Large-N_c Methods for Baryons: Nucleon-Nucleon Interactions

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Why NN interactions?

- Nuclei in test of SM and search for BSM physics
  - Neutrino oscillations
  - Neutrinoless double beta decay
  - Dark matter direct detection
  - …
- Fundamental ingredient for all processes involving two or more nucleons
- Interesting problem in nonperturbative QCD

Approaches to the NN interactions

- Phenomenological models
  - Parametrize in terms of meson-exchange, Gaussians,…
- Pionless/chiral effective field theories
  - Connection to QCD through symmetries
  - Ordering of terms based on power counting
  - Short-distance physics encoded in low-energy coefficients (LECs)
- Parameters/LECs
  - Experiment
  - Lattice QCD

Additional constraints from large-N_c analysis?

Large-N$_c$ QCD

- QCD with number of colors $N_c \to \infty$ and $g^2 N_c$ fixed
- Systematic expansion in $1/N_c$
- Phenomenologically successful
- Mesons
  - $\bar{q}q$
  - Stable
  - Weakly interacting $\sim 1/\sqrt{N_c}$

't Hooft (1974); Witten (1979)

Large-N$_c$ baryons

- Bound states of N_c quarks
- Completely antisymmetric in color: $\epsilon_{i_1 i_2 \cdots i_{N_c}} q^{i_1} q^{i_2} \cdots q^{i_{N_c}}$
- Baryon mass $M_B \sim N_c$
- In $N_c \to \infty$ limit: SU(2F) symmetry in baryon spectrum
- Application to NN interactions:
  - Determine large-N_c scaling of contributions to potential

Combine with EFTs to obtain dual expansion

  - Derive $1/N_c$ hierarchy among low-energy coefficients (LECs)

Witten (1979); Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

NN potential in large-N_c expansion

$$V(p_-, p_+) = \langle N_C(p_1'), N_D(p_2') \,|\, H \,|\, N_A(p_1), N_B(p_2) \rangle$$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q, \quad I^a = q^\dagger \frac{\tau^a}{2} q, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

- Coefficients v_{stu}
    - Momentum dependent
    - Constrained by symmetries

Dashen, Jenkins, Manohar (1994); Kaplan, Savage (1996); Kaplan, Manohar (1997); Cohen et al. (2002)

Large-N$_c$ scaling

- Nucleon matrix elements

$$\langle N' | G^{ia} | N \rangle \sim \langle N' | 1 | N \rangle \sim O(N_c)$$
$$\langle N' | S^{i} | N \rangle \sim \langle N' | I^{a} | N \rangle \sim O(1)$$

- Momenta (in t-channel)

$$p_- = (p_1' - p_2') - (p_1 - p_2) \sim O(1)$$
$$p_+ = (p_1' - p_2') + (p_1 - p_2) \sim 1/M_N \sim O(1/N_c)$$

- Coefficients (excluding momenta)

$$\tilde{v}_{stu} \sim 1$$

Dashen, Jenkins, Manohar (1994,95); Kaplan, Savage (1996); Kaplan, Manohar (1997)

Caveats

- Nuclear matter forms classical crystal for $N_c \to \infty$?
    - Assume that symmetries of NN interactions do not change
- Nucleon and Δ degenerate in large-N_c limit
- Δ plays important role in meson-baryon interactions
    - Ignore intermediate Δ states (for now)

Kaplan, Manohar (1997); Banerjee, Cohen, Gelman (2002)

Example: central potential

- General form

$$V_c = V_0^0 + V_\sigma^0 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 + V_0^1 \vec{\tau}_1 \cdot \vec{\tau}_2 + V_\sigma^1 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \vec{\tau}_1 \cdot \vec{\tau}_2$$

- Scaling

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 \sim \hat{S}_1 \cdot \hat{S}_2$$

$$\vec{\tau}_1 \cdot \vec{\tau}_2 \sim \hat{I}_1 \cdot \hat{I}_2$$

$$\vec{\sigma}_1 \cdot \vec{\sigma}_2 \, \vec{\tau}_1 \cdot \vec{\tau}_2 \sim \hat{G}_1 \cdot \hat{G}_2$$

