Ensemble Fluctuations in the UHECR Flux: Auger and JEM-EUSO Sensitivities

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Cosmic Frontier Workshop,
SLAC National Accelerator Laboratory
6-8 March, 2013

Outline

- What are ensemble fluctuations?
- Some examples
- Sensitivity of Auger and JEM-EUSO
- Conclusions

Markus Ahlers, LAA, and Andrew Taylor, PRD87 (2013) [arXiv:1209.5427]
Markus Ahlers, LAA, T. Paul, and Andrew Taylor, in preparation

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- Next best guess ☞ search for deviations from this mean prediction *i.e.* study the next statistical moment of the distribution
  ☞ which is just the ensemble variation

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- This mathematical regularization has a physical interpretation as the distance to the closest source ☞ and it is the contribution of these closeby sources that brings the largest contribution to spectral variations from the source distribution
- Variations of spectral indicies or emission rates are not in the form of oscillations (or wiggles) around the mean sources may conspire lifting the spectrum over a wide energy range

  BUT whatever the effect ☞ the mean deviation from the mean is given by the ensemble variation

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- Will Auger gather sufficient statistics to detect EF in GZK region? (*e.g.* if we run beyond 2015)
- What about JEM-EUSO?

Point-Source Flux

Imagine sitting at center of sphere with radius $r_\star$
with emission rate spectrum $\mathcal{Q}_A(E)/(4\pi r_\star^2)$

Point-source flux ☞ described by Boltzmann's equation

$$\frac{1}{r^2}\partial_r(r^2 F_{A,i}) \simeq \delta(r - r_\star)\frac{Q_{A,i}}{4\pi r^2} + \Gamma^{\rm CEL}_{A,i+1} F_{A,i+1} - \Gamma^{\rm CEL}_{A,i} F_{A,i} - \sum_{B<A} \Gamma_{(A,i)\to(B,i)} F_{A,i} + \sum_{B>A} \Gamma_{(B,i)\to(A,i)} F_{B,i}$$

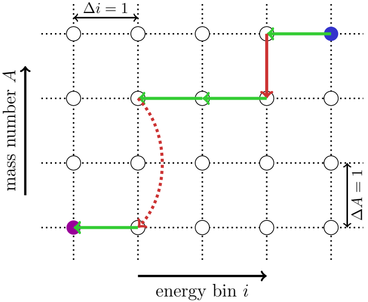
binned flux ☞ $F_{A,i} \equiv \Delta\epsilon_i \, A \, dF_A(A\epsilon_i)/dE$

emission rates ☞ $Q_{A,i} \equiv A\,\Delta\epsilon_i \, \mathcal{Q}_A(A\epsilon_i)$

interaction rates ☞

$$\Gamma^{\rm CEL}_{A,i} \equiv \frac{b_A(A\epsilon_i)}{A\Delta\epsilon_i}$$

$$\Gamma_{(A,i)\to(B,i)} \equiv \Gamma_{A\to B}(A\epsilon_i)$$



Ensemble-Averaged Flux

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- Mean total flux ☞ $\langle N_{\rm tot}(E) \rangle \equiv \sum_A \langle N_A(E/A) \rangle$

Relative Flux Variation

- Defining the residual $\delta X \equiv X - \langle X \rangle$ ☞ $\langle \delta X \, \delta Y \rangle = \langle XY \rangle - \langle X \rangle \, \langle Y \rangle$

- Covariance between relative flux of $\left\{ \begin{array}{c} \text{two particle species } A, B \\ \text{populating energy bins } i, j \end{array} \right.$ is

$$\langle \delta N_{A,i} \delta N_{B,j} \rangle \equiv \langle N_{A,i} N_{B,j} \rangle - \langle N_{A,i} \rangle \langle N_{B,j} \rangle$$

- Relative variation of total flux

☞ described by two-point density perturbations

$$\sigma^2_{\rm loc} = \sum_{A,B} \frac{\langle \delta N_A(E/A) \delta N_B(E/B) \rangle}{\langle N_{\rm tot}(E) \rangle^2}$$

Cosmic Variance Parameters

Estimate of ensemble fluctuations includes :

- source density
- source emission parameters
  γ and $E_{\max}$ ☞ same $\forall$ sources
- propagation effects
  M. Ahlers and A. Taylor, PRD 82 (2010)
- photopion (migration in energy bin)
- e^+e^- pairs (migration in energy bin)
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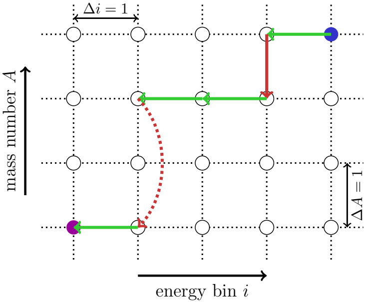

