THEORETICAL CHALLENGES FOR MODIFIED GRAVITY MODELS

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Cosmic Frontier Workshop
SLAC
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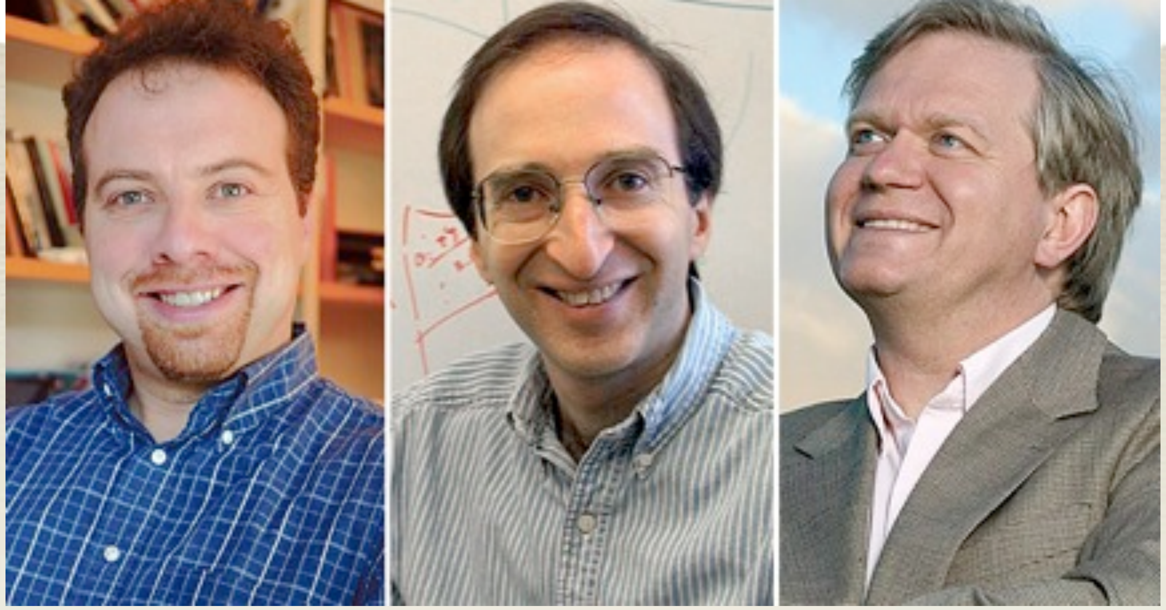
Overview

* Cosmic Acceleration - Dark Energy - Modified Gravity
* Categorizing Models of Modified Gravity/Dark Energy
* Example Theoretical Constructions and Theoretical Issues

Cosmic Acceleration



2011 Nobel Prize in Physics

Riess, Perlmutter, Schmidt

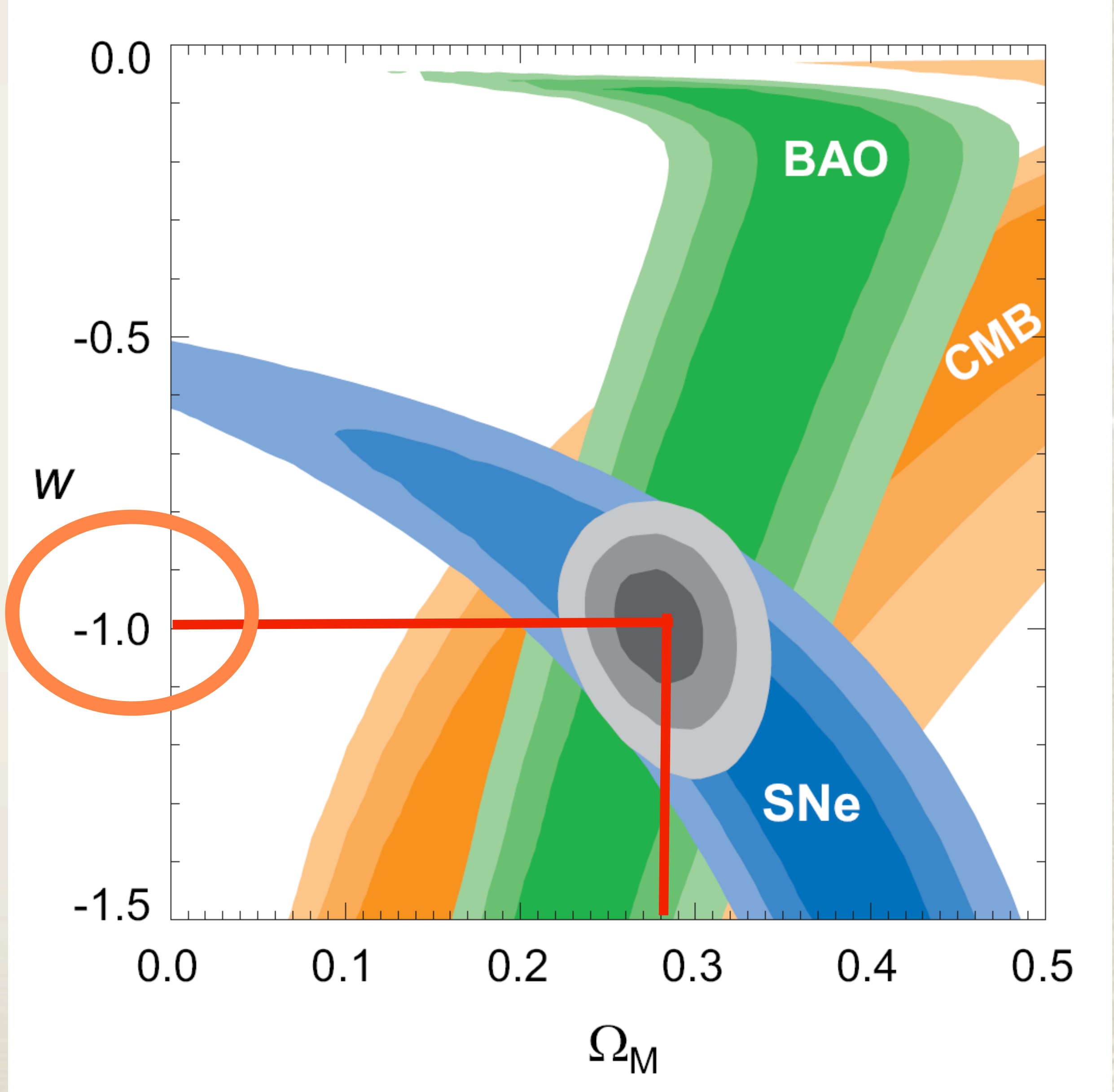
The Universe is Accelerating!
Acceleration can only occur if

$$w = \frac{p}{\rho} < -1/3$$

Data points tantalizingly close to

$$w = -1$$

$$w = -0.94 \pm 0.1$$



Frieman *et al.* (2008) Ann.Rev.Astron.Astrophys

Why are we so concerned?

Dark Energy?

Modified Gravity?

Cosmological constant?

New physics at Hubble scales?

New physics at a millimeter scales?

Cosmic Coincidence Problem

Cosmological Constant Problem

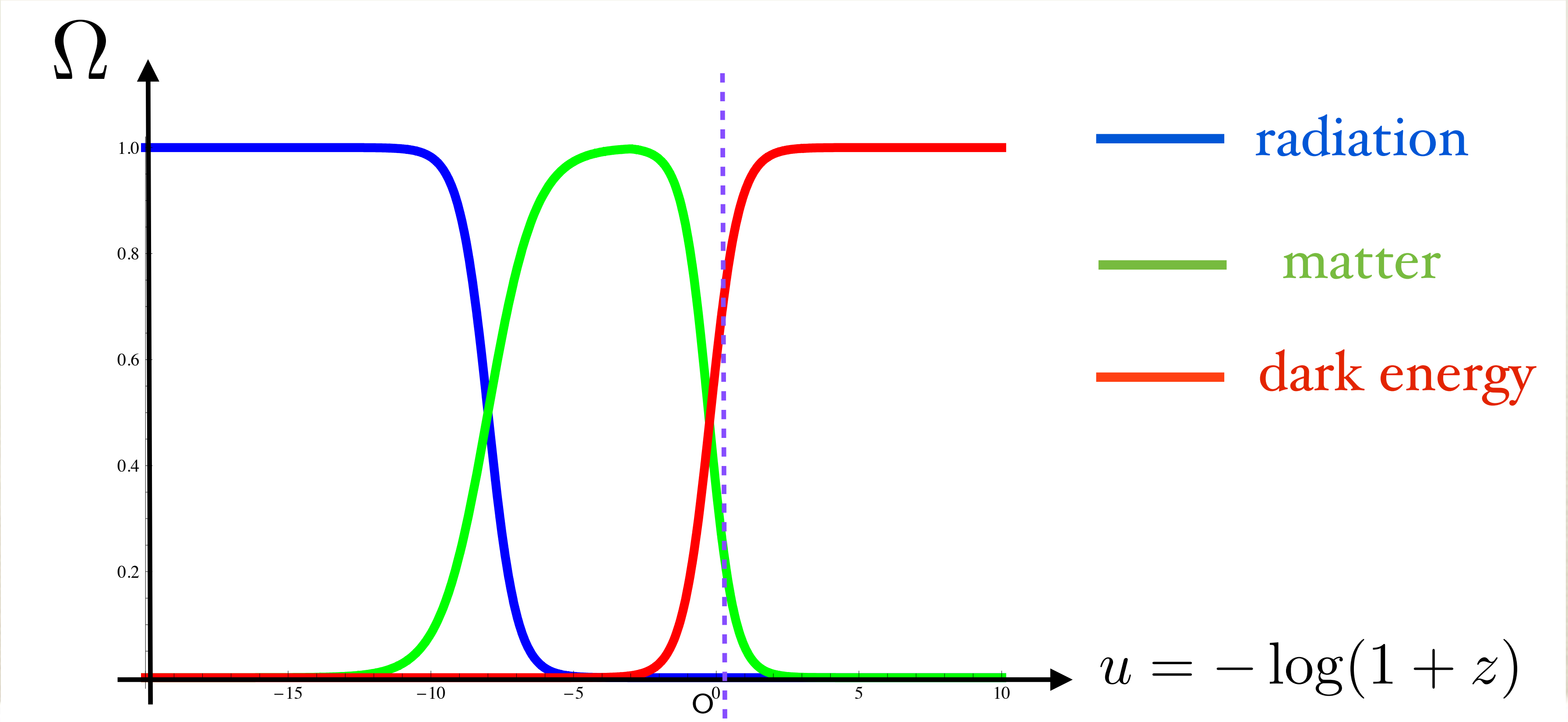
Cosmological constant problem

Why does dark energy come to dominate today?
around the time of structure formation?