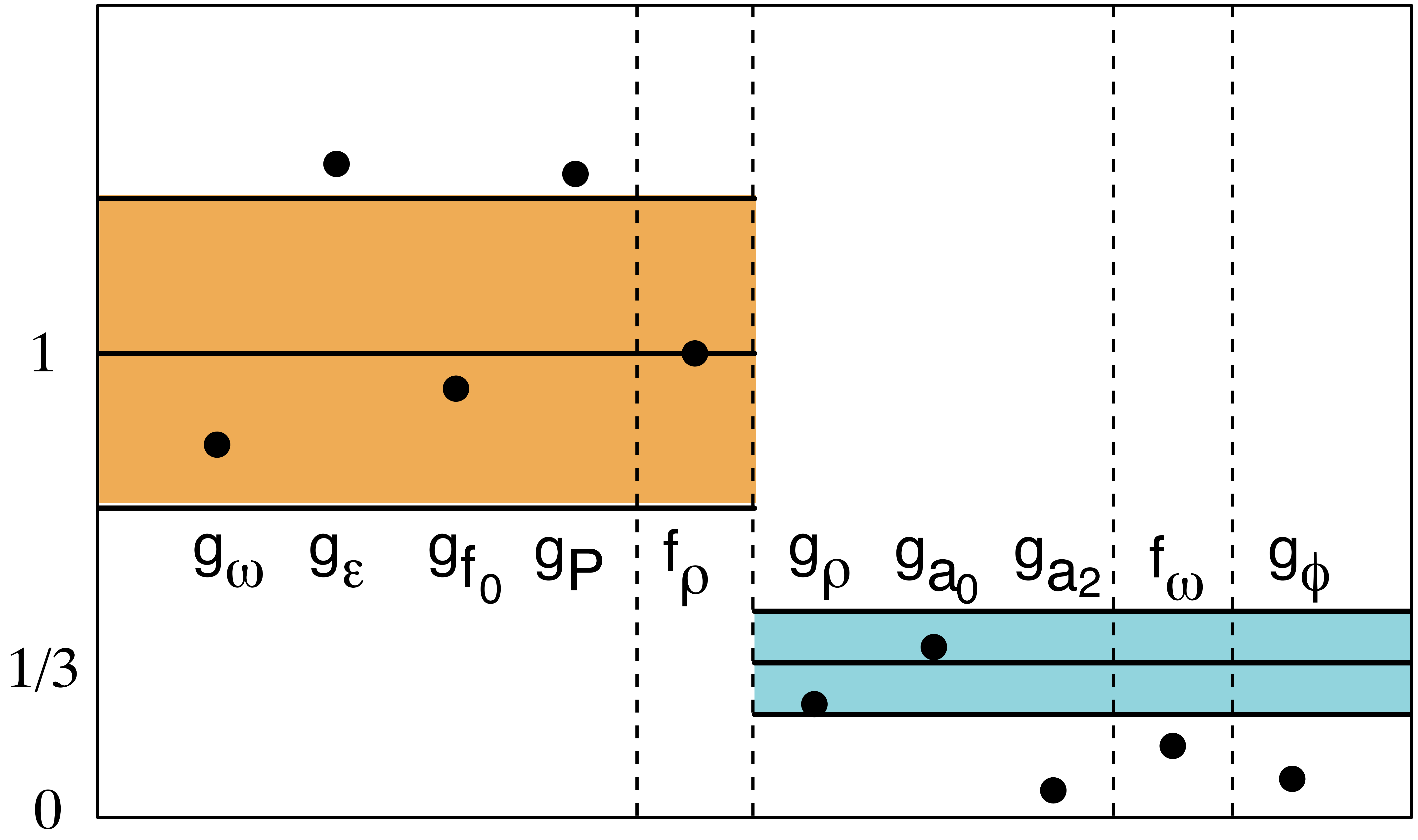
- Coefficients

$$V_0^0 \sim N_c \qquad V_\sigma^0 \sim 1/N_c$$

$$V_0^1 \sim 1/N_c \qquad V_\sigma^1 \sim N_c$$

Kaplan, Savage (1996); Kaplan, Manohar (1997)

Large-N_c scaling vs Nijmegen



Kaplan, Manohar (1997)

Pionless EFT

- Very low energies
- Only nucleons and external fields
- LO: *S*-wave interactions

$$\mathscr{L} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- LECs C_S, C_T assumed “natural” in EFT
- Alternatively: partial-wave basis

$$C^{({}^1S_0)} = (C_S - 3C_T), \quad C^{({}^3S_1)} = (C_S + C_T)$$

Pionless EFT and the large-N_c expansion

- Spin-isospin structure of operators

$$(N^\dagger N)(N^\dagger N) \sim 1_1 \cdot 1_2 \qquad (N^\dagger \sigma^i N)(N^\dagger \sigma^i N) \sim \hat{S}_1 \cdot \hat{S}_2$$