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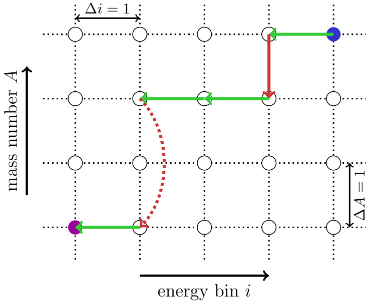
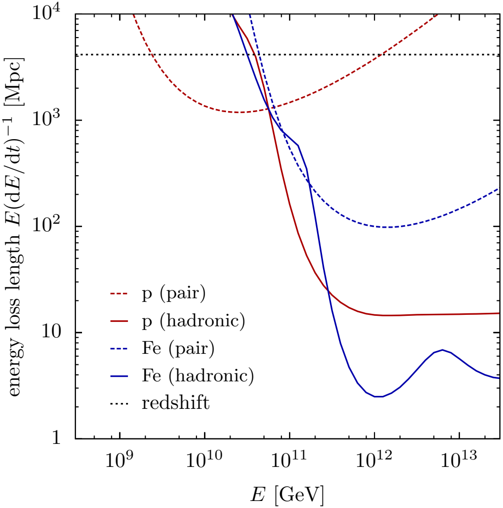


$\mathcal{H}_0 \sim 10^{-6} - 10^{-5}\ \mathrm{Mpc}^{-3}$ ☞ consistent with absence of repeaters

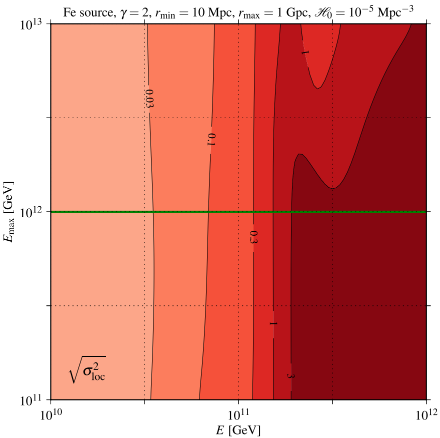
E. Waxman, K. B. Fisher, T. Piran ApJ 483 (1997)
T. Kashti and E. Waxman, JCAP 05 (2008)
H. Takami, S. Inoue, and T. Yamamoto, Astropart. Phys 35 (2012)
Pierre Auger Collaboration JCAP (submitted)

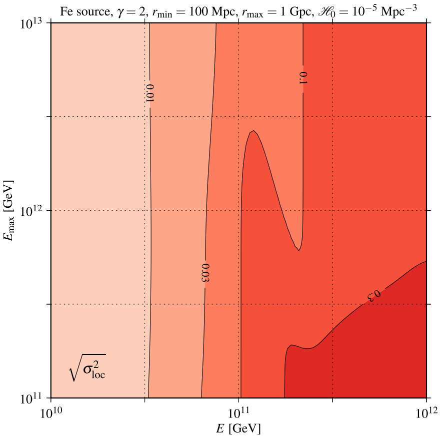
Energy Loss Lengths



Energy losses carve the average energy spectrum
and modulate its ensemble fluctuations

Relative Ensemble Fluctuation

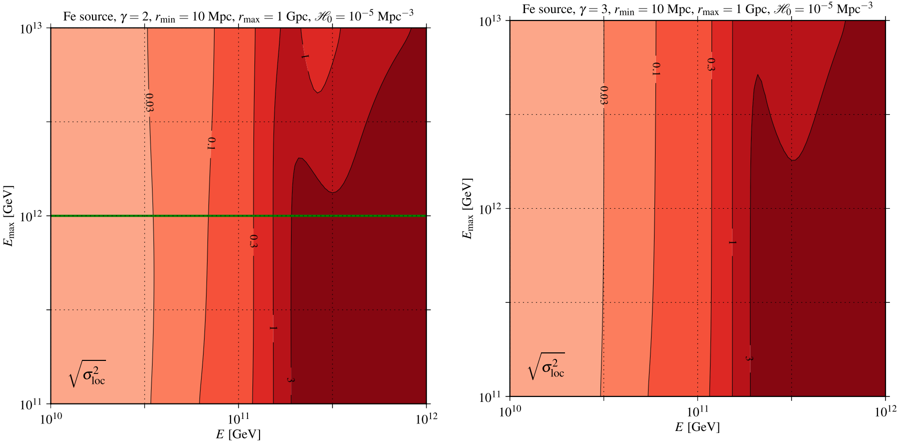




For $r_{\rm min} = 10$ Mpc ☞ relative ensemble fluctuation around mean flux increases with E and rises above the level of 10% at about $10^{10.8}$ GeV