Also seems coincidental that the amount of visible and dark matter are only a few orders of magnitude away from each other today

Universe began accelerating about redshift z~0.4 and age 10 Gyr

$$\frac{\Omega_{\mathrm{d.e.}}}{\Omega_{\mathrm{M}}} \sim a^3$$



Cosmological constant problem - loop corrections

C.C. is leading 'relevant operator' in action for gravity

Despite being most relevant operator: most UV sensitive!

$$\Delta\rho_\Delta = \frac{8\pi G}{3}\Delta\Lambda \sim \sum m_i^4 \ln(m_i/\mu)$$

$m_e^4/\rho \sim 10^{36}$ $m_W^4/\rho \sim 10^{56}$

Why doesn't Lambda pick up a large contribution from Phase Transitions?

Potential energy of Higgs field $V \sim (100GeV)^4$

QCD condensate energy in presence of qqbar bilinears
(chiral symmetry breaking) $V \sim (100MeV)^4$

What are the alternatives to General Relativity?

General Relativity is a beautiful well understood theory that has so far passed all known tests

But General Relativity does not naturally predict an accelerating universe! unless we put in some CRAZY matter/ energy

In recent years lots of work by cosmologists on possible extensions to general relativity -

e.g. change the way curvature responds to energy, extra dimensions, strings/branes,
new scalars particles (like the Higgs) which couple to matter ....

All of the proposed ideas fall into a small number of categories

The categories are based on their **screening** mechanisms

New Degrees of Freedom

Theorem: Cosmological constant is the `unique' large distance modification to GR that does not introduce any new degrees of freedom

Dynamical Models of Dark Energy or Modified Gravity will be distinguished by new degrees of freedom

= new particles - new fields - new gravitational waves - new forces - new dynamics

New degrees of freedom must necessarily by incredibly light!

$$m_{\rm d.e.} \leq 10^{-33} eV$$

$$m_{\rm d.e.} = \text{Hubble rate} = 1/\text{Age of universe}$$

Why are new d.o.f. *nearly* always scalars?

If theory Lorentz invariant, new d.o.f characterized by spin

Must be effectively bosonic to act at large distances
(even if fundamentally fermionic)

Massive spin 2 = Massless spin 2 + Massless spin 1 + Scalar
(=GR!)

Massive spin 1 = Massless spin 1 + Scalar

Massless spin 1 must coupled to conserved vector but

Energy and Momentum are conserved $\partial_\mu T^\mu{}_\nu = 0$

Range of energies for which every D.E./modified gravity theory looks like General Relativity plus scalars fields!

DARK ENERGY VERSUS MODIFIED GRAVITY

If new degrees of freedom are MINIMALLY coupled to gravity - we call the model DARK ENERGY

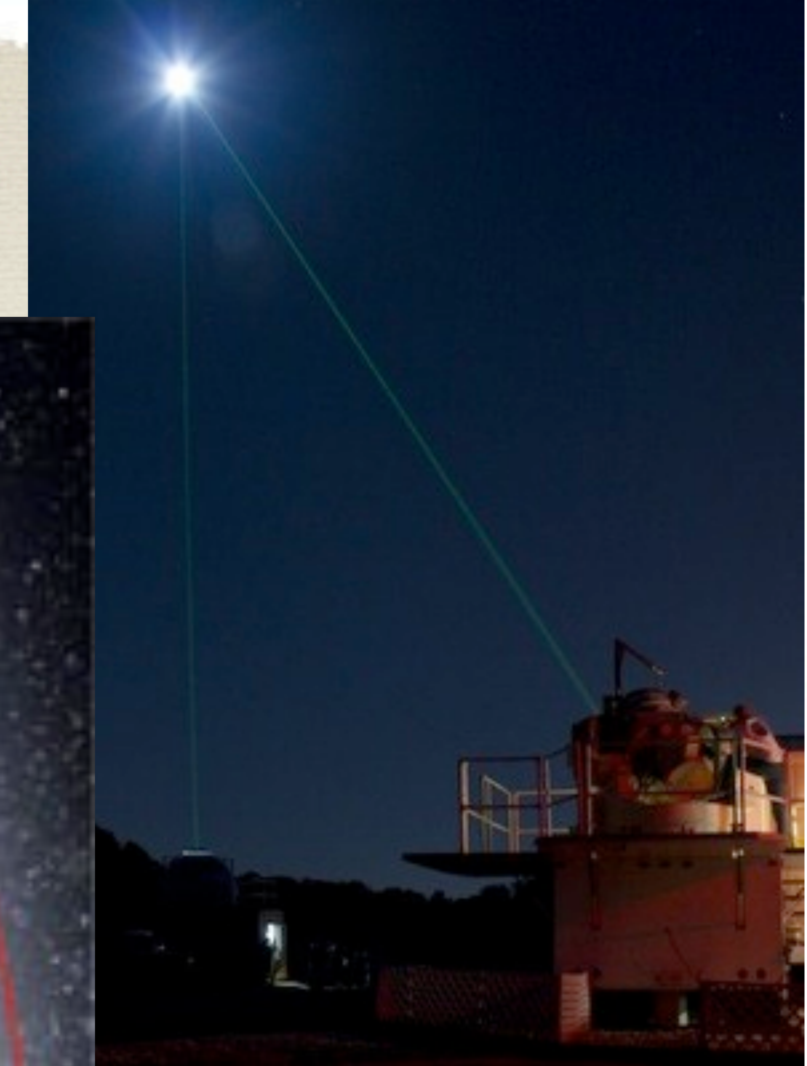
If new degrees of freedom are NON-MINIMALLY coupled to gravity - we call the model MODIFIED GRAVITY

Example - Brans-Dicke, Massive gravity, f(R) - can all be written as Einstein gravity with additional vector and scalar fields which are non-minimally coupled to matter

Even Brane-World models like DGP/Cascading Gravity can be reintepreted in this sense albeit with an infinite tower of extra degrees of freedom

What is NOVEL about modified gravity theories - is that the extra dynamical degrees of freedom have dynamics at cosmological scales, they are very light

New gravitational degrees of freedom that couple to matter (MODIFIED GRAVITY) are highly constrained



Fifth Forces (solar system)

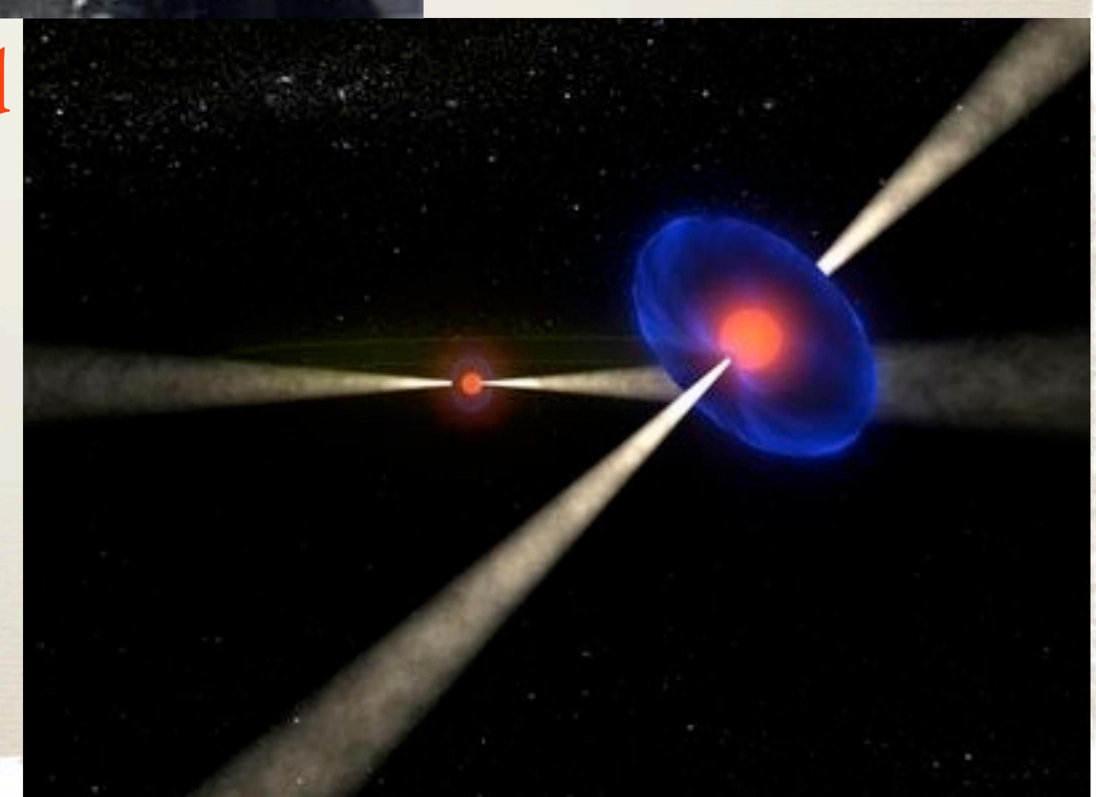
Equivalence Principle Tests etc.