- Large-N_c scaling of LECs

$$C_S \sim O(N_c) \qquad C_T \sim O(1/N_c)$$

- In large-N_c limit

$$C^{(^1S_0)} = C^{(^3S_1)}$$

- Wigner SU(4) symmetry (in *S* waves)

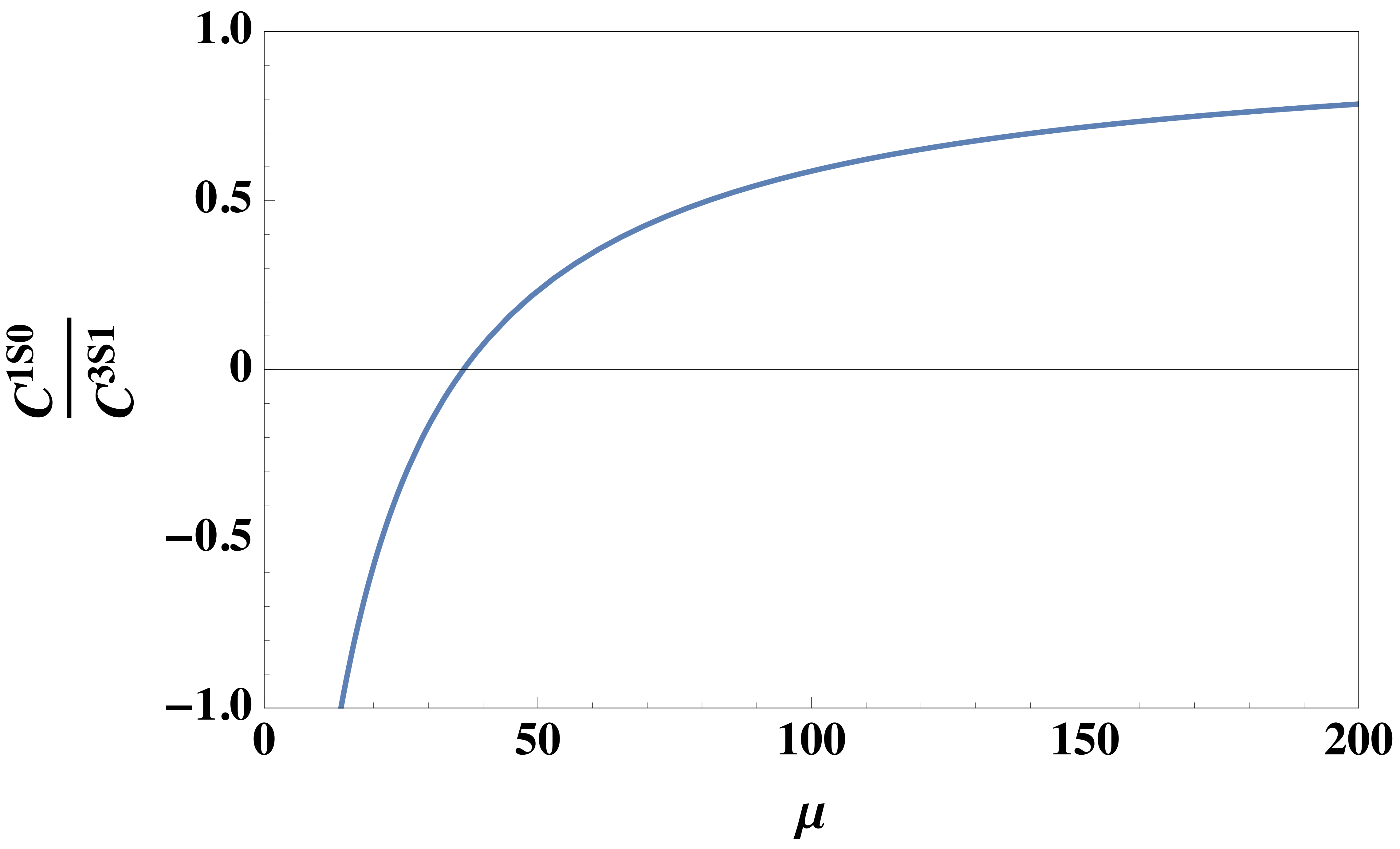
Kaplan, Savage (1996)

Renormalization-point dependence

- In PDS renormalization

$$\frac{C^{^1S_0}}{C^{^3S_1}} = \frac{1/a^{^3S_1} - \mu}{1/a^{^1S_0} - \mu} \xrightarrow{\mu \to 0} \frac{a^{^1S_0}}{a^{^3S_1}} \approx -4.4$$

