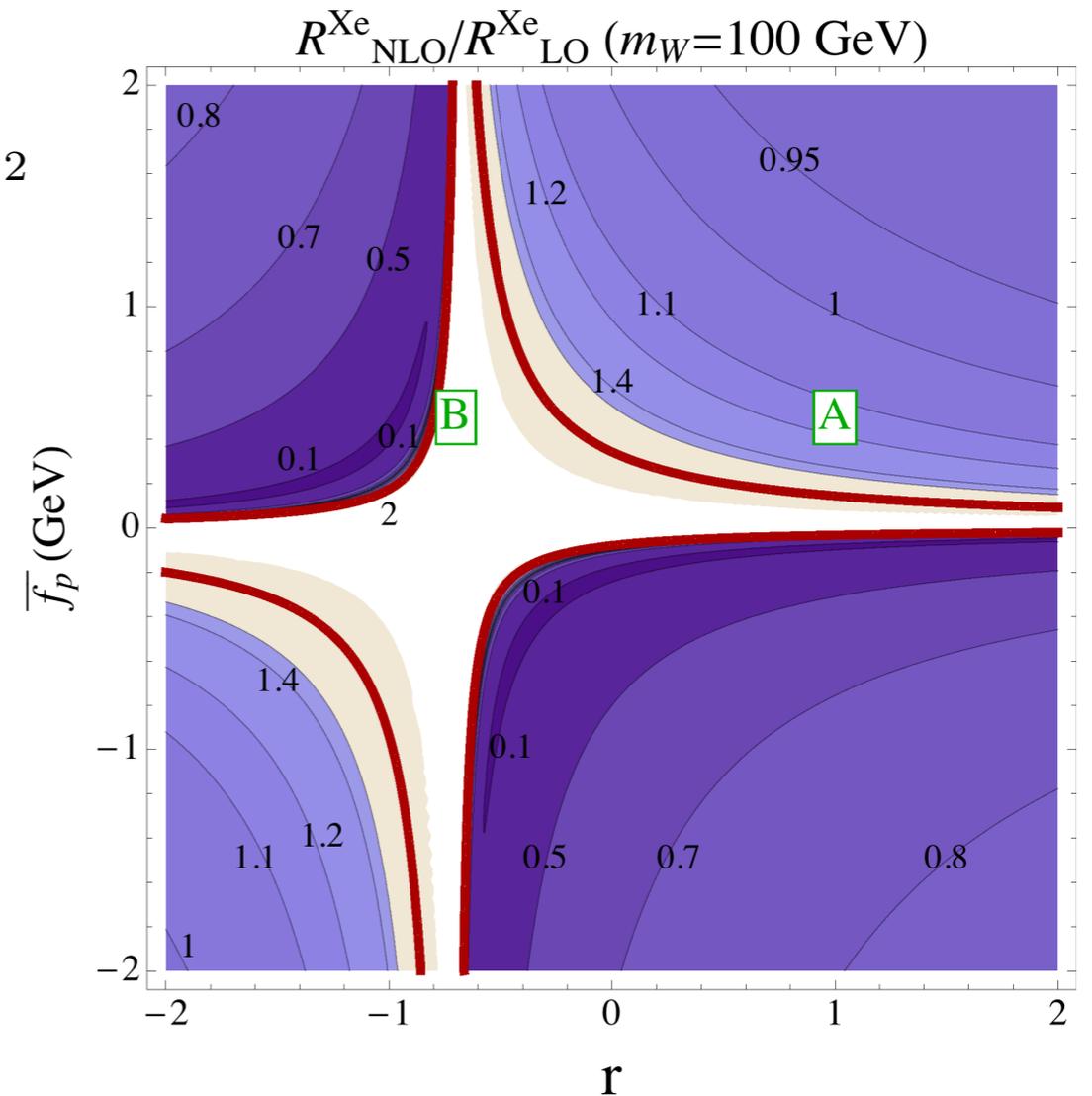
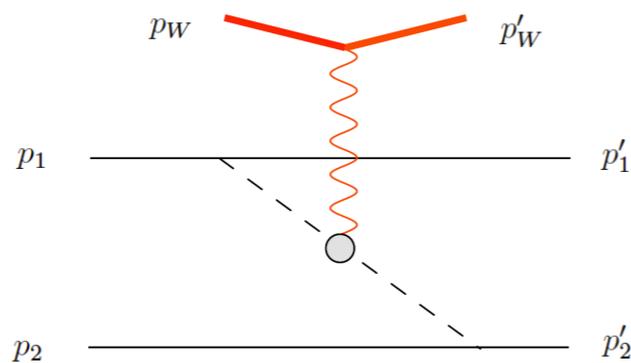
- beneficial to also have lattice constraints on charm scalar matrix element

Nuclear matrix elements

- simplest spin-independent, isospin-symmetric cross section uncontroversial

$$\sigma_{A,Z} \approx \frac{m_r^2}{\pi} |Z\mathcal{M}_p + (A-Z)\mathcal{M}_n|^2 \sim \frac{m_r^2 A^2}{\pi} |\mathcal{M}_p|^2$$

- two-body and higher operators break nucleon x nuclear factorization: can be significant when cancellations occur, e.g., large isospin violation



Cirigliano, Graesser, Ovanesyan, JHEP 1210 025 (2012)

summary

- WIMP paradigm a plausible extension of the SM
- circa Feb 2013, we know things now we didn't know then: (strong indication of SM-like higgs, nothing else yet definitive from LHC)
- heavy particle methods essential tool for controlled computations
 - illustrated with “wino”-like DM, extends to e.g., bino/wino/higgsino and other SM extensions
- careful analysis necessary to robustly connect models and cross sections, and to isolate universal behavior

**thanks for your
attention**

Universal mass shift induced by EWSB

$$-i\Sigma(p) = p \left[\begin{array}{c} W \\ \text{---} \\ \text{---} \end{array} \right] + \begin{array}{c} Z \\ \text{---} \\ \text{---} \end{array} + \begin{array}{c} \gamma \\ \text{---} \\ \text{---} \end{array} + \dots$$

$$-i\Sigma_2(v \cdot p) = -g_2^2 \int \frac{d^d L}{(2\pi)^L} \frac{1}{v \cdot (L+p)} \left[J^2 \frac{1}{L^2 - m_W^2} + J_3^2 \left(\frac{c_W^2}{L^2 - m_Z^2} - \frac{1}{L^2 - m_W^2} + \frac{s_W^2}{L^2} \right) \right] + \mathcal{O}(1/M)$$

heavy particle Feynman rules

$$\delta M = \Sigma(v \cdot p = 0) = \alpha_2 m_W \left[-\frac{1}{2} J^2 + \sin^2 \frac{\theta_W}{2} J_3^2 \right]$$

$$M_{(Q)} - M_{(Q=0)} = \alpha_2 Q^2 m_W \sin^2 \frac{\theta_W}{2} + \mathcal{O}(1/M) \approx (170 \text{ MeV}) Q^2$$

Different pole masses for each charge eigenstate in low-energy theory (or residual mass terms)