Binary Pulsar Timing

Nucleosynthesis/ Cosmology

Variation of fundamental constants

Need some kind of screening mechanism to hide extra d.o.f.



Interactions of new d.o.f.

Imagine a scalar $\phi = \phi_b + \delta\phi$

coupled to the energy density $\rho = \rho_b + \delta\rho$

Generic form of equation of motion for perturbations

$$Z(\phi_b, \rho_b)\left[\frac{d^2\delta\phi}{dt^2} - c_s^2\frac{d^2\delta\phi}{dx^2}\right] + m^2(\phi_b, \rho_b)\delta\phi = \beta(\phi_b, \rho_b)G_{\mathrm{Newton}}\delta\rho$$

kinetic term

mass term

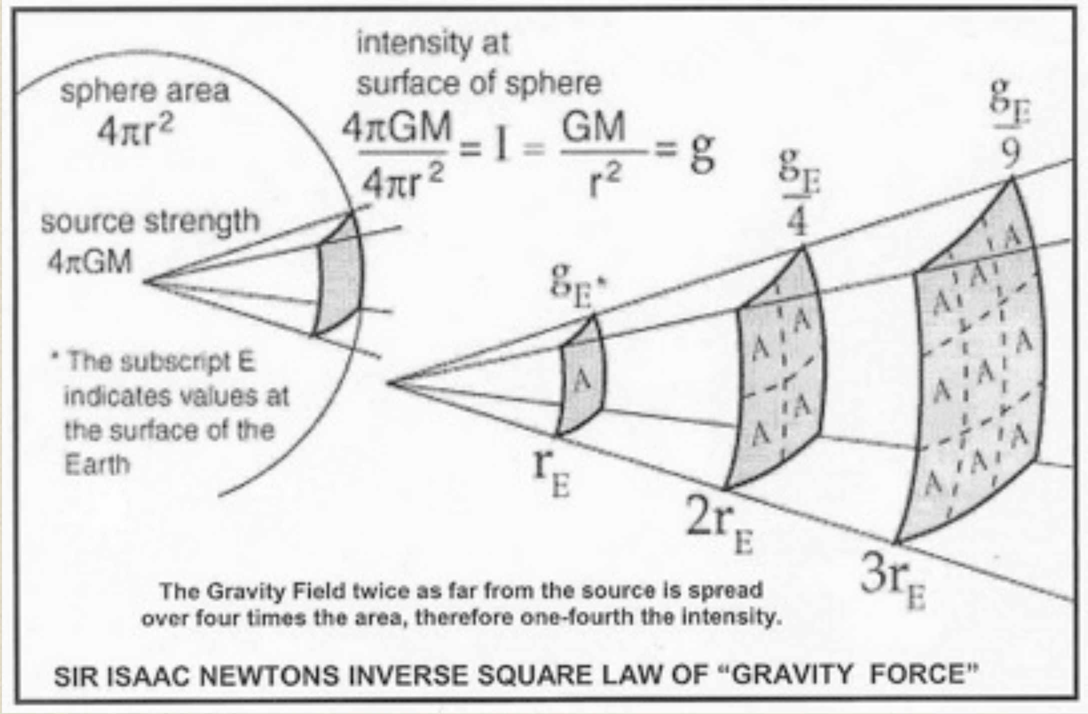
coupling to matter

gradient term

Fifth forces -

Forces beyond the Gravity, Electromagnetic, Weak Force, Strong Force

$$Z(\phi_b, \rho_b)\left[\frac{d^2\delta\phi}{dt^2} - c_s^2\frac{d^2\delta\phi}{dx^2}\right] + m^2(\phi_b, \rho_b)\delta\phi = \beta(\phi_b, \rho_b)G_{\mathrm{Newton}}\delta\rho$$



Force between two masses:

$$F \approx \frac{M_a M_b G}{r^2}\frac{\beta^2(\phi_b, \rho_b)}{\sqrt{Z}(\phi_b, \rho_b)c_s(\phi_b, \rho_b)}\exp(-m(\phi_b, \rho_b)r)$$

Fifth force constraints: screening

$$F \approx \frac{M_a M_b G}{r^2} \frac{\beta^2(\phi_b, \rho_b)}{\sqrt{Z}(\phi_b, \rho_b) c_s(\phi_b, \rho_b)} \exp(-m(\phi_b, \rho_b) r)$$

To ensure fifth forces are small

$$\frac{\beta^2(\phi_b, \rho_b)}{\sqrt{Z(\phi_b, \rho_b)} c_s(\phi_b, \rho_b)} \exp(-m(\phi_b, \rho_b) r)$$

Only three independent possibilities!

(a) Coupling is small $\beta(\phi_b, \rho_b) \ll 1$

(b) Mass is large $m(\phi_n, \rho_b) \gg \frac{1}{r_{exp}}$

(c) Kinetic term is large $Z(\phi_b, \rho_b) \gg 1$

I. Making the coupling small universally

$$\beta(\phi_b, \rho_b) \ll 1$$

Theoretical Models:

Quintessence and its multifarious generalizations!!!

Canonical Example: Scalar field with no direct coupling to matter

$$S = \int d^4x\sqrt{-g}\left(\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right) + S_{\rm m}$$

These are the *Vanilla* models of Dynamical Dark Energy

Dark energy contributes to the background evolution, and plays an indirect role in perturbations, additional isocurvature modes

Quintessence Theoretical Challenges

Typically not technically natural (Eta problem in Inflation) - significantly worse for Dark Energy

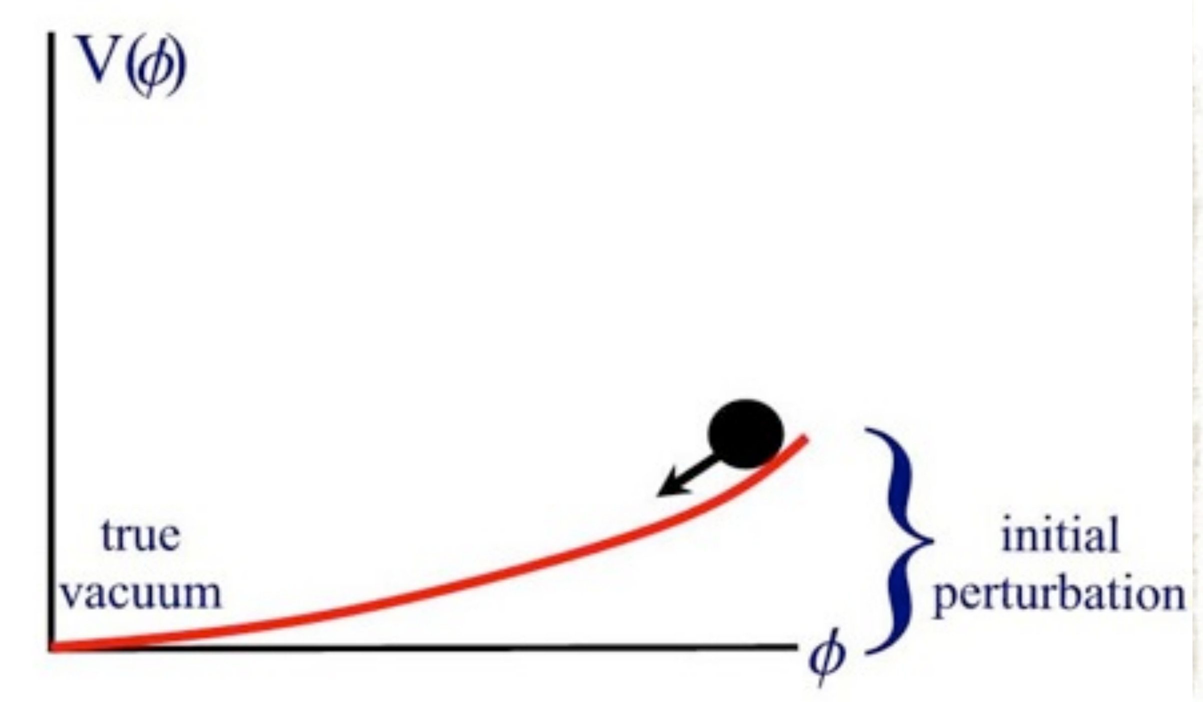
$$\Delta V \sim V(\phi)\frac{\phi^2}{M_{\rm pl}^2}$$

dim 6 operators

Closely akin to Higgs mass/gauge hierarchy problem

mass quadratically divergent, pick up mass comparable to heaviest particle

* Technically natural Scalar Field arises as a *pseudo-Nambu-Goldstone* field associated with an approximately broken continuous global symmetry



II. Making the coupling small environmentally

$$\beta(\phi_b, \rho_b) \ll 1$$

Theoretical Models:

Symmetron

Khoury and Hinterbichler 2010

Consider a scalar with

1. Symmetry
2. Symmetry breaking potential
3. Non-minimal coupling to matter density

example
$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\mu^2\phi^2 + \mathcal{L}_M(g_{\mu\nu}(1+\phi^2/M^2))\right)$$

Z2 symmetry

$$\phi \to -\phi$$

Broken symmetry vev

$$\phi^2 = \mu^2/\lambda$$

Symmetron - effective potential

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{4}\lambda\phi^4 + \frac{1}{2}\mu^2\phi^2 + \mathcal{L}_M(g_{\mu\nu}(1+\phi^2/M^2))\right)$$

As a result of non-minimal coupling, effective potential is

$$V_{\text{eff}}(\phi) = \frac{1}{2}\left(\frac{\rho}{M^2} - \mu^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 \qquad \beta \sim \frac{\phi M_{\text{PL}}}{M^2}$$