- Agreement with large-N_c expected errors for $\mu \gtrsim m_\pi$



Kaplan, Savage (1996)

Higher orders in pionless EFT

- Two-derivative operators: *S* waves, *P* waves, *S*-*D* mixing
- Fit to phase shifts and mixing parameter (μ=120 MeV)

$$C_{1\cdot 1} = (-0.58 \pm 0.11)\ \text{fm}^4\ , \quad C_{G\cdot G} = (0.40 \pm 0.05)\ \text{fm}^4\ , \quad C'_{G\cdot G} = (0.84 \pm 0.05)\ \text{fm}^4\ \Big\}\ \text{LO}$$

$$\left.\begin{aligned} &C_{\tau\cdot\tau} = (0.15 \pm 0.07)\ \text{fm}^4\ , \quad C_{\sigma\cdot\sigma} = (-0.39 \pm 0.07)\ \text{fm}^4\ , \quad C'_{\sigma\cdot\sigma} = (0.78 \pm 0.1)\ \text{fm}^4 \\ &\overset{\leftrightarrow}{C}_{1\cdot\sigma} = (-0.17 \pm 0.12)\ \text{fm}^4 \end{aligned}\right\}\ \text{NLO}$$

- Fit to phase shifts and increase mixing parameter by factor of 3

$$C_{1\cdot 1} = (-0.59 \pm 0.10)\ \text{fm}^4\ , \quad C_{G\cdot G} = (0.11 \pm 0.06)\ \text{fm}^4\ , \quad C'_{G\cdot G} = (1.72 \pm 0.13)\ \text{fm}^4\ \Big\}\ \text{LO}$$

$$\left.\begin{aligned} &C_{\tau\cdot\tau} = (0.16 \pm 0.07)\ \text{fm}^4\ , \quad C_{\sigma\cdot\sigma} = (-0.10 \pm 0.08)\ \text{fm}^4\ , \quad C'_{\sigma\cdot\sigma} = (-0.10 \pm 0.16)\ \text{fm}^4 \\ &\overset{\leftrightarrow}{C}_{1\cdot\sigma} = (-0.17 \pm 0.12)\ \text{fm}^4 \end{aligned}\right\}\ \text{NLO}$$

- Other physics can impact size of LECs

MRS, Singh, Springer (2018)

Magnetic couplings in pionless EFT

- NN contact terms coupled to magnetic field B

$$\mathcal{L} = eB_i \left[{}^{\not\pi}L_1 \left(N^T P_i N\right)^\dagger \left(N^T \bar{P}_3 N\right) - i\epsilon^{ijk} \, {}^{\not\pi}L_2 \left(N^T P_j N\right)^\dagger \left(N^T P_k N\right) \right] + \text{h.c.}$$

$P_i/\bar{P}_a$: projection onto ${}^3S_1/{}^1S_0$ partial waves

- Contributions to
  - ${}^{\not\pi}L_1$: radiative neutron capture $np \to d\gamma$
  - ${}^{\not\pi}L_2$: deuteron magnetic moment

Chen, Rupak, Savage (1999)

Naturalness?

- Extracted values at $\mu = m_\pi$

$$^{\not\pi}L_1 = 7.24\,\mathrm{fm}^4\,, \quad ^{\not\pi}L_2 = -0.149\,\mathrm{fm}^4$$

- $^{\not\pi}L_2$ “significantly smaller than the naively estimated size of ~ 1 fm^4”

- Simultaneously natural?

$$\left|\frac{^{\not\pi}L_2}{^{\not\pi}L_1}\right|_{\mathrm{exp}} \approx 0.021$$

Chen, Rupak, Savage (1999); Beane et al. (2000)

Large-N_c scaling

- Scaling not manifest in partial-wave basis

- "Large-N_c" basis

$$\mathcal{L} = eB^i \left[C_s^{(M)} \left(N^\dagger \sigma^i N\right) \left(N^\dagger N\right) + C_v^{(M)} \epsilon^{ijk} \epsilon^{3ab} \left(N^\dagger \sigma^j \tau^a N\right) \left(N^\dagger \sigma^k \tau^b N\right) \right]$$

- Large-N_c scaling of LECs
    - Isoscalar: $C_s^{(M)} \sim O(N_c^0)$
    - Isovector: $C_v^{(M)} \sim O(N_c)$