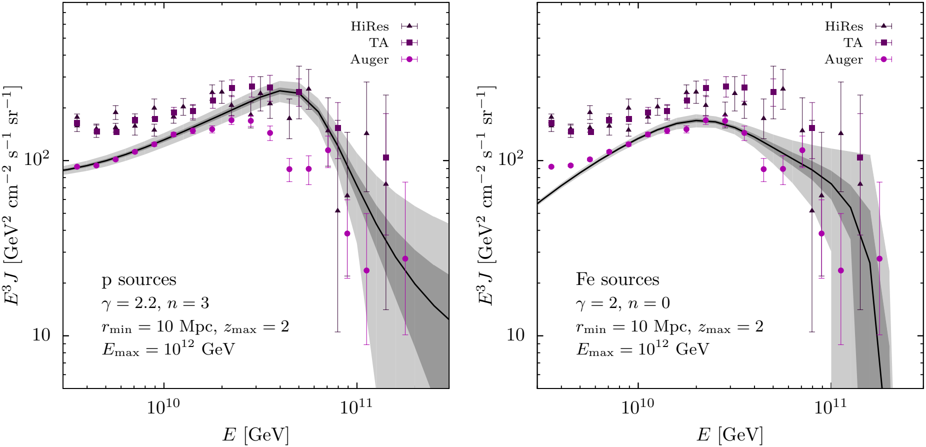
For $r_{\rm min} = 100$ Mpc ☞ ensemble fluctuations are smaller by factor ~ 3

Relative Ensemble Fluctuation



These results do not strongly depend on the spectral index

Spectral Wiggles ☞ $r_{\min} = 10$ Mpc

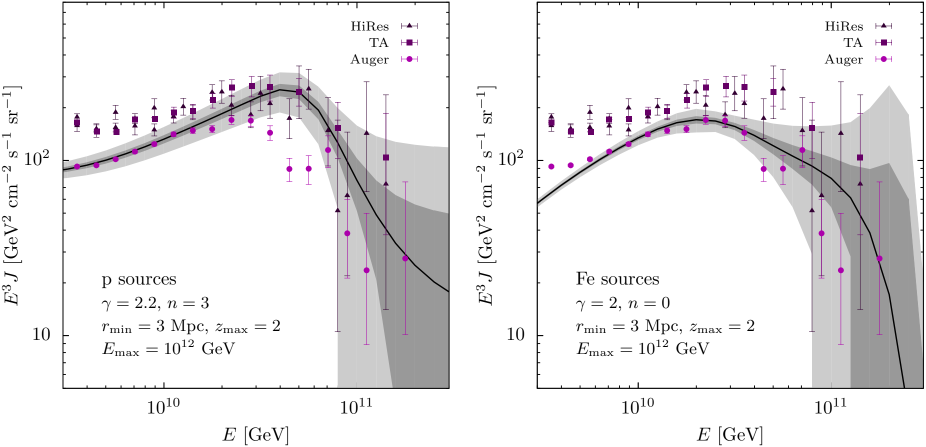


Approximate variation of the flux assuming a local source distribution:

$\mathcal{H}_0 = 10^{-5}$ Mpc^{-3} (dark gray band)

$\mathcal{H}_0 = 10^{-6}$ Mpc^{-3} (light gray band)

Spectral Wiggles ☞ $r_{\min} = 3$ Mpc



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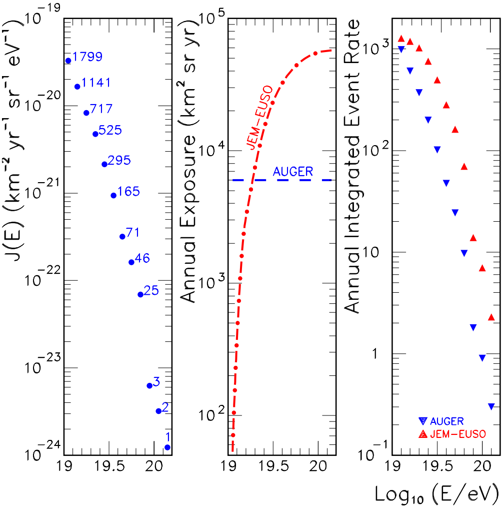
Exposures