At low densities symmetry broken, coupling large

$$\rho < \mu^2 M^2 \qquad \phi \sim \mu^2/\lambda \qquad \beta \sim \frac{\mu^2 M_{\text{Pl}}}{\lambda M^2}$$

At high densities symmetry recovered, coupling small

$$\rho > \mu^2 M^2 \qquad \phi \sim 0 \qquad \beta \sim 0$$

$$M \leq 10^{-3} M_{\text{Pl}} \qquad \mu^{-1} \sim Mpc$$

III. Making mass large environmentally

Theoretical Models: Chameleon, Generalized Branes-Dicke models, f(R)

starts with same idea: $m(\phi_n, \rho_b) \gg \dfrac{1}{r_{exp}}$

Khoury and Weltman, 2003

$$S_{\text{cham}} = \int d^4x\sqrt{-g}\left(\frac{M_{\text{Pl}}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right) + S_{\text{matter}}\left[g\ \ e^{2\beta\phi/M_{\text{Pl}}}\right]$$

$$V_{\text{eff}}(\phi) = V(\phi) + \rho\ e^{\beta\phi/M_{\text{Pl}}}$$

$$m_{\text{eff}}^2 = V_{,\phi\phi}(\phi_{\text{min}}) + \frac{\beta^2}{M_{\text{Pl}}^2}\rho\ e^{\beta\phi/M_{\text{Pl}}}$$

Chameleon effect

$$m_{\text{eff}}^2 = V_{,\phi\phi}(\phi_{\text{min}}) + \frac{\beta^2}{M_{\text{Pl}}^2}\rho\, e^{\beta\phi/M_{\text{Pl}}}$$

Conditions necessary for chameleon mechanism to take place: $\beta > 0$

Balance	Stability	*m* increase with density
$V_{,\phi} < 0$	$V_{,\phi\phi} > 0$	$V_{,\phi\phi\phi} < 0$

easy to satisfy, e.g. $V(\phi) \sim \frac{M^{4+n}}{\phi^n}$

To satisfy fifth force

$$M < 1meV$$

Chameleons Theoretical Challenges

Naively not *technically natural* (see Amol talk)

$$m_\phi \sim \beta \frac{\Lambda_{UV}^2}{M_{\rm pl}} \sim 1meV$$

when $\beta \sim O(1)$

$\Lambda_{UV} \sim TeV$

Adiabatic Instability (for strongly coupled chameleons) Bean et al. 2007

$c_s^2 < 0$ Type of Jeans instability, exponential growth of small scale modes

IV. Making the kinetic term large environmentally

$$Z(\phi_b, \rho_b) \gg 1$$

Theoretical Models:

Vainshtein (or kinetic chameleon) mechanism:

Massive Gravity, DGP, Cascading Gravity, Galileon models and their generalizations!

Mechanism relies on a nontrivial reorganization of effective field theory to allow for large kinetic terms - arguably only natural in the context of massive gravity/DGP/Cascading

Vainshtein (Kinetic Chameleon) effect

$$Z = 1 + \frac{\rho}{\Lambda^3 M_{\rm Pl}} \qquad\qquad \Lambda^3 \sim m^2 M_{\rm Pl}$$

Allow in the action Irrelevant kinetic operators

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{2}(\partial\phi)^2 - \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 + \frac{\phi}{M_{\rm pl}}\rho\right)$$

Expanding around background solution, generates large kinetic term

schematically: $\Box\phi \sim \dfrac{\rho}{M_{\rm pl}} \longrightarrow Z \approx 1 + \dfrac{\rho}{\Lambda^3 M_{\rm Pl}}$

$Z(\phi_b, \rho_b) \gg 1$ *when* $\rho_b \gg \Lambda^3 M_{\rm Pl} \sim m^2 M_{\rm Pl}^2$

Galileon - a model that relies on Vainshtein

Logic: write down every term in action consistent with symmetry

$$\pi \to \pi + c$$
$$\pi \to \pi + v_\mu x^\mu$$

$$\begin{aligned}
\mathcal{L}_1 &= \pi \\
\mathcal{L}_2 &= -\tfrac{1}{2}\,\partial\pi\cdot\partial\pi \\
\mathcal{L}_3 &= -\tfrac{1}{2}\,[\Pi]\,\partial\pi\cdot\partial\pi \\
\mathcal{L}_4 &= -\tfrac{1}{4}\big([\Pi]^2\,\partial\pi\cdot\partial\pi - 2\,[\Pi]\,\partial\pi\cdot\Pi\cdot\partial\pi - [\Pi^2]\,\partial\pi\cdot\partial\pi + 2\,\partial\pi\cdot\Pi^2\cdot\partial\pi\big) \\
\mathcal{L}_5 &= -\tfrac{1}{5}\big([\Pi]^3\,\partial\pi\cdot\partial\pi - 3[\Pi]^2\,\partial\pi\cdot\Pi\cdot\partial\pi - 3[\Pi][\Pi^2]\,\partial\pi\cdot\partial\pi + 6[\Pi]\,\partial\pi\cdot\Pi^2\cdot\partial\pi \\
&\quad +2[\Pi^3]\,\partial\pi\cdot\partial\pi + 3[\Pi^2]\,\partial\pi\cdot\Pi\cdot\partial\pi - 6\,\partial\pi\cdot\Pi^3\cdot\partial\pi\big)
\end{aligned}$$

$$\Pi^\mu_\nu = \partial^\mu\partial_\nu\pi$$

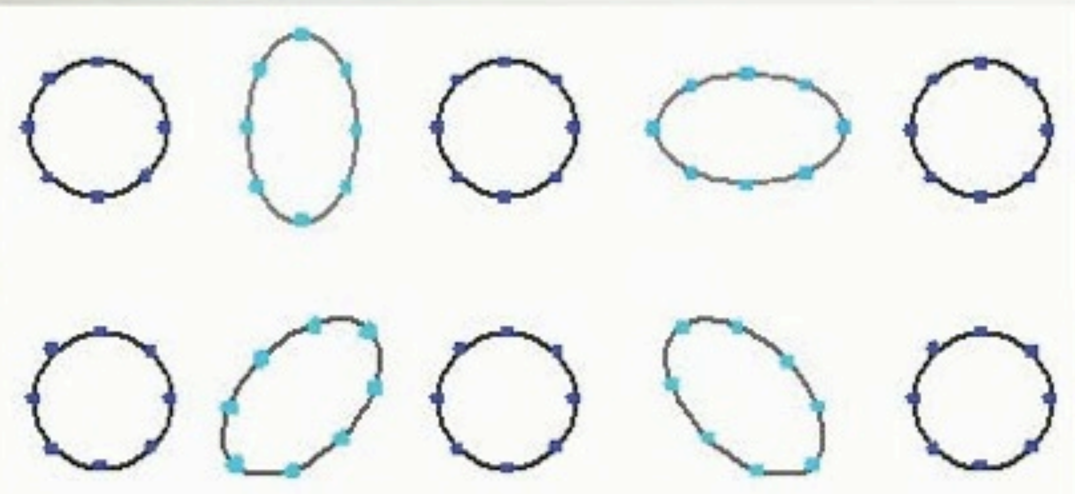
Nicolis et al. 0811.2197

Self-acceleration without ghosts!

Massive Gravity leads a scalar (helicity zero) field

Massive spin-2 field, has 5 dof

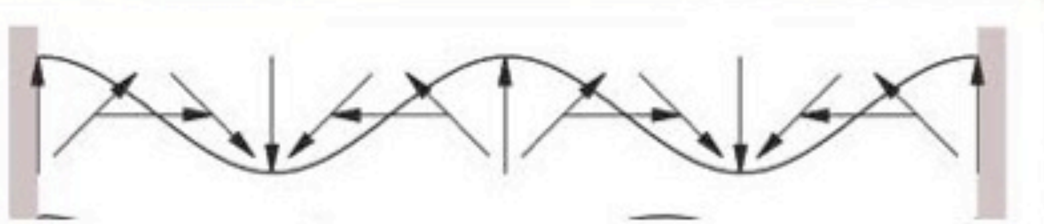
$$h_{\mu\nu} \sim \frac{G_N}{\Box_4 - m^2}\left(T_{\mu\nu} - \frac{1}{3} g_{\mu\nu} T\right)$$



tensor

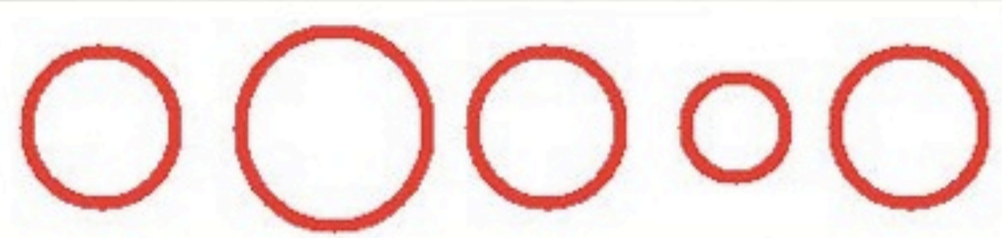
$$\left(\text{in GR its } h_{\mu\nu} \sim \frac{G_N}{\Box_4}\left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T\right)\right)$$