Richardson, MRS (2020); Detmold, Savage (2004)

Naturalness again

- LECs in different bases related by Fierz transformations

$$\not{\pi} L_1 = 8C_v^{(M)} \quad \not{\pi} L_2 = -C_s^{(M)}$$

- Numerical values at $\mu = m_\pi$

$$C_s^{(M)} = 0.149\,\mathrm{fm}^4 \quad C_v^{(M)} = 0.905\,\mathrm{fm}^4$$

- Assume $|C_v^{(M)}| \sim N_c |C_s^{(M)}|$:

$$\left|\frac{\not{\pi} L_2}{\not{\pi} L_1}\right|_{N_c} \approx \frac{1}{8N_c} \approx 0.042 \qquad \text{vs} \qquad \left|\frac{\not{\pi} L_2}{\not{\pi} L_1}\right|_{\exp} \approx 0.021$$

- Consistent to consider both $C_v^{(M)}$ and $C_s^{(M)}$ natural with large-N_c suppression

Richardson, MRS (2020)

Axial-NN LECs

- General axial external fields

$$\mathcal{L} = A^{ia} L_{1,A} \left(N^T P_i N\right)^\dagger \left(N^T \bar{P}_a N\right) + \text{h.c.} - 2i\epsilon^{ijk} L_{2,A} A^i \left(N^T P_j N\right)^\dagger \left(N^T P_k N\right)$$

  - $L_{1,A}$: proton-proton fusion, neutrino-deuteron reactions, tritium β-decay

$$L_{1,A}^{\text{NPLQCD}}(\mu = m_\pi) = 3.9(0.2)(1.0)(0.4)(0.9)\,\text{fm}^3$$

  - $L_{2,A}$: neutrino-deuteron reactions (negligible), deuteron strange axial form factor

- Assuming $|C_v^{(A)}| \approx N_c |C_s^{(A)}|$

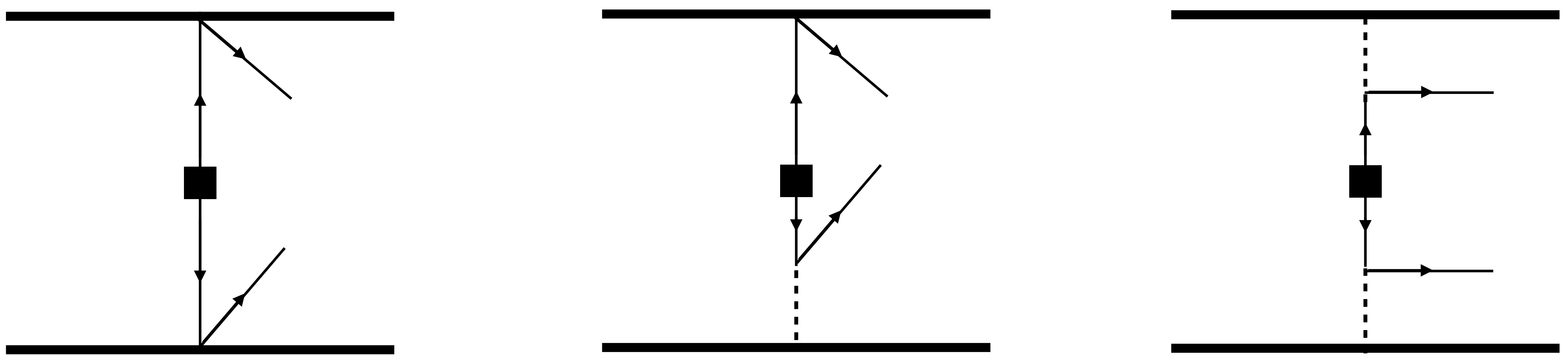
$$|L_{2,A}| \sim 0.16\,\text{fm}^3$$

Butler, Chen (2000); Butler, Chen, Vogel (2002); Ando et al (2008); De-Leon, Platter, Gazit (2019); Acharya, Bacca (2020); Savage et al (2017); Detmold, Shanahan (2021); Richardson, MRS (2020)

Neutrinoless double beta decay in chiral EFT

- Light Majorana neutrino exchange mechanism

- Long-range contributions to LO “neutrino potential”
    - direct neutrino exchange
    - neutrino exchange via pions



Prezeau, Ramsey-Musolf, Vogel (2003); Cirigliano et al. (2018)