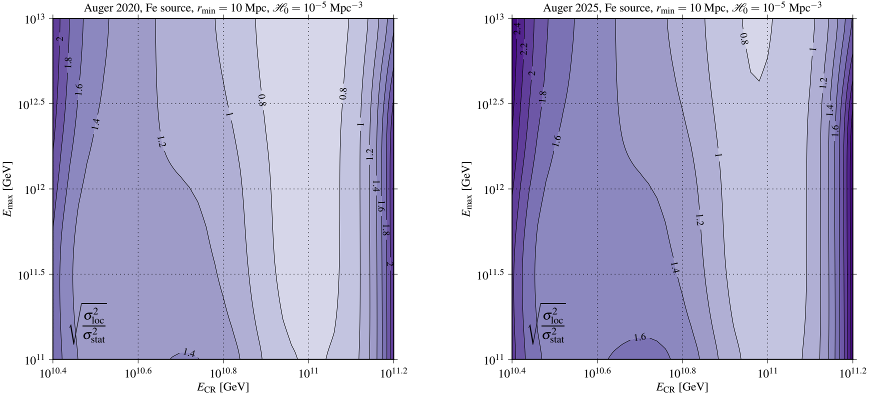


Pierre Auger Collaboration NIMA 613 (2010)

JEM-EUSO Collaboration Astropart. Phys. 44 (2013)

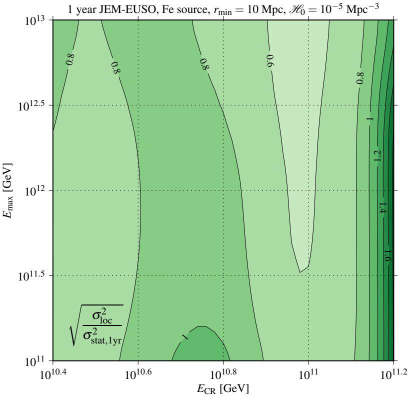
Ensemble Fluctuations vs. Statistics

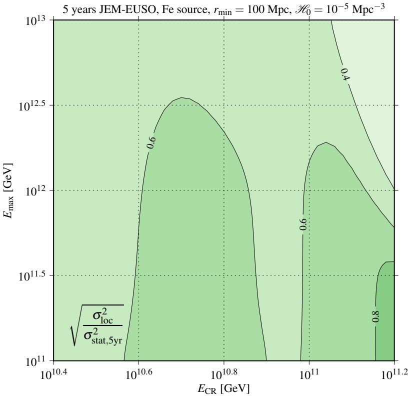
Exposure assumption:
21000 km^2 sr yr through 2010 ICRC + 6000 km^2 sr annual thereafter



Contour = 1 means the statistical error equals the ensemble fluctuation
(*i.e.* ”spectral wiggles” become discernable in the data)

Ensemble Fluctuations vs. Statistics (cont'd)

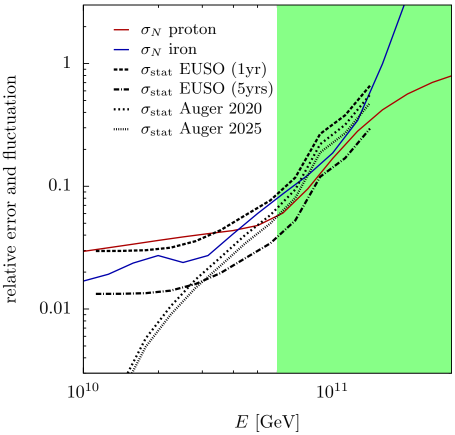


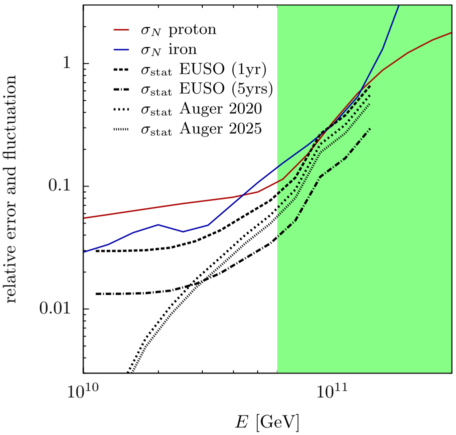


Though marginal ☞ with sufficient exposure
uncovering hints for $r_{\rm min} = 10$ Mpc not out of the question....

$r_{\rm min} = 100$ Mpc ☞ out of range

Summary





$r_{\rm min} = 10$ Mpc ☞ 3σ evidence

$r_{\rm min} = 3$ Mpc ☞ 5σ discovery

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  - injection parameters
  - lower bound on extragalactic magnetic field
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- UHECR exposures collected by 2025 will provide
  the required statistics for identification of ensemble fluctuations
  from the GZK suppression features on a statistical basis