$$2 \oplus 1 \oplus 2$$



vector

$$h_{\mu\nu} = h'_{\mu\nu} + \pi \eta_{\mu\nu}$$

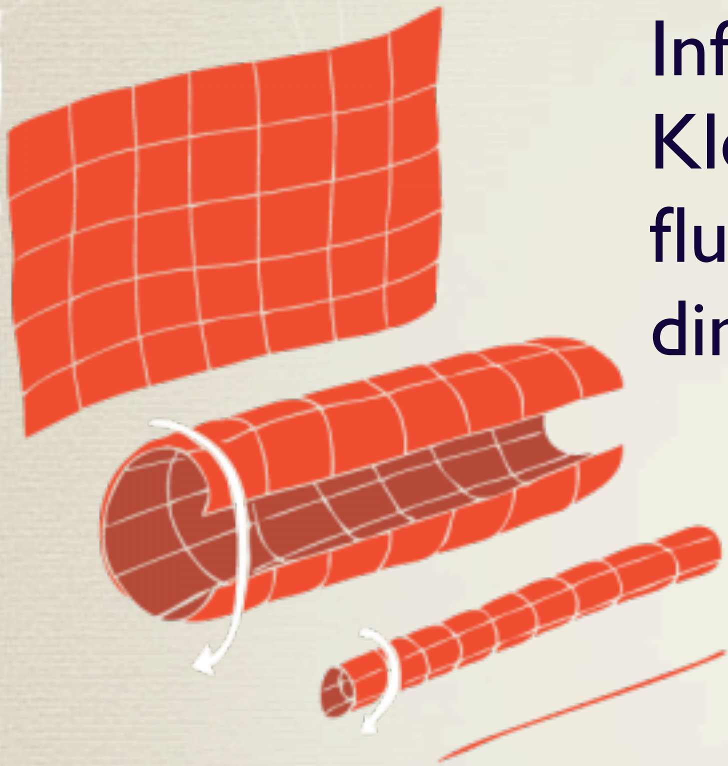


scalar

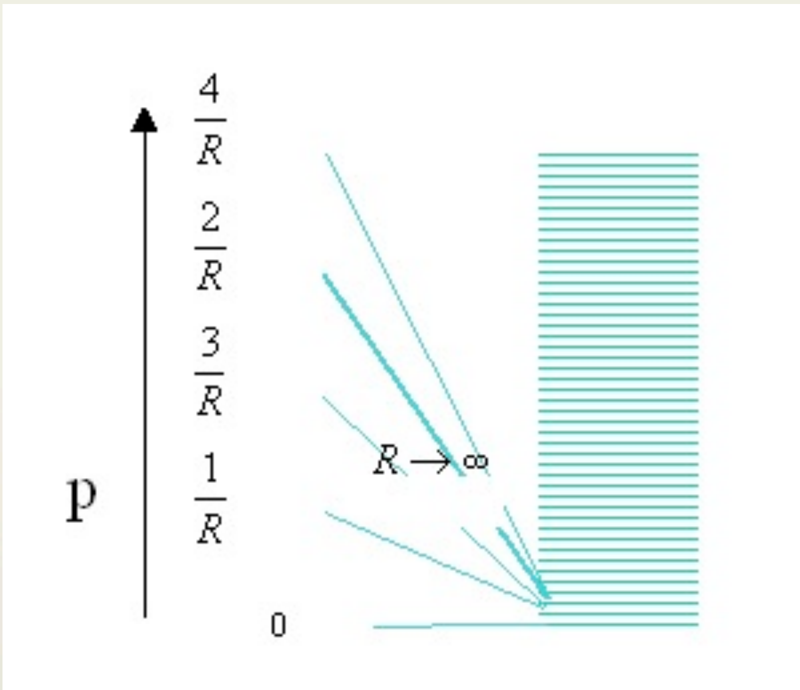
New scalar degree of freedom that couples to the trace of the stress energy momentum tensor

Giving a mass to a graviton is also not 'so' strange because thats what happens in extra dimensions and string theory!

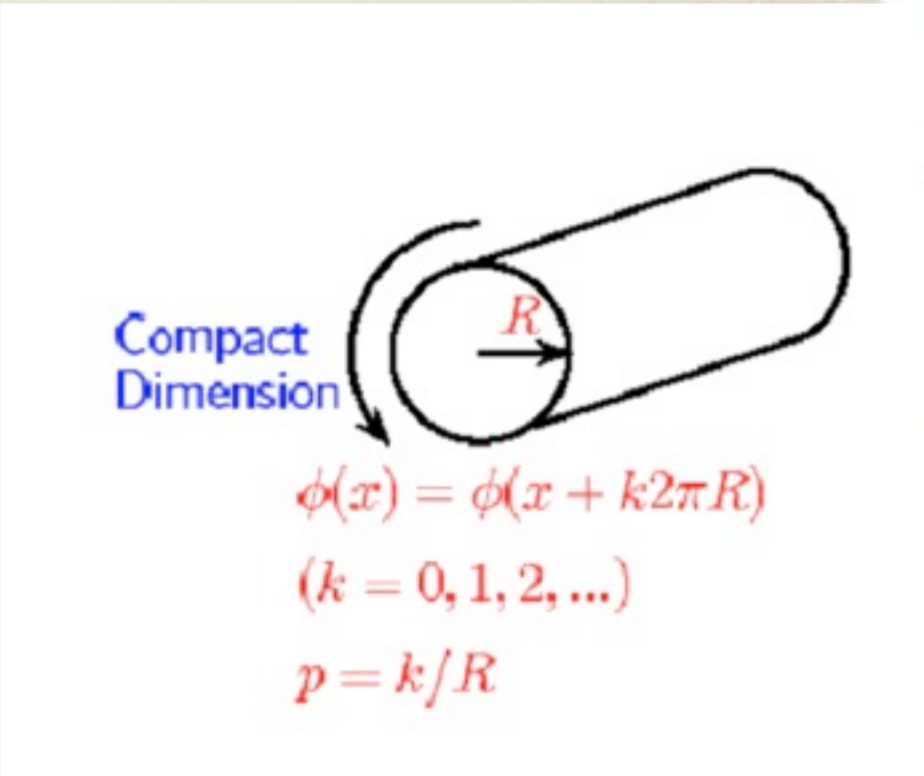
Extra dimensions - Kaluza-Klein theory



Infinite tower of massive Kaluza-Klein particles associated with fluctuations in the extra dimension



masses given by size of extra dimension



Kaluza and Klein

A nonlinear theory of Massive Gravity

Massive Gravity (de Rham-Gabadadze-Tolley)

free of ghosts!

$$S = \frac{M_{\rm Pl}^2}{2} \int d^4x \sqrt{-g} \left(R - \frac{1}{4} m^2 U(g,f) \right)$$

Many extensions and related models in the literature - bigravity, multi-vierbein, massive gravity coupled to DBI-Galileon, New Massive Gravity

Vainshtein effect

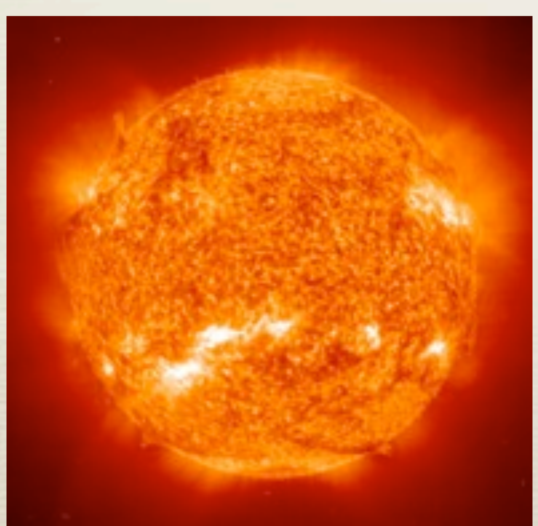
Characteristic radius from source
- Vainshtein radius
- helicity zero version of Schwarzschild radius

Screened region $\quad r \ll r_V \quad Z \gg 1$

$$r_V = (r_s m^{-2})^{1/3} \qquad \Lambda^3 \sim m^2 M_{\rm Pl}$$

Weak coupling region $\quad r \gg r_V \quad Z \sim 1$

For Sun



$$m^{-1} \sim 4000 Mpc$$

$$r_s \sim 3km$$

$$r_V \sim 250pc$$

Theoretical Challenges, open questions

* Do any of these models actually improve on the old cosmological constant problem?

* To what extend do the predictions of these models differ from LCDM ?
most focus on the existence of extra scalars - leads to fifth forces, new gravitational radiation, new dynamics

* How many of these models are simultaneously able to satisfy solar system and astrophysical tests and give interesting cosmological dynamics ?

* Do there exist natural models of chameleon/f(R), Brans Dicke that are stable under quantum corrections? (see Amol's talk)

* Are any of the chameleon, symmetron models embeddable within high energy physics? (eg Kurt Hinterbichler, Justin Khoury, Horatiu Nastase)

Theoretical Challenges, open questions

* All models that rely on the Vainshtein effect e.g. massive gravity - have a low strong coupling scale - does this signal new physics (Wilsonian completion) or can it be understood using novel QFT ideas - dual theory - classicalization ?

* Does the Vainshtein effect work at the quantum level - necessary to understand predictions for submillimeter gravity tests ?

* Einstein gravity is stable in the sense that it satisfies positive energy theorems - modifications to gravity may induce instabilities, ghosts, tachyons, gradient instabilities - how many of these models are sufficiently stable to be plausible frameworks for cosmology ?

* Einstein gravity also has build in well defined causal properties, i.e. well defined Cauchy problem - do all of these models have a well-defined Cauchy problem?

Theoretical Challenges, open questions

* Models with Vainshtein effect are fundamentally nonlinear - is there a simple analogue of the post-Newtonian or post-Friedmannian framework to understand their predictions for the evolution of large scale structure?