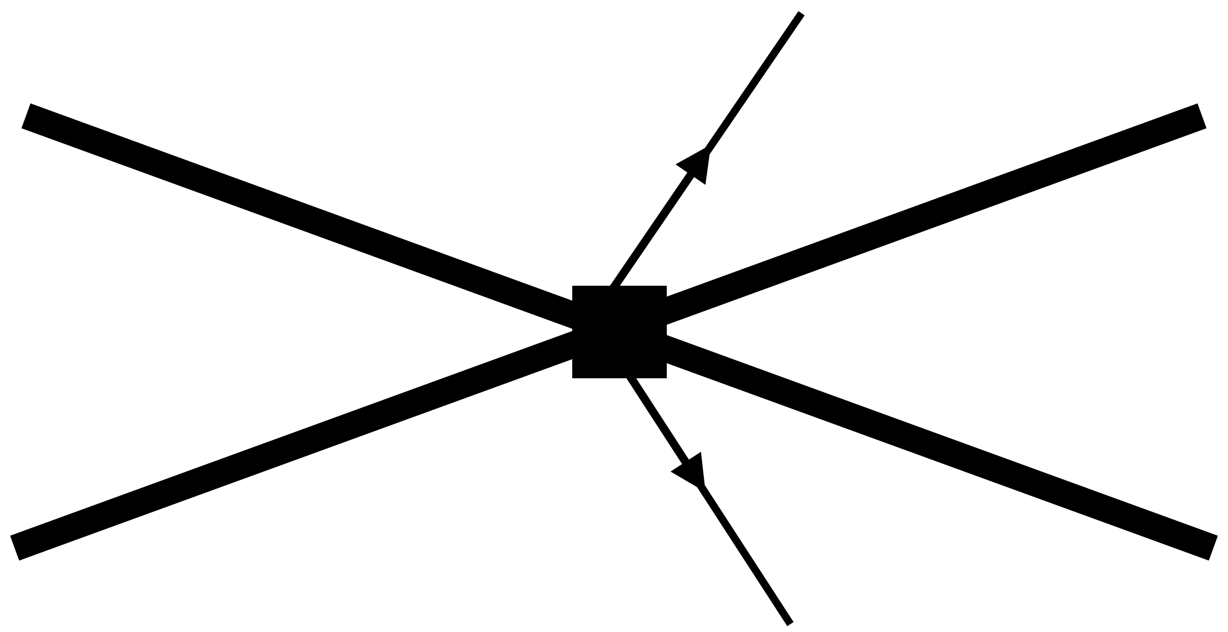
Contact term for neutrinoless double beta decay

- For $^1S_0 \rightarrow {}^1S_0$ transition

$$V_{\nu,L}(\mathbf{q}) = \frac{\tau_1^+ \tau_2^+}{\mathbf{q}^2} \left[1 + 2g_A^2 + \frac{g_A^2 m_\pi^4}{(\mathbf{q}^2 + m_\pi^2)^2} \right]$$

- Evaluation between $^1S_0$ nn and pp states → regulator dependence

- New short-range contribution $V_{\nu,S}$ with new LEC g_ν^{NN}



Cirigliano et al. (2018, 2019)

g_ν^{NN}

- Encodes short-range contribution from light Majorana exchange
- Previously not considered
- Determination would require data and/or lattice QCD
- Related to charge-independence-breaking (CIB) LEC C_1 by chiral symmetry
- C_1 currently not determined, only linear combination $C_1 + C_2$ with second CIB LEC C_2
- $C_1 - C_2$ sensitive to two-nucleon—multipion interactions
- Numerical impact estimated by assumption $g_\nu^{NN} \approx (C_1 + C_2)/2$

Detmold, Murphy (2020); Davoudi, Kadam (2020, 2021), Cirigliano et al. (2018); Cirigliano et al. (2018, 2019)

g_ν^{NN} and large Nc

- Contact term Lagrangian

$$\mathcal{L}_{\Delta L=2}^{NN} = \left(2\sqrt{2}G_F V_{ud}\right)^2 m_{\beta\beta}\bar{e}_L C\bar{e}_L^T \frac{g_\nu^{NN}}{4}\left[\left(\bar{N}u\tilde{Q}_L^w u^\dagger N\right)^2 - \frac{1}{6}\mathrm{Tr}(\tilde{Q}_L^w\tilde{Q}_L^w)\left(\bar{N}\tau^a N\right)^2\right] + \mathrm{H.c.}$$

with $\tilde{Q}_L^w = \tau^+ = (\tau^1 + i\tau^2)/2$

- Fierz relationship

$$\left(N^\dagger\sigma^i\tau^+N\right)^2 = -3\left(N^\dagger\tau^+N\right)^2$$

- g_ν^{NN} is LO in large-N_c expansion
- So what?