* Can we use symmetries or consistency requirements to restrict the number of plausible dark energy/modified gravity models to ease comparison with observations - for instance in massive gravity consistency restricts to a unique number of interactions make the theory practically unalterable without introducing yet further degrees of freedom

* Are there any quintessence of chameleon models which are embeddable to high energy physics frameworks for which the necessary small masses remain technically natural

* To what extend to dark energy/modified gravity models modify early universe physics

A nonlinear theory of Massive Gravity

de Rham-Gabadadze-Tolley (dRGT)
Massive Gravity

free of ghosts!

$$S = \frac{M_{\rm Pl}^2}{2}\int d^4x\sqrt{-g}\left(R - \frac{1}{4}m^2 U(g,f)\right)$$

Resummation of Massive Gravity
de Rham, Gabadadze, Tolley, PRL, 106, 231101 (2011)

dRGT model: allowed mass terms

de Rham, Gabadadze, Tolley, PRL, 106, 231101 (2011)

Construct the following matrix/tensor

$$K = \sqrt{f^{-1}g} \qquad\qquad K^{\mu}_{\nu} = \sqrt{f^{\mu\alpha}g_{\alpha\nu}}$$

Ghost-free mass terms are constructed from `characteristic polynomials'

$$U_0 = 1 \quad U_1 = Tr[K] \quad U_2 = \frac{1}{2!}\left[Tr[K]^2 - Tr[K^2]\right]$$

$$U_3 = \frac{1}{3!}\left[Tr[K]^3 - 3Tr[K]Tr[K^2] + 2Tr[K^3]\right]$$

$$U_4 = \frac{1}{4!}\left[Tr[K]^4 - 6Tr[K^2]Tr[K]^2 + 9Tr[K^3]Tr[K] + 3Tr[K^2]^2 - 6Tr[K^4]\right]$$

These come from expanding a determinant!

$$Det[1 + \lambda K] = 1 + \lambda U_1 + \lambda^2 U_2 + \lambda^3 U_3 + \lambda^4 U_4$$

Cosmology of dRGT model

Massive Cosmologies, Phys.Rev. D84 (2011) 124046
G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, A. J. Tolley

Perfect Homogeneous and Isotropic solutions (FRW) are forbidden in the simplest form of Massive Gravity

Possible to find inhomogeneous models that are locally indistinguishable from FRW over scales set by the graviton mass

COMPTON WAVELENGTH of GRAVITON = COHERENCE LENGTH

$$d \leq m^{-1}$$

In each bubble the Vainshtein mechanism ensures the cosmology is close to Einstein GR

Black Holes

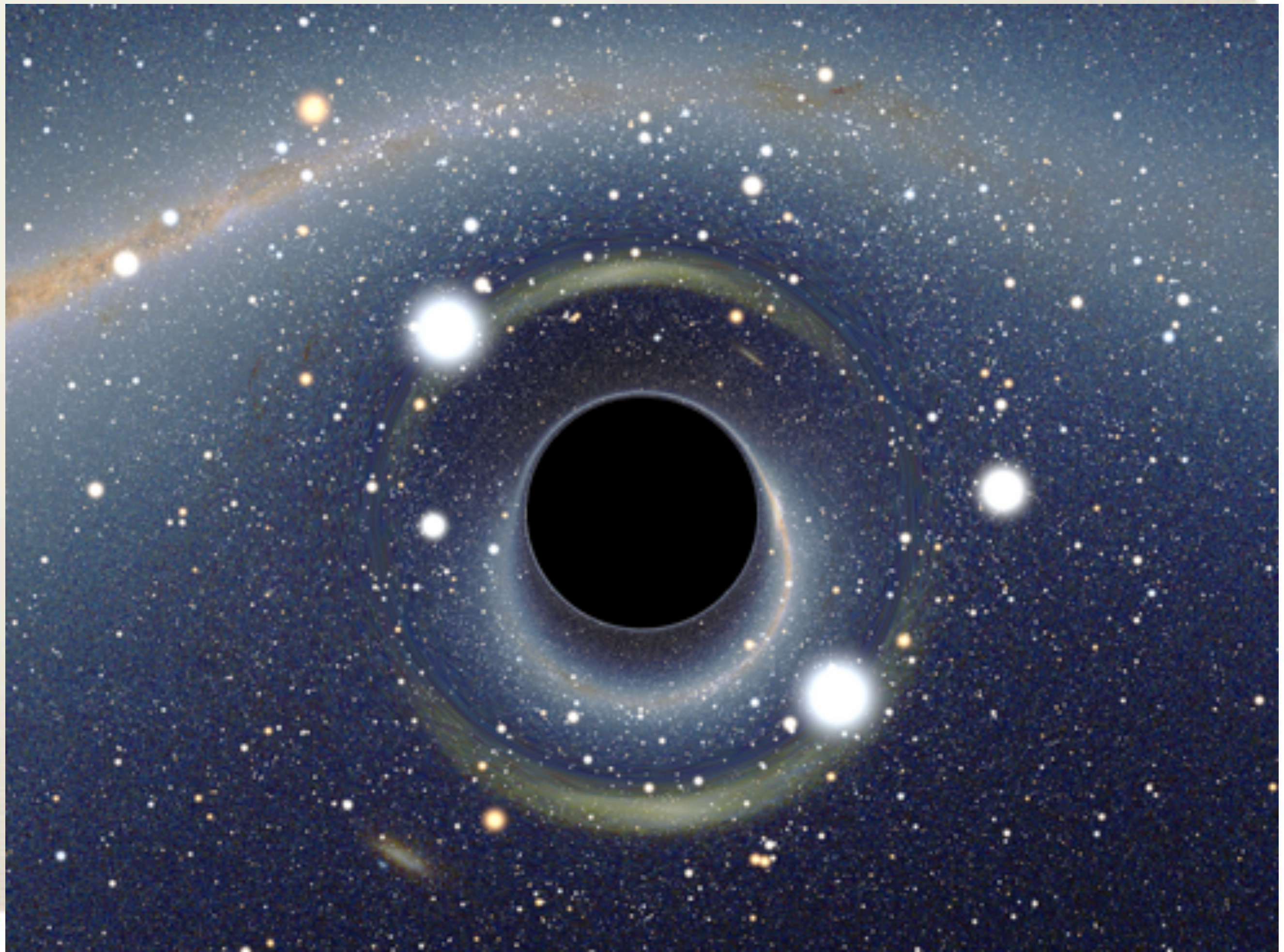
On Black Holes in Massive Gravity
L. Berezhiani,G. Chkareuli, G. Gabadadze, C. de Rham, Phys.Rev. D85 (2012) 044024

In Massive Gravity more than one effective metric:

waves travelling through a medium have a different velocity

Notion of causality is more subtle - scalar waves can travel faster or slower than tensor waves

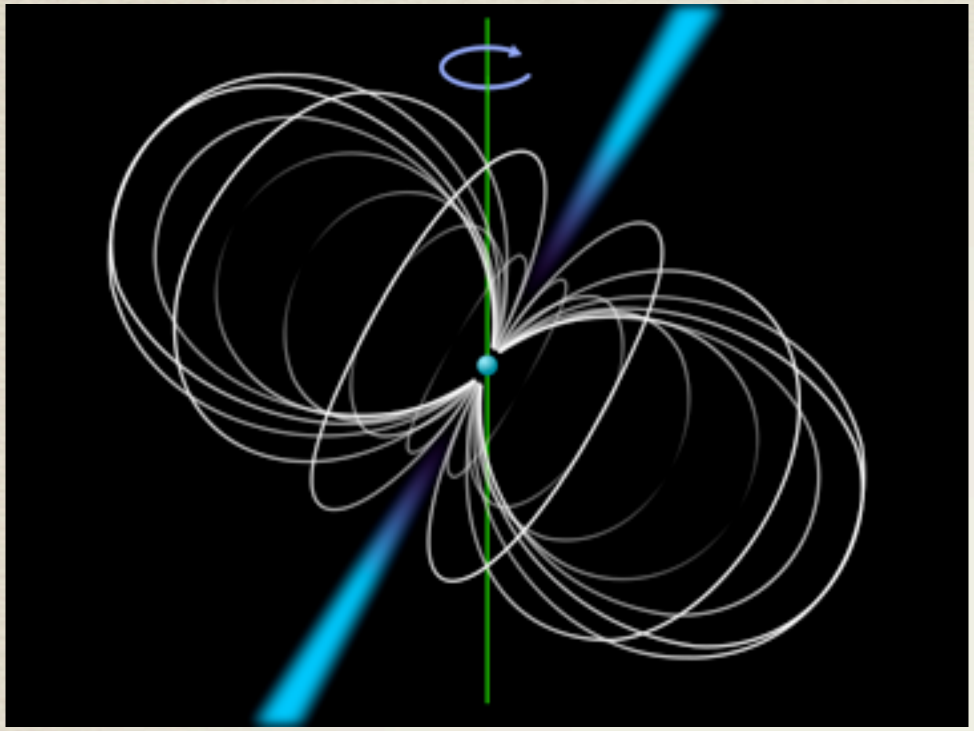
Black hole horizons
are more complex than in GR

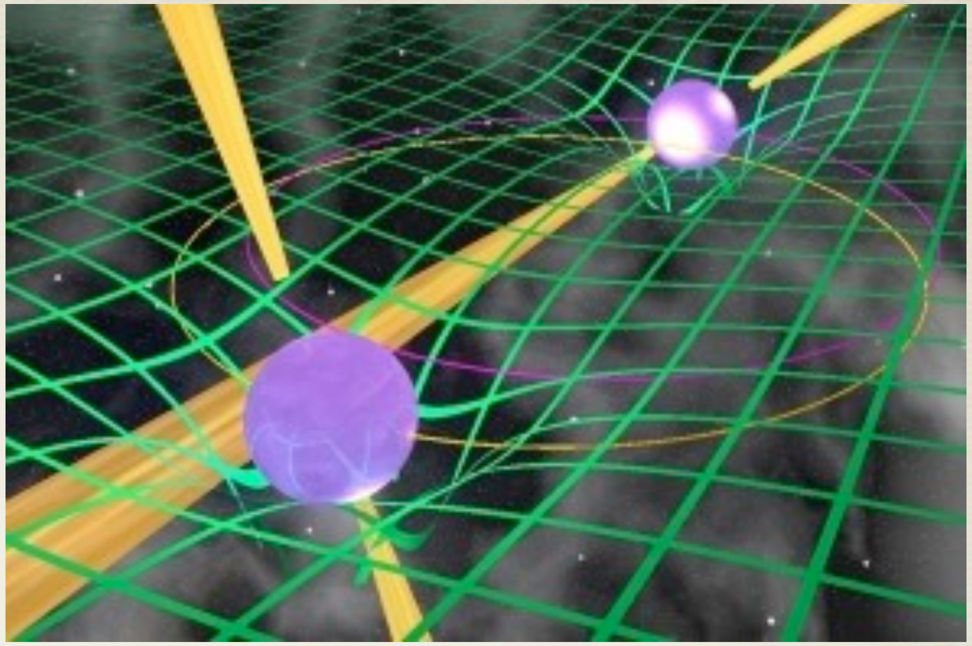


Near the BH horizon the Vainshtein mechanism ensures the geometry is close to Schwarzshild (general relativity)