Cirigliano et al. (2018, 2019), Richardson, MRS, Pastore, Springer (2021)

CIB Lagrangian and large N$_c$

- Construct most general CIB Lagrangian with two EM spurion insertions
- Determine large-N_c scaling of each term
- Reduce to minimal set using Fierz identities, trace identities, etc
- Isotensor Lagrangian at LO and NLO in N_c expansion

$$\mathcal{L}^{\Delta I=2}_{\text{LO-in-}N_c} = -3e^2\bar{\mathcal{C}}_3 \left[\left(N^\dagger \tilde{Q}_+ N\right)^2 - \frac{1}{6}\text{Tr}(\tilde{Q}_+^2)\left(N^\dagger \tau^a N\right)^2\right]$$

$$\mathcal{L}^{\Delta I=2}_{\text{NLO-in-}N_c} = -3e^2\bar{\mathcal{C}}_6 \left[\left(N^\dagger \tilde{Q}_- N\right)^2 - \frac{1}{6}\text{Tr}(\tilde{Q}_-^2)\left(N^\dagger \tau^a N\right)^2\right]$$

with $\tilde{Q}_\pm = \frac{1}{4}[u^\dagger \tau^3 u \pm u\tau^3 u^\dagger]$

Justification for approximation

- Comparison with Lagrangian of Cirigliano et al. gives

$$\mathcal{C}_1 = -3\bar{\mathcal{C}}_3 - 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$
$$\mathcal{C}_2 = -3\bar{\mathcal{C}}_3 + 3\bar{\mathcal{C}}_6 = -3\bar{\mathcal{C}}_3\left[1 + O(1/N_c)\right]$$

- Supports assumption $g_\nu^{NN} \approx (C_1 + C_2)/2$

- Also supported by independent calculation using analog to Cottingham formula

Richardson, MRS, Pastore, Springer (2021); Cirigliano et al. (2021)

Conclusions

- Large-N_c analysis
  - Captures nonperturbative QCD effects
  - Based on symmetry in baryon sector
  - Constraints in absence of data
  - Trends, not predictions
  - Only upper limits on size
  - Other scales can impact relative sizes

Conclusions

- Other applications
  - Three-nucleon interactions
  - Parity-violating NN interactions
  - Time-reversal-invariance-violating NN interactions
  - Two-nucleon EM and axial current operators
  - Dark matter couplings

Phillips, Schat (2013); Phillips, Samart, Schat (2015); Phillips, Samart, Schat, MRS (2016); MRS, Springer, Vanasse (2016); Nguyen, MRS, Springer (2021); Richardson, MRS (2020); Richardson, Lin, Nguyen (2022)

ANNUAL REVIEWS

Annual Review of Nuclear and Particle Science

Implications of Large-N_c QCD for the NN Interaction

Thomas R. Richardson,[1,2] Matthias R. Schindler,[3] and Roxanne P. Springer[2]

Parity-violating NN interactions

- Manifestation of PV quark interactions at hadronic level
- Interplay of weak and nonperturbative strong interactions
- Range of weak interactions ~ 0.002 fm
- Relative strength in NN case ~ $G_F m_\pi^2 \approx 10^{-7}$
- Very low energies (pionless EFT): five S-P transition operators

$$\mathcal{C}^{(^3S_1-^1P_1)}, \quad \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)}, \quad \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)}, \quad \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}, \quad \mathcal{C}^{(^3S_1-^3P_1)}$$

Danilov (1965, 1971); Zhu et al. (2005); Girlanda (2008); Phillips, MRS, Springer (2009)

PV NN interactions in the large-N_c expansion

- General operators structure

  - LO in N_c [$O(N_c)$]

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)[\tau_1 \tau_2]_2^{zz}$$

  - NLO in N_c [$O(N_c^0)$]

$$\mathbf{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3)$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)\ (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

$$\mathbf{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\ (\vec{\tau}_1 + \vec{\tau}_2)^3$$

$$\left[(\mathbf{p}_+ \times \mathbf{p}_-) \cdot \vec{\sigma}_1\, \mathbf{p}_- \cdot \vec{\sigma}_2 + (\mathbf{p}_+ \times \mathbf{p}_-) \cdot \vec{\sigma}_2\, \mathbf{p}_- \cdot \vec{\sigma}_1\right]\ (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

Phillips, Samart, Schat (2015)

Parity violation in pionless EFT + large N_c

- Comparison with pionless EFT Lagrangian → three terms at LO-in-N_c?

$$\mathcal{C}^{(^3S_1-^1P_1)} \sim N_c \quad \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \sim N_c \quad \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)} \sim N_c$$

$$\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} \sim N_c^0 \quad \mathcal{C}^{(^3S_1-^3P_1)} \sim N_c^0$$

- EFT Lagrangian forms minimal set of operators

- Different terms related by Fierz transformations

$$\mathcal{C}^{(^3S_1-^1P_1)} = 3\,\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} \left[1 + O(1/N_c^2)\right]$$

MRS, Springer, Vanasse (2016)

$1/N_c$ expansion of TV potential

- Leading order $O(N_c)$

$$\vec{p}_- \cdot (\vec{\sigma}_1\, \tau_1^z - \vec{\sigma}_2\, \tau_2^z)$$

- Next-to-leading order $O(N_c^0)$

$$\vec{p}_- \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}$$
$$\vec{p}_+ \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}$$
$$\vec{p}_+ \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2$$

Samart, Schat, MRS, Phillips (2016)

$1/N_c$ expansion of TV potential

- Next-to-next-leading order $O(N_c^{-1})$

$$
\begin{aligned}
&\vec{p}_- \cdot (\vec{\sigma}_1 \, \tau_2^z - \vec{\sigma}_2 \, \tau_1^z) \\
&\vec{p}_+^{\,2} \, \vec{p}_- \cdot (\vec{\sigma}_1 \, \tau_1^z - \vec{\sigma}_2 \, \tau_2^z) \\
&\vec{p}_+ \cdot (\vec{\sigma}_1 + \vec{\sigma}_2) \, (\vec{\tau}_1 \times \vec{\tau}_2)^z \\
&\vec{p}_+ \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \, (\vec{\tau}_1 + \vec{\tau}_2)^z \\
&[(\vec{p}_+ \times \vec{p}_-)\vec{p}_-]_2^{ij} \cdot [\sigma_1 \sigma_2]_2^{ij} \, (\vec{\tau}_1 - \vec{\tau}_2)^z \\
&[(\vec{p}_+ \times \vec{p}_-)\vec{p}_+]_2^{ij} \cdot [\sigma_1 , \sigma_2]_2^{ij} \, (\vec{\tau}_1 \times \vec{\tau}_2)^z
\end{aligned}
$$

- Can be multiplied by functions of $\vec{p}_-^{\,2}$

- No Fierz identities imposed

Chiral EFT for T violation

- LO TV potential

$$
\begin{aligned}
V_{\not{T}\not{P}}^{\mathrm{EFT}} = & - i\frac{\bar{C}_1}{2}\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)\cdot\vec{p}_- \\
& - i\left(\frac{g_A[\bar{g}_\pi^{(0)} - \bar{g}_\pi^{(2)}]}{2F_\pi}\frac{1}{(\vec{p}_-^2 + M_\pi^2)} + \frac{\bar{C}_2}{2}\right)\vec{\tau}_1\cdot\vec{\tau}_2\left(\vec{\sigma}_1 - \vec{\sigma}_2\right)\cdot\vec{p}_- \\
& - i\frac{g_A\bar{g}_\pi^{(1)}}{2F_\pi}\frac{1}{(\vec{p}_-^2 + M_\pi^2)}\left(\vec{\sigma}_1\tau_1^z - \vec{\sigma}_2\tau_2^z\right)\cdot\vec{p}_-
\end{aligned}
$$

- Large-N_c scalings

$$
\begin{aligned}
\bar{g}_\pi^{(1)} &\sim N_c^{1/2} \\
\bar{C}_1, \bar{C}_2 &\sim N_c^0 \\
\bar{g}_\pi^{(0,2)} &\sim N_c^{-1/2}
\end{aligned}
$$

Maekawa, Mereghetti et al. (2011); Bsaisou et al. (2015)

T violation summary

- One-pion exchange LO in N_c
- $\bar{g}_\pi^{(1)} > \bar{g}_\pi^{(0)} \sim \bar{g}_\pi^{(2)}$
- Suppression at NLO only $1/N_c$
- Expansion in given isospin sector in $1/N_c^2$
- Independent of/complementary to
  - Chiral suppressions
  - Hierarchy at quark level
  - …
- Also applied to meson-exchange models, pionless EFT

Samart, Schat, MRS, Phillips (2016); David, Vanasse (2019)