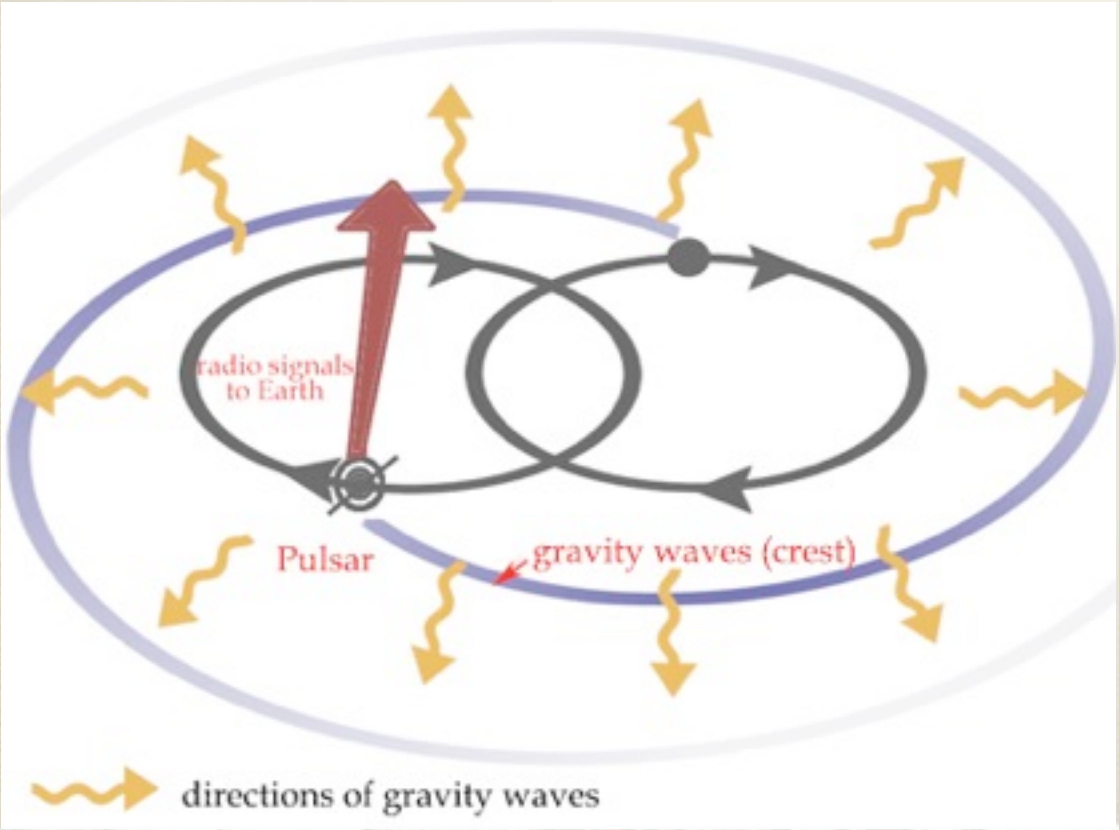
Binary Pulsars

Pulsar is a highly magnetized, rotating neutron star, which emits a beam of EM radiation - a PULSE!





The Nobel Prize in Physics 1993 to Russell A. Hulse and Joseph H. Taylor Jr. "for the discovery of a new type of pulsar, a discovery that has opened up new possibilities for the study of gravitation"



Binary Pulsars in Massive Gravity

Vainshtein mechanism in Binary Pulsars, de Rham, Tolley, Wesley. arXiv:1208.0580

Extra polarizations of graviton = extra modes of gravitational wave

Binary pulsars lose energy faster than in GR so the orbit slows down more rapidly

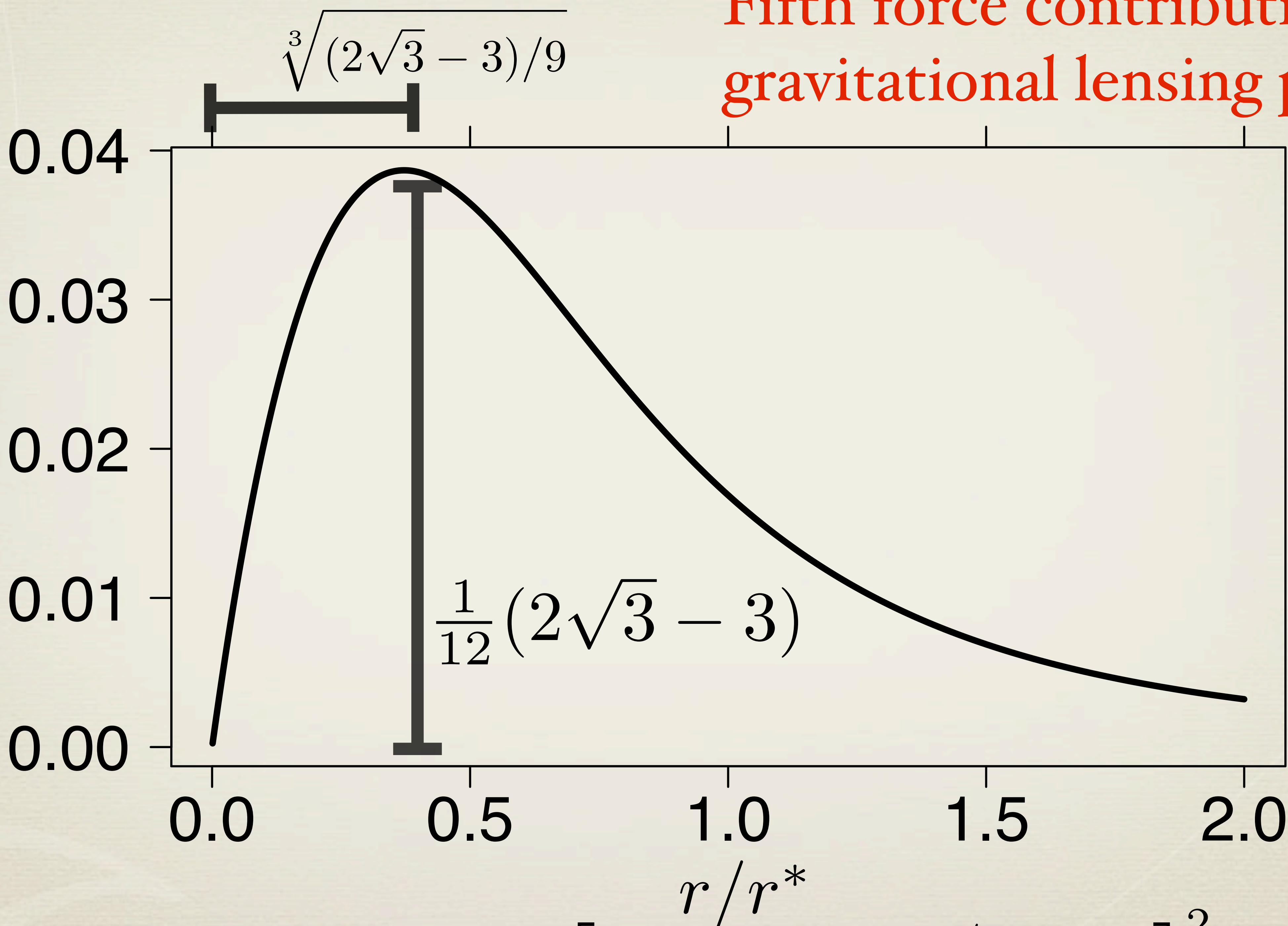
	A	B	C	D	E
Pulsar	1913+16 Taylor-Hulse	B2127+11	B1534+12	J0737–3039 double pulsar	J1738+0333
$M_1/M_\odot$	1.386	1.358	1.345	1.338	1.46
$M_2/M_\odot$	1.442	1.354	1.333	1.249	0.181
T_P/days	0.323	0.335	0.420	0.102	0.355
e	0.617	0.681	0.274	0.088	3.4×10^{-7}
$\frac{dT_P}{dt}\big\vert_\pi$ Monopole	9.8×10^{-22}	1.4×10^{-21}	1.1×10^{-22}	5.1×10^{-23}	8.1×10^{-24}
$\frac{dT_P}{dt}\big\vert_\pi$ Dipole	10^{-30}	10^{-32}	10^{-33}	10^{-32}	10^{-31}
$\frac{dT_P}{dt}\big\vert_\pi$ Quadrupole	9.1×10^{-21}	1.0×10^{-20}	6.1×10^{-21}	4.3×10^{-21}	1.1×10^{-21}
$\frac{dT_P}{dt}\big\vert_{GR}$	1.1×10^{-12}	1.7×10^{-12}	8.5×10^{-14}	5.6×10^{-13}	10^{-14}
σ	5.1×10^{-15}	1.3×10^{-13}	2.0×10^{-15}	1.7×10^{-14}	10^{-15}
Ref.	[29, 30]	[31]	[32, 33]	[34]	[35]

Table 1. The predicted contribution to the orbital period derivative $\dot{T}_P$ from π alone in the monopole, dipole and quadrupole channels (taking $m = 1.54 \times 10^{-33}$eV) for four known DNS pulsars (A to D) and one pulsar-white dwarf binary (E) with the GR result. The experimental uncertainty σ is given using [36].

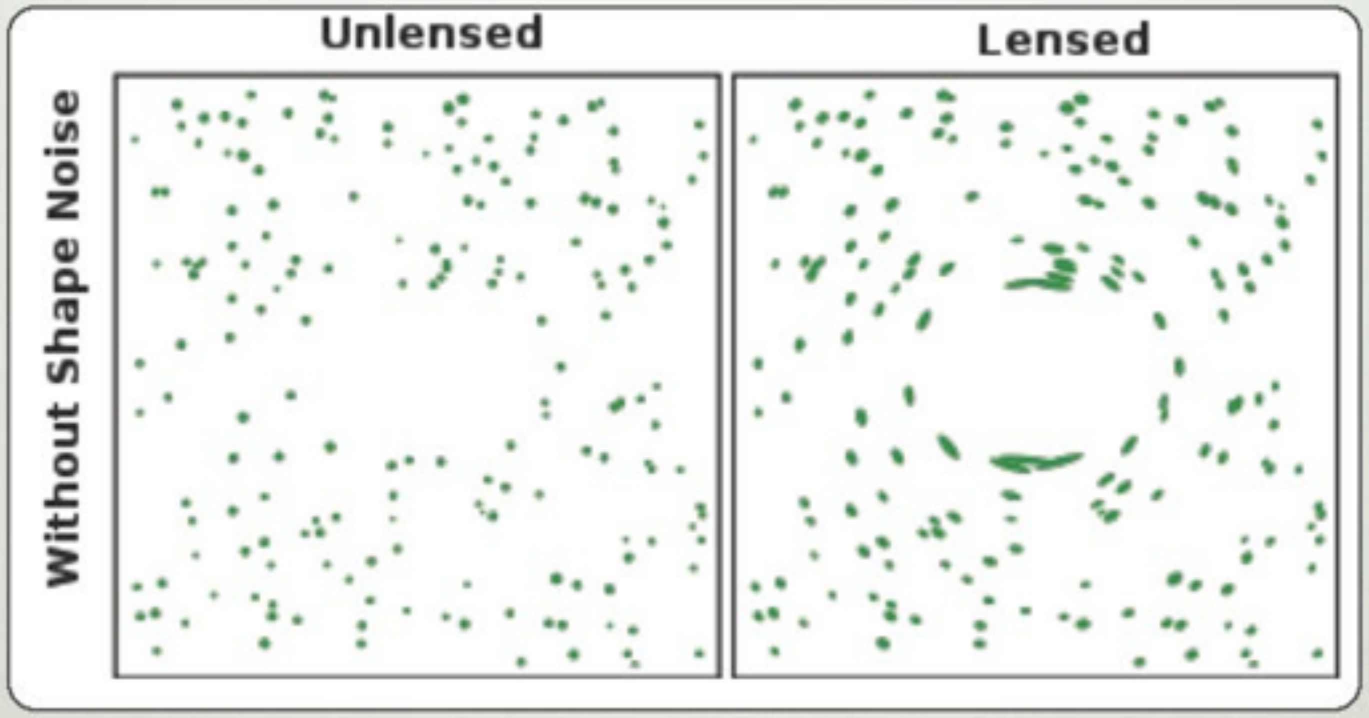
Gravitational Lensing

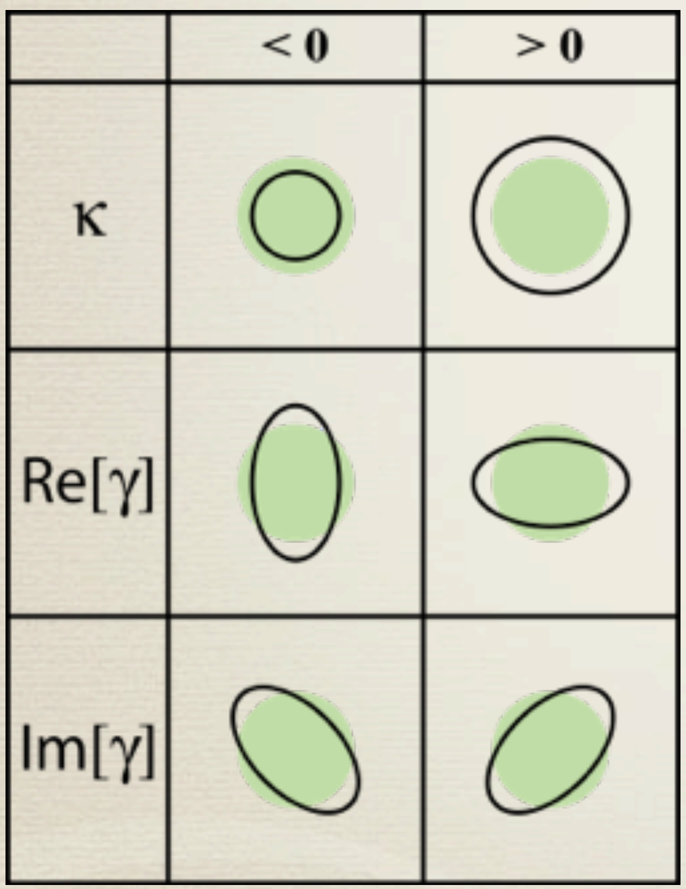
r* ~ kpc; for a typical galaxy r* ~ Mpc; and for a galaxy cluster, r* ~ 10 Mpc.

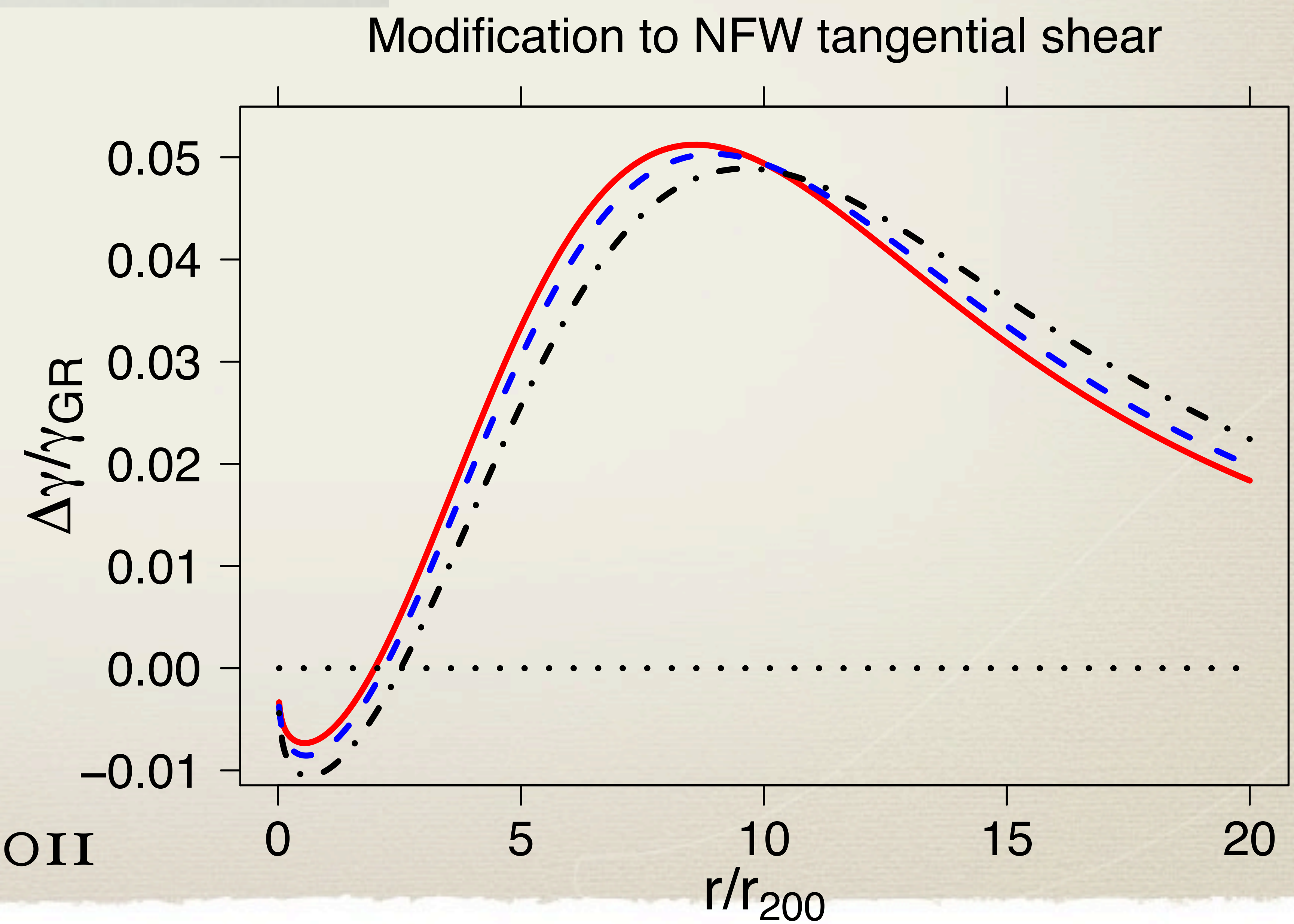
Fifth force contribution to gravitational lensing potential



$$\frac{r_c^2(\partial_r\varphi)^2}{\Phi_{Lens}[\mathrm{GR}]} = \frac{1}{2}\left(\frac{r}{r^*}\right)\left[\left(\left(\frac{r}{r^*}\right)^3+1\right)^{1/3} - \frac{r}{r^*}\right]^2$$







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